UCLA Anderson School of Management

Quizz #2

You have 45 minutes to finish this assignment. Please write legibly. You may use a cheat sheet as specified in the syllabus. No laptops or equipment that allows communication are allowed. Only a simple calculator.

Problem 1. Suppose that the stock follows the dynamics

$$dS_t = \mu S_t dt + \sigma S_t^{\gamma} d\overline{W}_t$$

and that the interest rate is constant and equal to r. Provide the Partial differential equation and the boundary condition for a European call option with strike price K and expiry date T. Derive the dynamics of S_t under the probability measure Q.

Problem 2. Make the Black Sholes pricing assumptions, i.e., a constant interest rate and the following dynamics for the stock:

$$dS_t = \mu S_t dt + \sigma S_t d\overline{W}_t, \ \mu \text{ and } \sigma \text{ constant.}$$

Assume that at time 0 the price of the stock is S_0. Find the time-0 price of a derivative that pays a constant K at time T, but only if at time $t_1 < T$ the value of S_{t_1} is higher than K^*. Otherwise the derivative is worthless at time T. Mathematically, the payoff of such a contract can be expressed as

$$\Phi\left(S_T\right) = 1_{\{S_{t_1} > K^*\}} K$$

Hint: derive first the price of the derivative security at time t_1 and then price a time-0 claim to that derivative security.

Problem 3. Make the Black Sholes pricing assumptions, i.e., a constant interest rate and the following dynamics for the stock:

$$dS_t = \mu S_t dt + \sigma S_t d\overline{W}_t, \ \mu \text{ and } \sigma \text{ constant.}$$

Assume that at time 0 the price of the stock is S_0. Find the arbitrage-free price of a forward on a call option. Specifically, suppose that we wish to price a forward contract that expires at time t_1, and the underlying of the forward is a European call option with strike price K that has an expiration date equal to $T > t_1$. Mathematically, find the value of K^* such that a contract paying

$$\Phi\left(S_{t_1}\right) = C\left(S_{t_1}; T, K\right) - K^*$$

has a value of 0 at time 0.

Hint: To solve this exercise, note that **all** assets have an expected rate of return equal to r under the probability measure Q.