Answers to Exercises Huanjie Guo ID： 10 655 496

Week 3

Q1 two worked examples of Euler's Theorem including all the details (i.e. m, a, EulerPhi[m], PowerMod[a,EulerPhi[m],m]) — one where the numbers are fairly small and one where the numbers are bigger;

small number
```
m = 5; a = 12;
EulerPhi[m]
4
PowerMod[a, EulerPhi[m], m]
1
```

bigger number
```
m = 11 010 111 211; a = 10 000 079;
EulerPhi[m]
10 002 455 040
PowerMod[a, EulerPhi[m], m]
1
```

Q2 an example of the working of ExtendedGCD, showing that recombining the multipliers with the two original numbers yields the GCD;

(-9)*121+5*220 = 11
```
ExtendedGCD[121, 220]
{11, {-9, 5}}
```

Q3 an example of the working of ChineseRemainder using moduli 13, 29, 64, determining u1, u2, u3,and checking that ChineseRemainder acting on {10,5,7} yields the same as 10u1 + 5u2 +7u3 mod 13 *29 *64;

calculate u1
```
ChineseRemainder[{1, 0, 0}, {13, 29, 64}]
7424
```
calculate u2
```
ChineseRemainder[{0, 1, 0}, {13, 29, 64}]
13 312
```
calculate u3
```
ChineseRemainder[{0, 0, 1}, {13, 29, 64}]
3393
```

```
ChineseRemainder[{10, 5, 7}, {13, 29, 64}]
19 783
```
10u1 + 5u2 + 7u3 mod 13 * 29 * 64 = 19783

Q4 the details of your Diffie-Hellman key exchange example;

choose p
```
p = Prime[1 000 001]
```

15 485 867

choose a

a = 2

Alice chooses as a random secret exponent m_A = 69 and Bob as a random secret exponent m_B = 103

```
ma = 69
mb = 103
```

get cA and cB

```
cA = PowerMod[a, ma, p]
14 482 528
cB = PowerMod[a, mb, p]
14 277 311
```

Alice can compute the common key with Bob by raising the publicly known c_B to the power m_A, which she only knows. She gets:

```
PowerMod[cB, ma, p]
14 224 808
```

Bob gets the same common key by raising c_A to the power m_B. Indeed, he gets:

```
PowerMod[cA, mb, p]
14 224 808
```

Q5 the details of your ElGamal public key cryptosystem example;

Alice choose q and a

```
q = Prime[1 000 003]
15 485 927
a = 2
2
```

Alice choose Private key and generate Public key

```
PrivateA = RandomInteger[{0, q - 1}]
8 579 339
PublicA = PowerMod[a, PrivateA, q]
4 099 103
```

Bob get PU = {q=15 485 927,a=2,PublicA = 4099103}

choose plaintext 5880

```
M = 5880
5880
k = RandomInteger[{0, q - 1}]
4 143 699
```

Bob generator one-time key K

```
K = PowerMod[PublicA, k, q]
6 629 221
```

Bob generate C1 and C2

```
C1 = PowerMod[a, k, q]
8 496 381
C2 = Mod[K * M, q]
1 741 221
```

Alice receive C1 and C2, use her Private Key to decrypt.

```
NewK = PowerMod[C1, PrivateA, q]
6 629 221
M = Mod[C2 * PowerMod[NewK, -1, q], q]
5880
```

Q6 the details of your ElGamal public key signature example;

Alice generate S1 and S2

```
q = Prime[1 000 003]
15 485 927
PrimitiveRootList[q]
{5, 7, 10, 13, 14, 15, 20, 21, 23, 26, 29, 30, 31, 39, 40, 41, 42, 43,
 45, 46, 52, 55, 56, 58, 60, 61, … 7 415 948 …, 15 485 877, 15 485 878,
 15 485 880, 15 485 883, 15 485 889, 15 485 890, 15 485 891, 15 485 892,
 15 485 894, 15 485 895, 15 485 900, 15 485 902, 15 485 903, 15 485 905,
 15 485 908, 15 485 909, 15 485 910, 15 485 911, 15 485 915, 15 485 916,
 15 485 918, 15 485 919, 15 485 921, 15 485 923, 15 485 924, 15 485 925}
large output | show less | show more | show all | set size limit...
a = 5
5
PrivateA = RandomInteger[{0, q - 1}]
8 346 382
PublicA = PowerMod[a, PrivateA, q]
3 551 317
M = 5880
5880
k = RandomInteger[{0, q - 1}]
3 312 473
S1 = PowerMod[a, k, q]
9 830 973
K = PowerMod[k, -1, q - 1]
6 904 381
S2 = Mod[K * Mod[M - PrivateA * S1, q - 1], q - 1]
11 346 278
```

Bob compare v1 and v2, if v1==v2, the message is valid.

```
v1 = PowerMod[a, M, q]
5 420 128
v2 = Mod[Power[PublicA, S1] * Power[S1, S2], q]
5 420 128
```

Q7 the details of RSA encryption and decryption starting from your own primes;

get nB

```
pB = Prime[1 000 005]
qB = Prime[1 000 018]
nB = pB * qB
phiB = (pB - 1) * (qB - 1)
15 485 941
15 486 181
239 818 085 281 321
239 818 054 309 200
```

get eB,dB, eB*dB = 1 mod nB

```
eB = RandomInteger[{1, nB}];
While[GCD[eB, phiB] ≠ 1, eB = RandomInteger[{1, nB}]];
eB
ExtendedGCD[eB, phiB]

32 554 268 125 351
{1, {90 641 561 823 751, -12 304 201 680 773}}

dB = 90 641 561 823 751;
```

encryption

```
m = 5 880 361;
c = PowerMod[m, eB, nB]
179 362 002 275 282
```

decryption

```
PowerMod[c, dB, nB]
5 880 361
```

Q8 the details of securely encrypting and signing a message from Alice to Bob, including the verification at Bob’s end.

get Alice’s Public Key and Private Key

```
pA = Prime[10 000 131]
qA = Prime[10 023 112]
nA = pA * qA
phiA = (pA - 1) * (qA - 1)
179 427 161
179 861 443
32 272 028 090 853 323
32 272 027 731 564 720
eA = RandomInteger[{1, nA}];
While[GCD[eA, phiA] ≠ 1, eA = RandomInteger[{1, nA}]];
eA
ExtendedGCD[eA, phiA]
25 767 449 002 289 833
{1, {-9 096 513 544 648 823, 7 263 068 525 164 923}}
dA = -9 096 513 544 648 823
-9 096 513 544 648 823
```

get Bob Public Key and Private Key

```
pB = Prime[1 000 005]
qB = Prime[1 000 018]
nB = pB * qB
phiB = (pB - 1) * (qB - 1)
15 485 941
15 486 181
239 818 085 281 321
239 818 054 309 200
eB = RandomInteger[{1, nB}];
While[GCD[eB, phiB] ≠ 1, eB = RandomInteger[{1, nB}]];
eB
ExtendedGCD[eB, phiB]
104 061 532 455 479
{1, {107 216 632 275 719, -46 523 299 054 632}}
dB = 107 216 632 275 719
107 216 632 275 719
```

Alice want to send “5880361”

she uses Bob’s public key to generate ciphertext c

she uses her private key to genenrate signature

```
mA = 5 880 361;
c = PowerMod[mA, eB, nB]
203 592 072 207 584
signature = PowerMod[c, dA, nA]
21 071 830 361 918 155
```

Bob receives c and signature of c

Bob uses Alice’s Public Key to check if the ciphertext was sent from Alice, and he finds that checkSign == c

```
checkSign = PowerMod[signature, eA, nA]
203 592 072 207 584
```

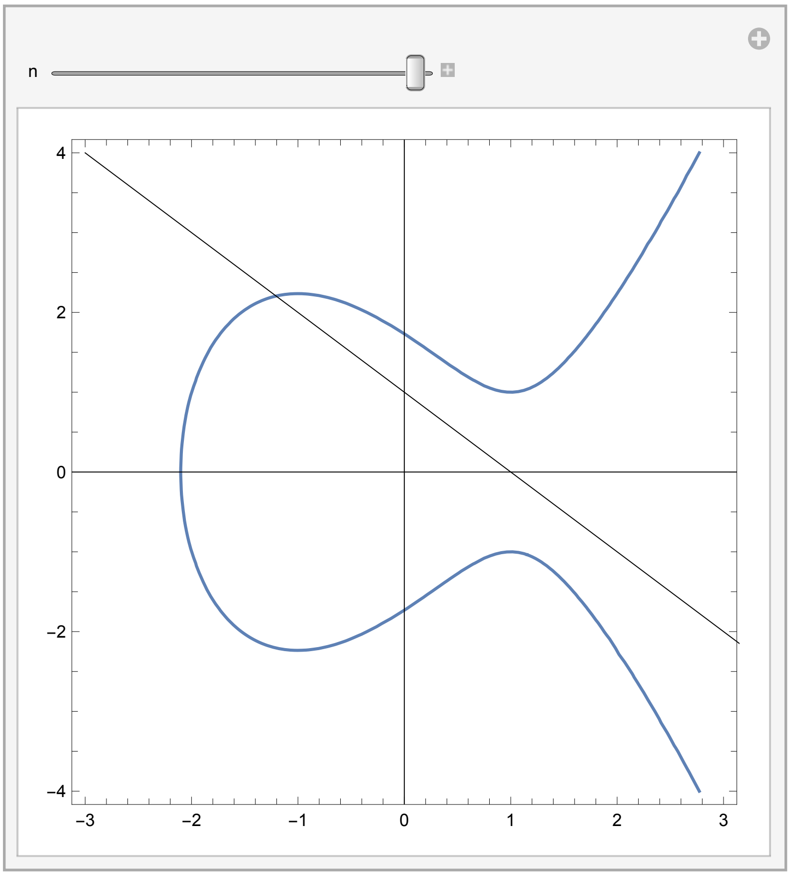
After check the signature, Bob uses his Private Key to decrypt the ciphertext and get the message mA

```
receiveMA = PowerMod[c, dB, nB]
5 880 361
```

Week 4

Q1 the ContourPlot and its output for the y2 =x3 - 5x + 3 elliptic curve example from Section 10.2, including the Epilog->Line, as the coefficient of x varies from -5 to -3;

```
Manipulate[ContourPlot[y^2 == x^3 + n x + 3, {x, -3, 3}, {y, -4, 4},
  Axes → True, Epilog → Line[{{-3, 4}, {4, -3}}]], {n, -5, -3}]
```



Q2 an answer to the question about straight lines intersecting elliptic curves in three places;

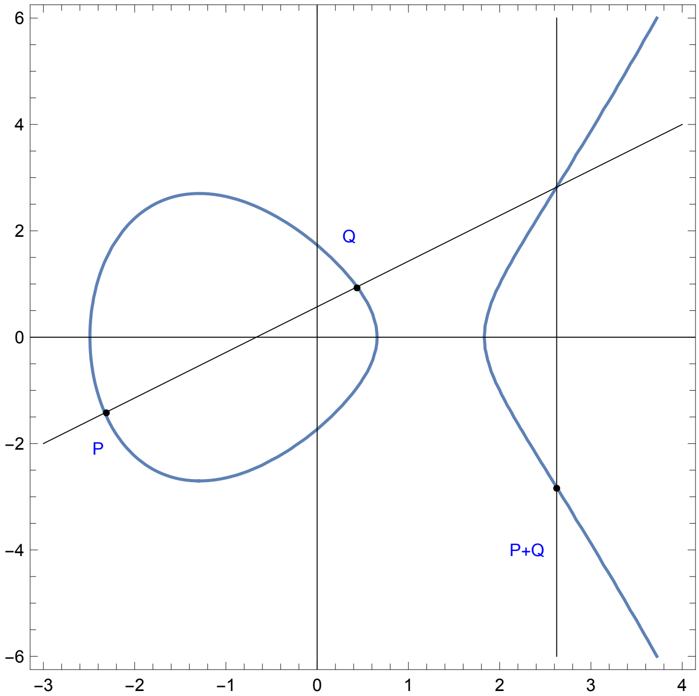
When the coefficient of x is -3, it only have one intersection.

```
NSolve[ {y^2 == x^3 - 3 x + 3, y == -x + 1}, {x, y}]
{{x → -1.20557, y → 2.20557}, {x → 1.10278 + 0.665457 i, y → -0.102785 - 0.665457 i},
 {x → 1.10278 - 0.665457 i, y → -0.102785 + 0.665457 i}}
```

Q3 the ContourPlot and its output for the example of addition of two points on a curve over the reals, where the first line is given by {{-3,-2},{4,4}} and including the bullet that represents the answer;

```
NSolve[ {y^2 == x^3 - 5 x + 3, 7 y == 6 x + 4}, {x, y}]
{{x → 2.62478, y → 2.82124},
 {x → -2.32767, y → -1.42371}, {x → 0.437585, y → 0.946501}}
```

```
ContourPlot[ y² == x³ - 5 x + 3, {x, -3, 4}, {y, -6, 6},
 Axes → True,    Epilog -> {Line[{{-3, -2}, {4, 4}}],
   Line[{{2.6247755634983756, -6}, {2.6247755634983756, 6}}],
   Text["\!\(\* StyleBox[\(P\),
      FontColor->RGBColor[0, 0, 1]]\)\!\(\* StyleBox[\(+\),
      FontColor->RGBColor[0, 0, 1]]\)\!\(\* StyleBox[\(Q\),
      FontColor->RGBColor[0, 0, 1]]\)", {2.3, -4}],
   Text["\!\(\* StyleBox[\(P\), FontColor->RGBColor[0, 0, 1]]\)",
    {-2.4, -2.1}],
   Text["\!\(\* StyleBox[\(Q\), FontColor->RGBColor[0, 0, 1]]\)",
    {0.35, 1.9}],          Text["•", {-2.31, -1.4}],
   Text["•", {0.43758485089023075, 0.9465013007630549}],
   Text["•", {2.6247755634983756, -2.821236197284322}]}]
```



Q4 the Table example (without the enclosing Flatten) that runs through the points of the y2 = x3 - 5x + 3
elliptic curve over Z11;

```
Clear[x, y];
p = 11;
Table[ Solve[ {y² == x³ - 5 x + 3, x == u}, {x, y}, Modulus -> p], {u, 0, p - 1}]
{{{x → 0, y → 5}, {x → 0, y → 6}}, {},
 {{x → 2, y → 1}, {x → 2, y → 10}}, {{x → 3, y → 2}, {x → 3, y → 9}},
 {{x → 4, y → 5}, {x → 4, y → 6}}, {{x → 5, y → 2}, {x → 5, y → 9}}, {},
 {{x → 7, y → 5}, {x → 7, y → 6}}, {}, {{x → 9, y → 4}, {x → 9, y → 7}}, {}}
```

Q5 the Solve example that finds the points of intersection of the straight line you chose with the y2 = x3 - 5x + 3 elliptic curve over Z11;

we choose (0,5) and (4,6) and get its formula. y = 5+x/4

```
InterpolatingPolynomial[{{0, 5}, {4, 6}}, t]
```

$5 + \frac{t}{4}$

calculate intersection

```
p = 11; Solve[ {y^2 == x^3 - 5 x + 3, y == 5 + x / 4}, {x, y}, Modulus -> p]
{{x → 0, y → 5}, {x → 4, y → 6}, {x → 5, y → 9}}
```

Q6 the result of EllipticAdd with the curve y2 = x3 - 5x + 3 and the points defining the straight line you chose for the previous question, and the relationship between the answer here and the previous answer;

we choose {0,5} , {4,6} ,which is on the same line with {5,9}, and we add the first two point and get the result which is {5,2}.

```
p = 11; a = 0; b = -5; c = 3; EllipticAdd[p, a, b, c, {0, 5}, {4, 6}]
{5, 2}
```

The third point is {5,9}. we can find that actually y = -(9) mod 11 = 2 and x is the same as Q.x which is 5.

Q7 the Table containing the first 10-20 doublings of {121,517}, the IntegerDigits of 432, and the confirmation that the relevant combination of these via EllipticAdd yields the point at infinity;

```
Table[P[n], {n, 1, 10, 1}]
{{630, 588}, {447, 354}, {69, 852}, {348, 843}, {539, 467},
 {572, 822}, {124, 198}, {804, 143}, {363, 372}, {533, 612}}

IntegerDigits[432, 2]
{1, 1, 0, 1, 1, 0, 0, 0, 0}
```

calculate p[8]+p[7]+p[5]+p[4] = {0} , and the order of {121,517} is 432.

```
EllipticAdd[p, a, b, c, EllipticAdd[p, a, b, c, {804, 143}, {124, 198}],
 EllipticAdd[p, a, b, c, {539, 467}, {348, 843}]]
{O}
```

Q8 the details of the Diffie-Hellman protocol starting from the primitive element that you chose, including: proving that the point lies on the curve y2 = x3 + 100x2 + 10x + 1 over Z863, deriving QAlice, QBob, QA, QB, as in the example, confirming that QA and QB are equal;

choose {48,357} , and check if it lies on the curve

```
p = 863;
a = 100;
b = 10;
c = 1;
x = 48;
y = 357;
Mod[y^2 - (x^3 + a * x^2 + b * x + c), p] == 0
```

```
True
```

check {48,357} order is bigger than 10

```
P = {48, 357}
f[1] = P;
f[n_] := f[n] = EllipticAdd[p, a, b, c, P, f[n - 1]]
Column[Table[f[n], {n, 1, 10, 1}]]
{48, 357}
{84, 681}
{712, 571}
{38, 608}
{211, 196}
{471, 262}
{566, 453}
{128, 78}
{787, 203}
{51, 377}
```

let Alice choose mA = 11, mB = 20 Then QAlice = {112, 82}, QBob = {792, 644}

```
QAlice = EllipticAdd[p, a, b, c, P[3], P[1]]
QBob = EllipticAdd[p, a, b, c, P[4], P[2]]
{112, 82}
{792, 644}
```

Alice can Compute the common key K.(A,B) with the calculation K.(A,B) = (m.A)*Q.B, where m.A is the 11

```
QA[0] = {792, 644};
QA[i_] := QA[i] = EllipticAdd[p, a, b, c, QA[i - 1], QA[i - 1]];
EllipticAdd[p, a, b, c, QA[3], QA[1]]
{548, 440}
```

Bob can Compute the common key K.(A,B) with the calculation K.(A,B) = (m.B)*Q.A, where m.b is the 20

```
QB[0] = {112, 82};
QB[i_] := QB[i] = EllipticAdd[p, a, b, c, QB[i - 1], QB[i - 1]];
EllipticAdd[p, a, b, c, QB[4], QB[2]]
{548, 440}
```

Q9 the preliminary calculations with finite fields of characteristic 2 with non-trivial extension degree, including the basic manipulations with dd and ee, and including the tabulation of powers of dd;

```
<< FiniteFields`

z16 = GF[2, 4]
GF[2, {1, 0, 0, 1, 1}]
dd = z16[{0, 0, 1, 1}]
{0, 0, 1, 1}_2
dd + dd
0
dd - dd
0
ee = z16[{1, 1, 0, 0}]
```

```
{1, 1, 0, 0}₂
dd ee
{1, 0, 1, 1}₂
dd / ee
{0, 0, 1, 0}₂

Table[dd^n, {n, 0, 15, 1}]
{1, {0, 0, 1, 1}2, {0, 1, 1, 0}2, {1, 1, 0, 0}2, {1, 0, 1, 1}2, {0, 1, 0, 1}2,
 {1, 0, 1, 0}2, {0, 1, 1, 1}2, {1, 1, 1, 0}2, {1, 1, 1, 1}2, {1, 1, 0, 1}2,
 {1, 0, 0, 1}2, {0, 0, 0, 1}2, {0, 0, 1, 0}2, {0, 1, 0, 0}2, {1, 0, 0, 0}2}

Table[dd^n, {n, 0, 30, 1}]
{1, {0, 0, 1, 1}2, {0, 1, 1, 0}2, {1, 1, 0, 0}2, {1, 0, 1, 1}2, {0, 1, 0, 1}2,
 {1, 0, 1, 0}2, {0, 1, 1, 1}2, {1, 1, 1, 0}2, {1, 1, 1, 1}2, {1, 1, 0, 1}2,
 {1, 0, 0, 1}2, {0, 0, 0, 1}2, {0, 0, 1, 0}2, {0, 1, 0, 0}2, {1, 0, 0, 0}2,
 {0, 0, 1, 1}2, {0, 1, 1, 0}2, {1, 1, 0, 0}2, {1, 0, 1, 1}2, {0, 1, 0, 1}2,
 {1, 0, 1, 0}2, {0, 1, 1, 1}2, {1, 1, 1, 0}2, {1, 1, 1, 1}2, {1, 1, 0, 1}2,
 {1, 0, 0, 1}2, {0, 0, 0, 1}2, {0, 0, 1, 0}2, {0, 1, 0, 0}2, {1, 0, 0, 0}2}
```

Q10 the derivation of the solution of y2 + xy = x3 + a x2 + c for the given parameters, and including the tabulation of PDoubles;

```
Column[Table[(dd^n)^3 + ee (dd^n)^2, {n, 0, 15, 1}]]
{0, 1, 0, 0}2
{1, 0, 0, 1}2
{1, 1, 0, 1}2
0
{1, 0, 0, 0}2
{1, 0, 1, 0}2
{0, 1, 0, 0}2
{1, 1, 0, 0}2
{0, 1, 0, 0}2
{1, 0, 1, 1}2
{0, 1, 1, 0}2
{0, 0, 0, 1}2
{1, 0, 1, 1}2
{1, 0, 1, 1}2
{0, 0, 1, 0}2
{0, 1, 0, 0}2
```

we can find that when n = 3, we get zero. so n=3

```
x = y = dd^3;
y^2 + x y
0
x^3 + ee x^2
0
```

```
P=.;
a=ee;c=0;
P[0]={dd^3,dd^3};
P[i_]:=P[i]=Z2mEllipticAdd[a,c,P[i-1],P[i-1]]
```

```
PDouble = Column[Table[P[n], {n, 0, 31, 1}]]
{{1, 1, 0, 0}2, {1, 1, 0, 0}2}
{{1, 0, 1, 0}2, {0, 1, 0, 1}2}
{{0, 0, 0, 1}2, {0, 1, 0, 1}2}
{{1, 1, 1, 1}2, {1, 1, 0, 0}2}
{{1, 1, 0, 0}2, 0}
{{1, 0, 1, 0}2, {1, 1, 1, 1}2}
{{0, 0, 0, 1}2, {0, 1, 0, 0}2}
{{1, 1, 1, 1}2, {0, 0, 1, 1}2}
{{1, 1, 0, 0}2, {1, 1, 0, 0}2}
{{1, 0, 1, 0}2, {0, 1, 0, 1}2}
{{0, 0, 0, 1}2, {0, 1, 0, 1}2}
{{1, 1, 1, 1}2, {1, 1, 0, 0}2}
{{1, 1, 0, 0}2, 0}
{{1, 0, 1, 0}2, {1, 1, 1, 1}2}
{{0, 0, 0, 1}2, {0, 1, 0, 0}2}
{{1, 1, 1, 1}2, {0, 0, 1, 1}2}
{{1, 1, 0, 0}2, {1, 1, 0, 0}2}
{{1, 0, 1, 0}2, {0, 1, 0, 1}2}
{{0, 0, 0, 1}2, {0, 1, 0, 1}2}
{{1, 1, 1, 1}2, {1, 1, 0, 0}2}
{{1, 1, 0, 0}2, 0}
{{1, 0, 1, 0}2, {1, 1, 1, 1}2}
{{0, 0, 0, 1}2, {0, 1, 0, 0}2}
{{1, 1, 1, 1}2, {0, 0, 1, 1}2}
{{1, 1, 0, 0}2, {1, 1, 0, 0}2}
{{1, 0, 1, 0}2, {0, 1, 0, 1}2}
{{0, 0, 0, 1}2, {0, 1, 0, 1}2}
{{1, 1, 1, 1}2, {1, 1, 0, 0}2}
{{1, 1, 0, 0}2, 0}
{{1, 0, 1, 0}2, {1, 1, 1, 1}2}
{{0, 0, 0, 1}2, {0, 1, 0, 0}2}
{{1, 1, 1, 1}2, {0, 0, 1, 1}2}
```

Q11 the derivation of the Diffie-Hellman protocol for the case cited.

Alice choose mA = 11, Bob choose mB = 20

Alice and bob generate their point and then sent it to each other

```
QAlice = Z2mEllipticAdd[a, c, PDouble[[3]], PDouble[[1]]]
{{0, 1, 0, 0}2, {1, 0, 1, 1}2}
QBob = Z2mEllipticAdd[a, c, PDouble[[4]], PDouble[[2]]]
{{0, 0, 1, 0}2, {1, 1, 0, 1}2}
```

Alice use bob’s point and her key mA = 11 to calculate

```
QAB = .; QAB[0] = QBob;
QAB[i_] := QAB[i] = Z2mEllipticAdd[a, c, QAB[i - 1], QAB[i - 1]];
Z2mEllipticAdd[a, c, QAB[3], QAB[1]]
{{1, 0, 1, 0}2, {1, 1, 1, 1}2}
```

Bob use Alice point and his key mB=20 to calculate

```
QBA = .; QBA[0] = QAlice;
QBA[i_] := QBA[i] = Z2mEllipticAdd[a, c, QBA[i - 1], QBA[i - 1]];
Z2mEllipticAdd[a, c, QBA[4], QBA[2]]
```
$\{\{1, 0, 1, 0\}_2, \{1, 1, 1, 1\}_2\}$

Week 5

Q1 the programs and outputs produced for your simulation of the BB84 QKD protocol, namely: AliceBasis AliceData, BobBasis, BobData, EqualBases, AgreedDataAlice, AgreedDataBob, (the code that produces, and the values of) AgreedKeyAlice, AgreedDataBob, CheckDigitsAlice, CheckDigitsBob ;

generate Alice Data and the direction of data
```
AliceBasis = Table[RandomInteger[1], {n, 1, 40, 1}]
{0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0,
 1, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0}
AliceData = Table[RandomInteger[1], {n, 1, 40, 1}]
{1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1,
 0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1, 1, 0}
```

Bob generate his direction and tell Alice
```
BobBasis = Table[RandomInteger[1], {n, 1, 40, 1}]
{0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1,
 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1}
```

Bob put the bit when direction of Alice and bob are the same. Otherwise she use random number.
```
BobData = Table[If[AliceBasis[[n]] == BobBasis[[n]],
   AliceData[[n]], RandomInteger[1]], {n, 1, 40, 1}]
{1, 0, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0,
 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0}
```

a table to tell which bit of two direction is the same.
```
EqualBases = Table[If[AliceBasis[[n]] == BobBasis[[n]], 1, 0], {n, 1, 40, 1}]
{1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 0, 1, 0,
 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0}

AgreedDataAlice = {};
AgreedDataBob = {};
```

Alice generate her data
```
For[n = 1, n < 41, n++, If[EqualBases[[n]] == 1,
  AgreedDataAlice = Append[AgreedDataAlice, AliceData[[n]]]]]
AgreedDataAlice
{1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, 0, 0, 1, 1}
```

Bob get his data
```
For[n = 1, n < 41, n++,
 If[EqualBases[[n]] == 1, AgreedDataBob = Append[AgreedDataBob, BobData[[n]]]]]
```

```
AgreedDataBob
{1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, 0, 0, 1, 1}

AgreedKeyAlice = {};
AgreedKeyBob = {};
CheckDigitsAlice = {};
CheckDightsBob = {};
```

we split data randomly

```
For[i = 1, i <= Length[AgreedDataAlice], i++,
 If[
   RandomInteger[1] == 1
  , {AgreedKeyAlice = Append[AgreedKeyAlice, AgreedDataAlice[[i]]],
    AgreedKeyBob = Append[AgreedKeyBob, AgreedDataBob[[i]]]}
  , {CheckDigitsAlice = Append[CheckDigitsAlice, AgreedDataAlice[[i]]],
    CheckDightsBob = Append[CheckDightsBob, AgreedDataBob[[i]]]}
 ]
]
AgreedKeyAlice
{0, 1, 1, 0, 0, 1, 1, 0, 1}
AgreedKeyBob
{0, 1, 1, 0, 0, 1, 1, 0, 1}
CheckDigitsAlice
{1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 1}
CheckDightsBob
{1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 1}
```

Q2 the programs and outputs produced for your simulation of the B 92 QKD protocol, namely: AliceData, AliceBasis, BobBasis, BobData, AgreedDataAlice, AgreedDataBob, (code etc., needed for) AgreedKeyAlice, AgreedKeyBob, CheckDigitsAlice, CheckDigitsBob ;

```
len = 40
40
AliceBasis = Table[RandomInteger[1], {, 1, len, 1}]
{0, 0, 1, 1, 1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1, 1, 0,
 1, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1}
AliceData = AliceBasis
{0, 0, 1, 1, 1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1, 1, 0,
 1, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1}
BobBasis = Table[RandomInteger[1], {, 1, len, 1}]
{1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 1,
 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0}
```

Bob put the bit when direction of Alice and bob are the same. Otherwise she use random number.

```
BobData = Table[If[AliceBasis[[i]] == BobBasis[[i]],
   AliceData[[i]], RandomInteger[1]], {i, 1, len, 1}]
{0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0,
 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1}
```

reliable data when BobBasis ≠ BobData

```
ReliableData = Table[If[BobBasis[[n]] ≠ BobData[[n]], 1, 0], {n, 1, len, 1}]
{1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1,
 0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1}

AgreedDataAlice = {};
AgreedDataBob = {};

For[ i = 1, i <= Length[ReliableData], i++ , If[ReliableData[[i]] == 1,
  {AgreedDataAlice = Append[AgreedDataAlice, AliceData[[i]]];
   AgreedDataBob = Append[AgreedDataBob, BobData[[i]]];
  }]]

AgreedDataAlice
{0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1}
AgreedDataBob
{0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1}

AgreedKeyAlice = {};
AgreedKeyBob = {};
CheckDigitsAlice = {};
CheckDightsBob = {};

# we split data randomly
For[i = 1, i <= Length[AgreedDataAlice], i++,
 If[
   RandomInteger[1] == 1
  , {AgreedKeyAlice = Append[AgreedKeyAlice, AgreedDataAlice[[i]]],
   AgreedKeyBob = Append[AgreedKeyBob, AgreedDataBob[[i]]]}
  , {CheckDigitsAlice = Append[CheckDigitsAlice, AgreedDataAlice[[i]]],
   CheckDightsBob = Append[CheckDightsBob, AgreedDataBob[[i]]]}
 ]
]
AgreedKeyAlice
{0, 1, 1, 1, 1, 1}
AgreedKeyBob
{0, 1, 1, 1, 1, 1}
CheckDigitsAlice
{0, 1, 0, 0, 0, 1, 1, 1, 1, 1}
CheckDightsBob
{0, 1, 0, 0, 0, 1, 1, 1, 1, 1}
```

Q3 the programs and outputs produced for your simulation of the E91 QKD protocol, namely: AliceBasis, BobBasis, EvesDrop, EqualBases, ReliableData, AliceData, BobData, AgreedDataAlice, AgreedDataBob, AgreedKeyAlice, AgreedKeyBob, CheckDigitsAlice, Check DigitsBob, and your comments about the last four pieces of data (lose a mark for no comment).

```
len = 80
```

```
80
AliceBasis = Table[RandomInteger[1], {, 1, len, 1}]
{0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 1, 0, 1,
 1, 1, 1, 0, 1, 1, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0,
 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0}
BobBasis = Table[RandomInteger[1], {, 1, len, 1}]
{0, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 1,
 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 0, 0, 0, 1,
 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0}
EvesDrop = Table[RandomInteger[1], {, 1, len, 1}]
{1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 0, 1, 1, 1, 0, 1, 1, 0, 0, 1, 1, 0,
 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 1, 1,
 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 1, 0, 0, 0, 1, 1, 0}
```

when AliceBasis[[n]] = BobBasis[[n]], set it 1 otherwise 0.

```
EqualBases = Table[If[AliceBasis[[n]] == BobBasis[[n]], 1, 0], {n, 1, len, 1}]
{1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1,
 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 0,
 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, 0, 0, 1}
```

the index which is reliable

```
ReliableData =
 Table[If[EqualBases[[n]] == 1 && EvesDrop[[n]] == 0, 1, 0], {n, 1, len, 1}]
{0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 1,
 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, 1, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1}

AliceData = Table[RandomInteger[1], {, 1, len, 1}]
BobData = Table[RandomInteger[1], {, 1, len, 1}]
```

set the data of Alice and Bob same as each other when ReliableData[[i]]==1

```
For[i = 1, i <= len, i++,
  If[ReliableData[[i]] == 1,
   AliceData[[i]] = BobData[[i]] = RandomInteger[1];
  ]
]
AliceData
{0, 1, 1, 1, 1, 0, 1, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0,
 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0,
 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 1, 0, 0, 0}
BobData
{1, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 1, 0,
 1, 1, 1, 0, 1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0,
 1, 1, 1, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0}

AgreedDataAlice = {};
AgreedDataBob = {};
```

Alice and Bob both think this data is ok

```
For[i = 1, i ≤ len, i++,
 If[EqualBases[[i]] == 1,
  {AgreedDataAlice = Append[AgreedDataAlice, AliceData[[i]]];
   AgreedDataBob = Append[AgreedDataBob, BobData[[i]]];}]


]
AgreedDataAlice
{0, 1, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 1, 1,
 1, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 0}
AgreedDataBob
{1, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 0, 1, 1,
 1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 1, 1, 1, 0, 1, 0, 1, 1, 1, 0}

AgreedKeyAlice = {};
AgreedKeyBob = {};
CheckDigitsAlice = {};
CheckDightsBob = {};

# split data randomly
For[i = 1, i <= Length[AgreedDataAlice], i++,
 If[
   RandomInteger[1] == 1
  , {AgreedKeyAlice = Append[AgreedKeyAlice, AgreedDataAlice[[i]]],
   AgreedKeyBob = Append[AgreedKeyBob, AgreedDataBob[[i]]]}
  , {CheckDigitsAlice = Append[CheckDigitsAlice, AgreedDataAlice[[i]]],
   CheckDightsBob = Append[CheckDightsBob, AgreedDataBob[[i]]]}
 ]
]

AgreedKeyAlice
{1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 1, 0}
AgreedKeyBob
{1, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0}
CheckDigitsAlice
{0, 1, 1, 1, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1}
CheckDightsBob
{1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 1, 1}
```

we can find that data of `AgreedKeyAlice` and AgreedKeyBob are not equal. CheckDigitsAlice and CheckDightsBob are not equal,too.
we calculate the error rate, if errorRate is higher than the threshold, we can assume that someone is trying to detect the key. So we should start again.

```
errorCount = 0;
For[i = 1, i ≤ Length[CheckDigitsAlice], i++,
 If[CheckDigitsAlice[[i]] ≠ CheckDightsBob[[i]], errorCount++]
]
errorRate = errorCount / Length[CheckDigitsAlice]
```

$\frac{4}{23}$