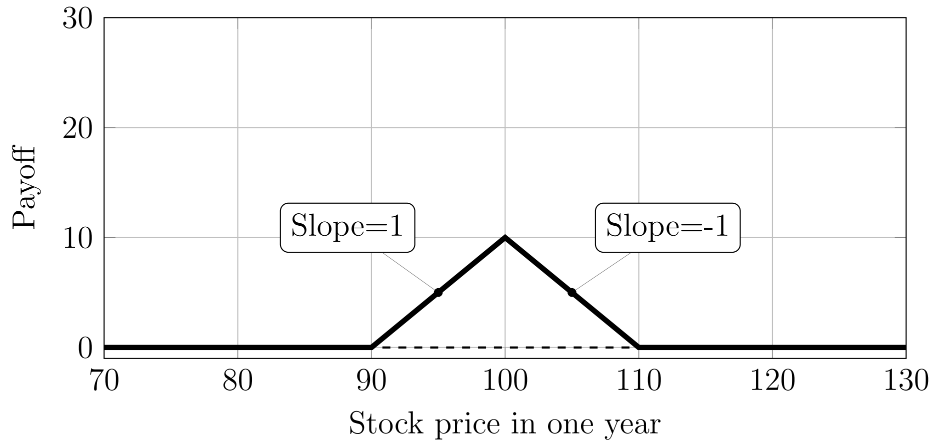
UCLA Anderson School of Management

Daniel Andrei, Derivative Markets MGMTMFE406, Winter 2018

Problem Set 1

These exercises do not need to be turned in for credit.

1 Below is a **payoff** diagram for a position. All options have 1 year to maturity and the stock price today is \$100. The yearly interest rate (**continuously compounded**) is 8%. The underlying asset (the stock) is not paying any dividends.



a. In the table below, record option quantities which construct the diagram. **Options given in the table have the corresponding strike in parenthesis**. Slopes are marked on the diagram. Denote a purchased position with "+" and a written position with "−".

Option	Call(90)	Call(100)	Call(110)
Position			

b. In the table below, record option quantities which construct the diagram. Options given in the table have the corresponding strike in parenthesis. Slopes are marked on the diagram. Denote a purchased position with "+" and a written position with "−".

Option	Put(90)	Put(100)	Put(110)
Position			

c. In the table below, record option quantities **and the position in the risk-free asset** which construct the diagram. Be clear about whether you are *borrowing* or *lending*. Options given in the table have the corresponding strike in parenthesis. Slopes are marked on the diagram. Denote a purchased option position with "+" and a written position with "−".

Option	Put(90)	Put(100)	Call(100)	Call(110)
Position				

Borrow/lend in the risk-free asset:

d. Can the portfolio corresponding to the above payoff have zero or negative initial premium? Why or why not?

e. How is this strategy commonly called? Discuss its benefits and disadvantages.

1 a. The position with calls is as follows:

Option	Call(90)	Call(100)	Call(110)
Position	+1	-2	+1

b. The position with puts is as follows:

Option	Put(90)	Put(100)	Put(110)
Position	+1	-2	+1

c. The position with options (calls and puts) is as follows:

Option	Put(90)	Put(100)	Call(100)	Call(110)
Position	+1	-1	-1	+1

This gives a butterfly that is below the X-axis by \$10. In order to elevate the payoff up to the level of the X-axis, you should also *lend* the present value of \$10, that is, $\$10 \times e^{-0.08} = \$9.23$.

d. No. Since the symmetric butterfly spread has non-negative payoff, it must have a positive value.

e. This strategy is called the symmetric butterfly spread. The benefit of the symmetric butterfly spread is that potential losses are never greater than its cost. The potential gain, however, is also limited. The largest profit is realized if the stock is at or very near the middle strike price (\$100 in this case) on expiration day. The butterfly spread is thus a conservative strategy with limited potential losses and profit. It profits from stocks that are stagnant or stocks that are trading within a very tight price range.

2 A 1M European put option on a non-dividend paying stock is currently selling for 2.50. The option has a strike of 50 and the underlying is currently worth 46. The interest rate is 10%. What should you do?

2 Recall that

$$\begin{aligned} P_t &\geq \max\left[0, PV(K) - S_t\right] \\ 2.5 &\geq \max\left[0, 49.59 - 46\right] \end{aligned}$$

Since the inequality is not satisfied, the put price is too low there is a possibility for arbitrage. Buy one option, buy one stock and borrow the necessary amount (48.5) for a zero value investment. The payoff at maturity is:

$$\max\left[0, 50 - S_T\right] + S_T - 48.5e^{0.1/12} = \max\left[0, 50 - S_T\right] + S_T - 48.91$$

If $S_T \geq 50$, the put is worthless and the payoff at maturity is $S_T - 48.91 > 0$. If $S_T < 50$, the put is in the money at maturity and the payoff is $50 - S_T + S_T - 48.91 = 1.09 > 0$. Thus, one can have a strictly positive payoff for a zero initial investment (arbitrage).

3 The price of a European call that expires in 6M and has strike of 30 is 2. The underlying stock price is 28 and a dividend of 0.50 is expected in 2M and again in 5M. Compute the price of a put with maturity 6M under the assumption that the interest rate is 10%.

3 Using the put-call parity we have:

$$\begin{aligned} P_t &= C_t - S_t + e^{-r(T-t)}K + PV(Div) \\ &= 2 - 28 + 30e^{-0.1\frac{1}{2}} + 0.5\left(e^{-0.1\frac{1}{6}} + e^{-0.1\frac{5}{12}}\right) \\ &= 3.508 \end{aligned}$$

4 Suppose that you are the manager and sole owner of company MFE. All the debt of the company matures in one year. If at that time the value of the company's assets is larger than the face value of debt you will pay off that face value, otherwise you will declare bankruptcy and the debt holders will own the assets of the company.

a. Express your position as an option on the firm's assets.

b. Express the position of the debt holders as a portfolio of securities.

4 a. As an equity holder you will be paid only if something is left after the firm has paid off its debt. If A_T denotes the liquidation value of the firm's assets and D the **face** value of its debt then you will receive $A_T - D$ if this amount is positive and zero otherwise since in that case the firm is liquidated and you are protected by limited liability. Your payoff is thus $\max[0, A_T - D]$ and corresponds to that of a European call with strike D written on the firm's assets.

b. Debt holders receive D if the liquidation value of the firm's assets is higher than D. If not, they take over the firm and liquidate it to obtain the payoff A_T. Combining the two shows that the terminal value of the firm's debt is

$$D_T = \min[D, A_T] = D - \max[0, D - A_T]$$

and corresponds to a long position in a zero-coupon with face value D and a short position in a European put option with strike D written on the firm's assets.

5 Show that, if C_0^A is the price of an American call with exercise price K and maturity T on a stock paying a dividend yield of δ, and P_0^A is the price of an American put on the same stock with the same strike price and exercise date, then

$$S_0 e^{-\delta T} - K < C_0^A - P_0^A < S_0 - K e^{-rT} \tag{1}$$

where S_0 is the stock price, r is the risk-free rate, and $r > 0$.

5 Let us focus first on the first inequality. Consider a portfolio with the following positions:

a. Long an European call option, C_0^E

b. Short an American put option, P_0^A

c. An amount K invested at the risk-free rate

d. Short $e^{-\delta T}$ units of the stock (with dividends being reinvested in the stock)

The possible values for this portfolio are:

	No exercise of the put option before maturity		Exercise of the put option before maturity (at some time $0 \leq t < T$)
	$S_T \geq K$	$S_T < K$	
Position a.	$S_T - K$	0	C_t^E
Position b.	0	$S_T - K$	$S_t - K$
Position c.	Ke^{rT}	Ke^{rT}	Ke^{rt}
Position d.	$-S_T$	$-S_T$	$-S_t e^{-\delta(T-t)}$
TOTAL	> 0	> 0	> 0

Since all the possible values are positive, we have

$$C_0^E - P_0^A + K - S_0 e^{-\delta T} > 0 \tag{2}$$

$$C_0^E - P_0^A > S_0 e^{-\delta T} - K \tag{3}$$

and thus

$$C_0^A - P_0^A > S_0 e^{-\delta T} - K \tag{4}$$

Focus now on the second inequality. Consider a portfolio with the following positions:

a. Short an American call option, C_0^A

b. Long an European put option, P_0^E

c. An amount Ke^{-rT} borrowed at the risk-free rate

d. Long one unit of the stock (with dividends being reinvested in the stock)

The possible values for this portfolio are:

<table>
<tr><td rowspan="2"></td><td colspan="2">No exercise of the call option before maturity</td><td rowspan="2">Exercise of the call option before maturity (at some time $0 \leq t < T$)</td></tr>
<tr><td>$S_T \geq K$</td><td>$S_T < K$</td></tr>
<tr><td>Position a.</td><td>$K - S_T$</td><td>0</td><td>$K - S_t$</td></tr>
<tr><td>Position b.</td><td>0</td><td>$K - S_T$</td><td>P_t^E</td></tr>
<tr><td>Position c.</td><td>$-K$</td><td>$-K$</td><td>$-Ke^{-r(T-t)}$</td></tr>
<tr><td>Position d.</td><td>$S_T e^{\delta T}$</td><td>$S_T e^{\delta T}$</td><td>$S_t e^{\delta t}$</td></tr>
<tr><td>TOTAL</td><td>> 0</td><td>> 0</td><td>> 0</td></tr>
</table>

Since all the possible values are positive, we have

$$-C_0^A + P_0^E - Ke^{-rT} + S_0 > 0 \tag{5}$$
$$C_0^A - P_0^E < S_0 - Ke^{-rT} \tag{6}$$

and thus

$$C_0^A - P_0^A < S_0 - Ke^{-rT} \tag{7}$$

6 Suppose that you are long the **non-dividend-paying** S&P 500, currently trading at \$100. The effective annual risk-free rate is 10% and you can trade the following set of options with one-year maturity:

Strike	80	100	115.4	120
Calls	29.15	16.492	–	8.436
Puts	1.88	–	14.8	17.527

To insure your position, you will need a put option with a one-year maturity and strike price K=100. Since none is available, you need to synthesize one. Describe the strategy that replicates the put and provide the cost of this strategy.

6 The put-call parity (for a non-dividend-paying underlying asset) tells us that a put is equivalent to

$$P_t = C_t - S_t + PV(K) \tag{8}$$

In words, to synthesize a put option we need to purchase a call option, take a short position in the underlying, and lend $K/(1+r)$. We obtain that the replicating cost of the put is

$$P_t = 16.492 - 100 + \frac{100}{1.1} = 7.4 \tag{9}$$

7 Suppose that you can trade 3 stocks, A, B, and C, each of which has a current price of \$100:

- A is a non-dividend-paying stock
- B pays a discrete \$10 dividend in 3 months
- C pays dividends at a continuously compounded dividend yield of 3%

The continuously compounded risk-free rate is 5%.

a. Consider a forward contract that expires in 6 months. What is the forward price for stock A? What is the forward price for stock B? What is the forward price for stock C?

b. Suppose you read the *Wall Street Journal* and figure that WSJ quotes a \$100.5 forward price for stock A. You conclude that there is an arbitrage opportunity. Explain the transactions you undertake to obtain a zero profit at time 0 and a positive profit in 6 months.

c. Do the same for stock B and C (assuming that WSJ quotes \$100.5 forward prices for both of them).

7 a. For stock A, we obtain:

$$F_{0,T}^{A} = S_0 e^{rT} = \$100 e^{0.05\times 6/12} = \$102.532 \tag{10}$$

For stock B, we need to take the discrete dividend into account:

$$F_{0,T}^{B} = (S_0 - De^{-rt})e^{rT} = (\$100 - \$10e^{-0.05\times 3/12})e^{0.05\times 6/12} = \$92.4057 \tag{11}$$

For stock C, we obtain

$$F_{0,T}^{C} = S_0 e^{(r-\delta)T} = \$100 e^{(0.05-0.03)\times 6/12} = \$101.005 \tag{12}$$

b. The forward contract is cheap: buy the forward contract, sell short one unit of the stock, and lend the proceeds. This transaction is called the *reverse cash-and-carry*:

Transaction	Time 0	Time 6 months
Long forward	0	$S_T - 100.5$
Short stock	\$100	$-S_T$
Lend \$100	-\$100	$\$100e^{0.05\times 0.5}$
Total	0	\$2.03151

c. In the case of stock B the forward is expensive. The arbitrage transaction is called the *cash-and-carry*:

Transaction	0	3 months	6 months
Short forward	0	0	$100.5 - S_T$
Long stock	-100	10	S_T
Borrow 100 over 6 months	100	0	-100$e^{0.05\times 0.5}$
Borrow PV(Dividend) over 3 months	10$e^{-0.05\times 0.25}$	-10	0
Lend PV(Dividend) over 6 months	-10$e^{-0.05\times 0.25}$	0	10$e^{0.05\times 0.25}$
Total	0	0	8.09427

Lastly, in the case of stock C, the forward is cheap (*reverse cash-and-carry*):

Transaction	0	6 months
Long forward	0	$S_T - 100.5$
Short tailed position in stock ($e^{-0.03\times 0.5}$ units)	$100e^{-0.03\times 0.5}$	$-S_T$
Lend $100e^{-0.03\times 0.5}$	$-100e^{-0.03\times 0.5}$	$100e^{(0.05-0.03)\times 0.5}$
Total	0	0.505