UCLA ANDERSON SCHOOL OF MANAGEMENT

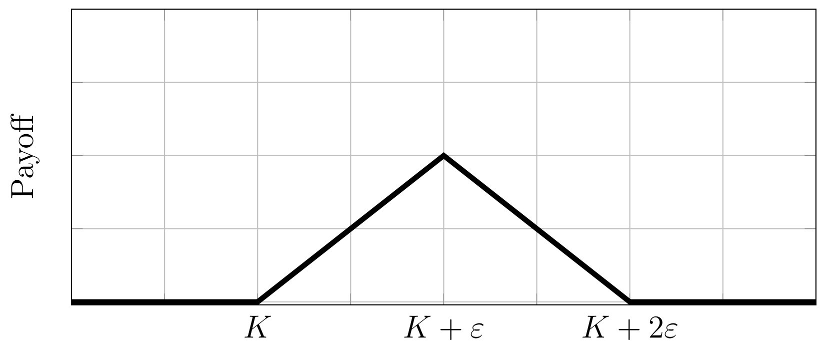
Daniel Andrei, Derivative Markets MGMTMFE 406, Winter 2018

Problem Set 3

These exercises do not need to be turned in for credit.

1 Butterfly Spread & Risk-Neutral Density

A butterfly spread consists of a portfolio of options yielding the following payoff:



All options involved in this payoff are assumed to have the same time to maturity and the same underlying. The stock is not paying dividends ($\delta = 0$).

a. Indicate how this payoff profile may be obtained using only call options.

b. How many units of this portfolio need to be bought so that the surface determined by the payoff of the butterfly spread is always equal to 1?

c. Using iteratively the approximation of a derivative whereby, for some function $f(x)$ and some small ε,

$$\frac{\partial f}{\partial x} \sim \frac{f(x+\varepsilon) - f(x)}{\varepsilon}$$

obtain that in the limit of ε converging to 0, the Butterfly spread equals $\partial^2 C / \partial K^2$.

d. For the Black-Scholes call option, compute this expression to establish a link between the second derivative of call options and the risk neutral density denoted $p(S_t, T \mid S_t, t)$.

e. We assume that $S_0 = 100$, $T - t = 0.5$. Use the following call prices and hedge ratios

$$C(S_0, T-t, K=95) = 10.97,\ \frac{\partial C}{\partial S_t}(S_0, T-t, K=95) = 0.63$$
$$C(S_0, T-t, K=100) = 8.45,\ \frac{\partial C}{\partial S_t}(S_0, T-t, K=100) = 0.54$$
$$C(S_0, T-t, K=105) = 6.39,\ \frac{\partial C}{\partial S_t}(S_0, T-t, K=105) = 0.45$$

We also assume the interest rate equals zero.

(i) This table indicates that all things remaining equal, an increase in the strike price lowers the hedge ratio. What is the economic intuition for this?

(ii) What is the initial value of the butterfly spread?

(iii) For one unit of butterfly spread sold, indicate the amount of risky asset that needs to be traded (bought or short-sold). Indicate the amount of risk-free asset that needs to be borrowed or placed as to start a dynamic trading strategy.

2 Portfolio of Options, Greeks

A financial institution currently holds the following portfolio of over-the-counter options on the stock XYZ:

	Position	Δ	Γ	Vega
Call A	-1,000	0.50	2.20	1.80
Call B	-500	0.80	0.60	0.20
Put C	-2000	-0.40	1.30	0.70
Call D	-500	0.70	1.80	1.40

In addition it is assumed that a traded option on XYZ is available with a delta of 0.6, a gamma of 1.5, and a vega of 0.8.

a. What position in the traded option and the stock of XYZ would make the portfolio both gamma and delta neutral?

b. What position in the traded option and the stock of XYZ would make the portfolio both vega and delta neutral? What is the gamma of the global portfolio?

c. Now assume that another option is made available with a delta of 0.1, a gamma of 0.5, and a vega of 0.6. How can the portfolio be made delta, gamma, and vega neutral?

3 Exploring the Black-Scholes formula

In this exercise we consider European options written on an asset with constant volatility σ and constant dividend yield δ. The interest rate is r and the time to expiration of all options is given by τ.

a. Compute the unique level of the strike such that $C = P$.

b. Compute the Δ-symmetric strike, that is, the unique level of the strike such that $\Delta_C = -\Delta_P$.

c. An $x\Delta$ call is a call whose delta is equal to $x\%$. Show that for fixed τ and S the strice of the $x\Delta$ call is given by

$$K_C(x) = Se^{\left(r-\delta+\sigma^2/2\right)\tau-\sigma\sqrt{\tau}\Phi^{-1}\left(e^{\delta\tau}x/100\right)} \tag{1}$$

where Φ^{-1} is the inverse of the normal cdf. How does $K_C(x)$ vary with x? What is the formula for the strike of the $x\Delta$ put?

d. Show that

$$\text{Vega}_C(t, S) = v(t, \Delta_C(t, S)) \tag{2}$$

for some function v to be determined. Plot vega as a function of delta for the case where $\delta = 0$, $T = 1$, $K = 1$ and interpret your result.

e. Under what condition(s) is it possible to make a European call on a stock both gamma and vega neutral by adding a position in only one other option?

4 Pricing Options & Risk-Neutral Probabilities

The current value of a stock is $S_0 = 85$. The stock pays dividends at a continuously-compounded rate of $\delta = 1\%$ and the risk free interest rate is $r = 4\%$.

Consider a set of 3 European options: 1 call and 1 put both with strike price $K = 80$ and a 6-month maturity, and 1 call with strike price $K = 90$, a 1-year maturity and a Black-Scholes price $C_{0,K=90} = 5.69957$.

a. What is the implied volatility of the stock?

b. Compute the price of the 80-strike 6-months call using the Black-Scholes formula.

c. Compute the put price using the put-call parity relation.

d. Using the Black-Scholes model, compute the risk-neutral probability that the 80-strike 6-months call ens up in-the-money. What about the put?

e. Using the Black-Scholes model, compute the risk-neutral probability that the stock price ends up **below its mean** at maturity.

f. Suppose a contract offers to pay \$1 if the stock price ends up between 80 and 90 in 6 months. How much would you be willing to pay for this contract?

5 Hedging

The current value of a stock is $S_0 = 85$. The stock pays dividends at a continuously-compounded rate of $\delta = 1\%$ and the risk free interest rate is $r = 4\%$.

Consider a set of 3 European options: 1 call and 1 put both with strike price $K = 80$ and a 6-month maturity, and 1 call with strike price $K = 90$, a 1-year maturity and a Black-Scholes price $C_{0,K=90} = 5.69957$.

Suppose you are the writer of 50 calls with $K = 80$ and $T = 0.5$ and you wish to hedge your position. Assume that the stock price increases to 89 tomorrow and decreases to 81 in 2 days.

a. Compute the daily profits on your **naked position**.

b. Suppose you want to hedge your position by taking an offsetting position in shares. Explain how you delta-hedge your position.

c. You wish to correct your delta-hedged position to be gamma-neutral. To do so, suppose you use the 90-strike 1-year call. Compute the number of calls you need to buy for every 80-strike 6-months call you sell.

d. Assuming you do not dynamically re-adjust the number you computed under (c), compute the daily profits of your delta-hedged gamma-neutral strategy.