UCLA ANDERSON SCHOOL OF MANAGEMENT

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Problem Set 2

These exercises do not need to be turned in for credit.

1 Consider a binomial model with $T = 4$, $r = 0.02$, $h = 0.25$, $u = e^{rh+0.2\sqrt{h}}$, $d = e^{rh-0.2\sqrt{h}}$, and $S_0 = 100$. Consider also a put with maturity T and strike $K = 90$ and denote by $v_t(s)$ the price of this option at date t ($0 \le t \le T$) in the state where the stock price is s.

a. Compute the risk-neutral probability of an increase in the stock price.

b. Develop an algorithm for computing v_t recursively. In particular, write a formula for v_t in terms of v_{t+1}.

c. Apply the algorithm you developed in the above question to compute the value of the put option at the initial date and at all points on the tree.

d. Provide a formula for the number of shares $\Delta_{t+1}(s)$ that should be held in the replicating portfolio between t and $t + 1$ when the stock is worth s at date t.

e. Compute the value at date 3 and when the stock price equals 91.851, i.e., $v_3^{\text{call}}(91.851)$, of a call option with maturity T and strike $K = 90$.

2 Consider a binomial model with $T = 3$, $r = 0.02$, $h = 0.25$, $u = e^{rh+0.1\sqrt{h}}$, $d = e^{rh-0.1\sqrt{h}}$, and $S_0 = 100$.

a. Compute the risk-neutral probability of an increase in the stock price.

b. Determine the initial price P_0^a of an American put with maturity T and payoff function $g_P(s) = \max[112 - s, 0]$.

c. Determine the initial price C_0^a of an American call with maturity T and payoff function $g_C(s) = \max[s - 112, 0]$.

d. Determine the initial price S_0^a of an American straddle with maturity T and payoff function $g_S(s) = g_P(s) + g_C(s)$. Explain why you find $S_0^a < P_0^a + C_0^a$.

e. Determine the initial price A_0^a of an American Asian put option which pays off the nonnegative amount

$$G_t = \max\left[100 - \frac{1}{t+1}\sum_{k=0}^{t} S_k, 0\right]$$

when exercised at time t. What portfolio should you hold between times zero and one in order to hedge such a derivative?

3 Suppose $S_0 = 55$, $K = 50$, $r = 0.06$, $u = 1.285233$, and $d = 0.809822$. A **barrier option** has a payoff that depends upon whether the price of the underlying asset reaches a specified level (over the whole life of the option), called a **barrier** B. The stock does not pay dividends ($\delta = 0$). Consider a 3-period binomial tree.

a. Assuming the barrier is $B = 90$, find the price of the **up-and-in call**: a call option that comes into existence if the barrier is touched.

b. Is the up-and-in call more or less expensive than a European call? Comment.

c. Assuming the barrier is $B = 40$, find the price of a **down-and-out put**: a put option that goes out of existence if the barrier is touched.

d. A **lookback put** pays $S_T^* - S_T$ at maturity, where $S_t^* = \max_{\tau \leq t} S_\tau$ denotes the **running maximum** of the stock price. With such an option the holder achieves perfect timing. This option requires that you keep track of both the current stock price S and its maximum S^*. Find the option price.

e. Is the lookback put more or less expensive than an American put? Explain.

4 Suppose we want to price options on a non-dividend paying stock, $S(t)$. The initial stock price at $t_0 = 0$ is $S(t_0) = 1$ and the continuously compounded interest rate is $r = 0.05$. We simulate 8 stock price paths under the risk-neutral measure, which are given Table 1

Path	$t_1 = 1$	$t_2 = 2$	$t_3 = 3$
1	0.80	0.93	1.20
2	1.07	1.20	1.05
3	0.91	0.85	0.77
4	1.03	1.17	1.32
5	0.95	1.12	0.87
6	1.10	0.87	1.13
7	1.12	1.03	0.81
8	0.89	0.97	1.23

Table 1: Stock price paths

a. Compute the price of an up-and-in call option with strike $K = 1$, barrier at $S_b = 1.20$, and expiration at t_3. The barrier is monitored at times t_1, t_2, and t_3.

b. Compute the price of a down-and-out put option with strike $K = 1$, barrier at $S_b = 0.80$, and expiration at t_3. The barrier is monitored at times t_1, t_2, and t_3.

c. Compute the price of an Asian call option with strike $K = 1$ and a payoff at t_3 of $\max\left[\frac{1}{3}\sum_{i=1}^{3} S(t_i) - K; 0\right]$, i.e. one with discrete arithmetic averaging.

5 Suppose we want to price options on a non-dividend paying stock, $S(t)$. The initial stock price at $t_0 = 0$ is $S(t_0) = 1$ and the instantaneous interest rate is $r = 0.05$. We simulate 8 stock price paths under the risk neutral measure, which are given in Table 2.

Path	$t_1 = 1$	$t_2 = 2$	$t_3 = 3$
1	1.18	1.28	1.51
2	0.89	1.18	1.32
3	1.12	1.43	1.34
4	0.78	0.74	0.87
5	1.07	0.97	1.33
6	0.91	0.95	0.94
7	1.24	1.06	1.05
8	0.94	0.85	1.04

Table 2: Stock price paths

a. Compute the price of a standard European put option with strike $K = 1$ expiring at t_3.

b. Compute the price of an American put option with strike $K = 1$, final expiration date t_3, and which is exercisable at times t_1, t_2, and t_3. Use the LSM approach of Longstaff and Schwartz (2001). This involves running a cross-sectional regression at times t_1 and t_2 of the realized discounted cash flows from continuation, which we denote as Y, on a constant, $S(t)$ and $S(t)^2$. The resulting conditional expectations functions are

$$\mathbb{E}_{t_1}[Y] = 23.73 - 54.56S(t_1) + 31.35S(t_1)^2 \tag{1}$$

$$\mathbb{E}_{t_2}[Y] = 3.93 - 8.77S(t_2) + 4.90S(t_2)^2 \tag{2}$$