

Solutions to Sample Quizz 2 (a)

1. The PDE is given by

$$\begin{aligned}\frac{\partial F}{\partial t} + r \frac{\partial F}{\partial S} S + \frac{\partial^2 F}{\partial S^2} \sigma^2 S^{2\gamma} - rF &= 0 \\ F(T, S_T) &= \max(S_T - K, 0)\end{aligned}$$

The risk neutral dynamics are given by

$$dS_t = rS_t dt + \sigma S_t^\gamma dW_t$$

2. Under the risk neutral measure we have that

$$\frac{dS_t}{S_t} = r_t dt + \sigma dW_t$$

which leads to the solution

$$S_T = S_0 e^{r_1 \frac{T}{2} + r_2 (T - \frac{T}{2}) dt + \sigma (W_T - W_0)}$$

Forming expectations gives

$$E(S_T) = S_0 e^{\frac{T}{2}(r_1 + r_2)}$$

Hence

$$K = E(S_T) = S_0 e^{\frac{T}{2}(r_1 + r_2)}$$

3. Let $C(S; K, T, r, \sigma)$ denote the Black Scholes formula when the underlying stock is equal to S , and K is the strike price, T is the time to expiration, r is the short rate, and σ is the volatility of the stock.

The price of the call in the presence of dividends is equal to

$$C((1 - \delta_1)(1 - \delta_2)S; K, T, r, \sigma)$$