

Problem Set 5

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Question 1

$$1. \text{ Ito's lemma: } dM = \frac{\partial M}{\partial t} dt + \frac{\partial M}{\partial S_1} dS_1 + \frac{\partial M}{\partial S_2} dS_2 + \frac{1}{2} \frac{\partial^2 M}{\partial S_1^2} (dS_1)^2 + \frac{1}{2} \frac{\partial^2 M}{\partial S_2^2} (dS_2)^2 + \frac{1}{2} \frac{\partial^2 M}{\partial S_1 \partial S_2} dS_1 dS_2$$

\therefore only consider delta approach, we assume $\Gamma = \frac{\partial^2 M}{\partial S^2} = 0$ and assume $dt = 1$

$$\therefore dM = \Delta_1 dS_1 + \Delta_2 dS_2 = \Delta_1 u_1 S_1 dt + \Delta_1 \sigma_1 S_1 dW_1 + \Delta_2 u_2 S_2 dt + \Delta_2 \sigma_2 S_2 dW_2$$

$$= (\Delta_1 u_1 S_1 + \Delta_2 u_2 S_2) dt + \Delta_1 \sigma_1 S_1 dW_1 + \Delta_2 \sigma_2 S_2 dW_2$$

$$E(dM) = \Delta_1 u_1 S_1 + \Delta_2 u_2 S_2 \quad \text{Var}(dM) = \Delta_1^2 \sigma_1^2 S_1^2 dt + \Delta_2^2 \sigma_2^2 S_2^2 dt + 2 \Delta_1 \Delta_2 \sigma_1 \sigma_2 S_1 S_2 \rho dt$$

$$= \Delta_1^2 \sigma_1^2 S_1^2 + \Delta_2^2 \sigma_2^2 S_2^2 + 2 \Delta_1 \Delta_2 \sigma_1 \sigma_2 S_1 S_2 \rho$$

$$P(M < M_0 - \text{VaR}) = 0.01$$

$$P(dM < -\text{VaR}) = 0.01$$

$$\mu - 2.326\sigma = -\text{VaR}$$

$$\text{VaR} = 2.326\sigma - \mu =$$

$$2.326 \sqrt{(\Delta_1^2 \sigma_1^2 S_1^2 + \Delta_2^2 \sigma_2^2 S_2^2 + 2 \Delta_1 \Delta_2 \sigma_1 \sigma_2 S_1 S_2 \rho)} - \Delta_1 u_1 S_1 - \Delta_2 u_2 S_2$$

$$2. dM = \Delta_1 dS_1 + \Delta_2 dS_2 + \frac{1}{2} \Gamma_1 (dS_1)^2 + \frac{1}{2} \Gamma_2 (dS_2)^2 + \Gamma_{12} dS_1 dS_2$$

$$= \Delta_1 u_1 S_1 dt + \Delta_1 \sigma_1 S_1 dW_1 + \Delta_2 u_2 S_2 dt + \Delta_2 \sigma_2 S_2 dW_2 + \frac{1}{2} \Gamma_1 \sigma_1^2 S_1^2 dt + \frac{1}{2} \Gamma_2 \sigma_2^2 S_2^2 dt + \Gamma_{12} \sigma_1 \sigma_2 S_1 S_2 \rho dt$$

$$E(dM) = \Delta_1 u_1 S_1 + \Delta_2 u_2 S_2 + \frac{1}{2} \Gamma_1 \sigma_1^2 S_1^2 + \frac{1}{2} \Gamma_2 \sigma_2^2 S_2^2 + \Gamma_{12} \sigma_1 \sigma_2 S_1 S_2 \rho$$

$$\text{Var}(dM) = \Delta_1^2 \sigma_1^2 S_1^2 + \Delta_2^2 \sigma_2^2 S_2^2 + 2 \Delta_1 \Delta_2 \sigma_1 \sigma_2 S_1 S_2 \rho$$

$$\text{VaR} = 2.326 \cdot \sigma - \mu$$

$$= 2.326 \sqrt{(\Delta_1^2 \sigma_1^2 S_1^2 + \Delta_2^2 \sigma_2^2 S_2^2 + 2 \rho \Delta_1 \Delta_2 \sigma_1 \sigma_2 S_1 S_2)} - \Delta_1 u_1 S_1 - \Delta_2 u_2 S_2 - \frac{1}{2} \Gamma_1 \sigma_1^2 S_1^2 - \frac{1}{2} \Gamma_2 \sigma_2^2 S_2^2 - \Gamma_{12} \sigma_1 \sigma_2 S_1 S_2 \rho$$

$$- \Delta_1 u_1 S_1 - \Delta_2 u_2 S_2 - \frac{1}{2} \Gamma_1 \sigma_1^2 S_1^2 - \frac{1}{2} \Gamma_2 \sigma_2^2 S_2^2 - \Gamma_{12} \sigma_1 \sigma_2 S_1 S_2 \rho$$