MANCHESTER 1824

The University of Manchester

Image Registration 3:

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Handouts & Lecture Notes

- **Report in Scientific American (June 2014):**

  "In each study, however, those who wrote out their notes by hand had a stronger conceptual understanding and were more successful in applying and integrating the material than those who used [sic] took notes with their laptops."

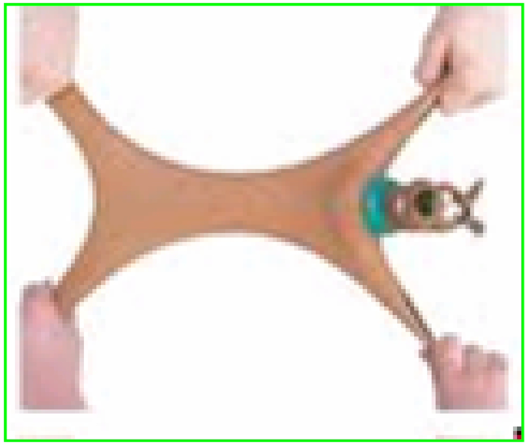
  The Pen Is Mightier Than the Keyboard

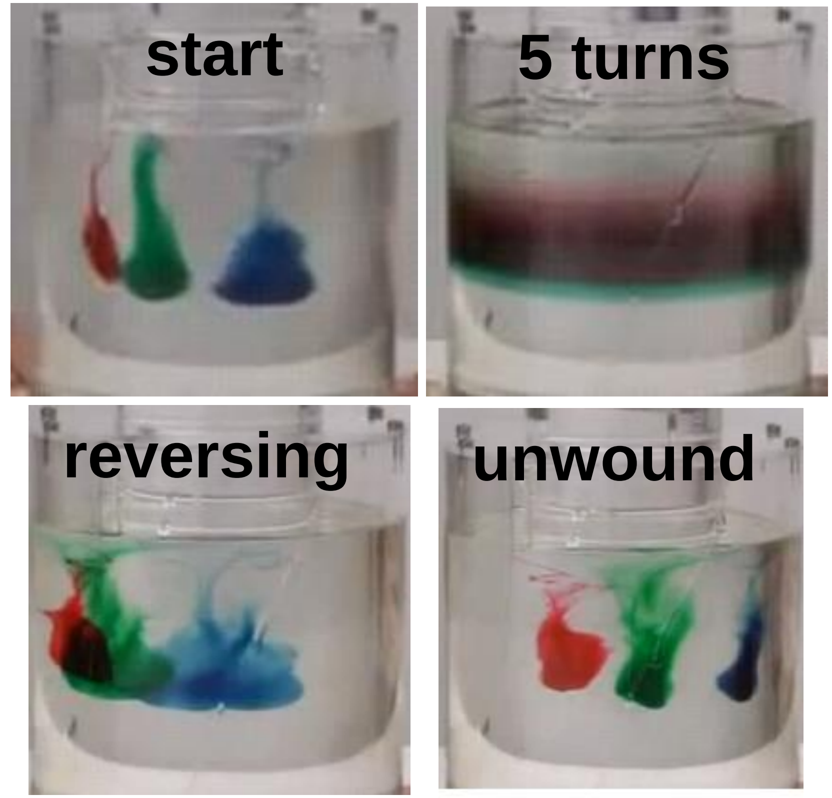
  P. A. Mueller, D. M. Oppenheimer, *Psychological Science,* Vol 25,Issue 6, pp. 1159 – 1168, April-23-2014.

- **Handouts are to aid note taking, not a total replacement for note taking**
- **Podcasts, slides, pdfs etc on BlackBoard**

From Elastic Solids to Fluids

Physics Based Models:





Elastic Registration:

- **Elasticity regularises warp**
- **Penalizes large deformations**
- **Can't deal with extreme cases**

Fluid:

- **Extremely large deformation**
- **Still unmixes!**
- **Viscosity and flow over time**

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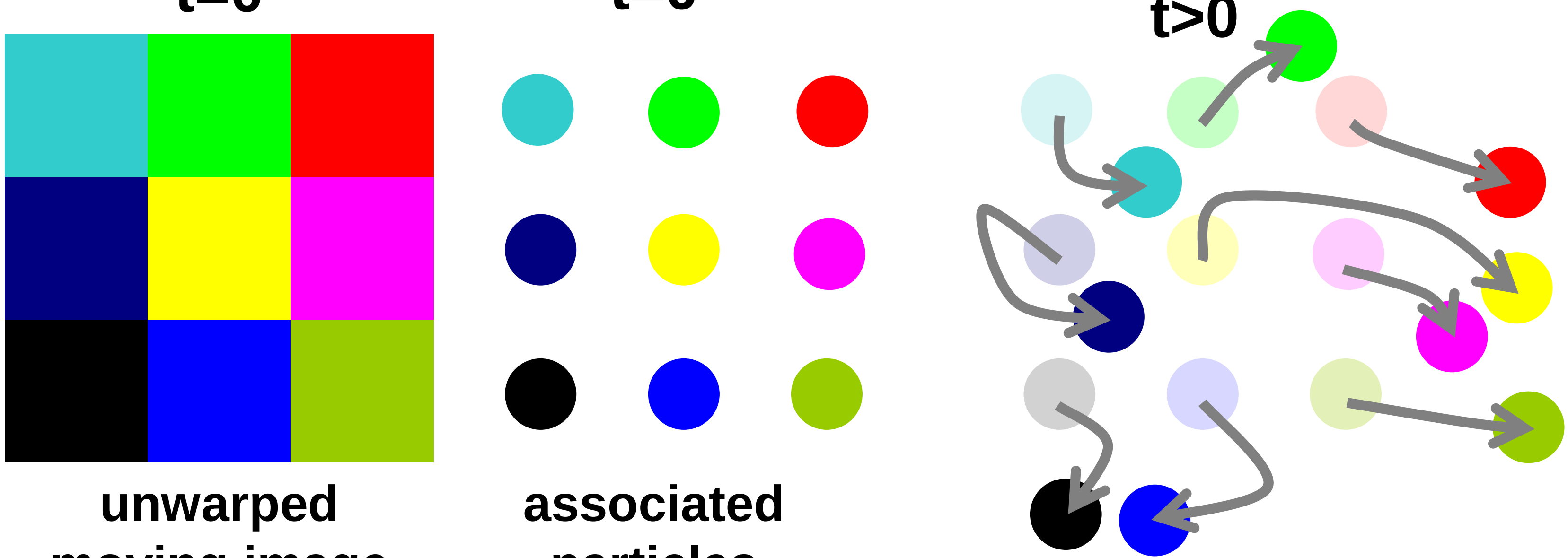
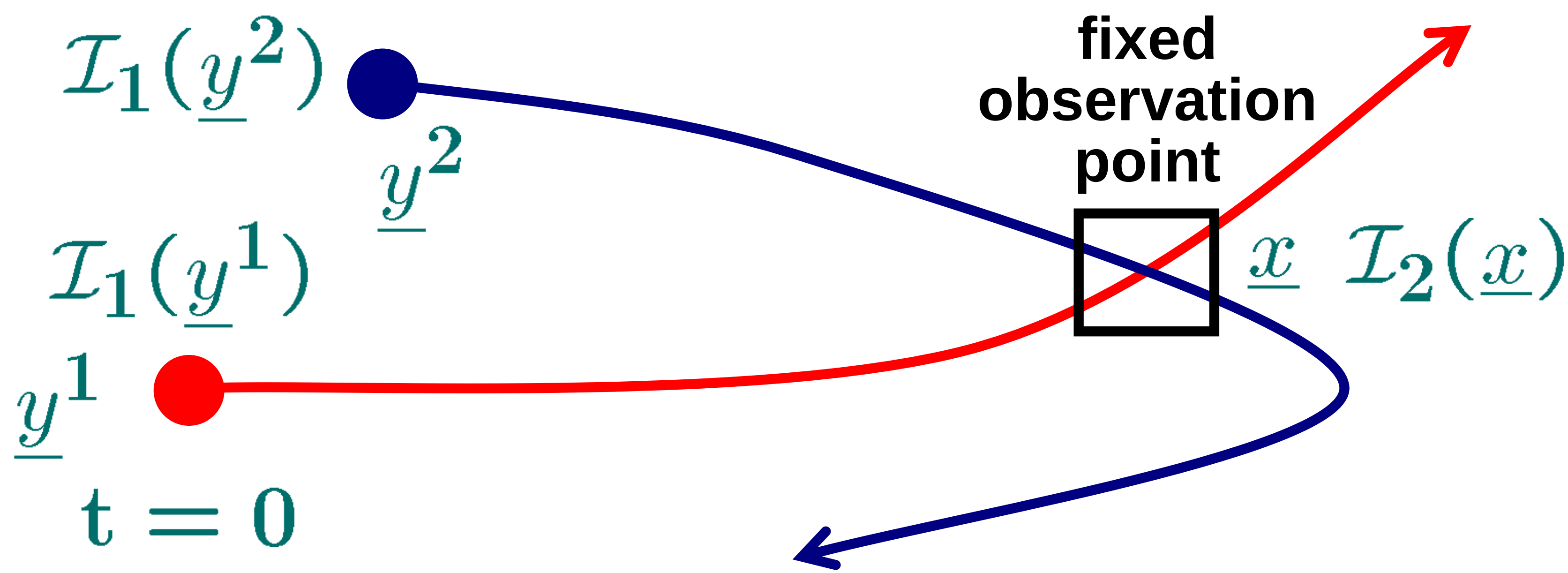
Moving Images using Fluid Flow



- Added time dimension allows greater flexibility
- Regulariser = fluid forces between moving particles

Fluid Picture: Notation



Trajectories: $\phi(\underline{y}^{1,2}, \mathrm{t}),\ \phi(\underline{y}^1, 0) = \underline{y}^1$

- **Follow just two particles**
- **Different particles pass observation point at different times, with different speeds, & different image values**
- **Trajectories can cross, just can't have two particles in the same place at the same time**

Fluid Picture: Notation

- Fluid particles that move in time,

carry the moving image information with them

- Particle that starts at $\underline{y}$ at $\mathbf{t} = \mathbf{0}$

carries image information $\mathcal{I}_1(\underline{y})$

$$\boxed{\phi(\underline{y}, \mathbf{0}) = \underline{y}}$$

- When reaches a point $\underline{x}$ at time t

Compare $\mathcal{I}_1(\underline{y})$ with value $\mathcal{I}_2(\underline{x})$

$$\boxed{\phi(\underline{y}, \mathbf{t}) = \underline{x}}$$

- Forces acting:

Image comparison forces $\mathcal{I}_1(\underline{y})$ vs $\mathcal{I}_2(\underline{x})$

Fluid regulariser:

Viscous forces between moving particles

Fluids & Particle Displacements

- **Define (backwards) displacement field:**

  Particle passing point $\underline{x}$ at time t
  started from $\underline{x} - \underline{u}(\underline{x}, \mathrm{t})$ at $\mathrm{t} = 0$

- **Particle velocity as it passes this point** **[Reg2, slide 19]** **:**

$$\underline{v}(\underline{x}, \mathrm{t}) = \frac{\partial \underline{u}(\underline{x}, \mathrm{t})}{\partial \mathrm{t}} + (\underline{v}(\underline{x}, \mathrm{t}) \cdot \overrightarrow{\nabla})\underline{u}(\underline{x}, \mathrm{t})$$

- **Regulariser forces: in elastic case (no flow)**

$$\underline{F}_{\mathbf{elas}}(\underline{x}) = \mu \overrightarrow{\nabla}^{2}\underline{u}(\underline{x}) + (\lambda + \mu)\overrightarrow{\nabla}(\overrightarrow{\nabla} \cdot \underline{u}(\underline{x}))$$

- **Fluids: similar, but Eulerian velocity rather than displacement:**

$$\boxed{\underline{F}_{\mathbf{visc}}(\underline{x}, \mathrm{t}) = \mu \overrightarrow{\nabla}^{2}\underline{v}(\underline{x}, \mathrm{t}) + (\lambda + \mu)\overrightarrow{\nabla}(\overrightarrow{\nabla} \cdot \underline{v}(\underline{x}, \mathrm{t}))}$$

massless & inertialess visco-elastic fluid

Fluid Registration:

- **Driving force:**

  Body forces = Image difference forces
- **Fluid massless and inertialess:**

  Flows so that viscous forces counteract applied forces
- **Penalises flow not displacement (which can be large)**
- **When reaches match, body forces vanish, flow stops**

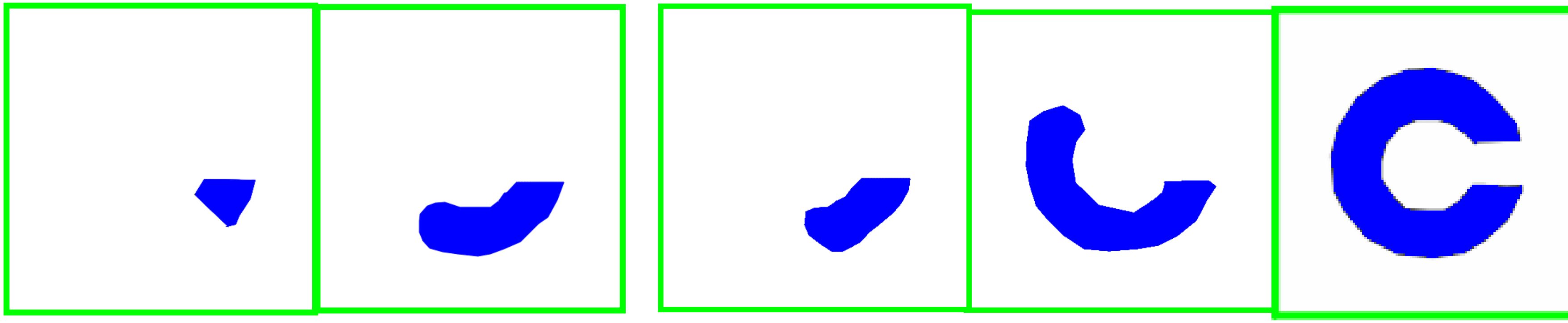
Elastic Solution:

- **Body forces balance displacement force**

Fluid Solution:

- **Flows until body forces vanish**

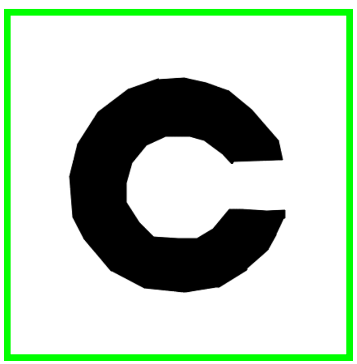
Blob to C:



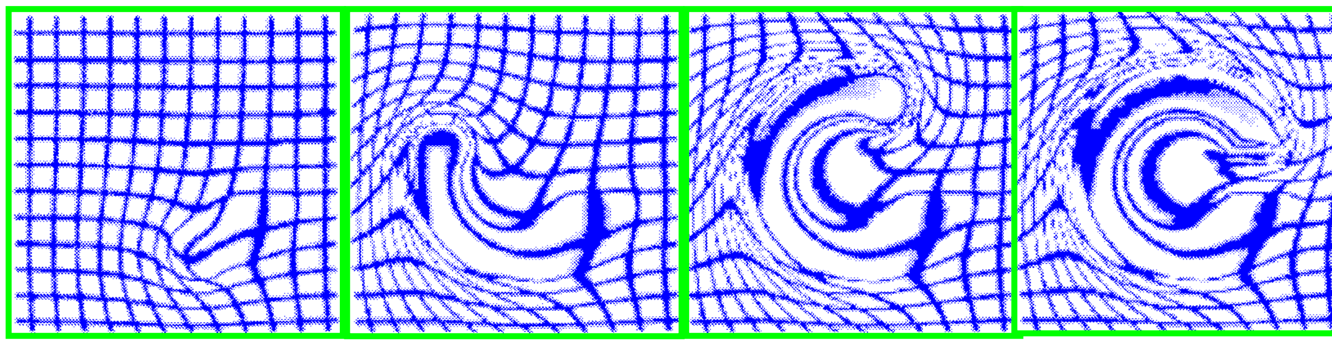
moving image

Elastic result

Fluid evolution



target



Grid deformation

Boundary is fixed

Fluid: Solution

Christensen, Rabbitt, and Miller
Deformable Templates Using Large Deformation Kinematics,
IEEE Transactions on Image Processing, 5(10), pages 1435—1447, (1996).

- **Start from zero displacement**
- **Evaluate image-match body forces**
- **Viscous forces balance - hence find flow fields** $\underline{v}(\underline{x}, \mathrm{t})$
- **Update displacements:** $\underline{u}(\underline{x}, \mathrm{t}) \mapsto \underline{u}(\underline{x}, \mathrm{t} + \Delta\mathrm{t})$
- **Compute new body forces**
- **Repeat until body forces vanish**

Fluid Registration: Summary

- **Like elastic, greater flexibility than parametric warps**
- **Like elastic, simplified and viscosity fixed across image**
- **Unlike elastic, can generate very large displacements**
- **As toy example shows, does so without tears or folds**
- **Computationally intensive, but suitable where considerable fine-scale matching, as in brains**

Other Flow-Based Methods:

- **Large Deformation Diffeomorphic Metric Mapping (LDDMM)**

limited time flow

Trajectory: $\phi(\underline{y}, \mathrm{t})$, from $\mathrm{t} = 0$ to $\boxed{\mathrm{t} = 1}$

Velocity field: $\underline{v}(\underline{x}, \mathrm{t})$ as before

- **Objective function:**

$$\mathcal{E} = \boxed{\mathcal{D}(\mathcal{I}_1, \mathcal{I}_2; \phi(\cdot, 1))} + \boxed{\lambda\mathcal{R}(\phi)}$$

image match at t=1 **trade-off warp term**

$$\mathcal{R}(\phi) = \int_0^1 \|\underline{v}(\cdot, \mathrm{t})\|^2 \mathrm{dt}$$ **depends on whole of flow**

$$\|\underline{v}(\cdot, \mathrm{t})\|^2 = \int \mathrm{d}\underline{x}(\mathbf{L}\underline{v}(\underline{x}, \mathrm{t}))^2$$ **norm on velocity fields**

Differential operator: $\mathbf{L} = \left(-\alpha\vec{\nabla}^2 + \gamma\right)^\beta$

Pairwise Registration:

- Large number of methods of pairwise registration:

  Image similarity term

  Warp representation (parametric, non-parametric)

  Exact regulariser

  2D, 3D or time series
- Different methods for different tasks

BUT:

- What if we want to register a whole population?
- Groupwise image registration?

Groupwise Image Registration

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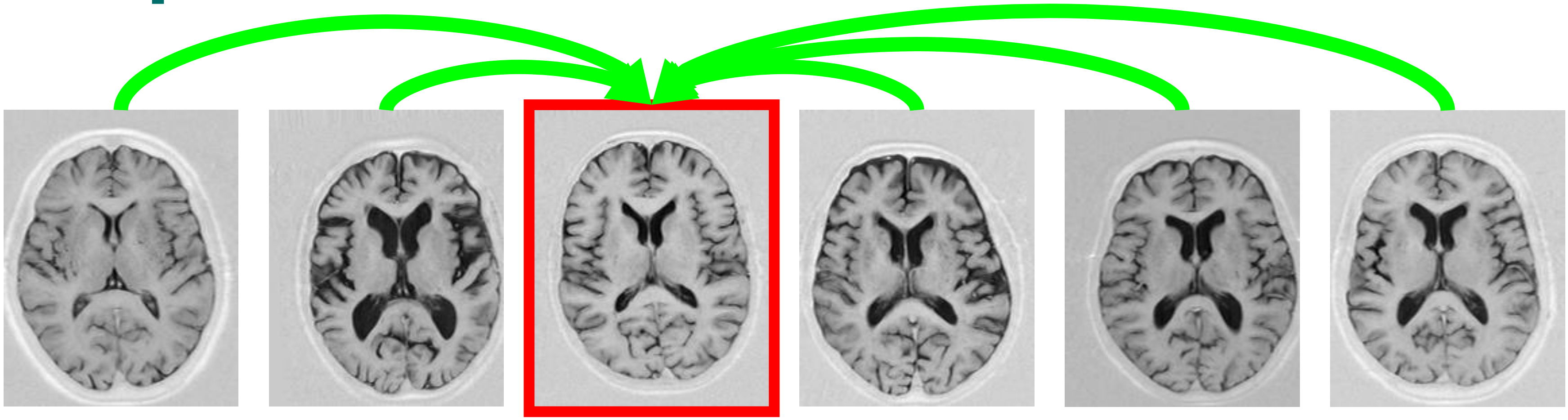
Repeated Pairwise



- Large set of images

Repeated Pairwise:

Chose a reference image, register all to this, 1-by-1

- Depends which reference you choose
- Doesn’t use all available image information
- Group can help resolve ambiguities

Groupwise Registration:

- Suppose we are warping whole set to common frame
- Image Matching:
- Pairwise:

  Difference of two aligned images, pixel-by-pixel
- Groupwise:

  Whole distribution of differences, pixel-by-pixel

  Some regions may be harder to match than others
- Use statistics of that distribution to evaluate groupwise image matching term
- Shows improvements over repeated pairwise

Link between Registration & Modelling

Image & Shape Models (AAM, ASM etc):

- Dense groupwise correspondence for training set

Registration:

- Output is such a correspondence

Groupwise Registration:

- Use quality of model to evaluate quality of registration

  Model built using bad registration generates images unlike actual training images
- Information-theoretic approach:

  Minimum Description Length (MDL): Rissanen

Minimum Description Length (MDL):

Objective function for registration:

- Build appearance model from correspondence
- Use model to encode all training images
  (i.e., generate them from the model)
- Transmit model as binary message
  more modes = longer message
- Transmit training set encoded using the model
  better the model fit to data, shorter the encoding
- Total length of message is objective function
- Simple to describe, harder to make work!

MDL for Shapes & Images:

- **MDL Shape Models:**

Davies, Twining & Taylor
Statistical Models of Shape: Optimisation & Evaluation
Springer, 2008 (electronic access via library)

- **MDL Appearance models & Groupwise registration:**

Stephen Marsland, Carole J. Twining and Chris J. Taylor
A minimum description length objective function for groupwise non-rigid image registration
Image and Vision Computing, Volume 26(3), Pages 333-346, 2008

Recent Research Issue:

- Lots of different methods of registration
- Many in routine use

Issue:

- Proper evaluation & comparison

  Ground-truth annotation difficult if not impossible
- Goal: evaluation without ground-truth
- Paper in progress at the moment……….

In Conclusion:

- **Image Registration:**

  Intellectually-challenging area

  Requires mathematical and computational sophistication

- **Uses:**

  Making sense of vast quantity of medical image data

  Practical applications: image-guided surgery

  Research applications:

  Many interesting questions that you couldn’t attempt to answer without image registration