# MFE 409 Lecture 3
# Risk for Options

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# Lecture Objectives

**Risk management for option trading**

- What are the risks of option strategies?

- How to quantify these risks?

# Trading Derivatives and Risk Management

- Two broad levels of risk management inside financial institutions
    - Trader level: (hard) risk limits
    - Institution level: aggregate positions and construct broad measures of risk

# Trading Derivatives and Risk Management

- Two broad levels of risk management inside financial institutions
  - Trader level: (hard) risk limits
    - Often expressed in terms of Greeks
  - Institution level: aggregate positions and construct broad measures of risk
    - Often around VaR

# Delta

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- Delta ($\Delta$) of a portfolio: change in portfolio price in response to a change in underlying price

$$\Delta = \frac{\partial P}{\partial S}$$

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- Buying $-\Delta$ of the underlying protects the portfolio against local changes in underlying price

- Can also hedge with another option

# Linear Products

- If the value of the portfolio is linear in the price of the underlying, delta-hedging eliminates all risk

- Examples: forwards, futures, fixed promises in foreign currency, ...

- Static hedging works perfectly: "hedge and forget"

# Example

- A U.S. company has a receivable of EUR 10mil in one year.
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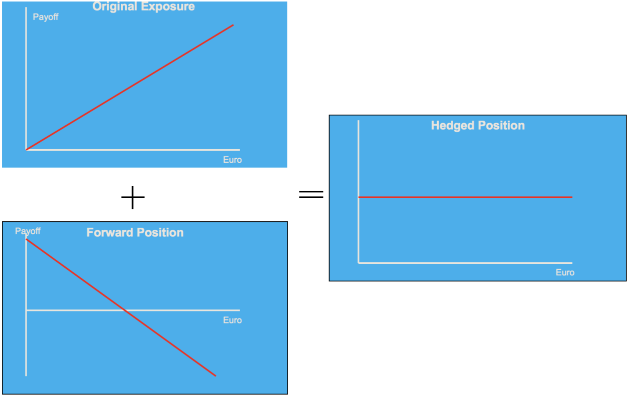

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  - Continuous delta-hedging eliminates all risk
  - By no arbitrage, can be used to find option prices

# VaR for Options: Delta Approach

$\Pi_t$ : exchange rate    $P_t$ : put price

- Portfolio:
  - Long EUR10m, EUR/USD $= 1.436$, volatility of EUR/USD $0.65\%$)
  - Short 10m ~~\$~~ puts to sell euros in 6m; opts $\Delta = -0.5044$
- ~~1~~ 99% VaR?

$$R_{t+1} = 10m \times (\Pi_{t+1} - \Pi_t) - 10m\,(P_{t+1} - P_t)$$

$$= 14.36 \times \left(\frac{\Pi_{t+1} - \Pi_t}{\Pi_t}\right) - 10m\,(P_{t+1} - P_t)$$

$$\approx 14.36 \times \left(\frac{\Pi_{t+1} - \Pi_t}{\Pi_t}\right) - 10m \times \Delta \times (\Pi_{t+1} - \Pi_t)$$

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$$\approx (14.36 - 14.36\,\Delta) \times \frac{\Pi_{t+1} - \Pi_t}{\Pi_t} \quad \mathcal{N}(0, 0.65\%)$$

$$\text{VaR} = 2.32\,(14.36 - 14.36\,\Delta) \times \sigma$$

Portfolio / Option contract $O(S_t)$

price of the option

price of the underlying

Portfolio gain: $O(S_{t+1}) - O(S_t)$

we know something about $S_{t+1} - S_t$
(normally distributed)

Approximate: $O(S_{t+1}) \approx O(S_t) + \underbrace{\frac{\partial O}{\partial S_t}(S_t)}_{\Delta} \times (S_{t+1} - S_t)$

$$O(S_{t+1}) - O(S_t) \approx \Delta \times S_t \times \frac{S_{t+1} - S_t}{S_t}$$

VaR: $2.32 \times \Delta \times S_t \times \sigma_S$

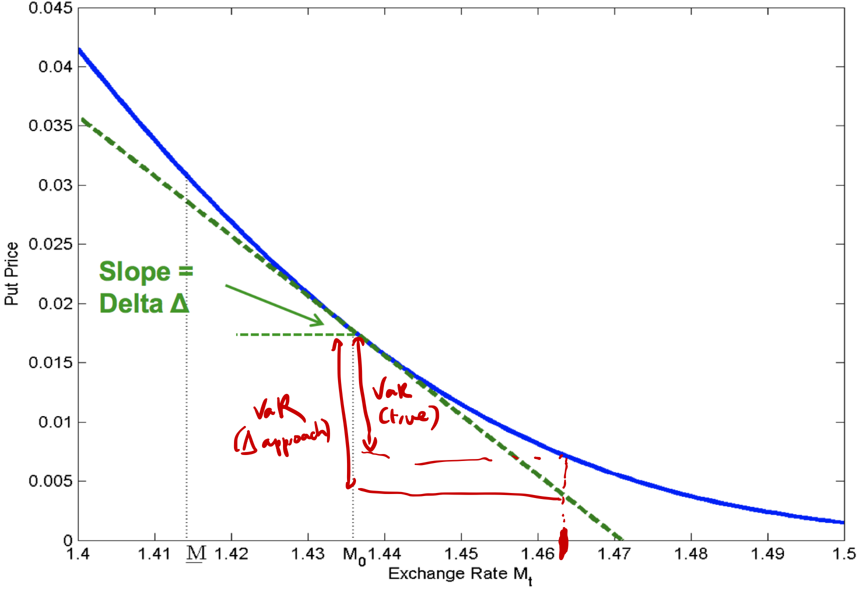
# VaR for Options: Delta Approach

- Portfolio:
  - Long EUR10m, EUR/USD $= 1.436$, volatility of EUR/USD $0.65\%$)
  - Short 10m in puts to sell euros in 6m, $\Delta = -0.5044$
- 1% VaR?
  - Put price $p_t = G(M_t)$, where
  - Approximately:

    $$p_{t+1} - p_t \approx G'(M_t) \times (M_{t+1} - M_t) = \Delta \times (M_{t+1} - M_t)$$
  - Portfolio gain:

    $$\begin{aligned} V_{t+1} - V_t &= 10\text{m} \times (M_{t+1} - M_t) + 10\text{m} \times (p_{t+1} - p_t) \\ &\approx 10\text{m} \times (1 + \Delta) \times (M_{t+1} - M_t) \\ &\approx \$14.36\text{m} \times (1 + \Delta) \times R_{M,t} \end{aligned}$$
  - 99% 1-day VaR $= 0.4956 \times \$217,204 = \$107,604$

# Put Price: Delta Approximation



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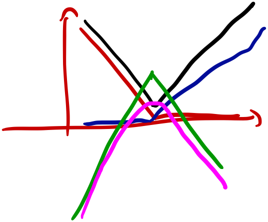
- Nick Leeson, 1995 Barings Bank 
- Short puts and Calls with the same strike price on Nikkei Index



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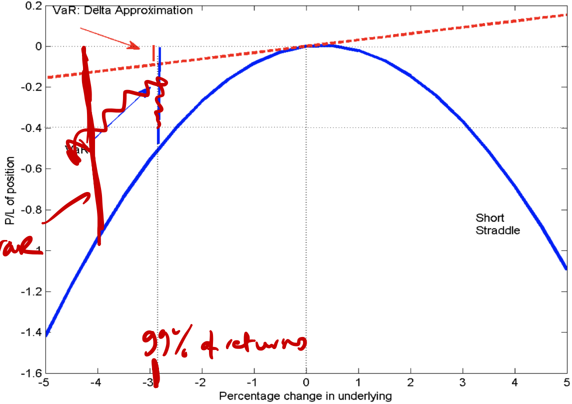

# Gamma

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- Also rate of change of Delta with respect to the price of the underlying asset:

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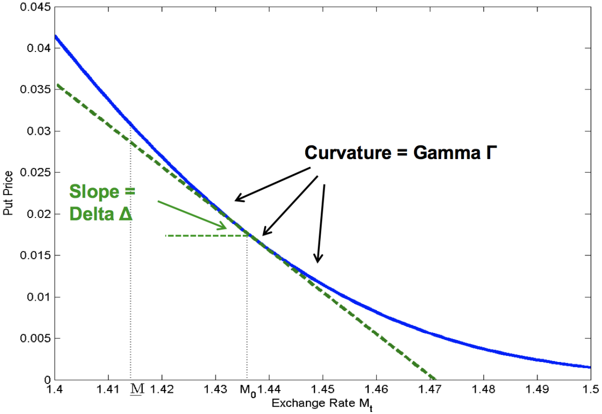
  - Delta-Gamma hedging does better with less frequent readjustments

# Put Price: Delta Gamma Approximation

$$P(S) \approx P(S_0) + \underbrace{P'(S_0)}_{\Delta}(S - S_0) + \frac{1}{2}\underbrace{P''(S_0)}_{\Gamma}(S - S_0)^2$$



# VaR for Options: Delta-Gamma Approach

$S_{t+1} - S_t \sim \mathcal{N}(\mu, \sigma)$ $S_t$ $S\mu$ $S^2\sigma^2$

- Assume change in underlying price $S_{t+1} - S_t \sim \mathcal{N}(\mu_S, \sigma_S^2)$

- Change in portfolio value:

$$P_{t+1} - P_t = \Delta \times (S_{t+1} - S_t) + \frac{1}{2}\Gamma \times (S_{t+1} - S_t)^2$$

$var(X) = E(X^2) - E(X)^2$

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- Can compute moments of $R_S = S_{t+1} - S_t$:

$$\mathbb{E}[R_s] = \mu_s$$
$$\mathbb{E}[R_s^2] = \mu_s^2 + \sigma_s^2$$
$$\mathbb{E}[R_s^3] =$$
$$\mathbb{E}[R_s^4] =$$

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$$\mathbb{E}[R_s^3] = \mu_S^3 + 3\mu_S\sigma_S^2$$

$$\mathbb{E}[R_s^4] = \mu_S^4 + 6\mu_S^2\sigma_S^2 + 3\sigma_S^4$$

$\mathbb{E}(P_{t+1} - P_t)$ $\quad$ $Var(P_{t+1} - P_t)$

# VaR for Options: Delta-Gamma Approach

- Obtain mean and variance of $P_{t+1} - P_t$:

$$\mathbb{E}[P_{t+1} - P_t] = \Delta \mu_s + \frac{1}{2}\Gamma\left(\sigma_s^2 + \mu_s^2\right)$$

$$\text{var}[P_{t+1} - P_t] = \Delta^2 \sigma_s^2 + \frac{1}{4}\Gamma^2\left(\mu_s^4 + 6\mu_s^2\sigma_s^2 + 3\sigma_s^4 - \sigma_s^4 - \mu_s^4 - 2\sigma_s^2\mu_s^2\right)$$

$$+ \Delta\Gamma \underbrace{\text{cov}\left(R_s, R_s^2\right)}_{E(R_s^3) - E(R_s)E(R_s^2)}$$

# VaR for Options: Delta-Gamma Approach

- Obtain mean and variance of $P_{t+1} - P_t$:

$$\begin{aligned} \mathbb{E}[P_{t+1} - P_t] &= \Delta \mathbb{E}[R_s] + \frac{1}{2}\Gamma \mathbb{E}[R_s^2] \\ &= \Delta \mu_S + \frac{1}{2}\Gamma(\mu_S^2 + \sigma_S^2) \\ \mathsf{var}[P_{t+1} - P_t] &= \Delta^2 \mathsf{var}[R_s] + \frac{1}{4}\Gamma^2 \mathsf{var}[R_s^2] + \frac{1}{2}\Delta\Gamma \mathsf{cov}[R_s, R_s^2] \\ &= \Delta^2 \sigma_S^2 + \frac{1}{2}\Gamma^2 \sigma_S^2 (2\mu_S^2 + \sigma_S^2) + \Delta\Gamma \mu_S \sigma_S^2 \end{aligned}$$

# VaR for Options: Delta-Gamma Approach

- Obtain mean and variance of $P_{t+1} - P_t$:

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\mathbb{E}[P_{t+1} - P_t] &= \Delta \mathbb{E}[R_s] + \frac{1}{2}\Gamma \mathbb{E}[R_s^2] \\
&= \Delta \mu_S + \frac{1}{2}\Gamma(\mu_S^2 + \sigma_S^2) \\
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\end{aligned}$$

- Plug in the estimator for the normal distribution:

$$\mathsf{VaR}(c) = -\mathbb{E}[P_{t+1} - P_t] + z(c)\sqrt{\mathsf{var}[P_{t+1} - P_t]}$$

# VaR for Options: Delta-Gamma Approach

- Obtain mean and variance of $P_{t+1} - P_t$:

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\mathbb{E}[P_{t+1} - P_t] &= \Delta \mathbb{E}[R_s] + \frac{1}{2}\Gamma \mathbb{E}[R_s^2] \\
&= \Delta \mu_S + \frac{1}{2}\Gamma(\mu_S^2 + \sigma_S^2) \\
\mathsf{var}[P_{t+1} - P_t] &= \Delta^2 \mathsf{var}[R_s] + \frac{1}{4}\Gamma^2 \mathsf{var}[R_s^2] + \frac{1}{2}\Delta\Gamma \mathsf{cov}[R_s, R_s^2] \\
&= \Delta^2 \sigma_S^2 + \frac{1}{2}\Gamma^2 \sigma_S^2 (2\mu_S^2 + \sigma_S^2) + 2\Delta\Gamma \mu_S \sigma_S^2
\end{aligned}$$

- Plug in the estimator for the normal distribution:

$$\mathsf{VaR}(c) = -\mathbb{E}[P_{t+1} - P_t] + z(c)\mathsf{var}[P_{t+1} - P_t]$$

- Can also deal with portfolio of options with more than one risk

# Cornish-Fisher Expansion

- With this approach, we could also compute any moments of the portfolio: skewness, kurtosis, ...
- How to incorporate into VaR calculation?

# Cornish-Fisher Expansion

- With this approach, we could also compute any moments of the portfolio: skewness, kurtosis, ...
- How to incorporate into VaR calculation?
- **Cornish-Fisher expansion**: asymptotic expansion for the quantile of a distribution
  - Skewness: $\xi_P = \mathbb{E}[(R_P - \mu_P)^3]/\sigma_P^3$
  - Quantile $1-c$:

$$\mu_P + \left( z(1-c) + \frac{1}{6}\left( z(1-c)^2 - 1 \right) \xi_P \right) \sigma_P$$

  - Can also include kurtosis and higher moments

# Vega

- Vega ($\nu$): derivative of option value with respect to the volatility of the underlying asset

$$\nu = \frac{\partial P}{\partial \sigma}$$

- Under the assumptions of Black-Scholes, there is no risk of change in volatility ... but in practice volatility can move

- We can add changes in volatility to our previous calculations:

$$P_{t+1} - P_t = \Delta \times (S_{t+1} - S_t) + \frac{1}{2}\Gamma \times (S_{t+1} - S_t)^2 + \nu(\sigma_{t+1} - \sigma_t) + ...$$

# Other Greeks

- Theta ($\Theta$): change of the value of the portfolio due to passage of time:

$$\Theta = \frac{\partial P}{\partial t}$$

  - Often ignored for risk management (same as means)
- Rho: change of the value of the portfolio due to a parallel shift in all interest rates in a particular country

$$\text{Rho} = \frac{\partial P}{\partial r}$$

  - Particularly relevant for interest rate and exchange rate products

# In practice

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- Traders must keep Gamma and Vega within limits set by risk management
  - Adjust whenever the opportunity arises
- Delta can be adjusted by trading the underlying
- Gamma and Vega need trading of other options

# Takeaways

- When trading options, identify the key risks and hedge them
- Think one step ahead and about potential large shocks: gamma-hedging
- For risk management: crucial to take into account the non-linearity of option contracts