Problem 2.

1. According to pigeonhole principle: for natural numbers k and m. if $n=km+1$. objects are distributed among m sets. then at least one of the sets will contain at least $k+1$ object.

$$8 > 2\times 3+1, \quad k=2, \ m=3.$$

at least $k+1=3$ of them are born within the same one-year period

2. Assume $W-W_0 \sim N(0,\sigma)$, and Losses in successive days are independent.

98% $VaR_5 = (z(0.98)\cdot\sigma - 0)\cdot\sqrt{5} = 10\,m.$

$$\sigma \approx 2.17755$$

$$VaR_{10} = \sqrt{10}\times(z(0.99)\cdot\sigma - 0) \approx 16.019 \text{ million}$$

3. 22 trading days in one month. $c=0.99$. $n=22$

$$P(\text{Exception} > 1) = 1 - \sum_{k=0}^{1} C_k^{22}(1-c)^k \cdot c^{n-k}$$

$$= 0.0202$$

For $c=0.99$, the expect NO. of exception is the ~~trading~~ window times 0.01. Then we can use a two-tailed test:

$$-2\ln[c^{n-m}(1-c)^m] + 2\ln[(1-m/n)^{n-m}(m/n)^m] \sim \chi^2(1)$$

If p-value is ~~large~~, small. we can reject null.

Therefore, there is bunching,

4. We can use cross-sectional data to calculate their daily VaR first. Assume Losses in successive days are independent.

Annual VaR = 1-day. VaR $\times\sqrt{252}$

Uber IPO 1-year 99% VaR at 10 million = 10 million × Annual VaR.

Cross-sectional data should have the same background as Uber. and should have been IPO in recent years.