Investments

Topic 8: Options

UCLA | Fall 2018

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Overview of Topic 8

1. Option Contracts
2. Option quotes and trading
3. Basic Trading Strategies
4. Static vs. Dynamic Replication
5. The Black-Scholes formula
6. Hedging and Market-Making
7. The Black-Scholes in the Real World
8. The Volatility Smile
9. Corporate Bond Valuation

1. Option Contracts

Options and their main types

- Options, as their name indicates, give the holder the right, but not the obligation, to do something in the future
  - A real option: the right to cut down a forest in some point in the future, or the right to invest in a fund or start-up
  - A stock option: the right, but not the obligation, to buy or sell a share of stock at a time in the future for a given price

- Types of options by payoff:
  - Call options: the right, but not the obligation, to buy a share of stock at a given price
  - Put Options: the right, but not the obligation, to sell a share of stock at a given price
  - Exotic options:
    - Knock-out (or in) options: a call or put option that ceases to exist if the underlying price crosses a certain barrier
    - Digital option: pays off only if underlying is in a certain range.
    - Options on options: compound or chooser options

- Types of options by exercise: European/American

Option contract specifications

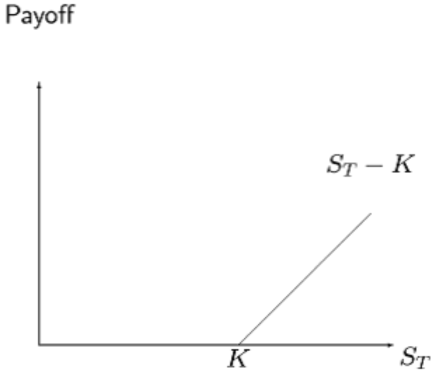
- Underlying asset
  - Usually equity, could be a bond, an interest rate or a currency
- Exercise price
  - Price at which the financial security can be bought or sold
- Expiration date: date on which the option expires
- Exercise style: when can you exercise your option?
- Payoff/settlement
  - What is received upon exercise? If an individual equity options is exercised, shares of stock are actually exchanged.
  - Other contracts would settle in cash
    - Index (Nasdaq 100 and S&P 500) options
    - Interest rate options
    - Commodity options

Payoffs from a Call Option

- Consider a call with a strike price of $K = 100$
- Let S_T denote the price of the underlying at maturity
- Gross payoffs to long and short positions at maturity:

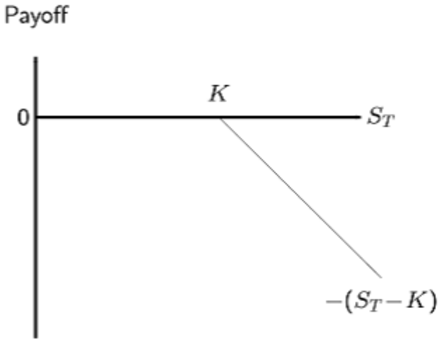
	S_T	Long Call Payoffs	Short Call Payoffs
	70	0	0
Out-of-the-money	80	0	0
	90	0	0
At-the-money	100	0	0
	110	10	-10
In-the-money	120	20	-20
	130	30	-30

Long Call with Strike K



$$\text{Payoff} = \begin{cases} 0, & \text{if } S_T < K \\ S_T - K, & \text{if } S_T \geq K \end{cases}$$

Short Call with Strike K



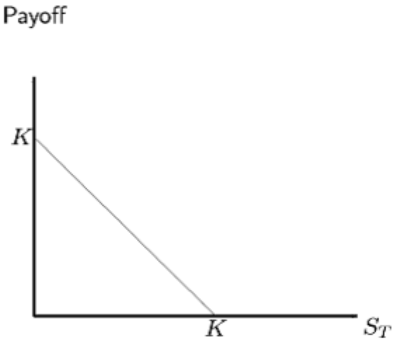
$$\text{Payoff} = \begin{cases} 0, & \text{if } S_T < K \\ -(S_T - K), & \text{if } S_T \geq K \end{cases}$$

Payoffs from a Put Option

- Consider a call with a strike price of $K = 100$
- Let S_T denote the price of the underlying at maturity
- Gross payoffs to long and short positions at maturity:

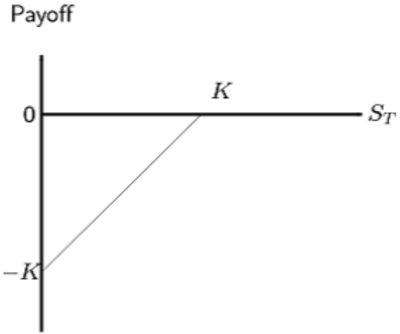
	S_T	Long Put Payoffs	Short Put Payoffs
	70	30	-30
In-the-money	80	20	-20
	90	10	-10
At-the-money	100	0	0
	110	0	0
Out–of-the-money	120	0	0
	130	0	0

Long Put with Strike K



$$\text{Payoff} = \begin{cases} K - S_T, & \text{if } S_T \leq K \\ 0, & \text{if } S_T > K \end{cases}$$

Short Put with Strike K



$$\text{Payoff} = \begin{cases} S_T - K, & \text{if } S_T < K \\ 0, & \text{if } S_T \geq K \end{cases}$$

Determinants of an option value

- The underlying value, S_t
- The strike price K
- An option's moneyness, K/S_t, captures the relation
  - "a 6% OTM put" means a put with $K/S_t = 0.94$
- If you are negative on the underlying, would you
  - Sell a call
  - Buy a put

Option Value and Volatility

Example: Consider a call option with $K = 100$.
Compare two distributions for S_T:

$$S_T = \begin{cases} 110, & \text{with prob } 1/2 \\ 90, & \text{with prob } 1/2 \end{cases}$$
$$S_T = \begin{cases} 120, & \text{with prob } 1/2 \\ 80, & \text{with prob } 1/2 \end{cases}$$

- Second distribution is more volatile
- It also yields superior payoffs to the option holder because of the asymmetry in call payoffs
- Without the asymmetry, the increase in volatility may not be attractive

Options and Views on Volatility

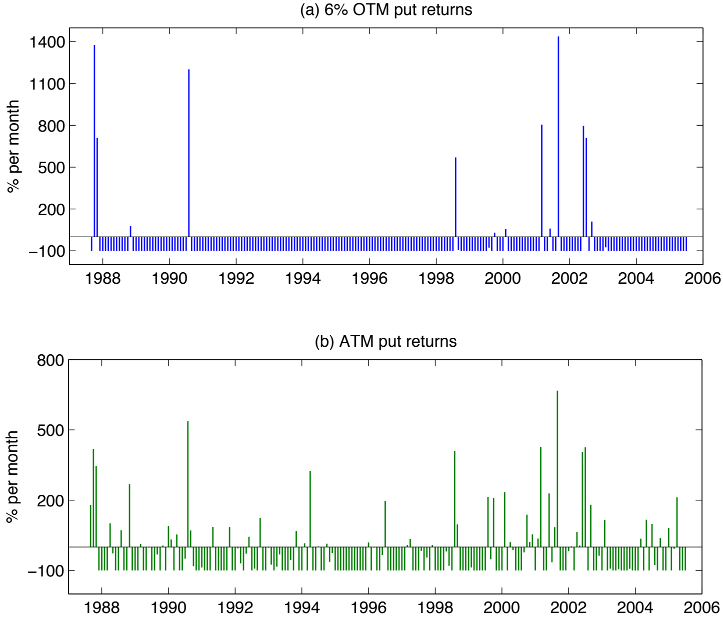
- Every option position (long/short put/call) embodies a view on direction
- From the options – volatility relationship, it can also be seen that every option also embodies a view on <u>volatility</u>
- Long option position (calls or puts): <u>bullish</u> view on volatility
  - Such a position increases in value when volatility increases, and decreases in value when volatility decreases.
- Short option position (calls or puts): <u>bearish</u> on volatility
  - Such a position increases in value when volatility decreases, and vice versa.

Leverage

- Out-of-the-money options are cheap and have a lot of embedded leverage
- Consider a stock of Chernov Educational Services (CES) trading at \$666 today
- The cost of a call struck at \$680 with four months to expiration is \$39
- Should I buy the stock or a call?
  - Buy a stock. Suppose the price goes to \$730 in 4 months. My return is 9.6%
  - Buy a call. The return is:

$$\frac{730 - 680 - 39}{39} = 28\%$$

SPX 1-month option returns



2. Option quotes and trading

Option Listing

- Exchange-traded options:
  - Stocks (American)
  - Futures (American)
  - Indices (European and American)
  - Currencies (European and American)
- OTC options
  - Vanilla (standard calls/puts as defined above)
  - Exotic (everything else - e.g., Asians, barriers)
- Others (e.g., embedded options)

Exchanges (Stocks/Indexes)

- 100 shares of the underlying equity/\$100 times the index
- Premiums stated in points. One point equals \$100
- Strike intervals: \$2.50 increments for strikes below \$25, \$5 increments between \$25 and \$200, \$10 above \$200
- American/European or American
- Delivery is $T + 2$ business days following exercise/Cash
- Expiration months: two near-term months plus two additional months in the January, February or March quarterly cycle
- Expiration is on the third Friday of the expiration month

Types of Options

- A dizzying array of exchange-traded contracts even if you limit to SPX as underlying
- LEAPS (**L**ong Term **E**quity **A**ntici**P**ation **S**ecurity), maturity > 12 months, American
- Binary options, European
- FLEX (**FL**exible **EX**change) Options offer a choice of
  - The underlying Index (e.g, S&P 100, S&P 500, Nasdaq 100, Russell 2000, or DJIA 30)
  - Call or Put.
  - Expiration date – up to 15 years from creation.
  - Strike price – may be specified as an index level, as a percentage, or any other readily understood method for deriving an index level
  - American or European
  - Settlement Value – settlement may be based on either the opening settlement value or closing settlement value
- Jumbo/Mini options (1000/10 shares per contract)
- Weekly options (one week or less to expiry)

Options clearing

- Options Clearing Corporation (OCC) acts as guarantor between clearing parties ensuring that the obligations of the contracts are fulfilled

- Central counterpatrty clearing is a process mitigating counterparty risk
  - novating trades between counterparties, e.g., a trade between member firm A and firm B becomes two trades: A-OCC and OCC-B
  - netting offsetting transactions between multiple counterparties
  - requiring collateral (margin)
  - providing a guarantee fund that can be used to cover losses that exceed a defaulting member's collateral

Closing positions

- You may exercise an option
- If you cannot or do not want to exercise, take an offsetting position
  - Standardized times of maturity help
- When would you want to offset instead of exercising?

Volume and open interest

Quotes

- Volume
  - number of option contracts being exchanged between buyers and sellers
- Open interest
  - number of option contracts that are still open and held by traders and investors
  - to close a position one must take an offsetting position (buy or sell "to close") or exercise
  - a measure of liquidity
- Most trading is OTM/ATM

The option market size

- The Chicago Board Options Exchange (CBOE) is the largest market for stock options.
- CBOE's market share is 31.6% for options on equities and exchange-traded products and 99.2% for index options.

CBOE Annual Contract Volume, mil

	2017	2016	2015	2014
Option Contracts				
Equities & ETPs	1,165	691	714	868
Indexes	496	433	408	406
Total Options Volume	1,662	1,124	1,122	1,275
Futures	74	60	52	51
Total Contract Volume	1,736	1,185	1,174	1,325

3. Basic Trading Strategies

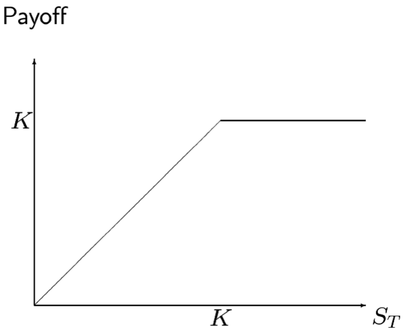
Basic Trading Strategies

- A "trading strategy" refers to a portfolio that consists of options on a given asset and possibly the asset itself

1. Covered calls & Protective puts
2. Spreads
   - Bull spreads
   - Bear spreads
3. Combinations
   - Straddles
   - Strangles

Covered Call with Strike K

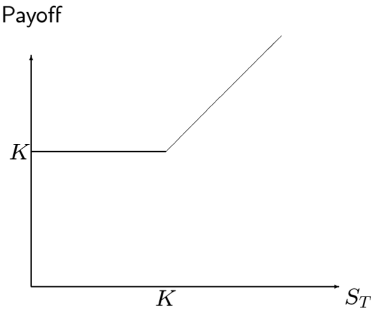
- Portfolio: Long stock, short call with strike K



$$\text{Payoff} = \begin{cases} S_T, & \text{if } S_T < K \\ K, & \text{if } S_T \geq K \end{cases}$$

Protective Put with Strike K

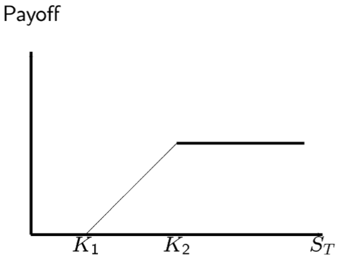
- Portfolio: Long stock, long put with strike K



$$\text{Payoff} = \begin{cases} K, & \text{if } S_T < K \\ S_T, & \text{if } S_T \geq K \end{cases}$$

Bullish Spread:

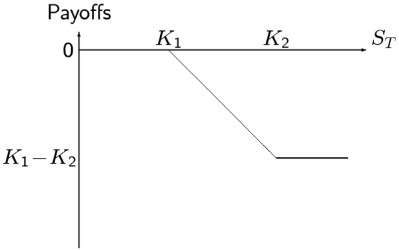
- Portfolio: Long call with strike K_1, short call with strike K_2, $K_1 < K_2$



$$\text{Payoff} = \begin{cases} 0, & \text{if } S_T \leq K_1 \\ S_T - K_1, & \text{if } K_1 < S_T \leq K_2 \\ K_2 - K_1, & \text{if } S_T > K_2 \end{cases}$$

Bearish Spread:

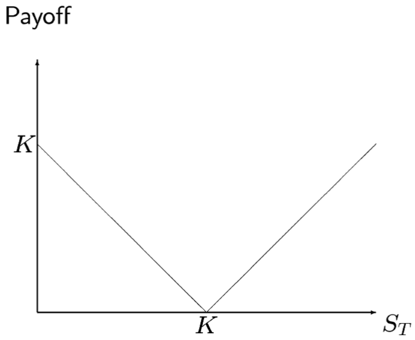
- Short call with strike K_1, long call with strike K_2



$$\text{Payoff} = \begin{cases} 0, & \text{if } S_T \leq K_1 \\ -(S_T - K_1), & \text{if } K_1 < S_T \leq K_2 \\ -(K_2 - K_1), & \text{if } S_T > K_2 \end{cases}$$

Straddle

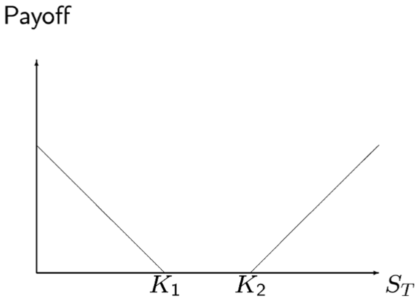
- Portfolio: Long call with strike K, long put with strike K



$$\text{Payoff} = \begin{cases} K - S_T, & \text{if } S_T < K \\ S_T - K, & \text{if } S_T \geq K \end{cases}$$

Strangle

- Portfolio: Long put with strike K_1, long call with strike K_2



$$\text{Payoff} = \begin{cases} K_1 - S_T, & \text{if } S_T \leq K_1 \\ 0, & \text{if } K_1 < S_T \leq K_2 \\ S_T - K_2, & \text{if } S_T > K_2 \end{cases}$$

4. Static vs. Dynamic Replication

Static vs. Dynamic Replication

- Why are we interested in understanding how to replicate derivatives?
  - That is how we find their value!
- For forwards and swaps, we found static replicating trading strategies
  - Benefit: "Model free", except assuming no trading frictions for large investors
- Do static replicating strategies exist for options?
  - Put-Call parity for European options
  - Otherwise, none
  - But, static considerations can give us no-arbitrage bounds
- For pricing of options, we rely on dynamic replicating trading strategies
  - The Binomial Model
  - Black-Scholes

Static No-Arbitrage Bounds for Option Prices

- First, note that a stock "payoff" is S_T (ex-dividend), then ...
- ... a call
  - payoff: $\max(S_T - K, 0)$
  - $C_t \geq \max(S_t - Ke^{-r(T-t)}, 0)$
  - $C_t \leq S_t$
- ... a put
  - payoff: $\max(K - S_T, 0)$
  - $P_t \geq \max(Ke^{-r(T-t)} - S_t, 0)$
  - $P_t \leq Ke^{-r(T-t)}$

European Put-Call Parity

- Note the similarity between the protective put and long call payoffs
- The difference in the payoffs is equal to K
- Let's buy a call (struck at K) and an asset which pays \$$K$ at maturity for sure, i.e., a ZCB with the face value of \$$K$
- By no-arbitrage, price (protective put) = price (call + bond):

$$\begin{aligned} P_t(S_t, K, T, r) + S_t &= C_t(S_t, K, T, r) + PV_t(K, T, r) \\ &= C_t(S_t, K, T, r) + Ke^{-r(T-t)} \end{aligned}$$

Using Put-Call Parity for Arbitrage

- If the parity is violated, then there is an arbitrage opportunity
- Example: $S_0 = \$100$, $K = \$100$, $T = 1$ year, $r = 7.69\%$, $C_0 = \$18$, and $P_0 = \$10$
- The parity implies

$$C_0 = P_0 + S_0 - PV_0(K, T, r) = 10 + 100 - 100e^{-0.0769 \cdot 1} = 17.40$$

- Sell the (relatively expensive) call at \$18, and replicate it for \$17.40. Make \$0.60

Action	Cashflow at $t = 0$	Cashflow at $t = T$ if $S_T \leq 100$	Cashflow at $t = T$ if $S_T > 100$
Sell a call	18	0	$-(S_T - 100)$
Buy stock	-100	S_T	S_T
Borrow PV of \$100	92.60	-100	-100
Buy a put	-10	$100 - S_T$	0
Net	0.60	0	0

5. The Black-Scholes formula

Main Assumptions of the Model

- Main assumption: Underlying asset price follows a *Geometric Brownian Motion.*

- In words, this effectively means two conditions must be satisfied:

    1. Stock prices are lognormally distributed (log returns are normally distributed).
    2. Asset prices are continuous and cannot "jump."

- In addition, the model's key assumptions are:

    1. Asset prices and trading strategies can change continuously, i.e., at each instant of time. (This requires techniques from stochastic calculus.)
    2. Volatility of asset returns is constant.
    3. Interest rates are constant.

Are These Assumptions Reasonable?

- The assumption of continuous trading and continuous changes in asset prices is clearly unreasonable but is in fact a *close approximation* to reality for many markets.
- The assumption of constant interest rates and constant volatility of returns is unrealistic and violated in practice.
- Also importantly, the fact that prices cannot "gap" is objectionable.
  - Price formation and news arrival process are inherently discrete (not continuous).
  - Prices often experience "jumps."
  - Prices often jump simply due to dividend payments, etc.
- To be revisited

Are These Assumptions Reasonable? (Cont'd)

- It turns out that interest rates and volatility can be made functions of time and current asset price. While the Black–Scholes formula will no longer be a valid valuation tool, the fundamental approach will be the same.

- Predictable dividend payments can be tackled easily.

- However, allowing volatility to have its own randomness ("stochastic volatility") and introducing jumps in asset prices leads to non–trivial complications.
  - Evidence suggests that both are required to generate the violations of normality observed in (log) returns: "fat tails" or "leptokurtosis" of asset returns.
  - Either in itself is unsatisfactory to explain the richness of option pricing dynamics.
  - Is it better to use a simpler model and be aware of its pitfalls than more complex ones which are also unsatisfactory? Many people think so.

To What End?

- Option prices in the Black–Scholes model can be expressed in *closed-form*, i.e., as explicit, readily calculated functions of underlying parameters.
- This makes computing option prices very easy.
- More importantly, it makes computing and understanding option price *sensitivities* to underlying parameters very easy.
- These sensitivities are very difficult to compute in the binomial model.
- Note, however, that binomial model is robust for calculation of variety of options including American options.

The Black-Scholes Formula

- The argument behind Black–Scholes is one of replication and no–arbitrage.

- To replicate a call, we must
  1. Take a long position in $\Delta > 0$ units of the underlying, and
  2. Borrow $-B > 0$ units at the risk–free rate.

- Then, the price of the call can be expressed as:

$$C_0 = \Delta \cdot S_0 + B$$

- Alternatively, the price can be expressed via risk-neutral expectation:

$$C_0 = e^{-rT} E_0^* C_T$$

The Black–Scholes Formula (Cont'd)

- The quantities Δ and B depend on many factors including:
  1. Depth–in–the–money of the option.
  2. Volatility.
  3. Time–to–maturity.
  4. Interest rate.

- The unique feature of the Black–Scholes formula is that it provides precise expressions for Δ and B in terms of the above parameters in the continuous limit case.

The Black–Scholes Formula (Cont'd)

- No–arbitrage price of a European call option in the Black–Scholes model:

$$
\begin{aligned}
C_0 &= S_0 \cdot N(d_1) - PV_0(K) \cdot N(d_1 - \sigma\sqrt{T}) \\
d_1 &= \frac{1}{\sigma\sqrt{T}}\left[\ln(S_0/K) + (r + \sigma^2/2)T\right]
\end{aligned}
$$

- S_0 : current asset price.
- K : strike price of the option.
- T : calendar time of option's maturity.
- 0 : current calendar time.
- σ : volatility, i.e., standard deviation of annualized asset returns.
- r : risk–free interest rate.
- $N(\cdot)$: cumulative standard normal distribution:

$$
N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx
$$

The Black–Scholes Formula (Cont'd)

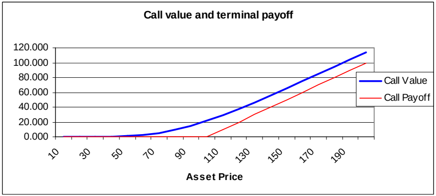
- Comparing the formula to the general expression, we see that
  1. $\Delta = N(d_1)$
  2. $-B = PV_0(K) \cdot N\left(d_1 - \sigma\sqrt{T}\right) \equiv PV_0(K) \cdot N(d_2)$.
- Thus, the Black–Scholes formula provides us with a precise composition of the replicating portfolio for the European call option.
- We also know that the call can be evaluated by the risk-neutral approach:

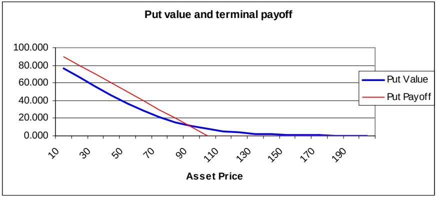
$$\begin{aligned} C_0 &= e^{-rT} E_0^* C_T = e^{-rT} E_0^* \max(S_T - K, 0) \\ &= e^{-rT} E_0^* (S_T - K | S_T > K) \\ &= e^{-rT} E_0^* (S_T | S_T > K) - PV_0(K) Q\{S_T > K\} \end{aligned}$$

- Thus, $N(d_2) = Q\{S_T > K\}$, and $N(d_1) = e^{-rT} E_0^* (R_T | R_T > K/S_0)$
- The put can be valued by put-call parity

The Black–Scholes Formula: Examples

- Let us first do an example of this calculation.If $K = 100, \sigma = 0.3, r = 0.1, T = 1.5$:
- Next, let us see a graph of this price function. Observe the non–linearity of the option price as a function of underlying spot price and the effect of moneyness of the option.

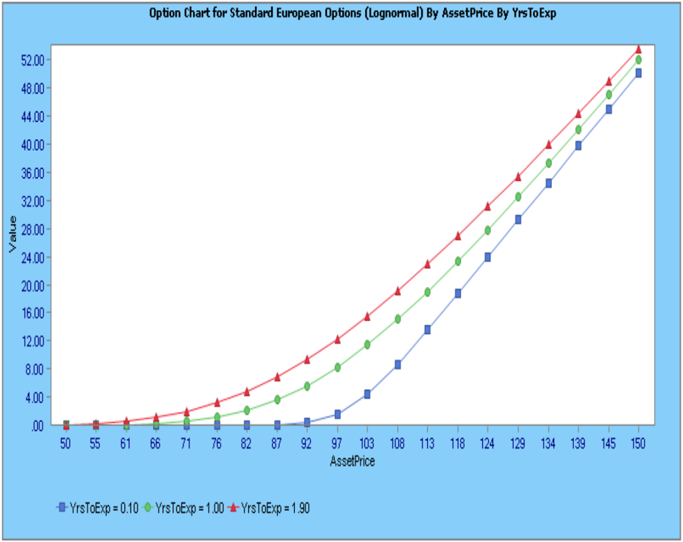




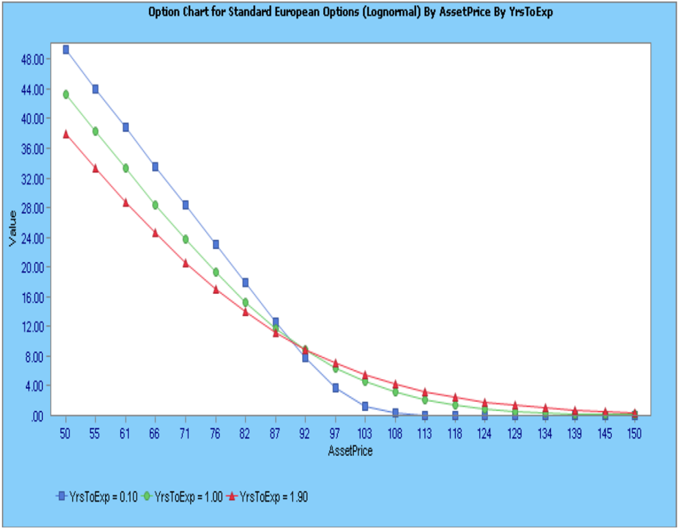
The Black–Scholes Formula (Cont'd)

- There are several remarkable features of these expressions:
  1. Option prices depend only on five variables: S, K, r, T, σ.
  2. Of these, only volatility σ is not directly observable.
  3. This makes the model relatively easy to implement in practice.
  4. They have been derived from principles of no–arbitrage and replication. If the model assumptions are correct, then deviation from these prices leads to arbitrage opportunity using the replicating portfolios that mimic perfectly the option payoffs at all points of time.
  5. They tell us the hedging strategy for option exposures: Delta hedge, i.e., if long the option, then sell Δ of the underlying.

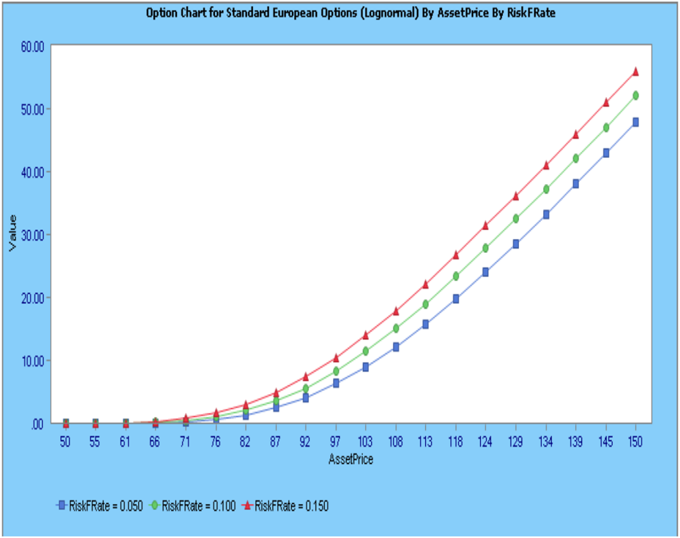
Call Value vs. T , $K = 100$



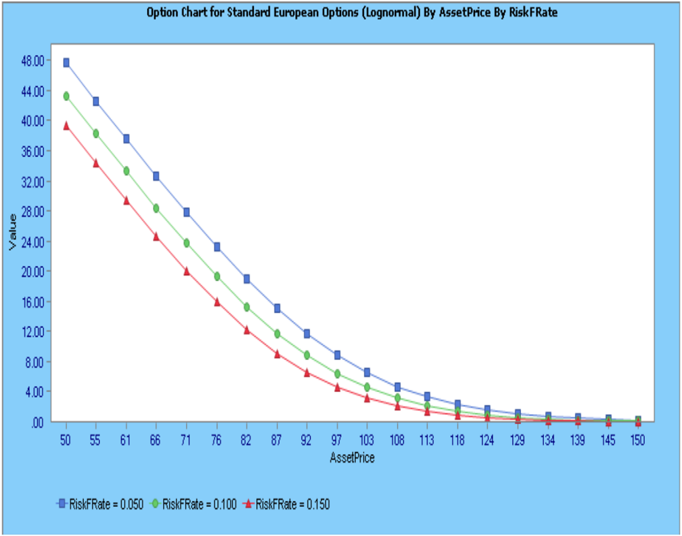
Put Value vs. T, K = 100



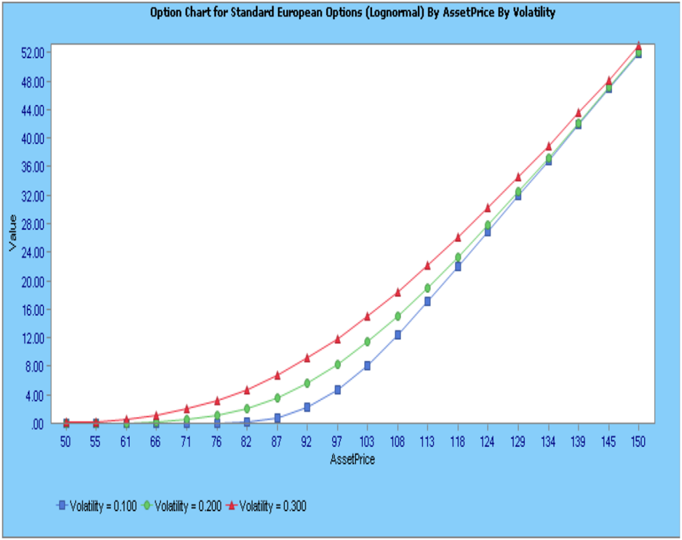
Call Value vs. r, K = 100



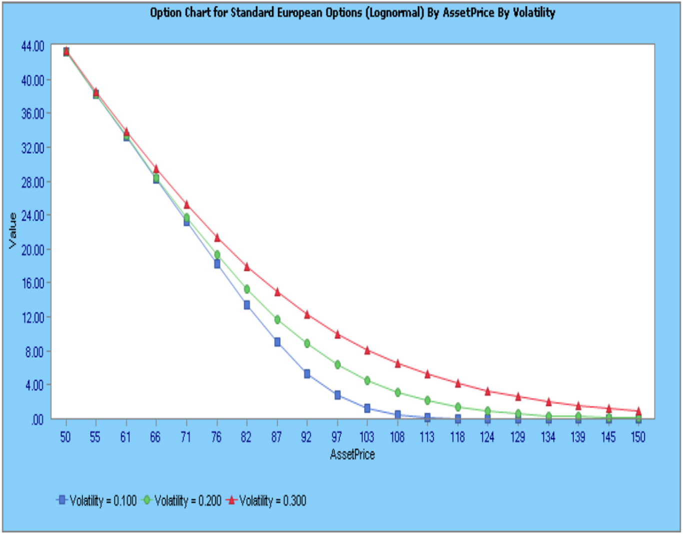
Put Value vs. r, K = 100



Call Value vs. Vol, K = 100



Put Value vs. Vol, K = 100



6. Hedging and Market-Making

The Need for Delta-Hedging

- Market-makers in options markets have a natural need for hedging option exposure
  - Provide immediacy: ready to sell or buy options at quoted prices.
  - Make their profits from the bid-ask spread.
  - Let's say a large number of buy orders on calls come through:
    - Market-maker is then short call options
    - Increase in price of underlying could bankrupt market maker
    - Market-maker delta-hedges by buying required units of underlying so as to not be exposed to movements in underlying
- Conceptually different from Proprietary trading
  - Often bet on view or take advantage of arbitrage opportunities
  - The market-maker's positions, on the other hand, are the result of customer order flow

The Need for Delta-Hedging (Cont'd)

- The Street is on average short options, so hedging in the underlying is in aggregate from the perspective of a short option position
    - i.e., buying underlying if a call and selling underlying if a put.
- Since investors are net buyers of insurance, hedging with other options is generally not feasible <u>in the aggregate</u>
- Another reason why market makers do not use other options to hedge is high transaction costs
    - Usually, the bid-ask spreads on the underlying are so much smaller that even with frequent trading it is desirable to delta-hedge
- This aggregate short volatility has been a reason for the success of volatility markets
    - E.g., variance swaps and, more recently, VIX futures and options

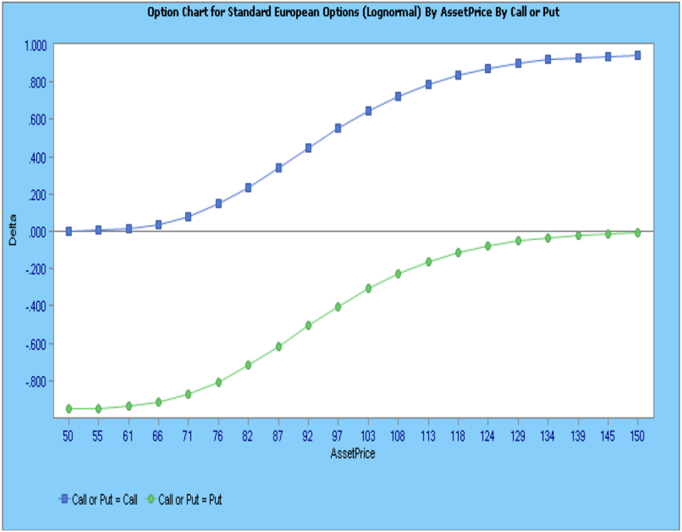
Our Perspective

- We will consider the case: Buy a European call
- We will assume you delta hedge the call, but only revise your hedge at discrete intervals (here we assume that's weekly))
- We will think about the Profit and Loss between each time-interval from the hedging strategy:
  - Buy the call (C_t), and delta hedge with the stock (short Δ_t units)zero.
  - The change in the value of your position between days is then:

$$P\&L_{t,t+\Delta t} = -\Delta_t \left(S_{t+\Delta t} - S_t\right) + \left(C_{t+\Delta t} - C_t\right)$$

  - Is this a perfect hedge?

Black-Scholes Put and Call Delta's



P&L of Delta Hedge if No Time Passes

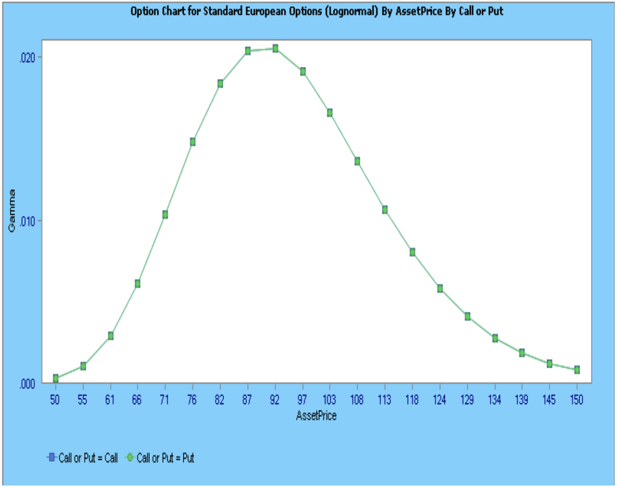
- Let's, for the sake of argument, assume the stock price changes but no time passes.
  - Thus, no interest loss and no time-value loss on call.
- The net position makes money unless the stock price stays constant
- Why? By Taylor formula,

$$C_{t+\Delta t} - C_t = \Delta_t \left(S_{t+\Delta t} - S_t\right) + \frac{1}{2}\Gamma_t \left(S_{t+\Delta t} - S_t\right)^2,$$

where $\Gamma = \frac{\partial^2 C}{\partial S^2}$ captures convexity of the option price

- The position is long convexity
  - Higher Γ means more convexity and larger average loss

Black-Scholes Call and Put Gammas



- Call's $\Gamma = \frac{1}{\sigma S_0 \sqrt{T}} N'(d_1)$
- They are the same! Not surprising since $\Delta_c = 1 + \Delta_p$

P&L of Delta Hedge: Effect of Theta

- Why would anyone consider delta hedging as a viable hedging strategy?
  - We assumed no time passes.
  - In reality, call must lose value in time...
- Theta, $\frac{\partial C}{\partial t} = \Theta$, measures the change in the value of the option when a unit of time passes, all else equal
- For calls, $\Theta < 0$, so the call will decrease in value.
  - From P&L: Assume the stock price stays constant

$$P\&L_{t,t+\Delta t} = \Theta_t \Delta t.$$

  - The net position loses due to the time-decay on the option
- The BS theta is for call/put

$$\Theta_0 = -\frac{\sigma S_0 N'(d_1)}{2\sqrt{T}} \mp rKe^{-rT} N(\pm d_2) \approx -\frac{1}{2}\Gamma_0 \sigma^2 S_0^2$$

(exact at $r = 0$)

Gamma versus Theta

- The net effect of time-decay and changes in the price of the underlying:

$$\begin{aligned} P\&L_{t,t+\Delta t} &= -\Delta_t \left(S_{t+\Delta t} - S_t\right) + \left(C_{t+\Delta t} - C_t\right) \\ &= \Theta_t \Delta t + \frac{1}{2}\Gamma_t \left(S_{t+\Delta t} - S_t\right)^2 \end{aligned}$$

- So, when does the $\Gamma-$effect cancel with the $\Theta-$effect?

$$\begin{aligned} P\&L_{t,t+\Delta t} &\approx -\frac{1}{2}\Gamma_t \sigma^2 S_t^2 \Delta t + \frac{1}{2}\Gamma_t \left(S_{t+\Delta t} - S_t\right)^2 \\ &= \frac{1}{2}\Gamma_t S_t^2 \left[\left(\frac{S_{t+\Delta t} - S_t}{S_t}\right)^2 - \sigma^2 \Delta t\right] \end{aligned}$$

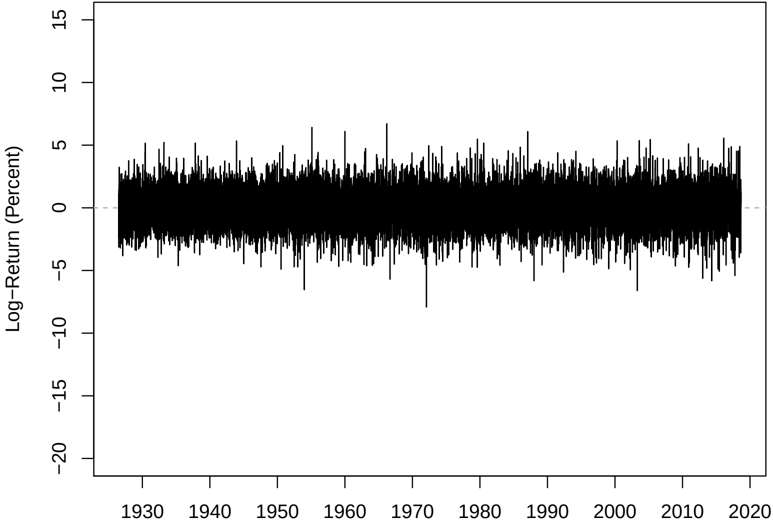
- Thus, the $\Gamma-$effect and the $\Theta-$effect cancel when the stock prices move by about 1 standard deviation.
- The hedged P&L is driven by the spread between realized and implied variance $\Rightarrow$ *volatility trading*

7. The Black-Scholes in the Real World

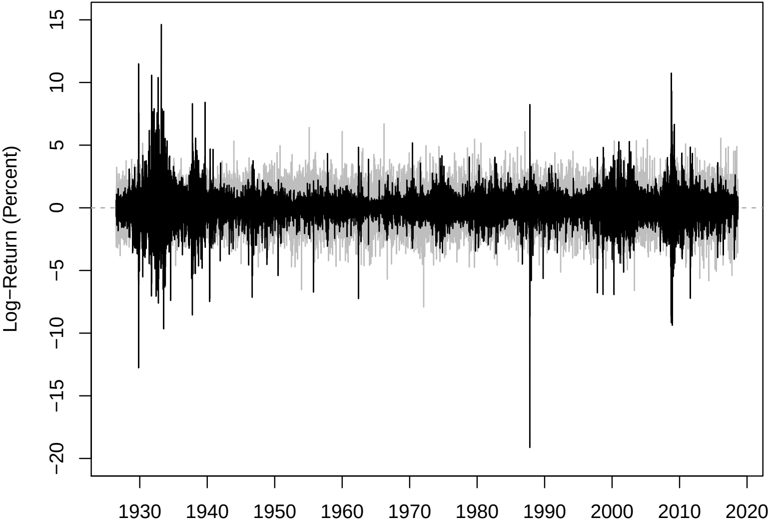
The Black-Scholes in the Real World

- If stock prices follow the Random Walk model, then we can accurately estimate μ and σ using a very long dataset
- Take the series of CRSP value-weighted market return beginning in 1927 and estimate the parameters via sample mean and volatility
- Simulate the returns given these parameters
- Compare with the actual returns
- Consider rolling estimates of volatility

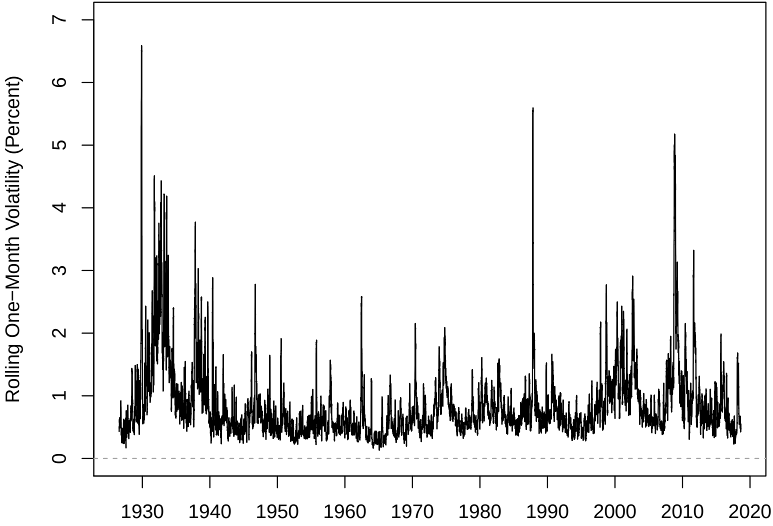
Simulated Log-Returns



Actual CRSP Market Log-Returns



Realized CRSP Market Volatility



Stock Return Distribution

- The easiest assumption to relax is the evolution of returns
- Add time-varying volatility and jumps:

$$d \log S_t = \mu_t dt + \sigma_t dW_t + dZ_t$$

- Details on jumps, Z_t :
  - $Z_t = N_t \cdot z$
  - N_t counts the number of events (jumps)
  - z is the jump magnitude
- How does this affect the usage of the Black-Scholes model?

Delta Hedging in Practice

- How do traders use the Black-Scholes formula?
- When writing a call, a trader wants to reduce the exposure to changes in the stock price
  - It is enough to make money on the huge bid-ask spread
- Use delta-hedging
- Let's see how well the delta-hedging strategy works

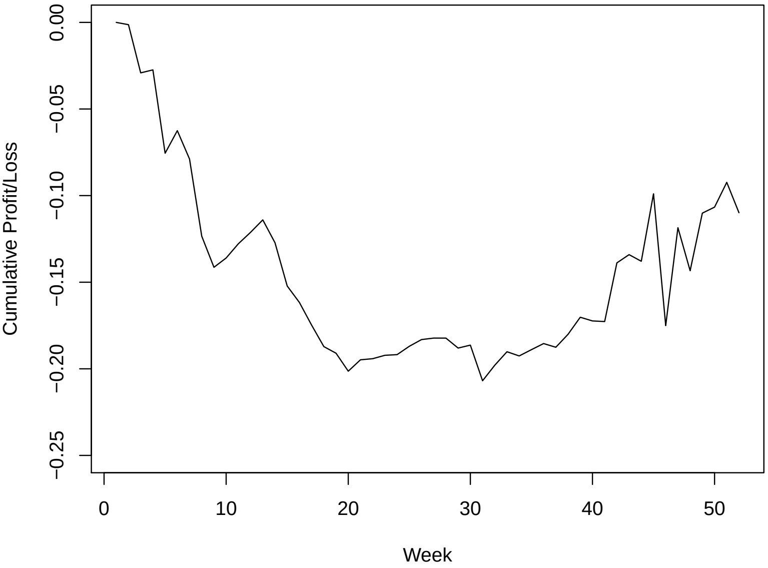
Dynamic Hedging

- Assume the trader sold a European call with 1 year to maturity and the strike $K = \$65$ with the current stock price of $50
- Because rebalancing can be potentially costly, she rebalances her delta-hedged portfolio once a week
- What is her portfolio?

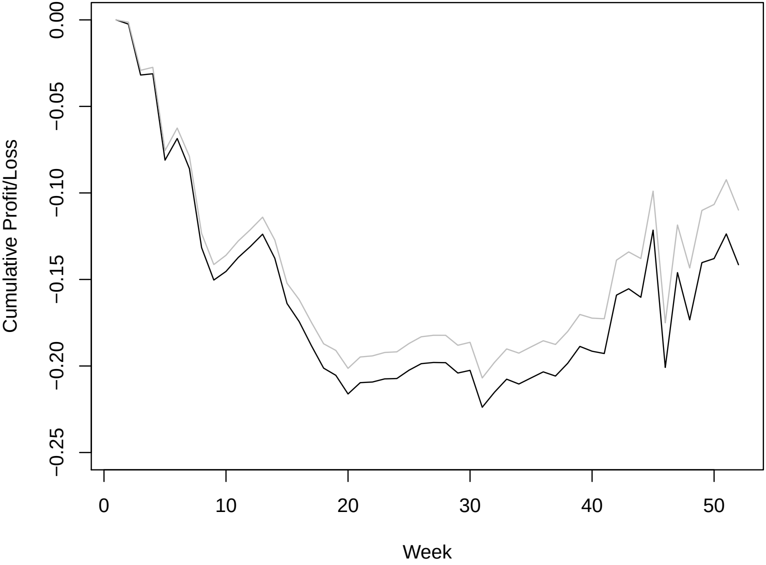
$$H_t = \Delta_t \cdot S_t - C_t + B_t$$

- Every week, she recomputes Δ and B according to the Black-Scholes formula
- Consider three scenarios:
  1. Stock price evolves according to the Black- Scholes Model
  2. In addition to case 1, allow for transaction costs, $0.03 per share
  3. Market crashes at a rate of 2 per year, $z_j \sim N(-0.03, 0.05^2)$

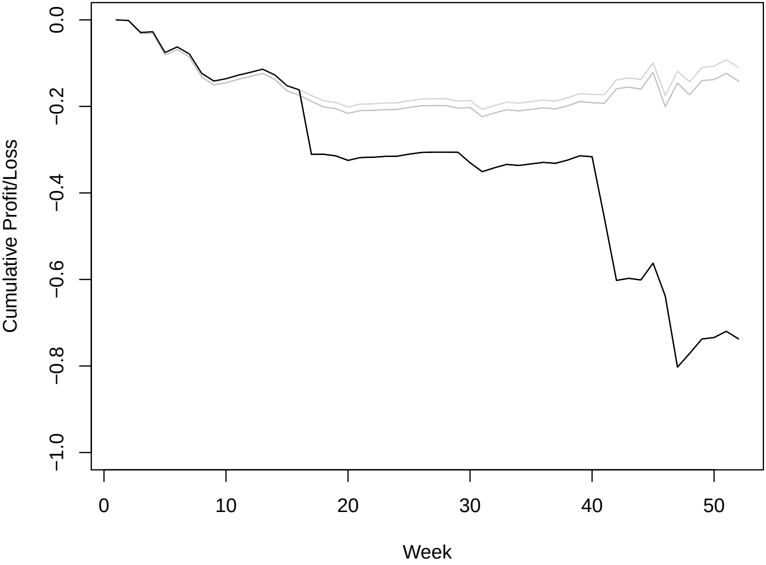
P&L 1: Black-Scholes



P&L 2: Black-Scholes with T-costs



P&L 3: Black-Scholes + Crashes



8. The Volatility Smile

The Volatility Smile

- The Black-Scholes model assumes constant volatility of returns

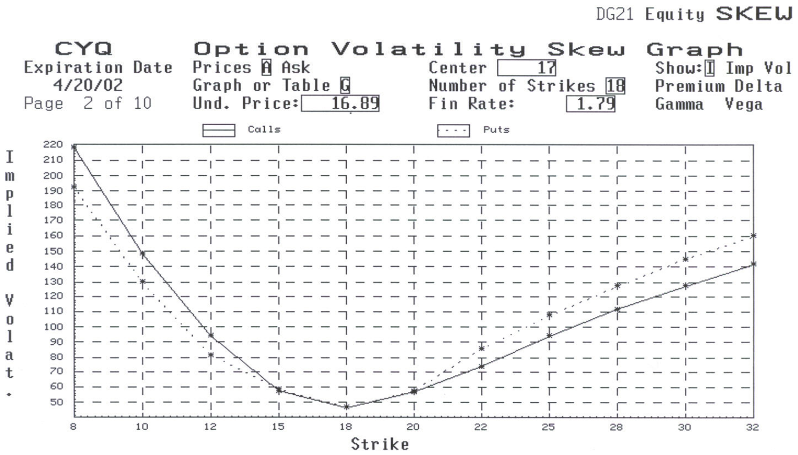
- What happens if you value options using historical volatility?
  - There is going to be a pricing error
  - Hard to characterize systematically because the magnitude of error depends on the stock price
  - Could be high or low even for the same stock

- Consider the following thought experiment:
  - Consider the observed market price of an option, O^M
  - Which value of volatility produces correct BS price?
  - Holding the option price fixed and solving for σ gives an "implied volatility".

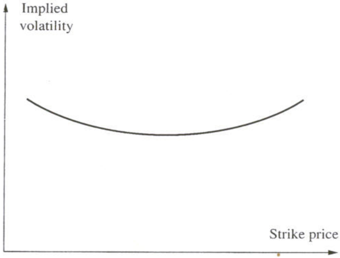
$$O_t^M = BS(S_t, K, T, r, \sigma_t^{IV})$$

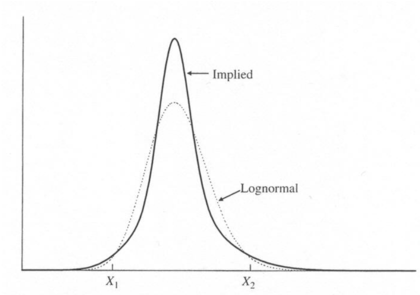
  - The relationship is one-to-one

Implied Volatility Smile

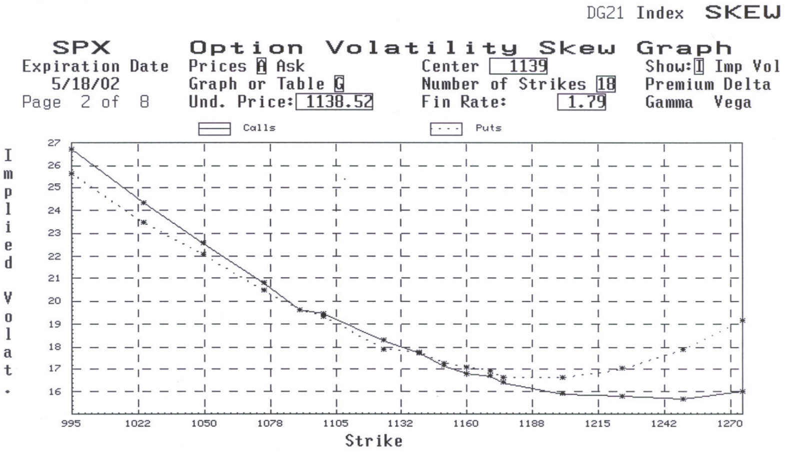


Smile Implied Distribution

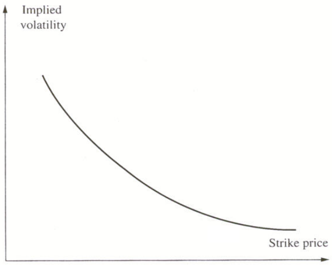


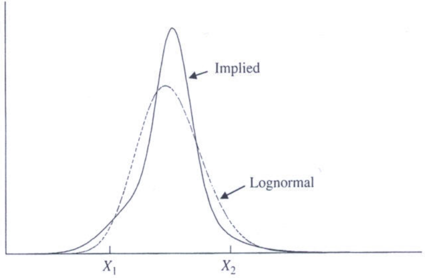


Implied Volatility Smirk



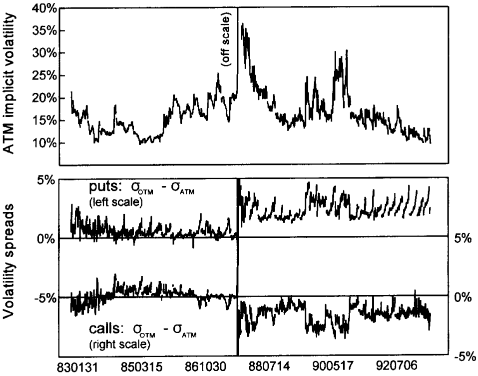
Smirk Implied Distribution





Reasons for Smiles and Smirks

- Leverage effect:
  - The Stock Price goes down
  - Leverage goes up
  - Volatility goes up
- Crashophobia
- Traders incorporate the possibility of Oct.87-like crash



9. Corporate Bond Valuation

Corporate Bond Valuation

- Merton proposed in 1974 to view shares and bonds as options on the firm's assets value
- Such a view allows for a unified approach to the firm?s capital structure
  - It is known as structural approach, or
  - contingent claims approach (CCA)
- We will discuss the simplest structural model:
  - A firm has one zero bond
  - A firm issued one share (claim) on its equity

A firm's balance sheet

- $V =$ the value of firm's assets is shared by debtholders and shareholders:
    - $S =$ shareholder's value, or equity value
    - $P =$ bondholder's value, or debt with face value F
- S can be viewed as a price of a call written on V with strike F :
    - Downside is limited to zero because of limited liability
    - Payoffs start after the debt of value F is paid out to bondholders, i.e., positive payoffs when $V > F$
- P resembles a price of a put on V struck at F. Why?

Valuation

- Suppose the bond matures at time T
- Debt payoff at maturity is

  $$P(V_T) = \min(V_T, F) = F + \min(V_T - F, 0) = F - \max(F - V_T, 0)$$

- Therefore, the value of debt at $t < T$ is

  $$P(V_t) = PV(F) - P(V_t, F, T)$$

- Equity payoff at maturity is

  $$S(V_T) = \max(V_T - F, 0)$$

- Therefore, the value of equity at $t < T$ is

  $$S(V_t) = C(V_t, F, T)$$

- Alternatively, we split the value of firms assets, V, between bondholders and shareholders

  $$S(V_t) = V_t - P(V_t) = V_t - PV(F) + P(V_t, F, T)$$

- Are these two answers the same?

Black-Scholes Value of Debt and Credit Spread

- Debt's value is $P(V_t) = PV(F) - P(V_t, F, T)$
- Black-Scholes formula gives us:

$$P(V_t) \;=\; Fe^{-r(T-t)} - [Fe^{-r(T-t)}N(-d_2) - V_t N(-d_1)]$$

- Re-write:

$$P(V_t) = Fe^{-r(T-t)}\left(1 - N(-d_2)\cdot\left[1 - \frac{V_t}{Fe^{-r(T-t)}}\frac{N(-d_1)}{N(-d_2)}\right]\right)$$

- Interpretation:
  - $N(-d_2)$ is default probability, *DP*
  - $[\ldots]$ is expected discounted loss given default per \$1, *LGD*
- Yield to maturity solves $P(V_t) = Fe^{-y(T-t)}$, that is,

$$y = -\frac{1}{T-t}\ln\frac{P(V_t)}{F}$$

- The credit spread $s = y - r$ reflects the key objects:

$$s = -\frac{1}{T-t}\ln(1 - DP\cdot LGD) \approx \frac{DP\cdot LGD}{T-t}$$

Ratings and Spreads

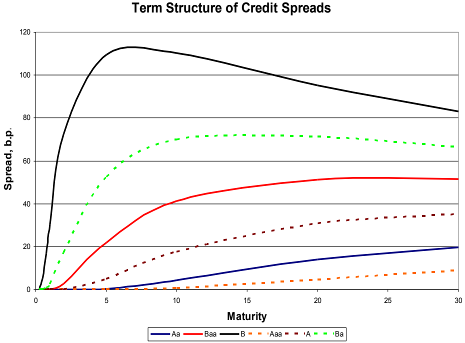
- We can use the Black-Scholes model to relate ratings to spreads
- F =total liabilities
- V =equity value + book value
- Use Ito's lemma to measure volatility, σ_V:

$$\sigma_S(t) = \sigma_V \cdot \frac{\partial S(V_t)}{\partial V_t} \cdot \frac{V_t}{S(V_t)} = \sigma_V N(d_1) \frac{V_t}{S_t}$$

Ratings and Spreads

- Typical numbers
  - $r = 2\%$
  - $\sigma_V = 23\%$
  - Leverage ratio F/V_t, %

Aaa	Aa	A	Baa	Ba	B
13	21	32	43	53	66



Observations:

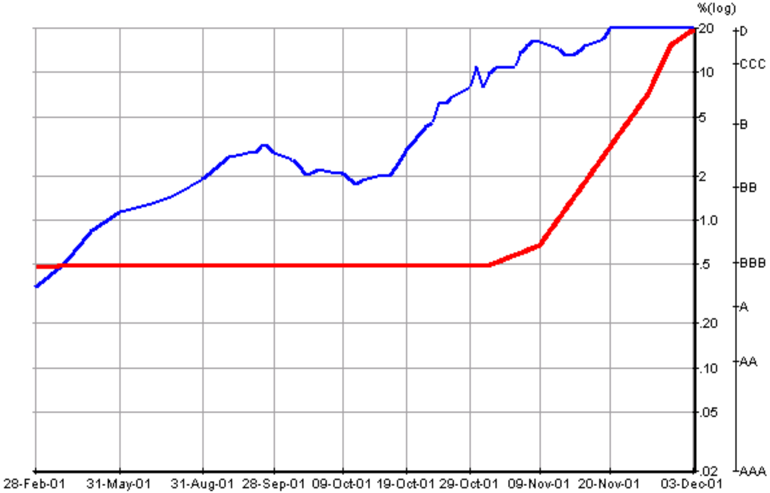
- Spreads are zero at short maturity
- Lower rated companies have a hump-shaped term structure
- Spreads are underestimated

10-yr srpead, bps	Aa	Baa	B
Data	91	194	408
Model	4	41	110

KMV

- Alternative implementation of the Merton models based on DP is offered by KMV
  - KMV was founded in 1989 by Stephen Kealhofer, John Andrew McQuown, and Oldrich Vasicek
  - KMV was acquired by Moody's in 2002 for $70 mil and became Moody's KMV
  - Moody's Analytics is formed in 2008
- KMV compute their proprietary measure of default probability, known as EDF - expected default frequency
- The measure is based on a similar formula and a comprehensive database, which tracks companies

Enron EDF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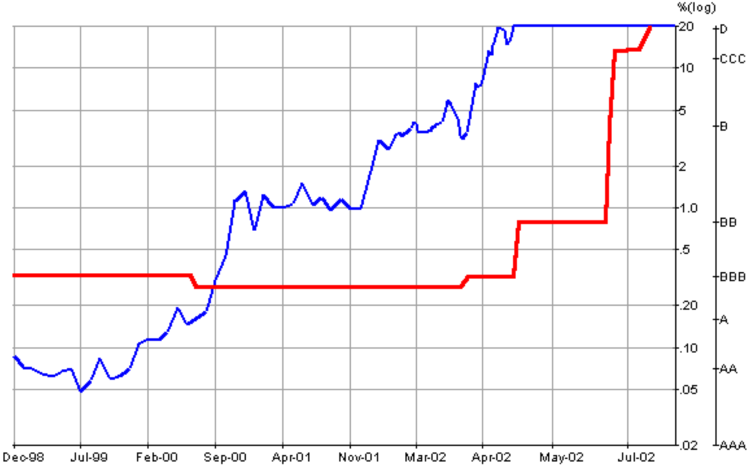
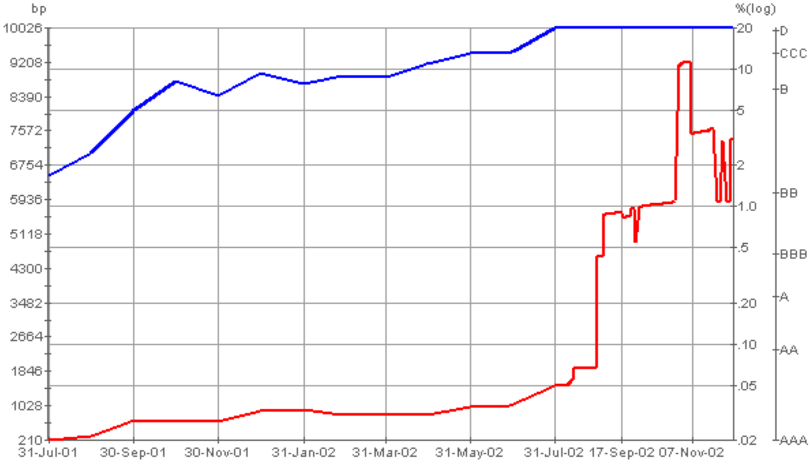
5
2
1.0
.5
.20
.10
.05
.02
D
CCC
B
BB
BBB
A
AA
AAA
28-Feb-01
31-May-01
31-Aug-01
28-Sep-01
09-Oct-01
19-Oct-01
29-Oct-01
09-Nov-01
20-Nov-01
03-Dec-01

WorldCom EDF



United EDF



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2
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D
CCC
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BB
BBB
A
AA
AAA
31-Jul-01
30-Sep-01
30-Nov-01
31-Jan-02
31-Mar-02
31-May-02
31-Jul-02
17-Sep-02
07-Nov-02

Moody's Analytics EDF