Quantitative Asset Management

Bernard Herskovic

UCLA Anderson SCHOOL of MANAGEMENT

Spring 2019

My background

- My research interests: Asset Pricing, Financial Economics, Economic Theory, Macroeconomics.
  - Stock returns and idiosyncratic volatility
  - Asset pricing and production networks
- Since July 2015: Tenure Track Assistant Professor of Finance at UCLA Anderson
- More information about me: www.bernardherskovic.com

Rules of the game

- Read the syllabus!
- Grades in this class will be assigned on the basis of
  - class participation (10%)
  - four problem sets (30%)
  - take-home final exam (35%)
  - final project (25%)
- Class attendance is required, and I will take attendance.
- No laptops/ cell phones.
- Study the required readings
- Class participation is extremely encouraged!!

Problem Sets, Final Exam and Final Project

- Problem sets
  - Individual answers
  - Can discuss with your classmates
  - Indicate whom you collaborated with!
  - Submit your own code
- Take-home final exam, starting on June 13 at 6pm
  - Due June 14 at 6pm (24 hours)
  - Similar to a Problem Set
- Final Project Presentations on Week 10
- The final exam and final project presentation cannot be rescheduled except for extreme circumstances.

Useful information

- Class materials
  - Slides, readings, and assignments: CCLE
  - No textbook
  - Mostly academic papers based on the best finance journals:

    Journal of Finance
    Journal of Financial Economics
    Review of Financial Studies
- My contact information
  - Office: C413
  - Email: bernard.herskovic@anderson.ucla.edu
- TA:

Quantitative Asset Management: course outline

Lecture 1	Mean-Variance Investing: Black-Litterman, $1/N$, Risk Parity
Lecture 2	Asset Growth, Profitability and 5 FF factors Due Sunday 4/14: *Problem Set 1*
Lecture 3	Momentum Due Sunday 4/21: *Problem Set 2*
Lecture 4	Time Series Momentum and Volatility
Lecture 5	Commodity, Short-selling, and Comomentum Due Sunday 5/5: *Problem Set 3*

Quantitative Asset Management: course outline

Lecture 6	Return Predictability
Lecture 7	BAB, QMJ, Currency Due Sunday 5/19: *Problem Set 4*
Lecture 8	Mutual Fund and Hedge Fund Performance
Lecture 9	Factor Zoo and Data Mining
Lecture 10	Final Project Presentations Due 6/4 8:30am: *Final project report*

Take-home Final Exam (June 13, 2019)

Lecture 1: Outline

Introduction: Contextualizing QAM

Problem Sets Overview

Annualizing return

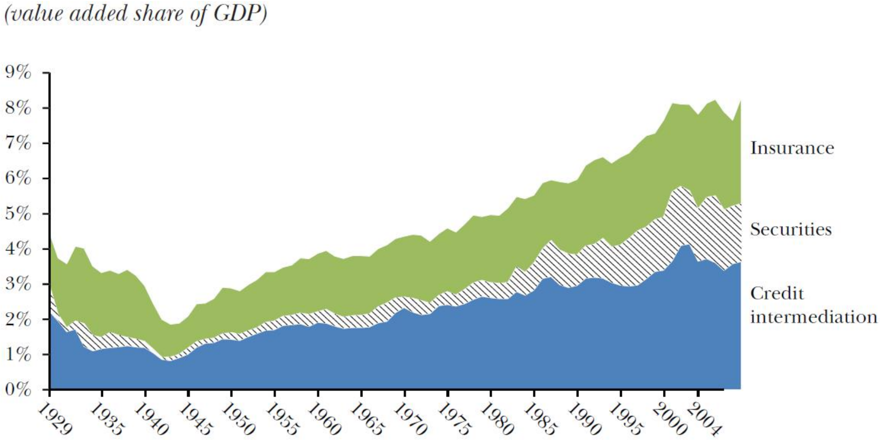
Market portfolio

Lecture

1. Review of Mean-Variance Analysis
   - Markowitz Model: No Risk-free Asset
   - Markowitz Model: With Risk-free Asset
   - Example: International Diversification Case Study
   - Pitfalls of Mean-Variance Analysis
2. $1/N$
3. Risk Parity
4. Black Litterman Approach

Introduction

The Growth of Financial Services



[Greenwood and Sharfstein](#) (2013, Journal of Economic Perspectives)

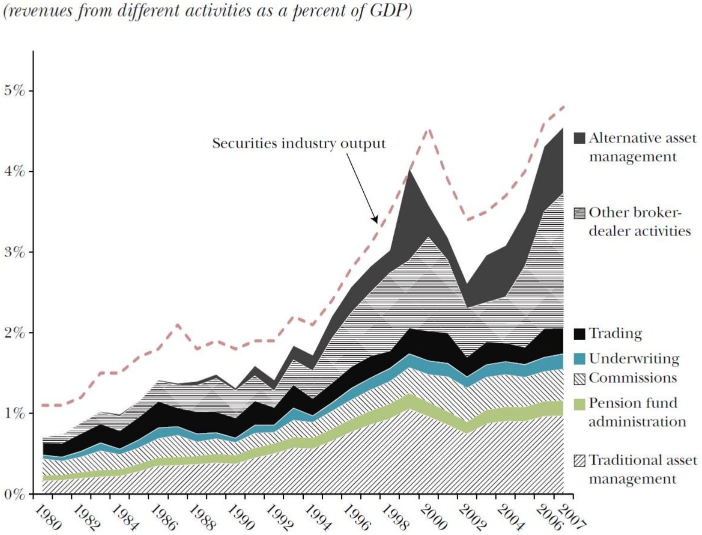
Investment Management Industry

- Financial services experienced enormous growth

  From 4.9% of GDP in 1980 to 7.9% of GDP in 2007
- Much of the growth of finance is associated with two activities:
  1. Asset management
  2. Provision of household credit.
- Large fraction of growth comes from fees earned in asset management

  As asset values increase, fee increased as a fraction of GDP

The growth of the Securities Industry, 1980–2007



Greenwood and Sharfstein (2013, Journal of Economic Perspectives)

Compensation in Finance

Compensation in finance has increased

- until 1990, the typical financial services employee earned about the same wages as his counterpart in other industries;
- by 2006, employees in financial services earned an average of 50% more (Phillippon and Reshef, 2012 QJE)
- Premium reaches 250% for top executives

Sustainable Increase?

Active Management and Fees

- Neoclassical finance view: there is no alpha.

  Is there any evidence that all actively managed funds in the aggregate outperform? Not really: buy a passive index stock fund and save yourself 67 bps per annum (French 2008, JF Presidential Address)

- Post-neoclassical view: there is alpha but not after fees

  in a world in which some managers have skill, but it's hard to tell, it makes sense to chase performance. In 'equilibrium', the fees will eat up all of the alpha (skill is not alpha). (Berk and Green 2004, JPE)

What about the data?

There is alpha relative to the Market

- There is alpha relative to the market portfolio
  - Value (stocks, currencies)
  - Momentum (stocks, currencies, bonds)
  - Carry trades (currencies, bonds)
- Is this really alpha or are we capturing new types of beta?
  - Probably mostly new types of beta
  - Risk is multidimensional
- Creates role for active management:
  - How much do you want to pay for this service?

    From the WSJ:
    "Investors Pull Cash From Hedge Funds"
    'Many pension funds, insurers and university endowments are pulling back from hedge funds, with industry results falling behind the total return of the S&P 500 for seven straight years and the high-fee structure raising eyebrows in several states.' (03/30/2016)

What about the data?

Asset Returns are Predictable

- Old notion of market efficiency: publicly available information does not predict returns
  - Clearly at odds with data
- Returns are predictable in
  - Stock markets: dividend yield, interest rates, price/earnings ratios all predict returns over longer holding periods (more than one year)
  - Bond markets: slope of the yield curve, forward spreads predict returns over the next year or so
  - Currency markets: interest rate spreads predict returns
- Creates role for active management

Portfolio Management

The CAPM does not 'work' anymore

- There are multiple priced factors (not just the market)
- Even if you only care about mean and variance, you should not hold the market in your basket of risky assets
- Portfolio management becomes more much interesting and exciting
- Maybe we need a large portfolio management industry: lots of sources of risk; different investors have different tolerances for different types of risk

Problem Sets Overview

What are we learning?

Problem sets are 30% of final grade

PS1 (7.5%)

- Build the market portfolio
- Use CRSP stock data, construct portfolios

PS2 (7.5%)

- Risk parity portfolio
- Use CRSP stock and bond data, construct portfolios conditional on stock characteristics

What are we learning?

PS3 (7.5%)

- Replicate momentum
- One-way sorted portfolio

PS4 (7.5%)

- Replicate size and book-to-market sorted portfolios
- Replicate the Fama and French factors!
- Use Compustat data (CRSP-Compustat merged)
- Construct book values (tricky) and double sort stocks

Final Project (25%)

Take-home final exam (35%)

- 2017' exam: replicate volatility puzzle
- 2018' exam: replicate different hedging portfolios

Problem Sets

Submission

- Name files correctly
- Submit via CCLE only
- Submit two files (not a zip file)
- Data cleaning is part of the functions

Writeups

- Write your name
- Write names of whom you discussed the PS with
- This is an individual assignment
- Don't copy-paste code/data
- Explain what you do
- Report what is asked: moments, number of decimal digits
- Explain assumptions and justify
- Anyone should be able to replicate your work based on the writeup

Annualization of Returns

Annualization of Returns

- Monthly returns: $\{R_t\}_t$
- Average and variance: μ_M and σ_M^2
- Sample average and variance: $\hat{\mu}_M$ and $\hat{\sigma}_M^2$
- Standard annualization:

$$
\begin{aligned}
\hat{\mu}_A &= 12 \times \hat{\mu}_M \\
\hat{\sigma}_A^2 &= 12 \times \hat{\sigma}_M^2 \\
\hat{\sigma}_A &= \sqrt{12} \times \hat{\sigma}_M \\
SR_A &= \sqrt{12} \times SR_M
\end{aligned}
$$

- This is the annualization you should use in all problem sets

Annualization of Returns

- Formulas make sense if monthly returns are i.i.d. and a sum of monthly returns (e.g. log returns)
- However, annual return are given by

$$R_A = (1 + R_1)(1 + R_2) \dots (1 + R_{12}) - 1$$

Annualization of Returns

- If returns are i.i.d.
  - Average

$$\begin{aligned}\mu_A \equiv \mathbb{E}[R_A] &= \mathbb{E}(1+R_1)\mathbb{E}(1+R_2)\ldots\mathbb{E}(1+R_{12}) - 1\\ &= (1+\mu_M)^{12} - 1\end{aligned}$$

  - Variance

$$\sigma_A^2 \equiv \mathbb{V}[R_A] = \mathbb{V}[R_A+1] = \mathbb{E}[(R_A+1)^2] - \mathbb{E}[R_A+1]^2$$
$$\mathbb{E}[(R_A+1)^2] = \mathbb{E}[(1+R_M)^2]^{12} = \left[\sigma_M^2 + (1+\mu_M)^2\right]^{12}$$

therefore the annual variance is

$$\sigma_A^2 = \left[\sigma_M^2 + (1+\mu_M)^2\right]^{12} - (1+\mu_M)^{24}$$

and the annual volatility is

$$\sigma_A = \sqrt{\left[\sigma_M^2 + (1+\mu_M)^2\right]^{12} - (1+\mu_M)^{24}}$$

Annualization of Returns

- If you want to annualize properly, then

$$\hat{\mu}_{annualized} = (1 + \hat{\mu}_M)^{12} - 1$$

$$\hat{\sigma}_{annualized} = \sqrt{\left[\hat{\sigma}_M^2 + (1 + \hat{\mu}_M)^2\right]^{12} - (1 + \hat{\mu}_M)^{24}}$$

- More details about these expressions

  Morningstar paper: What's Wrong with Multiplying by the Square Root of Twelve

- This is still not 100% accurate because it ignores autocorrelations...

Annualization of Returns

- However, from now on, we standardize annualization as:

$$\hat{\mu}_{annualized} = 12 \times \hat{\mu}_M$$
$$\hat{\sigma}_{annualized} = \sqrt{12} \times \hat{\sigma}_M$$

  Why?
  - because that's the standard
  - gives a good idea about the annual magnitude anyways
  - allows you to compare monthly returns
  - easy to get t-statistics—e.g. use monthly data directly
- OK as long as you don't compare different frequencies
- Both methods ignore correlations
- If you are going to compare monthly and annual returns, then construct actual annual return based on monthly data
  - Statistics are annualized by construction

Problem Set 1: Market Portfolio

Market Portfolio

- In CRSP, for each stock i at month t, you have
  - Holding period returns: $r_{i,t}^{h}$
  - Delisting return: $r_{i,t}^{d}$
  - ...
- Check missing return codes, share codes, etc.
- Use cum-dividend total returns:

$$r_{i,t} = \begin{cases} r_{i,t}^{h} & \text{if } r_{i,t}^{d} \text{ missing} \\ r_{i,t}^{d} & \text{if } r_{i,t}^{h} \text{ missing} \\ (1 + r_{i,t}^{h})(1 + r_{i,t}^{d}) - 1 & \text{if both not missing} \end{cases}$$

Market Portfolio

- Market cap: price $\times$ shares outstanding $= me_{i,t}$
  - Check units and signs!
- Market Portfolio weights:

$$w_{i,t}^{mkt} = \frac{me_{i,t-1}}{\sum_i me_{i,t-1}}$$

- Market Portfolio

$$R^m kt_t = \sum_i w_{i,t}^{mkt} r_{i,t} = \sum_i \frac{me_{i,t-1}}{\sum_j me_{j,t-1}} r_{i,t}$$

Lecture 1: Outline

1. Review of Mean-Variance Analysis
    - Markowitz Model
    - Example: International Diversification Case Study
    - Pitfalls of Mean-Variance Analysis
2. $1/N$
3. Risk Parity
4. Black Litterman Approach

Mean-Variance Analysis

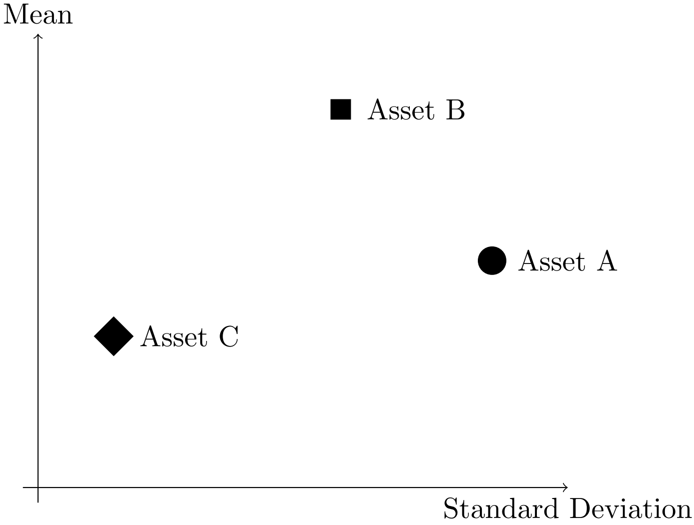
Notation

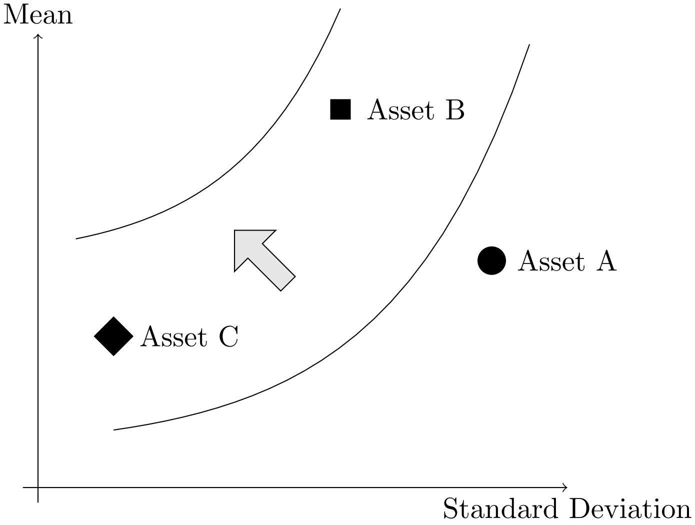
- Define
  - Number of assets: N
  - Column Vector of portfolio weights: w
  - Portfolio: $\omega' R$
  - Matrix variance covariance: $\mathrm{Var}(R) = \Sigma$
  - Expected returns: $\mathbb{E}[R] = \mu$
- The variance of the portfolio return:

$$w' \Sigma w$$

- The expected value of the portfolio return:

$$w' \mu$$


Mean
Asset B
Asset A
Asset C
Standard Deviation


Mean
Asset B
Asset A
Asset C
Standard Deviation

What do we need to know for mean-variance analysis?

Model inputs:

- Expected return
- Variance
- Co-variances (or correlations)

Mean-variance optimization

Optimization

$$\min_{w} \frac{1}{2} w' \Sigma w$$
$$\text{s.t.}$$
$$w'\mu \geq \bar{\mu}$$
$$w'\mathbf{1} = 1$$

where **1** is a column vector of ones

KKT conditions:

$$\Sigma w = \lambda_1 \mathbf{1} + \lambda_2 \mu$$
$$w'\mu \geq \bar{\mu}$$
$$w'\mathbf{1} = 1$$
$$\lambda_2 \geq 0$$
$$\lambda_2 (w'\mu - \bar{\mu}) = 0$$

See detailed derivations and proofs here

Mean-variance optimization (with risk aversion)

Optimization

$$\max_w w'\mu - \frac{\delta}{2} w'\Sigma w$$
$$\text{s.t.}$$
$$w'\mathbf{1} = 1$$

where $\mathbf{1}$ is a column vector of ones, δ is a risk aversion parameters

KKT conditions:

$$\delta \Sigma w = \mu + \lambda_1 \mathbf{1}$$
$$w'\mathbf{1} = 1$$

Using excess returns, weights on risky assets are

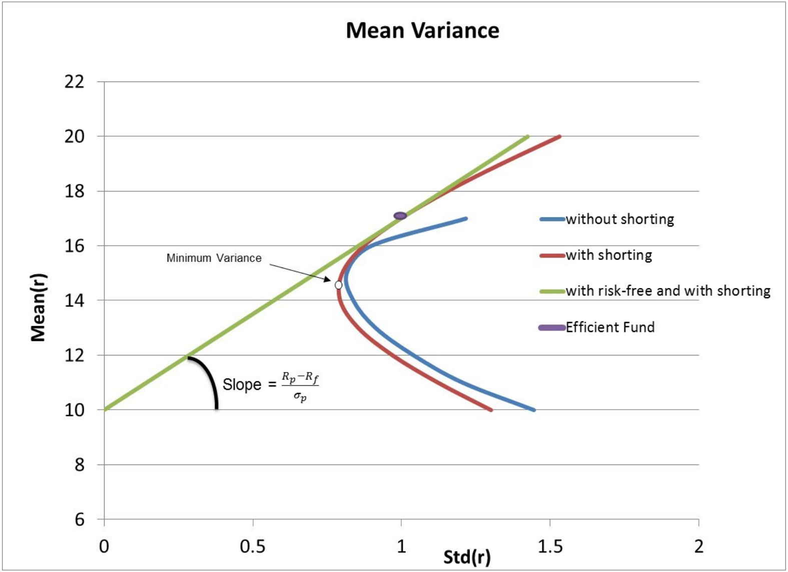
$$w^*_{ex} = \frac{1}{\delta} \Sigma^{-1}_{risky} \mu$$

One Fund Theorem

There is a single fund F of risky assets such that any efficient portfolio can be constructed as a combination of F and the risk-free asset

The slope of the line that connects the risk-free and F is the maximum Sharpe ratio:

$$slope = \frac{R_p - R_f}{\sigma_p}$$


Mean Variance
22
20
18
16
14
12
10
8
6
0
0.5
1
1.5
2
Std(r)
Mean(r)
Minimum Variance
Slope = $\frac{R_p - R_f}{\sigma_p}$
without shorting
with shorting
with risk-free and with shorting
Efficient Fund

Maximum Sharpe Ratio

To derive the maximum Sharpe ratio, we could simply maximize the slope..

$$slope = \frac{R_p - R_f}{\sigma_p} = \frac{\sum_{i=1}^{n} w_i R_i - R_f}{\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}}}$$

Maximize this slope by choosing the portfolio weights $w_i = 1, \ldots, n$ subject to the condition that the portfolio weights sum to one

Predictions from Mean-Variance Analysis

Portfolio Advice Summary

- All investors invest in the same fund of risky assets, regardless of risk aversion.
- Depending on risk aversion, they choose an appropriate mix of money market and risky fund investments
- What about the investment horizon?
  - Same portfolio advice applies for investors with longer horizons if we assume that investors are drawn from the same distribution each period
  - Your investment horizon (age) does not matter if returns are i.i.d. (independently and identically distributed over time)
- What about your labor income risk?
  - Does not matter
  - All you care about is mean and variance

International Diversification Case Study

International Diversification

- Broad asset allocation problem:
- Suppose we consider investing in
  1. U.S. Equities
  2. U.S. Bonds
  3. International Stocks
  4. Commodities

Measurement

- Huge measurement challenge
- We need to come up with estimates for expected returns, volatilities and correlations for all asset classes
  - backward looking estimates : Historical data
  - forward looking estimates: Other variables like price/earnings, price/dividend ratios, cyclically adjusted price/earnings

Historical Data

- Collect historical data on US Equities, US bonds and International Equities and Commodities (available online)
- Globalfinancialdata.com (good source of historical data):
  1. Total Return Stock Index S&P500
  2. Total Return Stock Index for World (excludes the U.S.)
  3. Total Return Bond Index for U.S. Government Bonds with maturity of 10 years
  4. Goldman Sachs Commodity Index
  5. 3-month T-bill as the risk-free

Estimates of Risk and Returns

	US Stocks	World Stocks	Bonds	Commodities
Average excess return	8.16	6.66	2.06	1.53
Standard deviation of excess return	20.37	18.53	7.46	23.26
Sharpe ratio	0.40	0.36	0.28	0.07

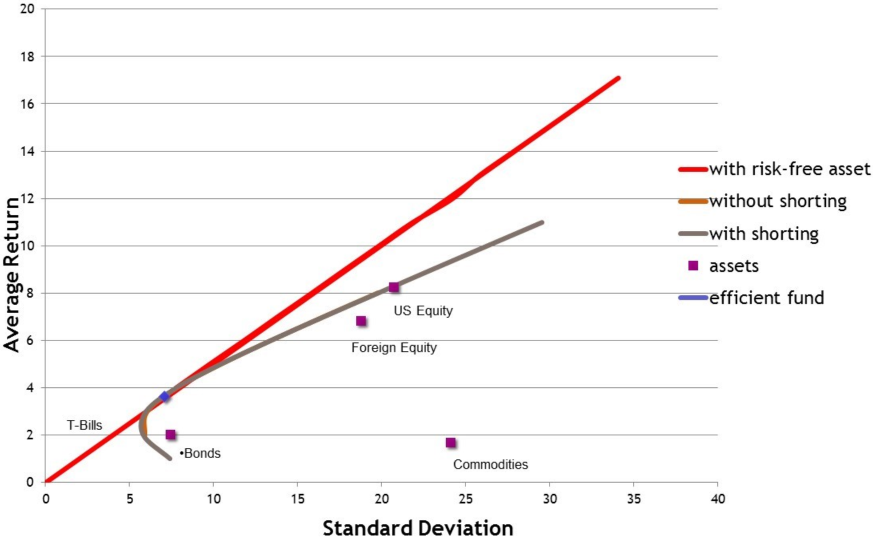
Sample: 1929-2012. Annual Data. Excess Returns.
Source: Globalfinancialdata.com

Correlation Matrix of Excess Returns

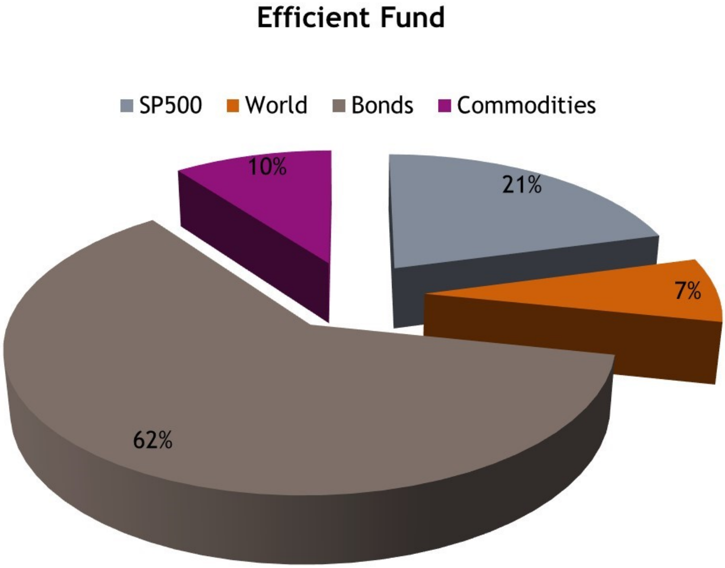
	US Stocks	World Stocks	Bonds	Commodities
US Stocks	1.00	0.86	0.02	0.01
World Stocks	0.86	1.00	-0.05	0.10
Bonds	0.02	-0.05	1.00	-0.28
Commodities	0.01	0.10	-0.28	1.00

Sample: 1929-2012. Annual Data. Excess Returns.
Source: Globalfinancialdata.com

Mean-Variance Analysis



Efficient Fund



Efficient Fund

- The maximum Sharpe Ratio is 50%!
- The Efficient Fund consists of:
  1. 21% in domestic equity
  2. 7% in foreign equity
  3. 62% in bonds
  4. 10% in commodities

- The benefits of international diversification are shrinking:
  - Stock returns correlations have increased
  - Bond returns correlations have increased

Pitfalls of Mean-Variance Analysis

Investment Management

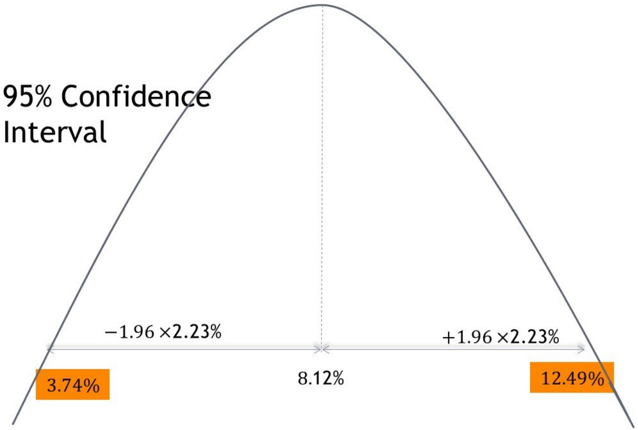
- Investment managers rarely apply the Markowitz framework
- Pension funds typically use a 60/40 equity/bond allocation
- Why not used more often?
  1. Requires specifying expected return assumption for entire universe of assets (investment managers focus on small subset)
  2. Investment managers tend to find the weights counterintuitive

Mean-Variance Analysis

- Has obvious appeal
- Gives clear answer to tough questions
- So... why not just use as your benchmark portfolio?
- Issues and solutions:
  - Model Uncertainty:
    - Expected returns are not precisely estimated
    - Mean-variance optimization is sensitive to inputs
  - Black-Litterman Approach to Mean-Variance Analysis

Uncertainty about the Expected Returns

- We estimated an expected excess return of 8.12% on U.S. equities (sample mean)
- How precise is that estimate?



Ill-behaved Optimization Problem

- Mean-Variance Analysis: Badly behaved optimization problem
- Small changes in expected returns generate large changes in optimized portfolio weights that come out of the mean-variance analysis
- Example:
  - Increase the expected return on U.S. equities by 2%
    - 47% in domestic equity
    - -17% in foreign equity
    - 58% in bonds
    - 11% in commodities
  - Decrease the expected return on U.S. equities by 2%
    - -5% in domestic equity
    - 32% in foreign equity
    - 64% in bonds
    - 8% in commodities

Black Litterman Approach (use a model!)

Market Weights

- We can compare to the implied portfolio weights to the market cap shares of different asset classes
- The market cap shares of different asset classes represent the equilibrium choices by the stand-in U.S. or world investor
- Good idea to compare the optimized weights to these equilibrium weights

Black Litterman

- N asset normally distributed

$$r \sim N(\mu, \Sigma)$$

- Equilibrium: w_{eq}
- Risk premium:

$$\Pi = \delta \Sigma w_{eq}$$

- Bayesian prior

$$\mu \sim N(\Pi, \tau\Sigma)$$

- Views (beliefs)

$$P\mu \sim N(Q, \Omega)$$

Black Litterman

- Applying Bayes' rule

  See Satchell and Scowcroft (2000)

$$\mu|\Pi \sim N\left(\bar{\mu}, \bar{M}^{-1}\right)$$

  where

$$\begin{aligned}
\bar{\mu} &= \left[(\tau\Sigma)^{-1} + P'\Sigma^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Sigma^{-1}Q\right] \\
\bar{M} &= \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]
\end{aligned}$$

Black Litterman

- Thus, posterior distribution of returns is given by

$$r \sim N\left(\bar{\mu}, \bar{\Sigma}\right)$$

  where $\bar{\Sigma} = \Sigma + \bar{M}^{-1}$
- Given risk aversion δ, mean-variance optimization yields

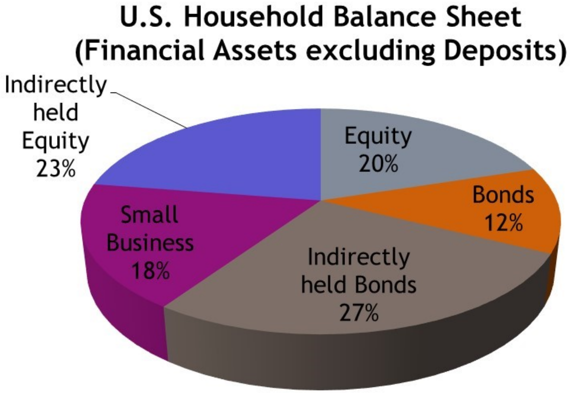
$$w^* = \frac{1}{\delta}\bar{\Sigma}^{-1}\bar{\mu}$$

- which implies

$$w^* = \frac{1}{1+\tau}\left(w_{eq} + P' \times \Lambda\right)$$

  where

$$\Lambda = \tau\Omega^{-1}Q/\delta - A^{-1}P\frac{\Sigma}{1+\tau}w_{eq} - A^{-1}P\frac{\Sigma}{1+\tau}P'\tau\Omega^{-1}Q/\delta.$$



Source: Federal Flow of Funds, Table B.100.e

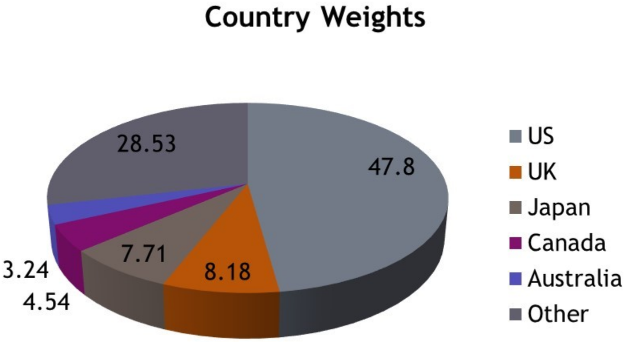
Black-Litterman Approach

Use market weights to infer expected returns, then consider small deviations and optimize

- Does not start from specifying assumptions about expected returns and volatilities
- Starts instead by backing out the expected returns that are consistent with the market cap weights (equilibrium expected returns)
- Then consider small deviations from these equilibrium expected returns to derive optimized weights

Global Equities

- Suppose we focus only on equities as an asset class
- Then the equilibrium weights can be readily obtained from MSCI
- The world investor invests 48% in U.S. equities, 8% in U.K. equities and the rest in a variety of different countries
- These are the market weights we can use a benchmark when we construct a portfolio of equities



MSCI ACWI All Cap Index Weights (August 2012)

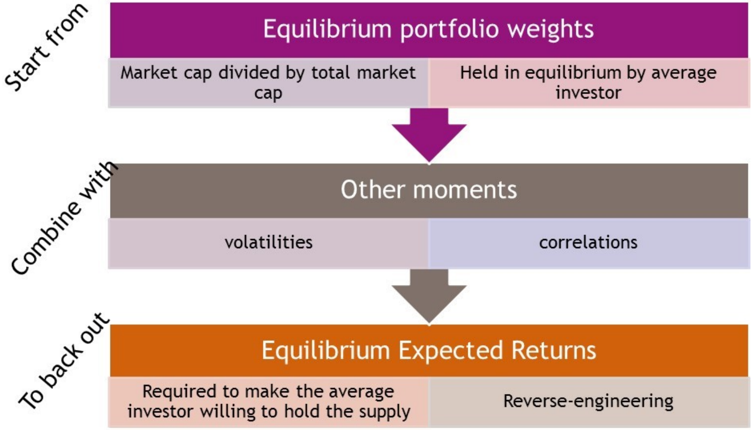
Example: Global Equities

- We're considering investments in Australian, Canadian, French, German, Japanese, U.K. and U.S equities
  - We set all of the expected returns equal to 7% at the outset
  - We do not have strong views
- Suppose we think German equities will outperform by 5%
  - Then we lower the expected returns on non-German equities by 2.5% and we increase the expected return on German equities by 2.5% to reflect our belief that German equities will outperform
- Mean-variance optimizer will dictate a short position of 95% in France

Black-Litterman Approach

- Does not start from specifying assumptions about expected returns and volatilities
- Starts instead by backing out the expected returns that are consistent with the market cap weights (equilibrium expected returns) $\rightarrow$ that's essentially equivalent to imposing the CAPM

Imposing the CAPM



Correlations

	Australia	Canada	France	Germany	Japan	UK
Canada	0.488					
France	0.478	0.664				
Germany	0.515	0.655	0.861			
Japan	0.439	0.310	0.355	0.354		
UK	0.512	0.608	0.783	0.777	0.405	
USA	0.491	0.779	0.668	0.653	0.306	0.652

	Equity Vol	Equilibrium Weight	Equilibrium Expected Return
Aus	16	1.6	3.9
Can	20.3	2.2	6.9
Fra	24.8	5.2	8.4
Germ	27.1	5.5	9.0
Jap	21.0	11.6	4.3
UK	20	12.4	6.8
US	18.7	61.5	7.6

Equilibrium expected returns

Start from returns that are consistent with the market weights (reflects the views of the average investor)

This is a neutral starting point

Introduce beliefs

Now, consider small deviations from these equilibrium returns (e.g., I'm 5% more bullish on France than the average investor)

Compute optimum portfolio

Re-compute the optimum portfolio weights

Global Equities: Black-Litterman approach

- Suppose we think German equities will outperform by 5%
  - We set the return on a long position in German equities and a short position in all other equities (weighted by market cap) equal to 5%
- We attach some confidence level to this view
- The new set of expected returns are a weighted average of the view and the equilibrium expected returns

	p	$\bar{\mu}$	w^*	$w^* - \frac{w_{eq}}{1+\tau}$
Australia	0.0	4.3	1.5	0.0
Canada	0.0	7.6	2.1	0.0
France	-29.5	9.3	-4.0	-8.9
Germany	100.0	11.0	35.4	30.2
Japan	0.0	4.5	11.0	0.0
UK	-70.5	7.0	-9.5	-21.3
USA	0.0	8.1	58.6	0.0

DeMiguel, Garlappi, Uppal (RFS, 2007)

Optimal Versus Naive Diversification: How Inefficient is the $1/N$ Portfolio Strategy?

What's the $1/N$ strategy?

- Equally-weighted portfolio
- This is a mean-variance solution if

$$\Sigma =$$

$$\mu =$$

- How would you test this strategy?

How to test the $1/N$ strategy?

- Compare naive strategy against 14 other strategies
- How are they going to compare?
  - Out-of-sample Sharpe ratio
  - Certainty-equivalent
  - Turnover
- Use different datasets (see Table 2)

- If mean variance is designed to deliver the highest Sharpe ratio, then how can the $1/N$ strategy outperform?

How to test the $1/N$ strategy?

Strategies (Table 1)

- Naive Portfolio: $1/N$
- Sample-based mean-variance
- Bayesian approach to estimation error
  - diffuse-prior
  - Bayes-Stein shrinkage
  - belief in asset-pricing model (data and model)
- Moment restrictions:
  - Minimum variance
  - value-weighted portfolio
  - asset-pricing model with unobservable factors (mp)
- Shortsale-constrained portfolios
  - Bayes-Stein constrained
  - Mean-variance constrained
  - Minimum-variance constrained
  - Generalized minimum-variance constrained
- Optimal combinations of portfolios (3)

Methodology

- Rolling windows: $M = 60$ or $M = 120$ months
- Measures of performance:
  1. Sharpe ratio
  $$SR_k = \frac{\hat{\mu}_k}{\hat{\sigma}_k}$$
  2. Certainty-equivalent (CEQ)
  $$\hat{\mu}_k - \frac{\gamma}{2}\hat{\sigma}_k^2$$
  3. Turnover
  $$\text{Turnover } = \frac{1}{T-M}\sum_{t=1}^{T-M}\sum_{j=1}^{N}|\hat{w}_{k,j,t+1} - \hat{w}_{k,j,t+}|$$

  Return-loss
  $$\text{return-loss}_k = \frac{\mu_{ew}}{\sigma_{ew}} \times \sigma_k - \mu_k$$

Results: Sharpe Ratios

Strategy	S&P sectors $N = 11$	Industry portfolios $N = 11$	Inter'l portfolios $N = 9$	Mkt/ SMB/HML $N = 3$	FF 1-factor $N = 21$	FF 4-factor $N = 24$
$1/N$	0.1876	0.1353	0.1277	0.2240	0.1623	0.1753
mv (in sample)	0.3848	0.2124	0.2090	0.2851	0.5098	0.5364
mv	0.0794	0.0679	−0.0332	0.2186	0.0128	0.1841
	(0.12)	(0.17)	(0.03)	(0.46)	(0.02)	(0.45)
bs	0.0811	0.0719	−0.0297	0.2536	0.0138	0.1791
	(0.09)	(0.19)	(0.03)	(0.25)	(0.02)	(0.48)
dm ($\sigma_\alpha = 1.0\%$)	0.1410	0.0581	0.0707	0.0016	0.0004	0.2355
	(0.08)	(0.14)	(0.08)	(0.00)	(0.01)	(0.17)
min	0.0820	0.1554	0.1490	0.2493	0.2778	−0.0183
	(0.05)	(0.30)	(0.21)	(0.23)	(0.01)	(0.01)
vw	0.1444	0.1138	0.1239	0.1138	0.1138	0.1138
	(0.09)	(0.01)	(0.43)	(0.00)	(0.01)	(0.00)
mp	0.1863	0.0533	0.0984	−0.0002	0.1238	0.1230
	(0.44)	(0.04)	(0.15)	(0.00)	(0.08)	(0.03)
mv-c	0.0892	0.0678	0.0848	0.1084	0.1977	0.2024
	(0.09)	(0.03)	(0.17)	(0.02)	(0.02)	(0.27)
bs-c	0.1075	0.0819	0.0848	0.1514	0.1955	0.2062
	(0.14)	(0.06)	(0.15)	(0.09)	(0.03)	(0.25)
min-c	0.0834	0.1425	0.1501	0.2493	0.1546	0.3580
	(0.01)	(0.41)	(0.16)	(0.23)	(0.35)	(0.00)
g-min-c	0.1371	0.1451	0.1429	0.2467	0.1615	0.3028
	(0.08)	(0.31)	(0.19)	(0.25)	(0.47)	(0.00)
mv-min	0.0683	0.0772	−0.0353	0.2546	−0.0079	0.1757
	(0.05)	(0.21)	(0.01)	(0.22)	(0.01)	(0.50)
ew-min	0.1208	0.1576	0.1407	0.2503	0.2608	−0.0161
	(0.07)	(0.21)	(0.18)	(0.17)	(0.00)	(0.01)

Results: Certainty-equivalent

Strategy	S&P sectors $N = 11$	Industry portfolios $N = 11$	Inter'l portfolios $N = 9$	Mkt/ SMB/HML $N = 3$	FF 1-factor $N = 21$	FF 4-factor $N = 24$
$1/N$	0.0069	0.0050	0.0046	0.0039	0.0073	0.0072
mv (in sample)	0.0478	0.0106	0.0096	0.0047	0.0300	0.0304
mv	0.0031	−0.7816	−0.1365	0.0045	−2.7142	−0.0829
	(0.28)	(0.00)	(0.00)	(0.31)	(0.00)	(0.01)
bs	0.0030	−0.3157	−0.0312	0.0043	−0.6504	−0.0362
	(0.16)	(0.00)	(0.00)	(0.32)	(0.00)	(0.06)
dm ($\sigma_\alpha = 1.0\%$)	0.0052	−0.0319	0.0021	−0.0084	−0.0296	0.0110
	(0.11)	(0.01)	(0.08)	(0.04)	(0.00)	(0.11)
min	0.0024	0.0052	0.0054	0.0039	0.0100	−0.0002
	(0.03)	(0.45)	(0.23)	(0.45)	(0.12)	(0.00)
vw	0.0053	0.0042	0.0044	0.0042	0.0042	0.0042
	(0.12)	(0.04)	(0.39)	(0.44)	(0.00)	(0.00)
mp	0.0073	0.0014	0.0034	−0.0026	0.0054	0.0053
	(0.19)	(0.05)	(0.17)	(0.04)	(0.09)	(0.10)
mv-c	0.0040	0.0023	0.0032	0.0030	0.0090	0.0075
	(0.29)	(0.10)	(0.29)	(0.28)	(0.03)	(0.42)
bs-c	0.0052	0.0031	0.0031	0.0038	0.0088	0.0074
	(0.36)	(0.15)	(0.23)	(0.46)	(0.05)	(0.44)
min-c	0.0024	0.0047	0.0054	0.0039	0.0060	0.0051
	(0.01)	(0.40)	(0.21)	(0.45)	(0.12)	(0.17)
g-min-c	0.0044	0.0048	0.0051	0.0038	0.0067	0.0070
	(0.04)	(0.41)	(0.28)	(0.40)	(0.17)	(0.45)
mv-min	0.0021	−0.2337	−0.0066	0.0044	−0.0875	−0.0318
	(0.07)	(0.00)	(0.01)	(0.28)	(0.00)	(0.07)
ew-min	0.0037	0.0052	0.0050	0.0039	0.0093	−0.0002
	(0.04)	(0.42)	(0.24)	(0.43)	(0.12)	(0.00)

Results: Turnover

Strategy	S&P sectors $N = 11$	Industry portfolios $N = 11$	Inter'l portfolios $N = 9$	Mkt/ SMB/HML $N = 3$	FF-1-factor $N = 21$	FF-4-factor $N = 24$
$1/N$	0.0305	0.0216	0.0293	0.0237	0.0162	0.0198
Panel A: Relative turnover of each strategy						
mv (in sample)	–	–	–	–	–	–
mv	38.99	606594.36	4475.81	2.83	10466.10	3553.03
bs	22.41	10621.23	1777.22	1.85	11796.47	3417.81
dm ($\sigma_\alpha = 1.0\%$)	1.72	21744.35	60.97	76.30	918.40	32.46
min	6.54	21.65	7.30	1.11	45.47	6.83
vw	0	0	0	0	0	0
mp	1.10	11.98	6.29	59.41	2.39	2.07
mv-c	4.53	7.17	7.23	4.12	17.53	13.82
bs-c	3.64	7.22	6.10	3.65	17.32	13.07
min-c	2.47	2.58	2.27	1.11	3.93	1.76
g-min-c	1.30	1.52	1.47	1.09	1.78	1.70
mv-min	19.82	9927.09	760.57	2.61	4292.16	4857.19
ew-min	4.82	15.66	4.24	1.11	34.10	6.80

Asness, Frazzini, and Pederson (2012)

Leverage aversion and risk parity.

Connection with MVE portfolios

Minimum-variance portfolios: mean-variance efficient portfolios under the assumption that all expected returns are equal.

Risk-parity portfolios: mean-variance efficient portfolios assuming that all assets are uncorrelated and have the same expected return.

Equal-weighted portfolios: mean-variance efficient portfolios under the assumption that all expected returns, variances and co-variances are equal across assets.

Traditional Asset Allocation

- Mean-variance analysis puts a lot of stock in our ability to accurately measure all of the variances, co-variances of returns as well as the expected returns
  - Results are very sensitive to small changes in assumptions
- Risk Parity Investing:
  - Takes a more agnostic approach.
  - Puts changes in volatility front and center

Traditional Asset Allocation

- Mean-variance analysis prescribes a 60/40 equity/bonds allocation
- However, most of the variation in returns is driven by the equity component, simply because stocks are much more volatile.
- So, at the end of the day, diversification gains are limited..
- Novel approach tries to avoid this: risk parity investing

What's going on?

- Lots of reasons for the empirical failure of mean-variance analysis (see previous lecture)
- In addition, high volatility assets have tended to underperform low volatility assets
  - That explains why the minimum variance portfolio typically does very well!
- Risk parity investing will exploit this fact

Risk Parity Investing

- Try to equalize the risk contribution of all asset categories
- Now we're diversified in terms of risk (rather than in terms of dollar investments)
- We'll invest more than 40% in bonds because bonds have much lower volatility than equities
- Risk is balanced across asset classes, but..
- Average return is lower
- Investors use leverage to increase the average return

Constructing Risk-parity portfolios

- "Risk Parity" is a portfolio that targets equal risk allocation across the available instruments
- To construct a risk-parity portfolio, we estimate volatilities of all the available asset classes and set the portfolio weight equal to:

$$w_{t,i} = k_t \sigma_{t,i}^{-1}, \quad i = 1, \ldots, n$$

  - If we want an unlevered portfolio, we choose $k_t = 1/\sum_i \sigma_{t,i}^{-1}$
  - If we want a levered portfolio, we simply choose a constant k over time; delivers constant volatility in each asset class
- Rebalancing is monthly
- RP monthly excess returns:

$$r_t^{RP} = \sum_i w_{t-1,i}(r_{t,i} - rf_t)$$

Two assets

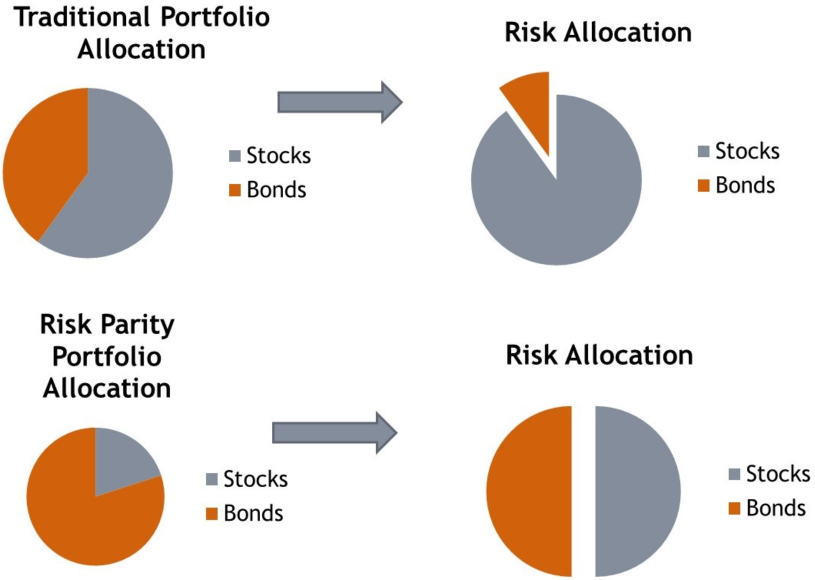
- Example with two assets:
  - $w_{1,t} = k_t \sigma_{1,t}^{-1}$
  - $w_{2,t} = k_t \sigma_{2,t}^{-1}$
  - In case without leverage, we set $k_t = \frac{1}{\sigma_{1,t}^{-1} + \sigma_{2,t}^{-1}}$
  - In case with leverage, we choose constant k and you short the risk-free $(w_{1,t} + w_{2,t} - 1)$

Levered Risk Parity Portfolio

- At the end of each calendar month, we set the portfolio weight in each asset class equal to the inverse of its volatility, estimated using 3-year monthly excess returns up to month $t - 1$, and
- these weights are multiplied by a constant to match the ex-post realized volatility of the Value-Weighted benchmark.

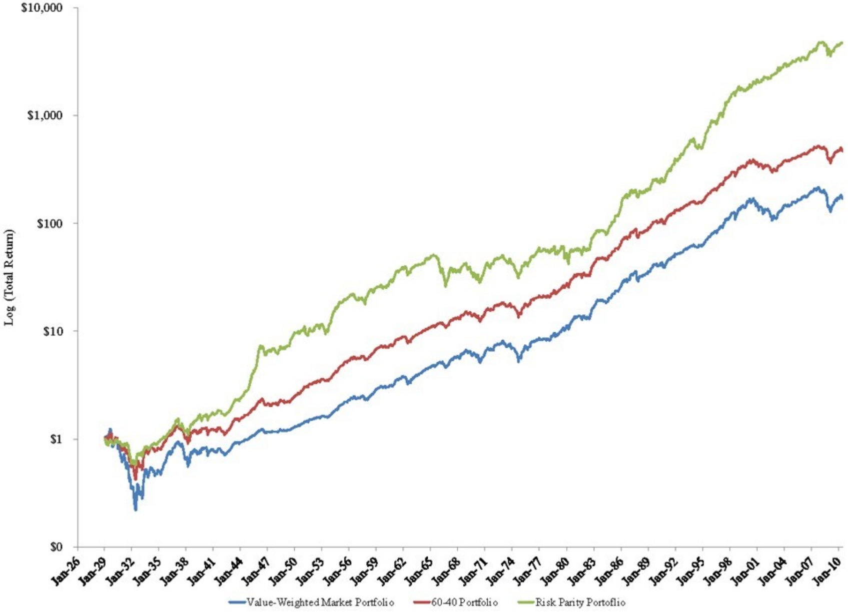
Traditional Portfolios

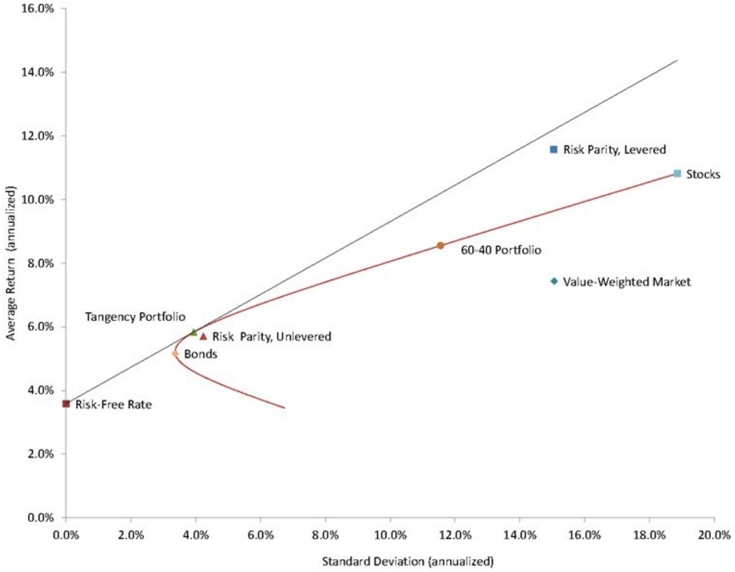
- "Value-Weighted Portfolio" is a market portfolio weighted by total market capitalization and rebalanced monthly to maintain value weights.
- "60-40" is a portfolio that allocates 60% in stocks and 40% in bonds, rebalanced monthly to maintain constant weights.


Traditional Portfolio Allocation
Stocks
Bonds
Risk Allocation
Stocks
Bonds
Risk Parity Portfolio Allocation
Stocks
Bonds
Risk Allocation
Stocks
Bonds

Risk Parity Performance

	Excess Return	t-Stat. of Excess Return	Alpha	t-Stat. of Alpha	Volatility	Sharpe Ratio	Skewness	Excess Kurtosis
A. Long sample (U.S. stocks and bonds, 1926–2010)								
CRSP stocks	6.71%*	3.18			19.05%	0.35	0.18	7.51
CRSP bonds	1.56*	4.28			3.28	0.47	–0.01	4.37
Value-weighted portfolio	3.84*	2.30			15.08	0.25	0.37	13.09
60/40 portfolio	4.65*	3.59			11.68	0.40	0.20	7.46
RP, unlevered	2.20*	4.67	1.39%*	4.44	4.25	0.52	0.05	4.58
RP	7.99*	4.78	5.50*	4.30	15.08	0.53	–0.36	1.92
RP minus value-weighted	4.15*	2.95	5.50*	4.30	12.69	0.33	–0.79	8.30
RP minus 60/40	3.34*	2.93	3.76*	3.33	10.31	0.32	–0.61	5.04


$10,000
$1,000
$100
$10
$1
$0
Log (Total Return)
Jan-26
Jan-29
Jan-32
Jan-35
Jan-38
Jan-41
Jan-44
Jan-47
Jan-50
Jan-53
Jan-56
Jan-59
Jan-62
Jan-65
Jan-68
Jan-71
Jan-74
Jan-77
Jan-80
Jan-83
Jan-86
Jan-89
Jan-92
Jan-95
Jan-98
Jan-01
Jan-04
Jan-07
Jan-10
Value-Weighted Market Portfolio
60-40 Portfolio
Risk Parity Portoflio



Sample: 1926-2010

Rationale

- Nothing wrong with large risk exposure to equities, unless of course per unit risk compensation is too low in the stock market, relative to what we see in other asset markets (such as the bond market!)
  - Low SR's in stock market
  - Higher SR's in bond market
  - Explains why the value-weighted market portfolio is far away from the tangency point
- That's the rationale behind risk-parity investing

Market is not Mean-Variance Efficient

- What accounts for the fact that risk-parity portfolios outperform?
  - Well, the market does not have the highest Sharpe ratio (at least not in this sample when we back-test)
  - The market is not mean-variance efficient
  - The risk-parity portfolio is close to mean-variance efficient!

What's behind this?

- Some investors cannot move up (northeast) along the CML (capital market line) because they are leverage constrained
- Instead, leverage-constrained investors will overweight riskier assets (substitute for leverage), driving up the price of riskier assets
- This invalidates the CAPM, which assumes everyone invests on the CML in the mean-volatility diagram
- Safer assets are underweighted by leverage-constrained investors and are underpriced as a result
- The mean-variance efficient portfolio will overweight safer assets

Leverage-constrained investors?

- Evidence of Leverage-constrained investors
  - Some investors are not allowed to borrow:
    - Mutual funds
    - Pension funds
  - Mutual funds provide asset allocations for low-to-high-risk-tolerant investors
    - High-risk-tolerant investors are told to concentrate in equities
  - Embedded leverage in ETFs

Caveats (1)

- RP does very well in sample of secularly declining long-term interest rates 1980-2012
- RP completely ignores valuations; only looks at volatility
  - Volatility tends to be low when prices are high
  - Pro-cyclical bias in investing.

Caveats (2)

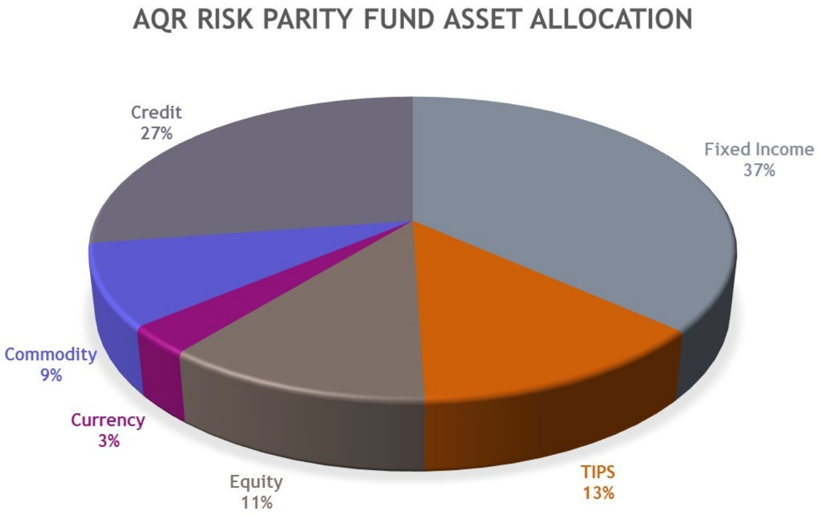
- Lots of leverage required
- Different investment technology
  - Risks associated with high leverage investing
    - LTCM (Long Term Capital Management) is a great example of the risks posed by highly levered portfolios during financial crises.
    - This risk is not reflected in standard volatility measures.

Leverage requires a different technology: 'Getting leverage requires getting financing, using derivatives, and establishing counterparty relations' Asness, Frazzini and Pedersen (2012, Financial Analysts Journal)

- We ignored the actual costs associated with leverage in the exercise
  - Financing spreads
  - Costs of deleveraging

AQR Risk Parity Fund

- On Feb. 2014
  - Net Asset Value: $ 863,202,986.00
  - Total Exposure: $ 2,724,182,816
  - Implied Leverage: 3.15



Source: AQR Risk parity Fund

AQR Risk Parity Fund Derivatives positions

- Equity: futures
- Fixed income: bond futures and interest rate swaps
- Currency: forwards and futures
- Commodities: forwards and futures
- Credit: Credit Default Swaps

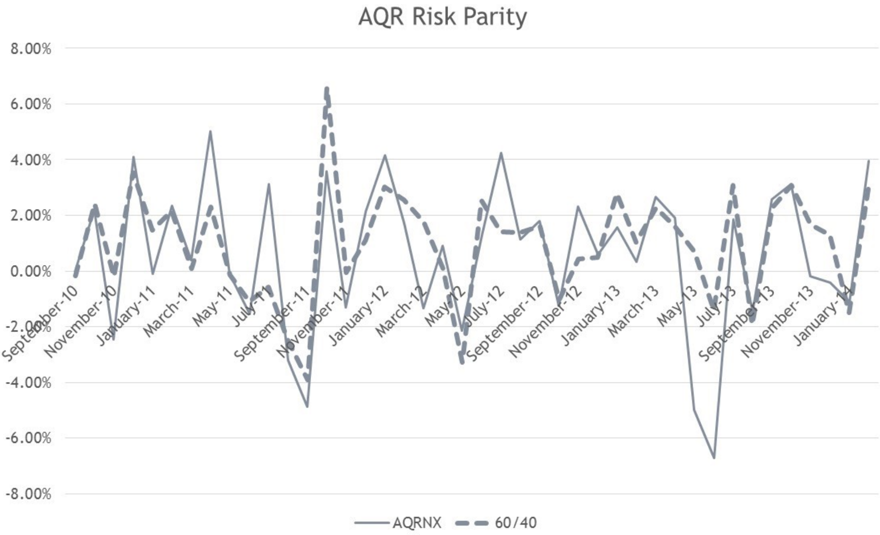
Fixed Income

		Exposure	Quantity
Fixed Income	Australia 10 Yr Bond Future	12,778,079	149
Fixed Income	Canada 10 Yr Bond Future	21,478,516	182
Fixed Income	Czech Republic Interest Rate Swap	19,307,020	370,000,000
Fixed Income	Euro Bund 10 Yr Bund Future	180,017,980	902
Fixed Income	Hong Kong Interest Rate Swap	39,379,347	299,000,000
Fixed Income	Hungary Interest Rate Swap	20,853,965	4,500,000,000
Fixed Income	Japan 10 Yr Bond Future	67,023,477	47
Fixed Income	Poland Interest Rate Swap	42,900,367	126,000,000
Fixed Income	Singapore Interest Rate Swap	76,518,961	99,500,000
Fixed Income	South Africa Interest Rate Swap	41,522,526	488,000,000
Fixed Income	South Korea Interest Rate Swap	81,455,037	86,000,000,000
Fixed Income	U.S. 10 Yr Treasury Note Future	394,390,469	3,167

Performance of RP Fund

	Risk Parity	Risk Parity	60/40
			Blended
Month-end Returns as of	Class N	Class I	S&P / Barc
February 28, 2014	AQRNX	AQRIX	Custom Index
1 Year	0.62%	0.88%	14.80%
3 Years	6.33%	6.57%	10.46%
5 Years	NA	NA	NA
10 Years	NA	NA	NA
Since Inception Cumulative Return	27.56%	28.69%	47.61%
Since Inception Annualized Return	7.38%	7.66%	12.06%
Since Inception Annualized Monthly Volatility	9.20%	9.15%	6.98%
Since Inception Sharpe Ratio	0.79	0.83	

60/40 Portfolio: Consists of 60% S&P 500 Index / 40% Barclays Capital Aggregate Bond Index.


AQR Risk Parity
8.00%
6.00%
4.00%
2.00%
0.00%
-2.00%
-4.00%
-6.00%
-8.00%
September-10
November-10
January-11
March-11
May-11
July-11
September-11
November-11
January-12
March-12
May-12
July-12
September-12
November-12
January-13
March-13
May-13
July-13
September-13
November-13
January-14
AQRNX
60/40

Chaves, Hsu, Li, and Shakernia (2011)

Risk Parity Portfolio vs. Other Asset Allocation Heuristic Portfolios

Risk Parity with more asset classes

Risk Parity vs. Other Portfolio Heuristics (with Nine Asset Classes), January 1980–June 2010

	Excess Return over T-bill	Volatility	Sharpe Ratio
60/40 S&P 500/BarCap Agg	5.1%	10.1%	0.50
U.S. Pension Model Portfolio (with 60/40 anchor)	5.1%	9.8%	0.52
Risk Parity Portfolio	3.8%	7.5%	0.51
Equal Weighting	4.5%	8.8%	0.51
Minimum Variance Weighting	1.6%	6.6%	0.24
Mean–Variance Optimal Weighting	4.4%	10.3%	0.43

Risk Parity in Subsamples

Subsample Analysis of Sharpe Ratios: Risk Parity vs. Other Portfolio Heuristics (with Nine Asset Classes), January 1980–June 2010

	Full Sample: Jan. 1980–Jun. 2010	Jan. 1980–Dec. 1989	Jan. 1990–Dec. 1999	Jan. 2000–Dec. 2009
60/40 S&P 500/BarCap Agg	0.50	0.56	0.99	0.04
U.S. Pension Model Portfolio (with 60/40 anchor)	0.52	0.63	0.89	0.15
Risk Parity Portfolio	0.51	0.39	0.69	0.54
Equal Weighting	0.51	0.49	0.64	0.48
Minimum Variance Weighting	0.24	–0.02	0.28	0.49
Mean–Variance Optimal Weighting	0.43	0.60	0.56	0.18

Take-away

Mean-variance analysis is useful but has significant drawbacks when it comes to implementation out-of-sample

Mean-variance analysis ignores tail risk (or assumes investors do not care about tail risk)

Alternatives have been proposed.