Fixed Income HW4

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In [1]:
```
import numpy as np
import pandas as pd
interests = np.array([[0.049,0.058,0.071,0.068,0.075,0.092],[0.049,0.044
,0.037,0.056,0.062,0.069],[0.049,0.031,0.026,0.051,0.067,0.048],[0.049,
0.05,0.061,0.069,0.069,0.06],[0.049,0.044,0.055,0.061,0.058,0.065]])
# 1
zcb = np.zeros(5)
for i in range(1,6):
    price = 0
    for j in range(5):
        rate = interests[j,:(i+1)].mean()
        price += 100 / (1 + rate) ** i
    zcb[i - 1] = price / 5
zcb_df = pd.Series(zcb,index=['One Year','Two Years','Three Years','Four
Years','Five Years'],name='Zero Coupon Bond')
print(zcb_df)
```

```
One Year       95.494448
Two Years      91.043297
Three Years    86.080349
Four Years     80.964933
Five Years     76.069183
Name: Zero Coupon Bond, dtype: float64
```

In [2]:
```
strike1 = 0.045
def caplet(strike, interest_path):
    caplet = 0
    for i in range(5):
        for j in range(1, 6):
            rate = interest_path[i, :j + 1].mean()
            payoff = max(0, interest_path[i, j] - strike) * 100
            caplet += payoff / (1 + rate) ** j
    caplet /= 5
    return caplet
caplet_price = caplet(strike1,interests)
print(caplet_price)
```

```
5.972029281625215
```

The price of a five year interest cap is 5.972

In [3]:
```
strike2 = 0.067
def floorlet(strike, interest_path):
    floorlet = 0
    for i in range(5):
        for j in range(1, 6):
            rate = interest_path[i, :j + 1].mean()
            payoff = max(0, strike - interest_path[i, j]) * 100
            floorlet += payoff / (1 + rate) ** j
    floorlet /= 5
    return floorlet
floorlet_price = floorlet(strike2,interests)
print(floorlet_price)
```

```
4.956964158672831
```

The price of a five year interest rate floor is 4.957

In [4]:
```
caplet_price4 = caplet(strike2,interests)
floorlet_price4 = floorlet(strike2, interests)
print(caplet_price4)
print(floorlet_price4)
```

```
0.6677004489183029
4.956964158672831
```

The price of caplet is 0.668, and the price of floorlet is 4.957. The floorlet is more valuable.

In [5]:
```
strike3 = 0.063
caplet_price5 = caplet(strike3, interests)
call_price = 0
average_rates = [x.mean() for x in interests]
for i in range(5):
    payoff = max(0, average_rates[i] - strike3) * 100
    call_price += payoff / (1 + average_rates[i]) ** 5
call_price /= 5
print(caplet_price5)
print(call_price)
```

```
1.2162937503769897
0.0836366909794678
```

The price of caplet is 1.2163, and the price of a call option is 0.0836. The caplet is more valuable.

In [6]:
```
year5_std = interests[:,5].std()
average_5year_std = np.std(average_rates)
print(year5_std)
print(average_5year_std)
```

```
0.014441606558828556
0.007766452071427328
```

The standard deviation of the short term rate in year 5 is 0.0144. And the standard deviation of average interest rate in 5 years is 0.0078.
Because the standard deviation of the average interest rate is much smaller than that of short term rate in year 5, and the price of options and caplets are positively related to the standard deviation of interest rates. Therefore, the price of the caplet is higher than that of call option, whose underlying is the average interest rate during all 5 years.