Solutions to Final Exam 2017, MGMT 237E – Empirical Methods in Finance

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Multiple Choice (80 points) 2 points per question. **All numbers are rounded to two decimal points. Some questions continue on the following page, make sure you see all answer options before answering.**

1. Which of the below statements best describes daily stock market returns?

   (a) Daily stock market returns have negative excess kurtosis and positive skewness

   (b) Daily stock market returns have negative excess kurtosis and negative skewness

   (c) Daily stock market returns have positive excess kurtosis and positive skewness

   (d) **Daily stock market returns have positive excess kurtosis and negative skewness**

   (e) None of the above

2. The Jarque-Bera test statistics is JB = (S(r))^2 / (6/T) + (K(r) - 3)^2 / (24/T). What is its asymptotic distribution (ignoring degrees of freedom considerations)?

   (a) Normal

   (b) Student-t

   (c) **Chi-square**

   (d) F

   (e) None of the above

3. Assume you have 81 observations of annual stock returns. The sample standard deviation of excess returns is 18%. The sample mean excess return is 5%. What is the standard error of the estimate of the unconditional mean based on this data?

   (a) 0.56%

   (b) 0.06%

   (c) **2.00%**

(d) 0.22%

(e) None of the above

4. Assume the log price follows a Random Walk with drift. Assume the drift term is 5% and the volatility of the shocks to price is 10%. What is the unconditional mean of this process?

   (a) 5%

   (b) 10%

   (c) 2%

   (d) 1%

   (e) **None of the above**

5. White standard errors for linear regression estimates account for:

   (a) Heteroskedasticity in the dependent variable

   (b) Heteroskedasticity in the independent variable

   (c) **Heteroskedasticity in the residuals**

   (d) None of the above

6. Assume the single factor Market Model with uncorrelated residuals. Assume the standard deviation of market returns is 20%. Firm A returns has beta of 0.5 and a residual standard deviation of 5%. Firm B returns has a beta of 1.5 and a standard deviation of 3%. What is the covariance of the returns to firm A and firm B?

   (a) 15%

   (b) **3%**

   (c) 3.34%

   (d) 23%

   (e) None of the above

7. Which of the below is **not** a reason to sort into portfolios when testing an expected return model?

   (a) It decreases the standard error of estimated factor betas

   (b) **It creates a greater spread in average returns**

(c) It ensures a balanced panel

(d) It makes it easier to estimate the variance-covariance matrix of residuals

(e) None of the above

8. Which of the below is **not** true about the HML factor in the Fama-French 3 factor?

   (a) A regression of the HML factor on the market factor yields a market beta less than 0.5

   (b) The underlying portfolio goes short growth firms and long value firms

   (c) The average return to HML has been positive in historical data

   (d) **HML has volatility less than half of the market portfolio due to its long-short nature**

   (e) None of the above

9. The fraction of overall variance explained by the first principal component equals

   (a) The first eigenvalue

   (b) The first eigenvector

   (c) **The first eigenvalue divided by the sum of all the eigenvalues**

   (d) The first eigenvector divided by the sum of all the eigenvectors

   (e) None of the above

10. In the APT, the assumption of no-arbitrage is needed because

    (a) Otherwise the residuals will be correlated across stocks

    (b) Otherwise the factors cannot be traded

    (c) **Otherwise the number of factors can be as many as the number of stocks**

    (d) Otherwise, factor sensitivities (betas) will vary over time

    (e) None of the above

11. In a time-series CAPM regression of excess returns to stock i on excess returns of the market, a positive alpha (ignoring statistical uncertainty) means that

    (a) The expected return on the stock must be higher than the expected return on the market portfolio

(b) The stock is over-valued

(c) The stock has the maximum Sharpe ratio

(d) The market has the maximum Sharpe ratio

(e) **None of the above**

12. Consider the Fama-French three factor model. The prices of risk are estimated to be: 6% (Mkt), 5% (HML), 3% (SMB). The risk-free rate is 2%. A firm with betas 0.7 (Mkt), 1.2 (HML), -0.3 (SMB) has expected excess return equal to:

    (a) **9.3%**

    (b) 11.3%

    (c) 6.1%

    (d) 8.1%

    (e) None of the above

13. The portfolio weights in the mean-variance efficient portfolio are

    (a) proportional to the product of the expected return vector and the variance-covariance matrix of returns

    (b) proportional to the product of the expected excess return vector and the variance-covariance matrix of returns

    (c) proportional to the product of the expected return vector and inverse of the variance-covariance matrix of returns

    (d) **proportional to the product of the expected excess return vector and inverse of the variance-covariance matrix of returns**

    (e) None of the above

14. If market returns are negatively autocorrelated

    (a) The variance ratio is increasing with the horizon

    (b) **The variance ratio is decreasing with the horizon**

    (c) The variance ratio is constant

    (d) None of the above

15. The GRS statistic in a test of the CAPM is a measure of the distance between the

(a) Expected return on the market and the expected return on the mean-variance efficient portfolio

(b) **Sharpe ratio of the market and the Sharpe ratio of the mean-variance efficient portfolio**

(c) Average alpha of the test assets and the average excess returns to the market

(d) Average R2 in the firms' time series regressions on the market, and zero

16. A regression of firm average excess returns on their market betas should, under the CAPM, yield a slope coefficient equal to

    (a) the risk-free rate

    (b) α

    (c) **the market risk premium**

    (d) the maximum Sharpe ratio

    (e) None of the above

17. For a covariance-stationary process, the unconditional covariance between r_t and r_{t+j}

    (a) **Is the same as that between r_{t-j} and r_t**

    (b) Depends on the current value of r_t

    (c) Depends on the current value of all state variables in a VAR

    (d) Is a function of time t

    (e) None of the above

18. The Ljung-Box test is a test of whether the m first autocorrelations of a series

    (a) are on average equal to zero

    (b) **are all equal to zero zero**

    (c) have one of the m autocorrelations being significantly different from zero

    (d) None of the above

19. A long-horizon investor should invest more in stocks than a short-horizon investor if

    (a) the variance ratio is increasing over time

(b) the risk premium is positive

(c) the standard deviation of returns is mean-reverting

(d) **none of the above**

20. Assume x_t follows an AR(1) with unconditional mean equal to 0.05 and an auto-correlation coefficient of 0.8. If the current value of x_t equals 0.1, what is the two periods ahead predicted value, $E_t[x_{t+2}]$?

    (a) 0.09

    (b) **0.082**

    (c) 0.064

    (d) 0.032

    (e) None of the above

21. When we use the I-GARCH(1,1) process with zero intercept in the vol specification for forecasting the variance at long horizons,

    (a) the forecast converges to zero

    (b) **the forecast is fixed at the conditional variance**

    (c) the forecast converges to the unconditional variance

    (d) None of the above

22. The first-order autocorrelation of the ARMA(1,1), $x_{t+1} = \phi_1 x_t - \theta_1 \varepsilon_t + \varepsilon_{t+1}$, with $\phi_1 = 0.9$ and $\theta_1 = 0.4$, is:

    (a) 0.90

    (b) 0.81

    (c) 0.52

    (d) 0.50

    (e) **none of the above (it's 0.727)**

23. To check the stationarity of an ARMA(p, q) process, we need to look at

    (a) the roots of the characteristic equation of the MA-component

    (b) **the roots of the characteristic equation of the AR-component**

(c) the roots of the characteristic equation of the MA-component and the AR-component

(d) None of the above

24. The main idea in statistical factor analysis is to

    (a) **to fight the curse of dimensionality by finding a couple of factors that account for most of the variation in a cross-section of N time series**

    (b) to find the true economic driving forces

    (c) to find uncorrelated macro-economic and financial variables

    (d) None of the above

25. A linear factor model works well if it

    (a) explains most of the time series variation in returns on all the test assets

    (b) explains most of the large, negative returns

    (c) explains both the time series variation in returns and the cross-sectional variation in average returns

    (d) **explains most of the cross-sectional variation in average returns on all the test assets**

26. The log-linearization of the return equation around the mean log price/dividend ratio delivers the following expression for log returns:

$$r_{t+1} = \Delta d_{t+1} + \rho p d_{t+1} + k - p d_t,$$

with a linearization coefficient ρ that depends on the mean of the log price/dividend ratio pd:$\rho = \frac{e^{pd}}{e^{pd}+1} < 1$. Assume that $\rho = 0.96$, that the volatility of annualized dividend growth is 12%, and that realized log dividend growth and returns are both i.i.d. Given this information, what is the volatility of returns?

    (a) **12.00%**

    (b) 16.97%

    (c) 24.00%

    (d) Cannot tell from the information given

(e) None of the above

27. For an AR(1) process $y_t = \phi_0 + \phi_1 y_{t-1} + \varepsilon_t$, the half-life of a shock h is given by

   (a) $h = 0.5/\log\phi_1$ periods.
   (b) $h = 0.5/\phi_1$ periods.
   (c) $h = \phi_1/0.5$ periods.
   (d) $\mathbf{h} = \log \mathbf{0.5}/\log|\boldsymbol{\phi}_1|$ **periods.**
   (e) None of the above.

28. Assume the following ARMA(1,1) process: $y_t = 0.9y_{t-1} - 0.8\varepsilon_{t-1} + \varepsilon_t$. If $y_t = 0.5$ and $\varepsilon_t = -0.3$, what is $E_t[y_{t+2}]$?

   (a) 0.00
   (b) 0.41
   (c) 0.56
   (d) **0.62**
   (e) 0.69
   (f) None of the above.

29. Choose the statement that is the most correct from the below. The Fama-French five-factor model predicts that:

   (a) **Firms with high profitability and low investment have high discount rates**
   (b) Firms with high profitability and high investment have high discount rates
   (c) Firms with low profitability and low investment have high discount rates
   (d) Firms with low profitability and high investment have high discount rates

30. The notion that high momentum stocks deliver high subsequent returns is consistent with

   (a) i.i.d. stock returns.
   (b) mean reversion in stock returns.
   (c) heteroskedasticity in stock returns

(d) **slow dissemination of information**

(e) None of the above.

31. Ross' arbitrage pricing theory (APT) says that, in a well-defined asset pricing model,

    (a) only traded assets are valid risk factors with a well-defined risk price

    (b) **even non-traded assets are valid risk factors with a well-defined risk price**

    (c) we can have multiple traded risk factors, but only for some test assets

    (d) the return on the market must be a risk factor

    (e) None of the above

32. Assume a forecasting regression of annual excess stock market returns on a one-year lagged predictive variable yields an R^2 of 9%. If stock return volatility is 20%, what is the volatility of the conditional annual risk premium, as estimated by this regression?

    (a) 0.0%

    (b) 1.2%

    (c) 1.8%

    (d) **6.0%**

    (e) None of the above

33. When we use the ARCH(1) model $\varepsilon_t = \sigma_t \eta_t;\ \ \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$ for forecasting, the optimal 2-period ahead forecast is given by:

    (a) $\boldsymbol{\sigma_t^2(2) = \alpha_0 + \alpha_1 \sigma_t^2(1)}$

    (b) $\sigma_t^2(2) = \alpha_0 + \alpha_1 \varepsilon_t^2$

    (c) $\sigma_t^2(2) = \alpha_0 + \alpha_1 \sigma_t^2$

    (d) None of the above

34. The unconditional variance of a GARCH(1,1) variable $\varepsilon_t = \sigma_t \eta_t;\ \ \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

    (a) $\alpha_0/(1 - (\alpha_1 + \beta_1)^2)$

(b) $\boldsymbol{\alpha}_0/(\mathbf{1}-\boldsymbol{\alpha}_1-\boldsymbol{\beta}_1)$

(c) α_0

(d) σ_t^2

(e) None of the above

35. The $q+1$-period ahead forecast of an MA(q) is

(a) zero

(b) the conditional mean

(c) **the unconditional mean**

(d) None of the above

36. RiskMetrics uses the 5% quantile of a normal to determine the Value-at-Risk at horizon k:

(a) VaR(k)= amount of position $\times 1.65 k \sigma_{t+1}^2$

(b) **VaR(k)= amount of position** $\times \mathbf{1.65}\sqrt{k}\boldsymbol{\sigma}_{t+1}$

(c) VaR(k)= amount of position $\times 1.96 k \sigma_{t+1}$

(d) VaR(k)= amount of position $\times 1.96\sqrt{k}\sigma_{t+1}$

(e) None of the above

37. To test a multi-factor asset pricing model, we can look at the intercepts in a multiple time-series regression of excess returns on the factors

(a) if the factors have a mean of zero

(b) if the factors are not traded returns

(c) **if the factors are traded returns**

(d) in none of these cases

38. Your analyst has run forecasting regressions using overlapping returns, but wonders how many lags he/she should use when applying Newey-West standard errors. The data is quarterly, the forecasting horizon is annual. How many lags is the most appropriate in this case?

(a) 0

(b) 1

(c) 3

(d) **More than 3 but less than 9**

(e) You should always use as many lags as possible (i.e., 9 or more)

39. The EGARCH and GJR-GARCH models can explain the following stylized fact of stock market volatility that the GARCH model cannot explain"

    (a) Stationarity

    (b) Clustering

    (c) **The leverage effect**

    (d) Persistence

    (e) None of the above

40. Both the autocorrelation and the partial autocorrelation functions of realized daily stock market return variance decay very slowly, significant even after 20 lags. Assume you will use 1 year of daily data to estimate your model. Given this information, which of the following models do you think will likely give the best out-of-sample forecasts of stock market variance?

    (a) An AR(1) model on realized variance

    (b) ARCH(1)

    (c) ARCH(20)

    (d) **GARCH(1,1)**

    (e) GARCH(20,20)

1. **Cash flow forecasting (25 pts)**: You are tasked with providing a forecast for market log return on equity ($roe_{t+1} = \ln\left(1 + \frac{\text{Net Income}_{t+1}}{\text{Book Value of Equity}_t}\right)$) for use in a valuation project. You have T annual time series observations of roe_t and the aggregate (market) log book-to-market ratio, bm_t. You believe that **both** the current bm and roe are important significant predictors of future roe. You further believe that it is not useful to use further lags of either variable or to consider non-linear variations of bm and roe in the forecasting exercise. Your boss wants the current 1 year ahead, 5 years ahead and 10 years ahead forecasts.

   (a) (**10 points**) Give the model you would use to answer this question. In particular, write down the model dynamics and assumptions on residuals. Make sure to clearly define all variables and take care to use appropriate time subscripts so the forecasting aspect of your model is clear.

   Solution:

   The question asks for a VAR(1) specification:

$$z_{t+1} - \boldsymbol{\mu} = \boldsymbol{\phi}\left(z_t - \boldsymbol{\mu}\right) + \varepsilon_{t+1}$$

   where

$$z_t = \begin{bmatrix} bm_t \\ roe_t \end{bmatrix}$$

   and $\boldsymbol{\mu} = [\mu_1 \;\; \mu_2]'$ is a 2×1 vector and $\boldsymbol{\phi}$ is a 2×2 matrix with elements:

$$\boldsymbol{\phi} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}.$$

   The residual vector $\varepsilon_t = [\varepsilon_{1t} \;\; \varepsilon_{2t}]$ has a 2×2 covariance matrix Σ. We could assume the residuals are jointly Normal and i.i.d., but this is not needed for consistency. You do need your residuals to be stationary, however. This could be implied from the assumptions you state. Your residual assumption is important for the discussion of the estimation below.

   (b) (**5 points**) How would you estimate your model and obtain standard errors for the relevant estimated parameters?

   Solution:

Estimate the model via OLS. That is, run the following two regressions

$$\begin{aligned} bm_{t+1} &= \phi_{01} + \phi_{11} bm_t + \phi_{12} roe_t + \varepsilon_{1,t+1} \\ roe_{t+1} &= \phi_{02} + \phi_{21} bm_t + \phi_{22} roe_t + \varepsilon_{2,t+1}, \end{aligned}$$

where

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = (I_2 - \boldsymbol{\phi})^{-1} \begin{bmatrix} \phi_{01} \\ \phi_{02} \end{bmatrix}$$

OLS is consistent even if the errors are non-normal and/or not i.i.d. In the case of heteroskedasticity and autocorrelation, the standard errors of the regression coefficients must be estimated accordingly using, e.g., Newey-West standard errors. The number of lags is unclear here and an empirical question. One could also estimate with maximum likelihood given a particular assumption on the residual distribution.

(c) (**5 points**) Assuming you have successfully estimated the parameters of your model, write down the equations that give the current (time T) estimate of 1-, 5-, and 10-years ahead forecasts for roe.

Solution:

Define the vector $e_{roe} = [0\ 1]$. Thus, $e_{roe} z_{t+k} = roe_{t+k}$. Then the 1-period ahead forecast is:

$$E_T \left[e_{roe} z_{T+1} \right] = e_{roe} \left(\boldsymbol{\mu} + \boldsymbol{\phi} \left(z_T - \boldsymbol{\mu} \right) \right).$$

The k-period ahead forecast is

$$E_t \left[e_{roe} z_{T+k} \right] = e_{roe} \left(\boldsymbol{\mu} + \boldsymbol{\phi}^k \left(z_T - \boldsymbol{\mu} \right) \right).$$

(d) (**5 points**) An earnings-based version of the Campbell-Shiller decomposition due to Ohlson (1995) implies that:

$$bm_t = \text{constant } + \sum_{j=1}^{\infty} \rho^{j-1} E_t \left[r_{t+j} \right] - \sum_{j=1}^{\infty} \rho^{j-1} E_t \left[roe_{t+j} \right]$$

where r_{t+j} denotes log market returns at time $t+j$ and ρ is a log-linearization coefficient close to but less than 1. Give the analytical expression for the cash flow component of the aggregate book-to-market ratio. That is, solve for $\sum_{j=1}^{\infty} \rho^{j-1} E_t [roe_{t+j}]$ using your forecasting model.

Solution:

$$\begin{aligned}
\sum_{j=1}^{\infty} \rho^{j-1} E_t [roe_{t+j}] &= \sum_{j=1}^{\infty} \rho^{j-1} e_{roe} \left(\boldsymbol{\mu} + \boldsymbol{\phi}^j (z_t - \boldsymbol{\mu}) \right) \\
&= e_{roe} \left(\sum_{j=1}^{\infty} \rho^{j-1} \boldsymbol{\mu} + \sum_{j=1}^{\infty} \rho^{j-1} \boldsymbol{\phi}^j (z_t - \boldsymbol{\mu}) \right) \\
&= e_{roe} \left(\frac{1}{1-\rho} \boldsymbol{\mu} + \boldsymbol{\phi} \sum_{j=1}^{\infty} \rho^{j-1} \boldsymbol{\phi}^{j-1} (z_t - \boldsymbol{\mu}) \right) \\
&= e_{roe} \left(\frac{1}{1-\rho} \boldsymbol{\mu} + \boldsymbol{\phi} (I_2 - \rho \boldsymbol{\phi})^{-1} (z_t - \boldsymbol{\mu}) \right).
\end{aligned}$$

2. **Cross-sectional Factor Models (25 pts)**: Assume you have been given three assets to invest in, in addition to the market portfolio. From a historical regression of excess asset returns on the excess market return for $t = 1, ..., T$, you have:

$$\begin{aligned}
R_{1t}^e &= 0.01 + 0.8 R_{mt}^e + \hat{\varepsilon}_{1t}, \\
R_{2t}^e &= -0.015 + 1.2 R_{mt}^e + \hat{\varepsilon}_{2t}, \\
R_{3t}^e &= 0.005 + 1.0 R_{mt}^e + \hat{\varepsilon}_{3t}.
\end{aligned}$$

The sample mean excess return on the market is, $\bar{R}_m^e = 0.05$; the sample standard deviation of excess market returns is 15%. Thus, the market Sharpe ratio is 1/3. Finally, the sample variance-covariance matrix of residual returns, $\varepsilon_t = [\varepsilon_{1t} \;\; \varepsilon_{2t} \;\; \varepsilon_{3t}]'$, is:

$$var(\hat{\varepsilon}_t) = \hat{\Sigma}_\varepsilon = \begin{bmatrix} 0.1^2 & 0 & 0 \\ 0 & 0.1^2 & 0 \\ 0 & 0 & 0.1^2 \end{bmatrix}.$$

(a) (**5 points**) Consider the three corresponding factor-hedged assets, where you hedge each asset's exposure to the market factor. What is the maximum Sharpe ratio you can obtain by combining these three hedged assets? Show the math behind your calculations.

Solution:

Hedged asset returns:

$$\begin{aligned} R^e_{1t} - 0.8R^e_{mt} &= 0.01 + \hat{\varepsilon}_{1t} \\ R^e_{2t} - 1.2R^e_{mt} &= -0.015 + \hat{\varepsilon}_{2t} \\ R^e_{3t} - 1.0R^e_{mt} &= 0.005 + \hat{\varepsilon}_{3t} \end{aligned}$$

The maximal squared Sharpe ratio is then given by:

$$[0.01 \quad -0.015 \;\; 0.005] \begin{bmatrix} 0.1^{-2} & 0 & 0 \\ 0 & 0.1^{-2} & 0 \\ 0 & 0 & 0.1^{-2} \end{bmatrix} \begin{bmatrix} 0.01 \\ -0.015 \\ 0.005 \end{bmatrix}$$

$$= \frac{0.01^2}{0.1^2} + \frac{0.015^2}{0.1^2} + \frac{0.005^2}{0.1^2} = 0.035.$$

Thus, the maximum Sharpe ratio is

$$\max SR = \sqrt{0.035} = 0.1871.$$

(b) (**5 points**) What is the maximum Sharpe ratio you can obtain by combining these three hedged assets with the market portfolio? Show the math behind your calculations.

Solution:

Here we use the fact that:

$$\begin{aligned} \mu' V^{-1} \mu &= \lambda' \Sigma_f^{-1} \lambda + \alpha' \Sigma_\varepsilon^{-1} \alpha \\ &= (1/3)^2 + 0.035 \\ &= 0.146 \end{aligned}$$

Thus, the maximal Sharpe ratio available is $\sqrt{0.146} = 0.382$.

(c) (**10 points**) Next, you run Fama-MacBeth regressions of the three asset returns on their market betas and an intercept. For your convenience, the cross-sectional average beta is 1 and cross-sectional variance of the betas is $\frac{2}{75}$. Give the mean return, standard deviation, and Sharpe ratio of the factor-mimicking portfolio that is implied by the regression.

Solution:

From the notes, we have that

$$\lambda_1 = \frac{1}{N}\frac{\beta' - E^I[\beta]}{var^I[\beta]}\bar{R}^e.$$

Thus, the weight in each of the three assets is:

$$\begin{aligned} w_1 &= \frac{1}{3}\frac{0.8-1}{2/75} = -2.5, \\ w_2 &= \frac{1}{3}\frac{1.2-1}{2/75} = 2.5, \\ w_3 &= \frac{1}{3}\frac{1-1}{2/75} = 0 \end{aligned}$$

The mean return is then:

$$\begin{aligned} E[R_P^e] &= -2.5(0.01 + 0.8\times 0.05) + 2.5(-0.015 + 1.2\times 0.05) \\ &= -1.25\% \end{aligned}$$

The variance is:

$$\begin{aligned} var(R_P^e) &= 2.5^2\left(0.8^2\times 0.15^2 + 0.1^2\right) + 2.5^2\left(1.2^2\times 0.15^2 + 0.1^2\right) \\ &\quad -2\times 2.5\times 2.5\times 0.8\times 1.2\times 0.15^2 \\ &= 0.1475. \end{aligned}$$

Thus, the standard deviation is $\sqrt{0.1475} = 38.406\%$.

The Sharpe ratio of this factor-mimicking portfolio is then

$$SR(R_P^e) = \frac{-1.25\%}{38.406\%} = -0.0325.$$

(d) (**5 points**) The correlation between this factor-mimicking portfolio and the market return is far less than 1, even ignoring any estimation error. One may have thought that what is recovered is exactly the factor return since we use the factor betas in the cross-sectional Fama-MacBeth regression. Why is it then, that the correlation is not perfect? Give a short and clear answer.

Solution:

The factor-mimicking portfolio has a much lower Sharpe ratio than the market, as firm alpha is negatively correlated with firm beta in the cross-section. The existence of alpha means there is a missing factor. Exposure to this factor is correlated with the exposure to the market factor, given the correlation between alpha and beta. Therefore, the factor-mimicking portfolio is a combination of the market factor and this missing factor.

3. **AR(p) processes (15 pts)**: Consider an AR(2) process with $\phi_1 = 1.1$ and $\phi_2 = -0.25$ (following the notation in the lecture notes).

   (a) (**5 points**) Is the process stationary? Show why or why not. No credit is given without a mathematical proof.

   Solution:

   Here, we need to find the characteristics roots of the equation:

   $$1 - 1.1x + 0.25x^2 = 0$$

   The roots of this polynomial are $\begin{bmatrix} 1.283\,5 \\ 3.116\,5 \end{bmatrix}$. Since the inverse of both roots are less than 1, the AR(2) process is stationary.

   (b) (**5 points**) Give the dynamic multiplier for a shock that occurred 2 periods ago. That is, calculate $\frac{\partial[r_{t+2}-\mu]}{\partial\varepsilon_t}$ (following the notation in the lecture notes).

   Solution:

$$\begin{aligned}
y_t &= \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t \\
&= \phi_1 \left(\phi_1 y_{t-2} + \phi_2 y_{t-3} + \varepsilon_{t-1}\right) + \phi_2 y_{t-2} + \varepsilon_t \\
&= \left(\phi_1^2 + \phi_2\right)\left(\phi_1 y_{t-3} + \phi_2 y_{t-4} + \varepsilon_{t-2}\right) + \phi_1\phi_2 y_{t-3} + \phi_1\varepsilon_{t-1} + \varepsilon_t \\
&= \left(\phi_1^2 + \phi_2\right)\phi_1 y_{t-3} + \left(\phi_1^2 + \phi_2\right)\phi_2 y_{t-4} + \left(\phi_1^2 + \phi_2\right)\varepsilon_{t-2} \ldots \\
&\quad +\phi_1\phi_2 y_{t-3} + \phi_1\varepsilon_{t-1} + \varepsilon_t.
\end{aligned}$$

Thus, the impact of a unit shock to ε_t on y_{t+2} is $\phi_1^2 + \phi_2$

$$\begin{aligned} \frac{\partial \left[r_{t+2} - \mu \right]}{\partial \varepsilon_t} &= \phi_1^2 + \phi_2 \\ &= 1.1^2 - 0.25 = 0.96 \end{aligned}$$

(c) (**5 points**) Now, instead assume $\phi_1 = 0.9$ and $\phi_2 = 0.8$. (i) Give the dynamic multiplier for a shock that occurred 2 periods ago. (ii) Is the process stationary? No credit is given without a mathematical proof.

In this case:

$$\begin{aligned} \frac{\partial \left[r_{t+2} - \mu \right]}{\partial \varepsilon_t} &= \phi_1^2 + \phi_2 \\ &= 0.9^2 + 0.8 = 1.61. \end{aligned}$$

Finding the roots of

$$1 - 0.9x - 0.8x^2 = 0$$

Roots $= \begin{bmatrix} 0.689\,06 \\ -1.814\,1 \end{bmatrix}$. In this case, the inverse of the first root is greater than 1 and thus the process is nonstationary.