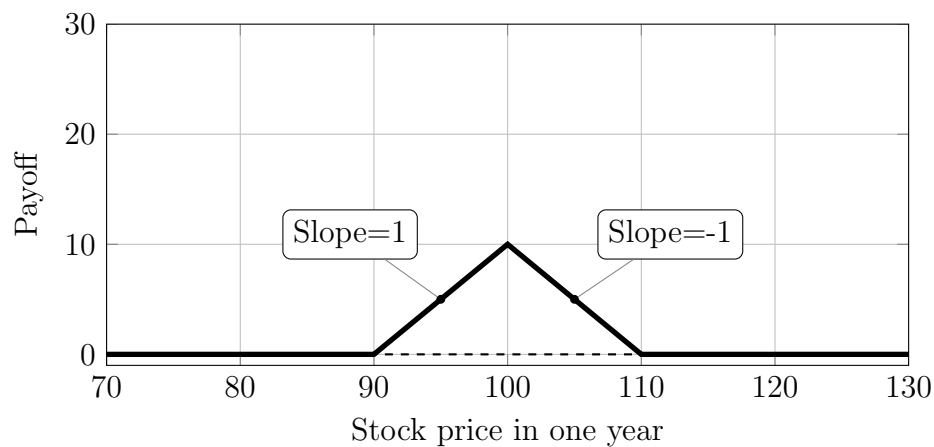# Problem Set 1

These exercises do not need to be turned in for credit.

1 Below is a **payoff** diagram for a position. All options have 1 year to maturity and the stock price today is $100. The yearly interest rate (**continuously compounded**) is 8%. The underlying asset (the stock) is not paying any dividends.

a. In the table below, record option quantities which construct the diagram. **Options given in the table have the corresponding strike in parenthesis**. Slopes are marked on the diagram. Denote a purchased position with "+" and a written position with "−".

Option	Call(90)	Call(100)	Call(110)
Position			

b. In the table below, record option quantities which construct the diagram. Options given in the table have the corresponding strike in parenthesis. Slopes are marked on the diagram. Denote a purchased position with "+" and a written position with "−".

Option	Put(90)	Put(100)	Put(110)
Position			

c. In the table below, record option quantities **and the position in the risk-free asset** which construct the diagram. Be clear about whether you are *borrowing* or *lending*. Options given in the table have the corresponding strike in parenthesis. Slopes are marked on the diagram. Denote a purchased option position with "+" and a written position with "−".

Option	Put(90)	Put(100)	Call(100)	Call(110)
Position				

Borrow/lend in the risk-free asset:

d. Can the portfolio corresponding to the above payoff have zero or negative initial premium? Why or why not?

e. How is this strategy commonly called? Discuss its benefits and disadvantages.

2 A 1M European put option on a non-dividend paying stock is currently selling for 2.50. The option has a strike of 50 and the underlying is currently worth 46. The interest rate is 10%. What should you do?

3 The price of a European call that expires in 6M and has strike of 30 is 2. The underlying stock price is 28 and a dividend of 0.50 is expected in 2M and again in 5M. Compute the price of a put with maturity 6M under the assumption that the interest rate is 10%.

4 Suppose that you are the manager and sole owner of company MFE. All the debt of the company matures in one year. If at that time the value of the company's assets is larger than the face value of debt you will pay off that face value, otherwise you will declare bankruptcy and the debt holders will own the assets of the company.

a. Express your position as an option on the firm's assets.

b. Express the position of the debt holders as a portfolio of securities.

5 Show that, if C_0^A is the price of an American call with exercise price K and maturity T on a stock paying a dividend yield of δ, and P_0^A is the price of an American put on the same stock with the same strike price and exercise date, then

$$S_0 e^{-\delta T} - K < C_0^A - P_0^A < S_0 - K e^{-rT} \tag{1}$$

where S_0 is the stock price, r is the risk-free rate, and $r > 0$.

6 Suppose that you are long the **non-dividend-paying** S&P 500, currently trading at \$100. The effective annual risk-free rate is 10% and you can trade the following set of options with one-year maturity:

Strike	80	100	115.4	120
Calls	29.15	16.492	–	8.436
Puts	1.88	–	14.8	17.527

To insure your position, you will need a put option with a one-year maturity and strike price K=100. Since none is available, you need to synthesize one. Describe the strategy that replicates the put and provide the cost of this strategy.

7 Suppose that you can trade 3 stocks, A, B, and C, each of which has a current price of $100:

- A is a non-dividend-paying stock

- B pays a discrete $10 dividend in 3 months

- C pays dividends at a continuously compounded dividend yield of 3%

The continuously compounded risk-free rate is 5%.

a. Consider a forward contract that expires in 6 months. What is the forward price for stock A? What is the forward price for stock B? What is the forward price for stock C?

b. Suppose you read the *Wall Street Journal* and figure that WSJ quotes a $100.5 forward price for stock A. You conclude that there is an arbitrage opportunity. Explain the transactions you undertake to obtain a zero profit at time 0 and a positive profit in 6 months.

c. Do the same for stock B and C (assuming that WSJ quotes $100.5 forward prices for both of them).