

Model Based Vision 2:

Statistical Shape Models (SSMs), Active Shape Models (ASMs), Active Appearance Models (AAMs)

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Handouts & Lecture Notes

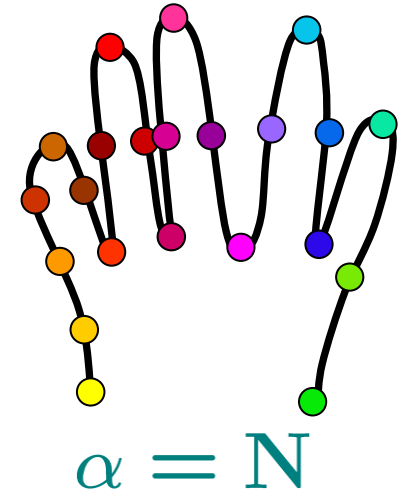
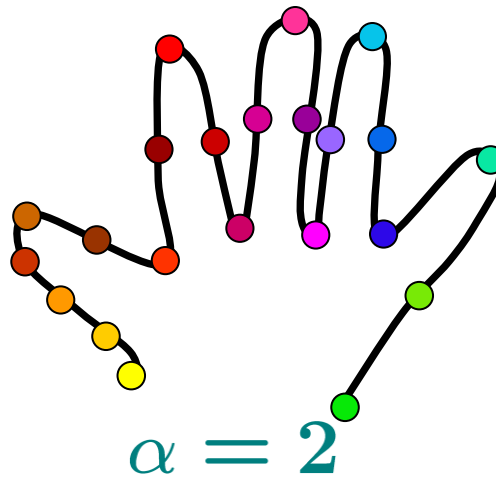
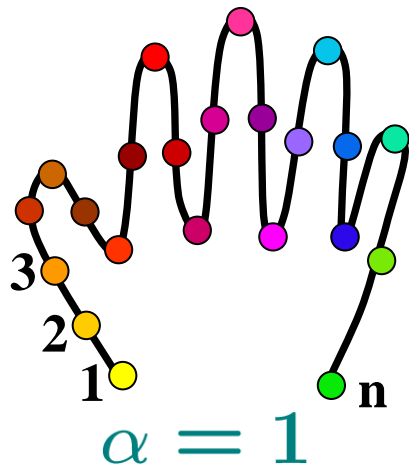
- Report in Scientific American (June 2014):
“In each study, however, those who wrote out their notes by hand had a stronger conceptual understanding and were more successful in applying and integrating the material than those who used [sic] took notes with their laptops.”

The Pen Is Mightier Than the Keyboard

P. A. Mueller, D. M. Oppenheimer, *Psychological Science*, Vol 25, Issue 6, pp. 1159 – 1168, April-23-2014.

- Handouts are to aid note taking, not a total replacement for note taking
- Podcasts, slides, pdfs etc on BlackBoard

Statistical Shape Models (SSMs)

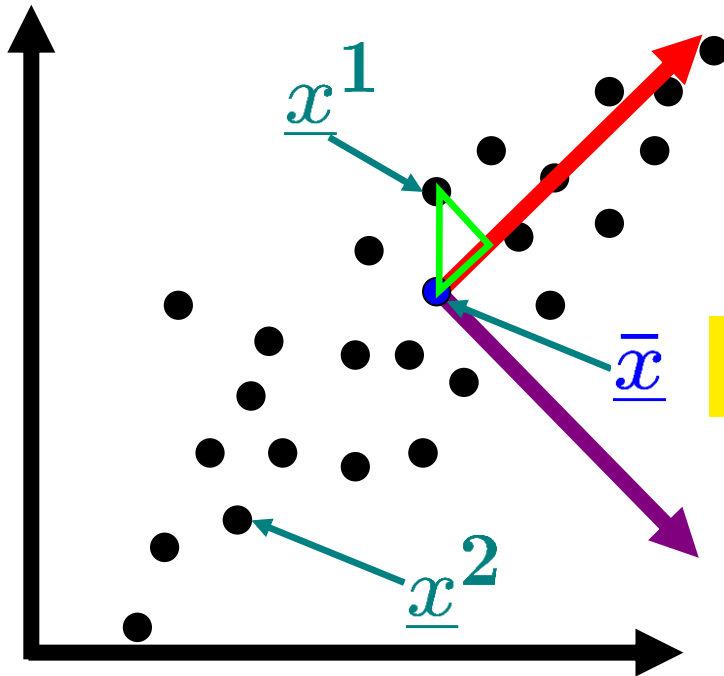


- Set of **aligned** shapes
- Shape vector: $\underline{x}^\alpha = (x_1^\alpha, y_1^\alpha, x_2^\alpha, y_2^\alpha, \dots, x_n^\alpha, y_n^\alpha)$
- Corresponding points on all shapes
- Entire **aligned** training set, set of shape vectors:

$$\{\underline{x}^\alpha : \alpha = 1, 2, \dots, N\}$$

Principal Component Analysis (PCA)

$$\{\underline{x}^\alpha : \alpha = 1, 2, \dots, N\}$$



Shape Space:
axes = coordinates
of every shape point

Mean shape:

$$\underline{\bar{x}} \doteq \frac{1}{N} \sum_{\alpha=1}^N \underline{x}^\alpha$$

$\underline{\hat{n}}$, unit axis vector

Maximize data projection:

$$\arg \max_{\underline{\hat{n}}} \sum_{\alpha=1}^N (\underline{\hat{n}} \bullet (\underline{x}^\alpha - \underline{\bar{x}}))^2$$

Repeat:

$$\arg \max_{\underline{\hat{m}}} \sum_{\alpha=1}^N (\underline{\hat{m}} \bullet (\underline{x}^\alpha - \underline{\bar{x}}))^2,$$

Constraint: $\underline{\hat{m}} \bullet \underline{\hat{n}} = 0$

PCA Solution: Covariance Matrix

Mean shape: $\bar{x} \doteq \frac{1}{N} \sum_{\alpha=1}^N \underline{x}^{\alpha}$

n: number of points on each shape
2: number of spatial dimensions

Covariance matrix:

$$C_{ij} \doteq \frac{1}{N} \sum_{\alpha=1}^N (\underline{x}^{\alpha} - \bar{x})_i (\underline{x}^{\alpha} - \bar{x})_j, \quad i, j = 1, \dots, 2n$$

Solve covariance matrix eigenproblem:

$$C \hat{n}^{\mu} = \lambda^{\mu} \hat{n}^{\mu}$$

(matrix \times vector = number \times vector)

Eigenvectors: $\{\hat{n}^{\mu}\}$, directions of new axes

Ordered eigenvalues: $\{\lambda^{\mu} : \lambda^1 \geq \lambda^2 \dots\}$,

how much variance in each direction

Generative Shape Models:

generated shape $\rightarrow \underline{x} = \underline{\bar{x}} + \underline{P} \underline{b}$

mean shape

shape parameters, coords wrt PCA axes

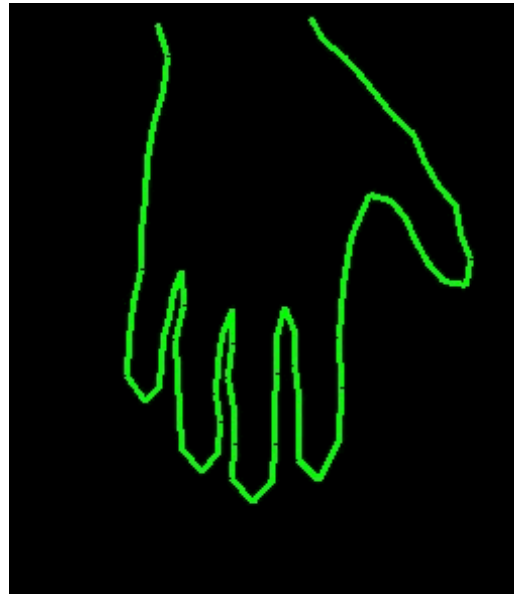
matrix, columns are eigenvectors

- New shape = mean plus weighted sum of eigenvectors
- PCA automatically finds relevant modes of variation

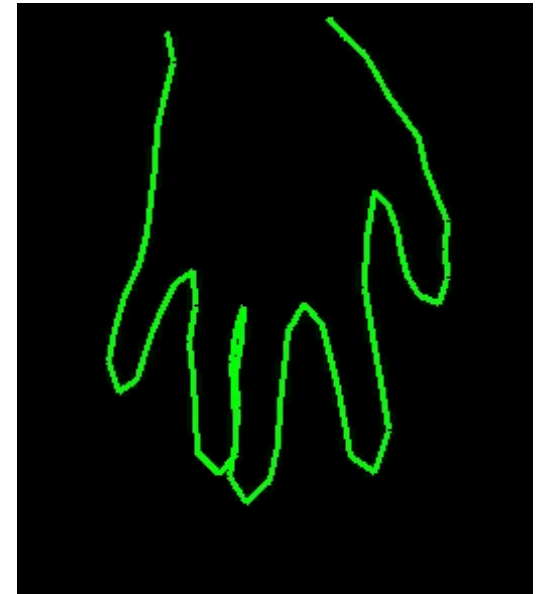
$b_1 \neq 0$ only



$b_2 \neq 0$ only



$b_3 \neq 0$ only



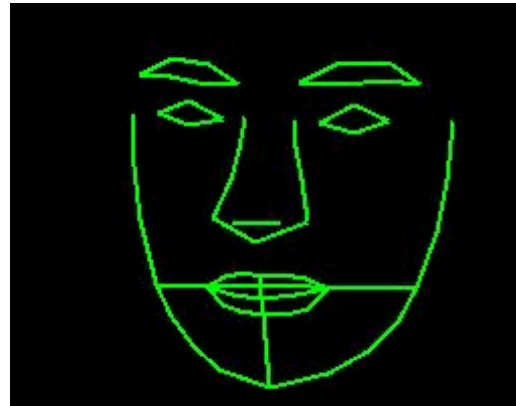
Generative Shape Models: Faces

First Mode



$b_1 \neq 0$ only

Second Mode



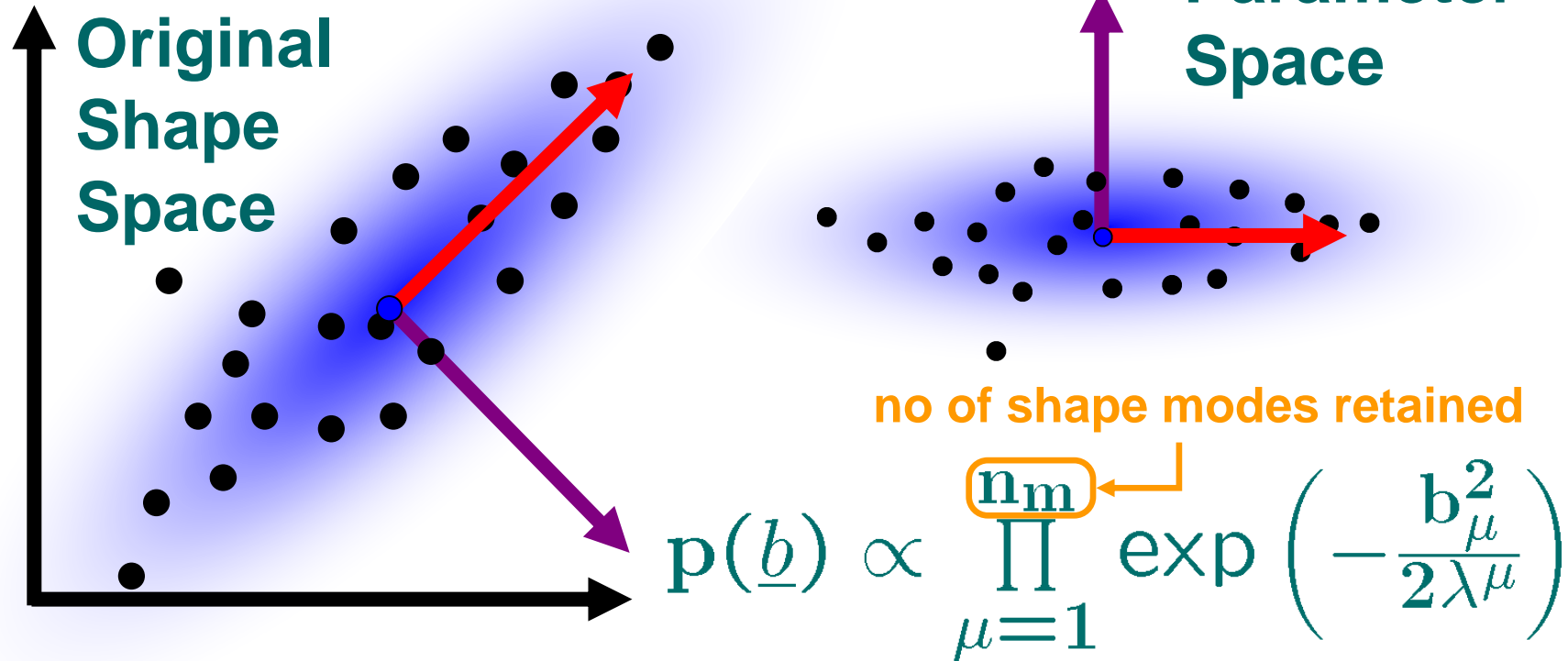
$b_2 \neq 0$ only

Third Mode



$b_3 \neq 0$ only

Statistical Shape Models:



- Multivariate Gaussian probability density function
- Aligned with PCA directions (eigenvectors)
- Matches variance seen in training set (eigenvalues)
- Product of Gaussians in parameter space

SSMs: Summary

Construction:

- Training set of shapes, corresponding landmarks
- Procrustes align shapes and compute mean shape
- Covariance matrix and solve PCA eigenproblem
- Shape parameters, modes of variation
- Construct gaussian probabilistic model

Results:

- **General**: modes of variation capture full variation
- **Specific**: modes capture only variation actually seen
- Assign probabilities to generated shapes

From SSMs to Active Shape Models

Task:

- Find shape in unseen image

Solution:

- Map from model frame to image frame

- Iterative localised search:

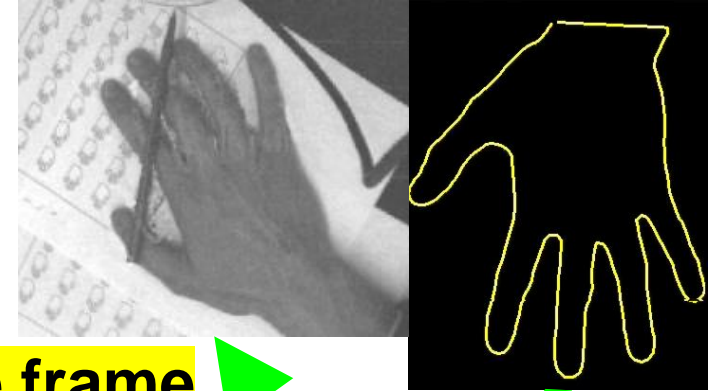
Search in neighbourhood of current points for new points

Fit model to suggested new shape

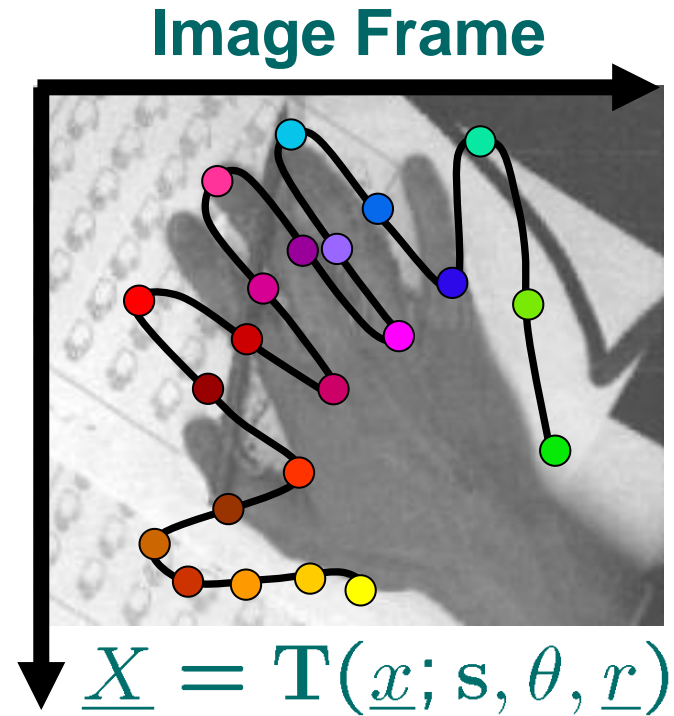
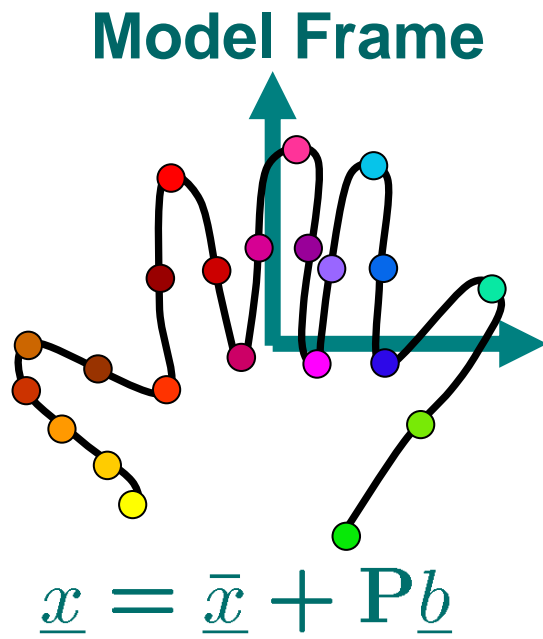
Apply **constraints** to shape based on learnt variation

Repeat until **convergence**

- Like ACM, moves towards edges, lines etc
- Unlike ACM, remains valid shape as it does so



Placing the Model in an Image



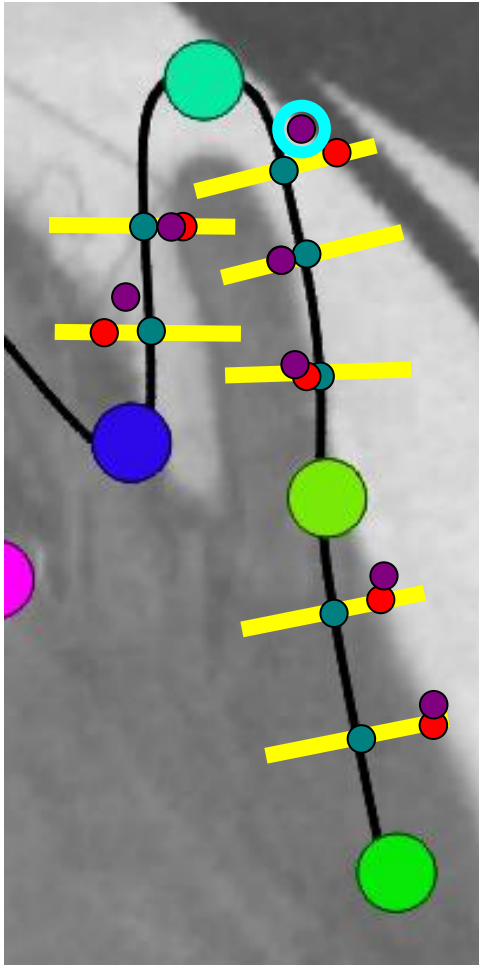
- **Scale, rotate, and translate to get to image frame:**

Pose transformation: $\underline{T}(\underline{x}; s, \theta, \underline{r}) = s\mathbf{R}(\theta)\underline{x} + \underline{r}$

- **Total set of parameters to define shape in an image:**

Pose parameters: s, θ, \underline{r} , Shape parameters: \underline{b}

ASM Local Iterative Search



- **Local search**
- **Initialise** near target
- Search along normals
- Look for strongest edge
- Gives **new set** of suggested shape points
 \underline{X}'
- Best-Fit model to shape:
 $\underline{X}' \approx \mathbf{T}(\underline{x}(\underline{b}); s, \theta, \underline{r})$
- Candidate shape:
 $\underline{X}'' = \mathbf{T}(\underline{x}(\underline{b}); s, \theta, \underline{r})$
- Still leaves some **errors**

Problems:

Shape model step: $\underline{X}'' = T(\underline{x}(\underline{b}); \mathbf{s}, \theta, \underline{r})$

- Even within model, can get extreme shapes

Edge-Finding:

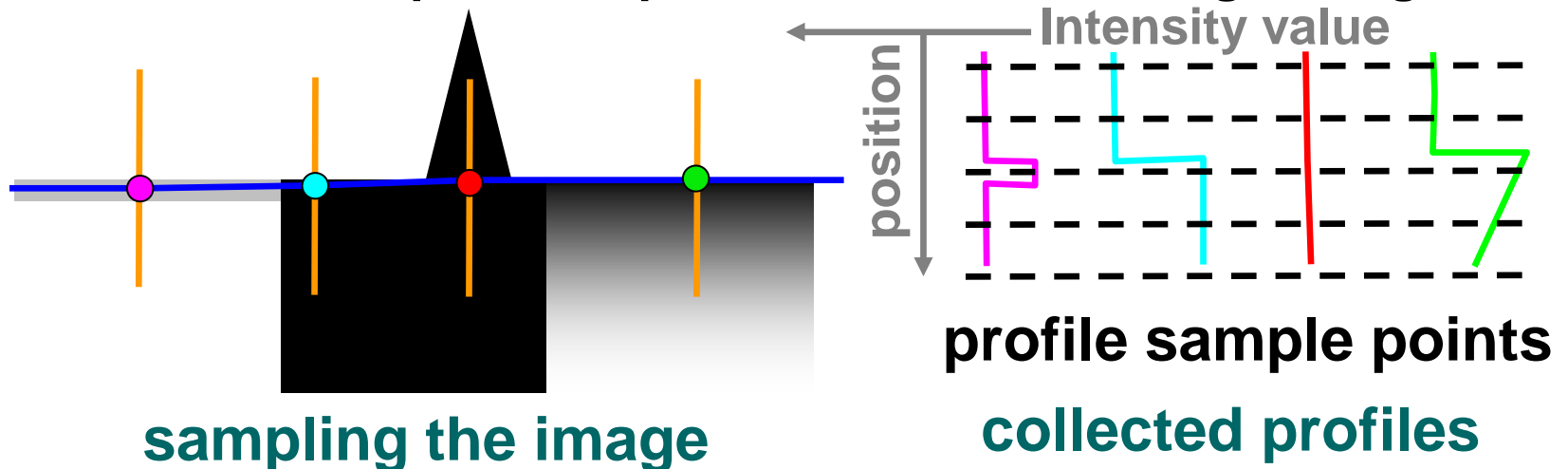
- Actual position not edge, or not strongest edge

Solution:

- Model profiles/appearance near points
- Apply shape and profile probability to control search

Profile Models

- For each shape point in each training image:
Sample image values along normals to **shape**
Normalise to eliminate illumination effects
- Build statistical model as for shape
Profile vector like shape vector
- Model assigns probability to each possible profile
Select most probable profile rather than strongest edge



Applying Shape Constraints in Search

- Sample as before, find best fit to profile model at each point
- Suggested new set of shape points: \underline{X}'

Hard Constraints:

Minimize : $|\underline{X}' - \mathbf{T}(\underline{x}(\underline{b}); \mathbf{s}, \theta, \underline{r})|^2$

with $p(\underline{b}) \geq p_{\text{thresh}}$ $\underline{X}'' = \mathbf{T}(\underline{x}(\underline{b}); \mathbf{s}, \theta, \underline{r})$

Soft Constraints:

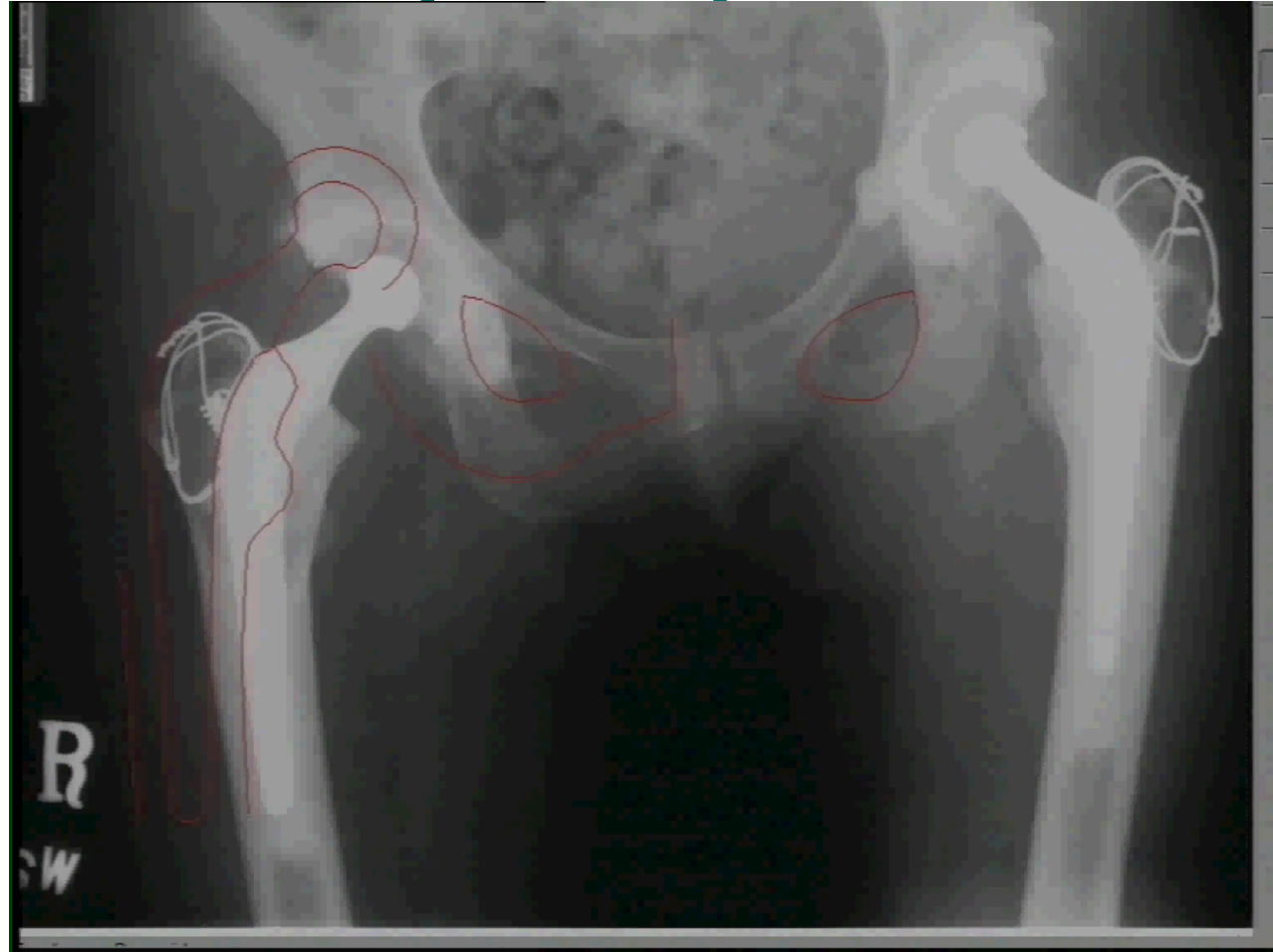
Minimize : $|\underline{X}' - \mathbf{T}(\underline{x}(\underline{b}); \mathbf{s}, \theta, \underline{r})|^2 - \alpha \log(p(\underline{b}))$

- Can also include profile probability as local quality of match into process – more extreme shapes provided quality of profile match merits it

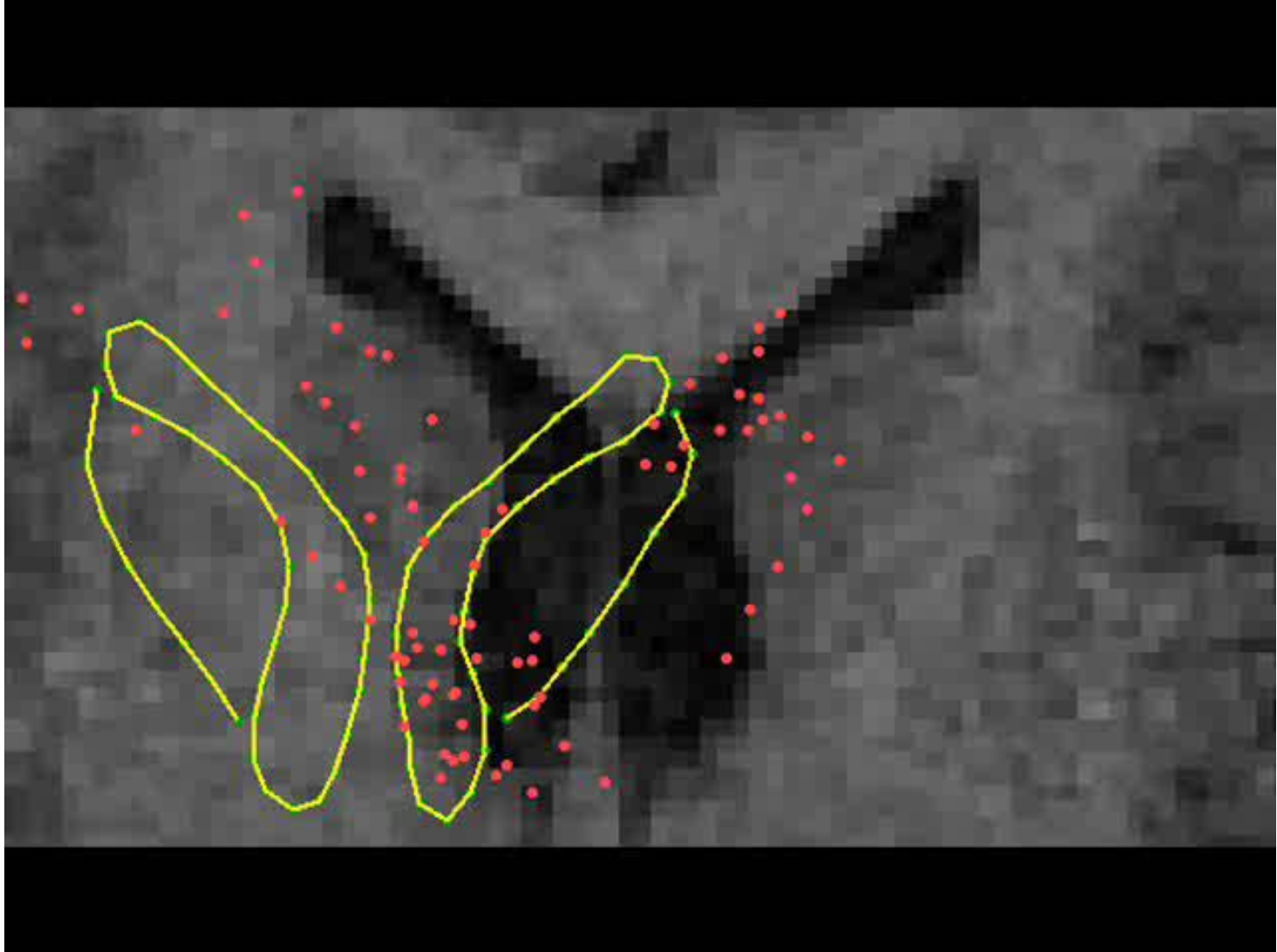
ASM Multi-Resolution Search

- **To increase basin of attraction, use multi-resolution**
 - Gaussian pyramid of training images**
 - Same shape points, but different profile models at each level**
- **Start search at coarse level, refine at finer level**
- **Similar to multi-resolution gaussian smoothing for edge-based Active Contours (previous lecture Slide 14)**

ASM Search Example: Hip



ASM Search Example: Brain



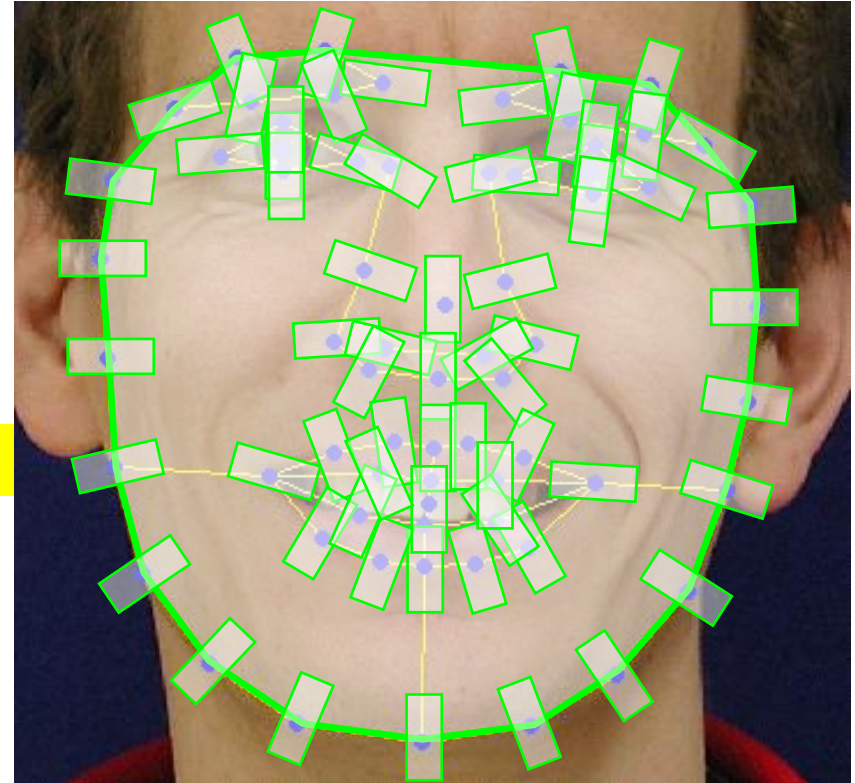
ASM: Summary

Advantages:

- Fast, **simple**, accurate
- Efficient extension to 3D

Disadvantages:

- Only **sparse use of image information**
- **Treats local profiles as independent**
- AAM: models all the texture



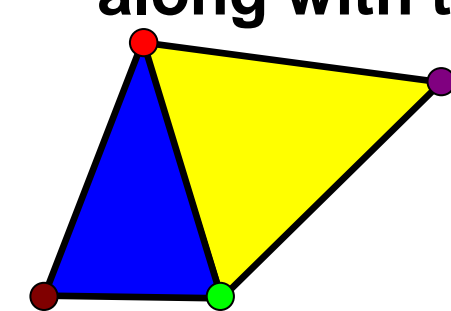
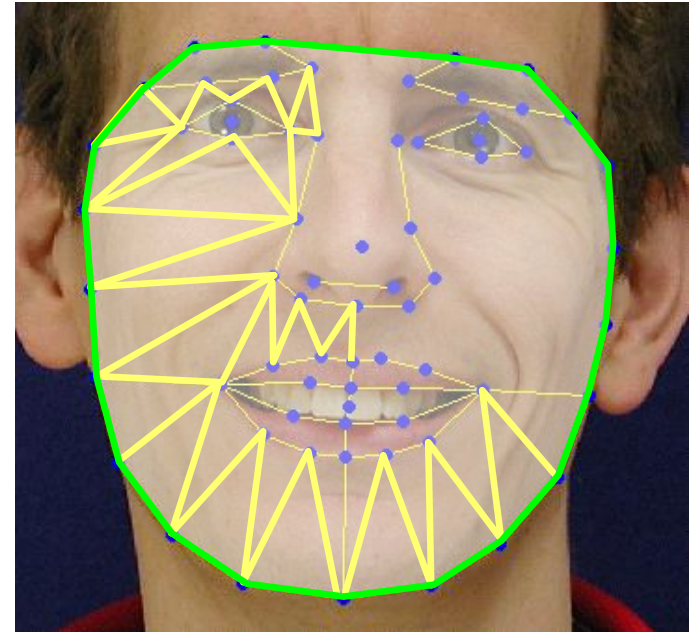
Active Appearance Models

Active Appearance Models

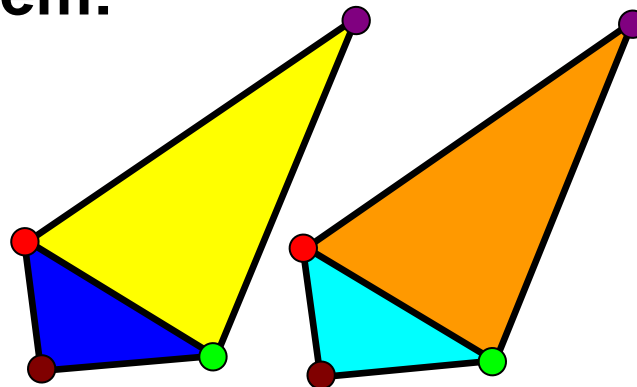
- Build SSM as before

$$\underline{x} = \underline{\bar{x}} + \underline{P}\underline{b}$$

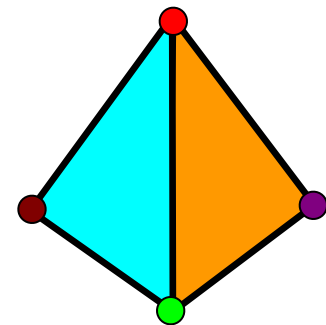
- Triangulate shapes
- Warp all shapes to mean shape
- Take image texture along with them:



Shape 1

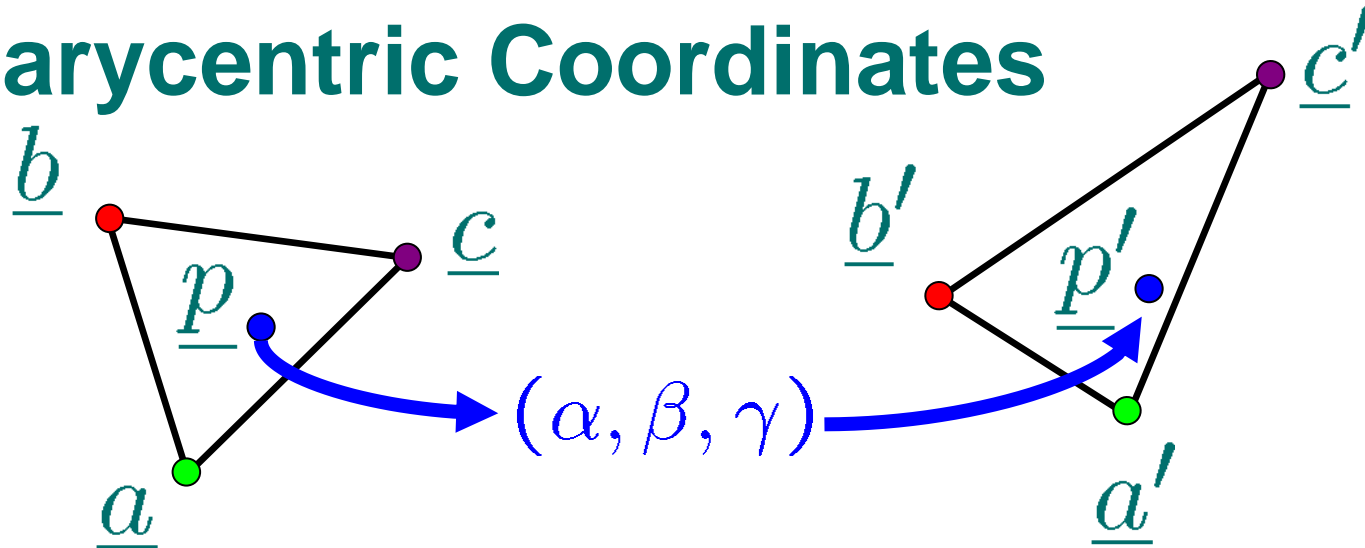


Common Frame



Shape 2

Barycentric Coordinates



$$\underline{p} = \alpha \underline{a} + \beta \underline{b} + \gamma \underline{c}$$

$$\alpha + \beta + \gamma = 1$$

$$\underline{p'} = \alpha \underline{a'} + \beta \underline{b'} + \gamma \underline{c'}$$

Inside: $\alpha, \beta, \gamma \geq 0$

- Map any point from training image to point within mean shape
- Map all textures: set of shape-free texture samples

Shape-Free Texture Model



Original
image



Frame of
mean shape

PCA Eigenmodel from
shape-free textures



First texture mode



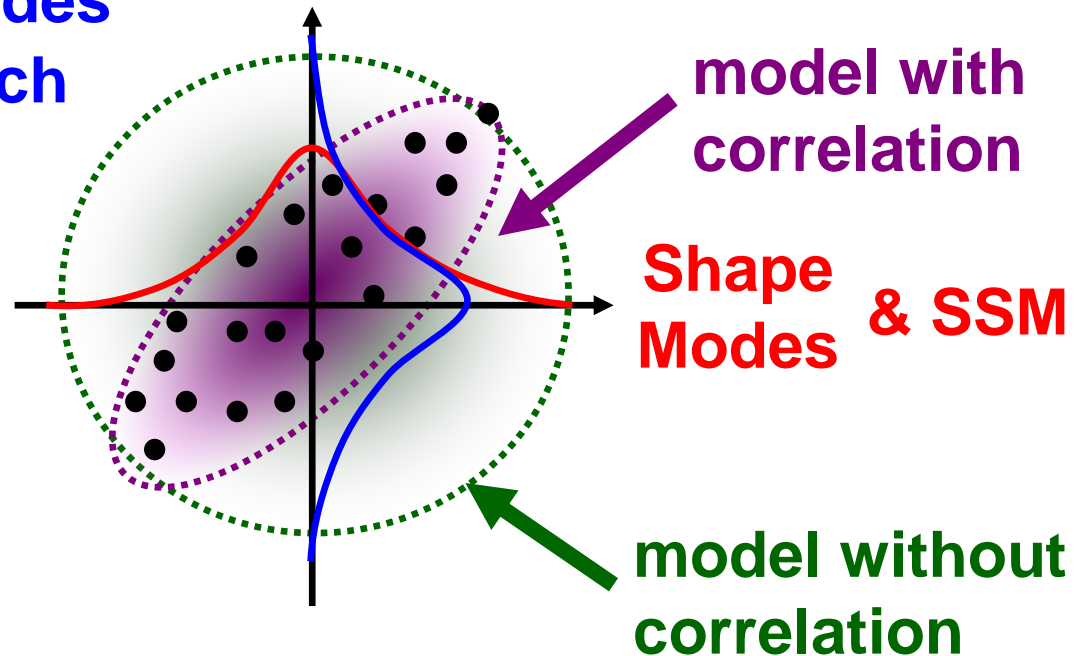
Second mode



Third mode

Combined Models:

Texture Modes
& eigenpatch
model



- Texture and shape don't vary independently
- Change of expression, shape changes & shadows
- Model with shape/texture correlation more specific & more compact (fewer parameters)

AAM Combined Models

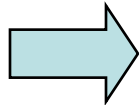
- Texture changes as expression (shape) changes



AAM Search:



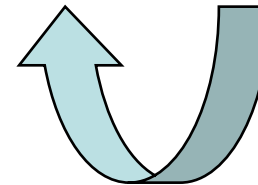
Place model
in image



Measure
Difference



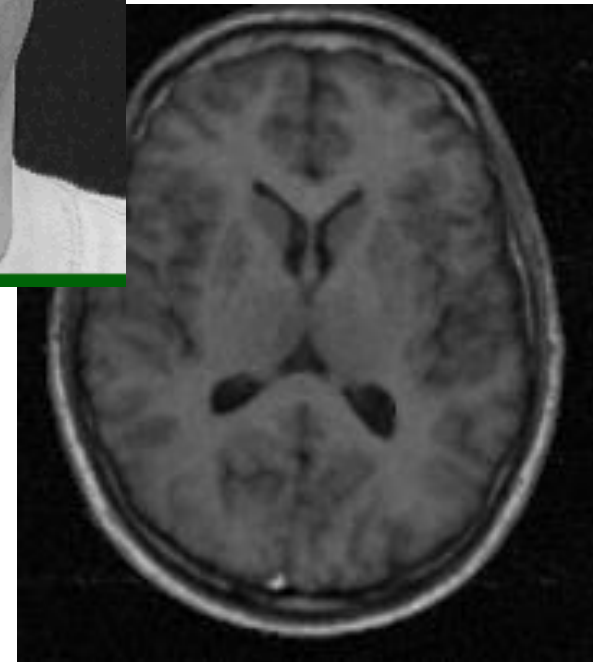
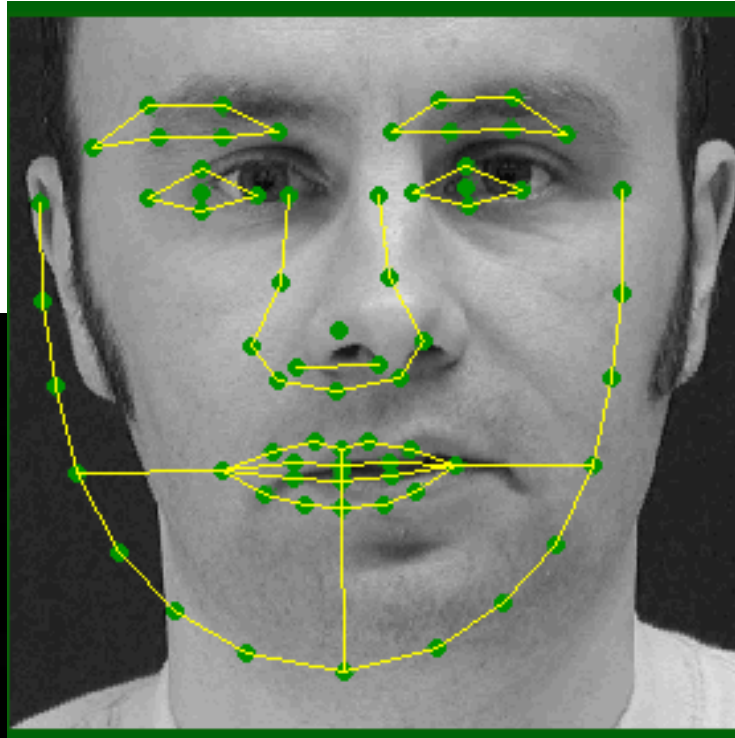
Update Model



Iterate

- **Synthesize model**
- **Measure difference**
- **Difference gives clues as to how to update model**
- **Learn from training set by displacement**
- **Iterate**

AAM Example:



Summary:

ASMs and AAMs:

- **General**: can encompass full range of variation
- **Specific**: generate only valid instances
- **Fast** and **efficient** search algorithms
- Applied in many different contexts
 - Faces (**identity/expression/sex/age/ethnicity**)
 - Medical images (**brains/knees/hips/spines/hands etc**)
- Extended to 3D (brains, kidneys, knees etc)

Research Issues:

- Annotation & Ordering not always possible
- Automatic location of correspondence
 - Geometry (e.g. curvature)
 - Automatic methods like MDL for surfaces** (see our book!)
 - Image registration** – Next set of lectures

Further Information:

- Mathematical details of PCA, correspondence problem

Electronic access via CAS/library to Springer ebooks:

Davies, Twining & Taylor, *Statistical Models of Shape*

- ASM, AAM etc
- Tim Cootes personal website
- Wikipedia: articles & links
- YouTube: many videos of ASM/AAM search & variants