Manifold Learning Overview

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Outline

Introduction

Manifold, manifold hypothesis and manifold learning

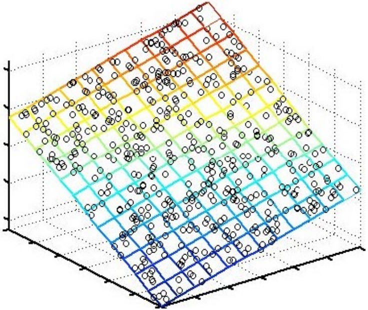
Rigid versus Non-Rigid Geometry

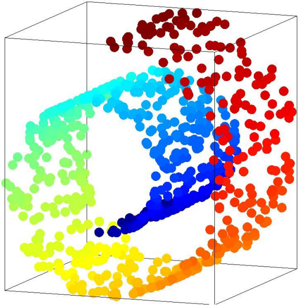
Difference between rigid and non-rigid shapes and their distance metrics

Illustrative Example

Manifold examples in perception and computer vision

- Manifold: a research area in mathematics of topology and differential geometry
  - Definition: A d-dimensional space is said to be a manifold if and only if at each point there exists a neighborhood that is homeomorphic to d-dimensional Euclidean space, $\mathbb{R}^d$.
- Manifold hypothesis: in context of ML, real-world high dimensional data often lie on low-dimensional (sub)manifolds embedded in the high-dimensional space.
- Manifold learning: discover and model low-dimensional manifolds via learning from data to form a low-dimensional latent embedding representation.





Introduction

- Although several topics related to manifold learning had been studied much much earlier, the term was not coined until 2000.

A Global Geometric Framework for Nonlinear Dimensionality Reduction

Tenenbaum, de Silva and Langford

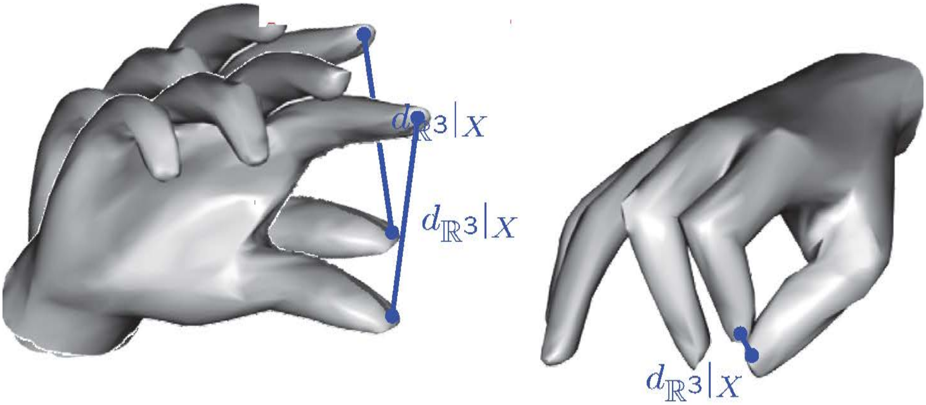
Science (Vol. 290, Dec 2000, 2319-2323)

Nonlinear Dimensionality Reduction by Locally Linear Embedding

Roweis and Saul

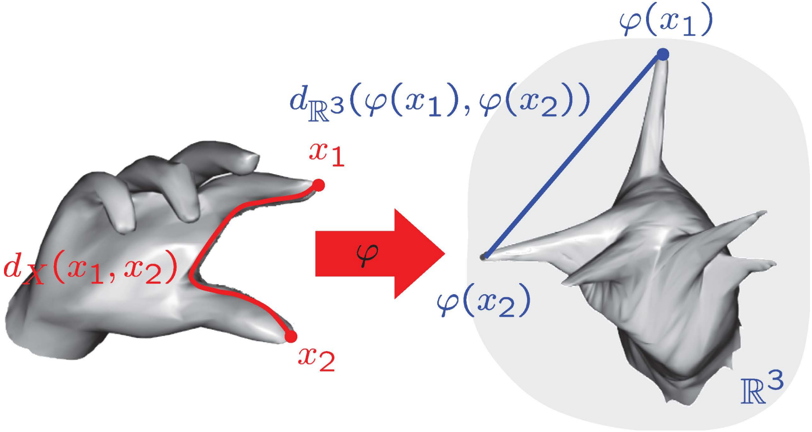
Science (Vol. 290, Dec 2000, 2323-2327)

- Rigid shapes are invariant in Euclidean space, while inelastic non-rigid shapes are variant in Euclidean space (extrinsic space).
- Extrinsic Euclidean distance does not work for non-rigid shapes.

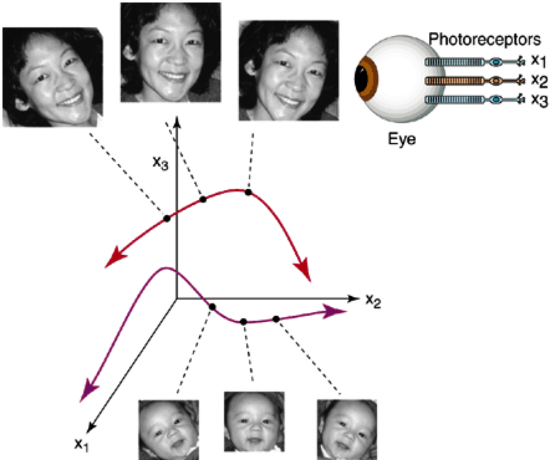


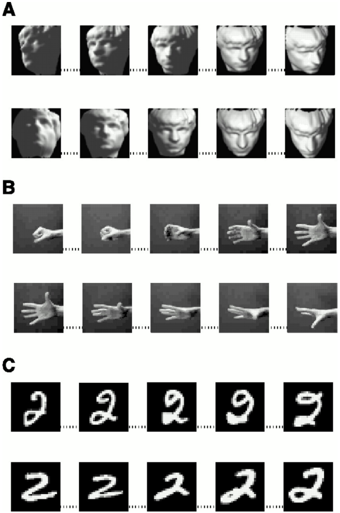
Rigid versus Non-Rigid Geometry

- There is intrinsic space for inelastic non-rigid shapes (nonlinear manifold) where its intrinsic distance is invariant for any data points in nonlinear manifold.
- Mapping the non-rigid shapes to a latent embedding space where extrinsic distance works to preserve the intrinsic distance, which is done by manifold learning in ML.



- Manifolds in visual perception
  Images of the same properties located in a low-dimensional manifolds



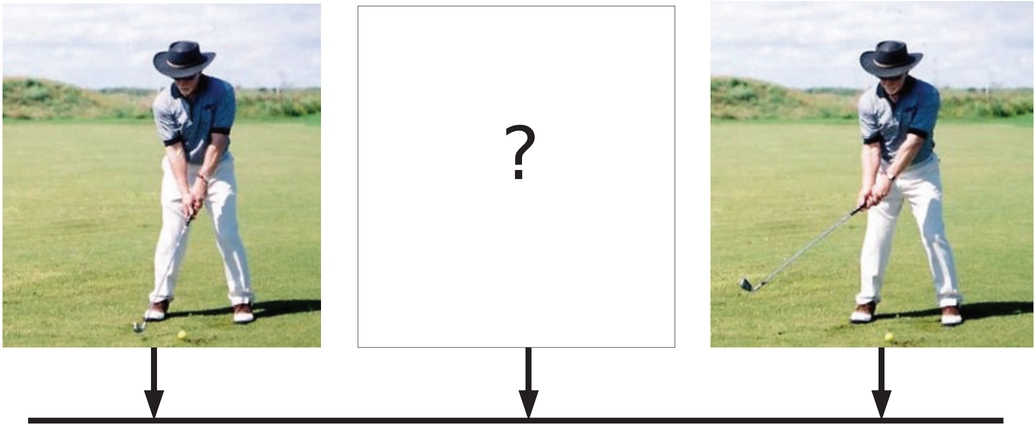


- Manifolds required to be captured in computer vision
  Many intrinsic properties underlying manifolds in computer vision

Appearance Variation

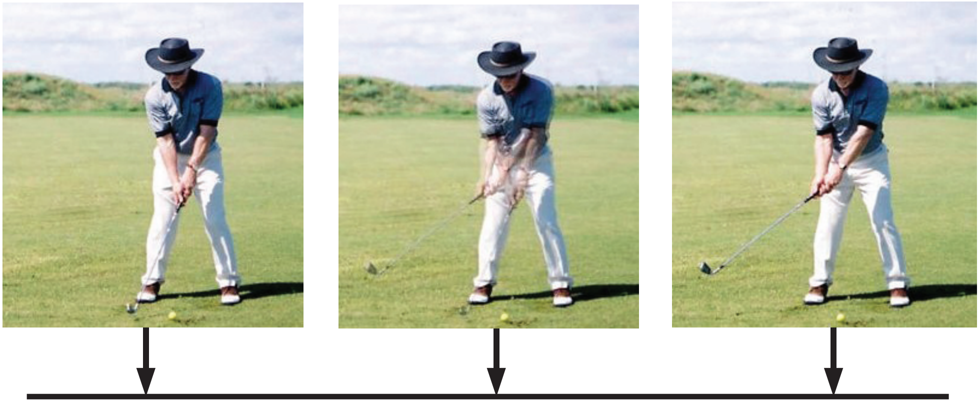
Shape Deformation

- Manifolds required to be captured in computer vision
  Missing frame in the manifold to be interpolated



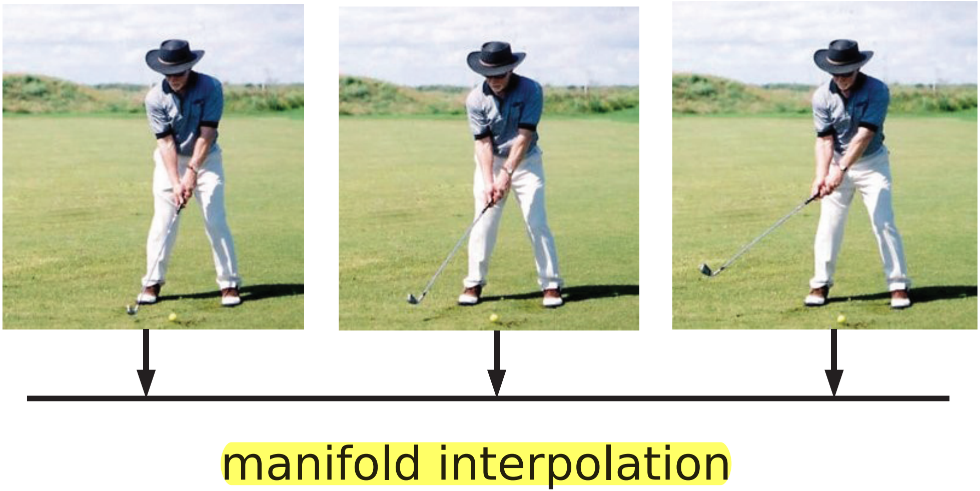
- Manifolds required to be captured in computer vision

  Missing frame in the manifold to be interpolated



linear interpolation

- Manifolds required to be captured in computer vision
  Missing frame in the manifold to be interpolated



Reference

If you want to deepen your understanding and learn something beyond this lecture, you can self-study the optional references below.

[Goodfellow et al., 2016] Goodfellow I., Bengio Y., and Courville A. (2016): *Deep Learning*, MIT Press. (Section 5.11.3)

[Seung & Lee, 2000] Seung H.S. and Lee D.D. (2000): The manifold ways of perception. *Science*, Vol. 290, 22nd Dec. 2000.