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“Momentum Crashes”

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- Abstract -

Momentum strategies have produced high returns and Sharpe ratios, and strong positive alphas relative to market models and other standard factors models. However, the returns to momentum strategies are highly skewed; they experience infrequent but strong and persistent strings of negative returns. These momentum “crashes” are forecastable: they occur following market declines, when market volatility is high, and contemporaneous with market “rebounds.” The low *ex-ante* expected returns associated with the crashes appear to result from a conditionally high premium attached to the option-like payoffs of the past-loser portfolios.

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Introduction

A momentum strategy is a bet that past returns will predict future returns. Consistent with this, a long-short momentum strategy is typically implemented by buying past winners and taking short positions in past losers.

Momentum appears pervasive: the academic finance literature has documented the efficacy of momentum strategies in numerous asset classes, from equities to bonds, from currencies to commodities to exchange-traded futures.¹ Momentum is strong: in US equities, where this investigation is focused, we see an average annualized return difference between the top and bottom momentum deciles of 16.5%/year, and an annualized Sharpe ratio of 0.82 (Post-WWII, through 2008).² This strategy's beta over this period was -0.125, and it's correlation with the Fama and French (1992) value factor was strongly negative.³ Momentum is a strategy employed by numerous quantitative investors within multiple asset classes and even by mutual funds managers in general.⁴

However, the consistent, strong positive returns of momentum strategies are punctuated with strong reversals, or "crashes." Like the returns to the carry trade in currencies, momentum returns are negatively skewed, and the crashes can be pronounced and persistent.⁵ In our 1927-2010 sample, the two worst months for the aforementioned momentum strategy are consecutive: July and August of 1932. Over these two months, the bottom momentum decile portfolio outperformed the top by 206%: over this two month period, the past-loser portfolio rose by 236%, while the past-winner portfolio saw a gain of only 30%. In a more recent crash, over the three-

¹A fuller discussion of this literature is given in Section 1

²Section 2 gives a detailed description of the construction of these value-weighted momentum portfolios, and summary statistics on their performance.

³Not surprisingly, it is not priced by the CAPM for the Fama and French (1993) three-factor model (see Fama and French (1996)). A Fama and French (1993) model augmented with a momentum factor, as proposed by Carhart (1997) is necessary to explain the momentum return. Also note that Asness, Moskowitz, and Pedersen (2008) argue that a three factor model (based on a market factor, and a value and momentum factor) is successful in pricing value and momentum anomalies in cross-sectional equities, country equities, commodities and currencies.

⁴Jegadeesh and Titman (1993) motivate their investigation of momentum with the observation that "...a majority of fund examined by Grinblatt and Titman (1989, 1993) show a tendency to buy stocks that have increased in price over the previous quarter."

⁵See Brunnermeier, Nagel, and Pedersen (2008), and others for evidence on the skewness of carry trade returns.

month period from March-May of 2009, the past-loser portfolio (decile 1) rose by 156%, while the past winner portfolio was up by only 6.5%.

We investigate the predictability of momentum crashes. At the start of each of the two crashes discussed above (July/August of 1932 and March-May of 2009), the broad US equity market (and, specifically, the CRSP VW index) was down significantly from earlier highs. Market volatility was high. Also, importantly, the market as a whole rebounded significantly in these momentum crash months.

This is consistent with the general behavior of momentum crashes: they tend to occur in times of market stress, specifically when the market has fallen and when *ex-ante* measures of volatility is high. They also occur when contemporaneous market returns are high. Note that our result here is consistent with that of Cooper, Gutierrez, and Hameed (2004), who find that the momentum premium falls to zero when the past three-year market returns has been negative.

These patterns are suggestive of the possibility that the changing beta of the momentum portfolio may partly be driving the momentum crashes. As documented earlier by Grundy and Martin (2001, GM), the betas of momentum strategies can fall significantly as the market falls. Intuitively, this result is straightforward, if not often appreciated: when the market has fallen significantly over the momentum formation period – in our case from 12 months ago to 1 month ago – there is a good chance that the firms that fell in tandem with the market were and are high beta firms, and those that performed the best were low beta firms. Thus, following market declines the momentum portfolio is likely to be long low-beta stocks (the past winners), and short high-beta stocks (the past losers).

We verify empirically that there is dramatic time variation in the betas of momentum portfolios. Using beta estimates based on daily momentum decile returns we find that, following major market declines, betas for the past-loser decile rises above 3 , and falls below 0.5 for past winners.

However, GM further argue that performance of the momentum portfolio is dramatically improved — particularly in the pre-WWII era, by dynamically hedging the market and size risk in the portfolio. While we replicate their results with a similar methodology, overall our empirical results do not support GM’s conclusion. The rea-

son for this is that, when GM create their hedged momentum portfolio, they calculate their hedging coefficients based on forward-looking measured betas.⁶ Therefore, their hedged portfolio returns are not an implementable strategy.

GM's procedure, while not technically valid, should not bias their estimated performance if their forward-looking betas are uncorrelated with future market returns. However we show that this correlation is present, is strong, and does bias GM's results.

The source of the bias is a striking correlation of the loser-portfolio beta with the return on the market. In a bear market, we show that the up- and down-market betas differ substantially for the momentum portfolio. Using Henriksson and Merton (1981) specification, we calculate up- and down-betas for the momentum portfolios.⁷ We show that, in a bear market, the momentum portfolio's up-market beta is about double its down-market beta (-1.53 *vs.* -0.74), and that this difference is highly statistically significant ($t = 5.0$).

More detailed analysis shows that most of the up- *vs.* down-beta asymmetry in bear market is driven by the past-loser decile: for this portfolio the up- and down betas differ by 0.6, while for the past-winner decile the difference is -0.2.

Our results are consistent with the following pattern: in a bear market, the firms which are in apparent distress (the high-beta loser firms) do respond somewhat more strongly to bad news, as one might expect. The loser down-beta is **1.8** times bigger than the winner portfolio's down-beta. But the response of the past losers to good news is dramatically different. The loser portfolio's up-beta is **3.3** times bigger than the up-beta for the winner portfolio. In the conclusion we briefly discuss potential explanations for this phenomenon, but fully understanding it is an area for future research.

The layout of the paper is as follows: In Section 1 we review the Literature we build upon in our analysis. Section 2 describes our data and portfolio construction. Section

⁶At the time GM undertook their study, only monthly CRSP data was available in the pre-1972 sample period. They therefore used a five-month forward-looking regression to determine the hedging coefficients.

⁷Following Henriksson and Merton (1981), the up-beta is defined as the market-beta conditional on the contemporaneous market return being positive, and the down-beta is the market beta conditional on the contemporaneous market return being negative.

3 describes our empirical analyses, and Section 4 speculates about the sources of the premia we observe, discusses areas for future research, and concludes.

1 Literature Review

A momentum strategy involves constructing a long-short portfolio, which purchases assets with strong performance, and sells assets with poor recent performance.

The performance of momentum strategies in U.S. common stock returns is documented in Jegadeesh and Titman (1993, JT). JT examine portfolios formed by sorting on past returns. For a portfolio formation date of t , their portfolios are formed on the basis of returns from $t - \tau$ months up to $t - 1$ month.⁸ JT examine strategies for τ between 3 to 12 months, and hold these portfolios between 3 and 12 months. Their data is from 1965-1989. For all horizons, the top-minus-bottom decile spread in portfolio returns is statistically strong. However, JT also note the poor performance of momentum strategies in pre-WWII US data.

Jegadeesh and Titman (2001) note the continuing efficacy of the momentum portfolios in common stock returns from the time of the publication of their original paper. However, as we document here, the performance of momentum strategies since the publication of their 2001 paper has been poor.

1.1 Momentum in Other Asset Classes

However, strong and persistent momentum effects are also present outside of US equities. Rouwenhorst (1998) finds evidence of momentum in equities in developed markets, and Rouwenhorst (1999) documents momentum in emerging markets. Asness, Liew, and Stevens (1997) demonstrates positive abnormal returns to a country timing strategy which buys a country index portfolio when that country has experienced strong recent performance, and sells the indices of countries with poor recent performance. Momentum is also present outside of equities: Okunev and White (2003) find

⁸The motivation for skipping the last month prior to portfolio formation is the presence of the short-term reversal effect (see Jegadeesh (1990))

momentum in currencies; Erb and Harvey (2006) in commodities; Moskowitz, Ooi, and Pedersen (2010) in exchange traded futures contracts; and Asness, Moskowitz, and Pedersen (2008) in bonds. Asness, Moskowitz, and Pedersen (2008) also attempt to integrate the evidence on within-country cross-sectional equity, country-equity, country-bond, currency, and commodity value and momentum strategies.

Among common stocks, there is evidence that momentum strategies perform well for industry strategies, and for strategies that are based on the firm specific component of returns (see Moskowitz and Grinblatt (1999), Grundy and Martin (2001).)

1.2 Sources of Momentum

The underlying mechanism responsible for momentum is as yet unknown. By virtue of the high Sharpe-ratios associated with the momentum effect, they are difficult to explain within the standard rational-expectations asset pricing framework. Following Hansen and Jagannathan (1991), In a frictionless framework the high Sharpe-ratio associated with zero-investment momentum portfolios implies high variability of marginal utility across states of nature. Moreover, the lack of correlation of momentum portfolio returns with standard proxy variables for macroeconomic risk (*e.g.*, consumption growth) sharpens the puzzle still further (see Daniel and Titman (2006)).

A number of behavioral theories of price formation purport to yield momentum as an implication. Daniel, Hirshleifer and Subramanyam (1998, 2001) propose a model in which momentum arises as a result of the overconfidence of agents; Barberis, Shleifer, and Vishny (1998) argue that a combination of representativeness; Hong and Stein (1999) model two classes of agents who process information in different ways; Grinblatt and Han (2005) argue that agents are subject to a disposition effect, and as a result are averse to recognizing losses, and are too quick to sell past winners.⁹ George and Hwang (2004) point to a related anomaly – the 52-week high – and argue that it is a result of anchoring on past prices.

1.3 Time Variation in Momentum Risk and Return

Grundy and Martin (2001) argue that, by their nature, momentum portfolios will

⁹Frazzini (2006) examines the presence of the disposition effect on the part of mutual funds.

have significant time-varying exposure to systematic factors. Because momentum strategies are bets on past winners, they will have positive loadings on factors which have had a positive realization over the formation period of the momentum strategy. For example, if the market went up over the last 12 months, a 12-month momentum strategy will be long high-beta stocks and short low-beta stocks, and will therefore have a high market beta.

GM further argue that the Fama and French (1993) market, value and size factors do not explain the returns to a momentum strategy. They show that hedging out a momentum strategy's dynamic exposure to size and value factors dramatically reduces the strategy's return volatility, increases the Sharpe ratio, and eliminates the momentum strategy's historically poor performance in January, and its poor record in the pre-WWII period. However, as we discuss in Section 3.4, their hedged portfolio is constructed based on forward-looking betas, and is therefore not an implementable strategy. In this paper, we show that this results in a strong bias in estimated returns, and that a hedging strategy based on *ex-ante* betas does not exhibit the performance improvement noted in GM.

Cooper, Gutierrez, and Hameed (2004) examine the time variation of average returns to US equity momentum strategies. They define UP and DOWN market states based on the lagged three-year return of the market. They find that in UP states, the historical mean return to a EW momentum strategy has been *0.93%/month*. In contrast in DOWN states the mean return has been *-0.37%/month*. They find similar results, controlling for market, size & value based on the unconditional loadings of the momentum portfolios on these factors.¹⁰

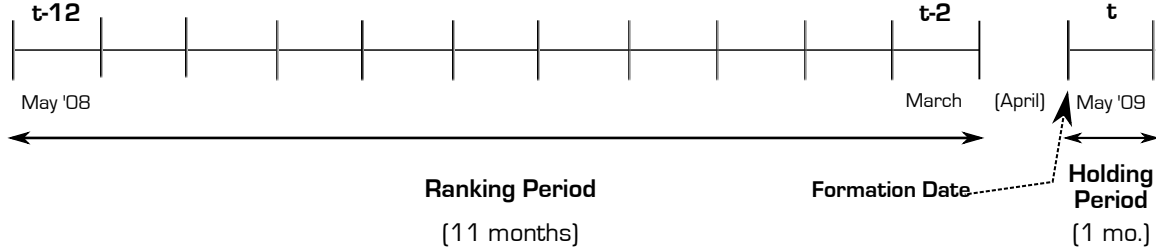
2 Data and Portfolio Construction

Our principal data source is CRSP. Using the monthly and daily CRSP data, we construct monthly and daily momentum decile portfolio. Both sets of portfolios are rebalanced only at the end of each month. Our universe is firms listed on NYSE, AMEX or NASDAQ as of the formation date. We utilize only the returns of common shares (with CRSP share-code of 10 or 11). We require that the firm have a valid share

¹⁰CGH control do not calculate conditional risk measures, *e.g.* using the instruments proposed by Grundy and Martin (2001).

Figure 1: **Momentum Portfolio Formation**

This figure illustrates the formation of the momentum decile portfolios. As of close of the final trading day of each month, firms are ranked on their cumulative return from 12 months before to one month before the formation date.



price and a valid number of shares as of the formation data, that there be a minimum of 8 valid returns over the 11 month formation period. Following convention and CRSP availability, all prices are closing prices, and all returns are from close to close.

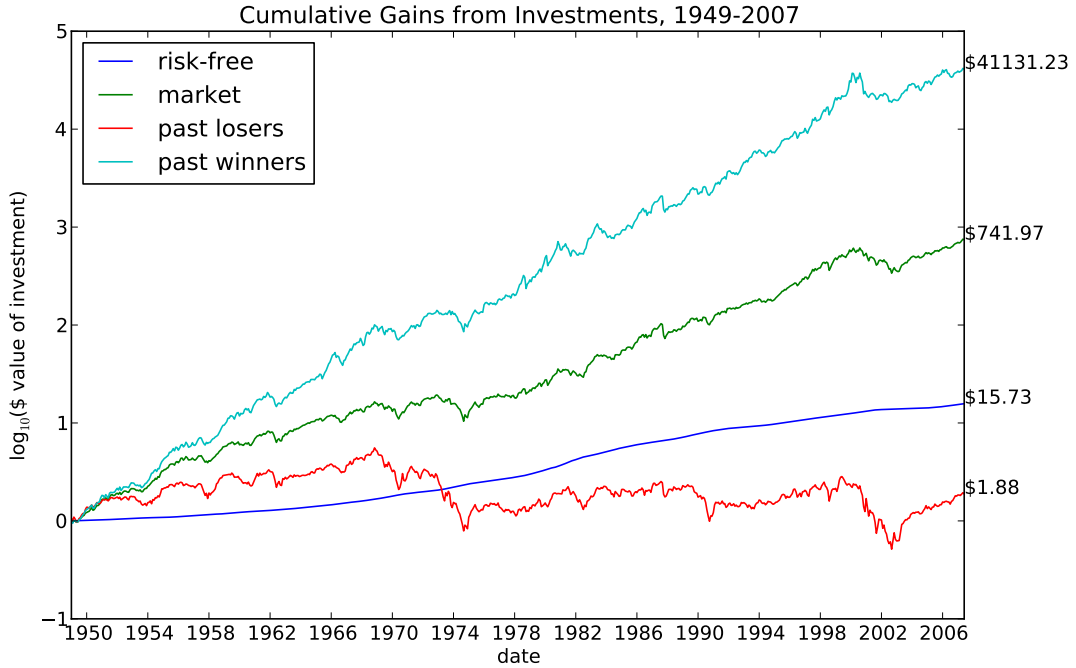
Figure 1 illustrates the portfolio formation process for the momentum returns in the month of May 2009. The ranking period returns are the cumulative returns from close of the last trading day of April 2008 through the last trading day of March 2009. We sort all firms meeting the data requirements into decile portfolios – labeled 1-10 – on the basis of their cumulative returns over the ranking period. The 10% of firms with the highest ranking period returns go into portfolio 10 – the “[W]inner” decile portfolio – and those with the lowest go into portfolio 1, the “[L]oser” portfolio. We also evaluate the returns for a zero investment Winner-Minus-Loser (WML) portfolio, which is the difference of the Winner and Loser portfolio each period.

The monthly returns of the decile portfolios are based on the value-weighted return from the closing price last trading day of March through the last trading day of April. Decile membership does not change within month, except in the case of delisting. As with the monthly returns, the daily returns are value weighted. This means that, in the absence of dividends, and other corporate actions, all portfolios are buy and hold portfolios.

The market return we used is the CRSP value weighted index. The risk free rate series is from the Ken French data library.¹¹

¹¹I convert the monthly risk-free rate series to a daily series, where necessary, by converting the risk-free rate at the beginning of each month to a daily rate, and assuming that that daily rate is valid through the month.

Figure 2: Momentum Components, 1949-2007



3 Empirical Results

3.1 Momentum Portfolio Performance

Figure 2 presents the cumulative monthly log returns for investments in (1) the risk-free asset; (2) the CRSP value-weighted index; (3) the bottom decile “past loser” portfolio; and (4) the top decile “past winner” portfolio. The y-axis of the plot gives the cumulative log return for each portfolio. On the right side of the plot, we present the final dollar values for each of the four portfolios.

Consistent with the existing literature, there is a strong momentum premium over this 50 year period. Table 1 presents return moments for the momentum decile portfolios over this period. The winner decile excess return averages 15.4%/year, and the loser portfolio averages -1.3%/year. In contrast the average excess market return is 7.5%. The Sharpe-Ratio of the WML portfolio is 0.83, and that of the market is 0.52. Over this period, the beta of the WML portfolio is slightly negative, -0.13, giving it an the WML portfolio an unconditional CAPM alpha of 17.6%/year ($t=6.8$). As one would expect given the high alpha, an *ex-post* optimal combination of the market and WML portfolios has a Sharpe ratio of 1.02, close to double that of the market. A

Table 1: **Momentum Portfolio Characteristics, 1947-2007**

This table presents characteristics of the monthly momentum portfolio excess returns over the 50 year period from 1947:01-2006:12. The mean return, standard deviation, alpha are in percent, and annualized. The Sharpe-ratio is annualized. The α , $t(\alpha)$, and β are estimated from a full-period regression of each decile portfolio's excess return on the excess CRSP-value weighted index. For all portfolios except wml, SK denotes the full-period realized skewness of the monthly log returns (not excess) to the portfolios. For wml, SK is the realized skewness of $\log(1+r_{wml})$

	Momentum Decile Portfolios										wml	Mkt
	1	2	3	4	5	6	7	8	9	10		
μ	-1.3	4.0	5.5	6.4	6.2	7.2	7.8	10.0	10.9	15.4	16.7	7.5
σ	23.6	19.0	16.3	15.2	14.2	14.7	14.6	15.0	16.0	20.2	20.1	14.5
α	-11.3	-4.3	-1.8	-0.6	-0.5	0.2	0.8	2.9	3.3	6.4	17.7	0
$t(\alpha)$	(-6.3)	(-3.3)	(-1.6)	(-0.7)	(-0.6)	(0.2)	(1.1)	(3.7)	(3.8)	(4.7)	(6.8)	(0)
β	1.33	1.11	0.97	0.94	0.90	0.94	0.93	0.95	1.01	1.20	-0.13	1
SR	-0.06	0.21	0.34	0.42	0.44	0.49	0.53	0.67	0.68	0.76	0.83	0.52
SK	-0.17	-0.21	-0.15	-0.33	-0.66	-0.67	-0.75	-0.51	-0.79	-0.74	-1.72	-1.34

pattern that we will explore further is the skeweness – note that the winner portfolios are considerably more negatively skewed than the loser portfolios, even over this relatively benign period.

3.2 Momentum Crashes

Since 1926, there have been a number of long periods over which momentum underperformed dramatically. Figures 3 and 4 show the cumulative daily returns to the same set of portfolios over the recent period from March 8 2009 through December 31, 2009, and over a period starting in 1932, and continuing through WWII. Over both of these two period, the loser portfolio strongly outperforms the winner portfolio.

Finally, Figure 5 plots the cumulative (monthly) log returns to the an investment in the WML portfolio.¹² An examination of Figures 3, 4 and 5 suggests several aspects of momentum underperformance:

1. While past winners have generally outperformed past loses, there are relatively long periods over which momentum experiences severe losses.
2. These “crashes” do not occur over a single day, but rather are spread out over the span of several months days.

¹²I describe the calculation of cumulative returns for long-short portfolios in Appendix A.1.

Figure 3: 2009 Momentum Performance

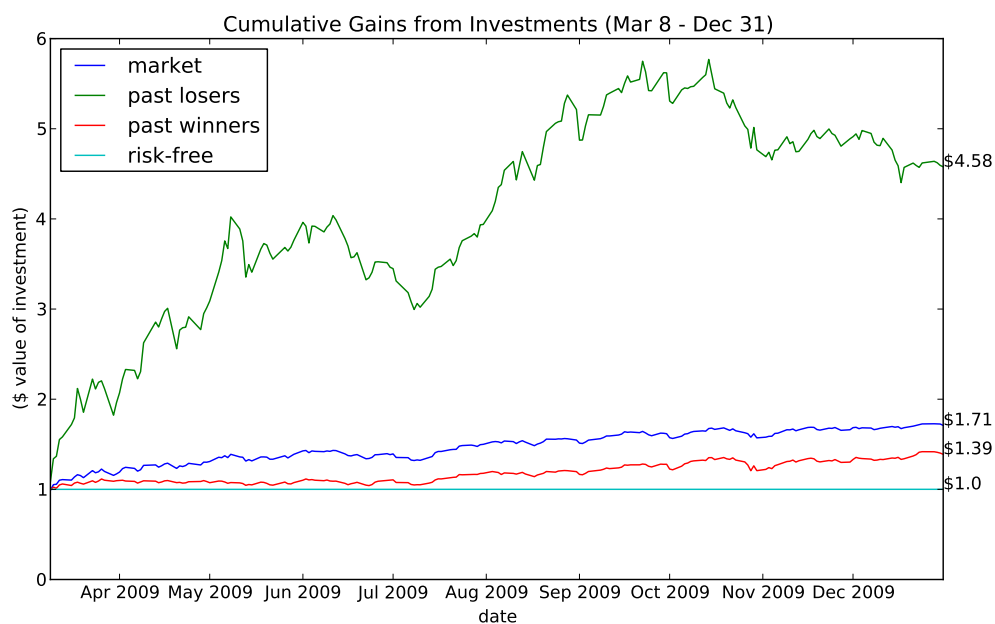


Figure 4: Momentum in the Great Depression

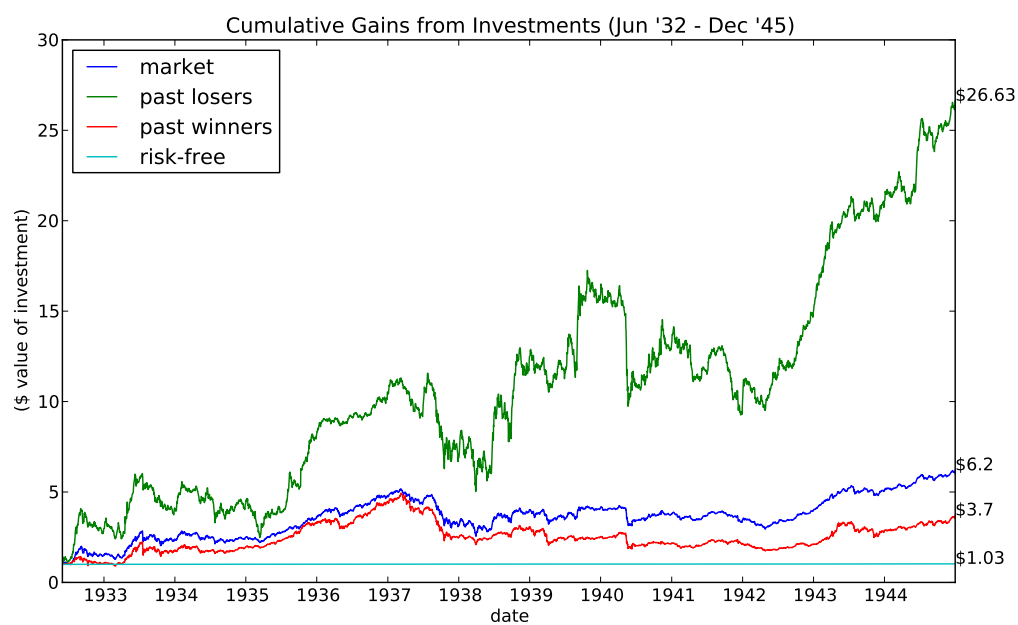
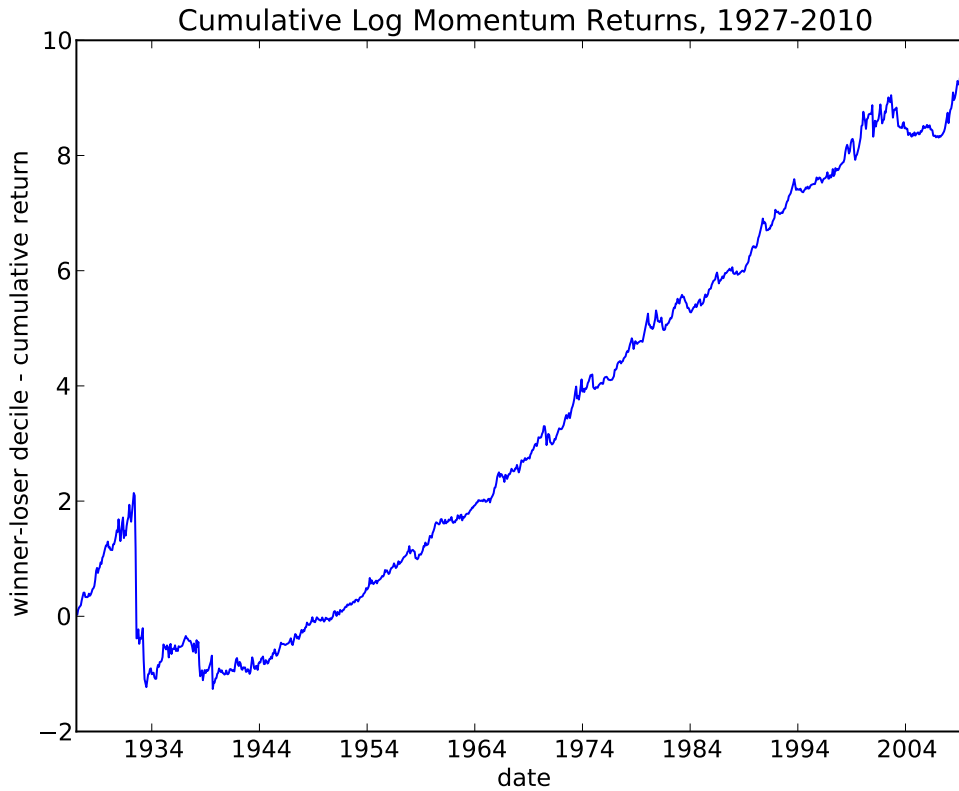


Figure 5: Cumulative Momentum Returns



3. Because of the magnitude of these losses, momentum strategies can experience long periods of underperformance.
4. However, the most severe momentum underperformance appears to occur in following market downturns, and when the market itself is performing well.

Table 3 presents the worst monthly returns to the WML strategy. In addition, this table gives the lagged two-year returns on the market, and the contemporaneous monthly market return. There are several points of note this Table, that we will examine more formally in the remainder of the paper:

1. All of these returns occur after severe market downturns, and during a month where the market rose, generally in a dramatic fashion.¹³
2. These returns are clustered: Note that the two worst are in July and August

¹³For January 2001, the past 2 year market returns is positive, but the market return since the high of March 2000, was very negative.

Table 2: Momentum Portfolio Characteristics, 1927-2009

The calculations for this table are identical to those in Table 1, except that the time period is 1927:01-2009:12.

	Momentum Decile Portfolios											
	1	2	3	4	5	6	7	8	9	10	wml	Mkt
μ	0.2	4.7	4.9	6.6	6.7	7.5	8.4	9.9	10.8	14.6	14.4	7.4
σ	34.4	28.7	24.7	22.6	21.0	20.4	19.5	18.8	19.8	22.7	27.7	18.9
α	-11.2	-5.1	-3.7	-1.4	-0.9	-0.0	1.3	3.1	3.7	7.2	18.4	0
$t(\alpha)$	(-5.8)	(-3.5)	(-3.2)	(-1.5)	(-1.1)	(-0.1)	(1.9)	(4.4)	(4.4)	(5.4)	(6.5)	(0)
β	1.56	1.34	1.18	1.10	1.03	1.03	0.97	0.93	0.96	1.01	-0.54	1
SR	0.01	0.17	0.20	0.29	0.32	0.37	0.43	0.53	0.54	0.64	0.52	0.39
sk	0.13	-0.05	-0.12	0.17	-0.05	-0.32	-0.65	-0.53	-0.81	-0.92	-6.32	-0.58

of 1932, following a market decline of roughly 90% from the 1929 peak. March and April of 2009 are ranked 7th and 3rd worst, and April and May of 1933 are the 5th and 10th worst. And three of the worst are from 2009 – over a three-month period in which the market rose dramatically and volatility fell. One was in 2001, and all of the rest are from the 1930s. At some level it is not surprising that the most extreme returns occur in periods of high volatility.

3. Closer examination shows that this poor performance is attributable to short side. For example, in July and August of 1932, the market actually rose by 82%. Over these two month, the winner decile rose by 30%, but the loser decile was up by 236%. Similarly, over the three month period from March-May of 2009, the market was up by 29%, but the loser decile was up by 156%. Thus, to the extent that the strong momentum reversals we observe in the data can be characterized as a crash, they are a crash where the short side of the portfolio – the losers – are crashing up rather than down.
4. The momentum crash – meaning the strong performance of the past losers – is not something which occurs over the span of minutes or days. It is not a Poisson jump. The temporal extent of the loser rebound (and the corresponding momentum crash) rather takes place slowly, over the span of multiple months.

Table 3: **Worst Monthly Momentum Returns**

This table presents the 11 worst monthly returns to the WML portfolio over the 1927:01-2009:12 time period. Also tabulated are MKT-2Y , the 2-year market returns leading up to the portfolio formation date, and MKT_t , the market return in the same month.

RANK	MONTH	WML_t	MKT-2Y	MKT_t
1	1932-08	-0.7896	-0.6767	0.3660
2	1932-07	-0.6011	-0.7487	0.3375
3	2009-04	-0.4599	-0.4136	0.1106
4	1939-09	-0.4394	-0.2140	0.1596
5	1933-04	-0.4233	-0.5904	0.3837
6	2001-01	-0.4218	0.1139	0.0395
7	2009-03	-0.3962	-0.4539	0.0877
8	1938-06	-0.3314	-0.2744	0.2361
9	1931-06	-0.3009	-0.4775	0.1380
10	1933-05	-0.2839	-0.3714	0.2119
11	2009-08	-0.2484	-0.2719	0.0319

3.3 Risk of Momentum Returns

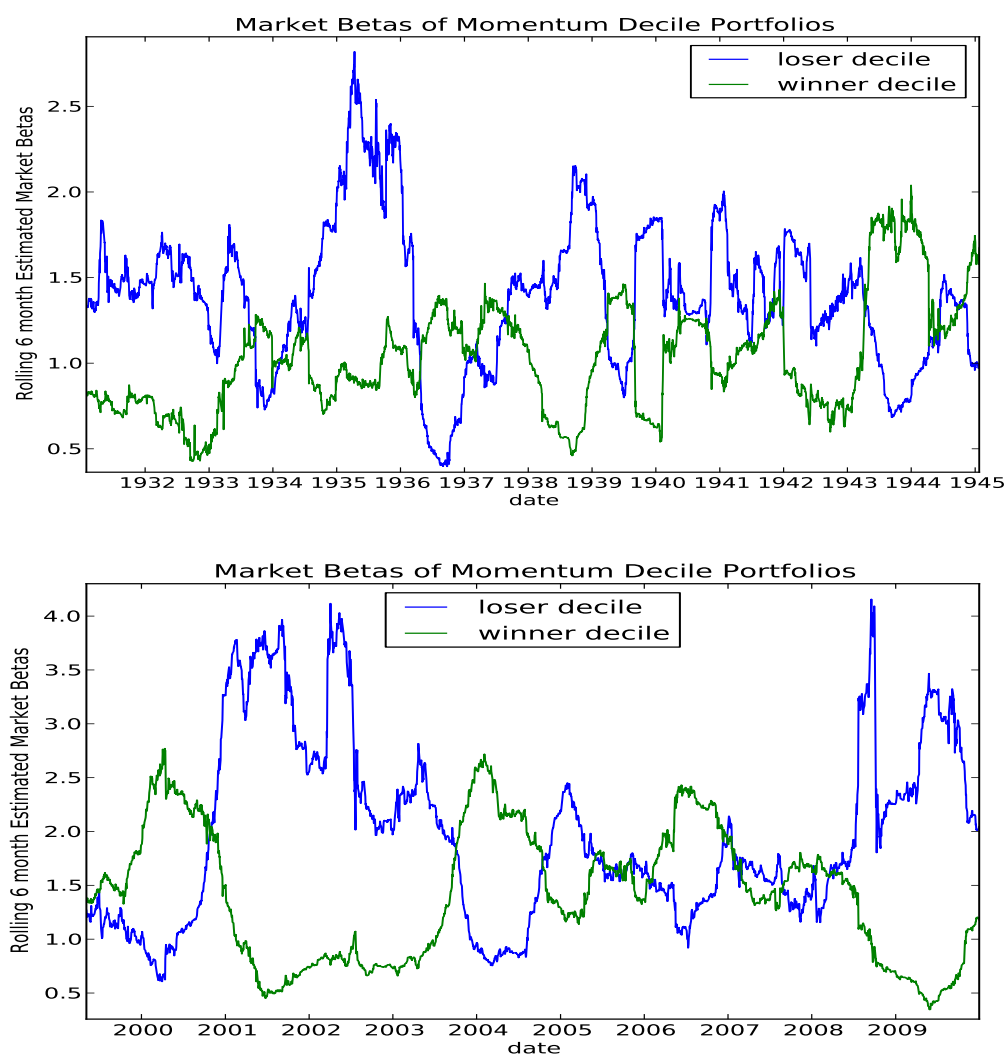
The data in Table 3, is suggestive that large changes in market beta may help to explain some of the large negative returns earned by momentum strategies.

For example, as of the beginning of March 2009, the firms in the Loser portfolio were, on average, down from their peak by over 80%. The loser firms at this point included the firms that were hit hardest in the financial crisis: Citigroup, Bank of America, Ford, GM, and International Paper (which was levered). In contrast, the past-winner portfolio was composed of defensive or counter-cyclical firms like Autozone. The loser firms, in particular, were often extremely levered, and at risk of bankruptcy. In the sense of the Merton (1990) model, their common stock was effectively an out-of-the-money option on the underlying firm value. This suggests that there were potentially large differences in the market betas of the winner and loser portfolios.

This is in fact the case. In Figure 6 we plot the market betas for the winner and loser momentum deciles, estimated using 126 day (≈ 6 month) rolling regressions with daily data, and using 10 daily lags of the market return in estimating the market.

Figure 6: **Market Betas of Winner and Loser Decile Portfolios**

These two plots present the estimated market betas over the periods 1931-1945, and 1999-2010. The betas are estimating by running a set of 128-day rolling regressions. Each regression uses 10 (daily) lagged market returns in the estimations of the beta as a way of accounting for the lead-lag effects in the data.



Specifically, we estimated a daily regression specification of the form:

$$\tilde{r}_{i,t}^e = \beta_0 \tilde{r}_{m,t}^e + \beta_1 \tilde{r}_{m,t-1}^e + \cdots + \beta_{10} \tilde{r}_{m,t-10}^e + \epsilon_{i,t} \quad (1)$$

and then report the sum of the estimated coefficients $\hat{\beta}_0 + \hat{\beta}_1 + \cdots + \hat{\beta}_{10}$. Particularly for the past losers portfolios, and especially in the Pre-WW-II period, the lagged coefficients are strongly significant, suggesting that the prices of firms in these portfolios respond slowly to market-wide information.

3.4 Hedging the Market Risk in the Momentum Portfolio

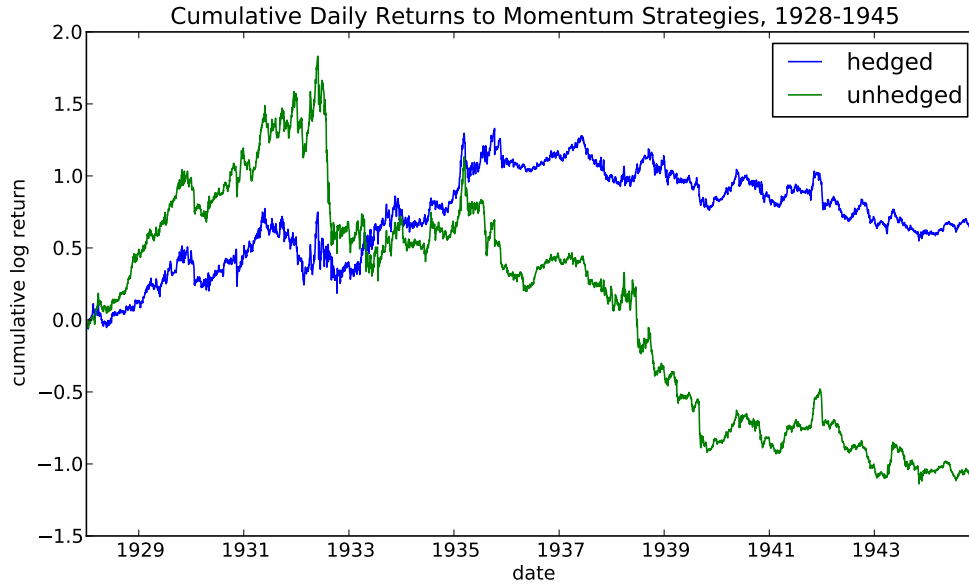
Grundy and Martin (2001) investigate hedging the market and size risk in the momentum portfolio. They find that doing so dramatically increases the returns to a momentum portfolio. They find that a hedged momentum portfolio has high returns and a high Sharpe-ratio in the pre-WWII period when the unhedged momentum portfolio suffers.

However, the hedging that Grundy and Martin (2001) do is based on an *ex-post* estimate of the portfolio's market and size betas, estimated over the coming 5 months. Part of the reason that Grundy and Martin (2001) employed this procedure is that, at the time they undertook their investigation, they only had monthly CRSP data available in the early part of the sample. With monthly data, *ex-ante* beta estimates are imprecise. However, to the extent that the future momentum-portfolio beta is correlated with the future return of the market, this will result in a biased estimate of the returns of the hedged portfolio. In Section 3.5, we will show there is in fact a strong correlation of this type which in fact does result in a large upward bias in the estimated performance of the hedged portfolio.

To begin the investigation, we first estimate the performance of a WML portfolio which hedges out market risk using an *ex-post* estimate of market beta, following Grundy and Martin (2001).¹⁴ We construct our *ex-post* hedged portfolio in a similar

¹⁴Note that Grundy and Martin (2001) also hedge out size risk. We do not. This presumably also increases the performance of their hedged portfolio. It is well known that (1) the momentum portfolio has a strongly positive size beta; and (2) that both the size portfolio and the momentum portfolio underperform in January. with their four month beta estimation period, the estimated size beta will tend to be larger in January. Thus, the ex-post hedged portfolio should upward biased

Figure 7: **Ex-post Hedged Momentum Portfolio Performance**



way, though using daily data. Specifically, the size of the market hedge is based on the future 42-day (2 month) realized market beta of the portfolio being hedged. Again, to calculate the beta we use 10 daily lags of the market return, as shown in equation (1).

The ex-post hedged portfolio exhibits considerably improved performance, consistent with the results of Grundy and Martin (2001). Figure 7 plots the performance of the *ex-post* hedged WML portfolio over the period form 1928-1945, and that of the unhedged portfolio.

3.5 Option-like Behavior of the WML portfolio

However, we now show that the strong performance of the *ex-post* hedged portfolio is a strongly upward biased estimate of the *ex-ante* performance of the portfolio. The reason, as alluded to earlier, is that the market beta of the WML portfolio is strongly *negatively* correlated with the contemporaneous realized performance of the portfolio.

To illustrate this, we run a set of monthly time-series regressions, the results of which

performance as well.

Table 4: **Market Timing Regression Results**

This table presents the results of estimating four specifications of a monthly time-series regressions run over the period August 1928 - December 2009 (992 months). The variables are described in the text.

Coeff.	Variable	Estimated Coefficients (<i>t</i> -statistics in parentheses)			
		(1)	(2)	(3)	(4)
$\hat{\alpha}_0$	1	0.017 (7.1)	0.016 (6.8)	0.016 (6.9)	0.022 (7.5)
$\hat{\alpha}_B$	I_B		-0.019 (-3.4)	0.007 (0.9)	
$\hat{\beta}_0$	$\tilde{R}_{m,t}^e$	-0.543 (-12.6)	0.038 (0.7)	0.054 (0.7)	0.054 (0.7)
$\hat{\beta}_B$	$I_B \cdot \tilde{R}_{m,t}^e$		-1.198 (-15.5)	-0.736 (-6.1)	-0.788 (-7.4)
$\hat{\beta}_{B,U}$	$I_B \cdot I_U \cdot \tilde{R}_{m,t}^e$			-0.794 (-5.0)	-0.695 (-6.0)
R_{adj}^2		0.136	0.321	0.339	0.339

are presented in Table 4. The variables used in the regressions are:

1. $\tilde{R}_{WML,t}$ is the WML return in month t .
2. $\tilde{R}_{m,t}^e$ is the excess CRSP value-weighted index return in month t .
3. I_B is an *ex-ante Bear-market* Indicator. It is 1 if the cumulative CRSP VW index return in the 24 months leading up to the start of month t is *negative*, and is zero otherwise.
4. \tilde{I}_L , is an *ex-ante bull-market* Indicator. is a a is 1 if the cumulative CRSP VW index return in the 24 months leading up to the start of month t is *positive*, and is zero otherwise. *Note that $I_L = (1 - I_B)$*
5. \tilde{I}_U is the contemporaneous – *i.e.*, not *ex-ante Up-Month* indicator variable. It is 1 if the excess CRSP VW index return is positive in month t , and is zero otherwise.

Regression (1) in Table 4 fits an unconditional market model to the WML portfolio:

$$\tilde{R}_{WML,t} = \alpha_0 + \beta_0 \tilde{R}_{m,t} + \tilde{\epsilon}_t$$

Consistent with the results in the literature, the market beta is somewhat negative, -0.543, and that the $\hat{\alpha}$ is both economically large (1.7%/month), and highly statistically significant.

Regression (2) in Table 4 fits a conditional CAPM with the bear market I_B indicator as a instrument:

$$\tilde{R}_{\text{WML},t} = (\alpha_0 + \alpha_B I_B) + (\beta_0 + \beta_B I_B) \tilde{R}_{m,t} + \tilde{\epsilon}_t.$$

This specification is an attempt to capture both expected return and market-beta differences in bear-markets. First, consistent with Grundy and Martin (2001), we see a striking change in the market beta of the WML portfolio in bear markets: it is -1.2 lower, with a t-statistic of almost -16 on the difference. The intercept is also lower: The point estimate for the alpha in bear markets – equal to the sum is now -0.2%/month, and the t-statistic on the difference in alpha in bear markets is -3.4.

Regression (3) introduces an additional element to the regression which allows us to assesses the extent to which the up- and down-market betas of the WML portfolio differ. The specification is similar to that used by Henriksson and Merton (1981) to assess market timing ability of fund managers:

$$\tilde{R}_{\text{WML},t} = [\alpha_0 + I_B(\alpha_B + \tilde{I}_U \alpha_{B,U})] + [\beta_0 + I_B(\beta_B + \tilde{I}_U \beta_{B,U})] \tilde{R}_{m,t} + \tilde{\epsilon}_t. \quad (2)$$

Now, if $\beta_{B,U}$ is different from zero, this suggests that the WML portfolio exhibits option-like behavior relative to the market. Specifically, a negative $\beta_{B,U}$ would mean that, in bear markets, the momentum portfolio is effectively short a call option on the market: in months when the realized market return is negative, the WML portfolio beta is -0.78. But when the market return is positive, the market beta is about twice as big – specifically, the point estimate is the sum of $\hat{\beta}_B$ and $\hat{\beta}_{B,U}$, about -1.5.

The predominant source of this *optionality* turns out to be the loser portfolio. Table 5 presents the results of the regression specification in equation (2) for each of the ten momentum portfolio. The final row of the table (the $\hat{\beta}_{B,U}$ coefficient) shows the strong up-market betas for the loser portfolios in bear markets. For the loser decile, the down-market beta is 1.539 ($= 1.252 + 0.287$) and the up-market beta is 0.602 higher (2.141). Also, note the slightly negative up-market beta increment for the

Table 5: **Momentum Portfolio Optionality in Bear Markets**

This table presents the results of a regressions of the excess returns of the 10 momentum portfolios and the Winner-Minus-Loser (wml) long-short portfolio on the CRSP value-weighted excess market returns, and a number of indicator variables. For each of these portfolios, the regression estimated here is:

$$\tilde{R}_{i,t}^e = [\alpha_0 + \alpha_B I_B] + [\beta_0 + I_B(\beta_B + \tilde{I}_U \beta_{B,U})] \tilde{R}_{m,t} + \tilde{\epsilon}_t$$

where R_m^e is the CRSP value-weighted excess market return, I_B is an *ex-ante* Bear-market indicator and I_U is a contemporaneous *UP*-market indicator, as defined in the text on page 17. The time period is 1928:08-2009:12.

Coef. Est.	Momentum Decile Portfolios – Excess Monthly Returns (<i>t</i> -statistics in parentheses)									
	1	2	3	4	5	6	7	8	9	10
$\hat{\alpha}_0$	-0.011 (-6.5)	-0.005 (-3.9)	-0.003 (-2.9)	-0.001 (-1.9)	-0.000 (-0.4)	-0.000 (-0.0)	0.001 (2.2)	0.003 (4.6)	0.003 (3.9)	0.005 (4.9)
$\hat{\alpha}_B$	-0.003 (-0.6)	-0.002 (-0.5)	-0.003 (-1.0)	-0.005 (-2.0)	-0.006 (-2.7)	-0.002 (-1.2)	-0.001 (-0.6)	-0.003 (-1.5)	0.005 (2.2)	0.002 (0.5)
$\hat{\beta}_0$	1.252 (32.8)	1.059 (38.0)	0.942 (42.3)	0.928 (50.5)	0.898 (54.4)	0.954 (67.5)	0.958 (63.8)	0.997 (66.7)	1.079 (63.7)	1.288 (50.2)
$\hat{\beta}_B$	0.287 (3.4)	0.350 (5.6)	0.351 (7.1)	0.158 (3.9)	0.150 (4.1)	0.075 (2.4)	0.028 (0.8)	-0.130 (-3.9)	-0.118 (-3.1)	-0.450 (-7.8)
$\hat{\beta}_{B,U}$	0.602 (5.3)	0.398 (4.8)	0.233 (3.5)	0.342 (6.2)	0.219 (4.4)	0.130 (3.1)	-0.003 (-0.1)	-0.002 (-0.0)	-0.214 (-4.2)	-0.197 (-2.6)

winner decile (= -0.197).

3.5.1 Asymmetry in the Optionality

It is interesting that the optionality associated with the loser portfolios that is apparent in the regressions in Table 5 is only present in bear markets. Table 6 presents the same set of regressions as in Table 5, only instead of using the Bear-market indicator I_B , we use the bull market indicator I_L . The key variables here are the estimated coefficients and t-statistics on $\beta_{L,U}$, presented in the last two rows of the Table. Unlike in Table 5, here any assymetry is mild, and the wml portfolio shows no statistically significant optionality. For loser decile portfolio, $\hat{\beta}_{L,U} = 0.019$, and the t-statistic is 0.2. Recall that, in bear markets, the estimated coefficient was 0.6, with a t-statistic of 5.3.

For the winner portfolios, we obtain the same slightly negative point estimate for the

Table 6: **Momentum Portfolio Optionality in Bull Markets**

This table presents the results of a regressions of the excess returns of the 10 momentum portfolios and the Winner-Minus-Loser (wml) long-short portfolio on the CRSP value-weighted excess market returns, and a number of indicator variables. For each of these portfolios, the regression estimated here is:

$$\tilde{R}_{i,t}^e = [\alpha_0 + \alpha_L I_L] + [\beta_0 + I_L(\beta_L + \tilde{I}_U \beta_{L,U})] \tilde{R}_{m,t} + \tilde{\epsilon}_t$$

where R_m^e is the CRSP value-weighted excess market return, I_L is an *ex-ante* bull-market indicator and I_U is a contemporaneous *UP*-market indicator, as defined in the text on page 17. The time period is 1928:08-2009:12.

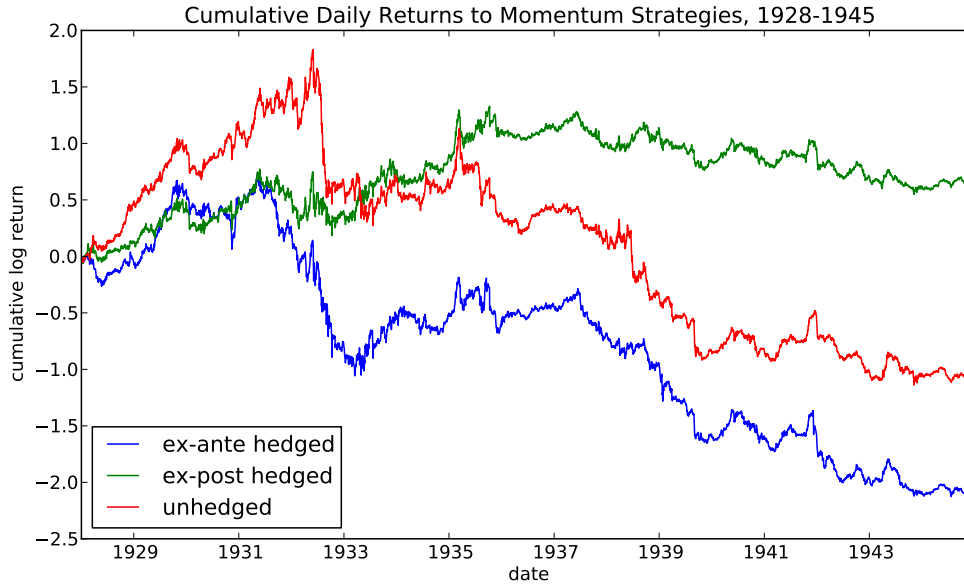
Coef. Est.	Momentum Decile Portfolios – Excess Monthly Returns (<i>t</i> -statistics in parentheses)										
	1	2	3	4	5	6	7	8	9	10	wml
$\hat{\alpha}_0$	0.005 (1.5)	0.006 (2.3)	0.002 (0.8)	0.004 (2.5)	0.001 (0.3)	0.002 (1.4)	0.000 (0.1)	-0.000 (-0.1)	0.001 (0.7)	0.001 (0.5)	-0.006 (-1.2)
$\hat{\alpha}_L$	-0.017 (-3.7)	-0.011 (-3.4)	-0.005 (-2.0)	-0.008 (-3.6)	-0.002 (-1.1)	-0.002 (-1.2)	-0.001 (-0.4)	0.002 (1.3)	0.003 (1.4)	0.008 (2.6)	0.023 (3.6)
$\hat{\beta}_0$	1.885 (47.9)	1.639 (57.1)	1.428 (62.7)	1.284 (67.5)	1.174 (69.2)	1.104 (76.4)	0.985 (64.7)	0.866 (57.0)	0.837 (48.2)	0.724 (27.7)	-1.161 (-21.1)
$\hat{\beta}_L$	-0.642 (-8.2)	-0.581 (-10.2)	-0.508 (-11.2)	-0.405 (-10.7)	-0.313 (-9.3)	-0.154 (-5.4)	-0.079 (-2.6)	0.104 (3.4)	0.267 (7.7)	0.653 (12.6)	1.302 (11.9)
$\hat{\beta}_{L,U}$	0.019 (0.2)	0.004 (0.0)	0.048 (0.7)	0.108 (1.9)	0.080 (1.5)	0.009 (0.2)	0.115 (2.5)	0.059 (1.3)	-0.055 (-1.0)	-0.194 (-2.4)	-0.216 (-1.3)

up-market beta increment. There is no apparent variation associated with the past market return.

3.6 *Ex-ante* Hedge of the market risk in the wml Portfolio

The results of the preceding section suggest that calculating hedge ratios based on future realized hedge ratios, as in Grundy and Martin (2001), is likely to produce strongly upward biased estimates of the performance of the hedged portfolio. As we have seen, the realized market beta of the momentum portfolio tends to be more negative when the realized return of the market is positive. Thus, the hedged portfolio – where the hedge is based on the future realized portfolio beta – will buy more of the market (as a hedge) in months where the market return is high.

Figure 8 adds the cumulative log return to the *ex-ante* hedged return to the plot from Figure 7. The strong bias in the ex-post hedge is clear here.

Figure 8: *Ex-Ante* Hedged Portfolio Performance

3.7 Market Stress and Momentum Returns

One very casual interpretation of the results presented in Section 3.5 is that there are option like payoffs associated with the past losers in bear markets, and the value of this option on the economy is not reflected in the prices of the past losers. This casual interpretation further suggests that the value of this option should be a function of the future variance of the market.

In this section we examine this hypothesis. Using daily market return data, we construct an *ex-ante* estimate of the market volatility over the next one month. In Table 7, we use this market variance estimate in combination with the bear-market indicator I_B previously employed to forecast future WML returns.

To summarize, we find that both estimated market variance and the bear market indicator independently forecast future momentum returns. The direction is as suggested by the results of the previous section: in periods of high market stress – bear markets with high volatility – momentum returns are low.

Table 7: **Momentum Returns and Estimated Market Variance**

This table presents estimated coefficients for the variations on the following regressions specification:

$$\tilde{r}_{WML,t} = \gamma_0 + \gamma_{Rm2y} \cdot I_B + \gamma_{\sigma_m^2} \cdot \hat{\sigma}_m^2 + \gamma_{int} \cdot I_B \cdot \hat{\sigma}_{m,t}^2 + \tilde{\epsilon}_t$$

Here, I_B is the bear market indicator described on page 17. σ_m^2 is an *ex-ante* estimator of market volatility over the next month.

	$\hat{\gamma}_0$	$\hat{\gamma}_B$	$\hat{\gamma}_{\sigma_m^2}$	$\hat{\gamma}_{int}$
1	0.0006 (5.59)	-0.0012 (-4.51)		
2	0.0008 (6.78)		-3.69 (-6.07)	
3	0.0009 (6.98)	-0.0006 (-2.04)	-3.07 (-4.54)	
4	0.0006 (6.06)			-4.75 (-7.17)
5	0.0006 (4.87)	-0.0004 (0.36)	-0.54 (-0.53)	-4.50 (-3.30)

4 Conclusions and Future Work

In “normal” environments, the market appears to underreact to public information, resulting in consistent price momentum. This effect is both statistically and economically strong. We see momentum manifested not only in equity markets, but across a wide range of asset classes.

However, in extreme market environments, the market prices of past losers embody a very high premium. When market conditions ameliorate, these losers experience strong gains, resulting in a “momentum crash.” We find that, in bear market states, and in particular when market volatility is high, the down-market betas of the past-losers are low, but the up-market betas are very large. This optionality does not appear to generally be reflected in the prices of the past losers. Consequently, the expected returns of the past losers are very high, and the momentum effect is reversed.

The effects are loosely consistent with several behavioral findings.¹⁵ In extreme situ-

¹⁵See Sunstein and Zeckhauser (2008), Loewenstein, Weber, Hsee, and Welch (2001), and Loewenstein (2000)

ations, where individuals are fearful, they appear to focus on losses, and probabilities are largely ignored. Whether this behavioral phenomenon is fully consistent with the empirical results documented here is a subject for further research..

Appendices

A Detailed Description of Calculations

A.1 Cumulative Return Calculations

The cumulative return, on an (implementable) strategy is an investment at time 0, which is fully reinvested at each point – *i.e.*, where no cash is put in or taken out, That is the cumulative arithmetic returns between times t and T is denoted $R(t, T)$.

$$R(t, T) = \prod_{s=t+1}^T (1 + R_s) - 1,$$

where R_s denotes the arithmetic return in the period ending at time t , where $r_s = \log(1 + R_s)$ denotes the log-return over period s ,

$$r(t, T) = \sum_{s=t+1}^T r_s.$$

For long-short portfolios, the cumulative return is:

$$R(t, T) = \prod_{s=t+1}^T (1 + R_{L,s} - R_{S,s} + R_{f,t}) - 1,$$

where the terms $R_{L,s}$, $R_{S,s}$, and $R_{f,s}$ are, respectively, the return on the long side of the portfolio, the short side of the portfolio, and the risk-free rate. Thus, the strategy reflects the cumulative return, with an initial investment of V_t , which is managed in the following way:

1. Using the $\$V_0$ as margin, you purchase $\$V_0$ of the long side of the portfolio, and short $\$V_0$ worth of the short side of the portfolio. Note that this is consistent with Reg-T requirements. Over each period s , the margin posted earns interest at rate $R_{f,s}$.
2. At then end of each period, the value of the investments on the long and the short side of the portfolio are adjusted to reflect gains to both the long and short side of the portfolio. So, for example, at the end of the first period, the

investments in both the long and short side of the portfolio are adjusted to set their value equal to the total value of the portfolio to $V_{t+1} = V_t \cdot (1 + R_L - R_S + R_f)$.

Note that, for monthly returns, this methodology assumes that there are no margin calls, etc, except at the end of each month. These calculated returns do not incorporate transaction costs.

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Table 8: **Market Model Regressions for Momentum Portfolios**

This table presents the results of a regressions of the excess returns of the 10 momentum portfolios on the CRSP value-weighted excess market returns:

$$\tilde{R}_{i,t}^e = \alpha + \beta \tilde{R}_{m,t}^e + \tilde{\epsilon}_t$$

where R_m^e is the CRSP value-weighted excess market return, as defined in the text on page 17. The time period is 1928:08-2009:12.

Momentum Decile Portfolios - Excess Returns										
Coef.	<i>(t-statistics in parentheses)</i>									
Vars.	1	2	3	4	5	6	7	8	9	10
$\hat{\alpha}$	-0.009 (-5.8)	-0.004 (-3.5)	-0.003 (-3.2)	-0.001 (-1.5)	-0.001 (-1.1)	-0.000 (-0.1)	0.001 (1.9)	0.003 (4.4)	0.003 (4.4)	0.006 (5.4)
$\hat{\beta}$	1.558 (53.0)	1.340 (60.5)	1.177 (66.5)	1.100 (75.4)	1.032 (82.2)	1.026 (99.6)	0.970 (91.6)	0.933 (86.7)	0.964 (75.2)	1.013 (49.9)

Table 9: **Timing Regressions for Momentum Portfolios**

This table presents the results of a regressions of the excess returns of the 10 momentum portfolios on the CRSP value-weighted excess market returns:

$$\tilde{R}_{i,t}^e = [\alpha_0 + \alpha_B I_B] + [\beta_0 + I_B \beta_B] \tilde{R}_{m,t}^e + \tilde{\epsilon}_t$$

where R_m^e is the CRSP value-weighted excess market return, and I_B is the bear-market indicator as defined in the text on page 17. The time period is 1928:08-2009:12.

Momentum Decile Portfolios										
Vars.	1	2	3	4	5	6	7	8	9	10
1	-0.011 (-6.5)	-0.005 (-3.9)	-0.003 (-2.8)	-0.001 (-1.8)	-0.000 (-0.4)	-0.000 (-0.0)	0.001 (2.2)	0.003 (4.6)	0.003 (3.8)	0.005 (4.9)
I_B	0.016 (4.1)	0.011 (3.8)	0.004 (2.0)	0.006 (3.1)	0.001 (0.5)	0.002 (1.2)	-0.001 (-0.9)	-0.003 (-2.1)	-0.002 (-1.0)	-0.004 (-1.7)
R_m^e	1.252 (32.4)	1.059 (37.6)	0.942 (42.1)	0.928 (49.6)	0.898 (53.8)	0.954 (67.3)	0.958 (63.9)	0.997 (66.8)	1.079 (63.2)	1.288 (50.0)
$I_B * R_m^e$	0.634 (11.5)	0.580 (14.4)	0.486 (15.2)	0.355 (13.3)	0.276 (11.6)	0.150 (7.4)	0.027 (1.2)	-0.131 (-6.2)	-0.242 (-9.9)	-0.564 (-15.4)