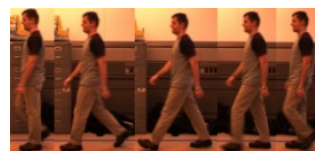
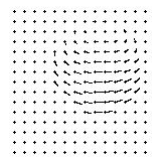


## Motion Analysis: Optical Flow

### Finally: Motion and tracking

Tracking objects, video  
analysis, low level motion



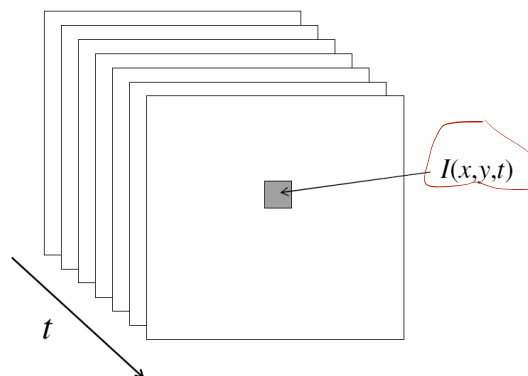
Tomas Izo

## Uses of motion

- Estimating 3D structure
- Segmenting objects based on motion cues
- Learning dynamical models
- Recognizing events and activities
- Improving video quality (motion stabilization)

## Video

- A video is a sequence of frames captured over time
- Now our image data is a function of space (x, y) and time (t)

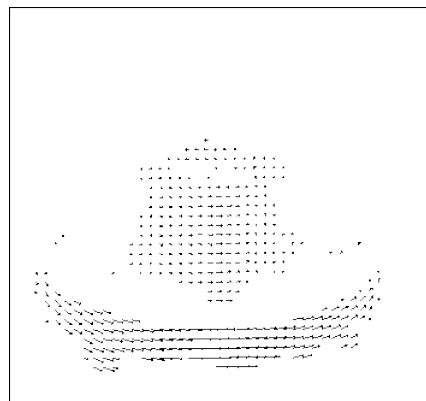
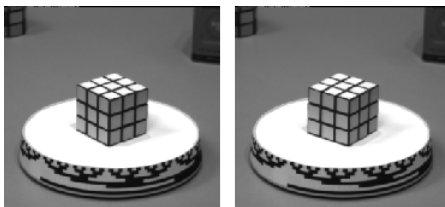


## Lecture's goals

- Motion Analysis
  - Optical flow

## Motion field

- The motion field is the projection of the 3D scene motion into the image



## Motion field + camera motion

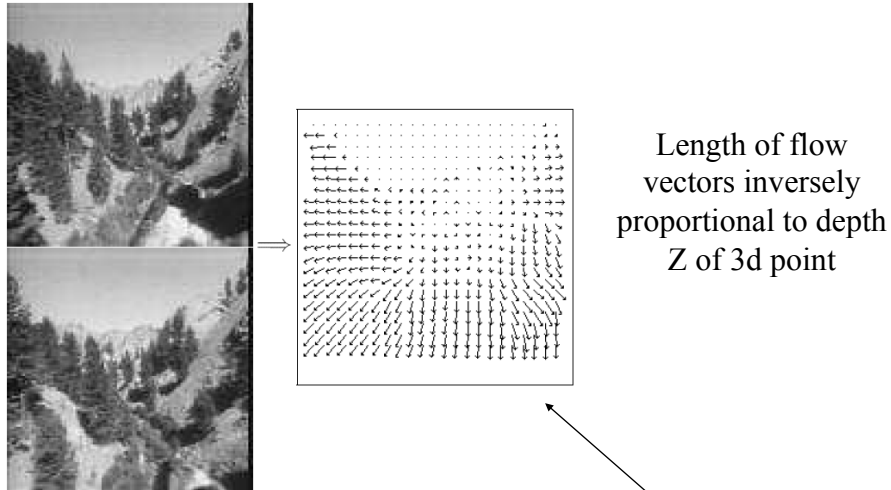


Figure 1.2: Two images taken from a helicopter flying through a canyon and the computed optical flow field.

points closer to the camera move more quickly across the image plane

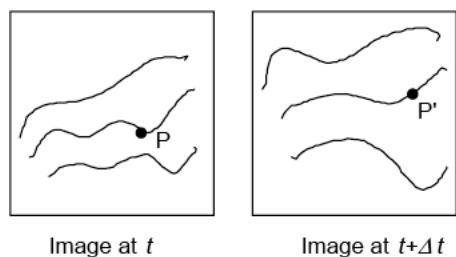
Figure from Michael Black, Ph.D. Thesis

## Optical Flow (Image Flow)

- Estimate velocity based on brightness pattern
- Originates from early psychological work on human vision
  - Cues used by pilots in landing aircraft.
- Local estimates of the velocity of the brightness field
  - Do not require image structure
  - Dense velocity maps

## Motion of the Brightness Pattern

- Motion field
  - Velocity vector at each point in the scene
- Optical Flow
  - Apparent motion of the brightness pattern
  - Components  $u$ ,  $v$  at each point on the image
- Mostly they correspond



Correspondence Problem

## The Optical Flow Equation

$$I(x, y, t) = I(x + dx, y + dy, t + dt)$$

Image structure doesn't change, just moves

$$I(x + dx, y + dy, t + dt) = I(x, y, t) + \frac{\partial I}{\partial x} dx + \frac{\partial I}{\partial y} dy + \frac{\partial I}{\partial t} dt + (h.o.t)$$

Taylor expansion

$$\frac{\partial I}{\partial x} dx + \frac{\partial I}{\partial y} dy + \frac{\partial I}{\partial t} dt = 0 \qquad \frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

$$\frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v + \frac{\partial I}{\partial t} = 0 \quad \text{Optical Flow Equation}$$

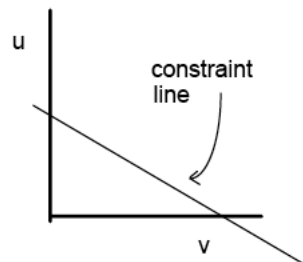
## Solving Optical Flow

$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} = 0$$

Image gradient

Rate of change of  
brightness with time

Equation of straight line



No unique solution for  $u$  and  $v$

## Constrained solution

- Smoothness
  - Should be small  $\rightarrow \begin{cases} u_x = \frac{\partial u}{\partial x}; & u_y = \frac{\partial u}{\partial y} \\ v_x = \frac{\partial v}{\partial x}; & v_y = \frac{\partial v}{\partial y} \end{cases}$  x and y derivatives of u and v
- Optical Flow constraint  $\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} = 0$
- Minimise  $\rightarrow \begin{cases} \sum \sum (u_x^2 + u_y^2) + (v_x^2 + v_y^2) \\ \sum \sum \left( \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} \right) \end{cases}$

## Optical Flow Due to Camera Translation

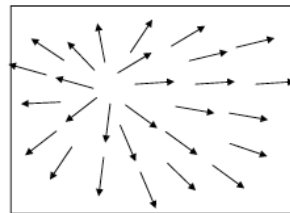


Intervening frames

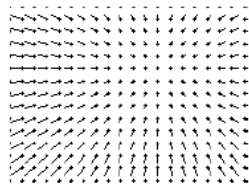


Focus of Expansion

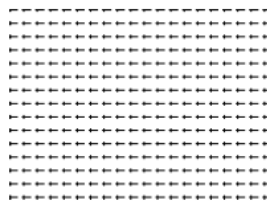
Defines instantaneous direction  
of camera motion



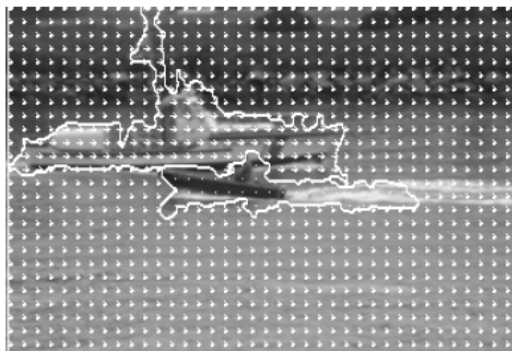
## Optical Flow



Focus of contraction



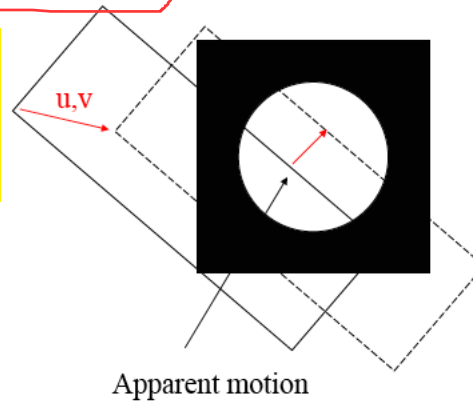
Panning



Motion-based segmentation.  
Camera panning left (following smaller boat)  
Larger boat moving right.

## The Aperture Problem

- Using local information (e.g. derivatives) we can only measure velocity vector *normal to local image gradient*.
- Need to apply further constraints
  - e.g. smoothness
  - Nearby points should have similar velocities



## Optical Flow

### When/where does this break down?

In what situations does the displacement of pixel patches not represent physical movement of points in space?

A uniform rotating sphere

Nothing seems to move, yet it is rotating

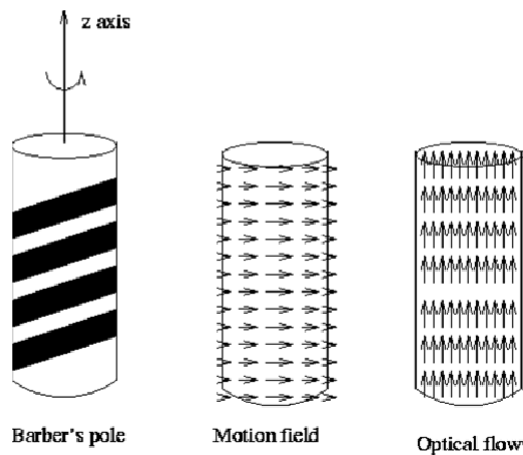
Changing directions or intensities of lighting can make things seem to move

For example, if the specular highlight on a rotating sphere moves.



# Aperture Problem

## Barber pole illusion



## Tracking

- Identify **significant** points
- Track from frame to frame
- Aperture problem avoided if we use **identifiable structure**.
- Simplest to match structure using cross-correlation
- No epipolar constraint
- May be able to predict motion.

