Fixed Income Homework 3
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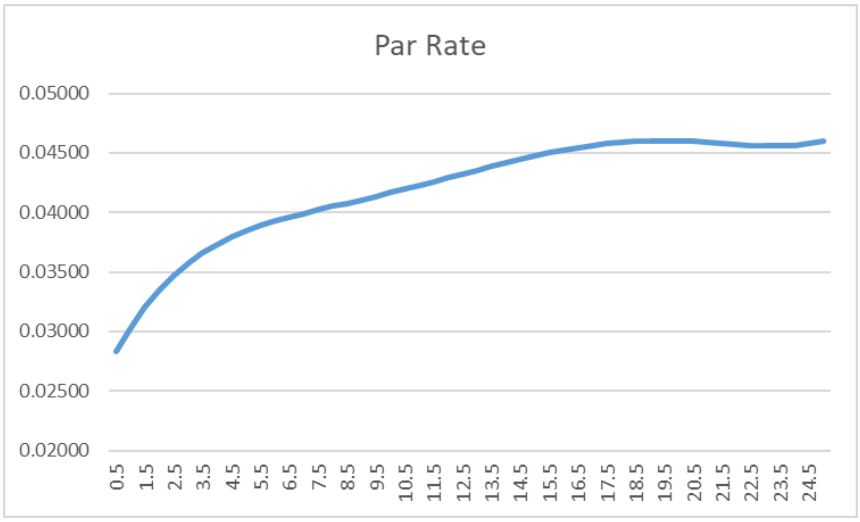
All **highlights** in questions refer to a **tab** in attached Excel file

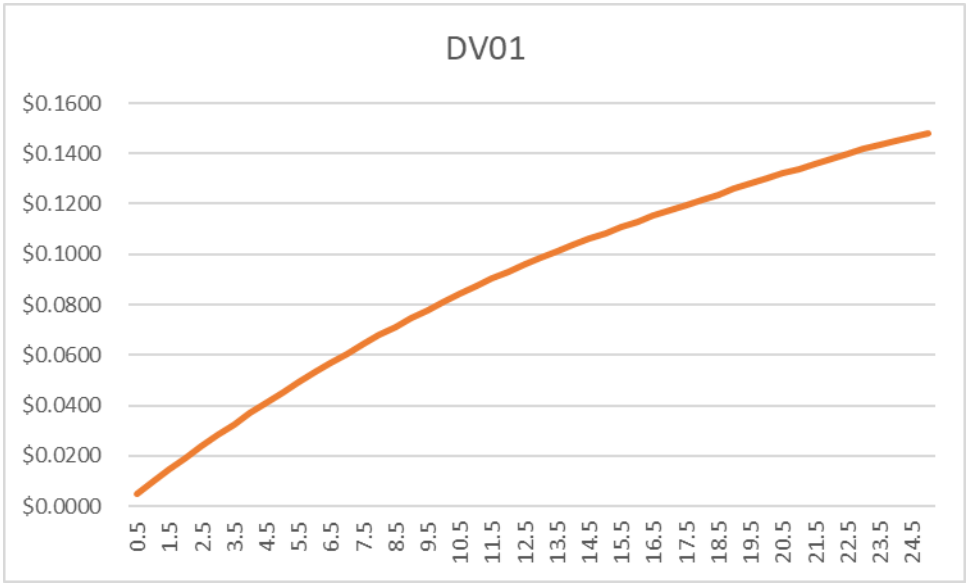
Question 1: Par Rate (Q1&2_Par & DV01 tab)



Question 2: DV01 (Q1&2_Par & DV01 tab)

To compute the DV01, I calculate the bond price with YTM = original yield – 0.01%.

DV01 = | initial price – price if yield decreases (increases) by 1 basis point |



Question 3: Duration (Q3_Duration tab)

Since the bonds we are evaluating are all priced at par, the yield-to-maturity would be equal to the par rate. Hence, we could get the duration by applying the simplified formula from the slide. Since we are dealing with semi-annual coupon bond, we need to divide yield by 2,

$$\mathrm{D} = \frac{1}{2}\left(1 + \frac{1}{\frac{y}{2}} + \left[100(2\mathrm{N} - 1) - \frac{1}{\frac{y}{2}}(100 + C \cdot 2N)\right]\frac{PV}{P}\right)$$

Where $\mathrm{PV} = \frac{1}{(1+\frac{y}{2})^{2N}}$, N = years to maturity, C is the semi-annual coupon payment, and P = 100.

And the modified duration is,

$$\text{Modified Duration} = \frac{D}{1 + \frac{y}{2}}$$

Semi-annual coupon	**Yield (=Par Rate)**	**Macauley Duration**	**Modified Duration**
1-Year Bond	0.030339	0.992529	0.977698
2-Year Bond	0.033504	1.951112	1.918966
3-Year Bond	0.035747	2.871345	2.820926
4-Year Bond	0.037323	3.752709	3.683960
5-Year Bond	0.038441	4.596345	4.509668

Question 4: You have a $5,000,000 liability due in 3 years. How much do you need to invest in a **3-year zero-coupon bond** to defend the liability? Use the same zero-coupon curve as in 1.

According to the zero-coupon curve in question 1, we get that the 3-year ZCB price is **$89.881** (**Q1&2_Par &DV01** tab)

To defend the liability of $5 million in 3 years, we need to invest in enough 3-year zero-coupon bond today so that the payoff in 3 years would match our liability. So we need to invest in 50,000 3-year zero-coupon bonds, which totals to $4,494,050.758 today.

$$\frac{5{,}000{,}000}{100} = 50{,}000\ bonds$$

$$50{,}000\ \times \$89.881 = \$4{,}494{,}050.758$$

Question 5: Convexity (Q5_Convexity tab)

$$\text{Convexity} = \frac{\sum_{i=1}^{N} i(i+1)CF(i)D(i)}{(1+\frac{y}{k})^2 \times k^2 \times Price}$$

Where k is the number of periods per year, which is 2 in our example.

	Convexity
1-Year Bond	1.441007
2-Year Bond	4.679181
3-Year Bond	9.556696
4-Year Bond	15.923195
5-Year Bond	23.640910

*The detailed cash flow tables for each bond is in **Appendix A**.

Question 6: Curve Shift (Q6_shifted curve and Summary tabs)

We can estimate the price change as,

$$\frac{\Delta P}{P} = -\text{MD} \times \Delta y \times 100 + \frac{1}{2} \times \text{Convexity} \times \Delta y^2 \times 100$$

	Yield/Par	**Modified Duration**	**Convexity**	**+100bp**	**-100bp**
1-year bond	0.030339	0.977698	1.441007	-$0.97049	$0.98490
2-year bond	0.033504	1.918966	4.679181	-$1.89557	$1.94236
3-year bond	0.035747	2.820926	9.556696	-$2.77314	$2.86871
4-year bond	0.037323	3.683960	15.923195	-$3.60434	$3.76358
5-year bond	0.038441	4.509668	23.640910	-$4.39146	$4.62787

Now comparing the estimated price with the actual price using shifted curve, we can see that the prices are very similar. The differences are in the third decimal points.

	Price after Changes		**Actual Price**	
	Price (+100bp)	**Price (-100bp)**	**+100bp**	**-100bp**
1-year bond	$99.02951	$100.98490	$99.02947	$100.98494
2-year bond	$98.10443	$101.94236	$98.10433	$101.94246
3-year bond	$97.22686	$102.86871	$97.22679	$102.86876
4-year bond	$96.39566	$103.76358	$96.39580	$103.76336
5-year bond	$95.60854	$104.62787	$95.60910	$104.62714

Appendix A: For calculating convexity in question 5

1-Year Coupon Bond			Maturity	0.992529
Par (Yield) =	0.030339		Convexity	1.441007
Period (t)	CashFlow	PVCF	T*(PVCF/Price)	T(T+1)*(PVCF/Price)
1	1.517	1.494	0.01494	0.02989
2	101.517	98.506	1.97011	5.91034
Total		100.000	**1.98506**	**5.94023**

2-Year Coupon Bond			Maturity	1.951112
Par (Yield) =	0.033504		Convexity	4.679181
Period (t)	CashFlow	PVCF	T*(PVCF/Price)	T(T+1)*(PVCF/Price)
1	1.675	1.648	0.01648	0.03295
2	1.675	1.620	0.03241	0.09723
3	1.675	1.594	0.04781	0.19125
4	101.675	95.138	3.80553	19.02763
Total		100.000	**3.90222**	**19.34907**

3-Year Coupon Bond			Maturity	2.871345
Par (Yield) =	0.035747		Convexity	9.556696
Period (t)	CashFlow	PVCF	T*(PVCF/Price)	T(T+1)*(PVCF/Price)
1	1.787	1.756	0.01756	0.03512
2	1.787	1.725	0.03450	0.10351
3	1.787	1.695	0.05085	0.20338
4	1.787	1.665	0.06660	0.33302
5	1.787	1.636	0.08179	0.49075
6	101.787	91.523	5.49139	38.43972
Total		100.000	**5.74269**	**39.60549**

4-Year Coupon Bond			Maturity	3.752709
Par (Yield) =	0.037323		Convexity	15.923195
Period (t)	CashFlow	PVCF	T*(PVCF/Price)	T(T+1)*(PVCF/Price)
1	1.866	1.832	0.01832	0.03664
2	1.866	1.798	0.03597	0.10790
3	1.866	1.765	0.05296	0.21186
4	1.866	1.733	0.06932	0.34662
5	1.866	1.701	0.08507	0.51041
6	1.866	1.670	0.10021	0.70149
7	1.866	1.640	0.11477	0.91818
8	101.866	87.860	7.02879	63.25908
Total		100.000	**7.50542**	**66.09218**

5-Year Coupon Bond			Maturity	4.596345
Par (Yield) =	0.038441		Convexity	23.640910
Period (t)	**CashFlow**	**PVCF**	**T*(PVCF/Price)**	**T(T+1)*(PVCF/Price)**
1	1.922	1.886	0.01886	0.03772
2	1.922	1.850	0.03700	0.11101
3	1.922	1.815	0.05446	0.21784
4	1.922	1.781	0.07124	0.35622
5	1.922	1.748	0.08738	0.52426
6	1.922	1.715	0.10287	0.72012
7	1.922	1.682	0.11776	0.94205
8	1.922	1.651	0.13204	1.18837
9	1.922	1.619	0.14574	1.45744
10	101.922	84.253	8.42533	92.67865
Total		100.000	**9.19269**	**98.23368**