Lecture 3:
Region Based Vision

Dr Carole Twining

Tuesday 10th March 2020

14:00pm – 15:00pm

Segmenting an Image

Assigning labels to pixels (cat, ball, floor)

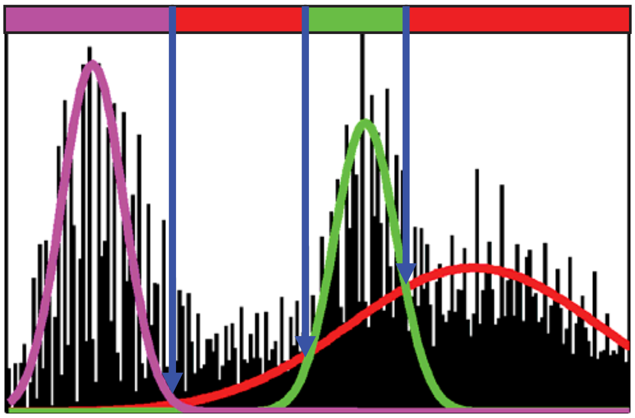
- **Point processing:**
  - colour or grayscale values, thresholding
- **Neighbourhood Processing:**
  - Regions of similar colours or textures
- **Edge information (next lecture)**
- **Prior information: (model-based vision)**
  - I know what I *expect* a cat to look like

Overview

- Automatic threshold detection
  - Earlier, we did this by inspection/guessing
- Multi-Spectral segmentation
  - Satellite & medical image data
- Split and Merge
  - Hierarchical, region-based approach
- Relaxation labelling
  - Probabilistic, learning approach
- Segmentation as optimisation

Automatic Threshold Selection

Automatic Thresholding: GMM

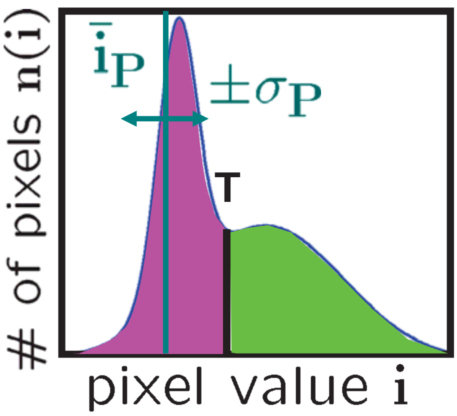


Segmentation Rule

Image Histogram

- Assume scene mixture of substances, each with normal/gaussian distribution of possible image values
- Minimum error in probabilistic terms
- But mixture of gaussians not easy to find
- Doesn't always fit actual distribution

Automatic Thresholding: Otsu's Method



Mean across purples:

$$\bar{i}_P = \frac{1}{N_P}\sum_{i=0}^{T} i \times n(i)$$

Variance for purples:

$$\sigma_P^2 = \frac{1}{N_P}\sum_{i=0}^{T} n(i)\left[i - \bar{i}_P\right]^2$$

Choose T to minimize:

$$N_P\sigma_P^2 + N_G\sigma_G^2$$

- Extend to multiple classes

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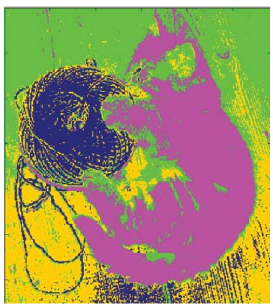

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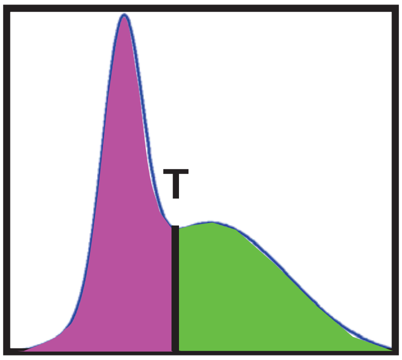
Automatic Thresholding: Max Entropy



For two sub-populations:

$$p_P(i) = \frac{n(i)}{N_P}, \quad i < T,$$

$$p_G(i) = \frac{n(i)}{N_G}, \quad i \geq T.$$

$$\boxed{\text{Entropy:} \quad -\sum p \ln p}$$

Two Entropies:

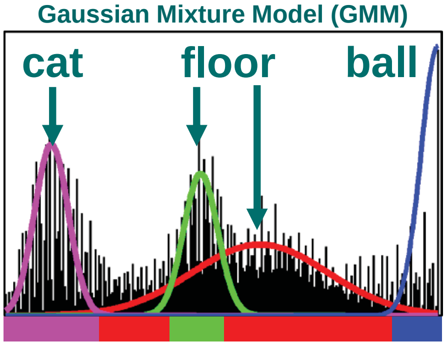
$$H_P = -\sum_{i<T} p_P(i) \ln p_P(i) \;\; \& \;\; H_G = -\sum_{i \geq T} p_G(i) \ln p_G(i)$$

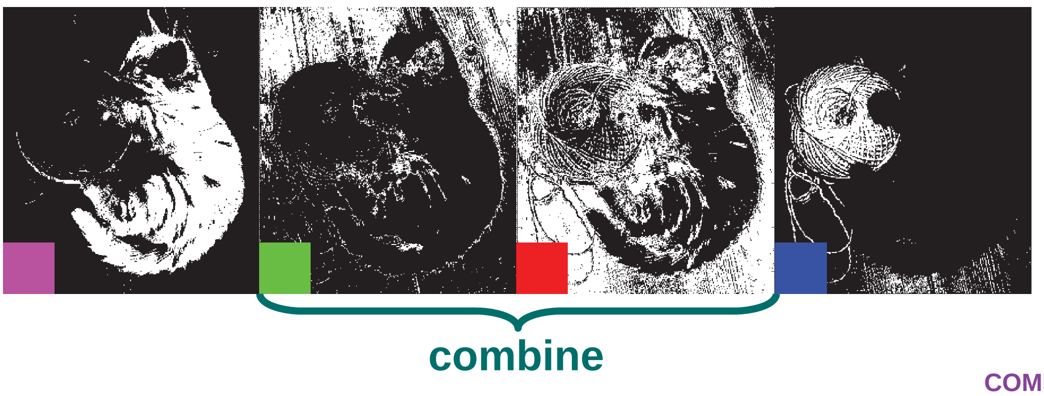
$$\boxed{\text{Minimise:} \;\; H_G + H_P \text{ to find } T.}$$

- Makes two sub-populations as peaky as possible

Automatic Thresholding: Example

Gaussian Mixture Model (GMM)





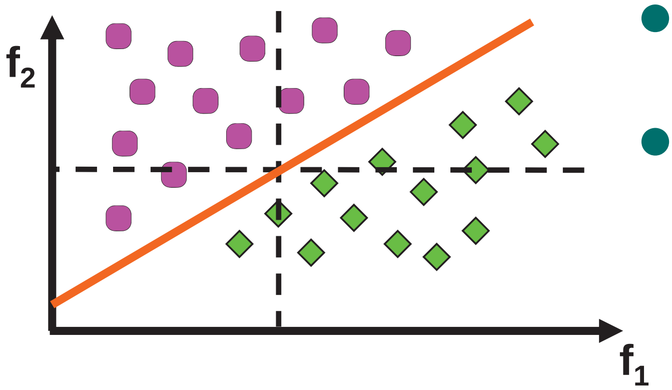
Automatic Thresholding: Summary

- Geometric shape of histogram (bumps, curves etc)
  - Algorithm or just by inspection
- Statistics of sub-populations
  - Otsu & variance
  - Entropy methods
- Model-based methods:
  - Sum of gaussians, gaussians & partial voluming etc.
- Detailed comparative evaluations for 40 methods
  - Sezgin M, Sankur B; Survey over image thresholding techniques and quantitative performance evaluation.
    Journal of Electronic Imaging, 13(1): pages 146-168, (2004).
- Fundamental limit on effectiveness:
  - Never be perfect if distributions overlap (two objects, shared colour!)
- Whatever method, need further processing

Multi-Spectral Segmentation

Multi-spectral Segmentation

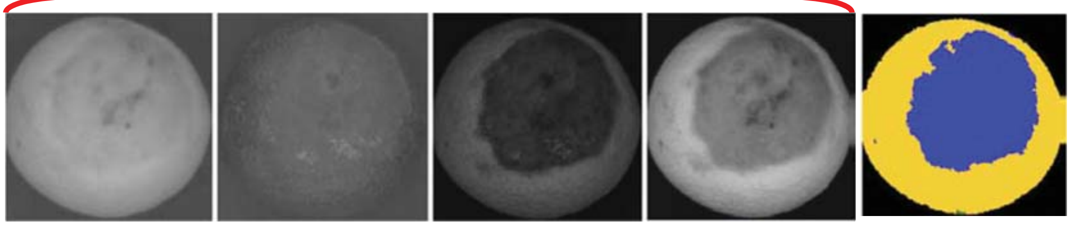
- Multiple measurements at each pixel:
  - Satellite remote imaging, various wavebands
  - MR imaging, various imaging sequences
  - Colour (RGB channels, HSV etc)
  - Multispectral imaging of historical documents (visible+IR+UV)
- Scattergram of pixels in vector space:



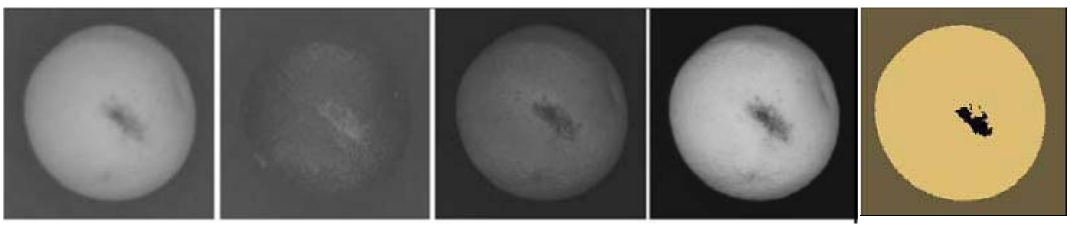
- Can't separate using single measurement
- Can using multiple

Multi-Spectral Segmentation:Example

Spectral Bands



Over-ripe Orange



Scratched Orange

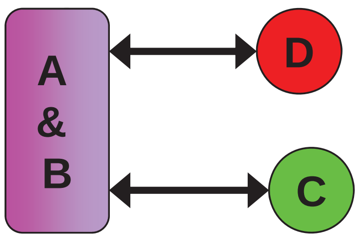
Multispectral Image Segmentation by Energy Minimization for Fruit Quality Estimation: *Martínez-Usó, Pla, and García-Sevilla,* Pattern Recognition and Image Analysis , 2005

Split and Merge

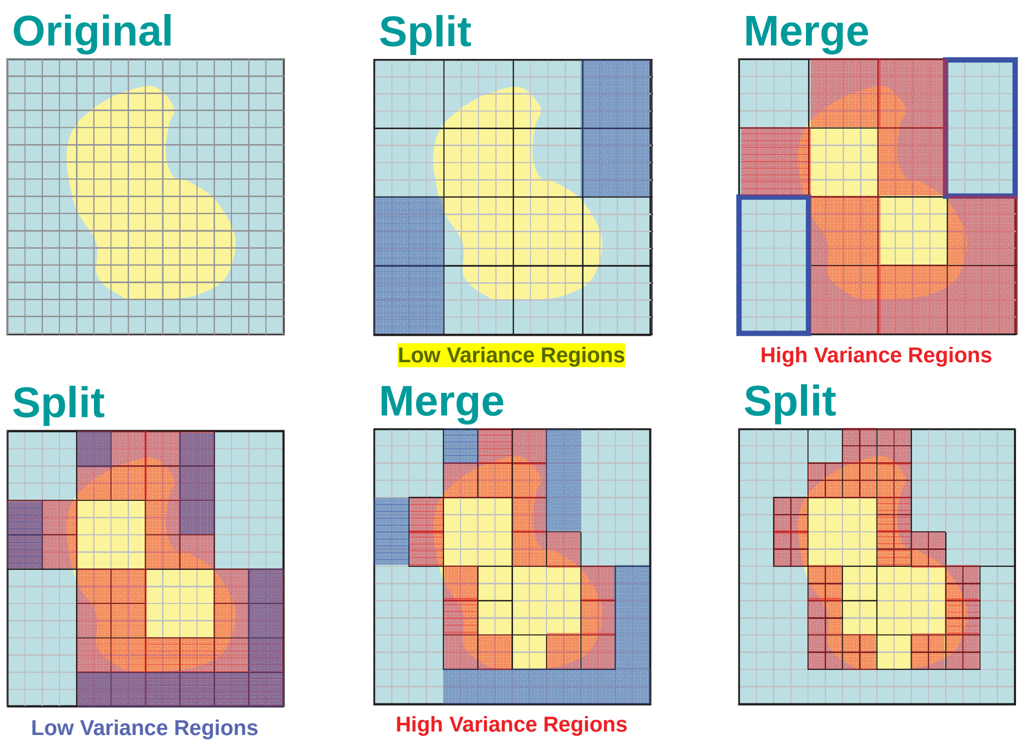
Split and Merge/Quadtree Segmentation

- Obvious approaches to segmentation:
  - Start from small regions and stitch them together
  - Start from large regions and split them

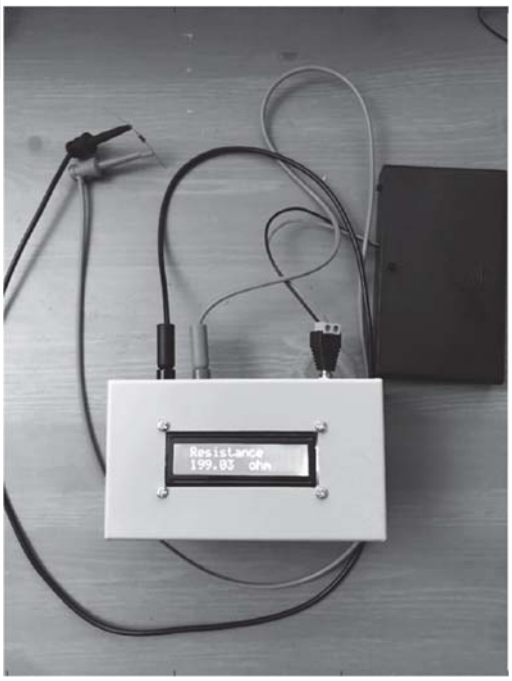
  Combine
- Start with large regions , split non-uniform regions
  - e.g. variance σ^2 > threshold
- Merge similar adjacent regions
  - e.g. combined variance σ^2 < threshold
- Region adjacency graph
  - housekeeping for adjacency as regions become irregular
  - regions are nodes, adjacency relations arcs
  - simple update rules during splitting and merging



Split and Merge/Quadtree Segmentation



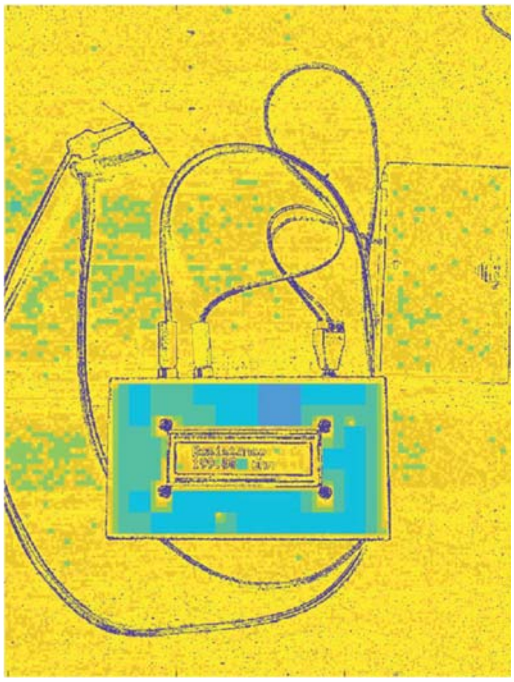
Split & Merge: Example



Original

Detail of Blocks

Result



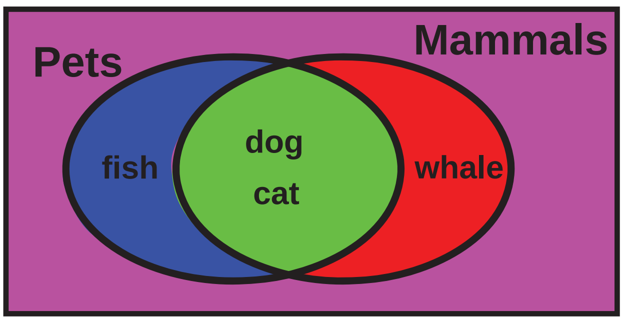
Relaxation Labelling

Aside: Conditional Probability

probability of A given that B is the case

$$P(A \mid B)$$

- P(pet) = $\dfrac{(\square+\square)}{\boxed{\text{ALL}}}$ etc
- P(pet | mammal) = $\dfrac{\square}{(\square+\square)}$
- P(mammal | pet) = $\dfrac{\square}{(\square+\square)}$



All Animal Species

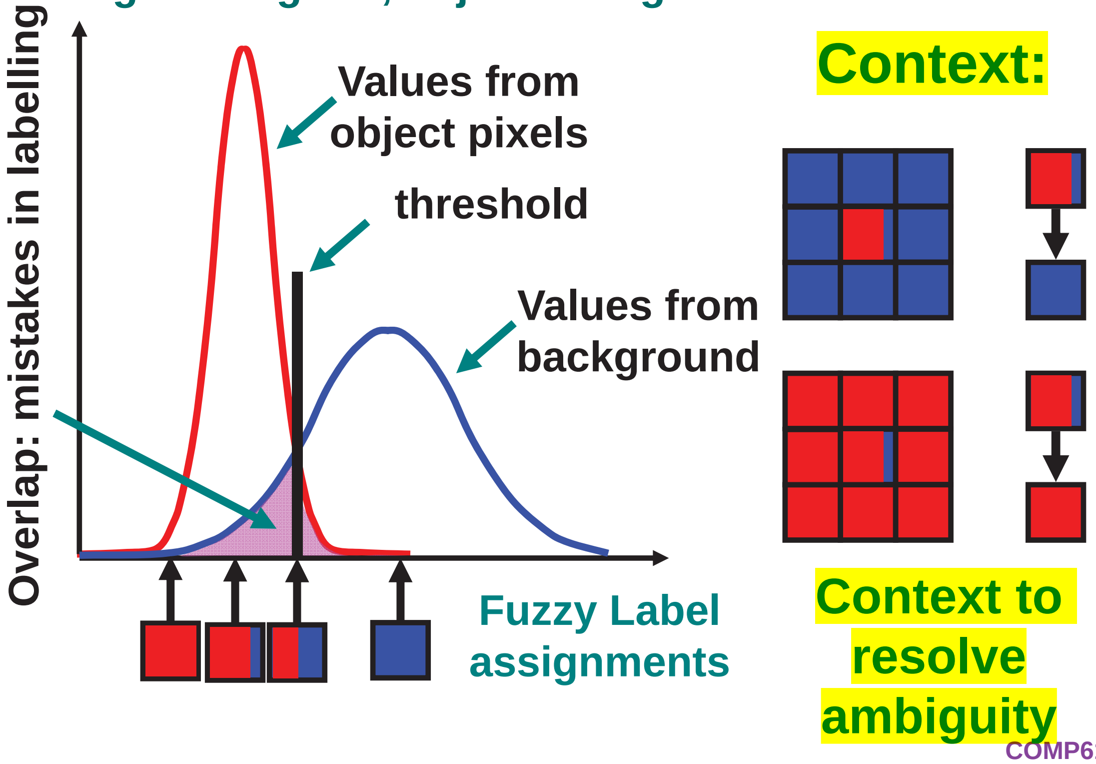
- Bayes Theorem:

$$\frac{\square}{(\square+\square)} \times \frac{(\square+\square)}{\boxed{\text{ALL}}} = \frac{\square}{(\square+\square)} \times \frac{(\square+\square)}{\boxed{\text{ALL}}}$$

- P(pet | mammal)P(mammal) = P(mammal | pet)P(pet)

Relaxation Labelling:

- Image histogram, object/background



Relaxation Labelling

- Evidence for a label at a pixel:
  - Measurements at that pixel (e.g., pixel value)
  - Context for that pixel (i.e., what neighbours are doing)
- Iterative approach, labelling evolves
- Soft-assignment of labels:

Possible labels: $\{l_\mu : \mu = \mathbf{1}, \ldots \mathbf{n}\}$

$\mathbf{P_i}(\mu)$: Probability that pixel i has label l_μ.

$\sum_\mu \mathbf{P_i}(\mu) \equiv \mathbf{1}$. normalised probability.

- Soft-assignment allows you to consider all possibilities
- Let context act to find stable solution

Relaxation Labelling

- **Compatibility:**

  Pixels i and j, labels μ and ν:

no effect $c_{i,j}(\mu, \nu) = 0$ **If not neighbours**

support (+ve) $c_{i,j}(\mu, \mu) = \alpha$ **Neighbours & same label**

oppose (-ve) $c_{i,j}(\mu, \nu) = -\alpha$ if $\mu \neq \nu$ **Neighbours & different label**

- **Contextual support for label μ at pixel *i* :**

$$s_i(\mu) \propto \boxed{\sum_{j \neq i}} \boxed{\sum_{\nu}} \boxed{c_{i,j}(\mu, \nu)} \boxed{P_j(\nu)}$$

look at all other pixels

all possible labels & how strong

degree of compatibility

Relaxation Labelling:

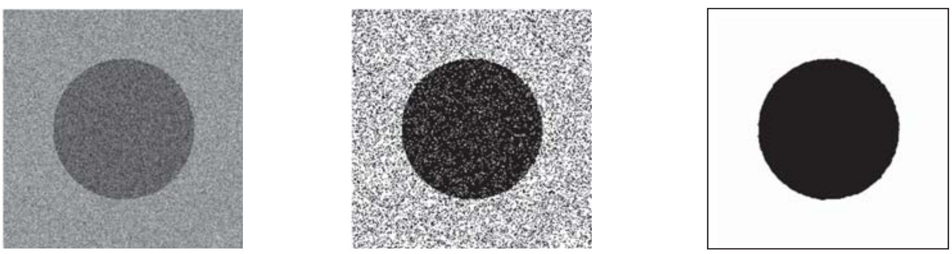
- **Update soft labelling given context:**

$$P_i(\mu) \Leftarrow A_i P_i(\mu)(1 + s_i(\mu))$$

A_i chosen so sums to 1 at i.

- **The more support, more likely the label**
- **Iterate**

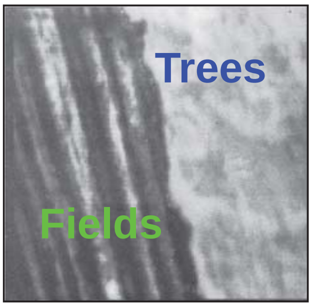
main idea!



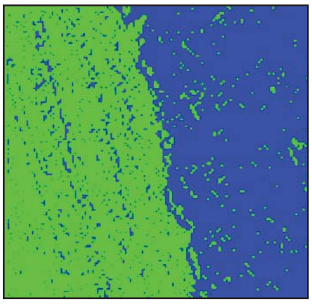
Noisy Image **Threshold labelling** **After iterating**

Relaxation Labeling:

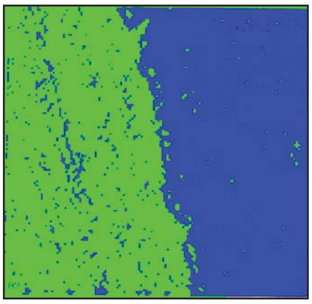
- Value of α alters final result



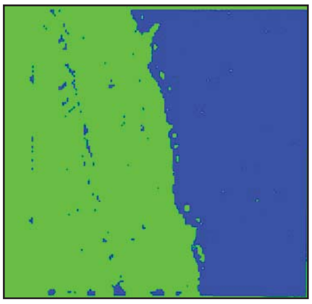
Initialisation



α = 0.75



α = 0.90



Segmentation as Optimisation

Segmentation as Optimisation

Image: $\mathcal{I}$, value at pixel i: $\mathcal{I}(i)$

Label Image: L, label at pixel i: $L(i)$

Label configuration in neighbourhood of i: $l(i)$

- **Maximise probability of labelling given image:**

$$P(L|\mathcal{I}) = \prod_i \underbrace{P(L(i)|\mathcal{I}(i))}_{\text{label at i given value at i}} \underbrace{P(L(i)|l(i))}_{\text{label at i given labels in neighbourhood of i}}$$

- **Re-write by taking logs, minimise cost function:**

$$C(L,\mathcal{I}) = \sum_i [\underbrace{-\log P(L(i)|\mathcal{I}(i))}_{\text{label-data match}} \underbrace{-\log P(L(i)|l(i))}_{\text{label consistency}}]$$

- **How to find the appropriate form for the two terms?**
- **How to find the optimum?**

Segmentation as Optimisation

$P(L(i)|l(i))$
label consistency

$P(L(i)|\mathcal{I}(i))$
label-data match

- **Exact form depends on type of data**
- **Histogram gives:** $p(\mathcal{I}(i))$
- **Model of histogram** $P(L(i)|\mathcal{I}(i))$

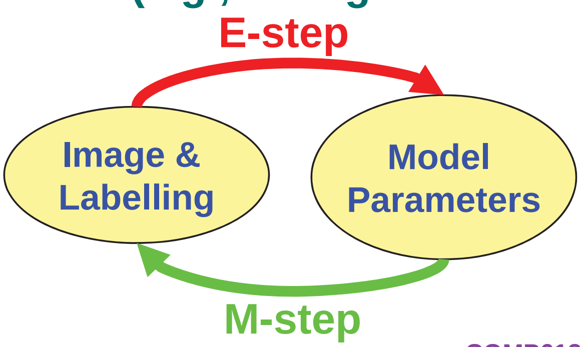
(e.g., sum of gaussians, relaxation case)

Learning approach:

- **Explicit training data (i.e., similar labelled images)**
- **Unsupervised, from image itself (e.g., histogram model):**

Expectation/Maximization

- **Given labels, construct model**
- **Given model, update labels**
- **Repeat**

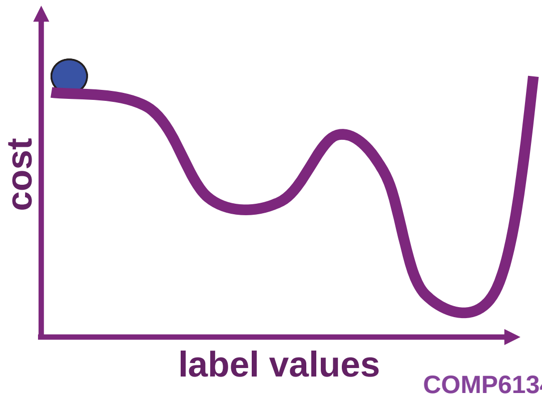


Segmentation as Optimisation

- **General case:**

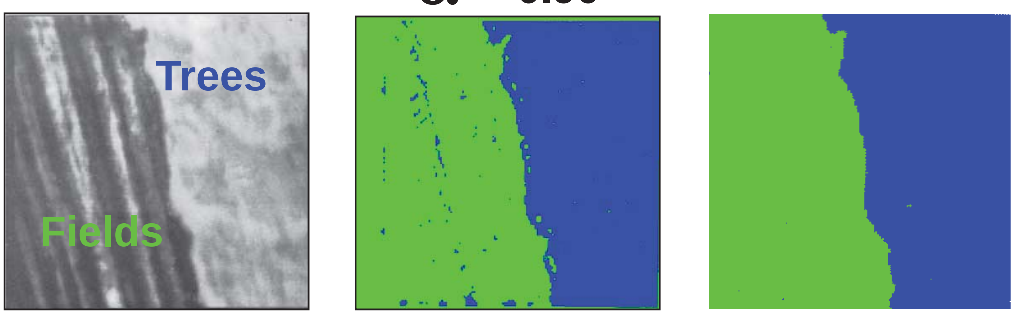
Cost function: $C(L, \mathcal{I}) = \boxed{\text{label-data match term}} + \boxed{\text{label consistency}}$

- **High-dimensional search space, local minima**
- **Analogy to statistical mechanics**
  - crystalline solid finding minimum energy state
  - stochastic optimisation
  - simulated annealing
- **Search:**
  - Downhill
  - Allow slight uphill



Segmentation as Optimisation

$\alpha = 0.90$



Original Relaxation Optimisation