Name: ____________________

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UCLA Anderson School of Management
Introduction to Stochastic Calculus Quizz 2
MGMTMFE403
Fall 2017

Honor Code Pledge:

During this exam you must adhere to the standards of the UCLA Student Conduct Code.

- You have 90 minutes to complete the examination.
- You may use a cheat sheet as specified in the syllabus. You may use scientific and financial calculators. No laptops.
- You are to answer these questions **without consulting anyone**.
- **Be neat and show your work**. Answers without work receive no credit. Wrong answers with partially correct work may receive partial credit.
- This exam consists of five (3) problems. You are to answer each in the space provided.
- **ATTENTION:** You are not allowed to discuss the contents of this exam with anyone until tomorrow. If you violate this rule, you will be found in violation of the UCLA Student Code of Conduct, and this will have severe consequences.
- **Good luck!**

1		/30
2		/30
3		/30
Total		/90

Problem 1. Exchange rates (30 Points)

The US-Euro exchange rate X_t follows the dynamics

$$dX_t = \mu_X X_t dt + \sigma_X X_t d\overline{W}_t$$

where μ_X, σ_X are constants and $\overline{W}_t$ is a standard Brownian motion. The dollar interest rate is constant and equal to r^d and the Euro interest rate is constant and equal to r^f. Suppose that we wish to obtain the arbitrage-free price of a derivative with payoff

$$\Phi\left(X_T\right) = \log\left(X_T\right)$$

at time T. Compute the price of such a derivative security at time 0 as a function of X_0.

Solutions to question 1

Problem 2. Delta Hedeging (30 Points)

Suppose that the price of a stock at time t is equal to $S_t = 100$, and the price of a European Call option is equal to $C_t = 10$. The Delta of the option is $N(d_1) = 0.2$.

a) How may shares of stocks should you be holding in a portfolio aimed to replicate the payoff of the option at time T? (This could be a fraction.)

b) How many dollars should you be borrowing in the replicating portfolio?

c) Suppose that

$$\frac{dS_t}{S_t} = 0.1dt + 0.25d\overline{W}_t.$$

Moreover, suppose that $r = 0.02$. Fill in the missing information in the dots bellow

$$\frac{dC_t}{C_t} = ...dt + ...d\overline{W}_t.$$

(Hint: The Call option's replicating portfolio consists of the stock and the bond. Compute the fraction of that portfolio that is invested in stocks and the fraction invested in bonds.)

Solutions to question 2

Problem 3. A "ratio" forward. (30 Points)

Suppose that the interest rate r is constant. Take two stocks $S_t^{(1)}$ and $S_t^{(2)}$ with dynamics

$$\begin{aligned} dS_t^{(1)} &= \mu_1 S_t^{(1)} dt + \sigma_1 S_t^{(1)} d\overline{W}_t^{(1)} \\ dS_t^{(2)} &= \mu_2 S_t^{(2)} dt + \sigma_2 S_t^{(2)} d\overline{W}_t^{(1)} \end{aligned}$$

Note that both stocks are affected by the same brownian motion.

a) Define:

$$z_t = \frac{S_t^{(1)}}{S_t^{(2)}}$$

Use Ito's Lemma to derive the dynamics of z_t under the risk neutral measure Q, i.e. fill the gaps in the expression below

$$dz_t = ...dt + ...dW_t^{(1)}.$$

Careful: First find the dynamics of $S_t^{(1)}$ and $S_t^{(2)}$ under the risk neutral measure Q and then use Ito's Lemma to find the dynamics of z_t under Q.

b) Compute the futures price for z_t, i.e., determine K so that

$$E_0^Q (z_T - K) = 0$$

Solutions to question 3

Solutions to question 3