Approximation of IMSE

$$MSE_n(x) = k(x,x) - k_n^T(x) \cdot K_n^{-1} \cdot k_n(x)$$

$$M_n = MSE_n(x).$$

$$M_{n+1} = M_n - \frac{1}{\sigma_n^2}\left(k(x, x_{n+1}) - k_n^T \cdot K_n^{-1} \cdot k_n(x)\right)^2$$

$$IMSE_n = \int_D MSE_n(x)\, dP(x)$$

$$I_n = IMSE_n$$

$$I_{n+1} = I_n - \frac{1}{\sigma_n^2}\int_D \left(k(x, x_{n+1}) - k_n^T K_n^{-1} \cdot k_n(x)\right)^2 dP(x)$$

$$\min_{x_{n+1}} -\frac{1}{\sigma_n^2}\int_D \left(k(x, x_{n+1}) - k_n^T K_n^{-1} \cdot k_n(x)\right)^2 dP(x)$$

$$f(x) = \left(k(x, x_{n+1}) - k_n^T \cdot K_n^{-1} \cdot k_n(x)\right)^2$$

$$\int_D f(x)\, dP(x) = E f(x)$$

$$\approx f(Ex) + \frac{1}{2} f''(Ex) \cdot Var(x)$$

$$g(x) = k(x, x_{n+1}) - k_n^T \cdot K_n^{-1} \cdot k_n(x)$$

$$f(x) = g^2(x)$$

$$f'(x) = 2\, g(x)\, g'(x)$$

$$f'(x) = 2 g(x)\cdot g'(x)$$

$$f''(x) = 2\cdot(g'(x))^2 + 2\, g(x)\cdot g''(x)$$

$$k(x, x_i) = \nu \cdot e^{-\frac{1}{\theta}(x-x_i)^2}$$

$$\begin{cases} \dfrac{\partial k(x, x_i)}{\partial x} = -\dfrac{2}{\theta}(x-x_i)\cdot k(x, x_i) \\[2ex] \dfrac{\partial k_n(x)}{\partial x} = -\dfrac{2}{\theta}\cdot \begin{bmatrix} x-x_1 & & 0 \\ & \ddots & \\ 0 & & x-x_n \end{bmatrix} \cdot k_n(x) \end{cases}$$

$$\begin{cases} \dfrac{\partial^2 k(x, x_i)}{\partial x^2} = -\dfrac{2}{\theta} k(x, x_i) + \dfrac{4}{\theta^2}(x-x_i)^2 \cdot k(x, x_i) \\[2ex] \dfrac{\partial^2 k_n(x)}{\partial x^2} = -\dfrac{2}{\theta} k_n(x) + \dfrac{4}{\theta^2}\cdot \begin{bmatrix} x-x_1 & & 0 \\ & \ddots & \\ 0 & & x-x_n \end{bmatrix}^2 \cdot k_n(x) \end{cases}$$

$$\mathrm{diag}(x-x_i) = \begin{bmatrix} x-x_1 & & 0 \\ & \ddots & \\ 0 & & x-x_n \end{bmatrix}$$

$$g'(x) = -\frac{2}{\theta}(x-x_{n+1})\cdot k(x, x_{n+1}) + \frac{2}{\theta}\cdot k_n^T \cdot K_n^{-1} \cdot \mathrm{diag}(x-x_i)\cdot k_n(x)$$

$$g''(x) = -\frac{2}{\theta} k(x, x_{n+1}) + \frac{4}{\theta^2}(x-x_{n+1})^2 \cdot k(x, x_{n+1})$$

$$+\frac{2}{\theta}\cdot k_n^T \cdot K_n^{-1}\, k_n(x)$$

$$-\frac{4}{\theta^2}\cdot k_n^T\cdot K_n^{-1}\cdot \mathrm{diag}^2(x-x_i)\cdot k_n(x)$$

$$Ef(x) \approx g^2(\mu) + \left((g'(\mu))^2 + g(\mu)\cdot g''(\mu)\right)\cdot \sigma^2$$

$$\max_{x_{n+1}} \frac{1}{\nu - k_n^T\cdot K_n^{-1}\cdot k_n}\cdot Ef(x) = h(x_{n+1})$$

$$\frac{\partial h(x_{n+1})}{\partial x_{n+1}} = \frac{2\cdot k_n^T\cdot K_n^{-1}\cdot \frac{\partial k_n}{\partial x_{n+1}}}{(\nu - k_n^T\cdot K_n^{-1}\cdot k_n)^2}\cdot Ef(x)$$

$$+\frac{1}{(\nu - k_n^T\cdot K_n^{-1}\cdot k_n)}\cdot \frac{\partial Ef(x)}{\partial x_{n+1}}$$

$$\frac{\partial Ef(x)}{\partial x_{n+1}} = 2\cdot g(\mu)\cdot \frac{\partial g(\mu)}{\partial x_{n+1}} +$$

$$\sigma^2\Big(2 g'(\mu)\cdot \frac{\partial g'(\mu)}{\partial x_{n+1}} +$$

$$\frac{\partial g(\mu)}{\partial x_{n+1}}\cdot g''(\mu) + g(\mu)\cdot \frac{\partial g''(\mu)}{\partial x_{n+1}}\Big)$$

$$\Bigg\{ \frac{\partial g(\mu)}{\partial x_{n+1}} = -\frac{2}{\theta}(x_{n+1}-\mu)k(x_{n+1},\mu) + \frac{2}{\theta}k_n^T(\mu)\cdot K_n^{-1}\cdot \mathrm{diag}(x_{n+1}-x_i)k_n$$

$$\frac{\partial g'(\mu)}{\partial x_{n+1}} = \frac{2}{\theta} k(\mu, x_{n+1}) - \frac{4}{\theta^2}(x_{n+1}-\mu)^2 \cdot k(\mu, x_{n+1})$$

$$- \frac{4}{\theta^2} k_n^T(\mu) \cdot \mathrm{diag}(\mu - x_i) \cdot K_n^{-1} \cdot \mathrm{diag}(x_{n+1} - x_i)\, k_n$$

$$\frac{\partial g''(\mu)}{\partial x_{n+1}} = \frac{4}{\theta^2}(x_{n+1}-\mu) \cdot k(\mu, x_{n+1})$$

$$+ \frac{4}{\theta^2} \cdot 2(x_{n+1}-\mu) \cdot k(\mu, x_{n+1})$$

$$- \frac{8}{\theta^3}(x_{n+1}-\mu)^3 \cdot k(\mu, x_{n+1})$$

$$- \frac{4}{\theta^2} k_n^T(\mu)\, K_n^{-1} \cdot \mathrm{diag}(x_{n+1} - x_i) \cdot k_n$$

$$+ \frac{8}{\theta^3} \cdot k_n^T(\mu) \cdot \mathrm{diag}^2(\mu - x_i)\, K_n^{-1} \cdot \mathrm{diag}(x_{n+1} - x_i) \cdot k_n$$