Agenda

- Libor Rates.
- Valuing Floating Rate Notes.
- Interest Rate Dynamics.
- Interest Rate Swaps.
- Swap Bond Math.
- Applications of Swaps.

Libor Rates

- USD Libor based on 16 A or AA rated banks.
- The trimmed average.
- Whose credit is reflected in Libor rates?
- Examples.
- Libor and zero-coupon bond prices.

$$D(0.25) = \frac{1}{1 + \frac{A}{360} L}$$

$$\Longrightarrow L = \left(\frac{1}{D(0.25)} - 1 \right) \times \frac{360}{A}$$

- Libor loan: Borrow \$1, in three months repay

$$1 + \frac{A}{360} L = 1 + \left(\frac{1}{D(0.25)} - 1 \right) = \frac{1}{D(0.25)}$$

Floating Rate Notes

- Cash flow structure.
- Setting in advance, paying in arrears.
- What is the PV of the FRN's cash flows?
- The two-stage discounting approach.
- Duration of a FRN.
- Convexity of a FRN.

Swap Intuition

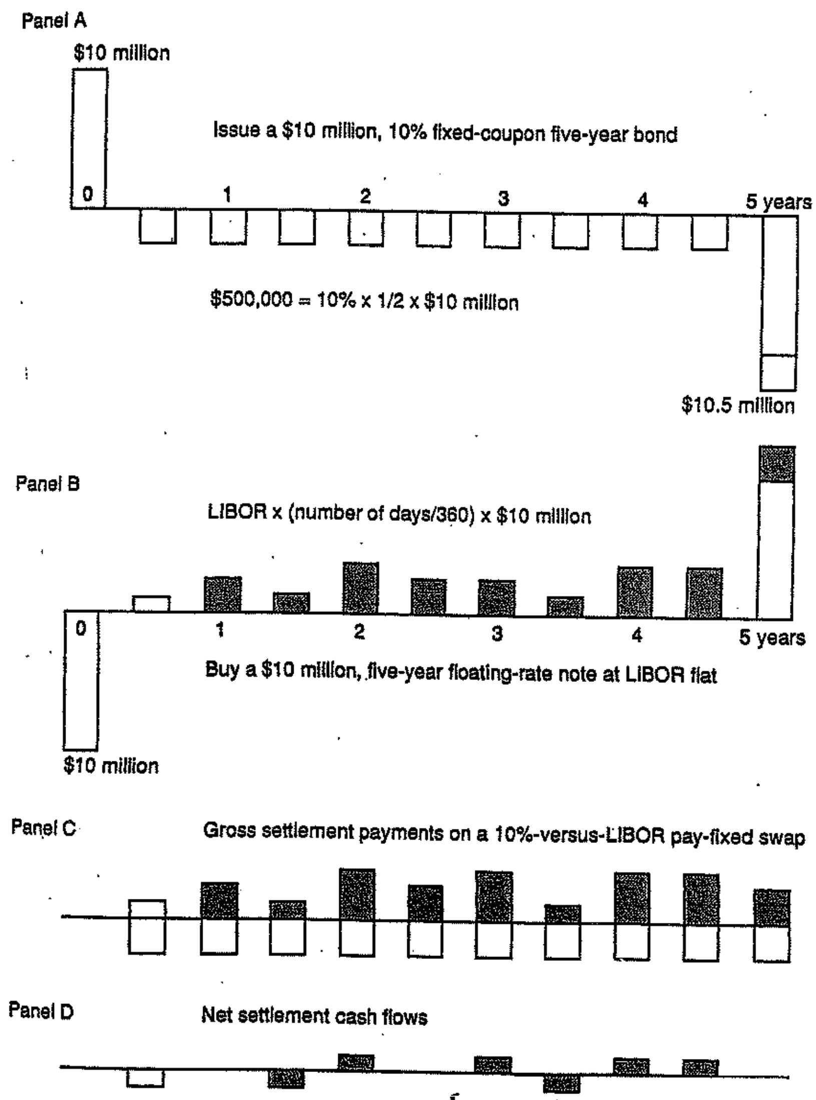
- We now have two securities that trade at par.
  - FRN.
  - Coupon bond with coupon set to par rate.
- Imagine that we exchange the FRN for a par coupon bond.

Time	Pay	Receive
0.25	$\frac{A}{360} L_0$	
0.50	$\frac{A}{360} L_{0.25}$	$\frac{c}{2}$
0.75	$\frac{A}{360} L_{0.50}$	
1.00	$1 + \frac{A}{360} L_{0.75}$	$1 + \frac{c}{2}$

Swaps

- Think of a swap as an exchange of a FRN for a par bond.
- Swap rate.
- Capital markets example.
- Terminology:
  - Receiving
  - Paying

EXHIBIT 5
Pay-Fixed Swap as a Combination of Capital Market Instruments



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<HELP> for explanation, <MENU> for similar functions.

United States | 99) Settings | 98) Excel | Interest Rate Swap Rates

Date Range: 11-Jan-2015 - 11-Feb-2015 1 Month

40) Semi Swaps 41) Sprds to Gov. 42) Ann Swaps 43) Ann Sprds 44) OIS Swaps 45) Combined 46) Muni Swaps

USD SemiAnnual 30/360 Swap Rates

Avg Now

Tenor	Bid	Ask	Mid	Change	Today	#SD Δ/day	Low	Range	High	Avg	+/-BPS	#SD
1) 1 YR	0.500	0.503	0.502	0.007			0.356		0.506	0.419	8.4	2.2
2) 2 YR	0.923	0.933	0.928	-0.001			0.647		0.937	0.771	16.2	2.2
3) 3 YR	1.265	1.289	1.277	0.002			0.930		1.289	1.082	20.8	2.3
4) 4 YR	1.509	1.545	1.527	0.003			1.148		1.545	1.315	23.0	2.3
5) 5 YR	1.691	1.721	1.706	0.004			1.314		1.721	1.490	23.0	2.3
6) 6 YR	1.830	1.840	1.835	-0.001			1.461		1.843	1.628	21.1	2.1
7) 7 YR	1.936	1.943	1.940	0.002			1.568		1.946	1.737	20.6	2.1
8) 8 YR	2.017	2.025	2.021	0.000			1.656		2.027	1.828	19.7	2.0
9) 9 YR	2.087	2.094	2.090	0.000			1.727		2.099	1.903	19.2	2.0
10) 10 YR	2.143	2.153	2.148	-0.001			1.789		2.158	1.966	18.8	1.9
11) 15 YR	2.335	2.346	2.340	-0.001			1.983		2.347	2.171	17.5	1.8
12) 20 YR	2.417	2.435	2.426	-0.001			2.078		2.435	2.270	16.5	1.8
13) 25 YR	2.461	2.471	2.466	-0.001			2.124		2.471	2.317	15.4	1.7
14) 30 YR	2.486	2.494	2.490	0.001			2.151		2.494	2.344	15.0	1.6

Swap Bond Math

- Swap value at time zero.
- Swap value at time t.

$$\frac{c}{2}\sum_{j=1}^{2(N-i)} D(i, i+j/2) + D(i, N) - 1$$

- DV01 of a swap.
- Convexity of a swap.
- Bloomberg examples.

<HELP> for explanation.
Enter all values and hit <Go>

91) Actions	92) Products	94) Data & Settings	95) Info	Swap Manager

3) Main 4) Curves 5) Cashflow 7) Details 10) Resets 11) Risk 13) Scenario 15) CVA 17) Matrix

Fixed Float Swap CCP OTC Cpty SWAP CNTRPART Deal ID 20) Properties

31) Load | 32) Save | 34) Ticket | 6) Trade Activit | 37) CCP Margin | 38) CVA | Send to VCON/

Leg 1	Receive Fixed		
Notional	10MM	Leg ID	
Currency	USD	Coupon	1.691191 %
Effective	02/13/2015	Calc Basis	Money Mkt
Maturity	02/13/2020	Day Count	ACT/ACT
Pay Freq	SemiAnnual	Unwind Cpn	1.691191 %
			61) Detail
MV	10,000,000.00	Accrued	0.00
Premium	100.00	DV01	5,068.10

Leg 2	Pay Float		
Notional	10MM	Leg ID	
Currency	USD	Index	US0003M
Effective	02/13/2015	Latest Index	0.25810
Maturity	02/13/2020	Tenor	3 Month
Reset Freq	Quarterly	Leverage	1.00000
Pay Freq	Quarterly	Spread	0.00 bp
		Day Count	ACT/360
			62) Detail
MV	-10,000,000.00	Accrued	0.00
Premium	-100.00	DV01	-247.06

Market CSA Coll Ccy N/A

Dscnt Curve 23 Bid USD Bloomberg Curve

OIS DC Stripping OFF

Dscnt Curve 23 Bid USD Bloomberg Curve
Fwd Curve 23 Bid USD Bloomberg Curve

Curve Date 02/11/2015 Valuation 02/13/2015

Valuation

Par Cpn	1.691191	Calculate	Premium		
Principal	0.00	Unwind Annuity	0.000000	PV01	4,821.01
Accrued	0.00	Unwind PV	0.00	DV01	4,821.04
Market Value	0.00	Premium	0.00000	Gamma (1bp)	2.71

<HELP> for explanation.
Enter all values and hit <Go>

91) Actions | 92) Products | 94) Data & Settings | 95) Info | Swap Manager

3) Main 4) Curves 5) Cashflow 7) Details 10) Resets 11) Risk 13) Scenario 15) CVA 17) Matrix

Fixed Float Swap CCP OTC Cpty SWAP CNTRPART Deal ID 20) Properties

31) Load | 32) Save | 34) Ticket | 16) Trade Activit | 37) CCP Margin | 38) CVA | Send to VCON/

Leg 1	Receive Fixed			Leg 2	Pay Float		
Notional	10MM	Leg ID		Notional	10MM	Leg ID	
Currency	USD	Coupon	1.691191 %	Currency	USD	Index	US0003M
Effective	02/13/2015	Calc Basis	Money Mkt	Effective	02/13/2015	Latest Index	0.25810
Maturity	02/13/2020	Day Count	ACT/ACT	Maturity	02/13/2020	Tenor	3 Month
Pay Freq	SemiAnnual	Unwind Cpn	1.750000 %	Reset Freq	Quarterly	Leverage	1.00000
				Pay Freq	Quarterly	Spread	0.00 bp
						Day Count	ACT/360
			61) Detail				62) Detail
MV	9,999,920.07	Accrued	0.00	MV	-10,000,000.00	Accrued	0.00
Premium	100.00	DV01	5,068.24	Premium	-100.00	DV01	-247.06

Market CSA Coll Ccy N/A | OIS DC Stripping OFF

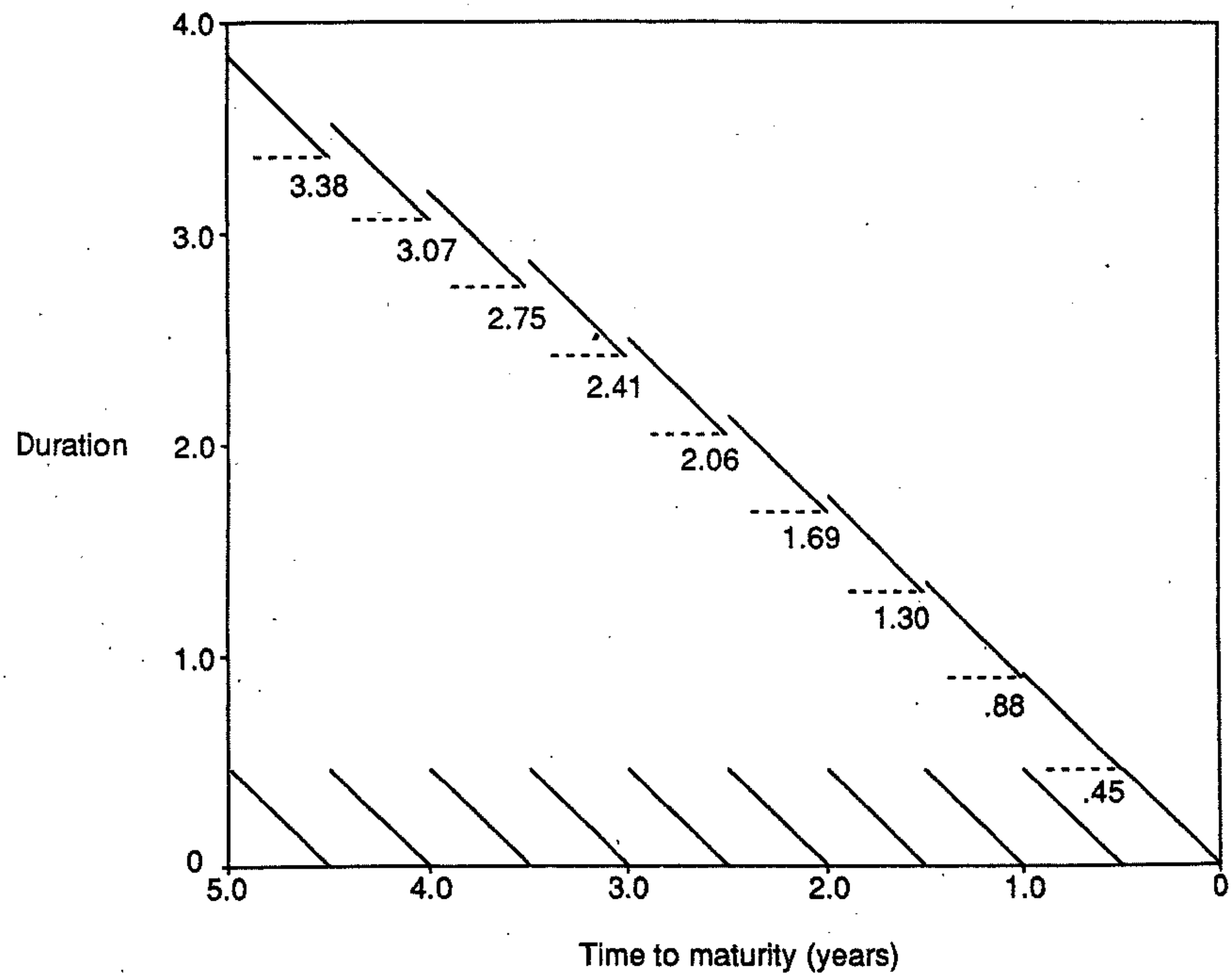
Dscnt Curve	23	Bid	USD Bloomberg Curve	Dscnt Curve	23	Bid	USD Bloomberg Curve
				Fwd Curve	23	Bid	USD Bloomberg Curve

Curve Date 02/11/2015 Valuation 02/13/2015

Valuation

Par Cpn	1.691357	Calculate	Premium		
Principal	-79.93	Unwind Annuity	-0.058809	PV01	4,820.54
Accrued	0.00	Unwind PV	-28,349.26	DV01	4,821.18
Market Value	-79.93	Premium	-0.00080	Gamma (1bp)	2.71

EXHIBIT 1
The Duration of a Swap over Time



The dotted lines indicate the duration of a swap during each time period. The duration of the interest rate swap is the duration of the fixed-rate side less the duration of the floating-rate side.

Risk Management Role of Swaps

d

- Balance sheet of a startup in the financial industry.
- Balance sheet after interest rate increase.
- Fundamental problem: mismatched DV01s of assets and liabilities.

Risk Management Role of Swaps: Case 1

3

- Firm issues 5 year FRN at Libor + 100bps.
- Net revenues tied to short-term rates.
- Assets and liabilities have similar DV01s.
- Two years later, assets have changed and now have 3 year duration.
- How to manage interest rate risk?
- Repurchase FRNs, issue fixed rate debt?
- Execute interest rate swap.

	1	2	3
FRN	-L	-L	-100-L
Swap	+L	+L	+L
	-F	-F	-F
Net	-F	-F	-F-100

Risk Management Role of Swaps: Case 1

- Swap diagram.
- All in cost of synthetic fixed rate debt.
- Swaping fixed into floating, floating into fixed.
- Savings and Loan Crisis of 1980s.

Suppose that two years ago Company X issued a five-year floating-rate note that pays semiannually a coupon rate equal to LIBOR + 1 percent. Assume further that when the firm issued this debt instrument its net revenues were closely tied to short-term interest rates.

Therefore, the FRN provided a natural internal hedge since the interest rate characteristics of its liabilities matched its assets. Using the concept of *duration*—a technical, statistical measure of a financial instrument's price elasticity relative to changes in yield—one would say that the company was *immunized* from interest rate risk since the short-duration FRN balanced the short-duration assets. Since both revenue and coupon interest cash flows would respond quickly to shifts in the level of interest rates, the market values of the assets and liabilities, and hence the value of the firm, were not very interest sensitive. Now suppose that there has been a fundamental change in the nature of the firm's assets such that net revenues are expected to be roughly constant in nominal terms over the next few years. Clearly, the company is exposed to the risk that interest rates will increase, thereby raising the interest expense of the FRN without any compensating rise in revenues. In terms of market value, the longer-duration assets would decline in price by more than the shorter-duration liabilities.[3]

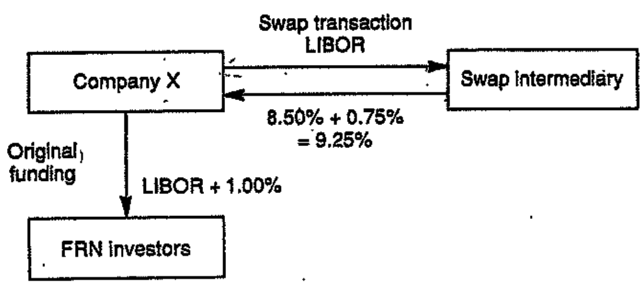
Two ways Company X can protect itself from rising rates are (*a*) issuing a new three-year fixed-coupon bond in an amount sufficient to buy back the remaining portion of the existing FRN or (*b*) combining the FRN with a three-year pay-fixed interest rate swap having semiannual settlement dates timed to match the bond's coupon payment dates. There are several reasons why the second solution is preferable from the firm's perspective. First, refinancing the existing debt issue requires a new capital market transaction and paying additional underwriting and registration fees. The interest rate swap, on the other hand, can be done with virtually no transaction costs. Second, in the absence of a call provision, an open market debt repurchase program is risky in terms of the total price to be paid and the total quantity of the notes that can be redeemed.[4] Finally, interest rate swaps can be reversed almost as easily as they can be implemented and, consequently, they can be thought of as a more flexible solution to a risk management problem that might be temporary.

The combination of the existing FRN and the pay-fixed interest rate swap is equivalent to a synthetic three-year fixed-rate bond with semiannual coupon payments. To see this, assume that the market maker's asking quote on a three-year swap against six-month LIBOR is 75 basis points over the Treasury yield, which currently stands at 8.50 percent. Once Company X agrees to these terms, two things will happen on each of its remaining coupon payment dates. First, since the original debt is still in place, the firm is responsible for making a coupon payment based on the prevailing LIBOR plus 100 basis points. Thus, observe that the swap agreement *supplements,* rather than replaces, the obligation that Company X has to its existing investors. Second, by the design of the swap, the firm receives from its counterparty payments based on the same six-month LIBOR that it must pay out on the FRN, while its swap obligations are payments based on the fixed rate of 9.25 percent.

These two steps are illustrated in Exhibit 7. This display also shows that, after adjusting for differences in rate quotation methods, the effective cost of the fixed-rate financing for Company X is 10.264 percent. An important caveat in interpreting this synthetic fixed rate is that it is not directly comparable with the fixed rate Company X would have obtained via the bond repurchase scheme since it involves credit exposure to the swap counterparty. Barring default on the part of the counterparty, however, the combination of the FRN and the pay-fixed swap has allowed the firm to hedge its exposure to rising rates.

Notice also that the plain vanilla swap has resolved the duration mismatch that arose between the assets and liabilities. Recall that it was a change in the nature of the asset base (and, in particular, that net revenues had become less sensitive to interest rates,

EXHIBIT 7
Hedging Application



Effective Annual Financing Cost (on a 365-Day Bond Basis)

FRN coupon payment: LIBOR + 1.00%(365/360)	= (LIBOR)(365/360) + 1.014%
Swap transaction: *(a)* Fixed-rate payment	= 9.25%
(b) Floating-rate receipt	= (LIBOR)(365/360)
Net cost of funds (9.25% + 1.014%)	= 10.264%

thereby lengthening the duration of assets) that created this mismatch. The pay-fixed swap effectively has lengthened the duration of the company's liabilities, so that once again it is in an immunized position. In the same vein, it is a simple matter to show that a receive-fixed swap has the effect of shortening the duration of fixed-coupon debt. These conclusions hold for swaps that are attached to liabilities, and the opposite holds for asset swaps: a pay-fixed swap shortens, and a receive-fixed swap lengthens, the duration of the asset. In general, *sensitizing* the cash flows of a security to interest rate changes in equivalent to shortening the duration; *desensitizing* the cash flows lengthens the duration.

Risk Management Role of Swaps: Case 2

8

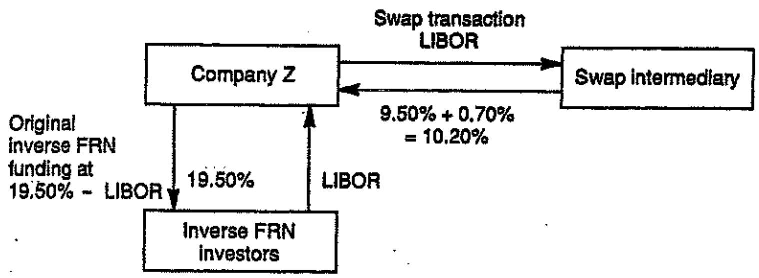
- Firm wants to raise funds for 5 year horizon by issuing fixed rate debt.
- Best cash market alternative is to issue Eurobond at 9.76 percent cost.
- Alternative: Issue inverse floater at 19.50 - Libor and then swap structure into fixed.
- Swap diagram.
- Arbitrage application.
- Lack of integration across markets creates opportunities.

The last two applications, while differing in their objectives for interest rate risk management, were similar in that each was motivated by the transactional efficiency with which the swap obtains the desired cash flows. But it is also possible that sufficient pricing discrepancies exist across markets to allow arbitrage profits as well. For instance, it is possible that the synthetic fixed-rate of 10.264 percent from the first example, obtained by combining the pay-fixed swap with the FRN, was considerably lower than Company X's best direct alternative in the fixed-rate bond market. However, as mentioned above, most arbitrage transactions at present are rather complex and require some unique features on either the capital market security or the swap. Here we shall show an example of an arbitrage strategy using an innovative security and a plain vanilla swap.

Suppose that Company Z is attempting to raise fixed-rate funds for the next five years and that its best capital market alternative would be a par-value Eurobond with an annually paid coupon rate of 10 percent. The Eurobond therefore would have a yield of 10 percent on an annually compounded basis or 9.76 percent for semiannual compounding. Suppose further that the choice of issuing an FRN and entering a pay-fixed swap does not dominate the Eurobond. However, the firm can issue a par-value *inverse* floating rate note, or "bull floater," paying a semi-annual coupon based on a formula of 19.50 percent *minus* six-month LIBOR. Notice that the inverse FRN differs from a traditional FRN in that coupon payments actually decrease as interest rates rise. Also, assume that the current quoted bid-ask spread for a five-year swap agreement involving six-month LIBOR is 70 and 76 basis points, respectively, and the on-the-run five-year Treasury yield is 9.50 percent.

If the firm issues the inverse FRN, it would then want to transform the coupon obligations into synthetic fixed-rate funding by means of a plain vanilla swap. Perhaps the best way to determine the requisite type of swap is to think of the inverse FRN as a

EXHIBIT 9
Arbitrage Application



Effective Annual Financing Cost
(on a 365-Day Bond Basis)

Inverse FRN coupon payment: (19.50% − LIBOR)(365/360)	= 19.77% − (LIBOR)(365/360)
Swap Transaction: (A) Fixed-rate receipt	= 10.20%
(B) Floating-rate payment	= (LIBOR)(365/360)
Net cost of funds (19.77% − 10.20%)	= 9.57%

combination of (*a*) two fixed-rate notes *issued* at 9.75 percent for a total of 19.50 percent and (*b*) one traditional FRN with a coupon of LIBOR held as an *asset*. Thus, to remove the presence of the floating rate altogether, the firm must pay LIBOR and receive the fixed rate in a supplementary swap agreement.[6] Exhibit 9 illustrates this process using the bid side of the fixed swap rate specified above and shows that the net cost of funds is 9.57 percent, assuming semiannually compounded rates, and that the swap fixed rate is on a 365-day basis, while LIBOR and the inverse FRN are on 360-day bases.

Exhibit 9 also indicates that the effective fixed-rate funding cost created by combining the inverse FRN with the receive-fixed plain vanilla swap is 19 basis points lower than the direct fixed rate issue (i.e., 9.76 percent − 9.57 percent). Thus, there is an apparent arbitrage opportunity due to a relative mispricing between the capital and swap markets. However, two important considerations mitigate against this conclusion. First, as mentioned in the previous examples and elaborated in the following section, any time a swap agreement is involved, there will be additional credit risk that must be considered. The synthetic fixed-rate financing scheme forces Company Z to accept the possibility that the swap intermediary will default. Of course, there is no such default exposure (from Company Z's viewpoint) with the direct issue of a fixed-rate Eurobond.

Second, and perhaps more important in this case, the effective yield of 9.57 percent does not represent a funding cost that is fixed with certainty. Since the coupon rate on the inverse FRN can never fall below zero, this instrument contains an implicit cap at LIBOR equal to 19.50 percent. Thus, Company Z is left unprotected against any movement in LIBOR above 19.50 percent since it would have to continue to make the higher net settlement payments on the swap without any offsetting reduction in its funding cost. To fully protect itself, Company Z would have to purchase an interest rate option (specifically, an interest rate *cap*) which would compensate it when LIBOR exceeds 19.50 percent.[7] Naturally, the cost of this option would have to be factored into the analysis of the synthetic strategy before a precise comparison can be made with the 9.76 percent yield available on the fixed-rate Eurobond. In any event, this additional expense makes the presence of any significant funding arbitrage more remote.

Table 19: Amounts outstanding of over-the-counter (OTC) derivatives

By risk category and instrument

In billions of US dollars

Risk Category / Instrument	Notional amounts outstanding					Gross market values				
	Jun 2012	Dec 2012	Jun 2013	Dec 2013	Jun 2014	Jun 2012	Dec 2012	Jun 2013	Dec 2013	Jun 2014
Total contracts	**641,309**	**635,685**	**696,408**	**710,633**	**691,492**	25,5[illegible]	**24,953**	**20,245**	**18,825**	**17,423**
Foreign exchange contracts	**66,672**	**67,358**	**73,121**	**70,553**	**74,782**	**2,249**	**2,313**	**2,427**	**2,284**	**1,722**
Forwards and forex swaps	31,395	31,718	34,421	33,218	35,190	773	806	957	824	571
Currency swaps	24,156	25,420	24,654	25,448	26,141	1,190	1,259	1,131	1,186	939
Options	11,122	10,220	14,046	11,886	13,451	286	249	339	273	213
Interest rate contracts	**496,215**	**492,605**	**564,673**	**584,799**	**563,290**	**19,216**	**19,038**	**15,238**	**14,200**	**13,461**
Forward rate agreements	65,181	71,960	86,892	78,810	92,575	52	48	168	108	126
Interest rate swaps	380,720	372,293	428,385	456,725	421,273	17,317	17,285	13,745	12,919	12,042
Options	50,314	48,351	49,396	49,264	49,442	1,848	1,706	1,325	1,174	1,292
Equity-linked contracts	**6,313**	**6,251**	**6,821**	**6,560**	**6,941**	**639**	**600**	**692**	**700**	**666**
Forwards and swaps	1,880	2,045	2,321	2,277	2,433	147	157	206	202	191
Options	4,434	4,207	4,501	4,284	4,508	492	443	486	498	475
Commodity contracts	**2,993**	**2,587**	**2,458**	**2,204**	**2,206**	**379**	**347**	**384**	**264**	**269**
Gold	523	486	461	341	319	51	42	80	47	32
Other commodities	2,471	2,101	1,997	1,863	1,887	328	304	304	217	237
Forwards and swaps	1,659	1,363	1,327	1,260	1,283					
Options	812	739	670	603	604					
Credit default swaps	**26,930**	**25,068**	**24,349**	**21,020**	**19,462**	**1,187**	**848**	**725**	**653**	**635**
Single-name instruments	15,566	14,309	13,135	11,324	10,845	715	527	430	369	368
Multi-name instruments	11,364	10,760	11,214	9,696	8,617	472	321	295	284	266
of which index products	9,723	9,656	10,163	8,746	7,939					
Unallocated	**42,185**	**41,815**	**24,986**	**25,496**	**24,810**	**1,849**	**1,808**	**779**	**724**	**670**
Memorandum Item:										
Gross Credit Exposure						3,692	3,612	3,784	3,033	2,842

Table 20A: Amounts outstanding of OTC foreign exchange derivatives

By instrument and counterparty

In billions of US dollars

Instrument / counterparty	Notional amounts outstanding					Gross market values				
	Jun 2012	Dec 2012	Jun 2013	Dec 2013	Jun 2014	Jun 2012	Dec 2012	Jun 2013	Dec 2013	Jun 2014
Total contracts	**66,672**	**67,358**	**73,121**	**70,553**	**74,782**	**2,249**	**2,313**	**2,427**	**2,284**	**1,722**
Reporting dealers	29,484	28,834	30,690	31,206	31,971	881	946	992	1,011	709
Other financial institutions	27,538	28,831	31,757	30,552	33,700	885	911	999	887	693
Non-financial customers	9,651	9,693	10,674	8,794	9,111	483	456	437	386	321
Outright forwards and foreign exchange swaps	**31,395**	**31,718**	**34,421**	**33,218**	**35,190**	**773**	**806**	**957**	**824**	**571**
Reporting dealers	11,576	11,083	11,846	11,647	11,931	282	295	360	325	209
Other financial institutions	14,023	14,860	16,441	16,506	18,245	337	351	421	359	263
Non-financial customers	5,796	5,775	6,134	5,066	5,014	153	160	175	140	99
Currency swaps	**24,156**	**25,420**	**24,654**	**25,448**	**26,141**	**1,190**	**1,259**	**1,131**	**1,186**	**939**
Reporting dealers	12,698	12,895	12,443	13,720	13,889	463	529	464	543	394
Other financial institutions	9,086	9,809	9,681	9,025	9,463	472	488	462	432	352
Non-financial customers	2,372	2,716	2,530	2,703	2,789	255	241	205	211	193
Options	**11,122**	**10,220**	**14,046**	**11,886**	**13,451**	**286**	**249**	**339**	**273**	**213**
Reporting dealers	5,211	4,856	6,401	5,840	6,151	135	123	167	143	106
Other financial institutions	4,429	4,162	5,635	5,022	5,992	76	71	116	96	77
Non-financial customers	1,482	1,203	2,010	1,025	1,308	75	55	56	35	29

Table 21A: Amounts outstanding of OTC single-currency interest rate derivatives

By instrument and counterparty

In billion of US dollars

Instrument / counterparty	Notional amounts outstanding					Gross market values				
	Jun 2012	Dec 2012	Jun 2013	Dec 2013	Jun 2014	Jun 2012	Dec 2012	Jun 2013	Dec 2013	Jun 2014
Total contracts	**496,215**	**492,605**	**564,673**	**584,799**	**563,290**	**19,216**	**19,038**	**15,238**	**14,200**	**13,461**
Reporting dealers	139,146	116,887	104,112	95,762	84,520	6,568	6,024	4,484	3,741	3,719
Other financial institutions	318,693	341,187	425,499	471,870	463,021	11,586	11,875	9,896	9,673	8,871
Non-financial customers	38,376	34,531	35,062	17,168	15,749	1,062	1,140	858	786	871
Forward rate agreements	**65,181**	**71,960**	**86,892**	**78,810**	**92,575**	**52**	**48**	**168**	**108**	**126**
Reporting dealers	23,113	11,505	7,681	5,577	4,872	21	14	32	30	24
Other financial institutions	41,030	57,998	75,334	72,213	87,065	27	32	117	68	92
Non-financial customers	1,038	2,458	3,877	1,019	638	3	2	18	9	10
Swaps	**380,720**	**372,293**	**428,385**	**456,725**	**421,273**	**17,317**	**17,285**	**13,745**	**12,919**	**12,042**
Reporting dealers	84,669	75,459	67,019	59,948	51,096	5,269	4,836	3,564	2,905	2,809
Other financial institutions	261,365	267,209	332,701	382,870	357,370	11,072	11,380	9,397	9,288	8,423
Non-financial customers	34,685	29,625	28,665	13,907	12,807	976	1,069	785	726	811
Options	**50,314**	**48,351**	**49,396**	**49,264**	**49,442**	**1,848**	**1,706**	**1,325**	**1,174**	**1,292**
Reporting dealers	31,364	29,923	29,413	30,236	28,552	1,278	1,174	888	806	886
Other financial institutions	16,298	15,981	17,464	16,786	18,586	487	463	382	317	356
Non-financial customers	2,653	2,448	2,520	2,242	2,303	83	69	55	50	50

Table 21B: Amounts outstanding of OTC single-currency interest rate derivatives

By currency

In billions of US dollars

Currency	Notional amounts outstanding					Gross market values				
	Jun 2012	Dec 2012	Jun 2013	Dec 2013	Jun 2014	Jun 2012	Dec 2012	Jun 2013	Dec 2013	Jun 2014
All currencies	**496,215**	**492,605**	**564,673**	**584,799**	**563,290**	**19,216**	**19,038**	**15,238**	**14,200**	**13,461**
Canadian dollar	7,380	7,507	9,342	10,385	10,471	186	166	146	139	126
Euro	180,469	189,702	229,989	241,668	221,855	8,038	9,263	7,407	6,989	7,362
Japanese yen	60,092	54,816	55,092	52,551	51,706	1,055	911	715	696	759
Pound sterling	39,916	42,256	46,346	52,626	60,823	1,462	1,616	1,104	1,294	1,079
Swedish krona	7,150	6,454	6,221	6,662	6,229	103	120	76	81	114
Swiss franc	5,494	5,357	5,583	5,750	5,343	161	149	113	121	113
US dollar	164,095	148,768	169,196	173,382	160,805	7,386	5,937	4,736	4,314	3,246
Other	31,618	37,745	42,904	41,777	46,059	825	876	941	566	661

23

Why Use Swaps?

- Leverage.
- Financing at Libor.
- Off-balance-sheet financing.
- Pure contractual; not securities.
- Easy to short by paying fixed.
- Very liquid, low transaction costs.
- Counterparty credit risk mitigated by marking to market and collateralization.

Forward Swaps

- We have been studying swaps that start immediately.
- Swaps can also start on a forward basis.
- If fixed let is worth par at time t, then the fixed leg is worth $D(t)$ at time zero.
- Solve for the forward swap rate from

$$D(t) = \frac{c}{2} \sum_{i=1}^{2N} D(t + i/2) + D(t + N)$$

$$FSR = \left[\frac{D(t) - D(t+N)}{\sum_{i=1}^{2N} D(t + i/2)} \right] \times 2$$

Fixed Income Arbitrage

- Table 1.
- How variable is the Libor - repo spreads?
- LTCM Case.
- Duarte, Longstaff, and Yu Case.

Table 1

Cash Flows from the Swap-Spread Arbitrage Strategy. This table illustrates the cash flows generated by a stylized form of the swap-spread arbitrage strategy. *CMS* and *CMT* denote the fixed swap and Treasury coupon rates. L_t denotes the Libor rate determined at time t, and r_t denotes the repo rate determined at time t. By market convention, floating rates paid at time t are set or determined at time $t-1$. *SS* denotes the swap spread and equals $CMS - CMT$. The term S_t denotes the difference $L_t - r_t$. For expositional simplicity, this table assumes that floating and fixed cash flows are paid each period.

	Cash Flow at Time						
Strategy	0	1	2	3	4	...	T
Receive Fixed Swap Rate	0	CMS	CMS	CMS	CMS	...	CMS
Pay Libor Rate on Swap	0	$-L_0$	$-L_1$	$-L_2$	$-L_3$	...	$-L_{T-1}$
Short Bond, Pay Coupons	+100	$-CMT$	$-CMT$	$-CMT$	$-CMT$	...	$-CMT - 100$
Receive Repo Rate on Margin Acct.	−100	r_0	r_1	r_2	r_3	...	$r_{T-1} + 100$
Total	0	$SS - S_0$	$SS - S_1$	$SS - S_2$	$SS - S_3$	...	$SS - S_{T-1}$

3

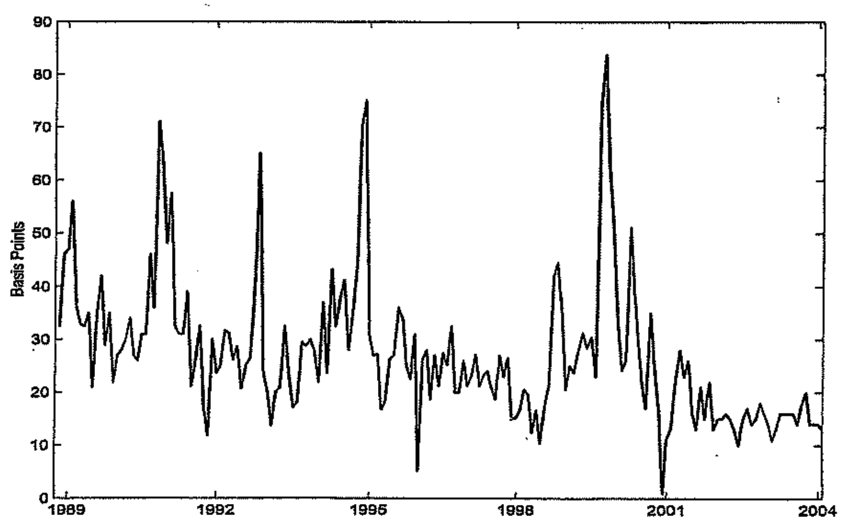


Figure 1. The Libor-Repo Spread. This graph plots the spread between the three-month Libor and general collateral repo rates.

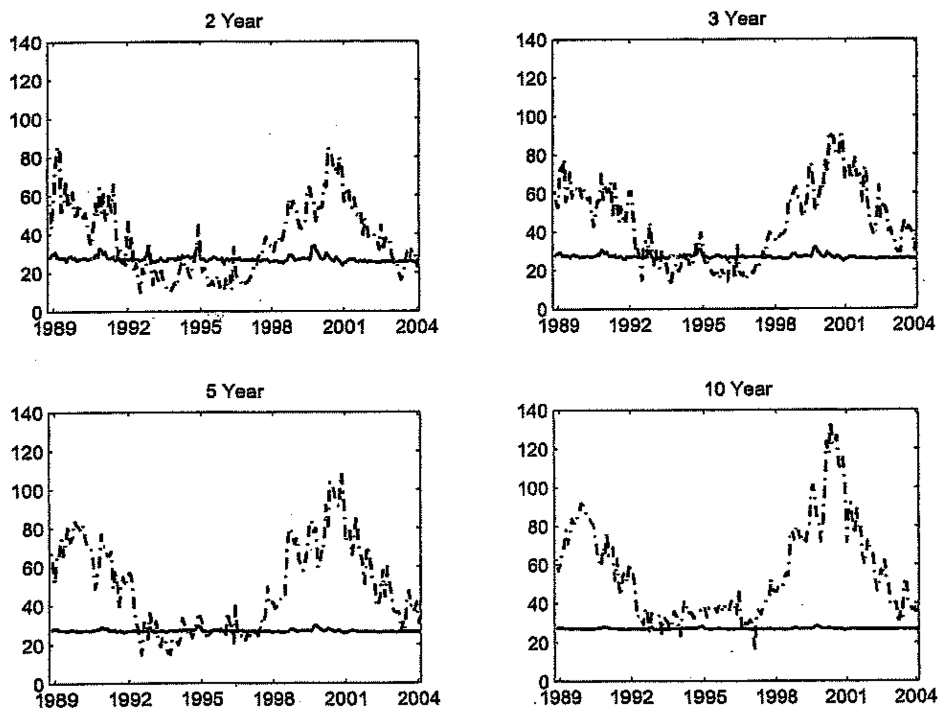
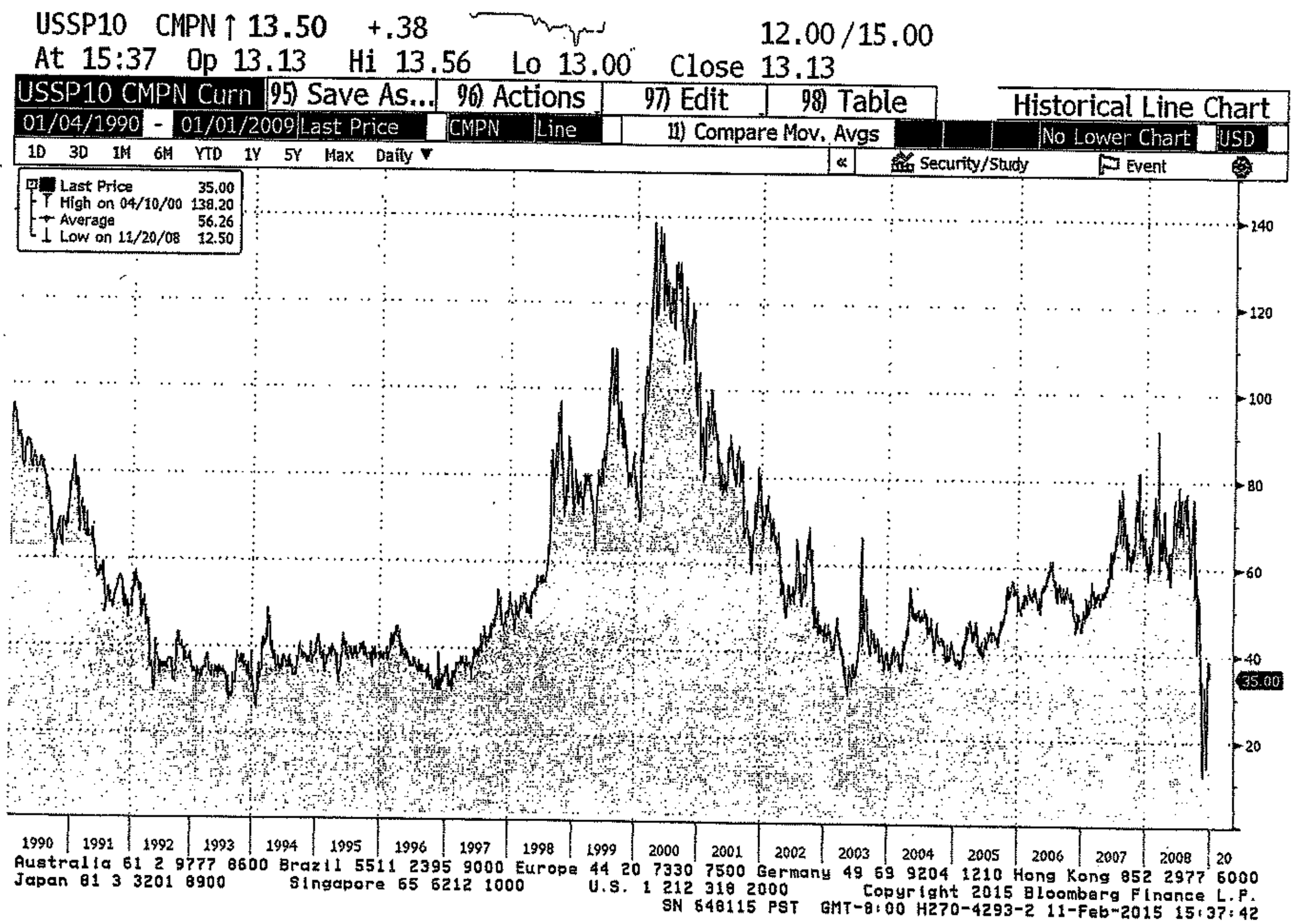


Figure 2. Swap Spreads and Expected Average Libor-Repo Spreads. These graphs plot the expected average value of the Libor-repo spread (solid line) and the corresponding swap spread (dashed-dotted line) for the indicated horizons. All spreads are in basis points.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12.00/15.00
At 15:37 Op 13.13 Hi 13.56 Lo 13.00 Close 13.13
USSP10 CMPN Curn
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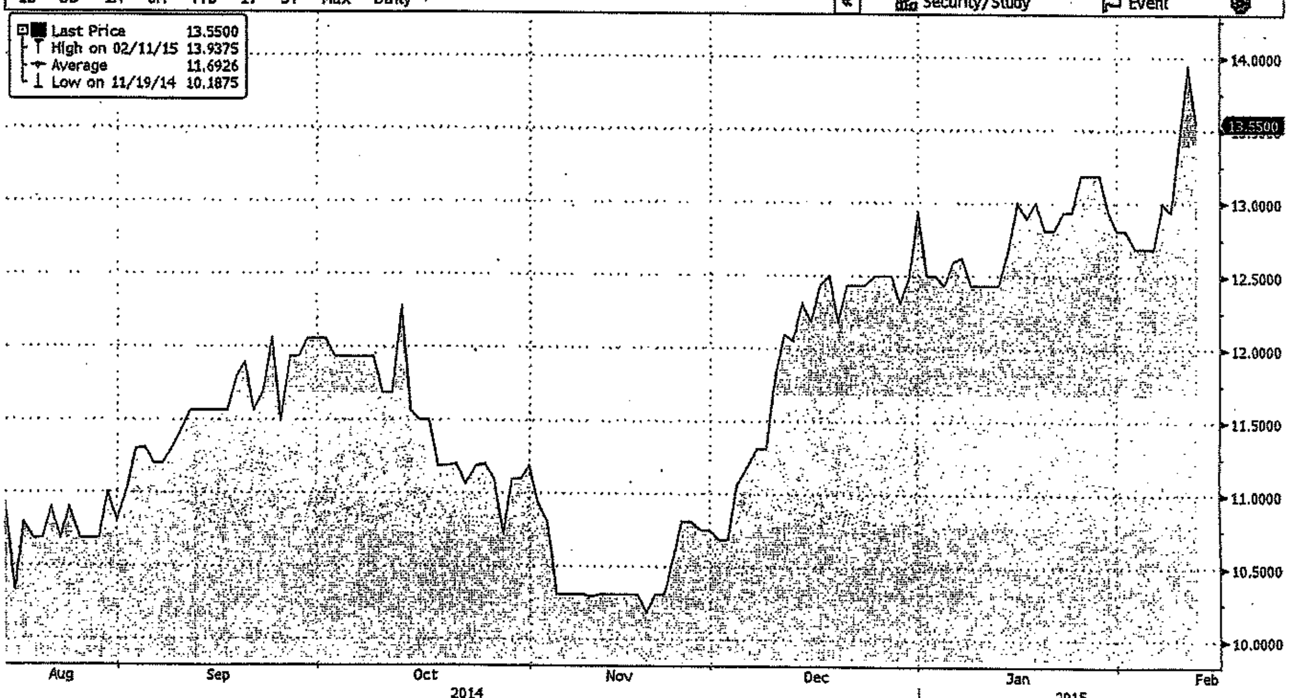
96) Actions
97) Edit
98) Table
Historical Line Chart
01/04/1990 - 01/01/2009 Last Price
CMPN
Line
11) Compare Mov. Avgs
No Lower Chart
USD
1D 3D 1M 6M YTD 1Y 5Y Max Daily ▼
Security/Study
Event
Last Price 35.00
High on 04/10/00 138.20
Average 56.26
Low on 11/20/08 12.50
140
120
100
80
60
40
35.00
20
1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 20
Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2015 Bloomberg Finance L.P.
SN 648115 PST GMT-8:00 H270-4293-2 11-Feb-2015 15:37:42

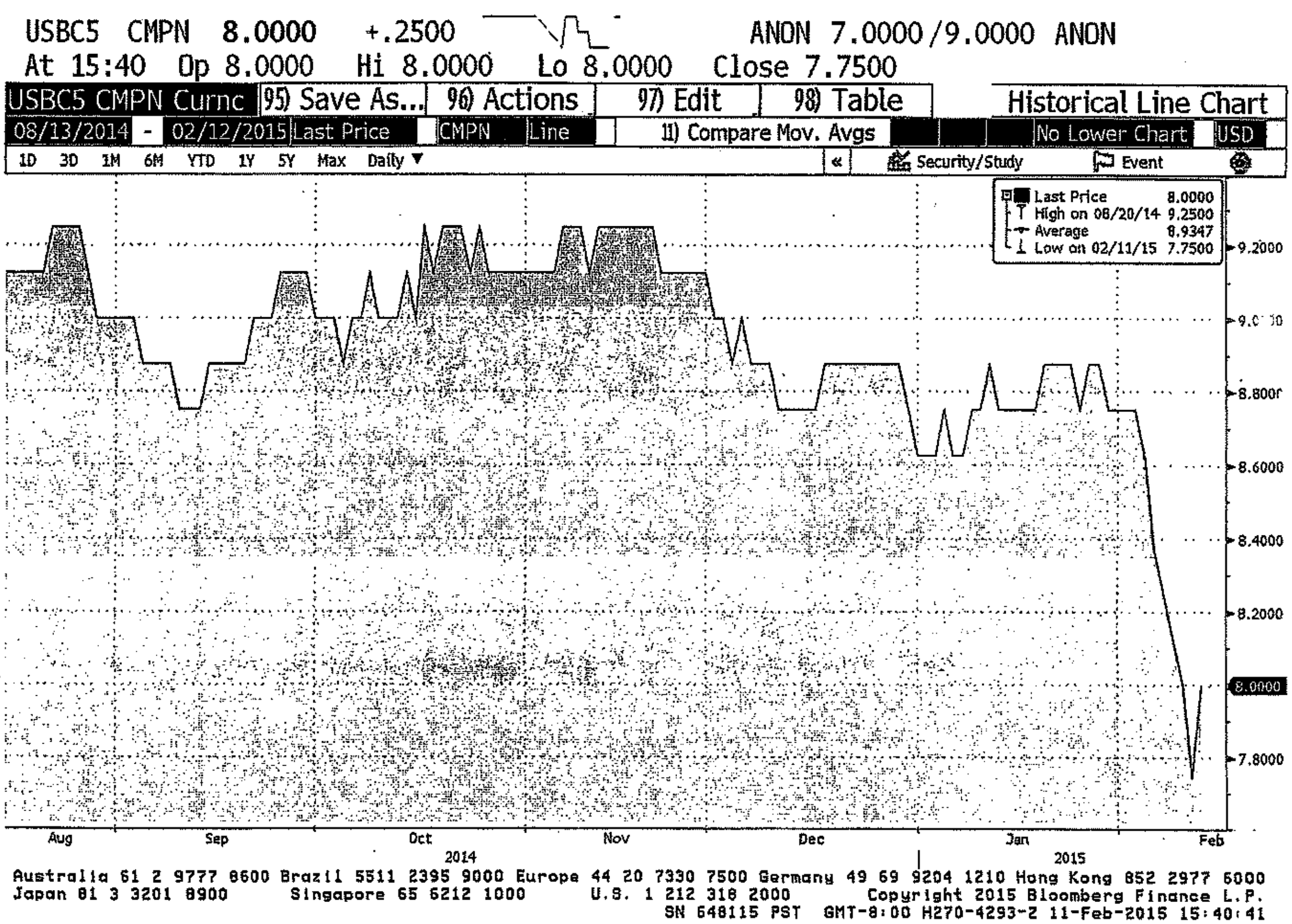
USBA5 CMPN 13.5500 -.3875 ANON 13.0000/14.1000 ANON
At 15:34 Op 14.0000 Hi 14.2500 Lo 13.5500 Close 13.9375

USBA5 CMPN Curnc | 95) Save As... | 96) Actions | 97) Edit | 98) Table | Historical Line Chart

08/13/2014 - 02/12/2015 Last Price | CMPN | Line | 11) Compare Mov. Avgs | No Lower Chart | USD

1D 3D 1M 6M YTD 1Y 5Y Max Daily ▼ « Security/Study Event

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At 15:40 Op 8.0000 Hi 8.0000 Lo 8.0000 Close 7.7500
USBC5 CMPN Curnc
95) Save As...
96) Actions
97) Edit
98) Table
Historical Line Chart
08/13/2014 - 02/12/2015 Last Price
CMPN
Line
11) Compare Mov. Avgs
No Lower Chart
USD
1D 3D 1M 6M YTD 1Y 5Y Max Daily
Security/Study
Event
Last Price 8.0000
High on 08/20/14 9.2500
Average 8.9347
Low on 02/11/15 7.7500
9.2000
8.8000
8.6000
8.4000
8.2000
8.0000
7.8000
Aug
Sep
Oct
Nov
Dec
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Feb
2014
2015
Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
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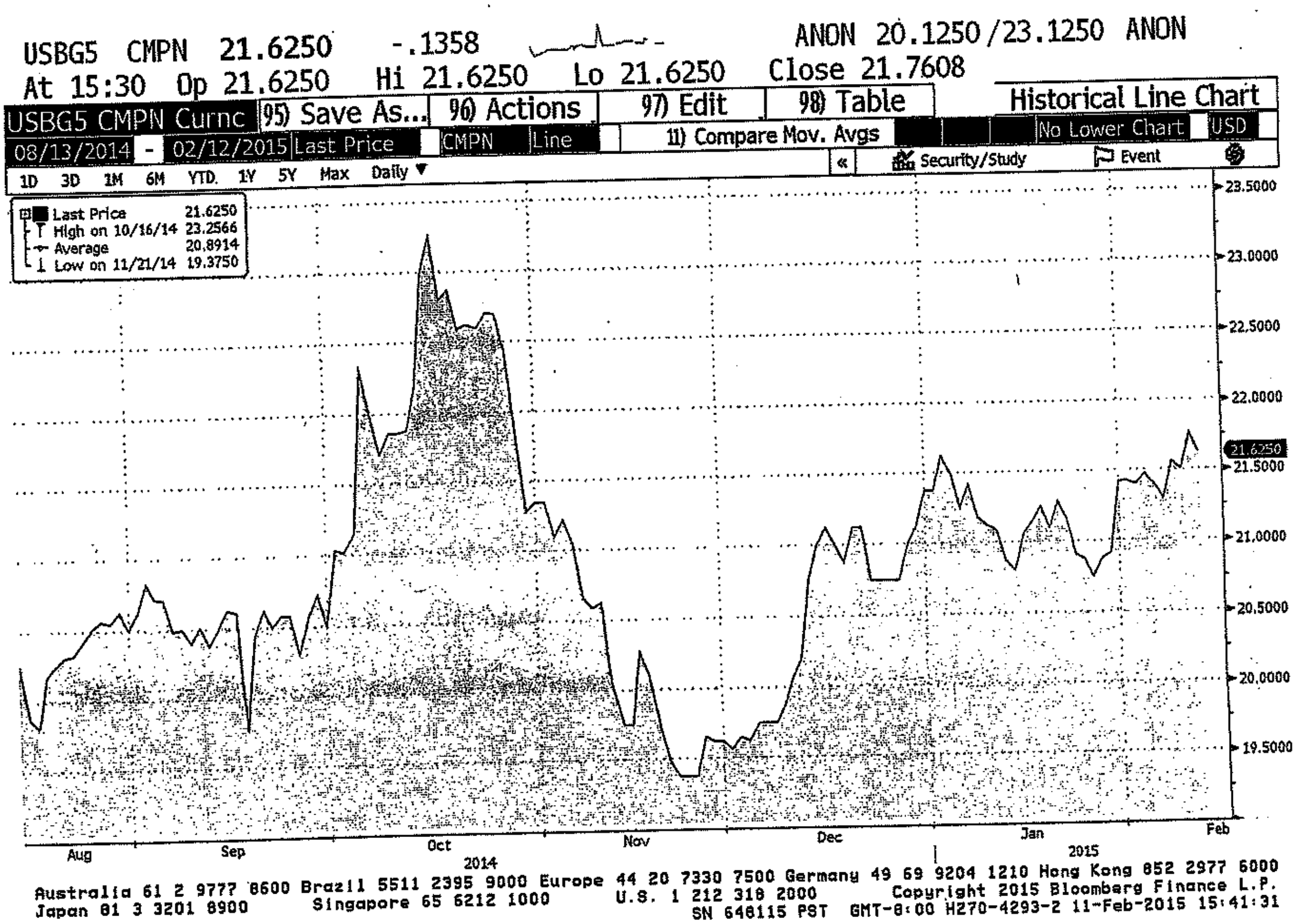

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ANON 20.1250/23.1250 ANON
At 15:30 Op 21.6250 Hi 21.6250 Lo 21.6250 Close 21.7608
Historical Line Chart
USBG5 CMPN Curncy
95) Save As...
96) Actions
97) Edit
98) Table
08/13/2014 - 02/12/2015 Last Price
CMPN
Line
11) Compare Mov. Avgs
No Lower Chart
USD
Security/Study
Event
1D 3D 1M 6M YTD 1Y 5Y Max Daily
Last Price 21.6250
High on 10/16/14 23.2566
Average 20.8914
Low on 11/21/14 19.3750
23.5000
23.0000
22.5000
22.0000
21.6250
21.5000
21.0000
20.5000
20.0000
19.5000
Aug
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Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
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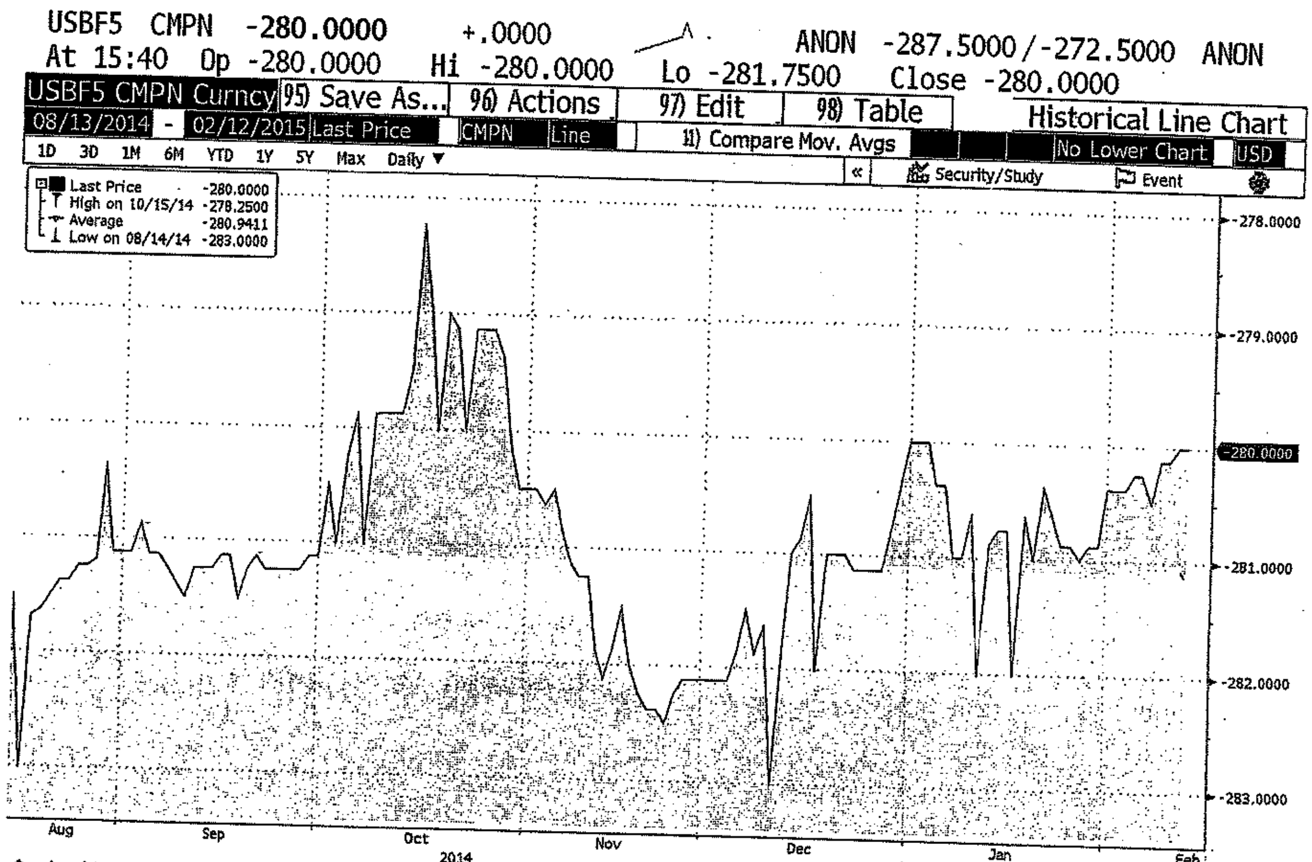
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At 15:40 Op -280.0000 Hi -280.0000 Lo -281.7500 Close -280.0000
USBF5 CMPN Curncy
95) Save As...
96) Actions
97) Edit
98) Table
Historical Line Chart
08/13/2014 - 02/12/2015 Last Price
CMPN
Line
11) Compare Mov. Avgs
No Lower Chart
USD
1D 3D 1M 6M YTD 1Y 5Y Max Daily
Security/Study
Event
Last Price -280.0000
High on 10/15/14 -278.2500
Average -280.9411
Low on 08/14/14 -283.0000
-278.0000
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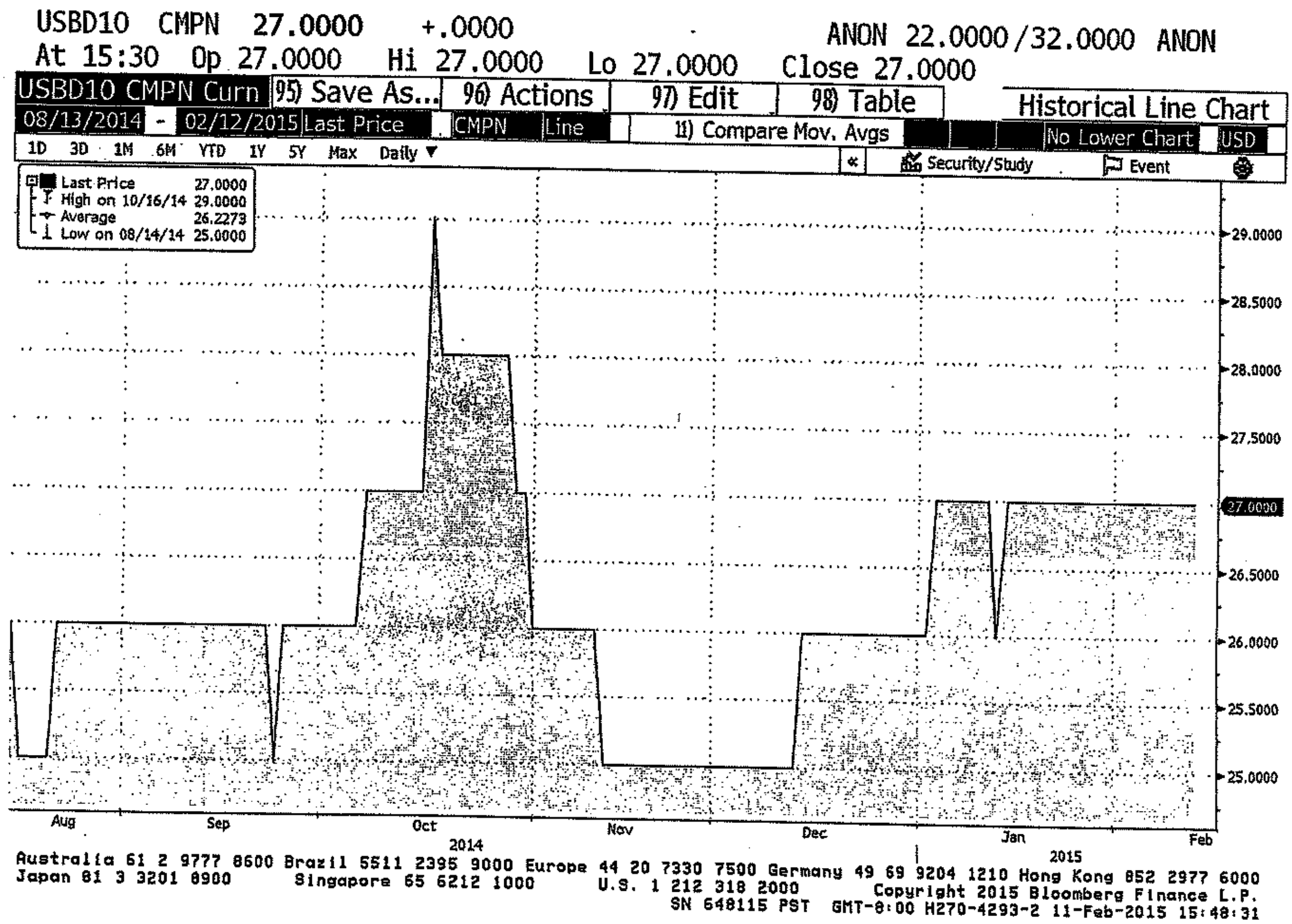
USBD10 CMPN 27.0000 +.0000
ANON 22.0000/32.0000 ANON
At 15:30 Op 27.0000 Hi 27.0000 Lo 27.0000 Close 27.0000
USBD10 CMPN Curn
95) Save As...
96) Actions
97) Edit
98) Table
Historical Line Chart
08/13/2014 - 02/12/2015 Last Price
CMPN
Line
11) Compare Mov. Avgs
No Lower Chart
USD
1D 3D 1M 6M YTD 1Y 5Y Max Daily
Security/Study
Event
Last Price 27.0000
High on 10/16/14 29.0000
Average 26.2273
Low on 08/14/14 25.0000
29.0000
28.5000
28.0000
27.5000
27.0000
26.5000
26.0000
25.5000
25.0000
Aug
Sep
Oct
2014
Nov
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Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2015 Bloomberg Finance L.P.
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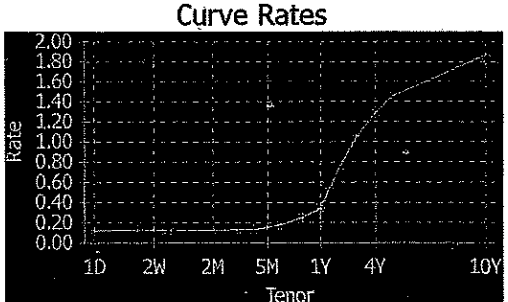
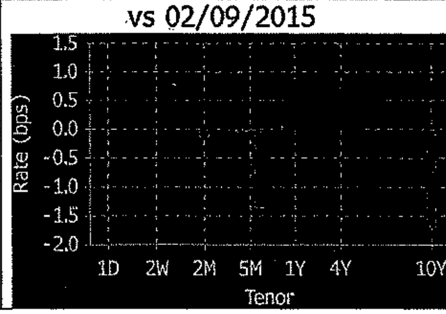
YCSW0042 Index DES

USD OIS Curve | Description





More...
1) Chart
2) Members
3) History
4) Related Curves
5) Spreads
66) Send As Link

Curve Properties

Entity	United States	Type	IRS
Currency	USD	Subtype	Par (Coupon)
Curve ID	S42 Corp	Calc Convention	Act/360
Bloomberg ID	YCSW0042 Index	Pricing Frequency	Intraday
BBGID	BBG006Z3S787	Pricing Source	Contributed/PCS
Rulemaker	Bloomberg	Member Update Freq	Bloomberg

Curve Details

This curve represents USD overnight index swaps. Payments are based on a fixed-rate versus a floating-rate overnight index with the fixed-rate portion on an annual, Actual/360 day-count basis and the floating-rate on an annual, Actual/360 day-count basis from the FED Funds Effective Rate {FEDL01 Index <GO>}. Pricing is a best bid/ask composite from latest quotes and the sources include both banks and brokers.

USD SWAP OIS:

An Overnight Indexed Swap (OIS) is a fixed/floating interest rate swap with the floating leg computed using a published overnight rate index, in the case of USD, the Fed Funds Effective Rate. Two parties agree to exchange at maturity the difference between interest acrrued at the fixed rate and interest acrrued at the compounded floating rate on the agreed notional amount of the swap. Net payment is made two business days after maturity.
The basis convention is Annual ACT/360.

1
200<Go> to view in Launchpad N1N121 Corp **TTSA**
13:49 **TULLETT & TOKYO** PAGE 1 / 2

USD BASIS SWAPS

	Maturity	Pay	Receive	Time
	1 Month vs 3 Month LIBOR			
1)	1 Year	-0.6250	3.3750	13:49
2)	2 Year	-1.0000	3.0000	6:01
3)	3 Year	-1.1250	2.8750	6:01
4)	4 Year	-1.1250	2.8750	6:01
5)	5 Year	-1.2500	2.7500	6:01
6)	7 Year	-1.2500	2.7500	9:18
7)	10 Year	-1.3750	2.6250	9:18
	1 Month vs 6 Month LIBOR			
8)	1 Year		4.0000	13:49
9)	2 Year	-1.3750	2.6250	6:01
10)	3 Year	-0.8750	3.1250	6:01
11)	4 Year	-1.0000	3.0000	6:01
12)	5 Year	-1.0000	3.0000	9:18
13)	7 Year	-1.1250	2.8750	9:18
14)	10 Year	-1.2500	2.7500	9:18

PAGE FWD FOR: T-BILLS vs 3M LIBOR
PRIME RATE vs 3M LIBOR
FED FUNDS vs 3M LIBOR

USD BASIS SWAPS

	Maturity	Pay	Receive	Time
	3 Month vs 6 Month LIBOR			
15)	1 Year	-1.5000	2.5000	13:49
16)	2 Year	-1.6250	2.3750	6:01
17)	3 Year	-1.7500	2.2500	6:01
18)	4 Year	-1.7500	2.2500	6:01
19)	5 Year	-1.7500	2.2500	9:18
20)	7 Year	-1.7500	2.2500	9:18
21)	10 Year	-1.7500	2.2500	9:18
	Commercial Paper vs. 3 Month LIBOR			
22)	1 Year	6.0000	10.0000	6:03
23)	2 Year	7.0000	11.0000	6:03
24)	3 Year	8.0000	12.0000	6:03
25)	4 Year	8.0000	12.0000	6:03
26)	5 Year	9.0000	13.0000	6:03
27)	7 Year	9.5000	13.5000	6:03
28)	10 Year	10.0000	14.0000	6:03

Page
200<Go> to view in Launchpad
N121 Corp **TTSA**

13:49 **TULLETT & TOKYO** PAGE 2 / 2

USD BASIS SWAPS

	Maturity	Pay	Receive	Time
	Treasury Bills vs 3 Month LIBOR			
1)	1 Year	9.0000	19.0000	6:03
2)	2 Year	15.0000	25.0000	6:03
3)	3 Year	20.0000	30.0000	6:03
4)	4 Year	25.0000	35.0000	6:03
5)	5 Year	30.0000	40.0000	6:03
6)	7 Year	31.0000	41.0000	6:03
7)	10 Year	33.0000	43.0000	6:03
	Prime Rate vs 3 Month LIBOR			
8)	1 Year	-288.5000	-282.5000	6:03
9)	2 Year	-287.5000	-281.5000	6:03
10)	3 Year	-287.0000	-281.0000	6:03
11)	4 Year	-286.0000	-280.0000	6:03
12)	5 Year	-286.0000	-280.0000	6:03
13)	7 Year	-284.5000	-278.5000	6:03
14)	10 Year	-284.0000	-278.0000	6:03

USD BASIS SWAPS

	Maturity	Pay	Receive	Time
	Fed Funds vs 3 Month LIBOR			
15)	1 Year	11.5000	15.5000	6:03
16)	2 Year	12.0000	16.0000	6:03
17)	3 Year	12.2500	16.2500	6:03
18)	4 Year	13.0000	17.0000	6:03
19)	5 Year	13.5000	17.5000	6:03
20)	7 Year	14.0000	18.0000	6:03
21)	10 Year	14.7500	18.7500	6:03

Risk and Return in Fixed-Income Arbitrage: Nickels in Front of a Steamroller?

Jefferson Duarte
University of Washington

Francis A. Longstaff
UCLA Anderson School and the NBER

Fan Yu
UC Irvine

We conduct an analysis of the risk and return characteristics of a number of widely used fixed-income arbitrage strategies. We find that the strategies requiring more "intellectual capital" to implement tend to produce significant alphas after controlling for bond and equity market risk factors. These positive alphas remain significant even after taking into account typical hedge fund fees. In contrast with other hedge fund strategies, many of the fixed-income arbitrage strategies produce positively skewed returns. These results suggest that there may be more economic substance to fixed-income arbitrage than simply "picking up nickels in front of a steamroller."

During the hedge fund crisis of 1998, market participants were given a revealing glimpse into the proprietary trading strategies used by a number of large hedge funds such as Long-Term Capital Management (LTCM). Among these strategies, few were as widely used—or as painful—as fixed-income arbitrage. Virtually every major investment banking firm on Wall Street reported losses directly related to their positions in fixed-income arbitrage during the crisis. Despite these losses, however, fixed-income arbitrage has since become one of the most popular and rapidly growing sectors within the hedge fund industry. For example, the Tremont/TASS (2005) Asset Flows Report indicates that total assets devoted to fixed-income arbitrage grew by more than $9.0 billion during 2005 and that the total

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We are grateful for valuable comments and assistance from Vineer Bhansali, Charles Cao, Darrell Duffie, Robert Jarrow, Philippe Jorion, Katie Kong, Haitao Li, Ravit Mandell, Bruno Miranda, Yoshihiro Mikami, Soetojo Tanudjaja, Abraham Thomas, Stuart Turnbull, Rossen Valkanov, Ryoichi Yamabe, Toshi Yotsuzuka, Eric Zivot, Gary Zhu, and seminar participants at the Bank of Canada, Case Western Reserve University, the Chicago Quantitative Alliance, the University of Kansas, the London School of Business, Nomura Securities, Simplex Asset Management, Pennsylvania State University, PIMCO, and the Workshop on Capital Structure Arbitrage at the University of Evry. We are particularly grateful for the comments and suggestions of Jun Liu, the Editor Yacine Aït-Sahalia, and of an anonymous referee. All errors are our responsibility. Address correspondence to: Francis Longstaff, Anderson School at UCLA, 110 Westwood Plaza, Los Angeles, CA 90095-1481, or e-mail: francis.longstaff@anderson.ucla.edu.

doi:10.1093/rfs/hhl026 Advance Access publication July 6, 2006

40

amount of hedge fund capital devoted to fixed income arbitrage at the end of 2005 is in excess of $56.6 billion.[1]

This mixed history raises a number of important issues about the fundamental nature of fixed-income arbitrage. Is fixed-income arbitrage truly arbitrage? Or is it merely a strategy that earns small positive returns most of the time, but occasionally experiences dramatic losses (a strategy often described as "picking up nickels in front of a steamroller")? Were the large fixed-income arbitrage losses during the hedge fund crisis simply due to excessive leverage, or were there deeper reasons arising from the inherent nature of these strategies? To address these issues, this article conducts an extensive analysis of the risk and return characteristics of fixed-income arbitrage.

Fixed-income arbitrage is actually a broad set of market-neutral investment strategies intended to exploit valuation differences between various fixed-income securities or contracts. In this analysis, we focus on five of the most widely used fixed-income arbitrage strategies in the market:

- Swap spread (SS) arbitrage.
- Yield curve (YC) arbitrage.
- Mortgage (MA) arbitrage.
- Volatility (VA) arbitrage.
- Capital structure (CS) arbitrage.

As in Mitchell and Pulvino (2001), our approach consists of following specific trading strategies through time and studying the properties of return indexes generated by these strategies. There are several important advantages to this approach. First, it allows us to incorporate transaction costs and hold fixed the effects of leverage in the analysis. Second, it allows us to study returns over a much longer horizon than would be possible using the limited amount of hedge fund return data available. Finally, this approach allows us to avoid potentially serious backfill and survivorship biases in reported hedge fund return indexes.

With these return indexes, we can directly explore the risk and return characteristics of the individual fixed-income arbitrage strategies. To hold fixed the effects of leverage on the analysis, we adjust the amount of initial capital so that the annualized volatility of each strategy's returns is 10%. We find that all five of the strategies generate positive excess returns on average. Surprisingly, most of the arbitrage strategies result in excess returns that are positively skewed. Thus, even though these strategies produce large negative returns from time to time, the strategies tend to generate even larger offsetting positive returns.

[1] The total amount of capital devoted to fixed-income arbitrage is likely much larger as the Tremont/TASS (2005) report covers less than 50% of the total estimated amount of capital managed by hedge funds. Also, many Wall Street firms directly engage in proprietary fixed-income arbitrage trading.

41

We study the extent to which these positive excess returns represent compensation for bearing market risk. After risk adjusting for both equity and bond market factors, we find that the SS and VA arbitrage strategies produce insignificant alphas. In contrast, the YC, MA, and CS arbitrage strategies generally result in significant positive alphas. Interestingly, the latter strategies are the ones that require the most "intellectual capital" to implement. Specifically, the strategies that result in significant alphas are those that require relatively complex models to identify arbitrage opportunities and/or hedge out systematic market risks. We find that several of these "market-neutral" arbitrage strategies actually expose the investor to substantial levels of market risk. We repeat the analysis using actual fixed-income arbitrage hedge fund index return data from industry sources and find similar results.

In addition to the transaction costs incurred in executing fixed-income arbitrage strategies, many investors must also pay hedge fund fees. We repeat the analysis assuming that hedge fund fees of 2/20 (a 2% management fee and a 20% slope above a Libor high water mark) are subtracted from the returns. While these fees reduce or eliminate the significance of the alphas of the individual strategies, we find that equally-weighted portfolios of the more "intellectual capital" intensive strategies still have significant alphas on a net-of-fees basis. On the other hand, however, our results indicate that fees in the fixed-income arbitrage hedge fund industry are "large" relative to the alphas that can be generated by these strategies.

Where does this leave us? Is the business of fixed-income arbitrage simply a strategy of "picking up nickels in front of a steamroller," equivalent to writing deep out-of-the-money puts? We find little evidence of this. In contrast, we find that most of the strategies we consider result in excess returns that are positively skewed, even though large negative returns can and do occur. After controlling for leverage, these strategies generate positive excess returns on average. Furthermore, after controlling for both equity and bond market risk factors, transaction costs, and hedge fund fees, the fixed-income arbitrage strategies that require the highest level of "intellectual capital" to implement appear to generate significant positive alphas. The fact that a number of these factors share sensitivity to financial market "event risk" argues that these positive alphas are not merely compensation for bearing the risk of an as-yet-unrealized "peso" event. Thus, the risk and return characteristics of fixed-income arbitrage appear different from those of other strategies such as merger arbitrage [see Mitchell and Pulvino (2001)].

This article contributes to the rapidly growing literature on returns to "arbitrage" strategies. Closest to our article are the important recent studies of equity arbitrage strategies by Mitchell and Pulvino (2001) and Mitchell, Pulvino, and Stafford (2002). Our article, however, focuses exclusively on fixed-income arbitrage. Less related to our work are a

42

number of important recent articles focusing on the actual returns reported by hedge funds. These papers include Fung and Hsieh (1997, 2001, 2002), Ackermann, McEnally, and Ravenscraft (1999), Brown, Goetzmann, and Ibbotson (1999), Brown, Goetzmann, and Park (2000), Dor and Jagannathan (2002), Brown and Goetzmann (2003), Getmansky, Lo, and Makarov (2004), Agarwal and Naik (2004), Malkiel and Saha (2004), and Chan, et al. (2005). Our article differs from these in that the returns we study are attributable to specific strategies with controlled leverage, whereas reported hedge fund returns are generally composites of multiple (and offsetting) strategies with varying degrees of leverage.

The remainder of this article is organized as follows. Sections 1 through 5 describe the respective fixed-income arbitrage strategies and explain how the return indexes are constructed. Section 6 conducts an analysis of the risk and return characteristics of the return indexes along with those for historical fixed-income arbitrage hedge fund returns. Section 7 summarizes the results and makes concluding remarks.

1. Swap Spread Arbitrage

Swap spread arbitrage has traditionally been one of the most popular types of fixed-income arbitrage strategies. The importance of this strategy is evidenced by the fact that swap spread positions represented the single-largest source of losses for LTCM.[2] Furthermore, the hedge fund crisis of 1998 revealed that many other major investors had similar exposure to swap spreads—Salomon Smith Barney, Goldman Sachs, Morgan Stanley, BankAmerica, Barclays, and D.E. Shaw all experienced major losses in swap spread strategies.[3]

The swap spread arbitrage strategy has two legs. First, an arbitrageur enters into a par swap and receives a fixed coupon rate *CMS* and pays the floating Libor rate L_t. Second, the arbitrageur shorts a par Treasury bond with the same maturity as the swap and invests the proceeds in a margin account earning the repo rate. The cash flows from the second leg consist of paying the fixed coupon rate of the Treasury bond *CMT* and receiving the repo rate from the margin account r_t.[4] Combining the cash flows from the two legs shows that the arbitrageur receives a fixed annuity of $SS=CMS-CMT$ and pays the floating spread $S_t=L_t-r_t$. The cash flows from the reverse strategy are just the opposite of these cash flows. There are no initial or terminal principal cash flows in this strategy.

[2] Lowenstein (2000) reports that LTCM lost $1.6 billion in its swap spread positions before its collapse. Also see Perold (1999).

[3] See Siconolfi et al. (1998), Beckett and Pacelle (1998), Dunbar (2000), and Lowenstein (2000).

[4] The terms *CMS* and *CMT* are widely used industry abbreviations for constant maturity swap rate and constant maturity Treasury rate.

45

Swap spread arbitrage is thus a simple bet on whether the fixed annuity of SS received will be larger than the floating spread S_t paid. If SS is larger than the average value of S_t during the life of the strategy, the strategy is profitable (at least in an accounting sense). What makes the strategy attractive to hedge funds is that the floating spread S_t has historically been very stable over time, averaging 26.8 basis points with a standard deviation of only 13.3 basis points during the past 16 years. Thus, the expected *average* value of the floating spread over, say, a five-year horizon may have a standard deviation of only a few basis points (and, in fact, is often viewed as essentially constant by market participants).

Swap spread arbitrage, of course, is not actually an arbitrage in the textbook sense because the arbitrageur is exposed to *indirect* default risk. This is because if the viability of a number of major banks were to become uncertain, market Libor rates would likely increase significantly. For example, the spread between bank CD rates and Treasury bill yields spiked to nearly 500 basis points around the time of the Oil Embargo during 1974. In this situation, a swap spread arbitrageur paying Libor on a swap would suffer large negative cash flows from the strategy as the Libor rate responded to increased default risk in the financial sector. Note that there is no *direct* default risk from banks entering into financial distress as the cash flows on a swap are not direct obligations of the banks quoting Libor rates. Thus, even if these banks default on their debt, the counterparties to a swap continue to exchange fixed for floating cash flows.[5]

In studying the returns from fixed-income arbitrage, we use an extensive data set from the swap and Treasury markets covering the period from November 1988 to December 2004. The swap data consist of month-end observations of the three-month Libor rate and midmarket swap rates for two-, three-, five-, seven-, and 10-year maturity swaps. The Treasury data consist of month-end observations of the constant maturity Treasury (CMT) rates published by the Federal Reserve in the H-15 release for maturities of two, three, five, seven, and 10 years. Finally, we collect data on three-month general collateral repo rates. The data are described in the Appendix. Figure 1 plots the time series of swap spreads against the expected average value of the short term spread over the life of the swap (based on fitting a simple mean reverting Gaussian process to the data).

To construct a return index, we first determine each month whether the current swap spread differs from the current value of the short term spread. If the

[5] In theory, there is the risk of a default by a counterparty. In practice, however, this risk is negligible as swaps are usually fully collateralized under master swap agreements between major institutional investors. Furthermore, the actual default exposure in a swap is far less than for a corporate bond as notional amounts are not exchanged. Following Duffie and Huang (1996), Duffie and Singleton (1997), Minton (1997), He (2000), Grinblatt (2001), Liu, Longstaff, and Mandell (2004), and many others, we abstract from the issue of counterparty credit risk in this analysis.

44

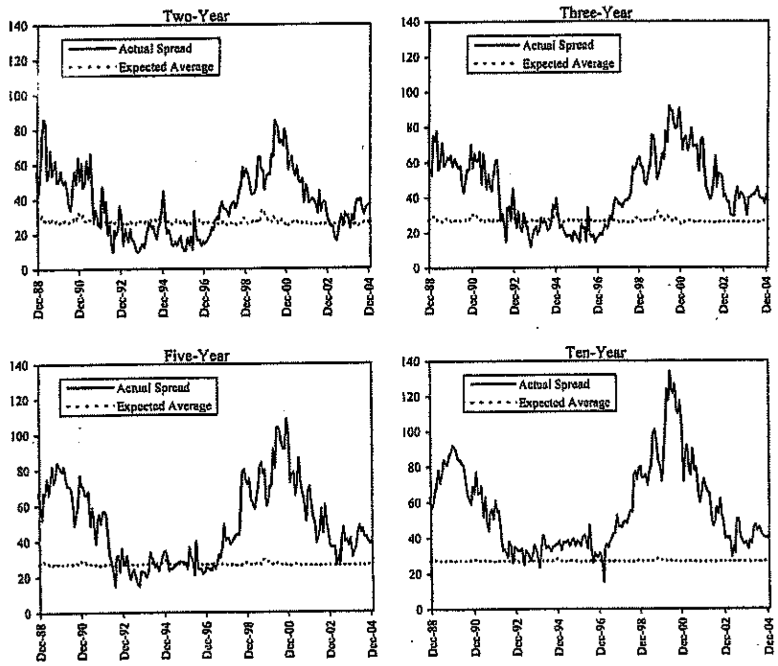

Figure 1
Swap spreads and expected average Libor-Repo spreads
These graphs plot the expected average value of the Libor-repo spread and the corresponding swap spread for the indicated horizons. All spreads are in basis points.

difference exceeds a trigger value of 10 basis points, we implement the trade for a \$100 notional position (receive fixed on a \$100 notional swap, short a \$100 notional Treasury bond, or vice versa if the difference is less than −10 basis points).[6] If the difference does not exceed the trigger, then the strategy invests in cash and earns an excess return of zero. We keep the trade on until it converges (swap spread converges to the short term spread) or until the maturity of the swap and bond. It is useful to think of this trade as a fictional hedge fund that has only a single trade. After the first month of the sample period, there could be one such hedge fund. After two months, there could be two hedge funds (if neither converges), etc. Each month, we calculate the return for each of these funds and then take the equally weighted average across funds as the index return for that month. In initiating and terminating positions, realistic transaction costs are applied (described in the Appendix). As with all strategies considered in

[6] We also implement the strategy with trigger values of five and 20 basis points and obtain very similar results.

this article, the initial amount of capital invested in the strategy is adjusted to fix the annualized volatility of the return index at 10% (2.887% per month). Observe that this swap spread arbitrage strategy requires nothing in the way of complex modeling to implement. Furthermore, as we compare current swap spread levels to current short term spread levels, there is no look-ahead bias in the strategy.

Table 1 provides summary statistics for the excess returns from the swap spread strategies. As shown, the mean monthly excess returns range from about 0.31 to 0.55%. All these means are significant at the 10% level, and two are significant at the 5% level. Three of the four skewness coefficients for the returns have positive values. Also, the returns for the strategies tend to have more kurtosis than would be the case for a normal distribution.

We also examine the returns from forming an equally weighted (based on notional amount) portfolio of the individual hedge fund strategies. As each individual strategy is capitalized to have an annualized volatility of 10%, the equally weighted portfolio will have smaller volatility if the returns from the individual strategies are not perfectly correlated. As shown, considerable diversification is obtained with the equally weighted portfolio as its volatility is only 82% of that of the individual strategies. The t-statistic for the returns from the equally weighted strategy is 2.78. The Bernardo and Ledoit (2000) gain/loss ratio for this equally weighted strategy is 1.643 and the Sharpe ratio is 0.597.[7]

Finally, note that the amount of capital per $100 notional amount of the strategy required to fix the annualized volatility at 10% varies directly with the horizon of the strategy. This reflects the fact that the price sensitivity of the swap and Treasury bond increases directly with the horizon or duration of the swap and Treasury bond.

2. Yield Curve Arbitrage

Another major type of fixed-income arbitrage involves taking long and short positions at different points along the yield curve. These yield curve arbitrage strategies often take the form of a "butterfly" trade, where, for example, an investor may go long five-year bonds and short two- and 10-year bonds in a way that zeros out the exposure to the level and slope of the term structure in the portfolio. Perold (1999) reports that LTCM frequently executed these types of yield curve arbitrage trades.

While there are many different flavors of yield curve arbitrage in the market, most share a few common elements. First, some type of analysis is applied

[7] We also investigate whether the inclusion of a stop-loss limit affects the results. The stop-loss limit is where an individual hedge fund is terminated upon the realization of a 20% drawdown. As the volatility of returns is normalized to 10% per year (2.887% per month), however, the stop-loss limit is almost never reached. Thus, the results when a stop-loss limit is included are virtually identical to those reported.

46

Table 1
Summary statistics for the swap spread arbitrage strategies

Strategy	Swap	n	Capital	Mean	t-Stat	Standard deviation	Minimum	Maximum	Skewness	Kurtosis	Ratio negative	Serial correlation	Gain/loss	Sharpe ratio
SS1	2 years	193	3.671	0.546	2.94	2.887	−9.801	11.552	0.449	2.454	0.326	−0.113	1.758	0.655
SS2	3 years	193	5.278	0.476	3.01	2.887	−8.482	11.209	0.178	2.002	0.326	−0.269	1.629	0.571
SS3	5 years	193	9.047	0.305	1.68	2.887	−10.663	10.163	−0.456	2.269	0.332	−0.135	1.372	0.366
SS4	10 years	193	15.795	0.313	1.69	2.887	−10.761	10.004	0.069	2.711	0.425	−0.114	1.381	0.376
EW SS	–	193	8.448	0.410	2.78	2.378	−8.569	8.439	−0.111	2.505	0.394	−0.148	1.643	0.597

This table reports the indicated summary statistics for the monthly percentage excess returns from the swap spread arbitrage strategies. Swap denotes the swap maturity used in the strategy. The EW SS strategy consists of taking an equally weighted (based on notional amount) position each month in the individual-maturity swap spread strategies. n denotes the number of monthly excess returns. Capital is the initial amount of capital required per $100 notional of the arbitrage strategy to give a 10% annualized standard deviation of excess returns. The t-statistics for the means are corrected for the serial correlation of excess returns. Ratio negative is the proportion of negative excess returns. Gain/loss is the Bernardo and Ledoit (2000) gain/loss ratio for the strategy. The sample period for the strategies is from December 1988 to December 2004.

4

to identify points along the yield curve, which are either "rich" or "cheap." Second, the investor enters into a portfolio that exploits these perceived misvaluations by going long and short bonds in a way that minimizes the risk of the portfolio. Finally, the portfolio is held until the trade converges and the relative values of the bonds come back into line.

Our approach in implementing this strategy is very similar to that followed by a number of large fixed-income arbitrage hedge funds. Specifically, we assume that the term structure is determined by a two-factor affine model. Using the same monthly swap market data as in the previous section, we fit the model to match exactly the one-year and 10-year points along the swap curve each month. Once fitted to these points, we then identify how far off the fitted curve the other swap rates are. Figure 2 graphs the time series of deviations between market and model for the two-year, three-year, five-year, and seven-year swap rates. For example, imagine that for a particular month, the market two-year swap rate is more than 10 basis points above the fitted two-year swap rate. We would

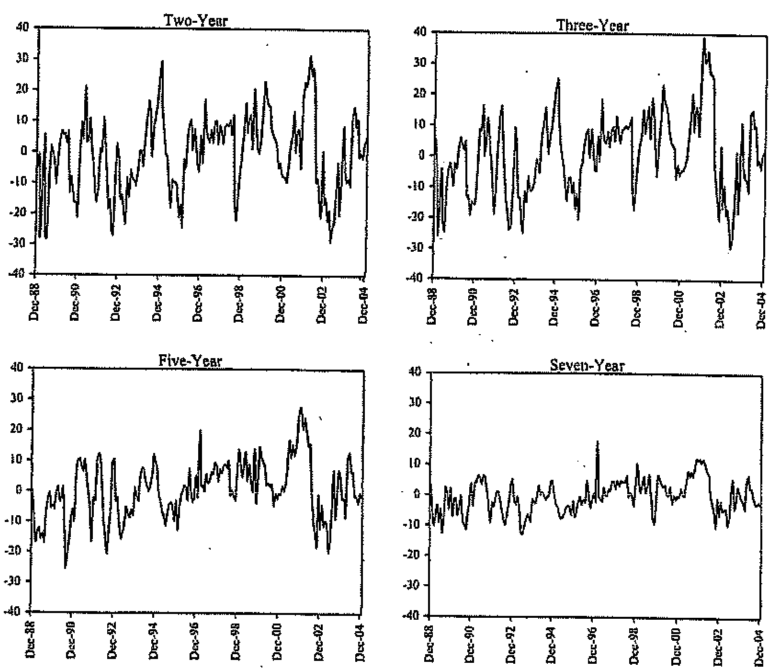

Figure 2
Deviations between market and model swap rates
These graphs plot the difference between the market swap rates for the indicated horizons and the corresponding values implied by the two-factor affine model fitted to match exactly the one-year and 10-year swap rates. All deviations are in basis points.

48

enter into a trade by going long (receiving fixed) $100 notional of a two-year swap and going short a portfolio of one-year and 10-year swaps with the same sensitivity to the two affine factors as the two-year swap. Thus, the resulting portfolio's sensitivity to each of the two factors would be zero. Once this butterfly trade was put on, it would be held for 12 months or until the market two-year swap rate converged to the model value. The same process continues for each month, with either a trade similar to the above, the reverse trade of the above, or no trade at all being implemented (in which case the strategy invests in cash and earns zero excess return), and similarly for the other swap maturities. Unlike the swap spread strategy of the previous section, this strategy involves a high degree of "intellectual capital" to implement as both the process of identifying arbitrage opportunities and the associated hedging strategies require the application of a multi-factor term structure model.

As before, we can think of a butterfly trade put on in a specific month as a fictional hedge fund with only one trade. Similarly, we can compute the return on this hedge fund until the trade converges. For a given month, there may be a number of these hedge funds, each representing a trade that was put on previously but has not yet converged. The return index for the strategy for a given month is the equally weighted average of the returns for all the individual hedge funds active during that month. As in the previous section, we include realistic transaction costs in computing returns and adjust the capital to give an annualized volatility of 10% for the index returns. The details of the strategy are described in the Appendix.[8]

Table 2 reports summary statistics for the excess returns from the yield curve strategies. We use a trigger value of 10 basis points in determining whether to implement a trade.[9] We implement the strategy separately for two-year, three-year, five-year, and seven-year swaps and also implement an equally weighted strategy (in terms of notional amount) of the individual-horizon strategies. As shown, the average monthly excess returns from the individual strategies as well as for the equally weighted strategy are all statistically significant and range from about 0.4 to 0.6%.

Table 2 also summarizes that the excess returns are highly positively skewed. The positive skewness of the returns argues against the view that this strategy is one in which an arbitrageur earns small profits most of the time but occasionally suffers a huge loss. As before, the excess returns display more kurtosis than would be the case for a normal distribution.

[8] At each date, we fit the model to match exactly the current one- and 10-year swaps. Thus, there is no look-ahead bias in the state variables of the model. While the parameters of the model are estimated over the entire sample, however, they are used only in determining the hedge ratios for butterfly trades. Thus, there should be little or no look-ahead bias in the results. As a diagnostic, we estimated the model using data for the first part of the sample period and then applied it to strategies for the latter part of the sample. The results from this exercise are virtually identical to those we report.

[9] Using a trigger value of five basis points gives similar results.

Table 2
Summary statistics for the yield curve arbitrage strategies

Strategy	Swap	n	Capital	Mean	t-Stat	Standard deviation	Minimum	Maximum	Skewness	Kurtosis	Ratio negative	Serial correlation	Gain/loss	Sharpe ratio
YC1	2 years	193	4.847	0.540	2.76	2.887	−6.878	10.056	0.569	0.902	0.301	−0.059	1.770	0.648
YC2	3 years	193	7.891	0.486	2.31	2.887	−6.365	11.558	0.591	1.172	0.337	0.014	1.643	0.583
YC3	5 years	193	7.794	0.615	3.29	2.887	−8.307	11.464	0.592	2.366	0.212	−0.108	2.102	0.738
YC4	7 years	193	4.546	0.437	2.46	2.887	−10.306	20.032	2.156	14.953	0.088	−0.158	2.355	0.524
EW YC	–	193	6.270	0.519	3.42	2.293	−5.241	11.329	0.995	3.269	0.347	−0.084	1.980	0.785

This table reports the indicated summary statistics for the monthly percentage excess returns from the yield curve arbitrage strategies. Swap denotes the swap maturity used in the strategy. The EW YC strategy consists of taking an equally weighted (based on notional amount) position each month in the individual-maturity yield curve strategies. n denotes the number of monthly excess returns. Capital is the initial amount of capital required per $100 notional of the arbitrage strategy to give a 10% annualized standard deviation of excess returns. The t-statistics for the means are corrected for the serial correlation of excess returns. Ratio negative is the proportion of negative excess returns. Gain/loss is the Bernardo and Ledoit (2000) gain/loss ratio for the strategy. The sample period for the strategies is from December 1988 to December 2004.

Finally, observe that the amount of capital required to attain a 10% level of volatility is typically much less than in the swap spread strategies. This reflects the fact that the yield curve trade tends to be better hedged as all the positions are along the same curve, and the factor risk is neutralized in the portfolio.

3. Mortgage Arbitrage

The mortgage-backed security (MBS) strategy consists of buying MBS passthroughs and hedging their interest rate exposure with swaps. A passthrough is a MBS that passes all the interest and principal cash flows of a pool of mortgages (after servicing and guarantee fees) to the passthrough investors. MBS passthroughs are the most common type of mortgage-related product and this strategy is commonly implemented by hedge funds. The Bond Market Association indicates that MBS now forms the largest fixed-income sector in the U.S.

The main risk of a MBS passthrough is prepayment risk. That is, the timing of the cash flows of a passthrough is uncertain because homeowners have the option to prepay their mortgages.[10] The prepayment option embedded in MBS passthroughs generates the so-called negative convexity of these securities. For instance, the top panel of Figure 3 plots the nonparametric estimate of the price of a generic Ginnie Mae (GNMA) passthrough with a 7% coupon rate as a function of the five-year swap rate (see the Appendix for details on the estimation procedure, data, and strategy implementation). It is clear that the price of this passthrough is a concave function of the interest rate. This negative convexity arises because homeowners refinance their mortgages as interest rates drop, and the price of a passthrough consequently converges to some level close to its principal amount.

A MBS portfolio duration hedged with swaps inherits the negative convexity of the passthroughs. For example, the bottom panel of Figure 3 plots the value of a portfolio composed of a \$100 notional long position in a generic 7% GNMA passthrough, duration hedged with the appropriate amount of a five-year swap. This figure reveals that abrupt changes in interest rates will cause losses in this portfolio. To compensate for these possible losses, investors require higher yields to hold these securities. Indeed, Bloomberg's option-adjusted spread (OAS) for a generic 7% GNMA passthrough during the period from November 1996 to February 2005 was between 48 and 194 basis points with a mean value of 112 basis points.[11]

[10] For discussions about the effects of prepayment on MBS prices, see Dunn and McConnell (1981a,b), Schwartz and Torous (1989, 1992), Stanton (1995), Boudoukh et al. (1997), and Longstaff (2005).

[11] OASs are commonly used as a way of analyzing the relative valuations of different MBSs. As opposed to static spreads, the OAS incorporates the information about the timing of the cash flows of a passthrough with the use of a prepayment model and a term structure model in its calculation. The OAS therefore adjusts for the optionality of a passthrough. For a discussion of the role of OAS in the MBS market, see Gabaix, Krishnamurthy, and Vigneron (2004).

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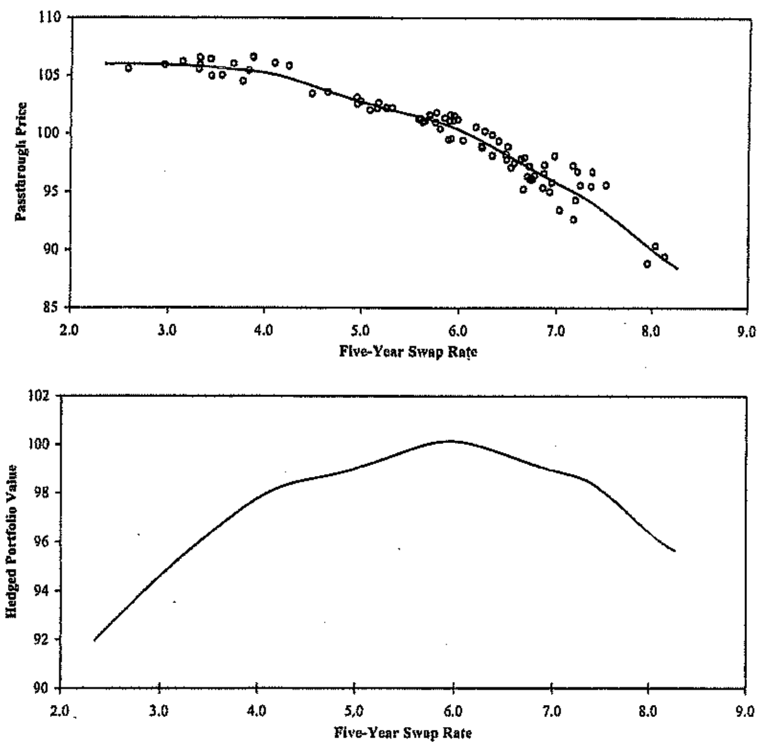

Figure 3
Passthrough price as a function of swap rates
The top panel of this figure displays the nonparametric estimate of the price of the 7% GNMA passthrough as a function of the five-year swap rate. Each point in this figure represents 25 daily observations. The bottom panel displays the value of a portfolio with $100 notional amount of this passthrough duration-hedge with a five-year swap. The hedge is initiated when the swap rate is 6.06%.

Long positions in passthroughs are usually financed with a form of repurchase agreement called a dollar roll. Dollar rolls are analogous to standard repurchase agreements in the sense that a hedge fund entering into a dollar roll sells a passthrough to a MBS dealer and agrees to buy back a similar security in the future at a predetermined price. The main difference between a standard repurchase agreement and a dollar roll is that with the roll, the dealer does not have to deliver a passthrough backed by exactly the same pool of mortgages. Unlike traditional repurchase agreements, a dollar roll does not require any haircut or over-collateralization [see Biby, Modukuri, and Hargrave (2001)]. Dealers extend favorable financing terms because dollar rolls give them the flexibility to manage their MBS portfolios. Assume, for instance, that a MBS dealer

52

wishes to cover an existing short position in the MBS market. To do so, the dealer can buy a passthrough from a hedge fund with the dollar roll. At the end of the roll term, the dealer does not need to return a passthrough backed by exactly the same pool of mortgages. As a result, dollar rolls can be used as a mechanism to cover short positions in the passthrough market.

The overall logic of the strategy of buying MBS passthroughs, financing them with dollar rolls, and hedging their duration with swaps is therefore two-fold. First, investors require larger yields to carry the negative convexity of MBS passthroughs. Second, the delivery option of the dollar rolls makes them a cheap source of MBS financing. To execute the strategy, it is necessary to specify which agency passthroughs are used [Ginnie Mae (GNMA), Fannie Mae (FNMA), or Freddie Mac (FHLMC)], the MBS coupons (trading at discount or at premium), the swap maturities used in the hedge, the model used to calculate the hedge ratios, the frequency of hedge rebalancing (daily, weekly or monthly), and the OAS level above which a long position in the passthrough is taken (the OAS trade trigger).

We use GNMA passthroughs because they are fully guaranteed by the U. S. Government and are consequently free of default risk.[12] The passthroughs we study are those with coupons closest to the current coupon as they are the most liquid. The passthroughs are hedged with five-year swaps. There is a large diversity of models that can be used to calculate hedge ratios. Indeed, every major MBS dealer has a proprietary prepayment model. Typically, these proprietary models require a high level of "intellectual capital" to develop, maintain, and use. We expect that some of these models used in practice deliver better hedge ratios than others. However, we do not want to base our results on a specific parametric model. Rather, we wish to have hedge ratios that work well on average. To this end, we adopt a nonparametric approach to estimate the hedge ratios. Specifically, we use the method developed by Aït-Sahalia and Duarte (2003) to estimate the first derivative of the passthrough price with respect to the five-year swap rate, with the constraint that passthrough prices are a nonincreasing function of the five-year swap rate. We use all the available sample of passthrough prices for this estimation.[13] The hedging rebalancing frequency is monthly. We expect that most hedge funds following this strategy estimate the duration of their portfolios at least daily and rebalance when the duration deviates substantially

[12] All the mortgage loans securitized by GNMA are federally insured. Among the guarantors are the FHA and VA.

[13] Even though the model is estimated over the entire sample, the current five-year swap rate is used to specify the hedging ratio. Thus, there is no look-ahead bias in the state variable of the model. To check whether look-ahead bias is induced by the parameter estimation procedure, we also estimated the model for the 6.5 and 7.0% coupons using only data available prior to the first day that the strategy is implemented. The returns obtained using the hedge ratios implied by this estimation procedure are virtually identical to those obtained when the entire sample period is used in the estimation.

53

from zero. For reasons of simplicity, we assume that all the trading in this strategy is done on the last trading day of the month. Trade triggers based on OAS may be used to improve the returns of mortgage strategies [see for instance Hayre (1990)]. We, however, take a long position on MBS passthroughs independently of their OAS as we want to avoid any dependence of our results on a specific prepayment model.

The strategy is implemented between December 1996 and December 2004 with a total of 97 monthly observations. The results of this strategy are displayed in Table 3. The first row of Table 3 displays the results of the strategy implemented with passthroughs trading at a discount. The second row displays the results of holding the passthrough with coupon closest to the current coupon, which can be trading at either a premium or a discount. The results using the premium passthroughs with coupons closest to the current coupons are in the third row. We again report results for an equally weighted portfolio (in terms of notional amount) of the individual strategies.

The excess returns of the MBS strategies can be either positively skewed (discount strategy) or negatively skewed (the premium strategy). The excess returns of the strategies are not significantly autocorrelated. The mean excess returns of the discount and par strategies are 0.691 and 0.466%. The mean returns of the discount and par strategies are statistically significant at roughly the 10% level. The performance of the premium passthrough strategy is considerably worse than that of the other strategies. The mean monthly return of the premium strategy is not different from zero at usual significance levels. The relatively poor performance of the premium passthrough strategy is partially caused by the strong negative convexity of the premium passthroughs. Indeed, the passthroughs in the premium strategy have an average convexity of -1.53 compared to -1.44 for the passthroughs in the par strategy and -1.11 for the passthroughs in the discount strategy.

4. Fixed-Income Volatility Arbitrage

In this section, we examine the returns from following a fixed-income volatility arbitrage strategy. Volatility arbitrage has a long tradition as a popular and widely used strategy among Wall Street firms and other major financial market participants. Volatility arbitrage also plays a major role among fixed income hedge funds. For example, Lowenstein (2000) reports that LTCM lost more than \$1.3 billion in volatility arbitrage positions before the fund's demise in 1998.

In its simplest form, volatility arbitrage is often implemented by selling options and then delta-hedging the exposure to the underlying asset. In doing this, investors hope to profit from the well-known tendency of implied volatilities to exceed subsequent realized volatilities. If the implied volatility is higher than the realized volatility, then selling options produces an

54

Table 3
Summary statistics for the mortgage arbitrage strategies

Strategy	Mortgage	n	Capital	Mean	t-Stat	Standard deviation	Minimum	Maximum	Skewness	Kurtosis	Ratio negative	Serial correlation	Gain/loss	Sharpe ratio
MA1	Discount	97	21.724	0.691	2.08	2.887	−6.794	11.683	0.882	2.929	0.383	0.128	1.999	0.830
MA2	Par	97	19.779	0.466	1.50	2.887	−7.600	11.676	0.330	2.263	0.402	0.059	1.565	0.560
MA3	Premium	97	16.910	0.065	0.23	2.887	−8.274	9.844	−0.274	1.452	0.402	−0.052	1.063	0.078
EW MA	–	97	19.471	0.408	1.39	2.750	−7.556	8.539	0.064	1.027	0.392	0.053	1.489	0.514

This table reports the indicated summary statistics for the monthly percentage excess returns from the mortgage arbitrage strategies. Mortgage denotes the type of mortgage-backed securities used in the strategy—discount, par, or premium. The EW MA strategy consists of taking an equally weighted (based on notional amount) position each month in the individual discount, par, and premium mortgage arbitrage strategies. n denotes the number of monthly excess returns. Capital is the initial amount of capital required per \$100 notional of the arbitrage strategy to give a 10% annualized standard deviation of excess returns. The t-statistics for the means are corrected for the serial correlation of excess returns. Ratio negative is the proportion of negative excess returns. Gain/Loss is the Bernardo and Ledoit (2000) gain/loss ratio for the strategy. The sample period for the strategies is from December 1996 to December 2004.

55

excess return proportional to the gamma of the option times the difference between the implied variance and the realized variance of the underlying asset.[14]

In implementing a fixed-income volatility arbitrage strategy, we focus on interest rate caps. Interest rate caps are among the most important and liquid fixed-income options in the market. Interest rate caps consist of portfolios of individual European options on the Libor rate [for example, see Longstaff, Santa-Clara, and Schwartz (2001)]. Strategy returns, however, would be similar if we focused on cap/floor straddles instead. At-the-money caps are struck at the swap rate for the corresponding maturity. The strategy can be thought of as selling a $100 notional amount of at-the-money interest rate caps and delta-hedging the position using Eurodollar futures. In actuality, however, the strategy is implemented in a slightly different way that involves a series of short-term volatility swaps. This alternative approach is essentially the equivalent of shorting caps but allows us to avoid a number of technicalities. The details of how the strategy is implemented are described in the Appendix.

The data used in constructing an index of cap volatility arbitrage returns consist of the swap market data described in Section 1, daily Eurodollar futures closing prices obtained from the Chicago Mercantile Exchange, and interest rate cap volatilities provided by Citigroup and the Bloomberg system. To incorporate transaction costs, we assume that the implied volatility at which we sell caps is 1% less than the market midpoint of the bid-ask spread (for example, at a volatility of 17% rather than at the midmarket volatility of 18%). Because the bid-ask spread for caps is typically less than 1% (or one vega), this gives us conservative estimates of the returns from the strategy. The excess return for a given month can be computed from the difference between the implied variance of a caplet at the beginning of the month, and the realized variance for the corresponding Eurodollar futures contracts over the month. The deltas and gammas for the individual caplets can be calculated using the standard Black (1976) model used to quote cap prices in this market. Although the Black model is used to compute hedge ratios, the strategy actually requires little in the way of modeling sophistication. To see this, recall that in the Black model, the delta of an at-the-money straddle is essentially zero. Thus, this strategy could be implemented almost entirely without the use of a model by simply selling cap/floor straddles over time. Note that there is no look-ahead bias in this implementation of the strategy.

Table 4 reports summary statistics for the volatility arbitrage return indexes based on the strategies for the highly liquid two-, three-, four-, and five-yeacap maturities as well as for the equally weighted (based on notional

[14] For discussions of the relation between implied and realized volatilities, see Day and Lewis (1988), Lamoureux and Lastrape (1993), and Canina and Figlewski (1993).

56

Table 4
Summary statistics for the fixed-income volatility arbitrage strategies

Strategy	Cap	n	Capital	Mean	t-Stat	Standard deviation	Minimum	Maximum	Skewness	Kurtosis	Ratio negative	Serial correlation	Gain/loss	Sharpe ratio
VA1	2 years	183	0.734	0.389	1.11	2.887	−9.720	6.550	−0.962	1.579	0.383	0.465	1.423	0.467
VA2	3 years	183	0.863	0.609	1.77	2.887	−9.675	6.851	−0.909	1.332	0.355	0.445	1.722	0.731
VA3	4 years	183	0.953	0.682	2.08	2.887	−10.295	7.087	−0.989	1.644	0.311	0.409	1.823	0.819
VA4	5 years	150	1.082	0.488	1.32	2.887	−9.997	6.654	−0.988	1.772	0.347	0.423	1.543	0.586
EW VA	–	183	0.908	0.584	1.79	2.280	−9.592	6.674	−0.925	1.392	0.344	0.425	1.709	0.720

This table reports the indicated summary statistics for the monthly percentage excess returns from the fixed-income volatility arbitrage strategy of shorting at-the-money interest rate caps of the indicated maturity. The EW VA strategy consists of taking an equally weighted (based on notional amount) position each month in the individual-maturity volatility arbitrage strategies. n denotes the number of monthly excess returns. Capital is the initial amount of capital required per \$100 notional of the arbitrage strategy to give a 10% annualized standard deviation of excess returns. The t-statistics for the means are corrected for the serial correlation of excess returns. Ratio negative is the proportion of negative excess returns. Gain/loss is the Bernardo and Ledoit (2000) gain/loss ratio for the strategy. The sample period for the strategies is from October 1989 to December 2004 (but is shorter for some strategies because cap volatility data for earlier periods are unavailable).

57

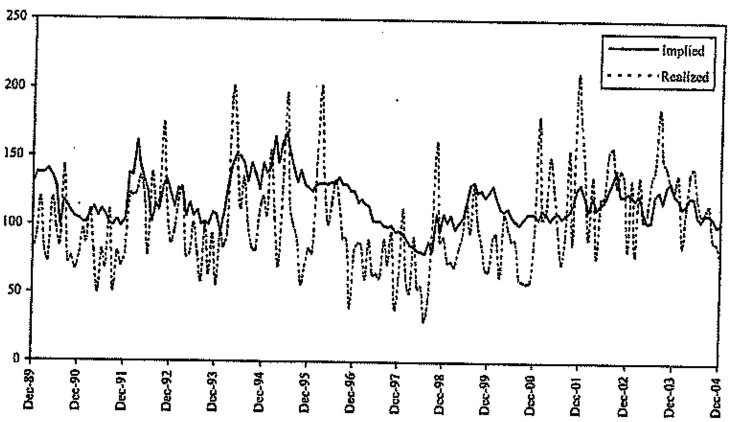

Figure 4
Implied and realized basis point volatility of four-year interest rate caps
This graph plots the implied annualized basis point volatility for a four-year interest rate cap along with the average annualized realized basis point volatility over the subsequent month of the Eurodollar futures contract corresponding to the individual caplets of the cap.

amount) strategy. As shown, the volatility arbitrage strategy tends to produce positive excess returns. The average excess returns range from about 0.40 to nearly 0.70% per month. The average excess return for the three-year cap strategy is significant at the 10% level, and the average excess return for the four-year cap strategy is significant at the 5% level. The average excess return for the equally weighted strategy is also significant at the 10% level. As an illustration of why this strategy produces positive excess returns, Figure 4 graphs the implied volatility of a four-year cap against the average (over the corresponding 15 Eurodollar futures contracts used to hedge the cap) realized Eurodollar futures volatility (both expressed in terms of annualized basis point volatility). In this figure, the implied volatility clearly tends to be higher than the realized volatility. Unlike the previous strategies considered, volatility arbitrage produces excess returns that are highly negatively skewed. In particular, the skewness coefficients for all the strategies are negative. Thus, these excess returns appear more consistent with the notion of "picking up nickels in front of a steamroller." The excess returns again display more kurtosis than would normally distributed random variables. These strategies require far less capital for a $100 notional trade than the previous strategies.

5. Capital Structure Arbitrage

Capital structure arbitrage (or alternatively, credit arbitrage) refers to a class of fixed-income trading strategies that exploit mispricing between a company's

58

debt and its other securities (such as equity). With the exponential growth in the credit default swap (CDS) market in the last decade, this strategy has become increasingly popular with proprietary trading desks at investment banks.[15] In fact, Euromoney reports that some traders describe this strategy as the "most significant development since the invention of the CDS itself nearly ten years ago" [Currie and Morris (2002)]. Furthermore, the *Financial Times* reports that "hedge funds, faced with weak returns or losses on some of their strategies, have been flocking to a new one called capital structure arbitrage, which exploits mispricings between a company's equity and debt" [Skorecki (2004)].

This section implements a simple version of capital structure arbitrage for a large cross-section of obligors. The purpose is to analyze the risk and return of the strategy as commonly implemented in the industry. Using the information on the equity price and the capital structure of an obligor, we compute its theoretical CDS spread and the size of an equity position needed to hedge changes in the value of the CDS or what is commonly referred to as the equity delta. We then compare the theoretical CDS spread with the level quoted in the market. If the market spread is higher (lower) than the theoretical spread, we short (long) the CDS contract, while simultaneously maintaining the equity hedge. The strategy would be profitable if, subsequent to initiating a trade, the market spread and the theoretical spread converge to each other.

More specifically, we generate the predicted CDS spreads using the CreditGrades (CG) model, which was jointly devised by several investment banks as a market standard for evaluating the credit risk of an obligor.[16] It is loosely based on Black and Cox (1976), with default defined as the first passage of a diffusive firm value to an unobserved "default threshold." For CDS data we use the comprehensive coverage provided by the Markit Group, which consists of daily spreads of five-year CDS contracts on North American industrial obligors from 2001 to 2004. To facilitate the trading analysis, we require that an obligor should have at least 252 daily CDS spreads no more than two weeks apart from each other.[17] After merging firm balance sheet data from Compustat and equity prices from Center for Research in Security Prices (CRSP), the final sample contains 135,759 daily spreads on 261 obligors. Details on the calibration of the model and the trading

[15] CDSs are essentially insurance contracts against the default of an obligor. Specifically, the buyer of the CDS contract pays a premium each quarter, denoted as a percentage of the underlying bond's notional value in basis points. The seller agrees to pay the notional value for the bond should the obligor default before the maturity of the contract. CDSs can be used by commercial banks to protect the value of their loan portfolios. For a more detailed description of the CDS contract, see Longstaff, Mithal, and Neis (2005).

[16] For details about the model, see Finkelstein et al. (2002).

[17] This criterion is consistent with capital structure arbitrageurs trading in the most liquid segment of the CDS market. On the practical side, it also yields a reasonably broad sample.

59

strategy are provided in the Appendix. This strategy clearly requires a high level of financial knowledge to implement.

To illustrate the intuition behind the trading strategy, we present the market spread, the theoretical spread, and the equity price for General Motors (GM) in Figure 5. First, we observe that there is a negative correlation between the CDS spread and the equity price. Indeed, the correlation between changes in the equity price and the market spread for GM is −0.32. Moreover, the market spread appears to be more volatile, reverting to the model spread over the long run. For example, the market spread widened to over 440 basis points during October 2002, while the model spread stayed below 300 basis points. This gap diminished shortly thereafter and completely disappeared by February 2003. The arbitrageur would have profited handsomely if he were to short CDS and short equity as a hedge during this period. Note, however, that if the arbitrageur placed the same trades two months earlier in August 2002, he would have experienced losses as the CDS spread continued to diverge. The short equity hedge would have helped to some extent in this case, but its effectiveness remains doubtful due to the low correlation between the CDS spread and the equity price.

Incidentally, a similar scenario played out again in May 2005 when GM's debt was on the verge of being downgraded. Seeing GM's CDS becoming ever more expensive, many hedge funds shorted CDS on GM and hedged their exposure by shorting GM equity. GM's debt was indeed downgraded shortly afterwards, but not before Kirk Kerkorian announced a $31-per-share offer to increase his stake in GM, causing the share price to soar. According to *The Wall Street Journal*, this "dealt

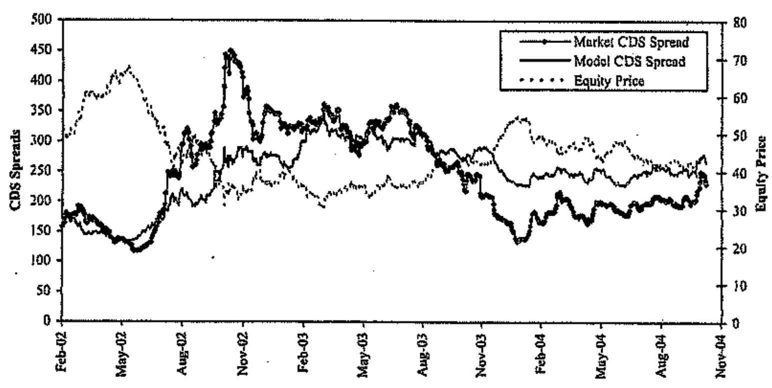

Figure 5
General Motors CDS spreads and equity price
This figure displays the market CDS spread, the model CDS spread, and the equity price for General Motors. The CDS spreads are in basis points.

60

the hedge funds a painful one-two punch: their debt bets lost money, and the loss was compounded when their hedge lost out as the stock price rose" [Zuckerman (2005)]. Overall, the GM experience suggests that the risk for individual trades is typically a combination of rapidly rising market spreads and imperfect hedging from the offsetting equity positions.

We implement the trading strategy for all obligors as follows. For each day t in the sample period of an obligor, we check whether $c_t > (1+\alpha)\ c_t'$, where c_t and c_t' are the market and model spreads, respectively, and α is called the trigger level for the strategy. If this criterion is satisfied, we short a CDS contract with a notional amount of \$100 and short an equity position as given by the CG model.[18] The positions are liquidated when the market spread and the model spread become equal, or after 180 days, whichever occurs first. We assume a 5% bid-ask spread for trading CDS. This is a realistic estimate of CDS market transaction costs in recent periods.

As there are 261 obligors in the final sample, we typically have thousands of open trades throughout the sample period. We create the monthly index return as follows. First, as the CDS position has an initial value of zero, we assume that each trade is endowed with an initial level of capital, from which the equity hedge is financed. All subsequent cash flows, such as CDS premiums and cash dividends on the stock position, are credited to or deducted from this initial capital. We also compute the value of the outstanding CDS position using the CG model and obtain daily excess returns for each trade. Then, we calculate an equally weighted average daily return across all open trades for each day in the sample and compound them into a monthly frequency. This yields 48 numbers that represent monthly excess returns obtained by holding an equally weighted portfolio of all available capital structure arbitrage trades. As all information used in implementing the strategy is contemporaneous, there is no look-ahead bias in strategy returns.

Table 5 summarizes the monthly excess returns for six strategies implemented for three trading trigger levels and for investment-grade or speculative-grade obligors. Also reported are the results for an equally weighted portfolio (based on notional amounts). First, we notice that the amount of initial capital required to generate a 10% annualized standard deviation is several times larger than for any of the previous strategies. This is an indication of the risk involved in capital structure arbitrage. In fact, results not presented here show that convergence occurs for only a small fraction of the individual trades. Furthermore, although Table 5 does not show any significant change in the risk and return of the

[18] We also consider the strategy of buying CDS contracts and putting on a long equity hedge when $c_t' > (1+\alpha)\ c_t$. This strategy yields slightly lower excess returns.

6

Table 5
Summary statistics for the capital structure arbitrage strategies

Strategy	Rating	Trigger	n	Capital	Mean	t-Stat	Standard deviation	Minimum	Maximum	Skewness	Kurtosis	Ratio negative	Serial correlation	Gain/ loss	Sharpe ratio
CS1	Investment	1.00	48	47.000	0.768	1.95	2.887	−8.160	10.570	0.223	5.337	0.271	−0.055	2.621	0.922
CS2		1.50	48	52.300	0.613	1.25	2.887	−8.020	12.770	0.266	8.682	0.375	0.162	2.435	0.735
CS3		2.00	48	44.900	0.731	1.30	2.887	−4.640	13.790	0.342	10.075	0.417	0.296	3.341	0.877
CS4	Speculative	1.00	48	86.900	0.709	2.30	2.887	−8.680	7.680	0.331	2.646	0.167	−0.298	2.513	0.851
CS5		1.50	48	90.500	0.669	2.17	2.887	−7.250	10.920	0.358	4.661	0.146	−0.306	2.921	0.802
CS6		2.00	48	75.900	0.740	1.03	2.887	−1.730	15.210	0.448	15.889	0.104	0.505	12.738	0.887
EW CS	–	–	48	66.250	0.705	1.70	2.029	−1.955	9.650	2.556	8.607	0.333	0.343	4.117	1.203

This table reports the indicated summary statistics for the monthly percentage excess returns from the capital structure arbitrage strategies. Rating denotes whether the strategy is applied to investment-grade or speculative-grade CDS obligors. Trigger denotes the ratio of the difference between the market spread and the model spread divided by the model spread, above which the strategy is implemented. The EW CS strategy consists of taking an equally weighted (based on notional amount) position each month in the individual capital structure arbitrage strategies. n denotes the number of monthly excess returns. Capital is the initial amount of capital required per \$100 notional of the arbitrage strategy to give a 10% annualized standard deviation of excess returns. The t-statistics for the means are corrected for the serial correlation of excess returns. Ratio negative is the proportion of negative excess returns. Gain/loss is the Bernardo and Ledoit gain/loss ratio for the strategy. The sample period for the strategies is from January 2001 to December 2004.

62

strategies when the trade trigger level is increased from 1 to 2, the mean return can in fact become 0 or negative at lower values of α, say 0.5. This suggests that the information content of a small deviation between the market spread and the predicted spread is low, and capital structure arbitrage becomes profitable only when implemented at higher threshold levels. Three of the six strategies have average monthly excess returns that are statistically significant at the 5% level. The equally weighted strategy has significantly lower volatility than the individual strategies, indicating that the individual strategies are not perfectly correlated with each other. Finally, these excess returns all display positive skewness and have more kurtosis than would a normally distributed random variable.

6. Fixed-Income Arbitrage Risk and Return

In this section, we study the risk and return characteristics of the fixed-income arbitrage strategies. In particular, we explore whether the excess returns generated by the strategies represent compensation for exposure to systematic market factors.

6.1 Risk-Adjusted Returns

The five fixed-income arbitrage strategies we study are often described in hedge fund marketing materials as "market-neutral" strategies. For example, as the swap spread strategy consists of a long position in a swap and an offsetting short position in a Treasury bond with the same maturity (or vice versa), this trade is often viewed as having no directional market risk. In actuality, however, this strategy is subject to the risk of a major widening in the Treasury-repo spread. Similar arguments can be directed at each of the other arbitrage strategies we consider. If the residual risks of these strategies are correlated with market factors, then the excess returns reported in previous tables may in fact represent compensation for the underlying market risk of these strategies.

To examine this issue, our approach will be to regress the excess returns for the various strategies on the excess returns of a number of equity and bond portfolios. For perspective, Figures 6 and 7 plot the time series of excess returns for the equally weighted SS, YC, MA, VA, and CS strategies. To control for equity-market risk, we use the excess returns for the Fama and French (1993) market (R_M), small-minus-big (SMB), high-minus-low (HML), and up-minus-down (momentum or UMD) portfolios (excess returns are provided courtesy of Ken French). Also, we include the excess returns on the S&P bank stock index (from the Bloomberg system). To control for bond market risk, we use the excess returns on the CRSP Fama two-year, five-year, and 10-year Treasury bond portfolios. As controls for default risk, we also use the excess returns for a portfolio of A/BBB-rated industrial bonds and for a

63

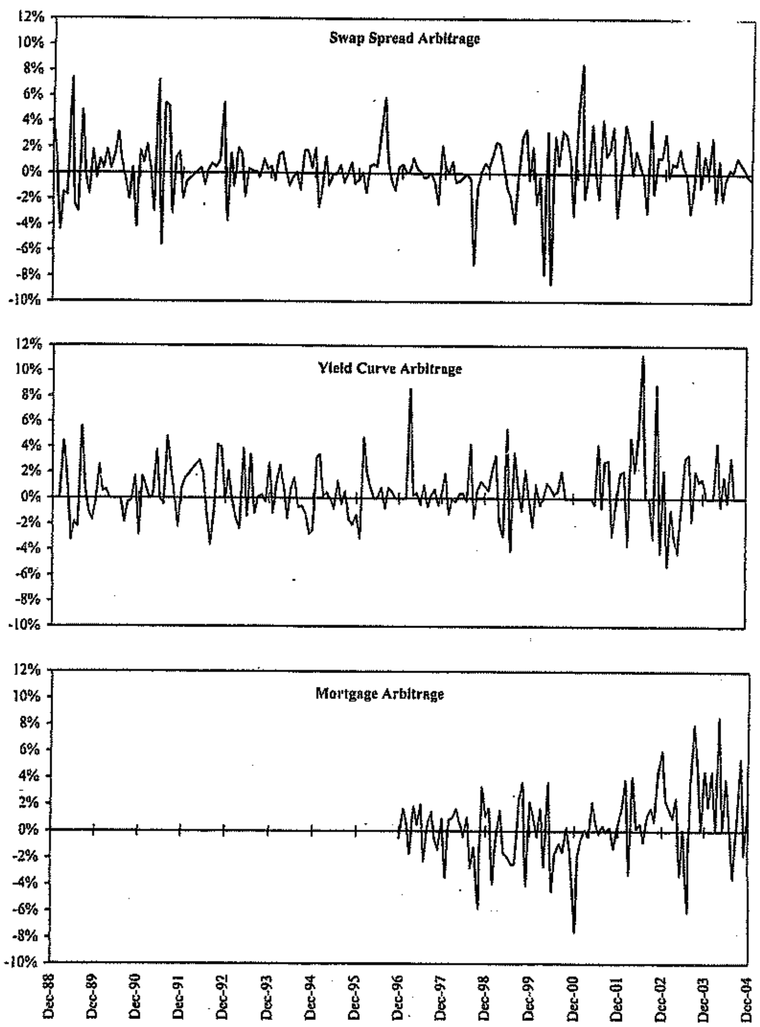

Figure 6
Monthly time series of excess returns
The top panel of this figure displays the monthly time series of excess returns for the equally weighted swap spread strategy. The middle panel displays the time series of excess returns for the equally weighted yield curve arbitrage strategy. The bottom panel displays the excess returns for the equally weighted mortgage strategy.

portfolio of A/BBB-rated bank sector bonds (provided by Merrill Lynch and reported in the Bloomberg system). Table 6 reports the regression results for each of the strategies, including the value of the alpha (the

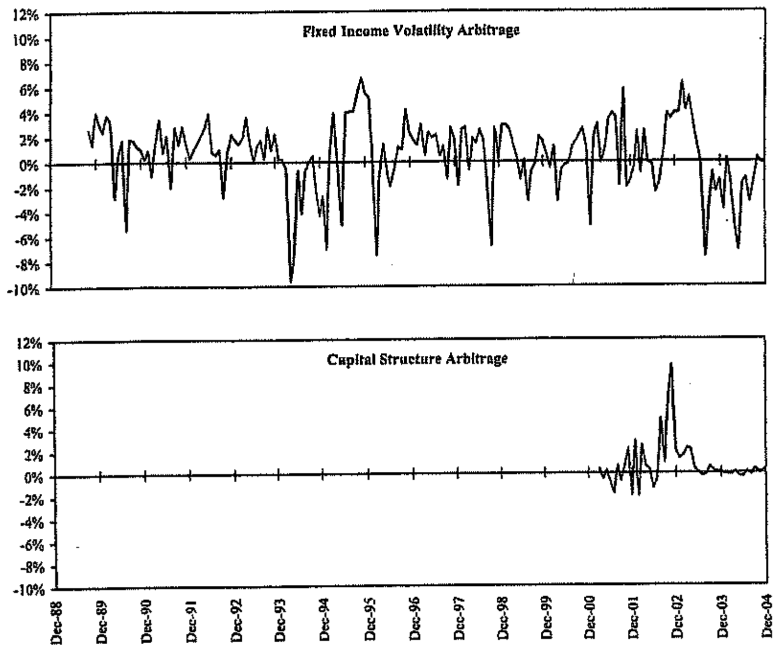

Figure 7
Monthly time series of excess returns
The top panel of this figure displays the monthly time series of excess returns for the equally weighted volatility strategy. The bottom panel displays the excess returns for the equally weighted capital structure arbitrage strategy.

intercept of the regression), along with the t-statistics for the alpha and the coefficients of the excess returns on the equity and fixed-income portfolios. Also reported are the R^2 values for the regressions.[19]

It is important to observe that a number of these factors are likely to be sensitive to major financial market "events" such as a sudden flight to quality or to liquidity [similar to that which occurred after the Russian Sovereign default in 1998 that led to the LTCM hedge fund crisis; see Dunbar (2000) and Duffie, Pedersen, and Singleton (2003)]. For example, Longstaff (2003) shows that the yield spread between Treasury and agency bonds is sensitive to macroeconomic factors such as consumer sentiment that portend the risk of such a flight. By including measures such as the excess returns on Treasury, banking, and general industrial bonds, or on banking stocks, we can control for the component of the

[19] We also explored specifications in which these explanatory variables appeared nonlinearly in the regression. The basic inferences about risk-adjusted excess returns were robust to these alternative specifications.

65

Table 6
Regression results

	Without fees		With fees		t Statistics										
Strategy	α	t-Stat	α	t-Stat	R_M	SMB	HML	UMD	R_S	R_2	R_5	R_{10}	R_I	R_B	R^2
SS1	0.350	1.44	0.115	0.59	1.57	0.25	0.41	0.90	−1.52	0.18	0.12	−0.97	−0.85	2.82	0.098
SS2	0.204	0.85	0.019	0.10	2.17	−0.14	1.19	1.04	−2.65	0.52	−0.03	−1.55	−0.02	0.42	0.105
SS3	−0.080	−0.36	−0.194	−1.04	2.56	−1.82	1.37	1.34	−2.64	1.94	−0.36	−3.53	1.84	2.95	0.212
SS4	−0.136	−0.62	−0.286	−1.45	2.71	−1.47	1.42	1.60	−1.81	2.48	1.44	−5.73	2.11	2.21	0.254
EW SS	0.084	0.45	−0.087	−0.55	2.78	−0.94	1.35	1.50	−2.67	1.53	0.34	−3.56	0.89	3.37	0.196
YC1	0.582	2.36	0.322	1.61	−0.81	1.25	−0.25	−0.16	1.03	1.44	0.10	0.23	−1.86	0.27	0.057
YC2	0.521	2.14	0.283	1.38	−1.04	0.93	−0.22	−0.14	0.88	1.98	−0.09	−0.00	−2.02	0.76	0.075
YC3	0.638	2.64	0.373	1.86	−0.85	1.78	0.33	0.86	0.62	0.84	−1.57	2.28	−3.10	2.31	0.094
YC4	0.653	2.74	0.387	1.95	−0.48	−0.56	−0.07	0.21	−0.27	1.27	−1.33	1.11	−2.30	1.44	0.117
EW YC	0.598	3.14	0.341	2.17	−1.01	1.09	−0.05	0.25	0.72	1.76	−0.91	1.14	−2.94	1.51	0.097
MA1	0.725	2.12	0.478	1.56	−1.42	−1.46	−1.33	−0.87	1.05	−0.74	−0.24	−0.39	2.52	−0.61	0.160
MA2	0.555	1.61	0.322	0.99	−1.64	−1.20	−1.68	−1.23	0.72	−0.23	−1.74	1.07	1.82	0.02	0.142
MA3	0.157	0.47	0.016	0.05	−2.08	−1.45	−1.61	−0.91	1.00	0.51	−2.68	1.18	2.41	−0.15	0.191
EW MA	0.479	1.47	0.272	0.89	−1.79	−1.43	−1.61	−1.05	0.96	−0.16	−1.62	0.64	2.35	−0.26	0.157
VA1	0.074	0.29	−0.098	−0.48	0.60	−0.71	0.39	0.92	−1.27	1.44	−0.78	−0.85	1.42	0.56	0.056
VA2	0.305	1.21	0.078	0.38	0.67	−1.29	0.22	0.93	−1.43	1.06	−1.01	−0.41	1.21	0.57	0.064
VA3	0.415	1.65	0.166	0.82	0.53	−1.56	0.03	0.95	−1.34	0.71	−0.93	−0.23	1.65	0.53	0.066
VA4	0.228	0.83	0.005	0.03	0.37	−1.59	0.06	1.09	−1.11	0.80	−0.97	−0.24	0.83	0.55	0.081
EW VA	0.308	1.26	0.084	0.42	0.56	−1.35	0.15	0.95	−1.38	0.92	−0.91	−0.41	1.50	0.58	0.063
CS1	1.073	1.66	0.734	1.35	0.58	−1.94	0.55	−0.59	0.59	0.52	−1.04	1.05	−0.30	−0.12	0.252
CS2	0.803	1.34	0.619	1.06	1.55	−2.06	0.85	−0.32	−0.73	0.32	−1.01	0.96	0.66	−0.68	0.352
CS3	1.076	1.70	0.787	1.41	1.45	−1.78	0.50	−0.38	−1.64	0.11	−0.48	0.44	0.98	−0.91	0.280
CS4	0.432	0.69	0.228	0.42	−0.61	−0.71	−1.23	−0.53	0.38	−0.35	−0.40	−0.70	1.80	0.43	0.303
CS5	1.150	1.67	0.817	1.30	−1.47	−0.38	−1.64	−1.46	0.48	−1.08	−0.35	0.11	0.96	−0.11	0.149
CS6	1.235	1.95	0.893	1.64	−0.72	−0.40	−0.50	−0.96	−1.36	−2.14	1.61	−1.03	2.50	−2.03	0.282
EW CS	0.961	2.11	0.680	1.69	0.14	−1.68	−0.38	−1.01	−0.51	−0.63	−0.38	0.19	1.53	−0.79	0.248
EW All	0.375	3.38	0.147	1.62	1.18	−1.41	0.49	0.95	−1.68	1.27	−1.29	−1.22	0.79	2.83	0.109
EW YC,MA,CS	0.525	3.56	0.275	2.28	−1.22	0.52	−0.58	−0.67	1.69	−0.13	−0.63	0.36	−0.16	0.38	0.054
CSFB	–	–	0.412	3.87	−0.80	0.79	−0.09	0.71	0.32	1.06	−2.30	0.17	−0.06	2.69	0.159
HFRI	–	–	0.479	4.22	−1.70	0.73	−0.59	−0.44	0.81	0.20	0.84	−2.76	1.52	0.40	0.139

This table reports the indicated summary statistics for the regression of monthly percentage excess returns on the excess returns of the indicated equity and bond portfolios. Results for the CSFB and HFRI fixed-income arbitrage hedge fund return indexes are also reported. R_M is the excess returns on the CRSP value-weighted portfolio. SMB, HML, and UMD are the Fama-French small-minus-big, high-minus-low, and up-minus-down market factors, respectively. R_S is the excess return on an S&P index of bank stocks. R_2, R_5, and R_{10} are the excess returns on the CRSP Fama portfolios of two-year, five-year, and 10-year Treasury bonds, respectively. R_I and R_B are the excess returns on Merrill Lynch indexes of A/BAA-rated industrial bonds and A/BAA-rated bank bonds, respectively. The sample periods for the indicated strategies are as reported in the earlier tables.

$$R_{it} = \alpha + \beta_1 R_{Mt} + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 UMD_t + \beta_5 R_{St} + \beta_6 R_{2t} + \beta_7 R_{5t} + \beta_8 R_{10t} + \beta_9 R_{It} + \beta_{10} R_{Bt} + \varepsilon_t$$

66

fixed-income arbitrage returns that is simply compensation for bearing the risk of major (but perhaps not-yet-realized) financial events. This is because the same risk would be present, and presumably compensated, in the excess returns from these equity and bond portfolios.

The excess returns from the various strategies presented in the previous sections include realistic estimates of the transactions costs involved with implementing the strategies. Thus, these returns are relevant from the perspective of an investor directly implementing these strategies or, equivalently, investing their own money in his or her own hedge fund. In general, however, many investors may not have direct access to these strategies and would instead invest capital in a fixed-income arbitrage hedge fund. Thus, hedge fund fees would need to be subtracted from the strategy returns to represent the actual returns these investors would achieve.

To address the implications of hedge fund fees in the analysis, we will also estimate the regression using an estimate of the net-of-fees excess returns from the various strategies as the dependent variable. Specifically, we assume that the investor must pay a standard 2/20 hedge fund fee (in addition to the transaction costs that are already incorporated into the strategy returns). This 2/20 fee structure means that the investor must pay an annual 2% fund management fee plus a 20% slope bonus for any excess returns (above a Libor-based high-water mark). This 2/20 fee structure is very typical in the hedge fund industry (although many funds are beginning to offer smaller fees in light of the increased competition and smaller returns in recent years). We note that as most of the strategies are above their high water marks throughout the sample period, this results in net-of-fees excess returns for the strategies that are nearly linear functions of the original excess returns (subtract 2% and multiply excess returns by 0.8). Thus, when the net-of-fees excess returns are regressed on the 10 explanatory variables, the t-statistics for these explanatory variables are virtually the same as when the original excess returns are regressed on these explanatory variables. Accordingly, to simplify the exposition, Table 6 reports the results for both the alpha based on the excess returns and the alpha based on the net-of-fees excess returns and the t-statistics for the explanatory variables from the excess return regressions.

We turn first to the results for the swap spread arbitrage strategies. Recall that each of these strategies generates significant (at the 10% level) mean excess returns. Surprisingly, Table 6 reports that after controlling for their residual market risk, none of the excess returns for the strategies results in a significant alpha. In fact, two of the individual swap spread strategies have negative alphas. When hedge fund fees are subtracted, the alphas are even smaller and even the equally weighted strategy results in a negative alpha.

Intuitively, the reason for these results is that the swap spread arbitrage strategy actually has a significant amount of market risk, and the excess returns generated by the strategy are simply compensation for that risk. Thus,

67

there is very little "arbitrage" in this fixed-income arbitrage strategy. This interpretation is strengthened by the fact that the R^2 values for the swap spread strategies range from about 10 to 25%. Thus, a substantial portion of the variation in the excess returns for the SS strategies is explained by the excess returns on the equity and bond portfolios. In particular, Table 6 reports that a number of the strategies have significant positive loadings on the market factor, significant negative loadings on the SMB and bank equity factors, significant loadings on the Treasury factors, and significant positive loadings on both the corporate bond factors.

The fact that these strategies have equity market risk may seem counterintuitive given that we are studying pure fixed-income strategies. Previous research by Campbell (1987), Fama and French (1993), Campbell and Taksler (2002), and others, however, documents that there are common factors driving returns in both bond and stock markets. Our results show that the same is also true for these fixed-income arbitrage strategies. These results are consistent with the view that the financial sector plays a central role in asset pricing. In particular, the swap spread strategy has direct exposure to the risk of a financial sector event or crisis. The commonality in returns, however, suggests that both the stock, Treasury, and corporate bond markets have exposure to the same risk. Thus, "financial-event" risk may be an important source of the commonality in returns across different types of securities.

Turning next to the yield curve arbitrage strategies, Table 6 reports that the results are almost the opposite of those for the swap spread arbitrage strategies. In particular, the excess returns for all four of the yield curve strategies, along with the excess returns for the equally weighted strategy, have significant alphas. These alphas are all in the range of 0.50 to 0.65% per month. In some cases, these alphas are even larger than the average value of the excess returns.

Turning to the net-of-fees excess returns for the yield curve strategies, Table 6 reports that at least two of the four individual strategies have alphas that are significant at the 10% level. Furthermore, the alpha for the equally weighted strategy is 0.341% and is significant at the 5% level. Thus, these strategies appear to produce significant risk-adjusted excess returns even after incorporating realistic hedge fund fees into the analysis.

In general, the R^2 values for the yield curve arbitrage strategies are small, ranging from about 6 to 12%. Interestingly, the only significant source of market risk in this strategy comes from a negative relation with the excess returns on general industrial corporate bonds (not from the bank sector bonds). One interpretation of this result may be that while the hedging approach used in the strategy is effective at eliminating the exposure to two major term structure factors, more than two factors drive the swap term structure. This interpretation is consistent with recent empirical

68

evidence about the determinants of swap rates such as Duffie and Singleton (1997) and Liu, Longstaff, and Mandell (2004).

The excess returns from the mortgage arbitrage strategies shown in Table 6 also appear to produce large alphas. The alpha for the discount mortgage strategy is 0.725% per month and is significant at the 5% level. Similarly, the alpha for the par strategy is 0.555% per month and is significant at the 10% level. The alpha for the premium strategy is not significant. When hedge fund fees are subtracted from the returns from these strategies, none of the alphas are significant (the alpha for the discount strategy, however, comes close with a t-statistic of 1.56).

Note that these mortgage strategies also have a substantial amount of market risk. In particular, the R^2 values for the regressions range from about 14 to 19%. For example, the strategies tend to have negative betas with respect to the market, but have positive loadings on the general industrial corporate bond factor.

The excess returns from the volatility arbitrage strategies appear to be substantially different from those of the other strategies. In particular, only the alpha from the four-year cap strategy is significant at the 10% level. Also, the strategies do not appear to have much in the way of market risk as the R^2 values are generally quite small. After subtracting out hedge fund fees, none of the alphas for the volatility arbitrage strategies is significant.

Finally, recall that we have only 48 months of excess returns for the capital structure arbitrage strategies as data on CDS contract before 2001 are not readily available because of the illiquidity of the market. Thus, one might expect that there would be little chance of detecting a significant alpha in this strategy. Despite this, Table 6 provides evidence that capital structure arbitrage does provide excess returns even after risk adjustment. Specifically, four of the six capital structure arbitrage strategies have excess returns that result in alphas that are significant at the 10% level. In addition, the t-statistic for the alpha for the equally weighted strategy's excess return is 2.11. In some cases, the alpha estimates are in excess of 1% per month. Thus, these alpha estimates are the largest of all of the fixed-income arbitrage strategies we consider.

Not surprisingly, the alphas for the capital structure arbitrage strategies are lower when we use the net-of-fees excess returns in the regression. Although all the alpha estimates are positive, only the CS6 strategy results in an alpha that is significant at about the 10% level. The alpha for the equally weighted strategy, however, is 0.680% which is significant at the 10% level.

Despite the large point estimates of the alphas for these capital structure arbitrage strategies, the R^2 values show that the strategies also have a large amount of market risk. These R^2 values are generally in the range of 15–35%. Interestingly, these strategies have significant positive loadings on the industrial bond factor and significant negative loadings on the SMB factor. As both the industrial bond and SMB factors are correlated

69

with corporate defaults, this suggests that there is an important business-cycle component to the returns on capital structure arbitrage.[20]

The results in this section so far have been either for individual strategies within the five broad classes of fixed-income arbitrage strategies or for equally weighted portfolios of the individual strategies. To extend the analysis, it is also useful to examine the returns to strategies that allocate capital over different types of fixed-income arbitrage.[21] To this end, we report results for the strategy that takes an equally weighted (based on notional) position in each of the 21 substrategies across all five broad classes of fixed-income arbitrage. As shown, this strategy benefits from being diversified over many different substrategies. Without including hedge fund fees, the alpha from this strategy is 0.375% with a t-statistic of 3.38. When hedge fund fees are included, however, the alpha is only 0.147% with a t-statistic of 1.62 (not quite significant at the 10% level).

As the previous results suggest that there may be economic returns to the strategies that require a higher level of "intellectual capital," we also consider a strategy that takes an equally weighted position in the 13 substrategies in the yield curve, mortgage, and capital structure arbitrage categories. As reported in Table 6, the alpha from this strategy when returns do not include hedge fund fees is 0.525% with a t-statistic of 3.56. When returns are taken net of hedge fund fees, the alpha for the strategy declines to 0.275%, but the t-statistic for the alpha of 2.28 is still significant at the 5% level.

To summarize, these results indicate that some, but not all, of the fixed-income arbitrage strategies generate significant risk-adjusted excess returns even after incorporating both transaction costs and hedge fund fees into the analysis. The strategies that appear to do the best are those that tend to require a higher level of "intellectual capital" in terms of the modeling requirements associated with the implementation of the strategies.

6.2 Historical Fixed-Income Hedge Fund Returns

We have focused on return indexes generated by following specific fixed-income arbitrage strategies over time rather than on the actual returns reported by hedge funds. As discussed earlier, there are a variety of important reasons for adopting this approach, including avoiding survivorship and backfill biases [see Malkiel and Saha (2004)], holding leverage fixed in the analysis, etc. To provide additional perspective, however, we repeat the analysis using actual fixed-income arbitrage hedge fund return data from several widely cited industry sources.

[20] For evidence about the relation between the SMB factor and default risk, see Vassalou and Xing (2004).

[21] We are grateful to the referee for suggesting this direction.

70

In particular, we obtain monthly return data from Credit Suisse First Boston (CSFB)/Tremont Index LLC for the HEDG fixed-income arbitrage index. The underlying data for this index is based on the TASS database. The sample period for these data is from January 1994 to December 2004. To be included in the index, funds must have a track record in the TASS database of at least one year, have an audited financial statement, and have at least $10 million in assets.[22] This index is value weighted. The TASS database includes data on more than 4500 hedge funds.

We also obtain monthly return data for the Hedge Fund Research Institute (HFRI) fixed-income arbitrage index. Although returns dating back to 1990 are provided, we only use returns for the same period as for the CSFB/Tremont Index to insure comparability. This index is fund or equally weighted and has no minimum fund size or age requirement for inclusion in the index. This data source tracks approximately 1500 hedge funds.

The properties of the fixed-income arbitrage hedge fund returns implied by these industry sources are similar in many ways to those for the return indexes described in the previous section. In particular, the annualized average return and standard deviation of the CSFB/Tremont fixed-income arbitrage index returns are 6.46 and 3.82%, respectively (excess return 2.60%). These values imply a Sharpe ratio of about 0.68 [which is close to the Sharpe ratio of 0.72 reported by Tremont/TASS (2004)]. The annualized average return and standard deviation for the HFRI fixed-income arbitrage index are 5.90 and 4.02%, respectively (excess return 2.05%). These values imply a Sharpe ratio of 0.51. On the other hand, there are some important differences between the CSFB/Tremont and HFRI indexes and our return indexes. In particular, the CSFB/Tremont and HFRI display a high level of negative skewness. The skewness parameters for the CSFB/Tremont and HFRI indexes are −3.23 and −3.07, respectively. Recall that with the exception of the volatility arbitrage strategies, most of our return indexes display positive (or only slight negative) skewness. Similarly, the CSFB/Tremont and HFRI indexes display significant kurtosis, with coefficients of 17.03 and 16.40, respectively.[23]

Although the correlations between the CSFB/Tremont and HFRI indexes and our return indexes vary across strategies, these correlations are typically in the range of about −0.10 to 0.30. In particular, the average correlations between the swap spread arbitrage returns and the CSFB/

[22] See Credit Suisse First Boston (2002) for a discussion of the index construction rules.

[23] The effects of various types of biases and index construction on the properties of fund return indexes are discussed in Brown et al (1992), Brooks and Kat (2002), Amin and Kat (2003), and Brulhart and Klein (2005).

Tremont and HFRI indexes are 0.12 and 0.18, respectively. The average correlations between the yield curve arbitrage returns and the two indexes are 0.02 and −0.02, respectively. The average correlations between the mortgage arbitrage returns and the two indexes are 0.22 and 0.30, respectively. The average correlations between the volatility arbitrage returns and the two indexes are 0.15 and 0.29, respectively. The average correlations between the capital structure arbitrage returns and the two indexes are −0.05 and 0.26, respectively. The reason for the slightly negative correlation between the indexes and the capital structure arbitrage returns is possibly because that this strategy is relatively new and may not yet represent a significant portion of the industry fixed-income arbitrage index. In summary, while the correlations are far from perfect, there is a significant degree of correlation between our return indexes and those based on reported hedge fund return data. Furthermore, these correlations are similar to the correlation of 0.36 reported by Mitchell and Pulvino (2001) between their return index and merger arbitrage returns reported by industry sources.

Table 6 also reports the results from the regression of the excess returns from the two indexes on the vector of excess returns described in the previous subsection. As these hedge fund return indexes are net of hedge fund fees, we interpret these results as being most compatible with the results in Table 6 based on net-of-fees excess returns. As shown, both the CSFB/Tremont and HFRI indexes appear to have significant alphas after controlling for equity and fixed-income market factors. The alpha for the CSFB/Tremont index is 0.412% per month; the alpha for the HFRI index is 0.479% per month. Both of these alphas are significant at the 5% level. It is worth reiterating the caution, however, that these indexes may actually overstate the returns of hedge funds. This is because of the potentially serious survivorship and backfill biases in these indexes identified by Malkiel and Saha (2004) and others. Thus, care should be used in interpreting these results. Furthermore, these biases (along with the heterogeneity of leverage across hedge funds and over time) may also be contributing factors in explaining the difference in the skewness between the CSFB/Tremont and HFRI indexes and the return indexes for our fixed-income arbitrage strategies. The CSFB/Tremont index appears to have significant exposure to the returns on five-year Treasuries and on the portfolio of bank bonds. This is consistent with Fung and Hsieh (2003) who find that fixed-income arbitrage strategy returns are highly correlated with changes in credit spreads. The HFRI index has significant exposure to the returns on 10-year Treasuries. The R^2 values for the regressions are similar to those for the individual fixed-income arbitrage strategy regressions.

72

7. Conclusion

This article conducts the most comprehensive study to date of the risk and return characteristics of fixed-income arbitrage. Specifically, we construct monthly return indexes for swap spread, yield curve, mortgage, volatility, and capital structure (or credit) arbitrage over extended sample periods.

While these are all widely used fixed-income arbitrage strategies, there are substantial differences among them as well. For example, very little modeling is required to implement the swap spread and volatility arbitrage strategies, while complex models and hedge ratios must be estimated for the other strategies. While attempting to be market neutral, some of the strategies have residual exposure to market-wide risk factors. For example, swap spread arbitrage is sensitive to a crisis in the banking sector, and mortgage arbitrage is sensitive to a large drop in interest rates triggering prepayments. These considerations motivate us to examine the risk and return characteristics of fixed-income arbitrage, both before and after adjusting for market risks.

We find a host of interesting results. To neutralize the effect of leverage, we choose a level of initial capital to normalize the volatility of the returns to 10% per annum across all strategies. We find that all five strategies yield positive excess returns. The required initial capital ranges from a few dollars per $100 notional for volatility and yield curve arbitrage to $50 or more for capital structure arbitrage. With the exception of volatility arbitrage, the returns have a positive skewness, contrary to the common wisdom that risk arbitrage generates small positive returns most of the time, but experiences infrequent heavy losses.

We also find that most of the strategies are sensitive to various equity and bond market factors. Besides confirming the role of market factors in explaining swap spread arbitrage and mortgage arbitrage returns, we find that yield curve arbitrage returns are related to a combination of Treasury returns that mimic a "curvature factor," and capital structure arbitrage returns are related to factors that proxy for economy-wide financial distress. Interestingly, we find that the three strategies that require the most "intellectual capital" to implement command positive excess returns even after adjusting for market risks and accounting for transaction costs and hedge fund fees.

Appendix A: Swap Spread Arbitrage

The swap data for the study consist of month-end observations of the three-month Libor rate and midmarket swap rates for two-, three-, five-, seven-, and 10-year maturity swaps. These maturities represent the most-liquid and actively traded maturities for swap contracts. All these rates are based on end-of-trading-day quotes available in New York to insure comparability of the data. In estimating the parameters, we are careful to take into account daycount differences among the rates as Libor rates are quoted on an actual/360 basis while swap rates are semiannual bond equivalent yields. There are two sources for the swap data. The primary source is the Bloomberg system which uses quotations from a number of swap brokers. The data for Libor rates and for

73

swap rates from the pre-1990 period are provided by Citigroup. As an independent check on the data, we also compare the rates with quotes obtained from Datastream and find the two sources of data to be very similar.

The Treasury data consist of month-end observations of the CMT rates published by the Federal Reserve in the H-15 release for maturities of two, three, five, seven, and 10 years. These rates are based on the yields of currently traded bonds of various maturities and reflect the Federal Reserve's estimate of what the par or coupon rate would be for these maturities if the Treasury were to issue these securities. CMT rates are widely used in financial markets as indicators of Treasury rates for the most actively traded bond maturities. As CMT rates are based heavily on the most recently auctioned bonds for each maturity, they provide an accurate estimate of yields for the most-liquid on-the-run Treasury bonds. As such, these rates are more likely to reflect actual market prices than quotations for less-liquid off-the-run Treasury bonds. Finally, data on three-month general collateral repo rates are obtained from Bloomberg as well as Citigroup.

We initiate the swap spread strategy whenever the current swap spread is more than ten basis points greater than (or less than) the current short-term Libor-general collateral repo spread. Once executed, the strategy is held until either the horizon date of the swap and bond or until the strategy converges. Convergence occurs when the swap spread for the remaining horizon of the strategy is less than or equal to (greater than or equal to) the short-term spread.

To calculate the returns from the strategy, we need to specify transaction costs and the valuation methodology. For transaction costs, we assume values that are relatively large in comparison to those paid by large institutional investors such as major fixed-income arbitrage hedge funds. In a recent article, Fleming (2003) estimates that the bid-ask spread for actively traded Treasuries is 0.20 32nds for two-year maturities, 0.39 32nds for five-year maturities, and 0.78 32nds for 10-year maturities. To be conservative, we assume that the bid-ask spread for Treasuries is one 32nd. Similarly, typical bid-ask spreads for actively traded swap maturities are on the order of 0.50 basis points. We assume that the bid-ask spread for swaps is one basis point. Finally, we assume that the repo bid-ask spread is 10 basis points. Thus, the repo rate earned on the proceeds from shorting a Treasury bond are 10 basis points less than the cost of financing a Treasury bond. This value is based on a number of discussions with bond traders at various Wall Street firms who typically must pay a spread of up to 10 basis points to short a specific Treasury bond. In some situations, a Treasury bond can trade special in the sense that the cost of shorting the bond can increase to 50 or 100 basis points or more temporarily [see Duffie (1996), Duffie, Gârleanu, and Pedersen (2002), and Krishnamurthy (2002)]. The effect of special repo rates on the analysis would be to reduce the total excess return from the strategy slightly.

Turning to the valuation methodology, our approach is as follows. For each month of the sample period, we first construct discount curves from both Treasury and swap market data. For the Treasury discount curve, we use the data for the constant maturity six-month, one-year, two-year, three-year, five-year, seven-year, and 10-year CMT rates from the Federal Reserve. We then use a standard cubic spline algorithm to interpolate these par rates at semiannual intervals. These par rates are then bootstrapped to provide a discount function at semiannual intervals. To obtain the value of the discount function at other maturities, we use a straightforward linear interpolation of the corresponding forward rates. In addition, we constrain the three-month point of the discount function to match the three-month Treasury rate. We follow the identical procedure in solving for the swap discount function. Treasury and swap positions can then be valued by discounting their fixed cash flows using the respective bootstrapped discount function.

74

Appendix B: Yield Curve Arbitrage

To implement this strategy, we assume that the riskless rate is given by $r_t = X_t + Y_t$, where X_t and Y_t follow the dynamics

$$dX = (\alpha - \beta X)dt + \sigma dZ_1, \tag{A1}$$

$$dY = (\mu - \gamma Y)dt + \eta dZ_2, \tag{A2}$$

under the risk-neutral measure, where Z_1 and Z_2 are standard uncorrelated Brownian motions. With this formulation, zero-coupon bond prices are easily shown to be given by the two-dimensional version of the Vasicek (1977) term structure model.

To estimate the six parameters, we do the following. We pick a trial value of the six parameters. Then, for each month during the sample period, we solve for the values of X_t and Y_t that fit exactly the one-year and 10-year points along the swap curve. We then compute the sum of the squared differences between the model and market values for the two-, three-, five-, and seven-year swaps for that month. We repeat the process over all months, summing the squared differences over the entire sample period. We then iterate over parameter values until the global minimum of the sum of squared errors is obtained. The resulting parameter estimates are $\alpha = 0.0009503$, $\beta = 0.0113727$, $\sigma = 0.0548290$, $\mu = 0.0240306$, $\gamma = 0.4628664$, and $\eta = 0.0257381$.

With these parameter values, we again solve for the values of X_t and Y_t that fit exactly the one-year and 10-year points along the swap curve. From this fitted model, we determine the difference between the model and market values of the two-, three-, five-, and seven-year swaps. If the difference exceeds the trigger level of 10 basis points, we go long (or short) the swap and hedge it with offsetting positions in one-year and 10-year swaps. The hedge ratios are given analytically by the derivatives of the swap values with respect to the state variables X_t and Y_t. Once implemented, the trade is held for 12 months, or until the market swap rate converges to its model value. The swap transaction costs used in computing returns are the same as those described above for the swap spread arbitrage strategy.

Appendix C: Mortgage Arbitrage

The MBS data used in the strategy are from the Bloomberg system. The mortgage data are for the period between November 1996 and December 2004. The data are composed of the current mortgage coupon, price, OAS, actual prepayment speed (CPR), and weighted-average time to maturity of generic GNMA passthroughs with coupons of 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, and 8.5%. The mortgage repo rates are the end-of-month one-month values. The mortgages in the pools are assumed to have initial terms of 30 years. Daily five-year swap rates are used to estimate hedge ratios. The assumed bid-ask spread for passthroughs is 1.28 32nds. This is the average bid-ask spread obtained from the Bloomberg system of generic GNMA passthroughs with coupons of six and seven percent. As before, the repo bid-ask spread is 10 basis points, and the swap bid-ask spread is one basis point. Most of the MBS passthrough trading is on a to-be-announced (TBA) basis. This means that at the time a trade is made, neither party to the trade knows exactly which pool of passthroughs will be exchanged. The TBA trades are settled once a month. The settlement dates are generally around the 21st of the month for GNMA passthroughs and are specified by the Bond Market Association. Settlement dates for trades from November 1996 and December 1999 are from the Bond Market Association

X5

newsletter. Settlement dates for trades from January 2000 to December 2004 are from the Bloomberg system.

The hedge ratios are estimated by a nonparametric regression of the prices of each passthrough on the five-year swap rate. The constrained nonparametric estimation follows the method developed by Aït-Sahalia and Duarte (2003), which is composed of an isotonic regression followed by a linear-kernel regression. In this method, passthrough prices are assumed to be a decreasing function of the level of the five-year swap rate. One regression is performed for each passthrough coupon. The kernel used is normal and the bandwidths are chosen by cross-validation over a grid of possible bandwidths. Because swap rates are very persistent, we follow a procedure similar to the one in Boudoukh et al. (1997) and perform the cross validation omitting all the data points in an interval. The bandwidth values are 0.001045 for the 4.5% passthrough, 0.0007821 for the 5.0% passthrough, 0.001384 for the 5.0% passthrough, 0.002926 for the 6.0 and 6.5% passthroughs, 0.002922 for the 7.0, 7.5, and 8.0% passthroughs, and 0.002473 for the 8.5% passthrough. The isotonic regression assumes that for each rate level there is only one observed price. In the sample, however, we observe various prices at the same rate level. To circumvent this problem, we take the average of the observed prices for each rate level before we run the isotonic regression. We note that the method developed by Aït-Sahalia and Duarte also allows for restrictions on the second derivative of the estimated function. In this application, however, we are only imposing restrictions on the first derivative because the price of passthroughs can be either a convex or concave function of the interest rate.

We implement the strategy in the following way. At the end of each month in the sample, a decision is made with respect to holding, buying, or selling a MBS passthrough. The decision is based on the current mortgage coupon and on the previous month's portfolio. Assume for instance that on the last trading day of the month, a hedge fund commits a certain amount of capital C_t to implement the MBS discount strategy. As part of this strategy, the hedge fund buys a \$100 notional amount of the MBS passthrough trading at a discount with coupon closest to the current mortgage coupon. At the same time, the hedge fund enters in a dollar roll and pays fixed in an interest rate swap. At the end of the next month, the hedge fund checks whether the passthrough purchased the previous month still satisfies the requirement of being at a discount with coupon closest to the current coupon. If so, the hedge fund continues to hold it, rebalances the hedge with a new five-year swap, and enters into a new dollar roll. If the passthrough does not satisfy this requirement, then the hedge fund sells it, closes the margin account, and restarts the strategy with a new MBS passthrough. The premium and the par passthrough strategies work in the same way.

The return calculation of the trading strategy is better clarified by means of an example. Assume that the hedge fund buys a \$100 notional amount of a MBS passthrough at P_t^{Ask} for settlement on the date S_1, and, to hedge its interest rate exposure, pays fixed on a five-year interest rate swap. To finance its long MBS position, the hedge fund uses a dollar roll in which the hedge fund agrees to deliver a \$100 notional amount of a MBS passthrough at S_1 in exchange for the dollar amount P_t^{Bid} and to receive a \$100 notional amount of a passthrough at the settlement date S_2 in exchange for the dollar amount P_t^{Roll}. At the end of the following month $t+1$, the hedge fund decides to sell the \$100 MBS position at price P_{t+1}^{Bid} for settlement at S_2 and unwind the five-year swap hedge. The net cash flows of the MBS transactions are $(-P_t^{Ask} - P_t^{Bid})$ at time S_1, and $(P_{t+1}^{Bid} - P_t^{Roll})$ at time S_2. The profit (or loss) of the MBS part of this trade is therefore $PV_{t+1}(P_{t+1}^{Bid} - P_t^{Roll}) - PV_t(P_t^{Ask} - P_t^{Bid})$, where PV_t is the time t value of the cash flows. In addition to the profits related to the MBS, the hedge fund also has profit from the swaps and from the capital invested in the margin account. The monthly return of this strategy is therefore the sum of the profits of all the parts of the strategy divided by C_t. Capital is allocated when a passthrough is purchased and is updated afterwards by the profits (or losses) of the strategy.

76

Note that the MBS return in the expression $PV_{t+1}(P^{Bid}_{t+1} - P^{Roll}_t) - PV_t(P^{Ask}_t - P^{Bid}_t)$ does not depend directly on the actual MBS passthrough prepayment because the counterparty of the hedge fund in the dollar roll keeps all of the cash flows of the passthrough that occur between S_1 and S_2. As a consequence, the value of P^{Roll}_t depends on the dealer forecast at time t of the prepayment cash flows between S_1 and S_2. The value of P^{Roll}_t is calculated as in the Bloomberg roll analysis [see Biby, Modukuri, and Hargrave (2001) for details about this calculation]. We assume that the implied cost of financing for the roll is the mortgage repo rate plus the bid-ask spread. In addition, as in Dynkin et al (2001), we assume that the forecast prepayment level is equal to the prepayment level of the month when the roll is initiated. In reality, the level of prepayments during the month when the roll is initiated is only disclosed to investors at the beginning of the subsequent month.

Appendix D: Volatility Arbitrage

Our approach for computing the returns from volatility arbitrage is based on entering into a sequence of one-month volatility swaps that pay the arbitrageur the difference between the initial implied variance of an interest rate caplet and the realized variance for the corresponding Eurodollar futures contract each month. This strategy benefits directly whenever the realized volatility is less than the implied volatility of interest rate caps and floors. This strategy is scaled to allow it to mimic the returns that would be obtained by shorting caps (and/or floors) in a way that keeps the portfolio continuously delta- and vega-hedged.

To illustrate the equivalence, imagine that the market values interest rate caplets using the Black (1976) model and that the implied volatility is constant (or that vega risk is zero). From Black, it can be shown that the price C of a caplet would satisfy the following partial differential equation,

$$\frac{\sigma^2 F^2}{2} C_{FF} - rC + C_t = 0, \tag{A3}$$

where F is the corresponding forward rate and σ^2 is the implied variance. Now assume that the actual dynamics of the forward rate under the physical measure are given by $dF = \mu_F dt + \hat{\sigma} F dZ$. Form a portfolio ($\Pi$) with a short position in a caplet hedged with a futures contract. Applying Ito's Lemma to the hedged portfolio gives

$$d\Pi = \left(\Pi_t + \frac{\hat{\sigma}^2 F^2}{2} \Pi_{FF}\right) dt. \tag{A4}$$

As the initial value of the futures contract is zero, its derivative with respect to time is zero, and its second derivative with respect to F is zero (we abstract from the slight convexity differences between forwards and futures), we obtain

$$d\Pi = \left(-C_t - \frac{\hat{\sigma}^2 F^2}{2} C_{FF}\right) dt. \tag{A5}$$

Substituting C_t from Equation (A3) in Equation (A5) gives

$$d\Pi = \left(\frac{(\sigma^2 - \hat{\sigma}^2)F^2}{2} C_{FF} + r\Pi\right)dt.$$

The value of this portfolio today is equal to the capital amount invested in this strategy. The excess profit of this strategy over a small period of time is approximately

$$\frac{(\sigma^2 - \hat{\sigma}^2)F^2}{2} C_{FF} dt. \tag{A7}$$

Thus, the instantaneous excess return on the strategy would be proportional to the gamma of the caplet times the difference between the implied and realized variance of the forward rate process. Note that this quantity is identical to the profit on a volatility swap where the notional amount is scaled by $F^2 C_{FF}/2$. This means that we can think of the trading strategy as either a volatility swap strategy or a short delta-hedged position in a caplet (holding implied volatility constant over the month).

We calculate the excess returns from the volatility arbitrage strategy by calculating the quantity in Equation (A7) for each individual caplet. As the implied volatility for the individual caplets within a cap, we use the market-quoted volatility for the cap. A 1% bid/ask spread represents a realistic value for interest rate caps and floors. Alternatively, a 1% transaction cost would also be realistic for a volatility swap (which can be approximated by an at-the-money-forward cap/floor straddle). As the realized volatility for each individual caplet, we use the volatility of the Eurodollar futures contract with maturity corresponding to the caplet. Using a one-month horizon for the strategy minimizes the effects of changes in the "moneyness" of the caps on the time series of returns.

Appendix E: Capital Structure Arbitrage

We provide a brief summary of the CG model, the selection of its parameters, and the use of the model in our capital structure arbitrage trading analysis. For details about the model and the associated pricing formulas, the reader is referred to Finkelstein et al. (2002, CGTD).

CG is a structural model in the tradition of Merton (1974), Black and Cox (1976), and Longstaff and Schwartz (1995). It assumes that the firm value is a diffusion, and default occurs when the firm value reaches a lower threshold called the "default barrier." Deviating slightly from the traditional structural models, however, CG assumes that the default barrier is an unknown constant that is drawn from a known distribution. This assumption helps to boost short-term credit spreads in a way similar to Duffie and Lando (2001).

To generate a predicted CDS spread, CG requires a set of seven inputs: the equity price S, the debt per share D, the mean default barrier as a percentage of debt per share $\bar{L}$, its standard deviation λ, the bond recovery rate R, the equity volatility σ_S, and the risk-free interest rate r. Consistent with the empirical analysis in the CGTD, we define D as total liabilities (taken from Compustat) divided by common shares outstanding, σ_S as the 1000-day historical equity volatility, r as the five-year constant maturity Treasury yield, and let λ be equal to 0.3. However, rather than setting $\bar{L}$ to be 0.5 and taking the bond recovery rate from a proprietary database as in the CGTD, we set R to be 0.5 and estimate the mean default barrier $\bar{L}$ by fitting the first 10 daily market spreads of an obligor to the CG model. This is consistent with the historical recovery rates on senior unsecured debt and the literature on endogenous bankruptoy. For example, in Leland (1994) and Leland and Toft

78

(1996), the default barrier is chosen by the manager with consideration for the fundamental characteristics of the company, such as the asset volatility and the payout rate.

The CG model is used in the trading analysis in three ways. Properly estimated with the above procedure, we first use it to calculate a time series of predicted CDS spreads for the entire sample period for each obligor. The comparison between the predicted spreads and the market spreads forms the basis of the trading strategy as explained in Section 5. Second, to calculate the daily returns on an open trade, we must keep track of the total value of the positions, notably the value of a CDS position that has been held for up to 180 days. The Markit CDS database used in this study, however, provides only the spreads on newly issued five-year contracts. We note that the value of an existing contract can be approximated by the change in five-year CDS spreads multiplied by the value of a five-year annuity, whose cash flows are contingent on the survival of the obligor. We use the term structure of survival probabilities from the CG model to mark to market the CDS position. Third, we numerically differentiate the value of the CDS position with respect to the equity price to identify the size of the equity hedge.

The trading analysis performed in Section 5 assumes a maximum holding period of 180 days, a CDS bid-ask spread of 5%, and a static equity hedge that is held fixed throughout a trade. It ignores the cost of trading equity because CDS market bid-ask spreads are likely to be the dominant source of transaction costs. We have experimented with setting different holding periods (30 to 360 days), updating the equity hedge daily, and computing the CDS market value using a reduced-form approach [such as Duffie and Singleton (1999)], all with results similar to those in Table 5. In addition, the average monthly excess returns remain positive even when the CDS bid-ask spread increases to 10%.

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79

80

Interest Rate Caps

- A caplet is European call option on Libor.
- Cash flow $= \frac{A}{360} \max(0, L_{T-1/4} - K)$.
- A cap is a portfolio of caplets, one for each maturity.
- A floor is a portfolio of floorlets.
- At the money cap has strike equal to swap rate.
- The Black model for a caplet:

$$D(T)\frac{A}{360}\left[F(0,\tau,T)N(d) - KN(d-\sqrt{\sigma^2\tau})\right]$$

$$d = \frac{\ln(F(0,\tau,T)/K) + \sigma^2\tau/2}{\sqrt{\sigma^2\tau}}$$

- Bloomberg examples.

Swaptions

- European option to enter a swap and receive fixed: Receivers.
- European option to enter a swap and pay fixed: Payers.
- Entering a swap and receiving fixed is equivalent to buying a bond and paying a strike price of par (paying floating leg of swap). Thus, receivers swaption is just a call option on a bond.
- In a swaption, the strike is fixed at par. Instead, the coupon and maturity determines the nature of the underlying bond.
- Structures: 2 into 10 swaption, 2 by 12 swaption.
- Bermudan swaption. 12 Noncall 2.
- At the money: coupon equals forward swap rate.
- Bloomberg examples.

Valuing Swaptions

- At the money swaption: Coupon equals forward swap rate.
- Black model for payers

$$\frac{A(0,\tau,T}{2}\left[FSR(0,\tau,T)N(d) - cN(d - sqrt\sigma^2\tau\right]$$

$$d = \frac{\ln(FSR(0,\tau,T)/c) + \sigma^2\tau/2}{\sqrt{\sigma^2\tau}}$$

- Special case of at the money swaption (both payers and receivers)

$$(D(\tau) - D(T))[2N(\sqrt{\sigma^2\tau}/2) - 1]$$

- Bloomberg examples.

82

<HELP> for explanation.

97) Regions | 98) Settings | 15:53:37 | Swaps Markets: United States

GV Ask/Chg			SW/GV		Swap Mid		FNMA	FN/GV		FN/SW		FHLMC	FH/GV		FH/SW	
2Y	0.664	+0.000	26.75	+0.50	0.927	-0.002	0.579	-9.5	-0.5	-15.8	+0.5	0.712	6.3	+0.5	-18.0	+1.0
3Y	1.066	+0.000	20.00	-0.63	1.270	-0.005	0.993	-5.4	-2.4	-12.7	+1.0	1.126	6.3	-2.3	-11.1	+0.9
4Y	1.380	+0.000	22.38	+0.42	1.516	-0.009	--	--	--	--	--	1.162	10.4	-1.9	-11.4	+0.8
5Y	1.542	+0.000	16.50	+0.50	1.691	-0.011	1.546	1.2	-0.3	-2.2	+1.5	1.590	8.8	+0.4	0.3	+1.7
7Y	1.846	+0.000	9.75	+0.69	1.925	-0.014	1.619	-22.2	-0.1	0.6	+1.7	1.691	-13.3	+0.7	0.6	+2.8
10Y	2.018	+0.000	14.00	+0.88	2.133	-0.016	2.309	28.8	+0.0	19.7	+1.9	1.995	-1.0	+0.1	9.8	+2.4
30Y	2.586	+0.000	-9.38	+0.38	2.481	-0.003	2.715	10.6	+0.0	35.2	+1.5	2.654	15.6	+0.5	37.2	+1.5

Dow Jones			S&P 500 Index			NASDAQ Composite Index			Bloomberg European 500		
DJIA	17862.14	-6.62	S&P 500	2068.53	-0.06	CCMP	4801.18	+13.54	BE500	252.88	-0.93

Cash Market	
1M LIBOR	0.17170
3M LIBOR	0.25810
6M LIBOR	0.37680
1Y LIBOR	0.66360
Fed Funds	0.07000
O/N Repo	0.15500
1W Repo	0.16500

Active Futures		
5 Year	119-22	- 01¾
10 Year	128-07+	- 04
LONG BOND	146-19	- 10
5Y Swap	102-18+	+ 01
10Y Swap	107-28+	+ 04
30Y Swap	122-18	+ 08

Swaption	1Y	3Y	5Y	7Y	10Y	Cap/Flr
1Y	56.360	47.630	42.400	40.750	36.490	67.430
2Y	47.530	43.320	38.740	36.760	34.450	63.960
3Y	44.590	42.260	38.560	35.330	33.400	58.500
4Y	41.480	38.230	36.110	34.480	32.750	54.490
5Y	37.330	37.100	35.320	33.740	32.390	51.820
7Y	37.140	35.100	33.260	31.980	30.650	46.050
10Y	34.100	31.840	30.400	29.230	28.750	43.450

30) Economic Releases (ECO)

	Date Time	C	A	M	R	Event	Period	Surv(M)	Actual	Prior	Revised
31)	02/11 04:00	US				MBA Mortgage Applications	Feb 6	--	-9.0%	1.3%	--
32)	02/11 11:00	US				Monthly Budget Statement	Jan	-$19.0B	-$17.5B	-$10.3B	--
33)	02/12 05:30	US				Retail Sales Advance MoM	Jan	-0.4%	--	-0.9%	--
34)	02/12 05:30	US				Retail Sales Ex Auto MoM	Jan	-0.5%	--	-1.0%	--
35)	02/12 05:30	US				Retail Sales Ex Auto and Gas	Jan	0.4%	--	-0.3%	--

<HELP> for explanation, <MENU> for similar functions.

91) Actions | 92) Products | 94) Data & Settings | 95) Info | Swap Manager

3) Main 4) Curves 5) Cashflow 7) Details 10) Resets 11) Risk 13) Scenario 17) Matrix

Cap CCP OTC Cpty CAP CNTRPARTY Deal ID 20) Properties

31) Load | 32) Save | 34) Ticket | 37) CCP Margin | 38) CVA |) Send to VCON/

Type	Cap		
Notional	10MM	Index	US0003M
Currency	USD	Tenor	3 Month
Effective	05/13/2015 3 MO X 5 YR	Leverage	1.00000
Maturity	02/13/2020	Spread	0.00 bp
Pay Freq	Quarterly ☑Single Look	Day Count	ACT/360
Reset Freq	Quarterly	Reset Type	In Advance
Strategy			
Cap Strike	1.72868 % Rcv X 1	☑Digital	%

61) Detail

Option

Position Long

Market		OIS DC Stripping	OFF
Curve Date	02/11/2015	Valuation	02/13/2015
Dscnt Curve	23 Bid USD Bloomberg Curve (	Fwd Curve	23 Bid USD Bloomberg Curve
Vol Cube	VCUB Bid USD BVOL Cube	Model	Black-Scholes

Valuation

				DV01	-2,693.82
				Delta (Hedge)	0.55407
Implied Vol	51.96	Calculate	Premium	Gamma (1bp)	9.26
ATM Strike	1.728679	Yield Value	64.001	Vega (1%)	4,136.30
Market Value	297,906.90	Premium	2.97907	Theta (1-day)	-70.28

<HELP> for explanation.

91) Actions | 92) Products | 93) Views | 94) Data & Setting | 95) Info | Swap Manager

1) Solver (Premium | 31) Load | 32) Save | 35) Trade | 38) CCP | 43) Send to TR

3) Main 4) Details 5) Curves 6) Cashflow 7) Resets 8) Scenario 10) Risk 12) Matrix

Deal Swaption Counterparty IRS CNTRPARTY | Deal ID 20) Properties...

Option

Style	European	Notional	10MM	Curve Date	02/11/2015
Position	Long Receiver	Currency	USD	Valuation	02/11/2015
Type	1Y X 5Y	Strike	2.089620 %	Model	Black-Scholes
Expiration	02/11/2016	Delivery	Price (Cash)	OIS DC Strip	OFF
Swap Start	02/16/2016	Prem at Expiry	Yes	CSA Coll Ccy	N/A
Swap End	02/16/2021	Fee(Pay)	0.00		
Notification Days	2 BD	Fee Date	02/11/2016		

Valuation Settings

60) Option Detail...

Underlying

Market

Valuation Results 22) Calculators 23) More Greeks...

ATM Strike	2.089620	Implied Vol (%)	42.61	DV01	2,145.81
Yield Value (bp)	35.250	Underlying Prem	0.00000	Gamma (1bp)	24.74
NPV Without Fee	166,594.81	Forward Prem	1.67438	Vega (1%)	3,850.66
NPV	166,594.81	Premium	1.66595	Theta (1-day)	-167.03

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The Relative Valuation of Caps and Swaptions: Theory and Empirical Evidence

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ABSTRACT

Although traded as distinct products, caps and swaptions are linked by no-arbitrage relations through the correlation structure of interest rates. Using a string market model, we solve for the correlation matrix implied by swaptions and examine the relative valuation of caps and swaptions. We find that swaption prices are generated by four factors and that implied correlations are lower than historical correlations. Long-dated swaptions appear mispriced and there were major pricing distortions during the 1998 hedge-fund crisis. Cap prices periodically deviate significantly from the no-arbitrage values implied by the swaptions market.

THE GROWTH IN INTEREST-RATE SWAPS during the past decade has led to the creation and rapid expansion of markets for two important types of swap-related derivatives: interest-rate caps and swaptions. These over-the-counter derivatives are widely used by many firms to manage their interest-rate risk exposure and collectively represent the largest class of fixed-income options in the financial markets. The International Swaps and Derivatives Association (ISDA) estimates that the total notional amount of caps and swaptions outstanding at the end of 1997 was over $4.9 trillion, which was more than 300 times the $15 billion notional of all Chicago Board of Trade Treasury note and bond futures options combined.

Caps and swaptions are generally traded as separate products in the financial markets, and the models used to value caps are typically different from those used to value swaptions. Furthermore, most Wall Street firms use a piecemeal approach in calibrating their models for caps and swaptions, making it difficult to evaluate whether these derivatives are fairly priced

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relative to each other. Financial theory, however, implies no-arbitrage relations that must be satisfied by cap and swaption prices. Specifically, a cap can be represented as a portfolio of options on individual forward rates. In contrast, a swaption can be viewed as an option on a portfolio of individual forward rates. Because of this, standard option pricing theory such as Merton (1973) implies that the relation between cap and swaption prices, or between different swaption prices, is driven primarily by the correlation structure of the forward rates. Given a unified valuation framework capturing these correlations, the no-arbitrage relations among cap and swaption prices can be tested directly.

This paper conducts an empirical analysis of the relative valuation of caps and swaptions using an extensive data set of interest-rate option prices. For the valuation framework, we use a string market model of the term structure of interest rates which blends the market-model framework of Brace, Gatarek, and Musiela (1997) and Jamshidian (1997) with the string-shock framework of Santa-Clara and Sornette (2001), Goldstein (2000), and Longstaff and Schwartz (2001). This approach has the important advantages of incorporating correlations directly into the model in a simple way and providing a unified framework for valuing fixed-income derivatives. The empirical approach taken in the paper consists of first solving for the covariance matrix implied by the market prices of all traded swaptions. This is the matrix equivalent of the familiar technique of solving for the implied volatility of an option. Once the implied covariance matrix has been identified, we can directly examine the implications for the relative values of caps and swaptions.

The empirical results provide a number of interesting insights into the fixed-income derivatives market. We find evidence of four statistically significant factors in the covariance matrix implied from market swaption prices. This contrasts with results based on historical covariance matrices which typically find only two to three factors, but is consistent with more recent evidence by Knez, Litterman, and Scheinkman (1994). Our results indicate that the market considers factors that contribute little to the unconditional volatility of term-structure movements, but represent a major source of conditional volatility during periods of market stress. Our results also indicate that the correlations among forward rates implied from swaption prices tend to be lower than those observed historically.

We then examine the relative valuation of swaptions and find that most swaptions tend to be valued fairly relative to each other. The major exception is during the 12-week period immediately following the announcement in September 1998 of massive trading losses at Long Term Capital Management. During this turbulent period, there is strong evidence of significant distortions in the quoted prices of many swaptions, a finding independently corroborated by interviews with many fixed-income derivatives traders. We also find that long-dated swaptions generally tend to be undervalued relative to other swaptions throughout the sample period.

Turning to the relative valuation of caps and swaptions, we find that the median differences between model and market cap prices are close to zero.

The distribution of differences, however, is skewed towards the right and all of the mean differences are positive and significant. This suggests that caps are typically valued fairly relative to swaptions, but that there are periodically large discrepancies between the two markets. This is particularly true during the hedge-fund crisis during late 1998. Alternatively, these results may imply that a more general model, such as one that allows a time-varying covariance structure, might be needed to capture fully the relative pricing of caps and swaptions.[1]

Finally, we contrast the hedging performance of the string market model with that of the standard Black model often used in practice. Despite using only four hedging portfolios to hedge all of the swaptions in the sample, the string market model performs slightly better than the Black model, which uses a different hedge portfolio for each of the 34 swaptions in our sample.

The remainder of this paper is organized as follows. Section II provides a brief introduction to cap and swaption markets. Section III describes the string market model framework used to value interest-rate derivatives. Section IV discusses the data. Section V presents the empirical results. Section VI compares the implications of the string model for fixed-income derivatives with those of the Black model. Section VII summarizes the results and makes concluding remarks.

I. The Caps and Swaptions Markets

This section provides a brief introduction to the caps and swaptions markets. We first describe the characteristics of caps and explain how they are used in the financial markets. We then discuss the features of swaptions and their uses.

A. The Caps Market

Many financial market participants enter into financial contracts in which they pay or receive cash flows tied to some floating rate such as Libor. To hedge the risk created by the variability of the floating rate, firms often enter into derivative contracts that are essentially calls or puts on the level of the Libor rate. These types of derivatives are known as interest-rate caps and floors.

Specifically, a cap gives its holder a series of European call options or caplets on the Libor rate, where each caplet has the same strike price as the others, but a different expiration date.[2] Typically, the expiration dates for the caplets are on the same cycle as the frequency of the underlying Libor rate. For example, a five-year cap on three-month Libor struck at six percent represents a portfolio of 19 separately exercisable caplets with quar-

[1] One example of this type of model is Collin-Dufresne and Goldstein (2000). We are grateful to the referee for this insight.

[2] For many currencies, the market convention is for the cap to be on the three-month Libor rate. In some markets, however, caps may be on the six-month Libor rate. For example, Yen caps with maturities greater than one year are usually on the six-month Libor rate.

terly maturities ranging from one-half to five years, where each caplet has a strike price of 0.06.[3] The cash flow associated with a caplet expiring at time T is $(a/360)\max(0, L(\tau,T) - K)$ where a is the actual number of days during the period from τ to T, $L(\tau,T)$ is the value at time τ of the Libor rate applicable from time τ to T, and K is the strike price. Note that while the cash flow on this caplet is received at time T, the Libor rate is determined at time τ, which means that there is no uncertainty about the cash flow from the caplet after Libor is set at time τ. The series of cash flows from the cap provides a hedge for an investor who is paying Libor on a quarterly or semiannual floating-rate note, where each quarterly or semiannual caplet hedges an individual floating coupon payment. In addition to caps, market participants often use interest-rate floors. These are similar to caps, except that the cash flow from an individual floorlet with expiration date T is $(a/360)\max(0, K - L(\tau,T))$. Thus, floors are essentially a series of European put options on the Libor rate. The market for interest-rate caps and floors is generically termed the caps market.

Market prices for caps and floors are universally quoted relative to the Black (1976) model. Specifically, let $D(t,T)$ denote the value at time t of a discount bond maturing at time T, and let $F(t,\tau,T)$ denote the value at time t for the Libor forward rate applicable to the period from time τ to T. Since $L(\tau,T) = F(\tau,\tau,T)$, a caplet can be viewed as an option on an individual Libor forward rate. Applying the Black model to this forward rate results in the following closed-form expression for the time-zero value of a caplet with expiration date T:

$$D(0,T)\,\frac{a}{360}\,[F(0,\tau,T)N(d) - K\,N(d - \sqrt{\sigma^2\tau/2})], \tag{1}$$

where

$$d = \frac{\ln(F(0,\tau,T)/K) + \sqrt{\sigma^2}\tau/2}{\sqrt{\sigma^2\tau}}$$

and

$$F(0,\tau,T) = \frac{360}{a}\left(\frac{D(0,\tau)}{D(0,T)} - 1\right)$$

and where σ is the volatility of changes in the logarithm of the forward rate. With this closed-form solution, the price of a cap is given by summing the values of the constituent caplets. Thus, a cap is simply a portfolio of indi-

[3] The standard market convention is to omit the first caplet since the cash flow from this caplet is set at time $t = 0$ and is not stochastic.

vidual options, each on a different forward Libor rate. The market convention is to quote cap prices in terms of the implied value of σ, which sets the Black model price equal to the market price. Note that the convention of quoting cap prices in terms of the implied volatility from the Black model does not necessarily mean that market participants view the Black model as the most appropriate model for caps. Rather, implied volatilities from the Black model are simply a more convenient way of quoting prices, because implied volatilities tend to be more stable over time than the actual dollar price at which a cap would be traded.

B. The Swaptions Market

The underlying instrument for a swaption is an interest rate swap. In a standard swap, two counterparties agree to exchange a stream of cash flows over some specified period of time. One counterparty receives a fixed annuity and pays the other a stream of floating cash flows tied to the three-month Libor rate. Counterparties are identified as either receiving fixed or paying fixed in the swap. Although principal is not exchanged at the end of a swap, it is often more intuitive to think of a swap as involving a mutual exchange of \$1 at the end of the swap. From this perspective, the cash flows from the fixed leg are identical to those from a bond with coupon rate equal to the swap rate, whereas the cash flows from the floating leg are identical to those from a floating rate note. Thus, a swap can be viewed as exchanging a fixed rate coupon bond for a floating rate note.[4]

At the time a swap is initiated, the coupon rate on the fixed leg of the swap is specified. Intuitively, this rate is chosen to make the present value of the fixed leg equal to the present value of the floating leg. To illustrate how the fixed rate is determined, designate the current date as time zero and the final maturity date of the swap as time T. The fixed rate at which a new swap with maturity T can be executed is known as the constant maturity swap rate and we denote it by $FSR(0,0,T)$, where the first argument refers to time zero, the second argument denotes the start date of the swap which is time zero for a standard swap, and T is the final maturity date of the swap. Once a swap is executed, then fixed payments of $FSR(0,0,T)/2$ are made semiannually at times $0.50, 1.00, 1.50, \ldots, T - 0.50$, and T. Floating payments are made quarterly at times $0.25, 0.50, 0.75, \ldots, T - 0.25$, and T and are equal to $a/360$ times the three-month Libor rate at the beginning of the quarter, where a is the actual number of days during the quarter. This feature is termed setting in advance and paying in arrears. Abstracting from credit issues, a floating rate note paying three-month Libor quarterly must be worth par at each quarterly Libor reset date. Because the initial value of

[4] For discussions about the economic role that interest-rate swaps play in financial markets, see Bicksler and Chen (1986), Turnbull (1987), Smith, Smithson, and Wakeman (1988), Wall and Pringle (1989), Macfarlane, Ross, and Showers (1991), Sundaresan (1991), Litzenberger (1992), Sun, Sundaresan, and Wang (1993), and Gupta and Subrahmanyam (2000).

a swap is zero, the initial value of the fixed leg must also be worth par. Setting the time-zero values of the two legs equal to each other and solving for the swap rate gives

$$FSR(0,0,T) = 2\left[\frac{1 - D(0,T)}{A(0,0,T)}\right], \tag{2}$$

where $A(0,0,T) = \sum_{i=1}^{2T} D(0,i/2)$ is the present value of an annuity with first payment six months after the start date and final payment at time T. Swap rates are continuously available from a wide variety of sources for standard swap maturities such as 2, 3, 4, 5, 7, 10, 12, 15, 20, 25, and 30 years.

For many swaptions, the underlying swap has a forward start date. In a forward swap with a start date of τ, fixed payments are made at time $\tau + 0.50$, $\tau + 1.00$, $\tau + 1.50, \ldots, T - 0.50$, and T and floating rate payments are made at times $\tau + 0.25$, $\tau + 0.50$, $\tau + 0.75, \ldots, T - 0.25$, and T. At the start date τ, the value of the floating leg equals par. Discounting this time-τ value back to time zero implies that the time-zero value of the floating cash flows is $D(0,\tau)$. Because the forward swap has a time-zero value of zero, the time-zero value of the fixed leg must also equal $D(0,\tau)$. This implies that the forward swap rate $FSR(0,\tau,T)$ must satisfy

$$FSR(0,\tau,T) = 2\left[\frac{D(0,\tau) - D(0,T)}{A(0,\tau,T)}\right]. \tag{3}$$

After a swap is executed, the coupon rate on the fixed leg may no longer equal the current market swap rate and the value of the swap can deviate from zero. Let $V(t,\tau,T,c)$ be the value at time t to the counterparty receiving fixed in a swap with forward start date $\tau \geq t$ and final maturity date T, where the coupon rate on the fixed leg is c. The value of this forward swap is given by

$$V(t,\tau,T,c) = \frac{c}{2}\sum_{i=1}^{2(T-\tau)} D(t,\tau + i/2) + D(t,T) - D(t,\tau), \tag{4}$$

where the first two terms in this expression represent the value of the fixed leg of the swap, and the third term is the present value of the floating leg, which will be worth par at time τ. For $t > \tau$, the swap no longer has a forward start date and the value of the swap on semiannual fixed coupon payment dates is given by the expression

$$V(t,\tau,T,c) = \frac{c}{2}\sum_{i=1}^{2(T-t)} D(t,\tau + i/2) + D(t,T) - 1. \tag{5}$$

Note that in either case, the value of the swap is just a linear combination of zero-coupon bond prices.

Swaptions or swap options allow their holder to enter into a swap with a prespecified fixed coupon rate, or to cancel an existing swap. Intuitively, swaptions can also be viewed as calls or puts on coupon bonds. Natural end users of swaptions are government agencies and firms coming to the capital markets to borrow funds. These entities use swaptions for the same reasons many firms issue callable or puttable debt—to cancel a swap with an above-market coupon rate or to enter into a new swap at a below-market coupon rate.

There are two basic types of European swaptions.[5] The first is the option to enter a swap and receive fixed payments. For example, let τ be the expiration date of the swaption, c be the coupon rate on the swap, and T be the final maturity date on the swap. The holder of this option has the right at time τ to enter into a swap with a remaining term of $T - \tau$, and receive the fixed annuity of c. Because the value of the floating leg will be par at time τ, this option is equivalent to a call option on a bond with a coupon rate of c and a remaining maturity of $T - \tau$ where the strike price of the call is \$1. This option is generally called a τ into $T - \tau$ receivers swaption, where τ is the maturity of the option and $T - \tau$ is the tenor of the underlying swap. This swaption is also known as a τ by T receivers swaption. Note that if the option holder is paying fixed at rate c in a swap with a final maturity date of T, then exercising this option has the effect of canceling the original swap at time τ since the two fixed and two floating legs cancel each other out. Observe, however, that when the option is used to cancel the swap at time τ, the current fixed for floating coupon exchange is made first.

The second type of swaption is the option to enter a swap and pay a fixed rate, and the cash flows associated with this option parallel those described above. An option that gives the option holder the right to enter into a swap at time τ with final maturity date at time T and pay fixed is generally termed a τ into $T - \tau$ or a τ by T payers swaption. Again, this option is equivalent to a put option on a coupon bond where the strike price is the value of the floating leg at time τ of \$1. A τ by T payers swaption can be used to cancel an existing swap with final maturity date at time T where the option holder is receiving fixed at rate c.

From the symmetry of the European payoff functions, it is easily shown that a long position in a τ by T receivers swaption and a short position in a τ by T payers swaption with the same coupon has the same payoff as receiving fixed in a forward swap with start date τ and coupon rate c. A standard no-arbitrage argument gives the receivers/payers parity result that at time t, $0 \leq t \leq \tau$, the value of the forward swap must equal the value of the

[5] For a discussion of the characteristics of American-style swaptions, see Longstaff, Santa-Clara, and Schwartz (2001). Callable bonds are also very similar to swaptions. For a discussion of callable bonds, see Bliss and Ronn (1998).

receivers swaption minus the value of the payers swaption. When the coupon rate c equals the forward swap rate $FSR(t,\tau,T)$, the forward swap is worth zero and the receivers and payers swaptions have identical values. In this case, the swaptions are said to be at the money forward.

As in the caps markets, the convention in the swaptions market is to quote prices in terms of their implied volatility relative to a standard pricing model. In swaption markets, prices are quoted as implied volatilities relative to the Black (1976) model as applied to the forward swap rate. Again, this does not mean that the market views this model as the most accurate model for swaptions. To illustrate how prices are quoted in the swaptions market, consider a τ by T European payers swaption where the fixed coupon rate equals c. Under the assumption that the forward swap rate follows a lognormal process under the annuity measure (the measure where the value of the annuity $A(t,\tau,T)$ is used as the numeraire), the Black model implies that the value of this swaption at time zero is

$$\frac{1}{2} A(0,\tau,T)[FSR(0,\tau,T)N(d) - cN(d - \sigma\sqrt{\tau})], \tag{6}$$

where

$$d = \frac{\ln(FSR(0,\tau,T)/c) + \sigma^2\tau/2}{\sigma\sqrt{\tau}},$$

where $N(\cdot)$ is again the cumulative standard normal distribution function and σ is the volatility of the logarithm of the forward swap rate. The value of the corresponding receivers swaption is given from the receivers/payers parity result. In the special case where the swaption is at-the-money forward, $c = FSR(0,\tau,T)$ and equation (6) reduces to

$$(D(0,\tau) - D(0,T))[2N(\sigma\sqrt{\tau}/2) - 1]. \tag{7}$$

Because this receivers swaption is at the money forward, the value of the corresponding payers swaption is identical. When an at-the-money-forward swaption is quoted at an implied volatility of σ, the actual price that is paid by the purchaser of the swaption is given by substituting σ into equation (7).[6]

In the previous section, we showed that caps are simple portfolios of options on individual forward rates. In contrast, swaptions can be viewed as

[6] Smith (1991) describes the application of the Black (1976) model to European swaptions. Brace et al. (1997), Jamshidian (1997), and others demonstrate that the Black model for swaptions can be derived within an internally consistent no-arbitrage model of the term structure in which the numeraire is the value of an annuity.

options on portfolios of forward rates. To see this, recall that a swaption is an option on the forward swap rate in the Black (1976) model. Furthermore, forward swap rates can be expressed as nearly linear functions of individual forward rates, where the weights are related to the durations of the cash flows from the fixed leg of the swap.[7] From this, it follows that the swaption can be thought of as an option on a linear combination or portfolio of forward rates. Merton (1973) presents a number of no-arbitrage propositions including the well-known result that the value of an option on a portfolio must be less than or equal to that of a corresponding portfolio of options. This inequality is strict if the assets underlying the individual options are not perfectly correlated. Although the forward swap rate is only approximately linear in the individual forward rates, the key implication of the Merton result, namely that the relative value of a portfolio of options and an option on a portfolio is determined by the correlations between the underlying assets, is directly applicable to caps and swaptions. This key implication motivates many of the empirical tests later in the paper. In particular, we solve for the correlation matrix among forwards implied by a set of swaption prices, and then examine the extent to which other fixed-income options satisfy the no-arbitrage restrictions imposed by the correlation structure of forwards.

Finally, while both caps and swaptions are quoted in terms of the Black (1976) model, it should be recognized that the Black model is being actually used in different ways in these markets. In particular, the caps market uses the forward short-term Libor rate as the underlying state variable in the Black model, whereas the swaptions market uses longer-term forward swap rates. Because forward swap rates are nearly linear in individual forward rates, the lognormality assumption implicit in the Black model cannot hold simultaneously for both individual forward rates and forward swap rates, since a linear combination of lognormal variates is not lognormal. This is the sense in which the two markets use different models; the inputs used in the Black model differ across the two markets. In addition, since the volatilities used in the Black model are for fundamentally different rates, direct comparisons between the quoted implied volatilities of caps and swaptions are invalid. This has important implications for the risk management of portfolios of caps and swaptions.

II. The Valuation Framework

In this section, we develop a general string market model for valuing fixed-income derivatives such as caps and swaptions. We then describe how to invert the model to solve for the implied covariance matrix that best fits observed market prices.

[7] This well-known rule of thumb or approximation can be obtained by differentiating the expression for the forward swap rate in equation (3) with respect to either spot or forward rates. For example, see Fabozzi (1997, Chapter 5).

A. The String Market Model

In a series of recent papers, Brace et al. (1997), Jamshidian (1997), and others develop term-structure models in which either Libor forward rates or forward swap rates are taken to be fundamental and their dynamics modeled directly using a Heath, Jarrow, and Morton (1992) framework. This class of models is often referred to as market models since they are based on the forwards of observable term rates in the market rather than on instantaneous forward rates. This approach has the advantage of solving some technical problems associated with continuously compounded lognormal rates as well as paralleling the standard practitioner approach of basing models on term rates. Libor-based and swap-based market models have been applied to a variety of interest-rate derivative valuation problems. Because the structure of these models is closely related to that of the Heath et al. framework, they share many of the same calibration issues and have typically only been implemented with a small number of factors.

In another recent literature, Kennedy (1994, 1997), Goldstein (2000), Longstaff and Schwartz (2001), and Santa-Clara and Sornette (2001) model the evolution of the term structure as a stochastic string. In this approach, each point along the term structure is a distinct random variable with its own dynamics, but which may be correlated with the other points along the term structure. Thus, string models are inherently high-dimensional models. Surprisingly, however, string models can actually be much easier to calibrate than models with fewer factors. The reason for this is that string models are directly parameterized by the correlation function for the points along the string. This direct approach is generally much more parsimonious than the standard approach of parameterizing the elements of a matrix of diffusion coefficients. The advantages of the string model approach to parameterization become increasingly important as the number of factors driving the term structure increases. Santa-Clara and Sornette show that the string model approach generalizes the Heath et al. (1992) framework for instantaneous forward rates while preserving its intuitive structure and appeal.

In this paper, we blend the market model setup with the string model approach of calibration to develop a valuation framework for fixed-income derivatives. This approach has the advantage of allowing us to develop the model in terms of the forward Libor rates that underlie the prices of caps and swaptions. At the same time, this approach makes it possible to directly model the correlation structure among Libor forwards in a simple way even when there are a large number of factors. Capturing the correlation structure is particularly important in this study; recall from earlier discussion that the correlation structure among forwards plays a central role in determining the relative valuation of caps and swaptions. We designate this valuation framework the string market model (SMM).

In this model, we take the Libor forward rates out to 10 years $F_i \equiv F(t,T_i,T_i + 1/2)$, $T_i = i/2$, $i = 1,2,\ldots,19$, to be the fundamental variables driving the term structure. Similarly to Black (1976), we assume that the risk-neutral dynamics for each forward rate are given by

$$dF_i = \alpha_i F_i dt + \sigma_i F_i dZ_i, \tag{8}$$

where α_i is an unspecified drift function, σ_i is a deterministic volatility function, dZ_i is a standard Brownian motion specific to this particular forward rate, and $t \leq T_i$.[8] Note that although each forward rate has its own dZ_i term, these dZ_i terms are correlated across forwards. The correlation of the Brownian motions together with the volatility functions determine the covariance matrix of forwards, Σ. This is different from traditional implementations of multifactor models that use several uncorrelated Brownian motions to shock each forward rate. This seemingly minor distinction actually has a number of important implications for the estimation of model parameters from market data.

To model the covariance structure among forwards in a parsimonious but economically sensible way, we make the assumption that the covariance between dF_i/F_i and dF_j/F_j is time homogeneous in the sense that it depends only on $T_i - t$ and $T_j - t$.[9] Furthermore, since our objective is to apply the model to swaps that make fixed payments semiannually, we make the simplifying assumption that these covariances are constant over six-month intervals. With these assumptions, the problem of capturing the covariance structure among forwards reduces to specifying a 19 by 19 time-homogenous covariance matrix Σ.

One of the key differences between this string market model and traditional multifactor models is that our approach allows the parameters of the model to be uniquely identified from market data. For example, if there are N forward rates, the covariance matrix Σ has only $N(N + 1)/2$ distinct parameters. Thus, market prices of fixed-income derivatives contain information on at most $N(N + 1)/2$ covariances, and no more than $N(N + 1)/2$ parameters can be uniquely identified from the market data. Since the string market model is parameterized by Σ, the parameters of the model are econometrically identified. In contrast, a typical implementation with constant coefficients of a traditional N-factor model of the form

$$dF_i = \alpha_i F_i dt + \sigma_{i1} F_i dZ_1 + \sigma_{i2} F_i dZ_2 + \ldots + \sigma_{iN} F_i dZ_N, \tag{9}$$

[8] We assume that the initial value of F_i is positive and that the unspecified α_i terms are such that standard conditions guaranteeing the existence and uniqueness of a strong solution to equation (8) are satisfied. These conditions are described in Karatzas and Shreve (1988, Chapter 5). In addition, we assume that α_i is such that F_i is nonnegative for all $t \leq T_i$.

[9] Although the assumption of time homogeneity imposes additional structure on the model, it has the advantage of being more consistent with traditional dynamic term-structure models in which interest rates are determined by the fundamental state of the economy. In addition, time homogeneity facilitates econometric estimation because of the stationarity of the model's specification. For discussions of the advantages of time-homogeneous models, see Andersen and Andreasen (2000) and Longstaff et al. (2001).

would require N parameters for each of the N forwards, resulting in a total of N^2 parameters. Given that there are only $N(N + 1)/2 < N^2$ separate covariances among the forwards, the general specification in equation (9) cannot be identified using market information unless additional structure is placed on the model. Similar problems also occur when there are fewer factors than forwards. By specifying the covariance or correlation matrix among forwards directly, the string market model avoids these identification problems. String models also have the advantage of being more parsimonious. For example, up to $N \times K$ parameters would be needed to specify a traditional K-factor model. In contrast, only $K(K + 1)/2$ parameters would be needed to specify a string market model with rank K.[10]

Although the string is specified in terms of the forward Libor rates, it is much more efficient to implement the model using discount bond prices. By definition,

$$F_i = \frac{360}{a}\left[\frac{D(t,T_i)}{D(t,T_i + 1/2)} - 1\right]. \tag{10}$$

Thus, the forward rates F_i can all be expressed as functions of the vector of discount bond prices with maturities $0.50, 1, \ldots, 10$. Conversely, these discount bond prices can be expressed as functions of the string of forward rates, assuming that standard invertibility conditions are satisfied.[11] Applying Itô's Lemma to the vector D of discount bond prices gives

$$dD = r\,D\,dt + J^{-1}\,\sigma\,F\,dZ, \tag{11}$$

where r is the spot rate, $\sigma\,F\,dZ$ is the vector formed by stacking the individual terms $\sigma_i(t,T_i)\,F_i\,dZ_i$ in the forward rate dynamics in equation (8), and J^{-1} is the inverse of the Jacobian matrix for the mapping from discount bond prices to forward rates. Since each forward depends only on two discount bond prices, this Jacobian matrix has the following simple banded diagonal form.[12]

[10] These types of identification problems parallel those which occur in general affine term-structure models. The specification and identification issues associated with affine term-structure models are discussed in an important recent paper by Dai and Singleton (2000).

[11] The primary condition is that the determinant of the Jacobian matrix for the mapping from discount bond prices to forward swap rates be nonzero. If this condition is satisfied, local invertibility is implied by the Inverse Function Theorem.

[12] For notational simplicity, discount bonds are expressed as functions of their maturity date in the Jacobian matrix. The Jacobian matrix represents the derivative of the 19 forwards $F_{0.50}$, $F_{1.00}, F_{1.50}, \ldots, F_{9.50}$ with respect to the discount bond prices $D(1.00), D(1.50), D(2.00), \ldots, D(10.00)$. Since $\sigma(T_i - t) = 0$ for $T_i \leq 0.50$, $D(0.50)$ is not stochastic and does not affect the diffusion term in equation (11).

$$J = \begin{bmatrix} -\frac{D(0.50)}{D^2(1.00)} & 0 & 0 & \cdots & 0 & 0 & 0 \\ \frac{1}{D(1.50)} & -\frac{D(1.00)}{D^2(1.50)} & 0 & \cdots & 0 & 0 & 0 \\ 0 & \frac{1}{D(2.00)} & -\frac{D(1.50)}{D^2(2.00)} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{D(9.50)} & -\frac{D(9.00)}{D^2(9.50)} & 0 \\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{D(10.00)} & -\frac{D(9.50)}{D^2(10.00)} \end{bmatrix}$$

It is important to observe that the drift term rD in equation (11) does not depend on the drift term α_i in equation (8). The reason for this is that discount bonds are traded assets in this complete markets setting and their instantaneous expected return is equal to the spot rate under the risk-neutral measure.[13] Thus, this string market model formulation has the advantage of allowing us to avoid specifying the complicated drift term α_i, making the model numerically easier to work with than formulations based entirely on forward rates. Again, since our objective is a discrete-time implementation of this model, we make the simplifying assumption that r equals the yield on the shortest maturity bond at each time period.[14]

The dynamics for D in equation (11) provide a complete specification of the evolution of the term structure. This string market model is arbitrage free in the sense that it fits the initial term structure exactly and the expected rate of return on all discount bonds equals the spot rate under the

[13] The bond market is complete in the sense that there are as many traded bonds as there are sources of risk. Thus, although no discount bond is a redundant asset, the market is complete and all fixed income derivatives can be priced under a risk-neutral measure in which the expected returns on all bonds equals the riskless rate. For a discussion of this point, see Santa-Clara and Sornette (2001).

[14] Extensive numerical tests indicate that this discretization assumption has little effect on the results; we find that this approach gives values for European swaptions that are virtually identical to those implied by their closed-form solutions.

risk-neutral pricing measure. Furthermore, the model allows each point along the curve to be a separate factor, but also allows for a general correlation structure through Σ. To complete the parameterization of the model, we need only specify Σ in a way that matches the market or the historical behavior of forward rates.

B. Implied Covariance Matrices

Rather than specifying the covariance matrix Σ exogenously, our approach is to solve for the implied matrix Σ that best fits the observed market prices of some set of market data. Specifically, we imply the covariance matrix from the set of all observed European swaption prices.

In solving for the implied covariance matrix, it is important to note that a covariance matrix must be positive definite (or at least positive semidefinite) to be well defined. This means that care must be taken in designing the algorithm by which the covariance matrix is implied from the data to insure than this condition is satisfied. Standard results in linear algebra imply that a matrix is positive definite if, and only if, the eigenvalues of the matrix are all positive.[15]

Motivated by this necessary and sufficient condition, we use the following procedure to specify the implied covariance matrix. First, we estimate the historical correlation matrix of percentage changes in forward rates H from a time series of forward rates taken from a five-year period prior to the beginning of the sample period used in our study.[16] We then decompose the historical correlation matrix into its spectral representation $H = U\Lambda U'$, where U is the matrix of eigenvectors and Λ is a diagonal matrix of eigenvalues. Finally, we make the identifying assumption that the implied covariance matrix is of the form $\Sigma = U\Psi U'$, where Ψ is a diagonal matrix with nonnegative elements. This assumption places an intuitive structure on the space of admissible implied covariance matrices.[17] Specifically, if the eigenvectors are viewed as factors, then this assumption is equivalent to assuming that the factors that generate the historical correlation matrix also generate the implied covariance matrix, but that the implied variances of these factors may differ from their historical values. Viewed this way, the identification assumption is simply the economically intuitive requirement that the market prices swaptions based on the factors that drive term-structure move-

[15] For example, see Noble and Daniel (1977).

[16] We implement this procedure using the historical correlation matrix rather than the covariance matrix to simplify the scaling of implied eigenvalues. We have also implemented this procedure using the historical covariance matrix. Not surprisingly, the eigenvectors from the historical covariance matrix are very similar to those obtained from the historical correlation matrix.

[17] This assumption is equivalent to requiring that the historical correlation matrix H and the implied covariance matrix Σ commute, that is, $H\Sigma = \Sigma H$. We are grateful to Bing Han for this observation.

ments. Extensive numerical tests suggest that virtually any realistic implied correlation matrix can be closely approximated by this representation.[18]

Given this specification, the problem of finding the implied covariance matrix reduces to solving for the implied eigenvalues along the main diagonal of Ψ that best fit the market data. Since there are typically far more swaptions than eigenvalues, we solve for the implied eigenvalues by standard numerical optimization where the objective function is the root mean squared error (RMSE) of the percentage differences between the market price and the model price, taken over all swaptions. Specifically, for a given choice of the elements of the diagonal matrix Ψ, we form the estimated covariance matrix $U\Psi U'$ and then simulate 2,000 paths of the vector of discount bond prices using the string market model dynamics in equation (11). In simulating correlated Brownian motions, we use antithetic variates to reduce simulation noise. The time homogeneity of the model is implemented in the following way. During the first six-month simulation interval, the full 19 by 19 versions of the matrices Σ and J are used to simulate the dynamics of the 19 forward rates. After six months, however, the first forward becomes the spot rate, leaving only 18 forward rates to simulate during the second six-month period. Because of the time homogeneity of the model, the relevant 18 by 18 covariance matrix is given by taking the first 18 rows and columns of Σ; the last row and column is dropped from the covariance matrix Σ. Similarly, the first row and column are dropped from the Jacobian since they involve derivatives with respect to the first forward, which has now become the spot rate. This process is repeated until the last six-month period, when only the final forward rate remains to be simulated.

Using the paths generated, we then value the individual at-the-money-forward European swaptions by simulation and evaluate the RMSE. In simulating the prices of swaptions, we use the following procedure. First, recall that since we simulate the evolution of the full vector of discount bond prices of all maturities ranging up to 10 years, these bond values are available at the expiration date τ of the swaption for each of the simulated paths of the term structure. From these discount bond prices at time τ, we can calculate the value of the underlying swap for each path. Specifically, the value of the swap $V(\tau,\tau,T,c)$ at time τ is given by the expression

$$V(\tau,\tau,T,c) = \frac{c}{2} \sum_{i=1}^{2(T-\tau)} D(\tau,\tau+i/2) + D(\tau,T) - 1, \tag{12}$$

[18] We note that there are alternative ways of specifying the correlation matrix. For example, Rebonato (1999) independently offers a method to construct correlation matrices among forward rates. In our framework, however, Rebonato's approach requires optimizing over a large set of parameters and is computationally infeasible. Additionally, we examined a variety of specifications where the covariance between the ith and jth forwards is of the form $e^{a+bT_i}e^{a+bT_j}e^{c|T_i-T_j|}$, where a, b, and c are calibrated to fit swaption prices based on the RMSE criterion. These types of specifications generally performed poorly relative to the specification used in this paper.

where c is the fixed coupon [illegible]ce of the swap which is equal to the forward swap rate $FSR(0,\tau,T)$ defined by equation (3). Thus, the value of the underlying swap at the expiration date τ of the swaption is easily calculated using the vector of discount bond prices. Once the value of the underlying swap at time τ is determined, the cash flow from the swaption at time τ is simply $\max(0, V(\tau,\tau,T,c))$ for a receivers swaption and $\max(0, -V(\tau,\tau,T,c))$ for a payers swaption. For each path, we then discount the cash flow from the option by multiplying by the compounded money-market factor $\prod_{i=0}^{2\tau-1} D(i, i+1/2)$. Finally, we average the discounted cash flows over all paths. Since at-the-money-forward receivers and payers swaptions have the same value, we use the average of the simulated receivers and payers swaptions as the simulated value of the swaption.

We iterate this entire process over different choices of the eigenvalues until convergence is obtained, using the same seed for the random number generator at each iteration to preserve the differentiability of the objective function with respect to the eigenvalue. Although 19 implied eigenvalues are required for the covariance matrix Σ to be of full rank, implied covariance matrices of lower rank can easily be nested in this specification by solving for the first N eigenvalues and then setting the remaining $19 - N$ equal to zero.[19]

III. The Data

In conducting this study, we use three types of data: Libor and swap data defining the term structure of interest rates, market-implied volatilities for European swaptions, and market-implied volatilities for Libor interest-rate caps. Together with the term-structure data, these implied volatilities define the market prices of swaptions and caps. The source of all data is the Bloomberg system, which collects and aggregates market quotations from a number of brokers and dealers in the OTC swap and fixed-income derivatives market.

The term-structure data consists of weekly observations (Friday closing) for the 6-month and 1-year Libor rates as well as midmarket 2-year, 3-year, 4-year, 5-year, 7-year, and 10-year par swap rates for the period from January 17, 1992, to July 2, 1999. These maturities are the standard maturities

[19] Although the numerical optimization is conceptually straightforward, there are a number of ways in which the search algorithm can be accelerated. For example, a least squares algorithm similar to Longstaff and Schwartz (2001) can be used to approximate forward swap rates as linear functions of the individual forward rates. Given a covariance matrix, this linear approximation then implies closed-form expressions for the variance of individual forward swap rates at the expiration dates of the swaptions, which can then be used to provide a closed-form approximation to the value of the swaption. This closed-form approximation can then be corrected for bias by an iterative process of comparing the simulated values given by the string market model to those implied by this approximation, and then adjusting the approximation. The implied eigenvalues can then be determined by optimizing the closed-form approximation rather than having to resimulate paths of the term structure at each iteration. With this type of algorithm, solving for the implied eigenvalues typically takes less than 10 seconds using a 750 MHz Pentium III processor.

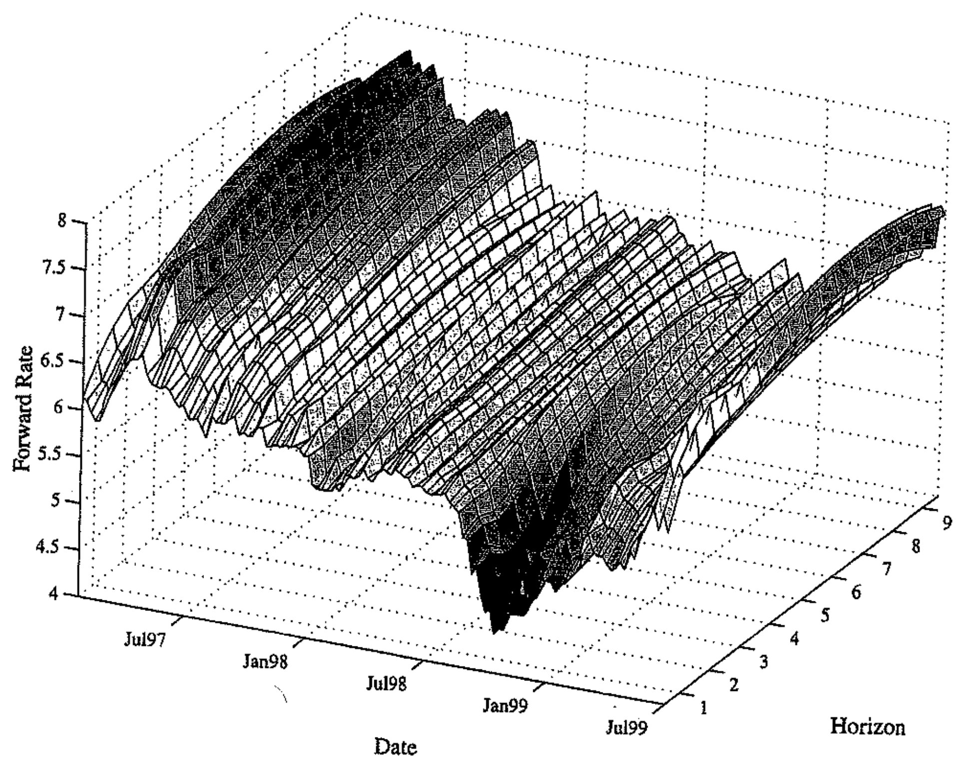

Figure 1. Time series of six-month Libor forward rates. The data set consists of weekly observations for six-month Libor forward rates starting at 0.5 to 9.5 years, for the period from January 24, 1997, to July 2, 1999. The forward rates are computed from the 6-month and 1-year Libor rates as well as the 2-year, 3-year, 4-year, 5-year, 7-year, and 10-year midmarket swap rates using a cubic spline to interpolate the par curve and then bootstrapping the forward curve. All data are obtained from the Bloomberg system. The weekly data for interest rates represent Friday closing rates. The forwards are denoted by the number of years until the beginning of the period covered by the 6-month forward rate. The total number of observations in the sample is 128.

for which swap rates are quoted in the market. From these rates, we solve for the term structure of six-month Libor forward rates out to 10 years in the following way. We first use the 6-month and 1-year Libor rates to solve for the 6-month and 1-year par rates. We then use a standard cubic spline algorithm to interpolate the par curve at semiannual intervals. Finally, we solve for 6-month forward rates by bootstrapping the interpolated par curve.[20] The term structures of Libor forward rates for the in-sample period from January 24, 1997, to July 2, 1999, are graphed in Figure 1. The term-

[20] Following the market convention, we discount cash flows using the swap curve as if it were the riskless term structure. Since the cash flows from both legs of a swap are discounted using this curve, however, this convention has little or no effect on valuation results.

structure data for the 5-year ex ante period from January 17, 1992, to January 17, 1997, is used to estimate the historical correlation matrix H from which the eigenvectors used in solving for the implied covariance matrices are determined. This ex ante correlation matrix is shown in Table I; all of the in-sample results are based on this ex ante correlation matrix. Note that the correlations are generally smooth monotonically decreasing functions of the distance between forward rates. One interesting exception is the correlation between the first and second forwards; the first two forwards display a significant amount of independent variation, hinting at money-market factors not present in longer-term forward rates.

The swaption data consists of weekly midmarket implied volatilities for 34 at-the-money-forward European swaptions for the in-sample period from January 24, 1997, to July 2, 1999. These 34 swaptions represent all of the standard quoted τ by T European swaption structures where the final maturity date of the underlying swap is less than or equal to 10 years, $T \leq 10$. As described earlier, the market convention is to quote swaption prices in terms of their implied volatility relative to the Black (1976) model for at-the-money-forward European swaptions given in equation (7); the market prices of these swaptions are given by substituting the implied volatilities into the Black model. Table II provides summary statistics for the implied volatilities. Figure 2 graphs the implied volatilities over time; Figure 3 shows a number of examples of the shape of the swaption implied volatility surface at different points in time during the sample period.

Observe that there is a significant spike in these implied volatilities during the fall of 1998. This spike coincides with the hedge-fund crisis precipitated by the announcement in early September 1998 of massive trading losses by Long Term Capital Management (LTCM). The sudden threat to the solvency of LTCM, which had been widely viewed as a premier client by many Wall Street firms, created a near panic in the financial markets. In the subsequent weeks, a number of other highly leveraged hedge funds also announced that they had experienced large trading losses on positions similar to those held by LTCM. Examples of these funds included Convergence Capital Management, Ellington Capital Management, D. E. Shaw & Co., and MKP Capital Management. In an effort to stabilize the market, the Federal Reserve Bank of New York persuaded a consortium of 16 investment and commercial banks to inject \$3.6 billion into LTCM in exchange for virtually all of the remaining equity in the fund. The prompt action by the Federal Reserve, announced to the markets on September 24, 1998, allowed LTCM to avoid insolvency and reduced the pressure on the fund to unwind trading positions at illiquid fire-sale prices, which would have exacerbated the problems at other hedge funds to which the consortium members had considerable risk exposure.

The interest-rate cap data consists of weekly midmarket implied volatilities for 2-year, 3-year, 4-year, 5-year, 7-year, and 10-year caps for the same period as for the swaptions data, January 24, 1997, to July 2, 1999. By market convention, the strike price of a T-year cap is simply the T-year

Table I

Correlation Matrix of Log Changes in Six-month Libor Forward Rates

The correlation matrix is based on weekly changes in the logarithm of individual six-month Libor forward rates for the ex ante period from January 17, 1992, to January 17, 1997. The forward rates are computed from the 6-month and 1-year Libor rates as well as the 2-year, 3-year, 4-year, 5-year, 7-year and 10-year midmarket swap rates using a cubic spline to interpolate the par curve and then bootstrapping the forward curve. All data are obtained from the Bloomberg system. The weekly data for interest rates represent Friday closing rates. The horizons of the 6-month forward rates used to compute the correlation matrix range from 0.50 years to 9.50 years forward, giving a total of 19 time series of forward rates. The total number of observations is 262.

	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50
0.50	1.000																		
1.00	0.340	1.000																	
1.50	0.579	0.950	1.000																
2.00	0.547	0.946	0.995	1.000															
2.50	0.544	0.904	0.964	0.983	1.000														
3.00	0.519	0.812	0.883	0.918	0.975	1.000													
3.50	0.476	0.771	0.841	0.884	0.954	0.996	1.000												
4.00	0.440	0.731	0.800	0.849	0.929	0.984	0.996	1.000											
4.50	0.410	0.692	0.762	0.815	0.901	0.967	0.986	0.997	1.000										
5.00	0.388	0.656	0.727	0.781	0.871	0.944	0.969	0.986	0.996	1.000									
5.50	0.373	0.621	0.693	0.747	0.838	0.914	0.943	0.966	0.983	0.995	1.000								
6.00	0.365	0.585	0.659	0.711	0.798	0.873	0.905	0.933	0.957	0.978	0.994	1.000							
6.50	0.360	0.546	0.621	0.668	0.747	0.816	0.850	0.882	0.912	0.942	0.970	0.991	1.000						
7.00	0.358	0.501	0.576	0.615	0.681	0.739	0.773	0.807	0.844	0.882	0.922	0.960	0.988	1.000					
7.50	0.355	0.448	0.522	0.551	0.600	0.643	0.674	0.709	0.749	0.796	0.847	0.901	0.950	0.986	1.000				
8.00	0.349	0.391	0.462	0.479	0.507	0.531	0.557	0.591	0.634	0.686	0.748	0.816	0.884	0.944	0.986	1.000			
8.50	0.340	0.333	0.400	0.404	0.411	0.414	0.434	0.465	0.508	0.564	0.633	0.712	0.797	0.879	0.945	0.987	1.000		
9.00	0.329	0.279	0.341	0.333	0.320	0.302	0.315	0.343	0.385	0.442	0.515	0.603	0.700	0.799	0.886	0.952	0.989	1.000	
9.50	0.317	0.233	0.290	0.271	0.239	0.202	0.209	0.232	0.271	0.329	0.404	0.497	0.603	0.714	0.819	0.903	0.961	0.991	1.000

Table II

Summary Statistics for OTC Market At-the-Money-Forward European Swaption Volatilities

The data set consists of 128 weekly observations from January 24, 1997, to July 2, 1999, of midmarket implied Black-model volatilities for the indicated N into M at-the-money-forward European swaption structure, where N denotes years until option expiration and M denotes the length of the underlying swap in years.

N	M	Mean	Standard Deviation	Minimum	Median	Maximum	Serial Correlation
0.50	1.00	14.60	3.37	9.70	13.50	27.00	0.926
1.00	1.00	16.10	2.98	11.80	15.35	11.80	0.945
2.00	1.00	16.66	2.27	13.10	16.20	13.10	0.940
3.00	1.00	16.42	1.99	13.00	16.10	13.00	0.932
4.00	1.00	16.08	1.72	13.00	15.95	13.00	0.941
5.00	1.00	15.73	1.48	12.90	15.70	12.90	0.933
0.50	2.00	15.35	3.10	10.40	14.45	10.40	0.919
1.00	2.00	16.08	2.59	12.20	15.50	12.20	0.943
2.00	2.00	16.31	2.05	13.00	16.00	13.00	0.936
3.00	2.00	16.02	1.74	12.90	15.85	12.90	0.939
4.00	2.00	15.70	1.51	12.80	15.70	12.80	0.931
5.00	2.00	15.38	1.34	12.70	15.50	12.70	0.925
0.50	3.00	15.34	2.93	10.60	14.65	10.60	0.913
1.00	3.00	15.89	2.35	12.20	15.30	12.20	0.937
2.00	3.00	16.00	1.84	12.90	15.70	12.90	0.933
3.00	3.00	15.72	1.57	12.80	15.65	12.80	0.927
4.00	3.00	15.42	1.37	12.70	15.50	12.70	0.929
5.00	3.00	15.09	1.23	12.60	15.20	12.60	0.920
0.50	4.00	15.32	2.73	10.80	14.70	10.80	0.906
1.00	4.00	15.67	2.14	12.10	15.30	12.10	0.930
2.00	4.00	15.72	1.64	12.80	15.65	12.80	0.933
3.00	4.00	15.45	1.42	12.70	15.50	12.70	0.930
4.00	4.00	15.14	1.27	12.60	15.20	12.60	0.918
5.00	4.00	14.80	1.14	12.50	15.00	12.50	0.917
0.50	5.00	15.30	2.57	11.00	14.70	11.00	0.897
1.00	5.00	15.42	1.93	12.00	15.20	12.00	0.926
2.00	5.00	15.45	1.49	12.70	15.40	12.70	0.924
3.00	5.00	15.18	1.32	12.60	15.25	12.60	0.932
4.00	5.00	14.85	1.18	12.50	15.00	12.50	0.919
5.00	5.00	14.48	1.04	12.50	14.70	12.50	0.909
0.50	7.00	15.09	2.41	11.00	14.60	11.00	0.887
1.00	7.00	15.10	1.71	12.00	15.00	12.00	0.914
2.00	7.00	15.03	1.38	12.40	15.05	12.40	0.921
3.00	7.00	14.77	1.24	12.30	14.90	12.30	0.913

swap rate. To parallel the features of swaptions and to simplify the analysis, we assume that caps are on the 6-month Libor rate rather than the 3-month rate.[21] The market prices of caps are then given by substituting the implied volatility into the Black model (1976) given in equation (1), where $T - \tau = 1/2$. Table III presents summary statistics for the market cap volatilities

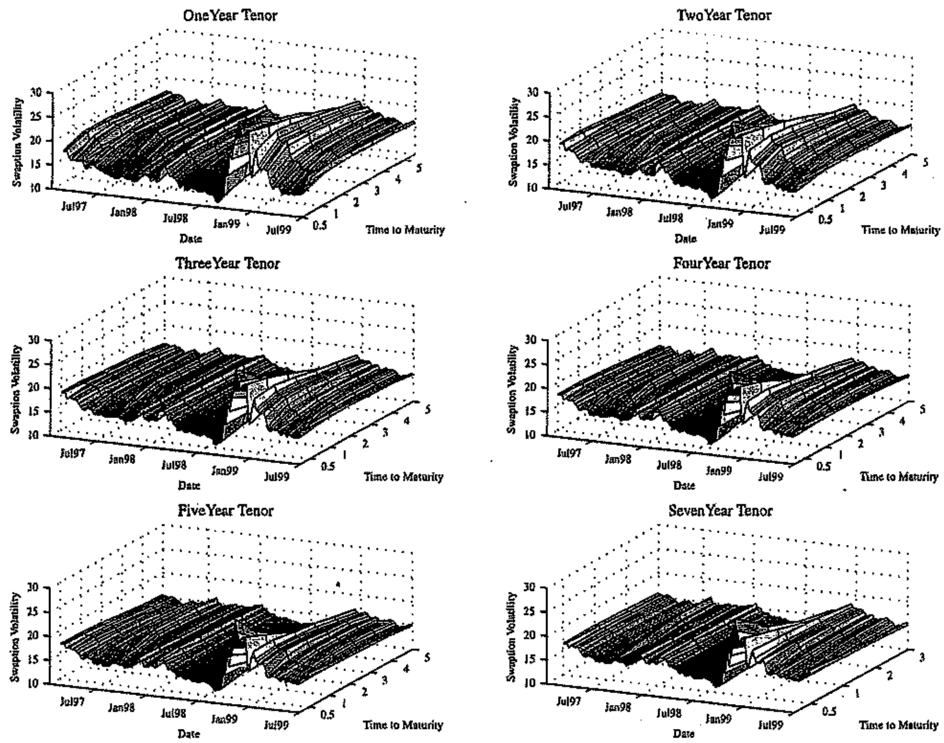

Figure 2. Time series of swaption volatilities. The data set consists of 128 weekly observations from January 24, 1997, to July 2, 1999, of midmarket implied Black-model volatilities for the indicated N into M at-the-money-forward European swaption structure, where N denotes years until option expiration (time to maturity) and M denotes the length of the underlying swap in years (the tenor). The subplots show, for each tenor, the implied volatilities of options with different times to maturity.

during the sample period. The implied volatilities display a time-series pattern similar to those observed for swaptions. Figure 4 also graphs the time series of cap volatilities.

IV. The Empirical Results

In this section, we report the empirical results from the study. First, we examine how many implied factors are required to explain the market prices of swaptions. We then study the relative valuation of swaptions in the string

[21] This assumption is relatively innocuous. We have spoken with several caps dealers who indicated that the implied volatilities for caps on six-month Libor would typically be equal to or an eighth to a quarter below the implied volatility for a cap on three-month Libor. Diagnostic tests presented later in the paper indicate that this assumption has virtually no effect on the empirical results.

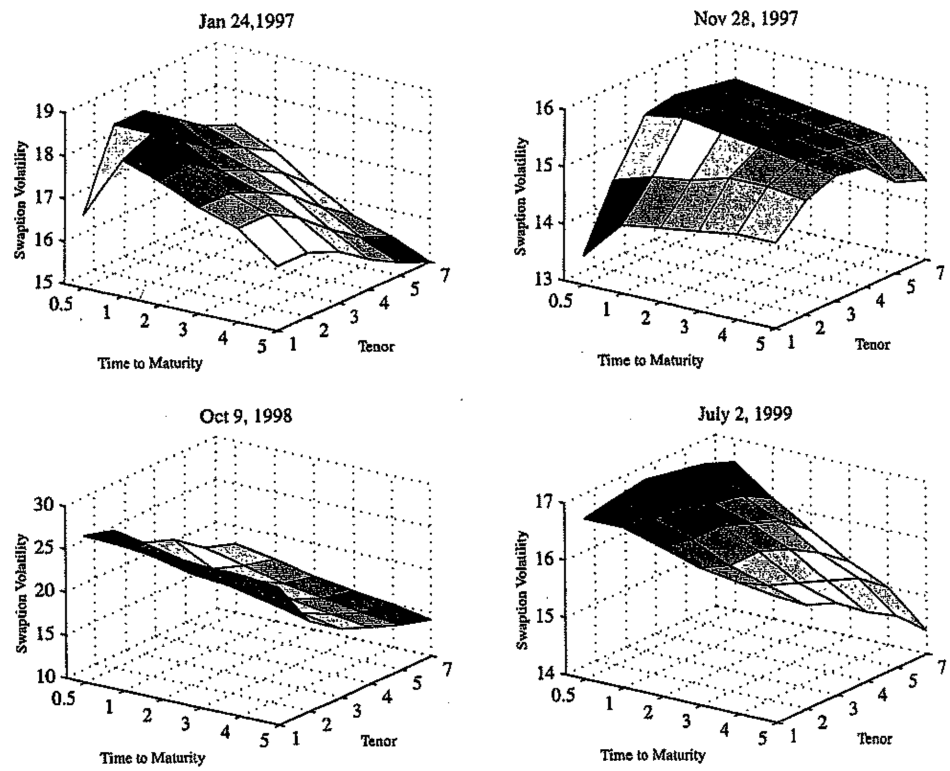

Figure 3. Examples of swaption volatility surfaces. This figure plots the quoted volatilities of swaptions on four different dates of the sample. Each figure shows quotes for swaptions with maturities between six months and five years on underlying swaps with horizons at the maturity of the options between one and seven years. Note that we do not use the four-into-seven and the five-into-seven swaptions in the empirical study since they are less liquid.

market model. Finally, we examine the relative valuation of both caps and swaptions in the string market model.

A. How Many Implied Factors?

Many researchers have studied the question of how many factors or principal components are needed to capture the historical variation in the term structure. For example, recent papers by Litterman and Scheinkman (1991) and others find that most of the variation in term-structure movements is explained by two or three factors. One important recent exception is Knez et al. (1994), who find evidence of a significant fourth factor affecting short-term interest rates.

An important advantage of our approach is that it offers a completely different perspective on this issue. Rather than focusing on the number of factors in historical term-structure data, we infer from swaption prices the

Table III

Summary Statistics for OTC Market Libor Interest-rate Cap Volatilities

The data set consists of 128 weekly observations from January 24, 1997, to July 2, 1999, of midmarket implied Black-model volatilities for the indicated cap maturities. All data are obtained from Bloomberg.

Cap Maturity	Mean	Standard Deviation	Minimum	Median	Maximum	Serial Correlation
2 Year	15.34	3.19	10.60	14.25	28.00	0.945
3 Year	16.43	2.75	12.10	15.60	25.20	0.940
4 Year	16.75	2.41	12.50	16.10	23.50	0.932
5 Year	16.84	2.21	12.90	16.30	22.75	0.941
7 Year	16.46	1.86	12.75	16.15	20.87	0.933
10 Year	15.97	1.55	12.60	15.90	19.75	0.919

actual number of factors that market participants view as important influences on the term structure. Since the implied factor structure is forward looking, the number of implied factors need not be the same as those obtained historically. Intuitively, this approach is analogous to the familiar technique of solving for the implied volatility in option prices; implied volatilities typically do not equal estimates of volatility based on historical data, and often provide more accurate forecasts of future volatility.[22]

We estimate the implied number of factors using an incremental likelihood ratio test based on all 128 weekly observations for each of the 34 European swaptions in the data set. Recall that when all but the first N eigenvalues in the diagonal matrix Ψ are equal to zero, the implied covariance matrix is of rank N, or equivalently, the implied covariance matrix is generated by N factors. For a given value of N, and for the ith week $i = 1,2,\ldots,128$, we use the procedure described in Section III.B to solve for the N-implied eigenvalues that minimize the sum of squared percentage swaption pricing errors, where the percentage errors are defined as the differences between the simulated and market values of each swaption, expressed as a percentage of the market value of the swaption. Note that these pricing errors arise because we are trying to fit 34 swaption prices with only $N < 34$ parameters. Thus, these errors have an interpretation very similar to that of the residuals from a nonlinear least squares regression. We repeat the process of solving for the N eigenvalues that minimize the sum of squared per-

[22] We note that other researchers have also used the approach of backing out factors from asset prices such as bonds. Important recent examples of this approach include Longstaff and Schwartz (1992), Chen and Scott (1993), Pearson and Sun (1994), Duffie and Singleton (1997), de Jong and Santa-Clara (1999), Dai and Singleton (2000), Duffee (2000), and many others. Our approach differs in that we use the information in swaption prices to address the question of the number of factors. Intuitively, it is clear that since swaptions have nonlinear payoffs, their prices may contain more information about market estimates of the conditional volatility of factors than can be recovered from bond prices alone.

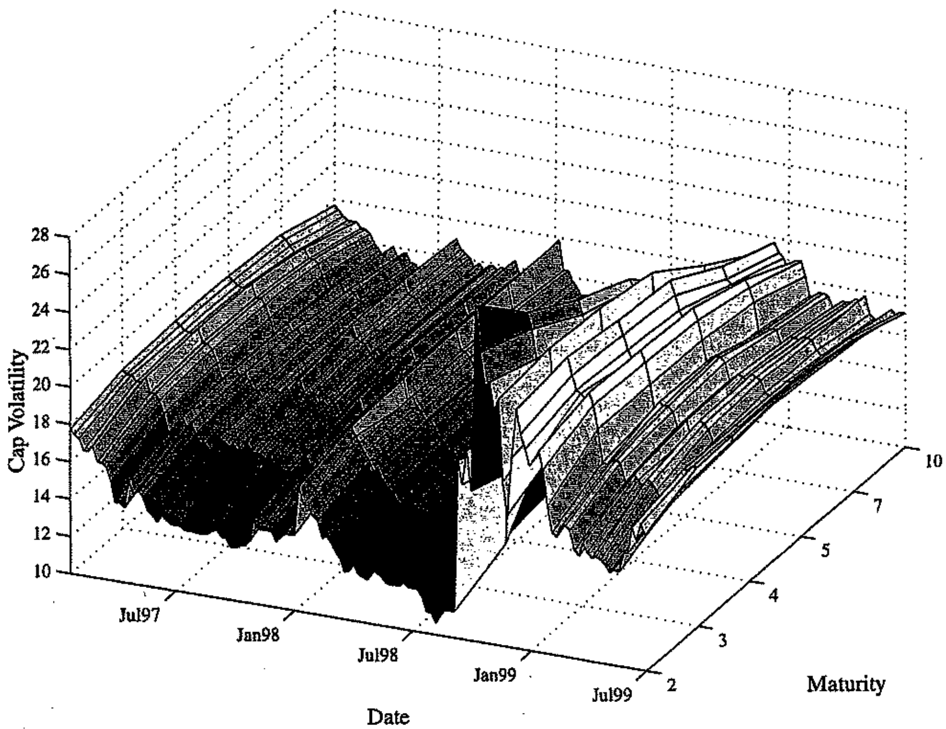

Figure 4. Time series of cap volatilities. The data set consists of 128 weekly observations from January 24, 1997, to July 2, 1999, of midmarket implied Black-model volatilities for the indicated cap maturities.

centage pricing errors for each of the 128 weeks in the sample period. Adding the sum of squared errors over all 128 weeks gives the total sum of squared errors. We then repeat this entire procedure for the case of $N + 1$ eigenvalues, where the same seed for the random number generator is used for all values of N to insure comparability in the results. Under the null hypothesis of equality, $128 \times 34 = 4{,}352$ times the difference between the logarithms of the sum of squared errors for N and $N + 1$ factors is asymptotically distributed as a chi-square variate with 128 degrees of freedom.

Table IV reports the results from the incremental pairwise comparisons as N ranges from one to seven. As shown, the pairwise comparisons are statistically significant for two versus one, three versus two, and four versus three factors, and are insignificant for all of the other comparisons. These results imply that there are four significant factors underlying the covariance matrix of forwards used by the market in the pricing of European swaptions. These results contrast with the earlier empirical work mentioned above, which finds only two to three factors in historical term-structure movements. It is important to mention, however, that most of these earlier studies focus on Treasury bonds whereas our results apply to the swap curve. Thus, it is

Table

Likelihood Ratio Tests for the Number of Implied Factors in European Swaption Prices

This table reports the likelihood ratio test statistics from pairwise incremental comparisons of the number of factors. In each case, we solve for the N-implied eigenvalues that minimize the sum of squared errors for the swaption prices and then compare with the sum of squared errors obtained by solving for the $N + 1$ implied eigenvalues that best fit the data. The difference between the sum of squared errors is asymptotically χ^2_{128} under the null hypothesis of equality for the full sample, and χ^2_{64} for the two half samples. For a given vector of eigenvalues, the sum of squared errors is given by first forming the implied covariance matrix from the diagonal matrix of eigenvalues and the historical eigenvectors, simulating 2,000 paths of evolution of the term structure using the string model, and then solving for the individual swaption values by simulation. In generating simulated paths, the same seed for the random number generator is used to insure comparability across the number of factors. The data set consists of 128 weekly observations of 34 swaption values for the period from January 24, 1997, to July 2, 1999, giving a total of 4,352 observations. The critical value of χ^2_{128} is 168.1332 at the 99 percent level. The critical value of χ^2_{64} is 93.2169 at the 99 percent level.

N Factors	$N + 1$ Factors	Test Statistic	p-Value
	A. Full Sample Period		
1	2	3352.183	0.000
2	3	2220.194	0.000
3	4	284.847	0.000
4	5	86.211	0.998
5	6	13.204	1.000
6	7	21.991	1.000
7	8	3.959	1.000
	B. First Half of the Sample Period		
1	2	4481.614	0.000
2	3	1699.481	0.000
3	4	76.549	0.135
4	5	137.540	0.000
5	6	24.084	1.000
6	7	24.516	1.000
7	8	7.324	1.000
	C. Second Half of the Sample Period		
1	2	2377.247	0.000
2	3	922.156	0.000
3	4	162.201	0.000
4	5	17.689	1.000
5	6	2.048	1.000
6	7	7.505	1.000
7	8	0.607	1.000

possible that the existence of a credit factor influencing swap rates but not Treasury rates could reconcile our results with those obtained by earlier researchers. Because of these results, all of our subsequent analysis is based on implied covariance matrices generated by four eigenvalues, resulting in four-factor or rank-four implied covariance matrices.

As a robustness check, we also conduct the incremental likelihood ratio tests using only the first half of the sample period (64 weeks) and also using only the second half of the sample period (64 weeks). Since the hedge-fund crisis of fall 1998 occurred entirely during the second half of the sample period, this diagnostic addresses whether the results about the number of factors are specific to this volatile period. As shown, however, the subperiod results are similar to those for the entire period. In both the first and second subperiods, the likelihood ratio tests find evidence of four statistically significant factors. Thus, the results about the number of factors are not artifacts of the hedge-fund crisis of fall 1998.[23]

To provide some insight into the four implied factors that market participants view as driving the term structure, Figure 5 graphs the first four eigenvectors, which define the weights of the first four factors, from the historical correlation matrix in Table I. As illustrated, these factors closely resemble those found in earlier papers. The first factor essentially generates parallel shifts in the term structure. The second factor generates shifts in the slope of the term structure. The third factor is a curvature factor that generates movements in the term structure where short-term and long-term rates move in opposite directions from the midterm rates. Finally, the fourth factor primarily affects the shape of the very short end of the term structure, possibly reflecting the influence of the Federal Reserve or other monetary authorities. Thus, this fourth factor has an interpretation very similar to the fourth factor found by Knez et al. (1994) in their study of short-term rates.

Since the eigenvectors used in solving for the implied covariance matrix have the interpretation of term-structure factors, the fitted eigenvalues can be viewed as the implied variances of the factors. To illustrate this, Figure 6 graphs the time series of fitted values for each of the four eigenvalues used to define the implied covariance matrix. The first eigenvalue shows the relative volatility over time of the parallel shift factor. The volatility of this factor was very stable during much of 1997, decreased somewhat during the early part of 1998, and then increased significantly during the fall of 1998 when the financial stability of a number of highly visible hedge funds was threatened by severe trading losses. The volatility of the term-structure slope factor decreased significantly during 1997, and was quite low during most of

[23] It is interesting to note that the four significant factors during the first half of the sample are the first, second, third, and fifth, while the four significant factors during the entire sample period and during the second half of the sample period are the first, second, third, and fourth. Thus, one could argue that as many as five factors could occasionally be needed to describe swaption prices. We take the more parsimonious view that there are only four significant factors based on the results for the full sample period.

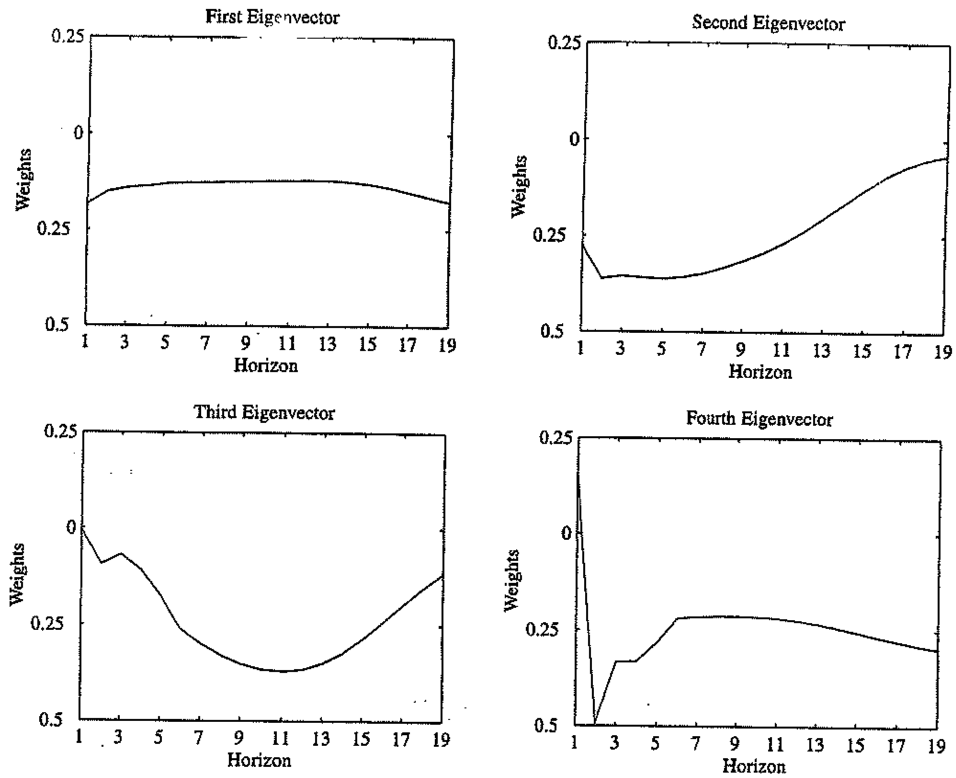

Figure 5. Eigenvector weights. The four subplots show the weights of the first four eigenvectors of the historical correlation matrix. The correlation matrix is based on weekly changes in the logarithm of individual six-month Libor forward rates for the ex ante period from January 17, 1992, to January 17, 1997. The forward rates are computed from the six-month and one-year Libor rates as well as the two-year, three-year, four-year, five-year, seven-year and 10-year midmarket swap rates using a cubic spline to interpolate the par curve and then bootstrapping the forward curve. The weekly data for interest rates represents Friday closing rates. The horizons of the six-month forward rates used to compute the correlation matrix range from 0.50 years to 9.50 years forward, giving a total of 19 time series of forward rates. The total number of observations is 262.

1998. In the fall of 1998, however, the volatility of this factor suddenly increased by a factor of nearly 10, but then quickly returned to levels near those at the beginning of the sample period. The volatility of the curvature factor shows a pattern similar to that of the slope factor; the volatility decreases significantly during 1997, is generally low during most of 1998, and then spikes dramatically during the fall of 1998. The behavior of the volatility of the short-term or fourth factor suggests one possible way of reconciling these results with the historical evidence on the number of factors. The implied volatility of this fourth factor is often quite small and can actually be zero. During periods of market stress such as the fall of 1998,

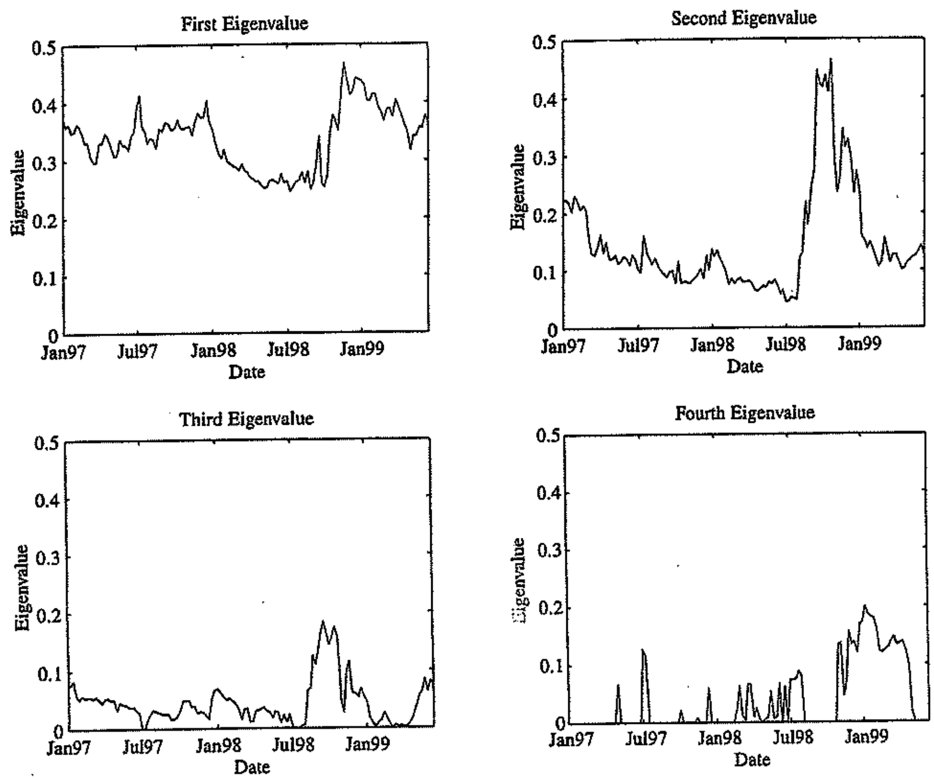

Figure 6. Time series of eigenvalues. The four subplots show the eigenvalues computed from the 128 weekly implied correlation matrices from January 24, 1997, to July 2, 1999, obtained by fitting the model to the swaption data, keeping fixed the eigenvectors of the historical correlation matrix.

however, the volatility of this factor can suddenly increase and become a major source of term-structure movements. Thus, the time-series pattern of the volatility of the fourth factor suggests that this may be more of an event-related factor that only becomes important in periods of extreme market stress. Since historical analysis of the number of factors is typically based on unconditional tests, factors that have time-varying volatilities that are usually small or zero may not show up in these types of standard tests. Despite this, these factors could represent a serious source of conditional volatility risk to market participants who would appropriately incorporate their effects into the market prices of swaptions. Recent papers by Hull and White (1999) and Jagannathan and Sun (1999) independently confirm that three factors are not sufficient to fully capture the pricing of interest rate caps and swaptions. Peterson, Stapleton, and Subrahmanyam (2000) find that going from one to two term-structure factors has a significant effect on the valuation of swaptions.

B. The Implied Correlation Matrix

As discussed, the implied eigenvalues uniquely determine the implied covariance matrix. In this sense, our approach is simply the matrix version of the familiar technique of inverting option prices to solve for the implied volatility of the underlying asset. One natural question that arises is how closely the implied correlation matrix matches the historical correlation matrix. To compare the two, we do the following. Based on the results of the likelihood ratio tests in the previous section, we set $N = 4$ and use the corresponding four implied eigenvalues for each week to define a diagonal matrix Ψ for each week. This diagonal matrix Ψ has the four implied eigenvalues as the first four elements along the diagonal, and zeros as the remaining diagonal elements. From Ψ and the historical matrix of eigenvectors U, the implied covariance matrix for that week is defined by $\Sigma = U\Psi U'$. Standardizing the covariance matrix gives the implied correlation matrix for that week. We repeat this process for all 128 weeks in the sample, resulting in a series of 128 implied correlation matrices.

To obtain summary measures of implied correlations, we then compute the matrix of average implied correlations by simply taking the time series average of each element in the implied correlation matrix over all 128 weeks. We then take the difference between the matrix of average implied correlations and the historical correlation matrix in Table I and report these differences in Table V. To provide some sense of the time-series variation in these differences, Table VI reports the matrix of standard deviations of the implied correlations.

As shown in Table V, there are clearly systematic differences between the historical and implied correlations. The differences along the main diagonal are all zero, of course, since the main diagonals of both the implied and historical correlation matrices consist of ones. As we move away from the main diagonal, however, the differences are almost all negative, which means that the implied correlations tend to be lower than the historical correlations. Most of the differences are on the order of 0.05 to 0.10, but a few are as large as 0.20. The largest differences are typically for the correlation of two-to-three-year forwards with seven-to-nine-year forwards. The only notable positive difference is for the correlation between the first and second forwards.

Table VI shows that there is a fair amount of time-series variation in the implied correlations, indicating that the implied correlation matrix is not constant over time. In general, however, the standard deviations do not appear to be excessively variable; most of the standard deviations range from 0.05 to 0.20. The largest standard deviation is for the correlation between the first and second forwards. Intuitively, however, it is this correlation that is likely to be the hardest to estimate since it only affects one of the swaptions; all of the other correlations affect multiple swaption values.

Table V

Differences Between the Average Implied Correlations and the Historical Correlations of Six-month Libor Forward Rates

These differences are calculated by averaging the 128 weekly implied correlation matrices from January 24, 1997, to July 2, 1999, obtained by fitting the model to the swaption data, and then subtracting the historical correlations shown in Table I.

	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50
0.50	0.000																		
1.00	0.054	0.000																	
1.50	0.002	0.007	0.000																
2.00	−0.000	0.008	0.000	0.000															
2.50	−0.012	−0.011	−0.006	−0.004	0.000														
3.00	−0.030	−0.047	−0.027	−0.021	−0.007	0.000													
3.50	−0.044	−0.058	−0.038	−0.032	−0.015	−0.002	0.000												
4.00	−0.057	−0.070	−0.049	−0.043	−0.023	−0.005	−0.001	0.000											
4.50	−0.068	−0.082	−0.061	−0.055	−0.033	−0.012	−0.006	−0.002	0.000										
5.00	−0.076	−0.095	−0.074	−0.068	−0.046	−0.022	−0.014	−0.007	−0.002	0.000									
5.50	−0.083	−0.112	−0.091	−0.085	−0.064	−0.039	−0.028	−0.018	−0.009	−0.003	0.000								
6.00	−0.088	−0.130	−0.111	−0.107	−0.088	−0.064	−0.051	−0.039	−0.026	−0.014	−0.005	0.000							
6.50	−0.090	−0.150	−0.133	−0.132	−0.118	−0.098	−0.085	−0.070	−0.053	−0.037	−0.020	−0.006	0.000						
7.00	−0.090	−0.168	−0.154	−0.156	−0.150	−0.136	−0.124	−0.109	−0.090	−0.069	−0.046	−0.023	−0.006	0.000					
7.50	−0.085	−0.177	−0.167	−0.172	−0.175	−0.170	−0.160	−0.145	−0.125	−0.103	−0.075	−0.047	−0.021	−0.004	0.000				
8.00	−0.080	−0.178	−0.174	−0.181	−0.189	−0.192	−0.183	−0.170	−0.152	−0.129	−0.101	−0.068	−0.036	−0.013	−0.003	0.000			
8.50	−0.073	−0.172	−0.172	−0.179	−0.193	−0.200	−0.193	−0.181	−0.164	−0.142	−0.114	−0.080	−0.046	−0.020	−0.005	−0.001	0.000		
9.00	−0.067	−0.159	−0.164	−0.171	−0.187	−0.196	−0.191	−0.181	−0.166	−0.146	−0.117	−0.084	−0.049	−0.020	−0.003	0.001	0.001	0.000	
9.50	−0.057	−0.142	−0.150	−0.156	−0.172	−0.183	−0.179	−0.170	−0.155	−0.137	−0.110	−0.078	−0.043	−0.013	0.003	0.008	0.005	0.002	0.000

Table VI

Standard Deviations of the Implied Correlations of Six-month Libor Forward Rates

This table reports the standard deviations of the individual elements of the 128 weekly implied correlation matrices from January 24, 1997, to July 2, 1999, obtained by fitting the model to the swaption data, and then subtracting the historical correlations shown in Table I.

	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50
0.50	0.000																		
1.00	0.602	0.000																	
1.50	0.404	0.054	0.000																
2.00	0.380	0.056	0.002	0.000															
2.50	0.251	0.097	0.014	0.007	0.000														
3.00	0.116	0.155	0.047	0.037	0.013	0.000													
3.50	0.120	0.146	0.072	0.054	0.026	0.004	0.000												
4.00	0.139	0.140	0.099	0.076	0.045	0.013	0.003	0.000											
4.50	0.159	0.139	0.124	0.099	0.065	0.025	0.010	0.002	0.000										
5.00	0.174	0.143	0.146	0.121	0.085	0.041	0.022	0.010	0.003	0.000									
5.50	0.183	0.151	0.166	0.142	0.107	0.061	0.039	0.024	0.012	0.004	0.000								
6.00	0.181	0.161	0.183	0.162	0.132	0.088	0.066	0.048	0.031	0.017	0.005	0.000							
6.50	0.169	0.174	0.196	0.181	0.160	0.123	0.103	0.083	0.064	0.044	0.024	0.007	0.000						
7.00	0.151	0.182	0.201	0.195	0.185	0.161	0.144	0.125	0.104	0.081	0.055	0.028	0.007	0.000					
7.50	0.139	0.183	0.197	0.199	0.200	0.189	0.177	0.161	0.142	0.118	0.089	0.056	0.026	0.006	0.000				
8.00	0.144	0.176	0.185	0.193	0.204	0.203	0.196	0.184	0.168	0.146	0.117	0.082	0.047	0.019	0.004	0.000			
8.50	0.163	0.165	0.169	0.180	0.196	0.203	0.200	0.193	0.180	0.161	0.134	0.101	0.064	0.032	0.012	0.003	0.000		
9.00	0.188	0.152	0.151	0.163	0.181	0.194	0.194	0.190	0.181	0.166	0.142	0.111	0.075	0.043	0.021	0.008	0.002	0.000	
9.50	0.214	0.140	0.135	0.146	0.165	0.180	0.184	0.183	0.177	0.165	0.145	0.117	0.083	0.053	0.030	0.016	0.006	0.001	0.000

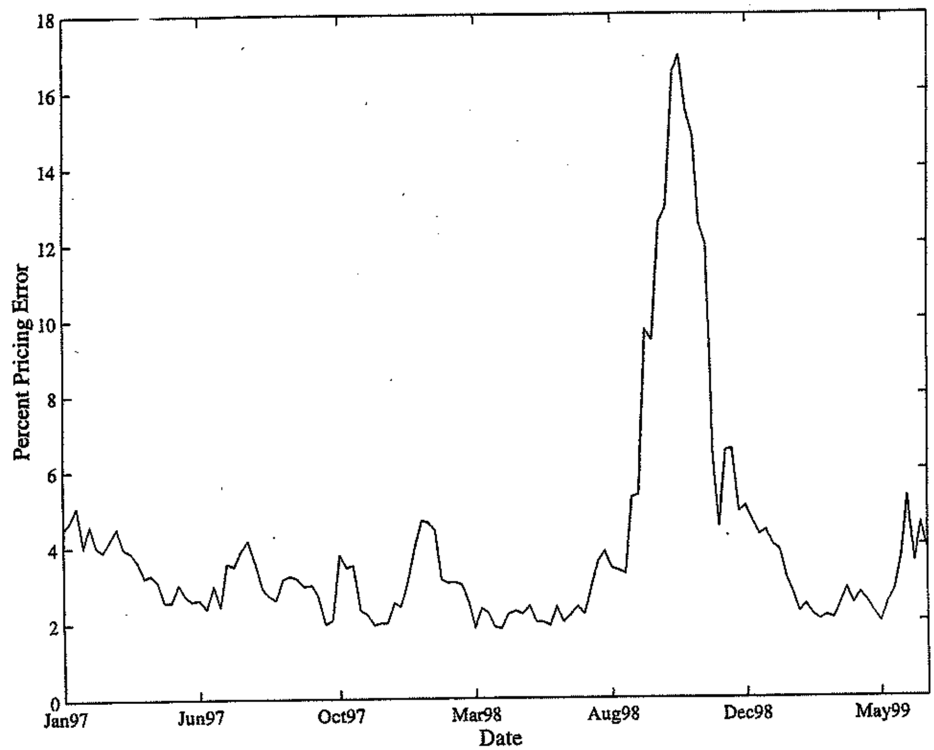

Figure 7. Time series of swaptions RMSE of four-factor string market model. The graph shows the RMSE of the differences between the fitted model swaptions prices and the market prices expressed as a percentage of the market prices. The data set consists of 128 weekly observations from January 24, 1997, to July 2, 1999, for 34 at-the-money-forward European swaption structures.

C. The Relative Valuation of Swaptions

The structure of the string market model imposes a number of constraints on the dynamic evolution of the term structure. Because of this, it is important to examine how well the model is able to describe the underlying structure of market swaption prices. Recall that the string market model is attempting to explain the cross section of 34 swaption prices using only four parameters each period. Thus, the model places a number of overidentifying restrictions of swaption prices and the pricing errors from fitting the string market model provide insights into how well these overidentifying restrictions are satisfied by the data.

To address this issue, Figure 7 graphs the RMSEs for the 34 swaptions in the sample for each of the 128 weeks in the sample period. Recall that these RMSEs are computed by first estimating the four eigenvalues that best fit the market swaption prices for that week, pricing the swaptions by simulating paths of the string market model, and then taking the percentage differences between the market and model prices. As illustrated, the RMSEs from this time-homogeneous string market model are generally very small; the model typically captures the shape of the swaption volatility surface quite closely. Leaving out the exceptional period in the fall of 1998, the RMSEs are generally between two to three percent. These RMSEs are roughly about one-third to one-half of the size of the bid-offer spread.[24] The median RMSE is 3.10 percent and the standard deviation of the RMSEs is 2.98 percent.

Although the string market model fits the swaptions market well during most of the sample period, the fall of 1998 is clearly a major outlier. During this period, the RMSE spikes up to as high as 16 percent. The period during which the RMSE exceeds five percent begins with the week of September 11, 1998. Interestingly, this is just a few days after the well-publicized letter from John Meriwether to the investors of LTCM informing them that the fund had lost 52 percent of its capital through the end of August due to major trading losses in a number of markets. The RMSEs remain consistently above five percent for the 10-week period from September 11, 1998, to November 13, 1998, which closely aligns with the period during which most of the uncertainty about the survival of many of the hedge funds involved in the crisis was being resolved.

The failure of the string market model to capture the shape of the swaptions volatility surface during this period raises two possibilities: either the assumption of time-homogeneity is too restrictive, or quoted prices in the swaptions market were inconsistent with the absence of arbitrage. Although we cannot completely resolve this classical "joint-hypothesis" problem, we have conducted extensive interviews with many swaptions traders who experienced this period. These traders generally made two points. First, because of the turbulence in the market, the liquidity in the swaptions market was less than typical, and the quality of the market quotations collected by Bloomberg could be questioned. Second, there was an almost uniform belief among traders that there were in fact arbitrage opportunities in the markets. Many traders during this period felt that the fear of a complete market meltdown prevented them from executing trades that otherwise would have been viewed as highly profitable during ordinary circumstances.[25]

Going beyond the overall RMSEs, it is also useful to examine the valuation errors for individual swaptions. Although the overall RMSEs are generally small and the fitting procedure requires pricing errors to have a mean close to zero, individual swaptions could still potentially display systematic

[24] Bloomberg reports that the typical bid-offer spread for these swaptions is about one unit of Black-model implied volatility; for a typical implied volatility of 16 percent, a 1 percent volatility bid-offer spread represents about 6 percent of the value of an at-the-money-forward swaption.

[25] Liu and Longstaff (2000) demonstrate that rational investors facing realistic margin constraints may actually choose to underinvest in arbitrages, or avoid investing in an arbitrage altogether, because of the risk that the arbitrage opportunity may widen further before it ultimately converges.

Table VII

Summary Statistics for Percentage Swaption Valuation Errors

The summary statistics reported are for the differences between the fitted model price and the market price expressed as a percentage of the market price. The data set consists of 128 weekly observations from January 24, 1997, to July 2, 1999, for the indicated N into M at-the-money-forward European swaption structures, where N denotes years until option expiration and M denotes the tenor or length of the underlying swap in years. The t-statistic for the mean and the standard deviation reported are adjusted for first-order serial correlation. The overall statistics are simple averages of the corresponding columns with the exception of the overall minimum, median, and maximum, which are computed from all observations.

N	M	Mean	t-Stat. Mean	Standard Deviation	Minimum	Median	Maximum	Serial Correlation
0.50	1.00	−2.456	−1.26	1.954	−19.623	−1.032	6.406	0.906
1.00	1.00	−1.123	−1.45	0.775	−12.922	−1.560	5.423	0.727
2.00	1.00	−2.124	−3.57	0.595	−8.583	−2.006	4.383	0.776
3.00	1.00	−0.924	−1.30	0.712	−5.146	−1.127	6.726	0.845
4.00	1.00	0.501	0.38	1.304	−4.030	−0.177	11.406	0.917
5.00	1.00	4.815	1.95	2.475	−1.209	3.470	24.683	0.935
0.50	2.00	−1.312	−1.06	1.238	−16.379	−1.033	5.057	0.853
1.00	2.00	−0.538	−1.43	0.375	−9.588	−0.273	3.654	0.575
2.00	2.00	−1.542	−3.24	0.476	−5.757	−1.577	2.163	0.809
3.00	2.00	0.289	0.42	0.685	−3.487	−0.131	9.523	0.855
4.00	2.00	1.906	1.35	1.411	−3.041	1.005	14.468	0.916
5.00	2.00	5.612	2.51	2.239	−0.995	4.289	24.027	0.924
0.50	3.00	−0.030	−0.01	2.332	−22.139	1.786	8.077	0.907
1.00	3.00	−1.696	−2.01	0.843	−13.825	−1.294	2.517	0.871
2.00	3.00	−1.920	−7.08	0.271	−4.701	−1.907	0.794	0.745
3.00	3.00	0.328	0.71	0.461	−6.406	0.112	6.868	0.793
4.00	3.00	2.033	1.69	1.201	−2.552	1.323	14.843	0.895
5.00	3.00	5.446	2.51	2.168	−0.193	3.976	22.971	0.934
0.50	4.00	−0.547	−0.19	2.815	−27.444	1.191	9.925	0.899
1.00	4.00	−2.523	−1.68	1.503	−16.314	−2.057	3.209	0.923
2.00	4.00	−2.209	−8.81	0.251	−5.382	−2.141	−0.034	0.763
3.00	4.00	0.151	0.40	0.373	−2.371	0.014	5.346	0.804
4.00	4.00	1.775	1.56	1.137	−2.494	1.124	12.965	0.912
5.00	4.00	5.409	2.64	2.045	−0.317	3.839	19.778	0.939
0.50	5.00	−1.529	−0.50	3.079	−32.517	0.387	9.693	0.897
1.00	5.00	−3.013	−2.62	1.149	−18.546	−2.631	6.900	0.814
2.00	5.00	−2.402	−6.42	0.374	−7.244	−2.148	−0.087	0.817
3.00	5.00	−0.446	−1.70	0.263	−3.433	−0.396	3.546	0.743
4.00	5.00	1.239	1.21	1.026	−1.963	0.383	10.447	0.915
5.00	5.00	5.615	2.79	2.013	−0.444	4.609	18.406	0.944
0.50	7.00	−3.604	−0.91	3.941	−40.981	−1.438	9.483	0.909
1.00	7.00	−5.365	−1.93	2.779	−26.164	−3.978	4.643	0.937
2.00	7.00	−5.096	−4.36	1.170	−16.876	−4.053	−2.015	0.909
3.00	7.00	−3.382	−15.35	0.220	−7.276	−3.154	−1.265	0.669
Overall		−0.255	−1.38	1.343	−40.981	−0.416	24.683	0.852

patterns of mispricing. To investigate this possibility, Table VII reports summary statistics for the pricing errors of individual swaption structures.

As shown, there are some clear patterns in the valuation errors. First, many of the valuation errors are highly serially correlated, implying that deviations between the model and market prices are persistent. Generally, the most persistent errors are for the swaptions with five years to maturity, while the least persistent errors occur for the swaptions with one, two, or three years to maturity.[26]

Table VII shows that although many of the means for the individual swaption valuation errors are significantly different from zero (after correcting the standard errors for serial correlation), the largest valuation errors occur for the swaptions with five years to maturity. In addition, the means for these five-year swaptions are all positive and greater than four percent. Note that the large positive means for these swaptions results in most of the other means being negative since there is an implicit adding-up-to-zero constraint imposed by the fitting procedure. Although smaller in magnitude, the mean differences for the swaptions with two years to maturity are also generally significantly different from zero. Another interesting feature of the valuation errors is that they tend to be skewed. This can easily be seen by comparing the mean valuation errors with the median errors. Note that the distribution of valuation errors tends to be skewed towards smaller values for short-maturity swaptions and towards larger values for long-maturity swaptions. Taken together, these results strongly suggest that there are significant and predictable valuation errors.

D. The Relative Valuation of Caps and Swaptions

In the string market model, cash flows from fixed-income derivatives can be expressed in terms of the fundamental forward rates defining the term structure. Thus, once the covariance matrix Σ has been estimated from the market prices of swaptions, the values of other fixed-income derivatives such as caps are uniquely determined by the string market model. In this sense, by parameterizing the model with swaption prices, which are essentially options on baskets of forwards, the model implies prices for caps, which can be viewed as baskets of options on individual forward rates. As in Merton (1973), the covariance matrix Σ determines the relation between the prices of options on portfolios and portfolios of options. It is important to note that the relation between swaption and cap prices implied by the model is a contemporaneous one; the prices of caps at time t in the model are implied from the prices of swaptions at time t. In this sense, the relative value relation implied by the model between caps and swaptions is similar to the put/call parity formula for options, which also places restrictions on the relative values of simultaneously observed call and put prices.

The main diagonal of the implied covariance matrix represents the implied variance of the individual forward rates as they roll down in maturity and become the spot rate. In particular, the implied variance of each forward

[26] It is important to note, however, that some of the persistence in these pricing errors may arise because the data consists of weekly observations of swaption prices where the maturities are typically multiple years. Thus, the overlapping nature of the data may induce serial correlation in the estimated pricing errors. We are grateful to the referee for pointing out this potential source of serial correlation in the pricing errors.

rate during the last period before it becomes the spot rate is the first element on the diagonal, the implied variance of each forward rate during the next-to-last period before it becomes the spot rate is given by the second element on the diagonal, and so forth. Since this provides a complete specification of the volatility of all forwards, the main diagonal uniquely determines the values of individual caplets (conditional on the number of eigenvalues fitted), which then determine the values of caps. Thus, once the model is fitted to the swaptions market, we can directly examine the implications for the valuation of caps. In the absence of arbitrage, the values of caps implied by the swaptions market should match the actual market prices of caps.

To examine the relative valuation of caps and swaptions, we use the main diagonal of the implied covariance matrix and solve for the implied values of 2-year, 3-year, 4-year, 5-year, 7-year, and 10-year at-the-money caps, using the Black model given in equation (1) and the initial term structure to value individual caplets. Since the Black model gives a closed-form expression for caplet prices, we do not need to solve for caps prices by simulation. The use of the Black model for pricing caplets is appropriate here since the lognormal dynamics for forward rates given in equation (8) imply that the Black model holds for individual caplets, since caplets are simple European options on individual forward rates. Note that the variance used in the Black model for valuing a caplet is simply the average variance for the corresponding forward rate from the present until the forward becomes the spot rate. Thus, the variance for the caplet maturing in 6 months is the first diagonal element, the variance for the caplet maturing in 12 months is the average of the first and second diagonal elements, and so forth. We repeat this procedure for each of the 128 weeks in the sample period and report summary statistics for the differences between the market and implied prices in Table VIII.

As illustrated, the hypothesis that market cap prices match the values implied by the swaption market is rejected for all of the maturities. The mean percentage pricing errors range from a high of 23.326 for the two-year caps down to 5.665 for the five-year caps. The positive means imply that the market cap prices are undervalued relative to swaptions. Note that these percentage pricing errors also tend to display a significant amount of persistence as evidenced by their first-order serial correlation coefficients.

A different perspective is obtained by focusing on the median values of the pricing differences. The median pricing errors are all within three percent of zero, and the overall median is only 0.862, which suggests that the caps and swaptions markets are usually consistent; the significant mean percentage pricing are primarily due to periodic large positive errors, resulting in a skewed, somewhat bimodal distribution of errors.

As an additional diagnostic, we also recompute the pricing errors under the assumption that the Black volatilities for caps on six-month Libor are 0.25 volatility points below those for caps on three-month Libor. Recall from the earlier discussion that there could be a slight difference in the quoted

Table VIII

Summary Statistics for Percentage Cap Valuation Errors

The summary statistics reported are for the percentage difference between the model price implied from fitting the string market model to the swaptions market and the market price expressed as a percentage of the market price. The data set consists of 128 weekly observations from January 24, 1997, to July 2, 1999, for the indicated maturities. The t-statistic for the mean and the standard deviation of the mean reported are adjusted for first-order serial correlation. The overall statistics are simple averages of the corresponding columns with the exception of the overall minimum, median, and maximum, which are computed from all observations.

Maturity	Mean	t-Stat. Mean	Standard Deviation	Minimum	Median	Maximum	Serial Correlation
2.00	23.326	3.50	6.669	−19.484	2.589	113.307	0.663
3.00	11.782	2.73	4.319	−11.359	−0.705	63.361	0.664
4.00	7.666	2.36	3.245	−9.739	−2.120	47.687	0.646
5.00	5.665	2.14	2.644	−9.191	−2.385	38.071	0.641
7.00	6.021	2.58	2.332	−8.941	−0.054	32.379	0.685
10.00	7.864	3.57	2.200	−4.592	2.883	32.578	0.734
Overall	10.387	2.81	3.568	−19.484	0.862	113.307	0.672

volatilities for caps on six-month Libor rather than on three-month Libor. The results, however, are virtually the same as those reported in Table VIII.

As another test of the Merton (1973) no-arbitrage bounds, we recompute the percentage pricing differences under the assumption that the correlations between all forwards equals one. This is done by fitting only a single implied eigenvalue to the market prices of the swaptions; all of the remaining eigenvalues are set equal to zero. This specification results in a rank-one covariance matrix, which in turn, implies perfect correlation among all forward rates. Following Merton, it is easily shown that the model price from the one-factor model should provide a lower bound for the value of a cap. This is directly an implication of the fact that the value of a portfolio of options should be greater than or equal to the value of an option on a portfolio. Thus, no-arbitrage considerations imply that the percent pricing differences from the one-factor model should all be positive. Virtually all of the cap prices satisfy this no-arbitrage bound. The mean and median values of the percentage pricing differences are now all negative. Of the $128 \times 34 =$ 4,352 observations, only 8 or 0.18 percent are positive.

In summary, the evidence suggests that while caps and swaptions almost always satisfy the strictest no-arbitrage restriction of Merton (1973), the values of caps and swaptions are frequently inconsistent with each other. This is consistent with Hull and White (1999) who independently find that a set of cap and swaptions prices for a single day in August 1999 cannot be reconciled within the context of a three-factor model. Similarly, Jagannathan and Sun (1999) find that caps and swaptions appear significantly mispriced in a three-factor Cox, Ingersoll, and Ross (1985) framework. Our results suggest the possibility that whereas buy-and-hold arbitrages may not be feasible, dynamic trading strategies exploiting inconsistencies in the relative valuation of caps and swaptions may be profitable. Of course, however, these results can also be interpreted as evidence that the model may too restrictive. For example, a time-varying covariance structure may be necessary to capture fully the relative values of caps and swaptions.

V. A Comparison to the Black Model

In this paper, we have examined the relative valuation of caps and swaptions using a multifactor string market model of the term structure. As an additional issue, it is also useful to contrast the performance of the multifactor string market model with the standard Black model often applied to caps and swaptions in practice.

Before making any comparisons, however, it is important to first understand the key differences between the two modeling approaches. The string market model is a unified multifactor framework in which the same calibration is used, for example, in pricing and hedging all of the swaptions in the sample. In contrast, 34 separate specifications of the Black model are needed to price and hedge the 34 swaptions in the sample, each specification with a different forward swap rate as the underlying factor, and each with a distinct volatility calibration. In this context, the Black model is more appropriately viewed as a collection of different univariate models, where the relationship between the underlying factors is left unspecified.[27] In contrast, the string market model provides a complete unified description of the multivariate relationships among all points along the term structure.

The piecemeal way in which the Black model is typically used results in many limitations to its applicability. Because the Black model requires a different calibration for each swaption, it does not place any overidentifying restrictions on swaption prices. For example, even if 34 different versions of the Black model are fitted exactly to the prices of the 34 swaptions in the sample, these calibrations tell us nothing about what the price of a 35th swaption would be if it were introduced into the sample. Thus, the Black model cannot be extended to other swaptions with different maturities, expiration dates, or strike prices. In practice, the volatilities or prices of the 34 original swaptions would be interpolated or extrapolated to price a 35th swaption. It is important to note, however, that it is the assumptions about interpolation or extrapolation that determine the pricing of the 35th swaption in this situation, not the Black model. In contrast, once calibrated, the string market model can be used to price any other fixed-income derivative such as a swaption with a different maturity, exercise date, or strike price,

[27] Because the 34 forward swaps underlying the 34 swaptions in our sample can be expressed in terms of just 19 distinct forward rates, it is tempting to argue that the dimensionality of the Black model cannot be higher than the number of forward rates. Since Black model volatilities can be specified arbitrarily, however, these volatilities may not be linked by the dependence of the forward swap rates on a common set of forward rates. Hence, the Black model is best viewed as a collection of models that may not be strictly compatible with each other.

interest-rate caps, exotic interest rate options with payoffs that depend on multiple forward rates, and even American-style swaptions.

Another problem with the way that the Black model is applied in practice is that it cannot be used to hedge portfolios of options. Since each swaption has its own underlying asset in the Black model, a swaption can only be hedged with its own associated forward swap. Thus, hedging the 34 swaptions in the sample requires 34 distinct hedging instruments. Since the relationships between different forward rates are left unspecified in the Black framework, there is no clear way in which the risks of different swaptions can be aggregated without making ad hoc auxiliary assumptions unrelated to the Black model itself. In a strict sense, it is not appropriate to aggregate the hedge ratios that are computed using different calibrations of the Black model since there is no guarantee that the different calibrations will be internally consistent. Thus, the Black model provides no guidance on how one swaption can be hedged with another. This implies that cross-hedging is not possible using only the Black model.[28] In contrast, the string market model implies that all risks can be hedged using four factors and that the risks of different types of fixed-income derivatives can be directly aggregated at a portfolio level.

Although the Black model places no overidentifying restrictions on prices, it is possible to contrast the two models in terms of their abilities to hedge fixed-income derivatives. The Black model implies that changes in swaption values are driven entirely by changes in the corresponding underlying forward swap rate.[29] This means that changes in 34 distinct forward swap rates would be needed to explain the variation in the prices of the 34 swaptions in the sample. In contrast, the string market model implies that the variation in the prices of the 34 swaptions can be explained in terms of the changes of only four factors. Thus, the string model attempts to explain the variation in swaption prices using far fewer state variables than this interpretation of the Black model.

To compare the two models, we do the following. Using only information available at time t, we solve for the hedge ratios for each swaption with respect to the state variables. In the Black model, the underlying state variable is the forward swap rate and the hedge ratio is given analytically by differentiating the pricing expression. In the string market model, the hedge ratios for the four factors are computed by simulation by varying the initial curve at time t and recomputing swaption prices. In doing this, we use the eigenvalues from time t and the eigenvectors determined from the ex ante period prior to the beginning of the sample period; all of the information used in computing hedge ratios at time t in the string market model is ob-

[28] In practice, the risk of fixed income derivative portfolios is often calculated by computing the sensitivity of Black model prices to changes in individual forward rates. Note, however, that this approach is much more consistent with the string market model than with the Black model.

[29] It is easily shown from the Black model expression in equation (6) that changes in the forward swap rate are spanned by the returns on two separate portfolios of zero-coupon bonds with values $D(t,\tau) - D(t,T)$ and $A(t,\tau,T)$, respectively.

servable at time t. We then solve for the pricing errors for both models by taking the change in swaption prices from time t to $t + 1$ and subtracting the change in the hedging portfolio over the same period, where the change in the hedging portfolio is given from the hedge ratios and the changes in the individual state variables. Since we only have data on at-the-money-forward swaptions rather than repeated observations on the prices of a specific swaption, we make the identifying assumption that the volatility for the swaption at time $t + 1$ is the same as the volatility for the at-the-money-forward swaption at time $t + 1$ in computing price changes.[30] These differences directly measure the hedging errors resulting from using the hedge ratios and hedging instruments implied by the two models over a one-week horizon.

Intuitively, one might suspect that the Black model would perform much better in this hedging analysis since it hedges with the 34 specific underlying forward swaps whereas the string market model uses only four hedges for all 34 swaptions. Surprisingly, however, the string market model actually performs slightly better than the Black model. For the case of receivers swaptions, the Black model explains 89.28 percent of the variability in the price changes for the 34 swaptions over the 127-week period. In contrast, the string market model explains 89.35 percent of the variability in the price changes.[31] For the case of payers swaptions, the Black model explains 92.46 percent of the variability in the price changes whereas the string market model explains 92.48 percent. We acknowledge, of course, that the differences in the explanatory power between the two models are economically quite small and only affect the fourth decimal place. Nevertheless, they are still impressive when one considers that the string market model is able to result in a slightly better hedge while using 30 fewer hedging instruments.

Intuitively, the reason why the common factors driving term-structure movements in the string market model have incremental power in explaining swaption price changes is not hard to understand. In the Black model, the volatility of forward swap rates is assumed to be constant. In the string market model, however, forward swap rates are essentially baskets of individual forward rates. As the composition of the basket changes over time, either as the option approaches maturity or as the swaption moves away from being at-the-money forward, the string market model captures the fact that the revised basket should have a different volatility. Thus, the string market model is better able to capture the variation in swaption-implied volatilities over time.[32] Several recent papers also confirm that the hedging performance of single-factor term-structure models is inferior to that of multi-

[30] This assumption is not very restrictive. We have also repeated the tests using only the subsample for which the swaption is at the money at both times t and $t + 1$. The results are very similar to those reported.

[31] The percent of variability is computed by simply taking one minus the ratio of the variance of the hedging errors divided by the variance of the actual price changes. This measure is essentially the R^2 for the hedge.

[32] We acknowledge, of course, that some of the variation in implied volatilities is probably due to stochastic volatility, which neither the Black nor the string market model incorporate.

factor models; for example, see Gupta (1999) and Driessen, Klaassen, and Melenberg (2000).

VI. Conclusion

Using a string market model framework calibrated to swaptions market data, we study the relative pricing of swaptions and interest-rate caps. We find evidence that the market considers the risk of four factors in the valuation of swaptions. This contrasts with earlier work documenting that two to three factors captures the historical behavior of term-structure movements. Our results suggest that the market may consider factors that may contribute little to the unconditional variance of term-structure movements, but can periodically affect the conditional variance of term-structure movements significantly.

Focusing on the valuation of swaptions, we find that swaption prices are generally well described by the time-homogeneous string market model with the exception of the short period during the hedge-fund crisis of late 1998. We also find evidence that long-dated swaptions in particular appear to be slightly undervalued by the market. Although we stop short of claiming that there are arbitrage opportunities in this market, the results clearly suggest the need for additional research.

Finally, we examine the relative valuation of caps and swaptions using the time-homogeneous string market model. Once the covariance structure among forwards has been implied from the market prices of swaptions, cap prices are determined from the main diagonal of the implied covariance matrix. This simple restriction is tested directly by comparing the prices of caps implied by the fitted string market model to their market prices. We find that although the median differences between the two markets are close to zero, there can be large differences between the two, particularly during periods of market stress. Again, since our results are based on market quotations rather than actual transactions, we cannot definitively conclude that there are arbitrage opportunities across the caps and swaptions market. These results, however, clearly indicate the possibility that differences in the way that models are calibrated and used in the caps market and the swaptions market may introduce a wedge between the relative prices of these important fixed-income derivatives.

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