Baseline PLS Procedure in Light et al. (2017)

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1 Baseline Model

Consider a set of N stocks whose characteristics are observed in at least two periods. The best predictor of returns on stock i at time t is expected return $\mu_{it} = E\left[R_{it+1}|\mathcal{F}_t\right]$ where $\mathcal{F}_t$ denotes public information. Econometricians lack some information contained in $\mathcal{F}_t$ and do not know μ_{it}. Instead they observe firm characteristics $X_{it}^a, a = 1, \ldots, A$ such that $\{X_{it-s}, s \geq 0, a = 1, \ldots, A, i = 1, \ldots, N\} \subseteq \mathcal{F}_t$.

Assume at each time period X_{it}^a have zero cross-sectional means and unit variances (demean, standardize). Assume

$$X_{it}^a = \delta_t^a \left(\mu_{it} - \bar{\mu}_t\right) + u_{it}^a \tag{1}$$

where δ_t^a measures the sensitivity of characteristics a to expected returns, and $\bar{\mu}_t$ is the cross-sectional average of expected returns at time $t.u_{it}^a$ are components of characteristics that are unrelated to stock returns and that can be arbitrary correlated among themselves.

The realized return on stock i can be written as

$$R_{it+1} = \mu_{it} + \varepsilon_{it+1} \tag{2}$$

where $E\left[\varepsilon_{it+1}|\mathcal{F}_t\right] = 0$ and unexpected returns ε_{it+1} are assumed to be orthogonal to $\mathcal{F}_t$. ε_{it+1} and ε_{jt+1} can be correlated.

2 PLS procedure

The PLS-based estimation of baseline model at time t can be implemented in two steps:

- **Step 1**: Run separate cross-sectional regressions of $R_{it}, i = 1, \ldots, N$, on each individual firm characteristic $X_{it-1}^a, i = 1, \ldots, N$ for $a = 1, \ldots, A$ and denote the obtained slopes a λ_t^a.

- **Step 2**: For each firm i, $i = 1, \ldots, N$ run a regression of X_{it}^a on $\lambda_t^a, a = 1, \ldots, A$, and denote the obtained slopes as $\hat{\mu}_{it}$.

Denote the number of stocks to be N and the number of firm characteristics to be $A\,(= 26)$. Two sets of regressions are run at time t with information from time t and time $t - 1$.

In Step 1, for $a = 1, \ldots, A$,

$$\underbrace{\begin{bmatrix} R_{1t} \\ R_{2t} \\ \vdots \\ R_{Nt} \end{bmatrix}}_{N\times 1} = \text{constant} + \underbrace{\begin{bmatrix} X^a_{1,t-1} \\ X^a_{2,t-1} \\ \vdots \\ X^a_{N,t-1} \end{bmatrix}}_{N\times 1} \lambda^a_t + \text{errors} \tag{3}$$

where 26 regressions yield 26 slopes $\{\lambda^a_t\}^A_{a=1}$. Then in Step 2, for each stock i, $i = 1, \ldots, N$,

$$\underbrace{\begin{bmatrix} X^1_{1t} \\ X^2_{1t} \\ \vdots \\ X^a_{1t} \end{bmatrix}}_{a\times 1} = \text{constant} + \underbrace{\begin{bmatrix} \lambda^1_t \\ \lambda^2_t \\ \vdots \\ \lambda^a_t \end{bmatrix}}_{a\times 1} \hat{\mu}_{it} + \text{errors} \tag{4}$$

where N regressions yield N slopes $\{\hat{\mu}_{it}\}^N_{i=1}$. λ^a_t in Equation 3 can be viewed as a proxy for δ^a_{t-1} in Equation 1 and $\hat{\mu}_{it}$ in Equation 4 is a consistent estimator for demeaned stock returns $(\mu_{it} - \overline{\mu}_t)$ up to a scaling factor in Equation 1 by Proposition 1.

References

Light, N., Maskov, D., and Rytchkov, O. (2017). Aggregation of information about the cross section of stock returns: a latent variable approach. *The Review of Financial Studies, Volume 30, Issue 4*, pages 1339–1381.