Performing Delayed Rejection Version for Monte Carlo Markov Chain

Based on Buffett Stock Holding Portfolio

Yilin Huang

**Introduction**

Stock market is always the popular choice for new investors to start their investment journey. People want to learn experiences in earning in stock market, like famous celebrities in stock markets investment Warren Buffett, who controls total $119 billion net worth so far (*Forbes: Warren Buffett* 2023). Based on gurofocus data, Buffett’s stock portfolio contains $313.26 billion values in total on December 14, 2023. Buffett’s stock portfolio is a popular sample for investors to analyze stock market strategy, and investors are interesting in prediction on Buffett’s next step action.

Monte Carlo Markov Chain is a tool for predicting stock data. As Mantas and Eimutis state in their paper *Modelling of Stock Prices By the Markov Chain Monte Carlo Method,* MCMC is flexible since it considers prediction based on any given point based on its properties of one-factor for the interest rate (Landauskas & Valakevičius, 2011). Stock price prediction is though volatile in a long-term run, but feasible through random walk model based on recent data in the short-term perspective. One of the most famous random walk models for prediction financial instrument values, especially stocks prices, is the Black Scholes Model (BSM). In this project, the prediction of Buffett’s stock portfolio of his total 51 stock choices is constructed using MCMC method combining with idea of Bayesian inference. The stock price prediction will basically follow the procedure of BSM, and the time range for prediction is in short term of 12 transaction days. The prediction is conducted based on 12 previous transaction days data.

**Data Collection**

Based on CNBC data, Buffett’s stock portfolio contained total number of 51 stocks holding before September 30, 2023 (*Berkshire Hathaway portfolio tracker* 2023). The list and holdings of shares data is presented in Appendix 1. Till September 30, 2030, Buffett owns $361 billion dollars stock holdings, and about 48.7% of portfolio total value is invested in Apple.INC. Those stocks 12 previous transaction days data are collected through yahoo finance.

**Methodology**

**Black Scholes Model**

Stock price prediction will be conducted based on random walk Black Scholes model. Black Scholes model is a Brownian motion model, which uses last time spot stock price data to predict the next time spot data price. Based on definition of black Scholes model in Investopedia (Hayes, 2023), BSM is a model first used for option pricing. By only knowing the risk-free rate, volatility, and previous stock price data, the new stock price data would be predicted. BSM uses idea of differential equation that the new price would be generated through the performance plus some random noise term. The formula for standard BSM is shown as formula (1)

(1)

Those inputs are collected from different procedures. Volatility is defined as the stock investing risk regarding to its price fluctuation (Hayes, 2023). Higher volatility refers to a higher probability of losing potential earnings during investing this stock. The volatility could be calculated through previous stock data based on formula (2):

(2)

After calculating the volatility of every stock, the risk-free rate is set to be some constant in BSM model. Risk-free rate is defined as the return rate that an investment that provides return with zero risk (Hayes, 2023). Often, the risk-free rate is set up based on US 10-year treasury bond, as it represents the current risk-free rate for investments safely. From this project perspective, the risk-free rate of BSM is set up to be equal to 10 percent in purpose of earning potential by following Buffett’s stock investment strategy.

From this point, all of variables in BSM are introduced and settled except the noise term variable “dW”. The noise term for BSM model is expected to be increase during increasing of time duration for prediction. The noise term in BSM contributes to the exponent term fluctuation rate on stock price prediction. Therefore, using Monte Carlo Markov Chain to predict noise term is needed for accuracy of stock price prediction model.

**Monte Carlo Markov Chain**

Monte Carlo Markov Chain (MCMC) is introduced to improve accuracy of model prediction by generating samples of noise term “dW” in BSM model. MCMC creates different chains for stock price prediction, and those chains differences are made based on variance of different noise term in BSM. Basically, the noise term is normally distributed with expected value of 0 and variance of 1. The state for noise term is updated by adding up previous noise term state with normal random variable with mean value of 0 and variance of 1 as formula (3) shown

(3)

After this basic structure set up, the Metropolis Hastings algorithm is introduced to filter the outlier of sigma, which may be far away from the standard normal distribution. The standard Metropolis Hastings algorithm gives the procedure to generate a accept rate based on Bayesian inference, and this accept rate is viewed as a threshold in MCMC to filter the sigma and exclude outliers.

**Metropolis Hasting Algorithm, Delayed Rejection, and Bayesian inference**

Metropolis Hasting algorithm (MH) is used for setting up threshold which used to determine whether accepting or rejecting the new proposal state. The MH sets up threshold by using accept rate, and MH exclude outliers by comparing the performance of previous state and new proposal state. The performance of state is related to normal probability density function calculation, and that basic MH algorithm is calculated as formula (4).

(4)

A uniform random number between 0 and 1 is created to compare with our accept rate. The decision making for whether accepting new proposal state is following in formula (5).

(5)

In this project, the total number of 100 MCMC chains will be produced for time from initial point to 12 predicted new transaction days. As stock’s natural property of big volatility, to maintain better convergence rate on our BSM, delayed rejection is introduced to improve performance of MH. The Delayed Rejection Metropolis Hasting (DRMH) is constructed based on normal MH, but with further consideration of rejection the new proposal state. DRMH request model to generate a new proposal state based on previous rejected state and analyze the performance for this new proposal state. Recalled that the previous state is rejected, and the new proposal state in DRMH is constructed with this rejection happening already. Hence, the probability theory of Bayesian inference comes in as formula (6).

(6)

In this case, θ represents the event of accepting or rejecting the new proposal state, and x represents the previous rejection state event. Bayesian inference provides theory for known prior data and new likelihood function to calculate the performance of new proposal state after the rejection on the previous state. At this point, the DRMH is conducted based on formula (7). In Zuev and Katafygiotis’s paper *Modified Metropolis-Hastings algorithm with delayed rejection*, there’s a better explanation on DRMH as shown in figure 1 (Zuev & Katafygiotis, 2011).

(6)

A math equations and formulas

Description automatically generated with medium confidence

*Figure 1. Metropolis Hastings Algorithm with Delayed Rejection*

**Results and Conclusion**

After DRMH is set up, the noise term for BSM is fixed, and the prediction is ready to go. All Buffett’s 51 stocks data are inputting into this model, and example result of Amazon stock price prediction is shown as Figure 2. After prediction, the average of 100 prediction chains for every stock is stored into Table 1. The expected return rate is calculated for every stock as shown in Appendix 2.

A graph with lines and numbers

Description automatically generated with medium confidence

*Figure 2. Amazon Stock Price Prediction with total 100 MCMC chain for 12 future transaction days after November 1st, 2023*

|  |  |  |
| --- | --- | --- |
| Date | Average Prediction (12days after) | Historical |
| 10/16/23 | 133.0899963 | 132.550003 |
| 10/17/23 | 134.5931988 | 131.470001 |
| 10/18/23 | 136.4191605 | 128.130005 |
| 10/19/23 | 138.4988058 | 128.399994 |
| 10/20/23 | 139.197702 | 125.169998 |
| 10/23/23 | 140.285851 | 126.559998 |
| 10/24/23 | 142.3746389 | 128.559998 |
| 10/25/23 | 142.586761 | 121.389999 |
| 10/26/23 | 145.5340453 | 119.57 |
| 10/27/23 | 143.4393772 | 127.739998 |
| 10/30/23 | 145.3925668 | 132.710007 |
| 10/31/23 | 146.4751615 | 133.089996 |

*Table 1: The 12 days average prediction transaction days and historical data for AMZN stock*

From the results shown in this project, the predicted 51 stock 12 predicted days returns mostly lies between 7 to 11 percentage, in which it’s considered as a fair return rate for stock investment portfolio in comparison to our 10% risk-free rate. There are limitations needs to be improved in the future. This project is limited for using MCMC to predict stock price. Prediction doesn’t cover the cashflow and other implicit financial, and this may be improved in the future for improve accuracy of this model. The 12 days of historical data may be less accurate. In the future, the time series combining with MCMC method will improve the accuracy of predicting stock price, since time series method will cover longer historical data behavior to improve prediction accuracy.

**Appendix**

**Appendix 1: Buffett’s stock portfolio at September 30th 2023**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Symbol | Holdings | Stake | Mkt. price | Value | Pct of portfolio |
| Ally Financial Inc | ALLY | 29,000,000 | 9.60% | $29.55 | $856,950,000 | 0.20% |
| Amazon.com Inc | AMZN | 10,000,000 | 0.10% | $144.52 | $1,445,200,000 | 0.40% |
| American Express Company | AXP | 151,610,700 | 20.80% | $167.51 | $25,396,308,357 | 7.00% |
| Aon PLC | AON | 4,100,000 | 2.00% | $319.58 | $1,310,278,000 | 0.40% |
| Apple Inc | AAPL | 915,560,382 | 5.90% | $192.32 | $176,080,572,666 | 48.70% |
| Atlanta Braves Holdings Inc Series C | BATRK | 223,645 | 0.40% | $35.54 | $7,948,343 | 0.00% |
| Bank of America Corp | BAC | 1,032,852,006 | 13.00% | $30.53 | $31,532,971,743 | 8.70% |
| BYD Co. Ltd | BYDDF | 87,613,142 | 8.00% | $27.45 | $2,404,980,748 | 0.70% |
| Capital One Financial Corp. | COF | 12,471,030 | 3.30% | $113.77 | $1,418,829,083 | 0.40% |
| Charter Communications Inc | CHTR | 3,828,941 | 2.60% | $361.98 | $1,386,000,063 | 0.40% |
| Chevron Corporation | CVX | 110,248,289 | 5.90% | $142.53 | $15,713,688,631 | 4.40% |
| Citigroup Inc | C | 55,244,797 | 2.90% | $47.86 | $2,644,015,984 | 0.70% |
| Coca-Cola Co | KO | 400,000,000 | 9.30% | $58.60 | $23,440,000,000 | 6.50% |
| DR Horton Inc | DHI | 5,969,714 | 1.80% | $133.61 | $797,613,488 | 0.20% |
| Davita Inc | DVA | 36,095,570 | 39.50% | $104.16 | $3,759,714,571 | 1.00% |
| Diageo plc | DEO | 227,750 | 0.00% | $139.58 | $31,789,345 | 0.00% |
| Floor & Decor Holdings Inc | FND | 4,780,000 | 4.50% | $100.28 | $479,338,400 | 0.10% |
| Globe Life Inc | GL | 831,014 | 0.90% | $123.12 | $102,314,444 | 0.00% |
| HP Inc | HPQ | 97,854,605 | 9.90% | $28.49 | $2,787,877,696 | 0.80% |
| Itochu Corporation | 8001.T | 118,331,800 | 7.50% | $39.54 | $4,678,697,841 | 1.30% |
| Jefferies Financial Group Inc | JEF | 433,558 | 0.20% | $36.21 | $15,699,135 | 0.00% |
| Kraft Heinz Co | KHC | 325,634,818 | 26.50% | $36.20 | $11,787,980,412 | 3.30% |
| Kroger Co | KR | 50,000,000 | 7.00% | $44.54 | $2,227,000,000 | 0.60% |
| Lennar Corp Class B | LEN-B | 152,572 | 0.40% | $123.76 | $18,882,311 | 0.00% |
| Liberty Latin America Series A | LILA | 2,630,792 | 6.10% | $6.86 | $18,047,233 | 0.00% |
| Liberty Latin America Series C | LILAK | 1,284,020 | 0.80% | $6.89 | $8,846,898 | 0.00% |
| Liberty Formula One Series C | FWONK | 7,722,451 | 3.70% | $61.65 | $476,089,104 | 0.10% |
| Liberty SiriusXM Series A | LSXMA | 20,207,680 | 20.60% | $26.28 | $531,057,830 | 0.10% |
| Liberty SiriusXM Series C | LSXMK | 43,208,291 | 19.80% | $26.39 | $1,140,266,799 | 0.30% |
| Liberty Live Series A | LLYVA | 5,051,918 | 19.80% | $31.86 | $160,954,107 | 0.00% |
| Liberty Live Series C | LLYVK | 11,132,590 | 17.50% | $33.18 | $369,379,336 | 0.10% |
| Louisiana-Pacific Corp | LPX | 7,044,909 | 9.80% | $62.85 | $442,772,531 | 0.10% |
| Markel Group Inc | MKL | 158,715 | 1.20% | $1,386.99 | $220,136,118 | 0.10% |
| Marubeni Corp | 8002.T | 141,000,200 | 8.30% | $15.58 | $2,196,756,991 | 0.60% |
| Mastercard Inc | MA | 3,986,648 | 0.40% | $410.38 | $1,636,040,606 | 0.50% |
| Mitsubishi Corp | 8058.T | 119,497,600 | 8.30% | $46.53 | $5,560,015,607 | 1.50% |
| Mitsui & Co | 8031.T | 125,022,300 | 8.10% | $35.41 | $4,427,105,621 | 1.20% |
| Moody’s Corp | MCO | 24,669,778 | 13.50% | $373.43 | $9,212,435,199 | 2.60% |
| Nu Holdings Ltd | NU | 107,118,784 | 2.30% | $8.15 | $873,018,090 | 0.20% |
| NVR Inc | NVR | 11,112 | 0.30% | $6,364.66 | $70,724,102 | 0.00% |
| Occidental Petroleum Corp | OXY | 228,051,027 | 25.90% | $56.48 | $12,880,322,005 | 3.60% |
| Paramount Global Class B | PARA | 93,730,975 | 15.30% | $15.22 | $1,426,585,440 | 0.40% |
| Sirius XM Holdings Inc | SIRI | 9,683,224 | 0.30% | $4.49 | $43,477,676 | 0.00% |
| Snowflake Inc | SNOW | 6,125,376 | 1.90% | $184.47 | $1,129,948,111 | 0.30% |
| SPDR S&P 500 ETF Trust | SPY | 39,400 | 0.00% | $454.76 | $17,917,544 | 0.00% |
| StoneCo Ltd | STNE | 10,695,448 | 3.40% | $16.27 | $174,014,939 | 0.00% |
| Sumitomo Corp | 8053.T | 101,210,400 | 8.20% | $21.17 | $2,142,901,828 | 0.60% |
| T-Mobile Us Inc | TMUS | 5,242,000 | 0.50% | $155.50 | $815,131,000 | 0.20% |
| Vanguard 500 Index Fund ETF | VOO | 43,000 | 0.00% | $417.86 | $17,967,980 | 0.00% |
| Verisign, Inc. | VRSN | 12,815,613 | 12.60% | $215.98 | $2,767,916,096 | 0.80% |
| Visa Inc | V | 8,297,460 | 0.40% | $254.29 | $2,109,961,103 | 0.60% |

***Appendix 2: Prediction result***

|  |  |
| --- | --- |
| Symbol | 12 days Return |
| ALLY | 9.99% |
| AMZN | 9.90% |
| AXP | 9.46% |
| AON | 8.84% |
| AAPL | 9.62% |
| BATRK | 9.70% |
| BAC | 8.53% |
| BYDDF | 10.04% |
| COF | 11.43% |
| CHTR | 8.30% |
| CVX | 11.08% |
| C | 8.88% |
| KO | 9.47% |
| DHI | 10.09% |
| DVA | 9.97% |
| DEO | 9.67% |
| FND | 9.93% |
| GL | 9.67% |
| HPQ | 10.04% |
| 8001.T | 9.81% |
| JEF | 9.64% |
| KHC | 9.84% |
| KR | 9.63% |
| LEN-B | 10.26% |
| LILA | 10.01% |
| LILAK | 9.26% |
| FWONK | 8.92% |
| LSXMA | 9.92% |
| LSXMK | 9.43% |
| LLYVA | 9.10% |
| LLYVK | 10.40% |
| LPX | 10.39% |
| MKL | 9.86% |
| 8002.T | 9.40% |
| MA | 9.69% |
| 8058.T | 10.12% |
| 8031.T | 10.11% |
| MCO | 9.03% |
| NU | 8.28% |
| NVR | 9.47% |
| OXY | 9.11% |
| PARA | 10.43% |
| SIRI | 9.59% |
| SNOW | 9.88% |
| SPY | 9.56% |
| STNE | 9.16% |
| 8053.T | 10.33% |
| TMUS | 8.47% |
| VOO | 9.76% |
| VRSN | 7.95% |
| V | 10.04% |

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