Problem 3.

1. Let $X_t = \log S_t$.

By Ito's lemma: $dX_t = \frac{\partial X}{\partial t}dt + \frac{\partial X}{\partial s}ds + \frac{1}{2}\frac{\partial^2 X}{\partial s^2}(ds)^2$

$$dX_t = (\mu - \tfrac{1}{2}\sigma^2)dt + \sigma dW_t.$$

$$\log S_T = X_T \sim N(X_0 + (\mu - \tfrac{1}{2}\sigma^2)T,\ \sigma^2 \cdot T).$$

If X is a random variable with quantile c equal to x_0, then the quantile c of $g(X)$ is $g(x_0)$ if g is a monotone function.

$$P(S_T < S_0 - VaR) = 1 - C. \iff$$

$$P(\log S_T < \log(S_0 - VaR)) = 1 - C = 0.01$$

$$\frac{\log(S_0 - VaR) - [X_0 + (\mu - \tfrac{1}{2}\sigma^2)T]}{\sigma\sqrt{T}} = -2.327. \quad T = \frac{10}{252},\ S_0 = 50$$

$$VaR = 50 \cdot (1 - e^{\frac{5}{126}(\mu - \frac{1}{2}\sigma^2) - 0.4635\sigma\sqrt{\cdot}})$$

Input $\mu = 0.07$ $\sigma = 0.16$. $\Rightarrow$ $VaR = 3.468$

2. ~~Number of stocks you can invest $= \frac{\text{Capital}}{VaR} \approx 28835063$.~~

~~Total stock value $= 28835063 \times 50 \approx 1441.75$ million~~

Let x denote the value you want to borrow by bonds.

$$\frac{100 - x \times 2\% \times \frac{10}{252}}{VaR} \times 50 = x \quad \Rightarrow \quad x \approx 1425.44 \text{ million}$$

Therefore, in your portfolio: Long 1425.44 million ~~bar~~ stock, short 1425.44 million bond, and keep 100 million capital.

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