**UCLA Anderson School of Management MGMTMFE403, Fall 2017**

**Quizz #1**

**NAME:**

**SECTION:**

You have 90 minutes to finish this assignment. Please write legibly. You may use a cheat sheet as specified in the syllabus. No laptops or equipment that allows communication are allowed. Only a simple calculator.

Please write the answer for each question in the space provided. If you need more space, you may use the back of your pages.

ATTENTION: You are not allowed to discuss the contents of this exam with anyone until tomorrow. If you violate this rule, you will be found in violation of the UCLA Student Code of Conduct, and this will have severe consequences.

**Problem 1. (30 Points)** Consider the proceses $M_t$ and $X_t$ defined by

$$\begin{aligned} dM_t &= aM_t dW_t, \\ dX_t &= 0.2dt + 0.5dW_t, \end{aligned}$$

where $a$ is a constant, and $W_t$ is a Brownian motion. Next consider the process

$$Z_t = M_t X_t$$

Provide the value of $a$, such that $Z_t$ is a (local) martingale.

Solutions to problem 1

**Problem 2. (40 Points)** Suppose that the function $F(t,x)$ satisfies the partial differential equation (PDE)

$$\begin{aligned} \frac{\partial F(t,x)}{\partial t} - \rho x \frac{\partial F(t,x)}{\partial x} + \frac{\sigma^2}{2} \frac{\partial^2 F(t,x)}{\partial x^2} - \beta F(t,x) &= 0 \\ F(T,x) &= e^{\lambda x} \end{aligned}$$

Provide an explicit formula for $F(0,x)$.

(Hint: Try a solution of the form: $F(t,x) = e^{a_0(t)+a_1(t)x}$, determine the appropriate functions $a_0(t)$, $a_1(t)$, and evaluate $F(t,x)$ at $t=0$).

Solutions to Problem 2

Solutions to Problem 2

**Problem 3. (30 Points)** Suppose that $X_t$ satisfies the SDE

$$dX_t = -\mu(X_t - \overline{X})dt + \sigma\sqrt{X_t}dW_t,\ X_0 = 1$$

Compute

$$E_0\left(X_T\right)$$

for some $T > 0$. (Hint: determine the dynamics of $d(e^{\mu t}X_t)$ and then integrate to compute $e^{\mu t}X_t$)

Solutions to Problem 3