HW8

Yue Yu, Yue Zhao, Jing Pu, Georgios Terzakis

1. Value of Caps

Year	Price of Cap
2	0.1631506
3	0.2779042
4	0.3839056
5	0.4952791
7	0.6980043
10	0.9840589

2. Price of at the money European Receiver Swaption

Structure	Price of Swaption
1 Into 1	0.002258492
1 Into 2	0.004487248
1 Into 3	0.006606333
1 Into 4	0.008606992
2 Into 1	0.003150624
2 Into 2	0.006146214
2 Into 3	0.008974396
5 Into 1	0.004248389
5 Into 2	0.008282629
5 Into 5	0.019013568

3. Sensitivity DV01 of the Swaption

Structure	DV01
1 Into 1	0.00000003213568
1 Into 2	0.00000005992730
1 Into 3	0.00000008381445
1 Into 4	0.00000010416660
2 Into 1	0.00000003928693
2 Into 2	0.00000007305439
2 Into 3	0.00000010182460
5 Into 1	0.00000003802496

5 Into 2	0.00000006925654
5 Into 5	0.00000012937050

4. Resettable Caps

Year	Resettable Caps
2	0.06460936
3	0.10988725
4	0.1559966
5	0.19934642
7	0.28463425
10	0.37991668

5. Value of 5 year CMS5 Cap

Year	Price of CMS5 Cap
2	0.006675497
3	0.010810242
4	0.014639353
5	0.01834579
7	0.024808286
10	0.032902549

Code for the Questions

Question 1

```
library(gdata)

library(dplyr)

library(data.table)

pfilea <- fread("Homework 7 pfilea.csv", header = FALSE,stringsAsFactors = F)
sigma <- fread("Homework 7 sigma.csv", header = FALSE,stringsAsFactors = F)
corchol <- fread("Homework 7 corchol.csv", header = FALSE,stringsAsFactors =
F)

colnames(pfilea)<- "DT"

pfilea[,time:=row_number(-DT) *0.5]
pfilea[,cumDT := order_by(time, cumsum(DT))]
pfilea[,par:= 2 * (1 - DT) / cumDT]


indx <- c(4, 6, 8, 10, 14, 20)
CMS <- pfilea$par[c(4, 6, 8, 10, 14, 20)]
```

```
N <- 1000
pv = matrix(nrow=length(CMS), ncol=N)

for (itn in 1:N) {
  DTmat = matrix(nrow = 20,ncol=20)
  DTmat[,1] = pfilea$DT[1:20];



  #string model: simulate D(T)
  for (i in  1:19){
    Z = rnorm(20 - i)
    r = 2 * (1/DTmat[i, i] - 1)
    corr =as.matrix(corchol[1:(20-i), 1:(20-i)])
    DTmat[(i+1):20,(i+1)] = DTmat[(i+1):20,i] + r * DTmat[(i+1):20,i]*1/2 + s
igma$V1[1:(20-i)] * corr %*% Z * sqrt(0.5) * DTmat[(i+1):20,i]
    }

  r = diag(2 * (1/DTmat - 1))
  discount = exp(-cumsum(r)/2)

  #find present value
  for (j in 1:length(CMS)) {
    pv[j,itn] = sum(pmax(0, r[1:indx[j]] - CMS[j]) *discount[1:indx[j]])
  }


}

cap1 <- rowMeans(pv)


cap1

## [1] 0.1631506 0.2779042 0.3839056 0.4952791 0.6980043 0.9840589
```

Question 2

```
path4 = "/Users/durgashankarb/Documents/Spring/Fixed Income/HW7/pfile.csv"
DT = read.csv(path4,header = F)

t <- seq(0.5,10,0.5)
dt0 <- DT[c(1:20),1]
dt = cbind(t,dt0)
par_seq = c(2,3,4,5,7,10)
par_seq1 = 2*par_seq
par_rates = rep(0,length(par_seq1))
for(i in 1:length(par_rates)){
dt1 = dt[,2][1:par_seq1[i]]
   par_rates[i] = 2*((1 - dt1[length(dt1)])/sum(dt1))
```

```
}
strike_rates = par_rates


sigma_0.5 = 0.1

atm_swaption = function(tow,T_mat,sigma_0.5){

  price = (dt0[(2*tow)] - dt0[2*(tow+T_mat)])*((2*pnorm(0.5*sigma_0.5*sqrt(to
w))) -1 )

  return(price)
}

tow = c(rep(1,4),rep(2,3),rep(5,3))
T_mat = c(seq(1,4),seq(1,3),c(1,2,5))
price_swaptions = rep(0,10)
for(i in 1:10){

  price_swaptions[i] = atm_swaption(tow[i],T_mat[i],sigma_0.5)
}
```

Question 3

```
r = rep(0,nrow(dt))
dt_up = rep(0,nrow(dt))
dt_down = rep(0,nrow(dt))
for(i in 1:length(r)){
  r[i] = 2*(((1/dt[i,2])^(1/(2*dt[i,1])))-1)
}
r_up = r+ 1/10000
r_down = r - 1/10000

for(i in 1:length(r)){
  dt_up[i] = (1/((1 + (0.5*r_up[i]))^i))
  dt_down[i] = (1/((1 + (0.5*r_down[i]))^i))
}


atm_swaption_up = function(tow,T_mat,sigma_0.5){

  price = (dt_up[(2*tow)] - dt_up[2*(tow+T_mat)])*((2*pnorm(0.5*sigma_0.5*sqr
t(tow))) -1 )

  return(price)
}
```

```
atm_swaption_down = function(tow,T_mat,sigma_0.5){

  price = (dt_down[(2*tow)] - dt_down[2*(tow+T_mat)])*((2*pnorm(0.5*sigma_0.5
*sqrt(tow))) -1 )

  return(price)
}

price_up  = rep(0,10)
price_down = rep(0,10)
dv01_swaption = rep(0,10)

for(i in 1:10){

  price_up[i] = atm_swaption_up(tow[i],T_mat[i],sigma_0.5)
  price_down[i] = atm_swaption_down(tow[i],T_mat[i],sigma_0.5)

 }

dv01_swaption = (-price_down + price_up)/200
```

Question 4

```
N <- 1000
pv = matrix(nrow=length(CMS), ncol=N)

for (itn in 1:N) {
  DTmat = matrix(nrow = 20,ncol=20)
  DTmat[,1] = pfilea$DT[1:20];



  #string model: simulate D(T)
  for (i in  1:19){
    Z = rnorm(20 -  i)
    r = 2 * (1/DTmat[i, i] - 1)
    corr =as.matrix(corchol[1:(20-i), 1:(20-i)])
    DTmat[(i+1):20,(i+1)] = DTmat[(i+1):20,i] + r * DTmat[(i+1):20,i]*1/2 + s
igma$V1[1:(20-i)] * corr %*% Z * sqrt(0.5) * DTmat[(i+1):20,i]
    }

  r = 2 * (1/DTmat - 1)
  discount = exp(-cumsum(diag(r))/2)

  #cash flow of resettable caps
  cf <- 0.5*pmax(0, c(r[col(r) == (row(r) - 1)], 0)- diag(r)) *(diag(r) > 0.0
7)
  #find present value
  for (j in 1:length(CMS)) {
```

```
    pv[j,itn] = sum(cf[1:indx[j]] *discount[1:indx[j]])
  }
}

cap2 <- rowMeans(pv)
```

Question 5

```
N <- 1000
pv = matrix(nrow=length(CMS), ncol=N)

for (itn in 1:N) {
  DTmat = matrix(nrow = 20,ncol=20)
  DTmat[,1] = pfilea$DT[1:20];

#string model: simulate D(T)
for (i in  1:19){
    Z = rnorm(20 -  i)
    r = 2 * (1/DTmat[i, i] - 1)
    corr =as.matrix(corchol[1:(20-i), 1:(20-i)])
    DTmat[(i+1):20,(i+1)] = DTmat[(i+1):20,i] + r * DTmat[(i+1):20,i]*1/2 + s
igma$V1[1:(20-i)] * corr %*% Z * sqrt(0.5) * DTmat[(i+1):20,i]
    }

  r = 2 * (1/DTmat - 1)
  discount = exp(-cumsum(diag(r))/2)


  #find present value
  for (j in 1:length(CMS)) {
    pv[j,itn] = sum(0.5 * max(0, CMS[1] - 0.05) * (diag(r)[1:indx[j]] > 0.06)
* discount[1:indx[j]])
  }
}

cap3 <- rowMeans(pv)
```