MFE 409: Midterm Review

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This list of topics is here to give you a guideline of the main ideas we studied in class as you prepare for the midterm.

1. Broad ideas
   (a) Reasons to manage risk: Modigliani-Miller theorem, reasons for regulation
   (b) Reasons to take risk, risk regulations as a constraint
2. Value-at-Risk
   (a) Definition of VaR
   (b) Rationale for VaR: necessary capital measure, tail risk measure
   (c) Issue 1 with VaR: not capturing the structure of tail risk
   (d) Definition of Expected Shortfall, why it helps with issue 1
   (e) VaR and ES for normal distributions
   (f) Role of time for VaR
   (g) VaR for a portfolio: normal distribution and approximation
   (h) DVaR, CVaR, and decomposition of VaR
   (i) RAROC and its use for capital allocation
   (j) Issue 2 with VaR: not always capturing diversification
   (k) Coherent risk measures, why VaR is not one, why ES is one
3. Back-testing
   (a) Definition
   (b) Distribution of the number of exceptions
   (c) Bunching
4. Historical simulation approach to compute VaR
   (a) Definition and implementation in the simplest case, computing ES.
   (b) Stressed VaR: definition and properties

(c) Estimation of the accuracy of VaR: parametric and bootstrap

(d) Tradeoff for choosing how much data to use

(e) Exponentially weighted VaR

(f) Implication of extreme value theory for tail of distributions

(g) Estimating a generalized Pareto distribution and using it to compute VaR and ES

5. Model-building approach to compute VaR

   (a) Normal model for a portfolio

   (b) Imperfect hedging and VaR reduction

   (c) Volatility: definition, best estimate, MLE, and estimator used in practice

   (d) Weighting schemes for volatility estimations

   (e) Applications: ARCH, EWMA, and GARCH

   (f) MLE estimation of these models

   (g) *Implied volatility: definition and construction (not on midterm)*

   (h) *How to use options to infer future moments of the data, limitations (not on midterm)*

   (i) Tradeoffs for choosing between model-building and historical simulation

$$\mathcal{L}(x_1, \ldots x_n \mid \Theta) = \mathcal{L}(x_1 \mid \Theta) \times \mathcal{L}(x_2 \mid x_1, \Theta) \times \cdots \mathcal{L}(x_n \mid x_1 \cdots x_n \Theta)$$

UCLA Anderson School of Management
Valentin Haddad, Financial Risk Management MFE 409, Spring 2018

Midterm

May 1, 2018

Date: ____________________

Name: ____________________

Signature: ____________________

- As a member of the UCLA Anderson academic community, the highest standards of academic behavior are expected of you. It is your responsibility to make yourself aware of these standards (specifically regarding plagiarism, individual work, and team work) and adhere to them. By signing the exam: (*i*) you certify your presence, and (*ii*) you state that you neither gave nor received help during the examination.
- This exam is close book. You can use a scientific calculator, but no computer or cell phone. You can use a one-page cheat sheet with notes (one page = one side of one sheet of paper). **Be sure to show your derivations and explain your work.**
- Please write your name on top of each page.
- You are not allowed to communicate with anyone (verbally, in writing, or electronically) during the exam period.
- You have **120 minutes** to finish the exam. The total score is **120 points**. One minute corresponds approximately to one point. Pace yourself wisely.

MIDTERM SOLUTIONS

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1 Short Answer Questions (72 Points)

1.1 Bank regulation (9 Points)

Why is the risk-taking of banks regulated?

SOLUTION: The rationale behind regulating banks is that the social costs of financial crises are big and exceed the sum of the private costs (i.e., there are externalities). The objective of financial regulation is to induce banks to internalize those costs.

1.2 Extreme value theory (9 Points)

Describe in details how you would use extreme value theory to compute the 1-day 99.9% VaR for investing \$100,000 in the S&P500 if you are given one year of daily data on the value of the index I_t.

SOLUTION:
Extreme value theory establishes the result that a wide range of distributions have a Pareto tail.
First form the returns, take the opposite to have losses, then select the top 5% of the data and call the threshold u. Third, estimate ξ and β using maximum likelihood and plug in. That is, solve for

$$\max_{\widehat{\xi},\widehat{\beta}} \sum_{i=1}^{n_u} \ln \left[\frac{1}{\widehat{\beta}} \left(1 + \frac{\widehat{\xi}\,(v_i - u)}{\widehat{\beta}} \right)^{-\frac{1}{\widehat{\xi}}-1} \right]$$

we plug the parameters into

$$\left\{ 1 - \left[\left(\frac{n}{n_u} \right) \alpha \right]^{-\widehat{\xi}} \right\} \frac{\widehat{\beta}}{\widehat{\xi}} + u = \widehat{VaR_\alpha}$$

-5%
2%
3%
7%
-10%
⋮
-70%

sort →

-20%
-10%
-5%
⋮

transform in losses
×(-1)

20%
10%
5%
$\boxed{2.7\%}$ u = 2.7%

95% of the data

Likelihood(ξ,β)
→ optimize
$\hat{\xi}, \hat{\beta}$
plug into the VaR formula

1.3 Confidence Interval of VaR (9 Points)

Suppose we use the historical method to estimate the 99% VaR of a portfolio from 1,000 observations and we get an estimate of $25 million. We approximate the empirical distribution of losses with a normal density function with $\mu = \$0.01m, \sigma = \$11m$. Calculate the 95% confidence interval of this estimate.

SOLUTION:

$$\begin{aligned} x &= \mu + \sigma\Phi^{-1}(0.01) = 0.01 - 2.36 * 11 \\ &= -25.57 \end{aligned}$$

$$f(x) = \frac{1}{11\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(-25.57 - 0.01)^2}{11^2}\right) = 0.00242$$

Standard Error

$$\frac{1}{0.00242}\sqrt{\frac{0.01 * 0.99}{1000}} = 1.298$$

which means the interval is $[22.46, 27.54]$

1.4 Mixing Views (9 Points)

You have the view that the gains of a project follow a uniform distribution between -2% and 6%. Your partner, instead, thinks gains follow a uniform distribution between -3% and 8%. Compute the 80% VaR assuming your view has a 0.5 chance of being correct and your partner's view has the other 0.5.

SOLUTION:

$$\Pr\left(W_0 - W \geq VaR_\alpha\right) = 1 - \alpha$$

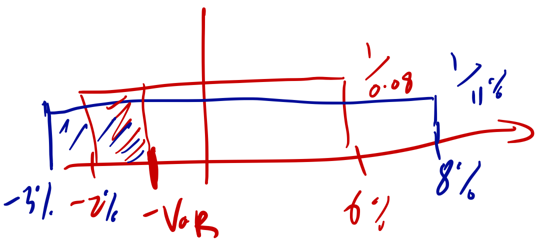
so

$$\Pr\left(G < -VaR_\alpha\right) = 1 - \alpha$$

So

$$0.5\frac{-VaR_\alpha + 0.02}{0.08} + 0.5\frac{-VaR_\alpha + 0.03}{\cancel{0.05}\ 0.11} = 1 - \alpha$$

solve for VaR_α and get 0.57%



$$\Pr\left(W_0 - W \geq VaR\right) = 20\%$$

$$\Pr\left(W \leq W_0 - VaR\right) = 20\%$$

↳ assume this is 0

$$\Pr\left(W \leq -VaR\right) = 20\%$$

$$= 0.5\ \frac{-VaR - (-2\%)}{8\%} + 0.5\ \frac{-VaR - (-3\%)}{11\%}$$

1.5 Risk and VaR (9 Points)

Give an example of two distributions where one is more risky than the other, yet they both have the same VaR. What are the practical implications of this observation?

SOLUTION:
Example from class would suffice. Practical implications: you can hide risk in the tail and avoid regulation.

1.6 Earthquake Risk (9 Points)

You are investing \$1 million in a real estate project in California, which is a seismic region. To evaluate the potential losses due to an earthquake over the next 5 years, you use am exponential distribution for the risk of this disaster. Specifically, you will lose over this period a fraction $e^{-d} \in [0,1]$ of your investment, where d follows an exponential distribution with intensity λ.

Compute the 5-year 99% VaR of your investment with respect to your initial invested capital of $W_0 = \$1m$.

Note: The CDF for an exponentially distributed random variable $X \geq 0$ is $F_X(x;\lambda) = \int_0^c \lambda e^{-\lambda u} du = 1 - e^{-\lambda x}$

SOLUTION:

$$\begin{aligned}
\Pr\left(W_0 - W \geq VaR_\alpha\right) &= 1-\alpha \\
\Pr\left(W_0 - W_0\left(1-e^{-d}\right) \geq VaR_\alpha\right) &= 1-\alpha \\
\Pr\left(W_0 e^{-d} \geq VaR_\alpha\right) &= 1-\alpha \\
\Pr\left(e^{-d} \geq VaR_\alpha\right) &= 1-\alpha \\
\Pr\left(-d \geq \ln VaR_\alpha\right) &= 1-\alpha
\end{aligned}$$

So

$$\Pr\left(d \leq -\ln VaR_\alpha\right) = \int_0^{-\ln VaR_\alpha} \lambda e^{-\lambda x} dx = 1 - e^{\lambda \ln VaR_\alpha}$$

which means

$$\begin{aligned}
1 - e^{\lambda \ln VaR_\alpha} &= 1-\alpha \\
VaR_\alpha^\lambda &= \alpha \\
\alpha^{\frac{1}{\lambda}} &= VaR_\alpha
\end{aligned}$$

The larger the intensity, the larger the VaR. Notice $\lim_{\lambda\to\infty} VaR_\alpha = 1$.

1.7 How Much Data? (9 Points)

When choosing how much data to use to compute VaR, what are the factors that enter your decision?

SOLUTION: Lecture 2 slide 23. Trade off. More precise: more data. But not all data because is obsolete.

1.8 Component VaR (9 Points)

Is the following statement TRUE or FALSE?

> Suppose you have a portfolio with three instruments with *uncorrelated returns*: a stock, a corporate bond, and a Treasury bond. You have \$2 million invested in the stock, \$10 million invested in the corporate bond, and \$50 million invested in the Treasury bond. The component 1-day 99% VaRs of this portfolio with respect to the stock, corporate bond, and Treasury bond are \$1.5 million, \$0.75 million, and \$2.5 million, respectively. If you decide to liquidate all your stock holdings, the 1-day 99% VaR of your new portfolio can be calculated by adding the aforementioned component VaRs, which is equal to \$0.75 million + \$2.5 million = \$3.25 million.

Explain your choice.

SOLUTION:
This statment is false. Component VaR depends on the portfolio. In particular, CVaR changes with the composition.

$$VaR = 2.32\sqrt{2^2\sigma_1^2 + 10^2\sigma_2^2 + 50\sigma_3^2}$$

2 Longer Question: Seasonal Volatility (24 Points)

We want to capture the idea that volatility varies throughout the year. To do so, we assume that monthly returns are independent and given by R_t distributed $\mathrm{N}(0,\sigma_t^2)$, where $\sigma_t = \sigma_0 f(t)$ with f a known deterministic function of the month t. We are interested in tracking the VaR for an investment of $200,000$ in the portfolio. Assume you have 5 years of past data.

1. Write down the log-likelihood of observing your data sample given the parameter σ_0. *(8 points)*
2. Derive the maximum likelihood estimator of σ_0. Comment on your result. *(8 points)*
3. Assume $f(t) = 1.2$ from April to September (included) and $f(t) = 1$ the rest of the year, and your estimator gave $\sigma_0 = 5\%$. What is your estimate for the *1-year* 99%-VaR for this asset. (Assume that returns compound arithmetically: $R_{1y} = R_{january} + \ldots + R_{december}$). *(8 points)*

SOLUTION:

1. For a give observation R_t
$$\phi\left(R_t\right) = \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{R_t^2}{2\sigma_t^2}\right)$$
so for the sequence of 5 years with monthly observations, the log-likelihood
$$\sum_{t=1}^{60} -\log\left(\sqrt{2\pi}\sigma_0 f\left(t\right)\right) - \frac{1}{2}\sum_{t=1}^{60}\left(\frac{R_t^2}{\sigma_0^2 f\left(t\right)}\right)$$
2. Take the first order conditions
$$\sigma_0 = \sqrt{\frac{1}{60}\sum_{t=1}^{60}\left(\frac{R_t^2}{f\left(t\right)}\right)}$$
Notice the estimation is a the standard sample volatility but weighting observations by f(t).
3. Since returns are independent normal, then the distribution has a standard deviation of $\sqrt{\left(0.05^2 + 0.06^2\right)6}$, so the VaR is -200000$\sqrt{\left(0.05^2 + 0.06^2\right)6}$. $(2.36) = 88388$.

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3 Longer Question: Trading options (24 Points)

Assume the IBM stock price can go up each month by 5% with probability 0.75 and down 15% with probability 0.25 with probability. Also assume that risk-neutral and actual probability coincide, and that the risk-free rate is 0%. The price of the IBM stock today is \$100 and you are investing for two months. You are the manager of a hedge fund investing in this stock and options related to it.

1. If there is no limited liability in this world, which trades have the highest expected returns? *(4 points)*
2. What is the 2-month 90% VaR for investing in one share of the stock? *(6 points)*
3. What is the 2-month 90% VaR for selling a two-month out-of-the-money put as a function of the strike K. Derive an expression and plot a graph. *(9 points)*
4. When your fund takes on short positions, it has to be able to repay what is due even in the (true) worst case scenario. So you decide to raise some money to trade. Assume you have \$1m of initial capital and somebody lent you \$9m with the constraint that your 2-month 90% VaR cannot exceed your capital. What is a profitable trade you can do by selling puts? Show it is and explain why it is profitable. *(5 points)*
5. *Bonus*: what is the most profitable trade you can do? *(10 points)*

SOLUTION:

1. All have the same expected return, equal to $r_f = 0$.
2. You can compute the cumulative loss distribution, and see that in the range of 0.0625<c=0.1<0.4375 the var is 10.75.
3. Similarly, compute payoffs of the correspondent puts: $-max(0,(K-72.25)), -max(0,(K-89.25)), -max(0,(K-110.25))$, with the associated cumulative probabilities so the 90% VaR is P_0-max(0,(K-89.25)). The plot is straightforward: VaR is P_0 until 89.25 and increases linearly for K>89.25.
4. If you can default, you can do profits by selling puts, and calibrating your portfolio to show your lender that you are outside of the VaR constraint. You are "hiding" the tail risk. This means that you are losing lender's money in bad states and making profits in good states.
5. You can optimize throughout moneyness.

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END OF EXAM

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