UCLA Anderson School of Management

Valentin Haddad, Financial Risk Management MFE 409, Spring 2019

Midterm

April 30, 2019

Date: ______________________

Name: ______________________

Signature: ______________________

- As a member of the UCLA Anderson academic community, the highest standards of academic behavior are expected of you. It is your responsibility to make yourself aware of these standards (specifically regarding plagiarism, individual work, and team work) and adhere to them. By signing the exam: (*i*) you certify your presence, and (*ii*) you state that you neither gave nor received help during the examination.
- This exam is close book. You can use a scientific calculator, but no computer or cell phone. You can use a one-page cheat sheet with notes (one page = one side of one sheet of paper). **Be sure to show your derivations and explain your work.**
- Please write your name on top of each page.
- You are not allowed to communicate with anyone (verbally, in writing, or electronically) during the exam period.
- You have **120 minutes** to finish the exam. The total score is **120 points**. One minute corresponds approximately to one point. Pace yourself wisely.

MIDTERM SOLUTIONS

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1 Short Answer Questions (69 Points)

1.1 VaR and Capital (7 Points)

Why is VaR a good measure of necessary capital?

SOLUTION: Slide deck 1b, pages 7-8

1.2 VaR and ES of Pareto-Distributed Random Variables (8 Points)

Let Z follow a Pareto distribution (i.e., $P(Z > z) = kz^{-\frac{1}{\xi}}$) with $\xi = 0.5$ and $k = 1$. Assume security X's returns are equal to $-Z + C$ given some constant C. Derive expressions for security X's 95% VaR and Expected Shortfall.

SOLUTION:

The VaR of $-Z$ is given by the following:

$$P(-Z \leq \mathsf{VaR}_Z) = \mathsf{VaR}_Z^{-2} = 0.05$$
$$\mathsf{VaR}_Z = \sqrt{20}$$

The Expected Shortfall of $-Z$ is given by the following:

$$\begin{aligned} ES_Z &= 40 \int_{-\infty}^{-\mathsf{VaR}} \frac{1}{Z^3} Z dZ \\ &= 40 \int_{-\infty}^{-\mathsf{VaR}} \frac{1}{Z^2} dZ \\ &= -40 \frac{1}{Z} \Big|_{-\infty}^{-\mathsf{VaR}} \\ &= -40 \left(-\frac{1}{\mathsf{VaR}} - 0 \right) = \frac{40}{\sqrt{20}} = 2\sqrt{20} \end{aligned}$$

Therefore, the VaR and the Expected Shortfall of X are the following:

$$\mathsf{VaR}_X = \sqrt{20} - C$$
$$ES_X = 2\sqrt{20} - C$$

1.3 Backtesting VaR (7 Points)

We want to back test our 99% daily VaR model. We have 600 observations of daily returns. Our model for the 99% daily VaR delivers 14 exceptions over the 600 day period. What do you conclude about our model? How would you test this formally (provide a precise mathematical formula, but you do not need to calculate the result)?

SOLUTION:

The probability of 14 or more exceptions is

$$1 - P(x \leq 13) = 1 - \sum_{i=0}^{13} \frac{600!}{i!(600-i)!}(0.01)^i(0.99)^{600-i} \approx 0.0034,$$

which can be interpreted as a p-value, indicating that we reject. The number of exceptions is higher than we would expect.

1.4 Coherent Risk Measures (7 Points)

Is VaR a coherent risk measure? Prove your answer.

SOLUTION: No, it fails the diversification criterion. You can prove this through a counterexample, see slide deck 1b, slides 28-29.

1.5 Exponential Weighting (8 Points)

When using exponential weighting to compute the VaR using the historical method, we typically choose $\lambda = 0.995$. When using exponential weighting to estimate volatility we typically choose $\lambda = 0.94$. Explain why these values differ.

SOLUTION: Different λ helps to adjust the weight of historical data (higher λ means less weight on recent data). Estimating the 0.01 quantile needs a lot of data (it only uses outliers), whereas estimating volatility is easier (it uses all the data), therefore we use a smaller λ.

1.6 VaR Approaches (8 Points)

You've been hired by the risk management department of Goldman Sachs. They ask you to calculate the 99% 10-day VaR for their portfolio of stocks. What would you do? Explain your choices.

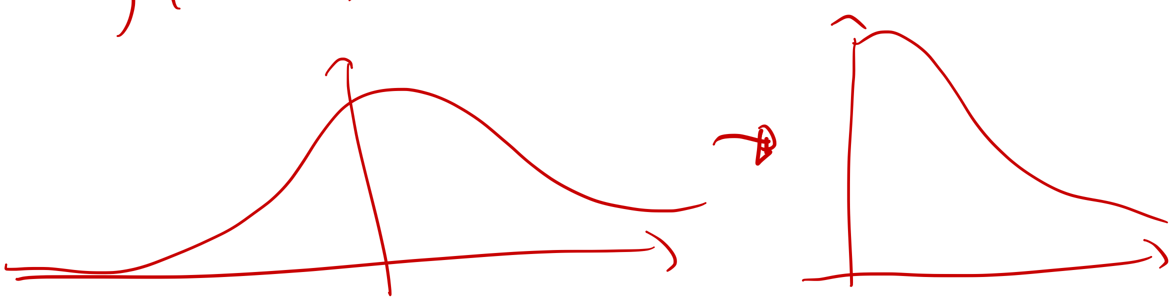
SOLUTION: Very open question, many approaches can be taken, e.g. historical approach or model-building approach. It is important, however, to explain why you are choosing a certain method.

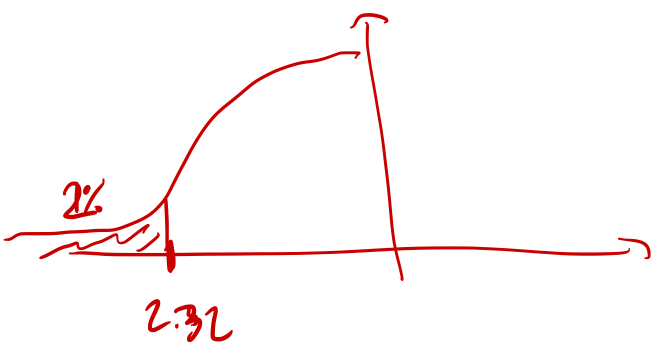
1.7 VaR for a "Straddle" (8 Points)

Assume that a stock trades at \$90 and has stock prices following a log-normal distribution with volatility 6% over 10 days. This corresponds to the log return following a normal distribution: $\log(R_{t+10}) \sim \mathcal{N}(0, 6\%^2)$. You are short a contract that pays $|\underbrace{\log(P_{t+10}/90)}|$ in 10 days. What is the 98% 10-day VaR of your position?

SOLUTION: The contract is really on absolute value of a normal. So you have to take the 99% percentile, $z = 2.33$, because both sides of the distribution give you losses (i.e., a short straddle). So for W_0, the VaR is $2.3\cancel{3}2 \times 0.06 = 13.96\%$ in terms of returns.

$\log(P_{t+10}/90) \sim \mathcal{N}(0, 6\%^2)$





1.8 Parameter Estimation in the ARCH(1) Model (8 Points)

The ARCH(1) model assumes that a return series $\{R_t\}$ follows:

$$R_t = \sigma_t \epsilon_t$$
$$\sigma_t^2 = \omega + \alpha R_{t-1}^2$$

where $\{\epsilon_t\}$ is standard normal and the parameters satisfy $\omega > 0$ and $0 \leq \alpha < 1$. Suppose you observe a sample of realizations $r_0, r_1, .., r_n$. Explain precisely how you would estimate the paramers ω and α.

SOLUTION:

$$R_t = \sigma_t \epsilon_t = \sqrt{\omega + \alpha R_{t-1}^2} \epsilon_t$$
$$E[R_t | R_{t-1} = r_{t-1}] = 0$$
$$Var(R_t | R_{t-1} = r_{t-1}) = \omega + \alpha R_{t-1}^2$$

$$\frac{1}{\sqrt{2\pi \sigma_t^2}} exp\left(- \frac{x^2}{2\sigma_t^2}\right)$$

Since R_t is conditionally normal, then

$$f_{R_t|R_{t-1}}(r) = \frac{1}{\sigma_t} \varphi\left(\frac{r}{\sigma_t}\right)$$
$$\varphi(r) = \frac{1}{\sqrt{2\pi}} \exp(-r^2/2)$$

Then the conditional likelihood is the joint conditional density is

$$f(r_n, r_{n-1}, ..., r_1 | r_0) = \frac{1}{\sigma_n} \varphi\left(\frac{r_n}{\sigma_n}\right) \times ... \times \frac{1}{\sigma_1} \varphi\left(\frac{r_1}{\sigma_1}\right)$$
$$= \frac{1}{(2\pi)^{n/2} \sigma_n ... \sigma_1} \exp\left(-0.5 \sum_{t=1}^{n} \frac{r_t^2}{\sigma_t^2}\right)$$

We maximize the likelihood function w.r.t. parameters ω and α. We do this either numerically or analyically through first-order conditions (i.e. setting partial derivatives of the likelihood function w.r.t. parameters ω and α to 0).

$$\mathcal{L}(r_1 \dots r_n | r_0) = \mathcal{L}(r_1 | r_0) \times \mathcal{L}(r_2 | r_0, r_1) \times \dots$$
$$= \mathcal{L}(r_1 | r_0) \times \mathcal{L}(r_2 | r_1) \times \mathcal{L}(r_3 | r_2) \dots$$

1.9 Accuracy of VaR (8 Points)

Explain how to use a bootstrap procedure to measure the accuracy of the Extreme Value Theory approach to compute VaR.

SOLUTION:

Assume we have T data points $x_0, \ldots, x_{T-1}$.

You can use a *parametric bootstrap*.

1. Estimate the parameters of the Pareto distribution via MLE using observations $x_0, \ldots, x_{T-1}$.
2. Draw N samples of size T from a Pareto distribution using the parameters estimated above.
3. For each of the samples, calculate the VaR as in the historical approach. This gives you N draws from the sampling distribution of the VaR, from which we can form confidence intervals.

2 Long Question 1: Managing a Currency Trading Desk (28 Points)

Deutsche Bank (DB) is a German bank that manages its book in EUR. Consider 2 desks in DB, one is long 150 million USD and the other is short 50 million GBP. The exchange rates are 1 USD = 0.9163 EUR and 1 GBP = 1.3599 EUR. The daily volatilities for changes in USD/EUR and GBP/EUR are 0.40% and 0.30%, respectively and means of 1 basis point and 1.5 basis points. The correlation between them is 0.5. For risk calculations, assume that the returns have mean zero and are normally distributed.

1. What is the 99% 1-day VaR for each desk? (5 Points)
2. What is the 99% 1-day VaR for the combined portfolio? (5 Points)
3. Consider an arbitrary portfolio with positions x_1 and x_2 in two assets. If you increase your position in x_1 by a small amount Δx_1, by how much do you need to change your position in asset 2 to keep your VaR constant? What is the effect of these changes on your expected profits? Obtain mathematical expressions as function of (some but not neccesarily all of) Δx_1, VaR, DVaR, CVaR, and expected returns. (9 Points)
4. How would you change the allocation of DB's trading desk? Give a quantitative argument. (9 Points)

SOLUTION:

1. Notice

$$
\begin{aligned}
R_{t+1}^{USDEUR} = \frac{E_{t+1}^{USDEUR}}{E_t^{USDEUR}} - 1 \quad &\sim \quad N\left(0.0001, 0.004^2\right) \\
R_{t+1}^{GBPEUR} = \frac{E_{t+1}^{GBPEUR}}{E_t^{GBPEUR}} - 1 \quad &\sim \quad N\left(0.00015, 0.003^2\right)
\end{aligned}
$$

Assume means are zero for risk management purposes. Then the VaR, measured in EUR,of the desk with USD is

$$\mathsf{VaR}_{0.99} = 2.326 \times 0.004 \times 137.45 = 1.279m.$$

The VaR, measured in EUR,of the desk with GBP is

$$\mathsf{VaR}_{0.99} = 2.326 \times 0.003 \times 67.995 = 0.475m.$$

2. The combined porfolio is bi-variate normal . Its volatility and VaR are

$$
\begin{aligned}
\sigma_P &= \sqrt{(137.45)^2 \sigma_1^2 + (67.995)^2 \sigma_2^2 - 2\,(67.995)\,(137.45)\,\sigma_1\sigma_2\rho} \\
&= \sqrt{0.2317} = 0.481 \\
\mathsf{VaR}_{0.99} &= 2.326\sigma_P = 1.120m.
\end{aligned}
$$

Notice the minus term in the portfolio volatility. This is because we short one of the two exchange rates.

3. Notice that

$$\mathsf{VaR} = CVaR_1 + CVaR_2 = x_1 \frac{\partial \mathsf{VaR}}{\partial x_1} + x_2 \frac{\partial \mathsf{VaR}}{\partial x_2} = x_1 DVaR_1 + x_2 DVaR_2.$$

Now for changes Δx_1 and Δx_2 we get $\tilde{\mathsf{VaR}}$

$$\tilde{\mathsf{VaR}} \approx (x_1 + \Delta x_1) DVaR_1 + (x_2 + \Delta x_2) DVaR_2 = \mathsf{VaR} + \Delta x_1 DVaR_1 + \Delta x_2 DVaR_2.$$

Equating $\mathsf{VaR} = \tilde{\mathsf{VaR}}$ gives

$$0 = \Delta x_1 DVaR_1 + \Delta x_2 DVaR_2,$$

and thus

$$\Delta x_2 = -\frac{DVaR_1 \Delta x_1}{DVaR_2}.$$

Expected returns then change by

$$\begin{aligned}\Delta\mathbb{E}[R] &= \Delta x_1 \mathbb{E}[R_1] + \Delta x_2 \mathbb{E}[R_2] \\ &= \Delta x_1 \Big(\mathbb{E}[R_1] - \frac{DVaR_1}{DVaR_2}\mathbb{E}[R_2]\Big).\end{aligned}$$

4. We want to maximize $RAROC = E(r_P)/\mathsf{VaR}$ of the entire portfolio. To do this, we can calculate $E(r_i)/DVaR_i$ ratios for both positions, and then adjust the portfolio towards the position with the higher ratio. This will improve $RAROC$.

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3 Longer Question 2: Bond portfolio (23 Points)

Assume you invest in two units of a bond that pays \$100 with probability 90%, and defaults with probability 10%. If the bond defaults, the amount in dollars that you recover is uniformally distributed on $[0, 50]$.

1. What is the 99% VaR of your investment in the bond? *(5 points)*
2. What is the 99% expected shortfall of your investment in the bond? *(5 points)*
3. Now assume there are two *independent* bonds, each of which has the same payment profile as defined above. You hold one unit of each bond. What is the 99% VaR of your investment in this bond portfolio? *(8 points)*
4. What is the 99% expected shortfall of your investment in the bond portfolio? *(5 points)*

SOLUTION:

Let $W_0 = \$200$.

1. We are in the default region with probability 10%, so for the 99% VaR we consider the 0.1 quantile of a uniform payoff in [0,50], which is 5 cents on the dollar. Consequently, the 99% VaR is \$190.
2. Because this is a uniform distribution, the expected shortfall will be the mid-point on [190,200], i.e. \$195.
3. Notice first that the probability of both bonds defaulting is $0.1^2 = 0.01$, and constitutes the worst 1% of days. Even though there is a cutoff point at the 0.01 quantile, both the limit from the right and the left is equal, such that the 99% VaR is \$100. In particular, if both firms default, which is the case in the worst 1% of days, the best payoff comes from recoveries $50 + 50 = 100$. Likewise, if one bond defaults, the worst that can happen is zero recovery for the other bond, and we have $100 + 0 = 100$ as well.
4. Consider X and Y the payoffs from the two bonds. The expected payoff in default is

$$\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y] = 25 + 25 = 50.$$

The expected shortfall is \$150.

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END OF EXAM

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