Price you buy : ask (offer) the market sell for

Price you sell : bid the market pays

bid = 49.75 , ask = 50 , Commission = 15

Buy : $100 \times 50 + 15 = 5015$

Sell : $100 \times 49.75 - 15 = 4960$

Transaction Cost : $5015 - 4960 = 55$

Short-Selling

	Day 0	Dividend Ex-Day	Day 90
Action	Borrow Shares Sell shares	– –	Return Shares Purchase shares
Cash	$+S_0$	$-D$	$-S_{90}$

3. Continuous Compounding all in T years

Effective annual rate r : $(1+r)^T$

Annual rate r , Compounded n times / year : $(1+r/n)^{nT}$

Annualized Continuous : $e^{rT} \equiv \lim_{n\to\infty} (1+r/n)^{nT}$

effective annual rate of return : $\left(\frac{S_T}{S_0}\right)^{\frac{1}{T}} - 1$

Continuous Compound: $\ln\left(\frac{S_T}{S_0}\right) \times \frac{1}{T}$

4. Payoff for a Forward Contract:

(Buyer) Long forward = Spot price at expiration - Forward Price

(Seller) Short forward = Forward price - Spot price

Forward price = $F_{0,T} = S_0 \cdot e^{(r-\delta)T}$

If $F_{0,T} > S_0 e^{(r-\delta)T} \Rightarrow$ Arbitrage

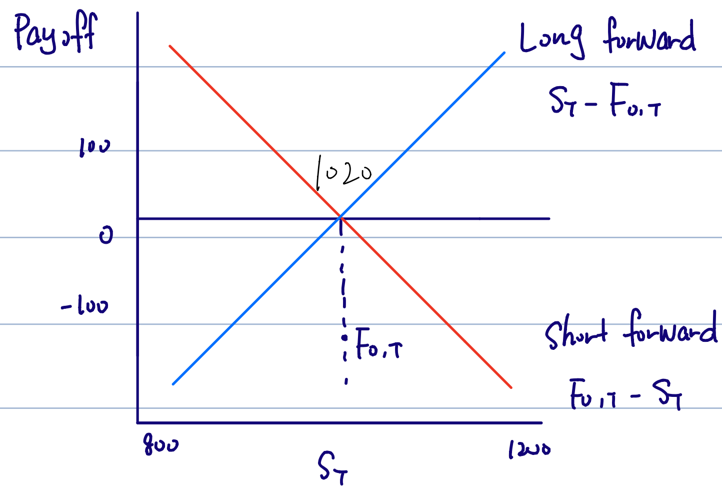
If $F_{0,T} < S_0 \cdot e^{(r-\delta)T}$

$\Downarrow$

all different signs.

Transaction	Time 0	Time T
Buy $e^{-\delta T}$ stock	$-S_0 \cdot e^{-\delta T}$	S_T
Borrow $S_0 \cdot e^{-\delta T}$	$+S_0 \cdot e^{-\delta T}$	$-S_0 e^{(r-\delta)T}$
Short Forward	0	$F_{0,T} - S_T$
Total	0	$F_{0,T} - S_0 \cdot e^{(r-\delta)T} > 0$

Today : spot price = 1000, 6-month forward price = 1020



5. Option

payoff of a call option $C_T = \max(S_T - K, 0)$

profit $= \max[S_T - K, 0]$ − future value of premium

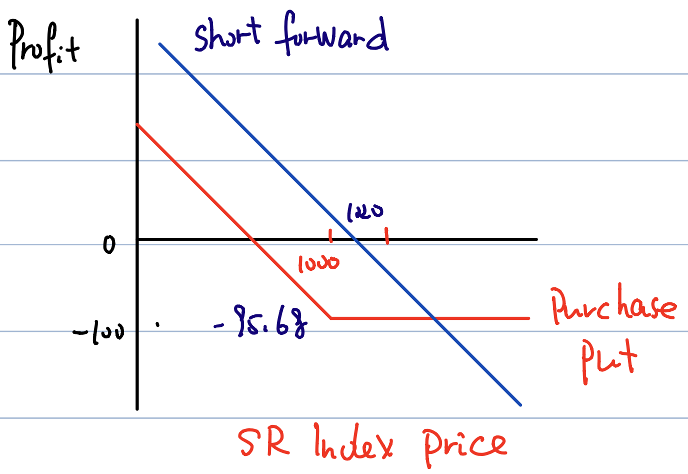
option premium = 95.68

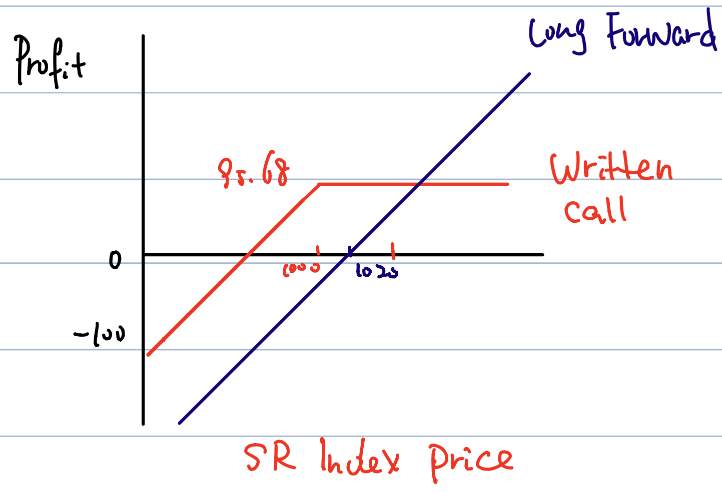
$\max[0, S_T - \$1000] - \$95.68$

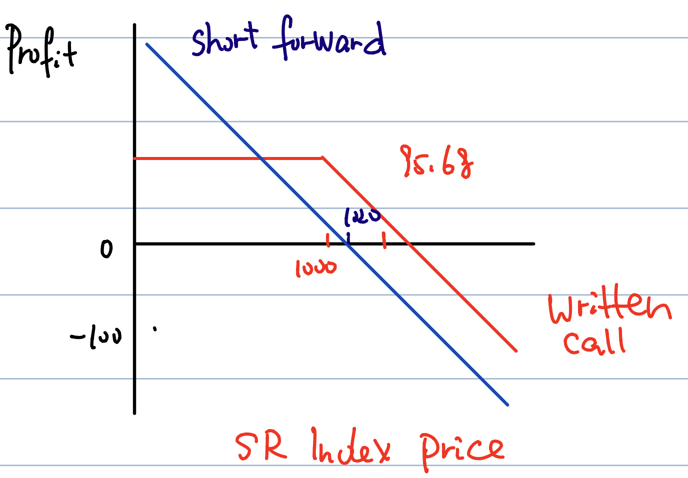
$P \geq \max[0, PV(K) - S_T]$

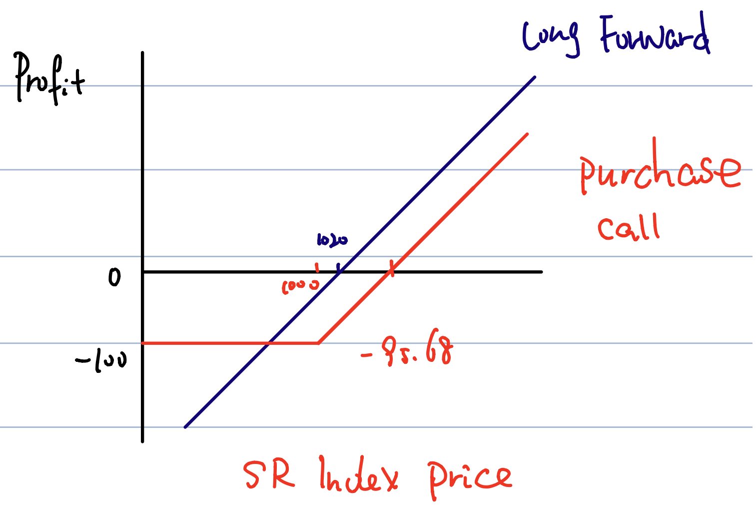
Payoff of a put option $P_T = \max[K - S_T, 0]$

Profit = $\max[K - S_T, 0]$ - future value of premium









Position	Max. Loss	Max. Gain
Long forward	- Forward Price	$+\infty$
Short forward	$+\infty$	Forward Price
Long call	- FV (premium)	$+\infty$
Short call	$+\infty$	FV (Premium)
Long put	- FV (premium)	Strike - FV (Premium)
Short put	FV (Premium) - Strike	FV (Premium)

Put - Call Parity

$$C_T - P_T = \max[S_T - K, 0] - \max[K - S_T, 0]$$

$$= \max[S_T - K, 0] + \min[S_T - K, 0] = S_T - K$$

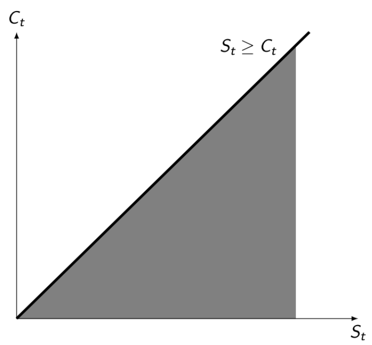
$$\Rightarrow \quad C_t - P_t = S_t - PV(K) > 0 \quad \text{for European Options}$$

For dividend paying stocks

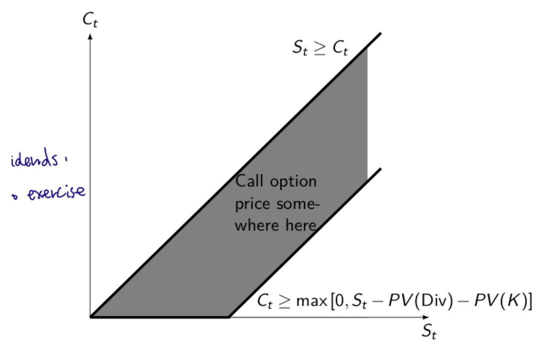
$$C_t - P_t = [S_t - PV(Div)] - PV(K)$$

$$C_t^{American} \geq C_t \geq S_t - PV(K) > S_t - K$$

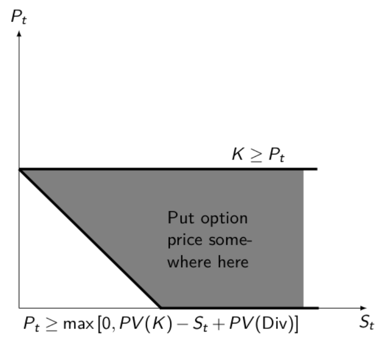
Maximum and Minimum Option Prices: Call Price



Maximum and Minimum Option Prices: Call Price



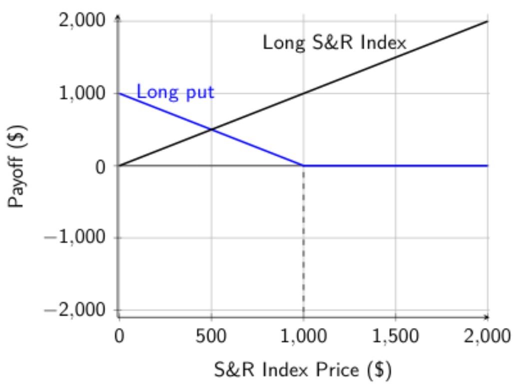
Maximum and Minimum Option Prices: Put Price

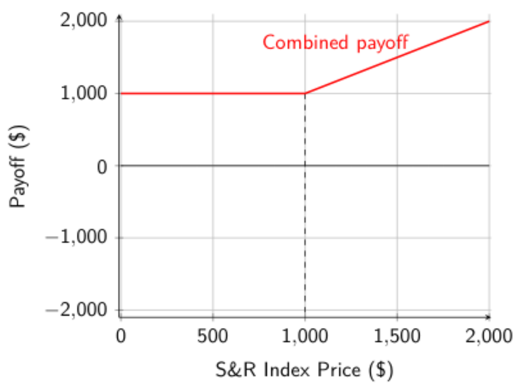


Floors

against fall in price

(long position)

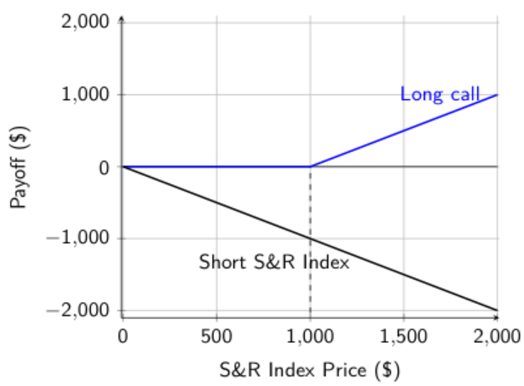


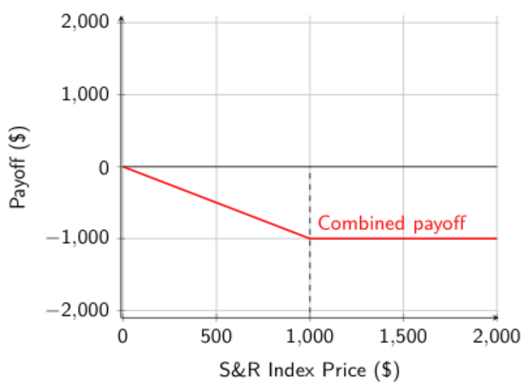


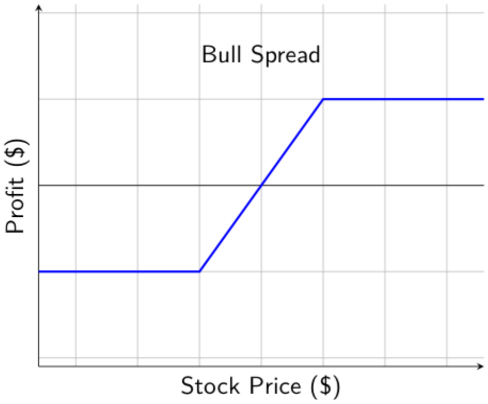
Caps

against increase in price

(short position)

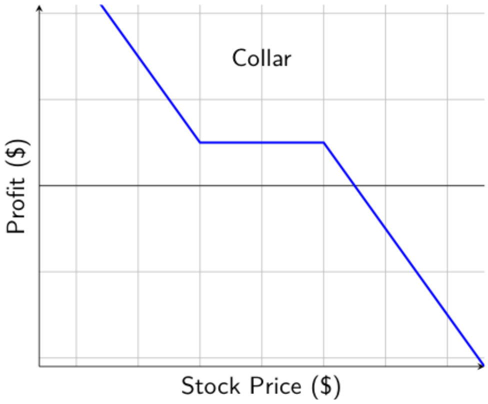






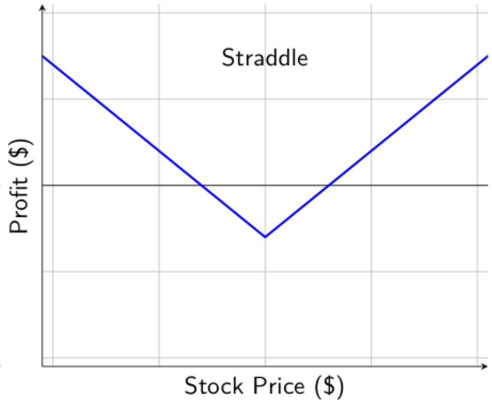
Bull Spread

	K_{low}	K_{ATM}	K_{high}
Call		Buy	Sell
Put	Buy		



Collar

	K_{low}	K_{ATM}	K_{high}
Call			Sell
Put	Buy		

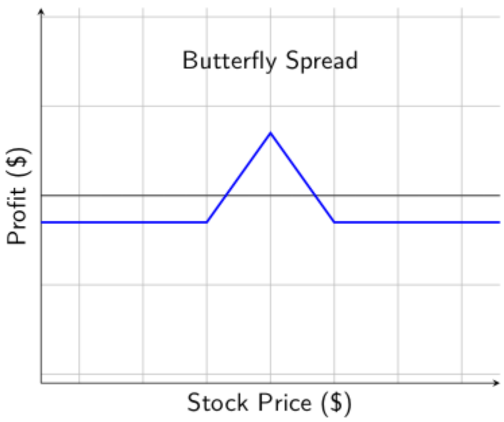


Straddle

	K_{low}	K_{ATM}	K_{high}
Call		Buy	
Put		Buy	

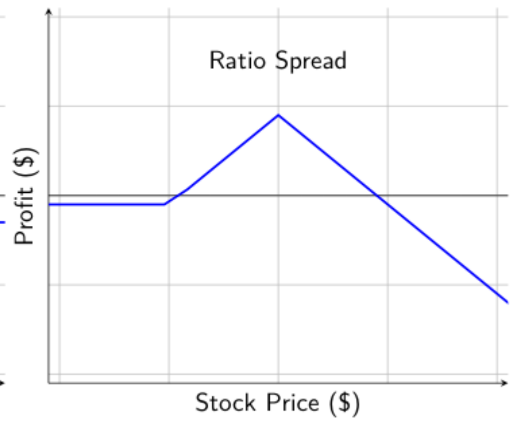
Strangle

	K_{low}	K_{ATM}	K_{high}
Call			Buy
Put	Buy		



Butterfly Spread

	K_{low}	K_{ATM}	K_{high}
Call	Buy	Sell (2)	Buy
Put			



Ratio Spread

	K_{low}	K_{ATM}	K_{high}
Call		Buy	Sell (n)
Put			

Bull Spread

- You believe a stock will appreciate $\Rightarrow$ buy a call option (forward position insured)
- You can lower the cost if you are willing to reduce your profit should the stock appreciate $\Rightarrow$ sell a call with higher strike
- Surprisingly, you can achieve the same result by buying a low-strike put and selling a high-strike put
- Opposite: **bear spread**

Collar

- A collar is fundamentally a short position (resembling a short forward contract)
- Often used for insurance when we own a stock (**collared stock**)
- The collared stock looks like a bull spread; however, it arises from a different set of transactions
- Opposite: **written collar**

Straddle

- A straddle can profit from stock price moves in both directions
- The disadvantage is that it has a high premium because it requires purchasing two options
- Opposite: **written straddle**

Strangle

- Opposite: **written strangle**
- To reduce the premium of a straddle, you can buy out-of-the-money options rather than at-the-money options.

Butterfly Spread

- A butterfly spread is a written straddle to which we add two options to safeguard the position: An out-of-the money put and an out-of-the money call.
- A butterfly spread can be thought of as a written straddle for the timid (or for the prudent!)
- Opposite: **long iron butterfly**

Ratio Spread

- Ratio spreads involve buying one option and selling a greater quantity (n) of an option with a more out-of-the money strike
- The ratio (i.e., "1 by n") is the number of short options divided by the number of long options
- The options are either both calls or both puts
- It is possible to construct ratio spreads with zero premium $\Rightarrow$ we can construct insurance that costs nothing if it is not needed!

	Volatility Will Increase	**No Volatility View**	**Volatility Will Fall**
Price Will Fall	Buy puts	Sell underlying	Sell calls
No Price View	Buy straddle	Do nothing	Sell straddle
Price Will Increase	Buy calls	Buy underlying	Sell puts

6. Binomial Tree

$uS = 59.954$

$u = 1.4623$

$C_u = \max[0, 59.954 - 40] = 19.954$

$S = 41$

$c = ?$

$d = 0.8025$

$dS = 32.903$

$C_d = \max[0, 32.903 - 40] = 0$

Replicating Portfolio:

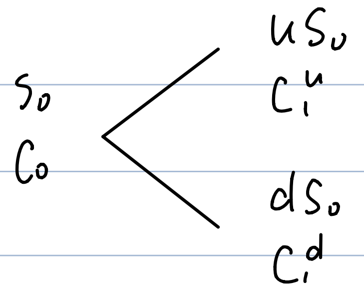
$$\begin{cases} 59.954\,\Delta + e^{0.08} B = 19.954 \\ 32.903\,\Delta + e^{0.08} B = 0 \end{cases}$$

Δ shares of Stock $= 0.738$

\$B risk free $= -22.405$

$C = 0.738 \times 41 - 22.405 = 7.853$

Mispriced option: Buy low and sell high



replicating with dividend

$$\begin{cases} \Delta \times S_0 \times u \times e^{\delta h} + B \times e^{rh} = C_1^u \\ \Delta \times S_0 \times d \times e^{\delta h} + B \times {}^{rh} = C_1^d \end{cases}$$

$$\Rightarrow \begin{cases} \Delta = e^{-\delta h} \cdot \dfrac{C_1^u - C_1^d}{S_0(u-d)} \\ B = e^{-rh} \cdot \dfrac{C_1^d \cdot u - C_1^u \cdot d}{u-d} = e^{-rh}(C_1^u - \Delta S_0 u e^{\delta h}) \end{cases}$$

$\underset{\text{continuous}}{\Delta S_0 \times u \times e^{\delta h}} \approx \underset{\text{discrete}}{\Delta S_0 \times u \times (1+\delta)}$

$$C_0 = \Delta S_0 + B$$

$$= e^{-rh} \cdot \left(C_1^u \cdot \frac{e^{(r-\delta)h} - d}{u-d} + C_1^d \cdot \frac{u - e^{(r-\delta)h}}{u-d} \right)$$

If $\delta = 0$ and $h = 1$

$$C_0 = \Delta S_0 + B = e^{-r} \cdot \left(C_1^u \frac{e^r - d}{u-d} + C_1^d \frac{u - e^r}{u-d} \right)$$

No arbitrage: $d < e^{(r-\delta)h} < u$

If $u < e^{(r-\delta)h}$

	$t=0$	state $= d$	state $= u$
Short stock	$+e^{-\delta h} S$	$-d \times S$	$-u \times S$
Lend money	$-e^{-\delta h} \cdot S$	$+e^{(r-\delta)h} \cdot S$	$+e^{(r-\delta)h} \cdot S$
Payoff	0	>0	≥ 0

If $d > e^{(r-\delta)h}$, all reverse signs above.

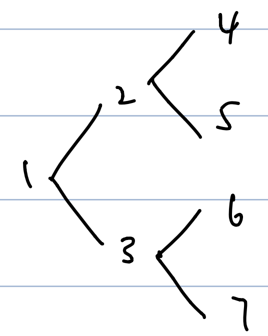
Asian Option

$$C_{uu} = \max[\tfrac{1}{2}(S_0 \cdot u \cdot u + S_0 \cdot u) - K, 0]$$

$$C_{ud} = \max[\tfrac{1}{2}(S_0 \cdot u \cdot d + S_0 \cdot u) - K, 0]$$

$$C_{du} = \max[\tfrac{1}{2}(S_0 \cdot d \cdot u + S_0 \cdot d) - K, 0]$$

$$C_{dd} = \max[\tfrac{1}{2}(S_0 \cdot d \cdot d + S_0 \cdot d) - K, 0]$$



一条 path 上的平均值

7. Uncertainty in the binomial model

$$uS_t = S_t \cdot e^{(r-\delta)h} \cdot e^{+\sigma\sqrt{h}}$$

$$dS_t = S_t \cdot e^{(r-\delta)h} \cdot e^{-\sigma\sqrt{h}}$$

$$\Rightarrow \begin{cases} u = e^{(r-\delta)h + \sigma\sqrt{h}} \\ d = e^{(r-\delta)h - \sigma\sqrt{h}} \end{cases}$$

$h = T/n$ $\quad \frac{\text{Time}}{\text{Steps}}$

$$C_0 = e^{-rh}[p^* C_1^u + (1-p^*) C_1^d]$$

$$p^* \equiv \frac{e^{(r-\delta)h} - d}{u - d}$$

risk-neutral probability under rN pricing

3-Period:

$$C_0 = e^{-3r} \cdot (p^3 \cdot S_{uuu} + 3p^2(1-p) S_{uud} + 3p(1-p)^2 S_{udd} + (1-p)^3 S_{ddd})$$

n-Period:

$$C_0 = e^{-rT} \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} p^{*k}(1-p^*)^{n-k} \max[S_0 \cdot u^k d^{n-k} - X, 0]$$

Call: $S \geq C_{Am} \geq C_{Euro} \geq \max[0, S - PV(D) - PV(K)]$

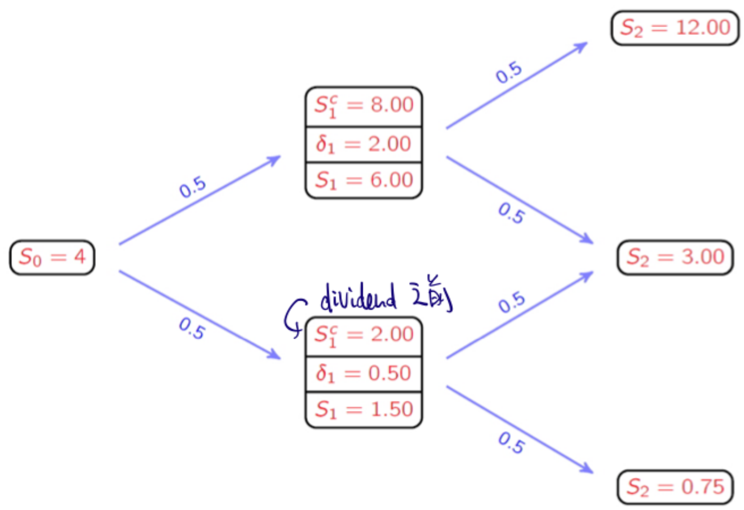
Put: $K \geq P_{Amer} \geq P_{Euro} \geq \max[0, PV(K) - S + PV(D)]$

American Call on a nondividend-paying stock should never be exercised prior to expiration

discrete dividend:

$$u = 1/d = e^{\sigma\sqrt{h}}, \quad p^* = \frac{e^{rh} - d}{u - d}$$

Two periods, $u = 1/d = 2$, $S_0 = 4$, $e^{rh} = 1.25$, $\delta = 0.25$, $\mathbb{D} = \{1\}$



$\left(\frac{u}{d}\right)$ $\frac{1}{u}$ $\frac{1}{u^2} = d$

Convertible Bond Credit Risk

$$P_u = \frac{e^{(r-\delta)h} - d \cdot e^{-\lambda h}}{u - d}$$

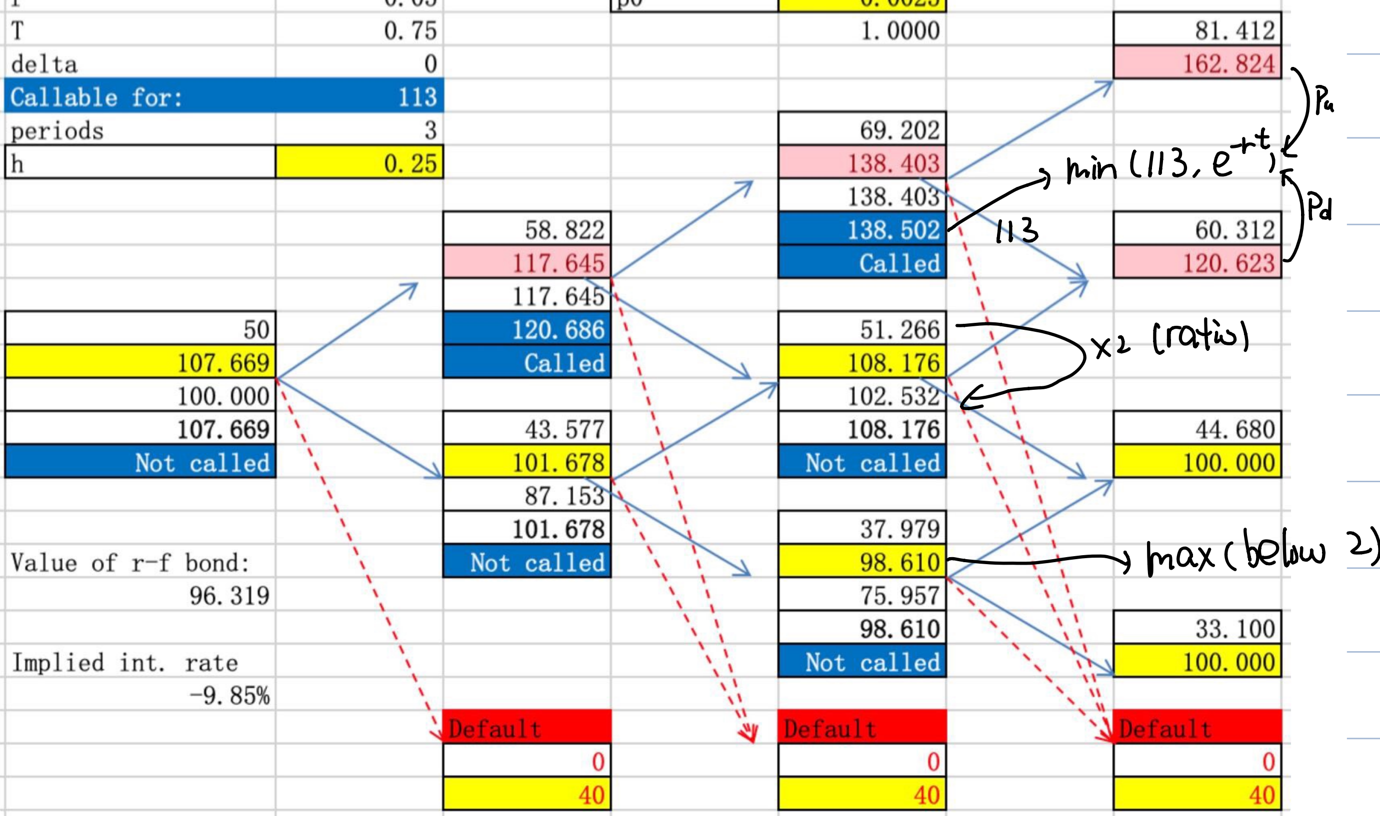
$$P_d = \frac{u \cdot e^{-\lambda h} - e^{(r-\delta)h}}{u - d}$$

Probability of default

$$P_0 = 1 - e^{-\lambda h}$$

S	50
Face value	100
Conversion ratio	2
σ (volatility)	0.3
λ (default intensity)	0.01
r	0.05
T	0.75
delta	0
Callable for:	113
periods	3
h	0.25

u	1.1764
d	0.8715
pu	0.4697
pd	0.5278
p0	0.0025
	1.0000



Value of r-f bond:
96.319

Implied int. rate
-9.85%

	Stock value	$\times u$ or $\times d$
	Convertible bond value	$\max(a, \min(\text{Call price}, b))$
a	Value if the bond is converted	Stock price $\times 2$
b	Value if the bond is not converted (but not yet called)	$(e^{-rt}(P_u \cdot B_u + P_d B_d))$
(Not) Called	Called (or not) by the issuer	Called if $b > 113$

Real Option: the right, but not the obligation to make a particular business decision

Investment Project		Call Option
Investment Cost	=	Strike Price
PV of Project	=	price of underlying asset

Perpetual growing annuity $\Rightarrow$ $PV = \dfrac{\text{Cash flow}}{\text{discount rate} - \text{growth rate}}$

Static NPV = PV − Initial Cost

$\delta = \ln(1 + CF/PV) \Rightarrow$ Find u, d

$$\begin{cases} u = e^{(r-\delta)+\sigma} \\ d = e^{(r-\delta)-\sigma} \end{cases} \qquad p = \frac{e^{r-\delta} - d}{u-d}$$

Ex: Initial investment = \$100. perpetual CF = \$18

$g = 3\%$, discount = 15%

$\Rightarrow$ PV = \$150, NPV = \$150 − \$100 = \$50 $\quad \delta = \ln(1 + 18/150) = 0.1133$

$u = 1.571$, $d = 0.5795$, $r = 6.766\%$, $\sigma = 50\%$

CF = \$18
PV = 150
Call$_{EX}$ = 50
Call$_{NO}$ = 56.19

PV = \$236.27
Call$_{EX}$ = \$136.27
Call$_{NO}$ = \$124.22

PV = \$86.92
Call$_{EX}$ = 0
Call$_{NO}$ = \$13.94

PV = \$372.15
Call = \$272.15

PV = \$136.91
Call = \$36.91

PV = \$50.36
Call = 0

View as a Amer. Call option where
PV = S_t
Investment = K.

\$56.19 > \$50

whether and when to invest in a Project ~ Call option

Shut down, restart, and 永久 abandon a project ~ project + Put option

invest in projects that may give rise to future options ~ Compound option

Switch between inputs, outputs, or tech ~ rainbow option

Longstaff and Schwartz

Stock Price Paths				
Path	t_0	t_1	t_2	t_3
1	1.00	1.09	1.08	1.34
2	1.00	1.16	1.26	1.54
3	1.00	1.22	1.07	**1.03**
4	1.00	.93	.97	**.92**
5	1.00	1.11	1.56	1.52
6	1.00	.76	.77	**.90**
7	1.00	.92	.84	**1.01**
8	1.00	.88	1.22	1.34

Cash Flow Matrix			
Path	t_1	t_2	t_3
1	–	–	–
2	–	–	–
3	–	–	.07
4	–	–	.18
5	–	–	–
6	–	–	.20
7	–	–	.09
8	–	–	–

$K = 1.1$

$r = 0.06$

Put

$T = t_2$: $\mathbb{E}[Y] = -1.070 + 2.983X - 1.813X^2$

Regression & Optimal Exercise				
Path	Y	X	EX	NO
1	$.00 \times 0.94176$	1.08	.02	< .0369
2	–	–	–	–
3	$.07 \times 0.94176$	1.07	.03	< .0461
4	$.18 \times 0.94176$	.97	.13	> .1176
5	–	–	–	–
6	$.20 \times 0.94176$	.77	.33	> .1520
7	$.09 \times 0.94176$	.84	.26	> .1565
8	–	–	–	–

Cash Flow Matrix			
Path	t_1	t_2	t_3
1	–	–	–
2	–	–	–
3	–	–	.07
4	–	.13	–
5	–	–	–
6	–	.33	–
7	–	.26	–
8	–	–	–

X: S_2 that are in the money at t_2

Y: Corresponding discounted CF No Ex.

NO: E(Y) plug X in function

Exercise if Ex > NO (E(Y))

$T = t_1$: $\mathbb{E}[Y] = 2.038 - 3.335X + 1.356X^2$

Regression & Optimal Exercise				
Path	Y	X	EX	NO
1	$.00 \times 0.94176$	1.09	.01	.0139
2	–	–	–	–
3	–	–	–	–
4	$.13 \times 0.94176$	.93	.17	.1092
5	–	–	–	–
6	$.33 \times 0.94176$	.76	.34	.2866
7	$.26 \times 0.94176$	.92	.18	.1175
8	$.00 \times 0.94176$	.88	.22	.1533

Cash Flow Matrix			
Path	t_1	t_2	t_3
1	–	–	–
2	–	–	–
3	–	–	.07
4	.17	–	–
5	–	–	–
6	.34	–	–
7	.18	–	–
8	.22	–	–

Path	$t_1 = 1$	$t_2 = 2$	$t_3 = 3$	Pay off at t_1		
1	1.18	1.28	1.51	–	–	–
2	0.89	1.18	1.32	0.11	–	–
3	1.12	1.43	1.34	–	–	–
4	0.78	0.74	0.87	0.22	0.26	0.13
5	1.07	0.97	1.33	–	0.03	–
6	0.91	0.95	0.94	0.09	0.05	0.06
7	1.24	1.06	1.05	–	–	–
8	0.94	0.85	1.04	0.06	0.15	–

	Expected at t_2	Exer.	Expected at t_1	Exer	Final Payoff
1	–	–	–	–	
2	–	–	0.004	0.11	0.01
3	–	–	–	–	
4	0.123	0.26	0.247	0.22	0.26
5	0.034	0.03	–	–	
6	0.021	0.05	0.041	0.09	0.09
7	–	–	–	–	
8	0.016	0.15	0.144	0.06	0.15

	No exercise of the put option before maturity		Exercise of the put option before maturity (at some time $0 \le t < T$)
	$S_T \ge K$	$S_T < K$	
Position a.	$S_T - K$	0	C_t^E
Position b.	0	$S_T - K$	$S_t - K$
Position c.	Ke^{rT}	Ke^{rT}	Ke^{rt}
Position d.	$-S_T$	$-S_T$	$-S_t e^{-\delta(T-t)}$
TOTAL	> 0	> 0	> 0

a. Long an European call option, C_0^E

b. Short an American put option, P_0^A

c. An amount K invested at the risk-free rate

d. Short $e^{-\delta T}$ units of the stock (with dividends

	No exercise of the call option before maturity		Exercise of the call option before maturity (at some time $0 \le t < T$)
	$S_T \ge K$	$S_T < K$	
Position a.	$K - S_T$	0	$K - S_t$
Position b.	0	$K - S_T$	P_t^E
Position c.	$-K$	$-K$	$-Ke^{-r(T-t)}$
Position d.	$S_T e^{\delta T}$	$S_T e^{\delta T}$	$S_t e^{\delta t}$
TOTAL	> 0	> 0	> 0

a. Short an American call option, C_0^A

b. Long an European put option, P_0^E

c. An amount Ke^{-rT} borrowed at the risk-free rate

d. Long one unit of the stock (with dividends being

分清 Profit 和 payoff 图！

profit 的 premium 是 FUTURE VALUE！

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$