In [1]:
```
import pandas as pd
import numpy as np
from scipy.special import comb
import matplotlib.pyplot as plt
SMALL = 0.000001
```

In [2]:
```
volatility_dat = pd.read_excel('/Users/huanyu/Desktop/FixedIncome/hw6/Ho
mework 6 voldat.xlsx',header=None)[0].values
structure_dat = pd.read_excel('/Users/huanyu/Desktop/FixedIncome/hw6/Hom
ework 6 pfilea.xlsx',header=None)[0].values
bdt_tree = pd.read_excel('/Users/huanyu/Desktop/FixedIncome/hw6/Homework
6 bdttree.xls',header=None)
result_tree = np.zeros((30,30))
tau = 0.5
tau_sqrt = np.sqrt(tau)
r = (1 / structure_dat[0] - 1) * 2
result_tree[0][0] = r
def discount_T(r,period,result_tree,volatility_dat):
    # period = 2 * T
    # cash_flow = np.full(period + 1, 1.0)
    cash_flow = np.zeros(period + 1)
    for i in range(period):
        cash_flow[i] = 1 / (1 + r * np.exp(-2 * i * tau_sqrt * volatilit
y_dat[period - 2]) / 2)
        # cash_flow[i] = 0.5
    for i in range(period - 1, 0, -1):
        for j in range(i):
            # print(cash_flow)
            cash_flow[j] = (0.5 * cash_flow[j] + 0.5 * cash_flow[j + 1])
/ (1 + result_tree[i - 1][j] / 2)
    return cash_flow[0]
for i in range(2,31):
    low = 0
    high = 1
    r_init = (low + high) / 2
    discount = discount_T(r_init,i,result_tree,volatility_dat)
    while abs(discount - structure_dat[i - 1]) > SMALL:
        if discount < structure_dat[i - 1]:
            high = r_init
        else:
            low = r_init
        r_init = (high + low) / 2
        discount = discount_T(r_init,i,result_tree, volatility_dat)
    for j in range(i):
        result_tree[i - 1][j] = r_init * np.exp(-2 * j * volatility_dat[
i - 2] * tau_sqrt)
result_tree_df = pd.DataFrame(result_tree.T)
```

In [3]:
```
print(result_tree_df)
```

```
           0         1         2         3         4         5         6  \
0   0.056605  0.063271  0.072571  0.084126  0.098328  0.114151  0.12912
0
1   0.000000  0.054927  0.061243  0.069504  0.079533  0.091035  0.10268
2
2   0.000000  0.000000  0.051684  0.057425  0.064331  0.072601  0.08165
8
3   0.000000  0.000000  0.000000  0.047444  0.052035  0.057899  0.06493
8
4   0.000000  0.000000  0.000000  0.000000  0.042089  0.046174  0.05164
2
5   0.000000  0.000000  0.000000  0.000000  0.000000  0.036824  0.04106
8
6   0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.03265
9
7   0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
8   0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
9   0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
10  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
11  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
12  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
13  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
14  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
15  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
16  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
17  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
18  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
19  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
20  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
21  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
22  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
23  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
24  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
25  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
26  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
27  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
```

```
0
28  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0
29  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.00000
0

           7         8         9    ...         20        21        22
23  \
0   0.146042  0.162491  0.180817  ...  0.425049  0.452148  0.479614  0.
499390
1   0.115811  0.129220  0.144201  ...  0.355171  0.378885  0.403039  0.
421442
2   0.091838  0.102762  0.115000  ...  0.296781  0.317493  0.338690  0.
355660
3   0.072828  0.081721  0.091713  ...  0.247990  0.266049  0.284615  0.
300146
4   0.057753  0.064989  0.073141  ...  0.207221  0.222940  0.239173  0.
253297
5   0.045798  0.051682  0.058330  ...  0.173153  0.186817  0.200987  0.
213761
6   0.036318  0.041100  0.046518  ...  0.144687  0.156546  0.168898  0.
180396
7   0.028800  0.032685  0.037098  ...  0.120901  0.131180  0.141931  0.
152238
8   0.000000  0.025992  0.029586  ...  0.101025  0.109925  0.119271  0.
128476
9   0.000000  0.000000  0.023595  ...  0.084416  0.092113  0.100228  0.
108423
10  0.000000  0.000000  0.000000  ...  0.070538  0.077188  0.084226  0.
091499
11  0.000000  0.000000  0.000000  ...  0.058942  0.064681  0.070778  0.
077217
12  0.000000  0.000000  0.000000  ...  0.049252  0.054201  0.059478  0.
065165
13  0.000000  0.000000  0.000000  ...  0.041155  0.045418  0.049982  0.
054993
14  0.000000  0.000000  0.000000  ...  0.034389  0.038059  0.042002  0.
046410
15  0.000000  0.000000  0.000000  ...  0.028735  0.031892  0.035296  0.
039166
16  0.000000  0.000000  0.000000  ...  0.024011  0.026725  0.029660  0.
033052
17  0.000000  0.000000  0.000000  ...  0.020064  0.022394  0.024925  0.
027893
18  0.000000  0.000000  0.000000  ...  0.016765  0.018766  0.020945  0.
023540
19  0.000000  0.000000  0.000000  ...  0.014009  0.015725  0.017601  0.
019865
20  0.000000  0.000000  0.000000  ...  0.011706  0.013177  0.014791  0.
016765
21  0.000000  0.000000  0.000000  ...  0.000000  0.011042  0.012429  0.
014148
22  0.000000  0.000000  0.000000  ...  0.000000  0.000000  0.010445  0.
011940
23  0.000000  0.000000  0.000000  ...  0.000000  0.000000  0.000000  0.
010076
24  0.000000  0.000000  0.000000  ...  0.000000  0.000000  0.000000  0.
```

```
000000
25  0.000000  0.000000  0.000000  ...  0.000000  0.000000  0.000000  0.
000000
26  0.000000  0.000000  0.000000  ...  0.000000  0.000000  0.000000  0.
000000
27  0.000000  0.000000  0.000000  ...  0.000000  0.000000  0.000000  0.
000000
28  0.000000  0.000000  0.000000  ...  0.000000  0.000000  0.000000  0.
000000
29  0.000000  0.000000  0.000000  ...  0.000000  0.000000  0.000000  0.
000000

          24        25        26        27        28        29
0   0.526489  0.553772  0.581055  0.608521  0.636108  0.664062
1   0.445570  0.469987  0.494538  0.519382  0.544466  0.570003
2   0.377087  0.398878  0.420904  0.443300  0.466026  0.489266
3   0.319130  0.338528  0.358233  0.378363  0.398887  0.419965
4   0.270081  0.287309  0.304894  0.322939  0.341421  0.360480
5   0.228571  0.243839  0.259496  0.275633  0.292233  0.309420
6   0.193440  0.206947  0.220859  0.235257  0.250132  0.265593
7   0.163709  0.175636  0.187974  0.200796  0.214096  0.227974
8   0.138548  0.149062  0.159985  0.171382  0.183252  0.195683
9   0.117253  0.126509  0.136164  0.146277  0.156851  0.167966
10  0.099232  0.107368  0.115890  0.124850  0.134254  0.144175
11  0.083980  0.091124  0.098634  0.106561  0.114912  0.123753
12  0.071073  0.077337  0.083948  0.090952  0.098357  0.106225
13  0.060149  0.065636  0.071449  0.077629  0.084187  0.091179
14  0.050904  0.055705  0.060810  0.066257  0.072059  0.078264
15  0.043081  0.047277  0.051756  0.056552  0.061677  0.067178
16  0.036459  0.040124  0.044050  0.048268  0.052792  0.057663
17  0.030856  0.034053  0.037491  0.041197  0.045186  0.049495
18  0.026113  0.028901  0.031909  0.035162  0.038676  0.042485
19  0.022100  0.024528  0.027158  0.030012  0.033104  0.036467
20  0.018703  0.020817  0.023114  0.025615  0.028335  0.031302
21  0.015828  0.017668  0.019672  0.021863  0.024253  0.026868
22  0.013396  0.014994  0.016743  0.018660  0.020759  0.023062
23  0.011337  0.012726  0.014250  0.015927  0.017768  0.019796
24  0.009594  0.010800  0.012128  0.013594  0.015208  0.016992
25  0.000000  0.009166  0.010323  0.011603  0.013017  0.014585
26  0.000000  0.000000  0.008786  0.009903  0.011142  0.012519
27  0.000000  0.000000  0.000000  0.008452  0.009537  0.010746
28  0.000000  0.000000  0.000000  0.000000  0.008163  0.009224
29  0.000000  0.000000  0.000000  0.000000  0.000000  0.007917

[30 rows x 30 columns]
```

In [4]:

```
expected_r = np.zeros(30)
for i in range(0,30):
    denominator = 2 ** i
    temp = 0
    for j in range(i + 1):
        expected_r[i] += result_tree[i][j] * comb(i,j) / denominator
forward_rate = np.zeros(30)
forward_rate[0] = (1 / structure_dat[0] - 1) * 2
for i in range(1,30):
    forward_rate[i] = (structure_dat[i - 1] / structure_dat[i] - 1) * 2
plt.plot(expected_r,label='Expected r')
plt.plot(forward_rate,label='Forward rate')
plt.legend()
plt.show()
```