Hw2

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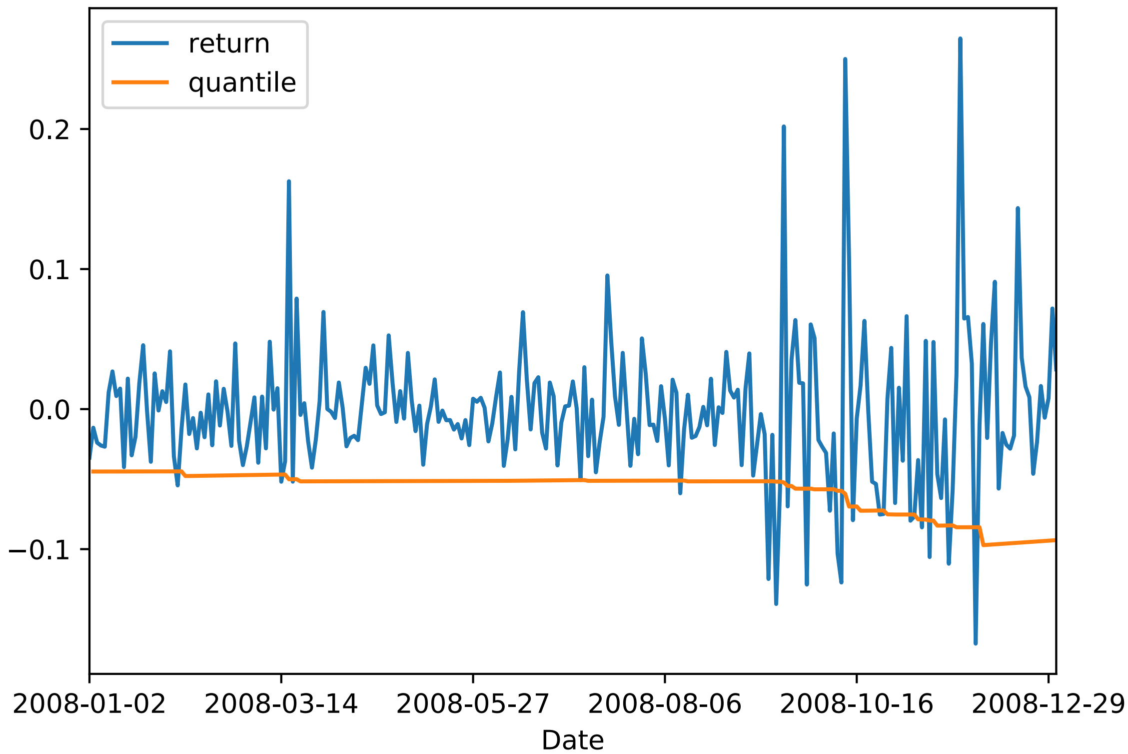
Problem 1

1. Technical used to calculate VaR: Historical method.
   Goldman Sachs use historical data to estimate VaR. And they weight the data to give greater weights to more recent observations.
   Time horizon: one day time horizon.
   Confidence level: 95%
   Number of VaR exceptions in 2008 (days where loss exceeded VaR): 13 days
   changes to VaR methodology made as a result of the financial crisis
   No changes in the report

2.

```
import pandas as pd
from scipy.stats import norm
import numpy as np
import matplotlib.pyplot as plt
from pandas_datareader import data
from pandas.plotting import register_matplotlib_converters
register_matplotlib_converters()
stock_data = pd.read_csv('/Users/huanyu/Desktop/RiskManagement/hw2/GS.csv')
days = len(stock_data)
stock_data['return'] = stock_data['Adj Close'] / stock_data['Adj Close'].shift(1) - 1
c = 0.99
z_c = norm.ppf(1 - c)
for i,v in enumerate(stock_data['Date'].values[502:]):
    quantile = stock_data.loc[stock_data['Date'] < v,'return'].quantile(0.01)
    #print(quantile)
    stock_data.loc[stock_data['Date'] == v,'VaR'] = stock_data.loc[stock_data['Date'] == v, 'return'] -
    stock_data.loc[stock_data['Date'] == v, 'quantile'] = quantile
stock_data['quantile'] = stock_data['quantile'].shift(1)
stock_data.loc[~stock_data['quantile'].isna(),'within_var'] = np.ceil(stock_data.loc[~stock_data['quant
stock_data.loc[stock_data['Date'] > '2007-12-31',('Date','return','quantile')].plot(x='Date',y=['return
plt.show()
```




```
print(len(stock_data[stock_data['within_var'] == 0]))
```

```
## 21
```

(b) By back test, there are 21 days where loss exceeded VaR. On a 252 trading days basis, $\frac{21}{252} \approx 8.33\% >> 1\%$. The risk is much higher than that measured by VaR.

(c) The firm is exposured to market volatility, so the return is more volatile and more returns accross the VaR.

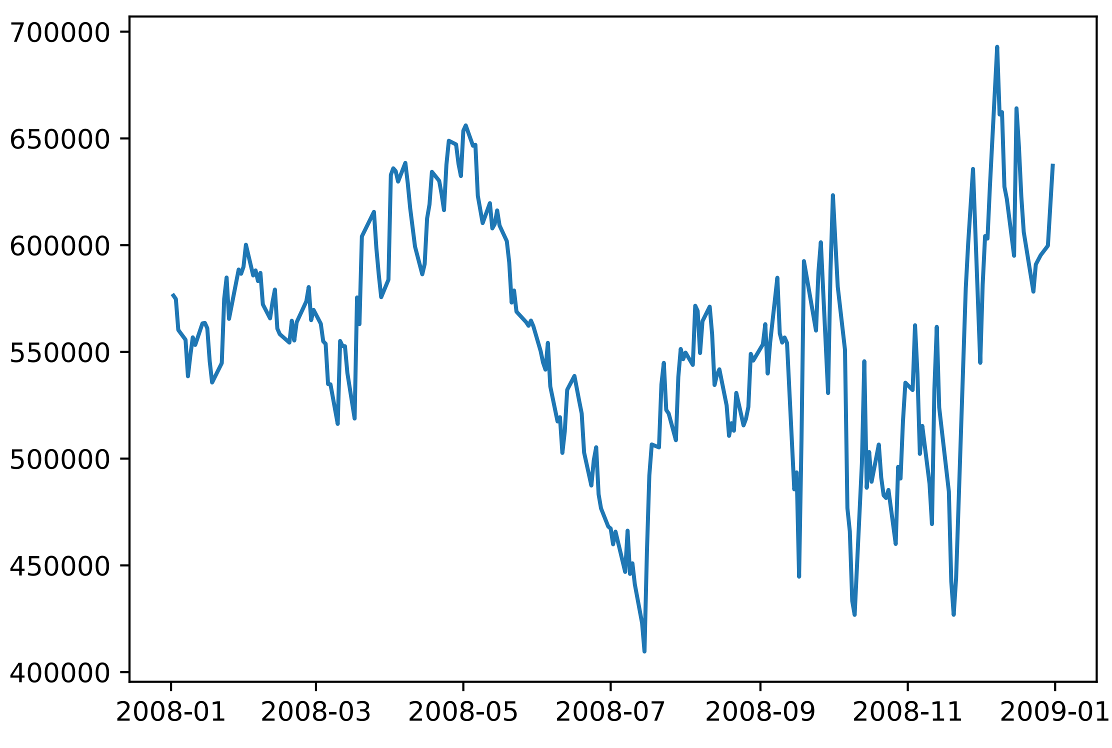
3.

(a)

```
start_date = pd.to_datetime('01/01/2006')
end_date = pd.to_datetime('12/31/2008')
gs = data.DataReader('GS','yahoo',start_date,end_date)
ubs = data.DataReader('UBS','yahoo',start_date,end_date)
jpm = data.DataReader('JPM','yahoo',start_date,end_date)
citi = data.DataReader('C','yahoo',start_date,end_date)
bcs = data.DataReader('BCS','yahoo',start_date,end_date)
ms = data.DataReader('MS','yahoo',start_date,end_date)
db = data.DataReader('DB','yahoo',start_date,end_date)
bac = data.DataReader('BAC','yahoo',start_date,end_date)
bnp = data.DataReader('BNPQY','yahoo',start_date,end_date)
cs = data.DataReader('CS','yahoo',start_date,end_date)
banks = [gs,ubs,jpm,citi,bcs,ms,db,bac,bnp,cs]
start_date = pd.to_datetime('01/02/2008')
```

```python
portfolio = 0
for i, v in enumerate(banks):
    if i % 2 == 0:
        v.loc[start_date,'value'] = 1000000
        share = 1000000 / v.loc[start_date,'Adj Close']
    else:
        v.loc[start_date, 'value'] = 2000000
        share = 2000000 / v.loc[start_date, 'Adj Close']

    for j, w in enumerate(v.index):
        v.loc[w,'value'] = share * v.loc[w,'Adj Close']
    portfolio += v.loc[:,'value']
portfolio = pd.DataFrame({'value':portfolio})
portfolio['return'] = portfolio['value'] / portfolio['value'].shift(1) - 1
for i, v in enumerate(portfolio.index[502:]):
    quantile = portfolio.loc[:v, 'return'].quantile(0.01)
    portfolio.loc[v, 'VaR'] = portfolio.loc[v, 'value'] * -quantile
plt.plot(portfolio['VaR'])
plt.show()
```




```python
lastday_var = portfolio.loc['2008-12-31','VaR']
print(lastday_var)
```

```
## 637202.5450970458
```

(b) DVAR

```python
portfolio_dvar = 0
result_dvar = dict()
banks_name = ['gs','ubs','jpm','citi','bcs','ms','db','bac','bnp','cs']
for i, v in enumerate(banks):
    for j, w in enumerate(banks):
        if j % 2 == 0:
            w['share'] = 1000000 / w.loc[start_date, 'Adj Close']
        else:
            w['share'] = 2000000 / w.loc[start_date, 'Adj Close']
    if i % 2 == 0:
        v['share'] = 1000001 / v.loc[start_date,'Adj Close']
    else:
        v['share'] = 2000001 / v.loc[start_date, 'Adj Close']
    portfolio_dvar = 0
    for j, w in enumerate(banks):
        w['value'] = w['share'] * w['Adj Close']
        portfolio_dvar += w.loc[:,'value']
    portfolio_dvar_df = pd.DataFrame(portfolio_dvar)
    portfolio_dvar_df['return'] = portfolio_dvar_df['value'] / portfolio_dvar_df['value'].shift(1) - 1
    for j, w in enumerate(portfolio_dvar_df.index[502:]):
        quantile = portfolio_dvar_df.loc[:w, 'return'].quantile(0.01)
        portfolio_dvar_df.loc[w, 'VaR'] = portfolio_dvar_df.loc[w, 'value'] * -quantile
    result_dvar[banks_name[i]] = portfolio_dvar_df.loc['2008-01-01':,'VaR'] - portfolio.loc['2008-01-01

dvar_df = pd.DataFrame(result_dvar)
print(dvar_df.loc['2008-12-31',:])
```

```
## gs      0.044388
## ubs     0.039194
## jpm     0.070150
## citi    0.029169
## bcs    -0.000139
## ms      0.064282
## db      0.041399
## bac     0.032932
## bnp     0.033671
## cs      0.058290
## Name: 2008-12-31 00:00:00, dtype: float64
```

CVAR

```python
portfolio_cvar = 0
result_cvar = dict()
for i, v in enumerate(banks):
    for j, w in enumerate(banks):
        if j % 2 == 0:
            w['share'] = 1000000 / w.loc[start_date, 'Adj Close']
        else:
            w['share'] = 2000000 / w.loc[start_date, 'Adj Close']
    if i % 2 == 0:
        v['share'] = 1010000 / v.loc[start_date,'Adj Close']
    else:
        v['share'] = 2020000 / v.loc[start_date, 'Adj Close']
    portfolio_cvar = 0
    for j, w in enumerate(banks):
```

```
        w['value'] = w['share'] * w['Adj Close']
        portfolio_cvar += w.loc[:,'value']
    portfolio_cvar_df = pd.DataFrame(portfolio_cvar)
    portfolio_cvar_df['return'] = portfolio_cvar_df['value'] / portfolio_cvar_df['value'].shift(1) - 1
    for j, w in enumerate(portfolio_cvar_df.index[502:]):
        quantile = portfolio_cvar_df.loc[:w, 'return'].quantile(0.01)
        portfolio_cvar_df.loc[w, 'VaR'] = portfolio_cvar_df.loc[w, 'value'] * -quantile
    result_cvar[banks_name[i]] = portfolio_cvar_df.loc['2008-01-01':,'VaR'] - portfolio.loc['2008-01-01

cvar_df = pd.DataFrame(result_cvar)
print(cvar_df.loc['2008-12-31',:])
```

```
## gs        443.877323
## ubs       783.902628
## jpm       701.456622
## citi      583.348750
## bcs        -1.356101
## ms       1285.650387
## db        414.002079
## bac       658.687853
## bnp       336.723815
## cs       1165.804553
## Name: 2008-12-31 00:00:00, dtype: float64
```

(c) The sum of CVaR divided by 1% should be total portfolio VaR. And the result $(444 + 784 + 701 + 583 + 1286 + 414 + 658 + 338 + 1166)/0.01 \approx 637203$

(d) We should allocate less capital on firms with large DVAR and CVAR.

Problem 2.

1. Use the formula in questions 2.

$$ES = \frac{1}{1-c} \int_c^1 VaR_\alpha \cdot d\alpha$$

$\because W - W_0 \sim N(\mu, \sigma^2)$

$$VaR_\alpha = Z(\alpha) \cdot \sigma - \mu$$

$$\int_c^1 VaR_\alpha d\alpha = \int_c^1 (Z(\alpha) \cdot \sigma - \mu) d\alpha = -\mu(1-c) + \sigma \int_c^1 Z(\alpha) \cdot d\alpha$$

$\cancel{\int_c^1 Z(\alpha) d\alpha} =$ $Z(x) = \Phi^{-1}(x)$. $\Phi'(x) = f(x)$, in which $\Phi(x)$ is cdf, $f(x)$ is pdf

$$\int_c^1 Z(\alpha) d\alpha = \int_{\Phi^{-1}(c)}^{\Phi^{-1}(1)} Z(\Phi(W)) \cdot f(W) dW = \int_{Z(c)}^{\infty} W \cdot f(W) \cdot dW$$

$\because f(W) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{W^2}{2}}$

$$\therefore \int_{Z(c)}^{\infty} W \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{W^2}{2}} dW = -\frac{1}{\sqrt{2\pi}} e^{-\frac{W^2}{2}} \Bigg|_{Z(c)}^{\infty}$$

$$= \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{Z^2(c)}{2}}$$

$$\therefore ES = \frac{1}{1-c} \int_c^1 VaR_\alpha d\alpha = -\mu + \frac{1}{\sqrt{2\pi} \cdot (1-c)} e^{-\frac{Z^2(c)}{2}}$$

Problem 2.

2. In the notes. $ES = W_0 - \dfrac{\int_{-\infty}^{W_0 - VaR} w \cdot f(w) \cdot dw}{\int_{-\infty}^{W_0-VaR} f(w)\, dw} = W_0 - \dfrac{\int_{-\infty}^{W_0-VaR} w f(w)\, dw}{1-c}.$

$$ES = \frac{1}{1-c} \left(W_0 (1-c) - \int_{-\infty}^{W_0-VaR} w f(w)\, dw \right)$$

The question is coverted to prove. $W_0(1-C) - \int_{-\infty}^{W_0-VaR} w f(w)\, dw = \int_c^1 VaR_\alpha d\alpha$

Let $F(W) = 1-\alpha$. $\quad F(W_0 - VaR_c) = 1-C$. $\quad F(-\infty) = 0$

Let $G(x) = 1 - F(x)$. $\quad G(W_0 - VaR_c) = c \quad G(-\infty) = 1 \quad G'(x) = -f(x)$

By Substitution rule for Definite Intergrals.

$$\int_a^b f(g(x)) \cdot g'(x)\, dx = \int_{g(a)}^{g(b)} f(u) \cdot du \quad u = g(x).$$

$G'(W) = -f(W) \quad G(W) = 1 - F(W) = \alpha. \quad W = G^{-1}(\alpha)$

$\because W = W_0 - VaR_\alpha = G^{-1}(\alpha)$

$$\therefore -\int_{-\infty}^{W_0-VaR} w f(w)\, dw = \int_{-\infty}^{W_0-VaR} G^{-1}(G(w))\, G'(w)\, dw$$

$$= \int_{G(-\infty)}^{G(W_0-VaR)} G^{-1}(\alpha) \cdot d\alpha = \int_1^c (W_0 - VaR_\alpha) \cdot d\alpha.$$

$$W_0 (1-C) - \int_{-\infty}^{W_0-VaR} w f(w)\, dw = W_0(1-C) + \int_1^c (W_0 - VaR_\alpha)\, d\alpha$$

$$= W_0(1-C) + W_0(C-1) + \int_c^1 VaR_\alpha d\alpha$$

$$= \int_c^1 VaR_\alpha\, d\alpha.$$