Problem 3.

1. let $X_t = \log S_t$.

By Ito's lemma: $dX_t = \frac{\partial X}{\partial t} dt + \frac{\partial X}{\partial S} dS + \frac{1}{2}\frac{\partial^2 X}{\partial S^2}(dS)^2$

$$dX_t = (\mu - \tfrac{1}{2}\sigma^2)\,dt + \sigma\, dW_t.$$

$$\log S_T = X_T \sim N(X_0 + (\mu - \tfrac{1}{2}\sigma^2)T,\ \sigma^2 \cdot T).$$

If X is a random variable with quartile c equal to x_0, then the quartile c of $g(X)$ is $g(x_0)$ if g is a monotone function.

$$P(S_T < S_0 - VaR) = 1 - c. \iff$$

$$P\{\log S_T < \log(S_0 - VaR)\} = 1 - c = 0.01$$

$$\frac{\log(S_0 - VaR) - \{X_0 + (\mu - \frac{1}{2}\sigma^2)T\}}{\sigma \cdot \sqrt{T}} = -2.327. \quad T = \frac{10}{252},\ S_0 = 50$$

$$VaR = 50 \cdot (1 - e^{\frac{5}{126}(\mu - \frac{1}{2}\sigma^2) - 0.4635 \cdot \sigma})$$

Input $\mu = 0.07$ $\sigma = 0.16$. $\Rightarrow$ $VaR = 3.468$

2. Let x denote the value you want to borrow by bonds.

$$\frac{100 - x \times (e^{0.02 \times \frac{10}{252}} - 1)}{VaR} \times 50 = x + 100 \quad \Rightarrow x \approx \cancel{1426.57}\ 1326.57 \text{ million}$$

Therefore, in your portfolio: Long 1426.57 million stock and Short 1326.57 million bonds.