Problem.1.

1. $P(W < W_0 - VaR) = 1 - c$.

$cdf = 1 - e^{-\lambda x} = 1 - c$.

$x = -\frac{1}{\lambda} \log c$.

$\because x = W_0 - VaR$.

$\therefore VaR = W_0 + \frac{1}{\lambda} \log(c)$

$\because \lambda = \frac{1}{W_0}$

$\therefore VaR = W_0 (1 + \log c)$.

$W_0 = 200, \quad c = 99.9\%$

$VaR = 199.8$

2. $P(W > W_0 + VaR) = 1 - c$

$cdf = 1 - e^{-\lambda x} = c$.

$x = -\frac{1}{\lambda} \log(1-c)$

$\because x = W_0 + VaR$

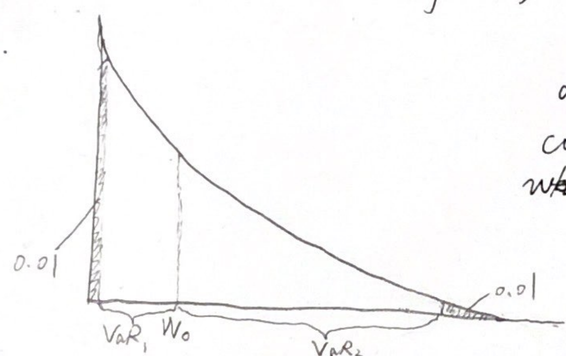
$\therefore VaR = -W_0 - W_0 \cdot \log(1-c)$

$= -W_0 (1 + \log(1-c))$

$W_0 = 200, \quad c = 99.9\%$

$VaR = 1181.55$

3.



Because of the asymmetry of exponential distribution. when you short the asset, its value could increase a lot more than it could decrease when you buy it. Therefore: $VaR_2 = 1181.55 >> 199.8$.

4. $ES = W_0 - E[W \mid W \le W_0 - VaR]$

$$= W_0 - \frac{\int_{-\infty}^{W_0 - VaR} W f(W)\, dW}{\int_{-\infty}^{W_0 - VaR} f(W) \cdot dW}$$

In which:

$\int_{-\infty}^{W_0 - VaR} f(W)\, dW = 1 - c$.

$\because$ For exponential distribution: $f(W) = 0$ for $W < 0$

$\therefore \int_{-\infty}^{W_0 - VaR} W \cdot f(W)\, dW = \int_{0}^{W_0 - VaR} W \cdot f(W) \cdot dW$.

$f(W) = \lambda \cdot e^{-\lambda W}$.

Denote $A = \int_0^{W_0 - VaR} W f(W) \cdot dW$

Integrating by parts:

$A = -W \cdot e^{-\lambda W} \Big|_0^{W_0 - VaR} + \int_0^{W_0 - VaR} e^{-\lambda W} dW$

$= (VaR - W_0) \cdot e^{\lambda(VaR - W_0)} + (-\frac{1}{\lambda}) \cdot e^{-\lambda W} \Big|_0^{W_0 - VaR}$

$= (VaR - W_0) \cdot e^{\lambda(VaR - W_0)} + \left[(-\frac{1}{\lambda}) \cdot e^{\lambda(VaR - W_0)} + \frac{1}{\lambda} \right]$

$= (VaR - W_0) \cdot e^{(VaR - W_0)/W_0} + W_0 - W_0 \cdot e^{(VaR - W_0)/W_0}$

4. $\therefore ES = W_0 - \dfrac{A}{1-C}$

in which $A = (VaR - 2W_0)e^{(VaR-W_0)/W_0} + W_0$

For question 1: $ES_1 = 199.9$

~~For question 2: $ES_2 =$~~

For question2: $ES = E[W \mid W \geq W_0 + VaR] - W_0$

$$ES = \frac{\int_{W_0+VaR}^{\infty} W \cdot f(W) \cdot dW}{1-C} - W_0$$

same as question 1 above.

$$A = -We^{-\lambda W}\Big|_{W_0+VaR}^{\infty} + \left(-\frac{1}{\lambda}\right) \cdot e^{-\lambda W}\Big|_{W_0+VaR}^{\infty}$$

$$= (2W_0 + VaR) \cdot e^{-(W_0+VaR)/W_0}$$

$\therefore ES_2 = \dfrac{A}{1-C} - W_0$

in which: $A = (2W_0 + VaR) \cdot e^{-(W_0+VaR)/W_0}$

$ES_2 = 1381.56$