MGMT MFE 431-3

Statistical Arbitrage

Lecture 07: Short-Term Alphas

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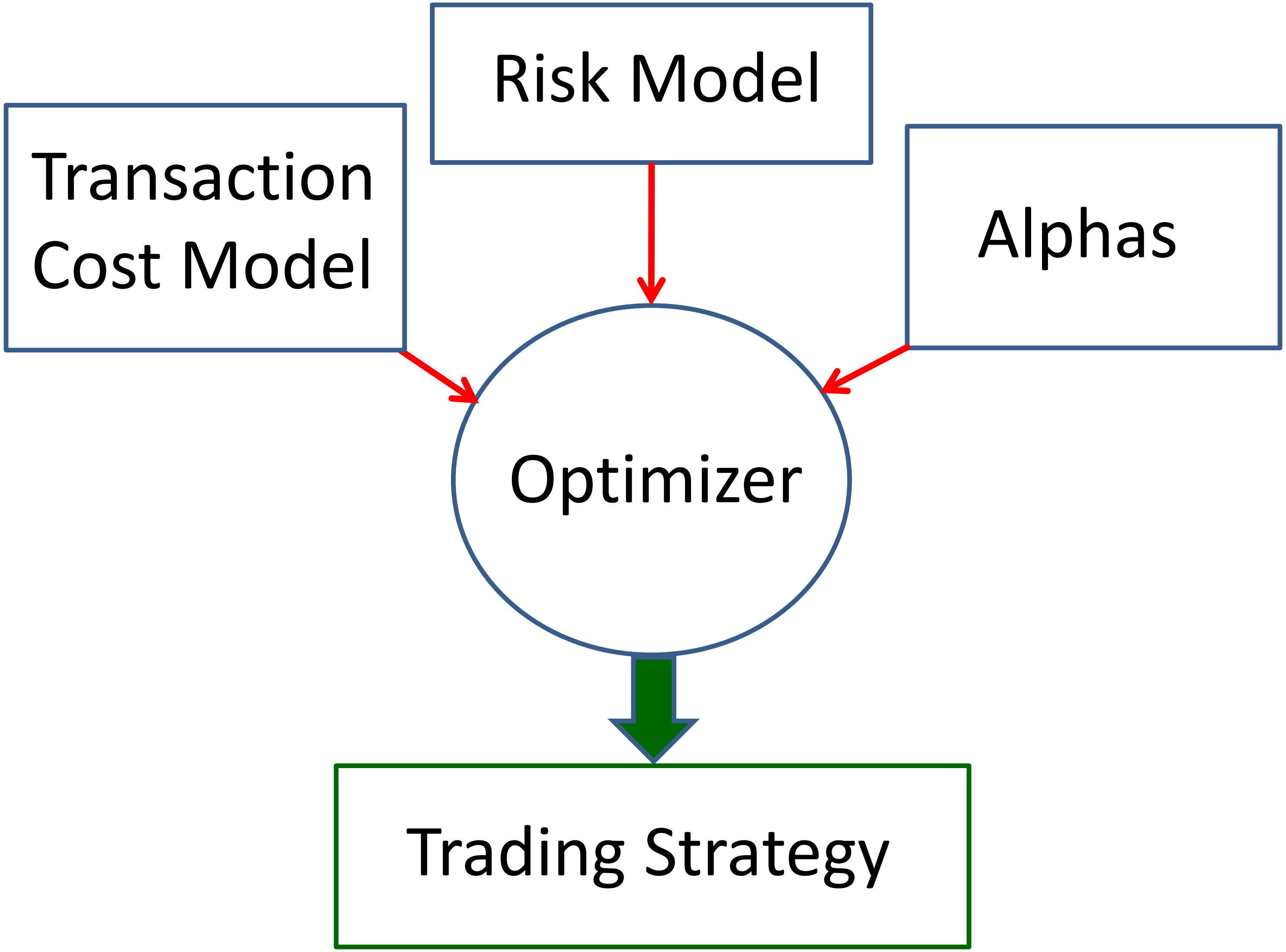
Overall Structure



What We've Seen So Far

- Risk Model: shrinkage estimator of the covariance matrix of stock returns
- Transaction Cost Model: 1bp + ½ bid-ask spread
- Alpha: Weighted blend of various standardized, windsorized alphas

Optimizer

- Inputs:
  - position as of close of business on day *t-1*
  - alphas using data observed up to day *t-1*
  - t-costs using data observed up to day *t-1*
  - risk model using data observed up to day *t-1*
  - constraints using data observed up to day *t-1*
- Output: trade to be executed on day *t*

$$\boxed{\text{final position}(t+1) = \text{final position}(t) + \text{trade}\ (t)}$$

Timeline

- Day *t-1*: most recent available data
- Day *t*: trade gets executed
- Day *t+1*: returns start to be earnt

Backtest Code

- Load all necessary data into memory
- Create the alphas
- Start from portfolio with zero dollar invested
- Loop over all days in backtest period
  - Every day: call optimizer to find optimal rebalancing trade given initial position
  - End-of-day position becomes initial position of next day
- Compute P&L

Notation

- x: (n × 1) vector of desired portfolio weights
- w: (n × 1) vector of initial portfolio weights
- Σ: (n × n) covariance matrix of stock returns
- α: (n × 1) vector of aggregate alphas
- β: (n × 1) vector of historical betas
- τ: (n × 1) vector of transaction costs

Objectives and Constraints

- Minimize risk: $x' \Sigma x$
- Maximize exposure to alpha: $\alpha' x$
- Neutralize exposure to beta: $\beta' x = 0$
- Minimize transaction costs: $\tau' |x-w|$
- Other constraints:
  - maximum trade size
  - maximum position size
  - maximum industry and country exposure

Optimization Problem

$$\max_x \; \alpha' x - \lambda \cdot \tau' |x-w| - \mu \cdot x' \Sigma x$$

<u>subject to:</u> $\beta' x = 0$

and other constraints:

- maximum trade size
- maximum position size
- maximum industry exposure
- maximum country exposure

Objective Function

controls turnover

controls book size

$$\max_x \; \alpha' x - \lambda \cdot \tau' |x-w| - \mu \cdot x'\Sigma x$$

maximize alpha exposure

minimize transaction cost

minimize portfolio variance

Maximum Trade Size

- 1% of Average Daily Volume (ADV)
  Can go to 2% if necessary (big book)

- Capped so liquid stocks do not dominate
- Example: cap at $150K
  Can go higher if necessary (big book)

Maximum Position Size

- Multiple of maximum trade size
- I want to be able to liquidate every position in how many days?
- 10 days $\Rightarrow$ 10 × max trade size
- Can be big relative to book size
- Keep balance between liquid and illiquid stocks

min(10 × max trade size, 2.5% of long side of book)

Merge the 2 Constraints

- Max trade size for ith stock: θ_i

$$\Rightarrow \quad w_i - \theta_i \le x_i \le w_i + \theta_i$$

- Max position size for ith stock: π_i

$$\Rightarrow \quad -\pi_i \le x_i \le \pi_i$$

- Enforce both constraints simultaneously:

$$\underbrace{\max(w_i - \theta_i, -\pi_i)}_{\gamma_i} \le x_i \le \underbrace{\min(w_i + \theta_i, \pi_i)}_{\delta_i}$$

Industry Constraints

- Sectors are a factor of risk
- Difficult to time sector performance
- Constrain industry exposure
- But not to zero (too much transaction cost)
- For $50 × 50M book size: r* = $300,000 limit

Industry Dummy

- ρ industries
- Boolean matrix R of dimension $(n \times \rho)$
- $R(i,j) = 1$ if i^{th} stock belongs to j^{th} industry
- $R(i,j) = 0$ if i^{th} stock is outside j^{th} industry
- Every row of matrix R has exactly one entry equal to 1; all other entries are equal to 0
- Constraint: $-r^* \cdot \mathbf{1} \leq R'x \leq r^* \cdot \mathbf{1}$
  where $\mathbf{1}$ = vector of ones of the right dimension

Country Constraints

- Countries are a factor of risk
- Difficult to time country performance
- Constrain country exposure
- But not to zero (too much transaction cost)
- For \$50 × 50M book size: f* = \$100,000 limit
- Tighter than industry exposure

Country Dummy

- φ countries
- Boolean matrix F of dimension $(n \times \varphi)$
- $F(i,j) = 1$ if i^{th} stock belongs to j^{th} country
- $F(i,j) = 0$ if i^{th} stock does not belong to j^{th} country
- Every row of matrix F has exactly one entry equal to 1; all other entries are equal to 0
- <u>Constraint:</u> $-f^* \cdot \mathbf{1} \leq F'x \leq f^* \cdot \mathbf{1}$

Overall Problem

$$\max_x \quad \alpha' x - \lambda \cdot \tau' |x-w| - \mu \cdot x'\Sigma x$$

<u>Subject to:</u>

- beta neutrality: $\beta' x = 0$
- max trade and position: $\gamma \le x \le \delta$
- industry constraint: $-r^* \cdot \mathbf{1} \le R'x \le r^* \cdot \mathbf{1}$
- country constraint: $-f^* \cdot \mathbf{1} \le F'x \le f^* \cdot \mathbf{1}$

Is this standard Quadratic Programming?

Quadratic Programming

- Quadratic programming (QP) is fast, efficient and guaranteed to converge
- Excellent off-the-shelf software
- Matlab optimization toolbox

- <u>Problem: </u>the absolute value in the transaction cost term is not *standard* quadratic programming: $\tau'|x-w|$

Split Variables

- Classic solution: split each variable into 2
- Drawback: twice as many variables
- Advantage: no need to use nonlinear programming
- Define:
  - $y = \max(x-w,0)$
  - $z = \max(w-x,0)$
- Then $y \geq 0$, $z \geq 0$, $x = w + y - z$ and $|x-w| = y+z$

Indeterminacy?

- Initial problem strictly convex
  $\Rightarrow$ unique solution in x
- Twice as many variables:
  solution still unique in y and z?
- Replace y by y+1 and z by z+1
  $\Rightarrow$ x = w + y – z remains unchanged!

- Still OK because |x-w| = y+z penalized

New Formulation

$\max_{y,z}\ \alpha'(w+y-z) - \lambda \cdot \tau'(y+z) - \mu \cdot (w+y-z)'\Sigma(w+y-z)$

Subject to:

- beta neutrality: $\beta\,'\,(w+y-z) = 0$
- max trade and position: $\gamma \le w+y-z \le \delta$
- industry constraint: $-\,r^* \cdot \mathbf{1} \le R'\,(w+y-z) \le r^* \cdot \mathbf{1}$
- country constraint: $-\,f^* \cdot \mathbf{1} \le F'\,(w+y-z) \le f^* \cdot \mathbf{1}$

Very close to standard Quadratic Programming

Standard Quadratic Programming

$$\min_u \; 0.5\, u' H u + g' u$$

Subject to:

- $A u \leq b$
- $C u = d$
- $LB \leq u \leq UB$

Rewrite Optimization Problem

$$\min_{y,z} -\alpha'(y\text{-}z) + \lambda \cdot \tau'(y+z) + 2\mu \cdot w'\Sigma(y\text{-}z) + \mu \cdot (y\text{-}z)'\Sigma(y\text{-}z) + \text{constant}$$

<u>Subject to:</u>

- beta neutrality: $\beta'(y\text{-}z) = -\beta' w$
- max trade and position: $\gamma - w \leq y\text{-}z \leq \delta - w$
- industries: $-r^* \cdot \mathbf{1} - R'w \leq R'(y\text{-}z) \leq r^* \cdot \mathbf{1} - R'w$
- countries: $-f^* \cdot \mathbf{1} - F'w \leq F'(y\text{-}z) \leq f^* \cdot \mathbf{1} - F'w$

Maps into standard Quadratic Programming

Mapping Objective Function

- $u = \begin{bmatrix} y \\ z \end{bmatrix}$

- $H = 2\,\mu \begin{pmatrix} \Sigma & -\Sigma \\ -\Sigma & \Sigma \end{pmatrix}$

- $g = \begin{pmatrix} 2\mu\,\Sigma w - \alpha + \lambda\,\tau \\ -2\mu\,\Sigma w + \alpha + \lambda\,\tau \end{pmatrix}$

Mapping Inequality Constraints

- $A = \begin{pmatrix} R' & -R' \\ -R' & R' \\ F' & -F' \\ -F' & F' \end{pmatrix}$
- $b = \begin{pmatrix} r^* \cdot \mathbf{1} - R'w \\ r^* \cdot \mathbf{1} + R'w \\ f^* \cdot \mathbf{1} - F'w \\ f^* \cdot \mathbf{1} + F'w \end{pmatrix}$

Mapping Equality Constraints

- $C = \begin{pmatrix} \beta' & -\beta' \end{pmatrix}$
- $d = -\beta' w$

Bounds on Optimization Variables

- Lower bound:
  LB = vector of zeros of dimension $(2n \times 1)$

- Upper bound:
  $$UB = \begin{pmatrix} \max(0,\min(\theta,\pi - w)) \\ \max(0,\min(\theta,\pi + w)) \end{pmatrix}$$

Matlab Quadratic Optimizer

- quadprog.m
- No starting point needed
- options = optimset('Algorithm','interior-point-convex')
- options = optimset(options,'Display','iter')

- [u,fval,exitflag,output] = quadprog(H,g,A,b,C,d,LB,UB,[],options)

Other Good Optimizers

- IBM: CPLEX
- FICO: Xpress
- Sunset: XA
- Stanford: QPOPT, SQOPT and MINOS
- Roger Fletcher: BQPD
- KNITRO

- Not cheap!

Plan of the Rest of Lecture 07

1. Complement on Portfolio Optimization

2. Short-Term Mean-Reversion Alpha

3. Analyst Recommendations

4. Audits and Controls

Follow-up on Optimization

- Dynamic Trading with Predictable Returns and Transaction Costs
- Nicolae Gârleanu, Finance Professor, Haas School of Business , University of California Berkeley
- Lasse Heje Pedersen, Finance Professor, Stern School of Business , New York University
- *Journal of Finance* (2013)

Non-Myopic Trading Rule

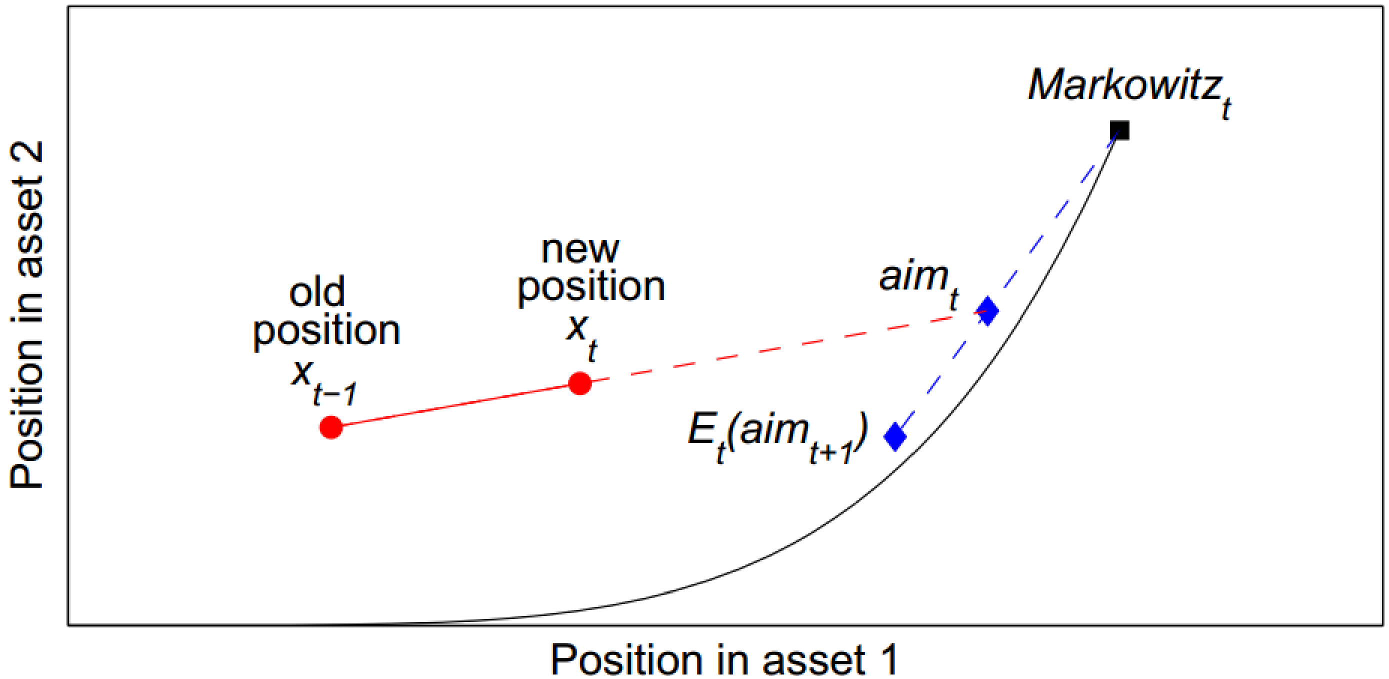
- Some alphas have a quick half-life (two weeks)
- Other alphas are very sticky, e.g., value (>1 year)

- Not just take into account the value of your alphas today
- Also consider the expected future evolution of your alphas

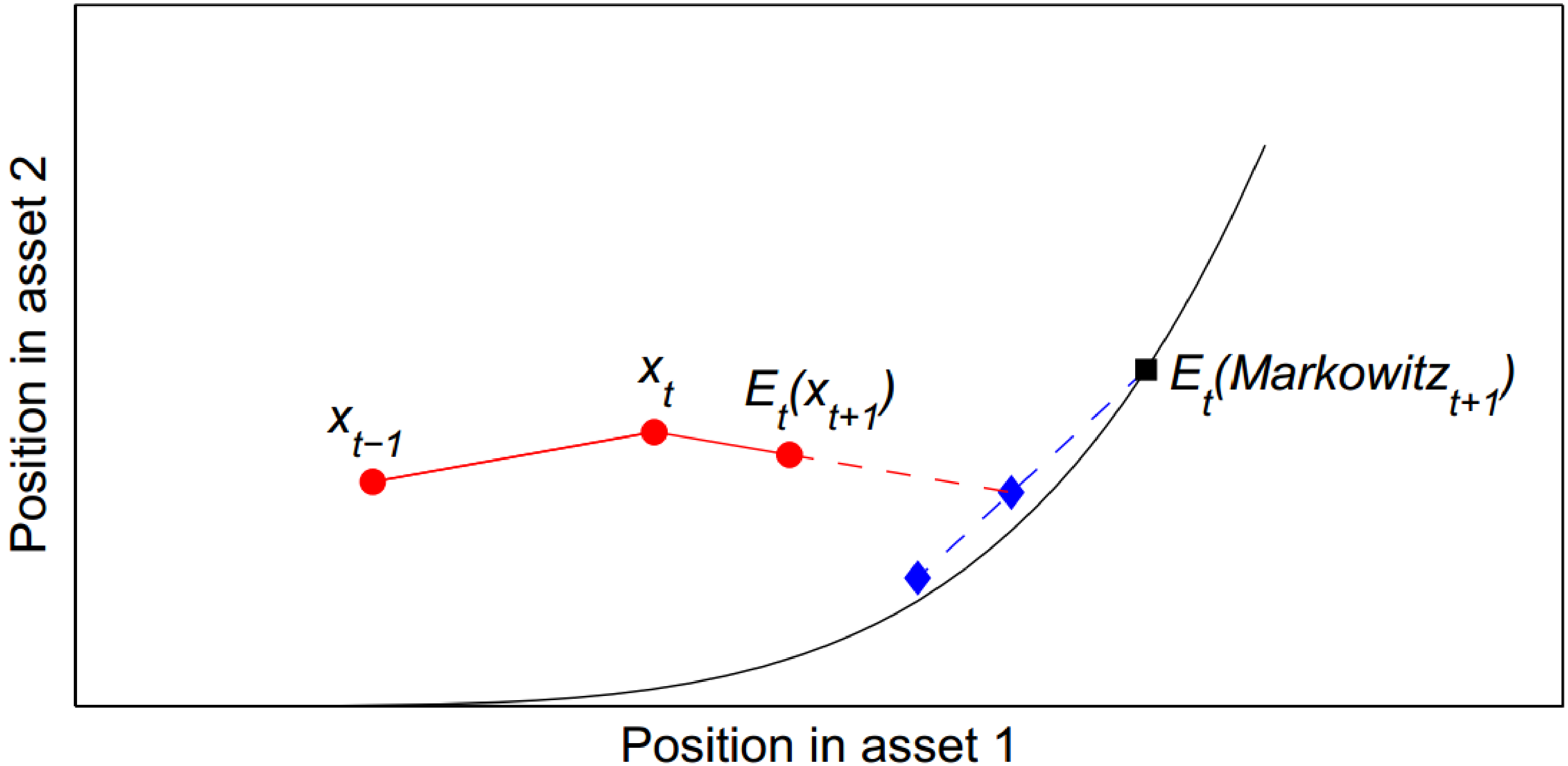
Hunter aims ahead of the duck

- The aim portfolio is a weighted-average of the current and future Markowitz portfolios

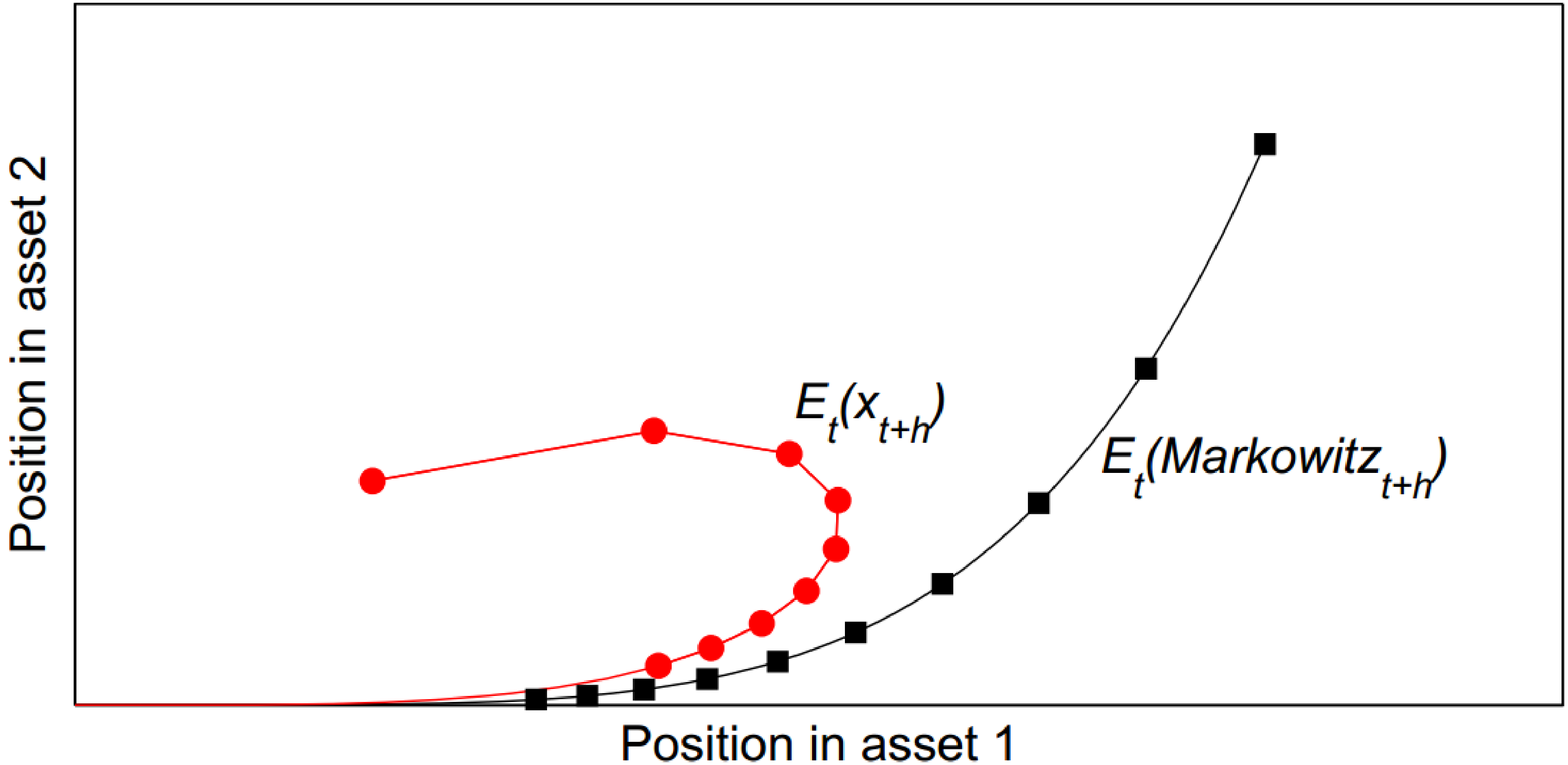
Panel A: Construction of Current Optimal Trade



Expected Next Optimal Trade



Expected Evolution of Portfolio



Bottom Line

- More advanced method is useful if you trade very big portfolio (>$500M)

- For small-to-medium size portfolios: Can be approximated by underweighting fast-decay signals

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Short-Term Mean Reversion Alpha

- Bruce Lehmann (1990) “Fads, Martingales and Market Efficiency” *Quarterly Journal of Economics*
- Stocks that went up (down) relative to their industry peers over the past 21 days will underperform (outperform) going forward
- High-turnover factor, but very strong!
- Works even better when there is high volatility

Differences with Pairs Trading

- Pairs trading looks at cointegration
- Reversion is relative to all the stocks in the same industry, not just one
- Pairs trading has entry/exit points
- Reversion uses all the stocks, whereas pairs trading selects certain pairs

Industry Dummy

- ρ industries
- Boolean matrix R of dimension $(n \times \rho)$
- $R(i,j) = 1$ if i^{th} stock belongs to j^{th} industry
- $R(i,j) = 0$ if i^{th} stock does not belong to j^{th} industry
- Every row of matrix R has exactly one entry equal to 1; all other entries are equal to 0

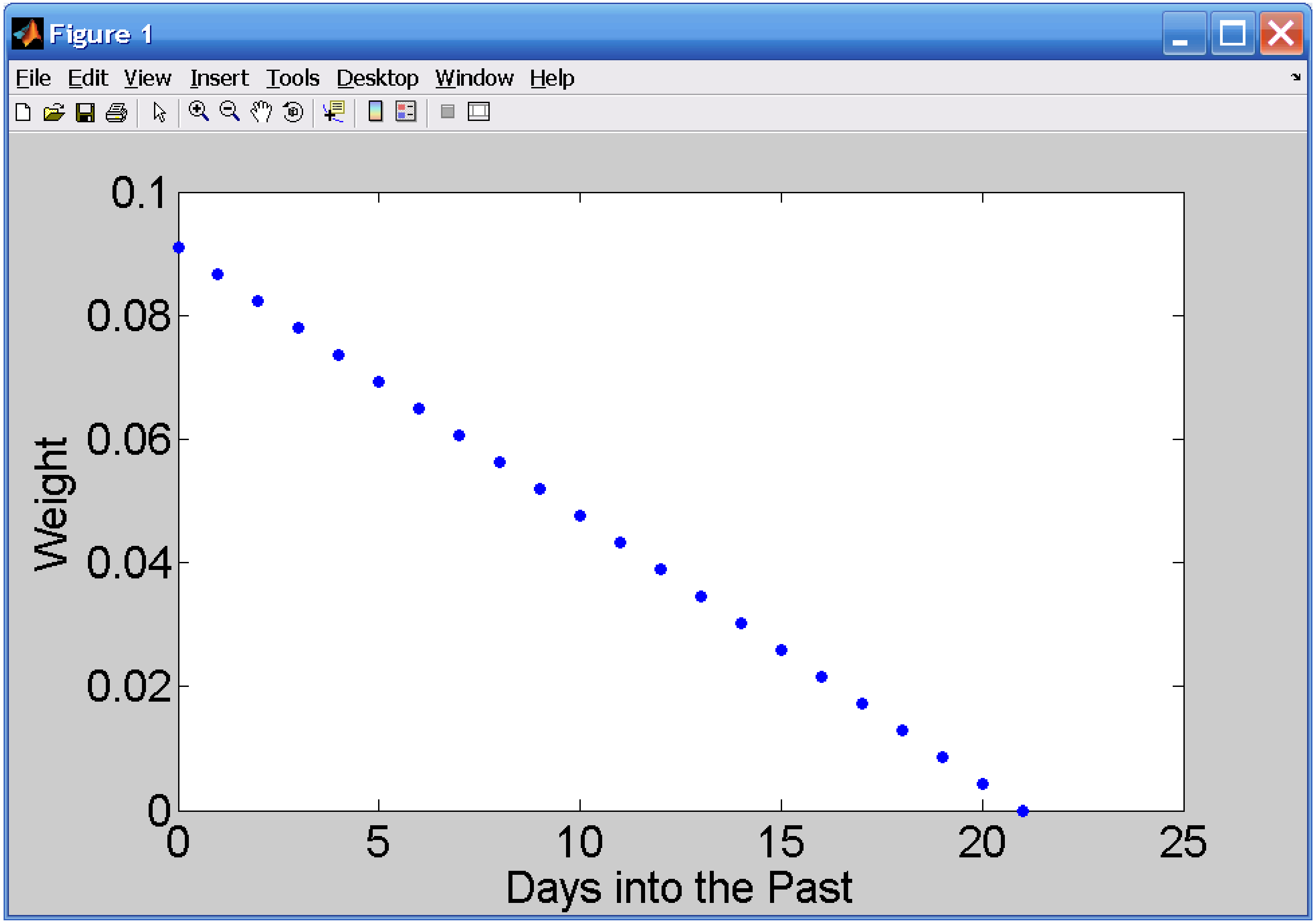
Mathematical Definition

- r_{ti} = arithmetic return on day t for stock i using Total Return Index
- $\underline{\alpha}_{ti} = -\left(r_{ti} + r_{t-1,i} + \ldots + r_{t-20,i}\right) / 21$
- $\underset{(1\times n)}{\alpha_t} = \underset{(1\times n)}{\underline{\alpha}_t} \cdot \underset{(n\times n)}{\left[\mathrm{I} - R\,(R'\,R)^{-1}\,R'\right]}$
- Cross-sectionally demean, standardize and windsorize every day

How to Improve It

- Triangular Decay Window
- Linearly decreasing weighting scheme so that weight on day t–21 is zero
- weight on day t–j: $w_j = a - b \cdot j$
- $w_{21} = a - b \cdot 21 = 0$
- Weights sum to one: $w_0 + w_1 + \ldots + w_{20} = 1$
- $a \cdot 21 - b \cdot (0+1+2+\ldots+20) = 1$
- Solution: $w_j = (1/11) - (1/231) \cdot j$ for $j=0,\ldots,21$

Underweight Distant Past



Improved Reversion Alpha

- r_{ti} = return on day t for stock i
- $\underline{\alpha}_{ti} = -\left(w_0 \cdot r_{ti} + w_1 \cdot r_{t-1,i} + \ldots + w_{20} \cdot r_{t-20,i}\right)$

- $\underset{(1\times n)}{\alpha_t} = \underset{(1\times n)}{\underline{\alpha}_t} \cdot \underset{(n\times n)}{\left[I - R\,(R'\,R)^{-1}\,R'\right]}$

- Cross-sectionally demean, standardize and windsorize every day

Other Refinements

- Truncate stock returns that are too extreme
- If stock outperformed equally weighted average return of all active stocks that day by more than 5%, say it only outperformed by 5%
- If stock underperformed equally weighted average return of all active stocks that day by more than -5%, say it only underperformed by 5%

Cleaning the Reversion Alpha

- b_{ti} = market beta of stock t on day i
- Beta-driven moves do not revert because the market index follows a random walk
- Can improve reversion alpha by cleaning it with respect to beta:
- Use $\underset{(1\times n)}{\alpha_t} \cdot \underset{(n\times n)}{\left[I - b_t\,(b_t'\,b_t)^{-1}\,b_t'\right]}$
- Can also clean reversion alpha w.r.t. momentum

Information Moves Do Not Revert

For the purpose of computing the reversion alpha, set the return on an earnings announcement day to zero

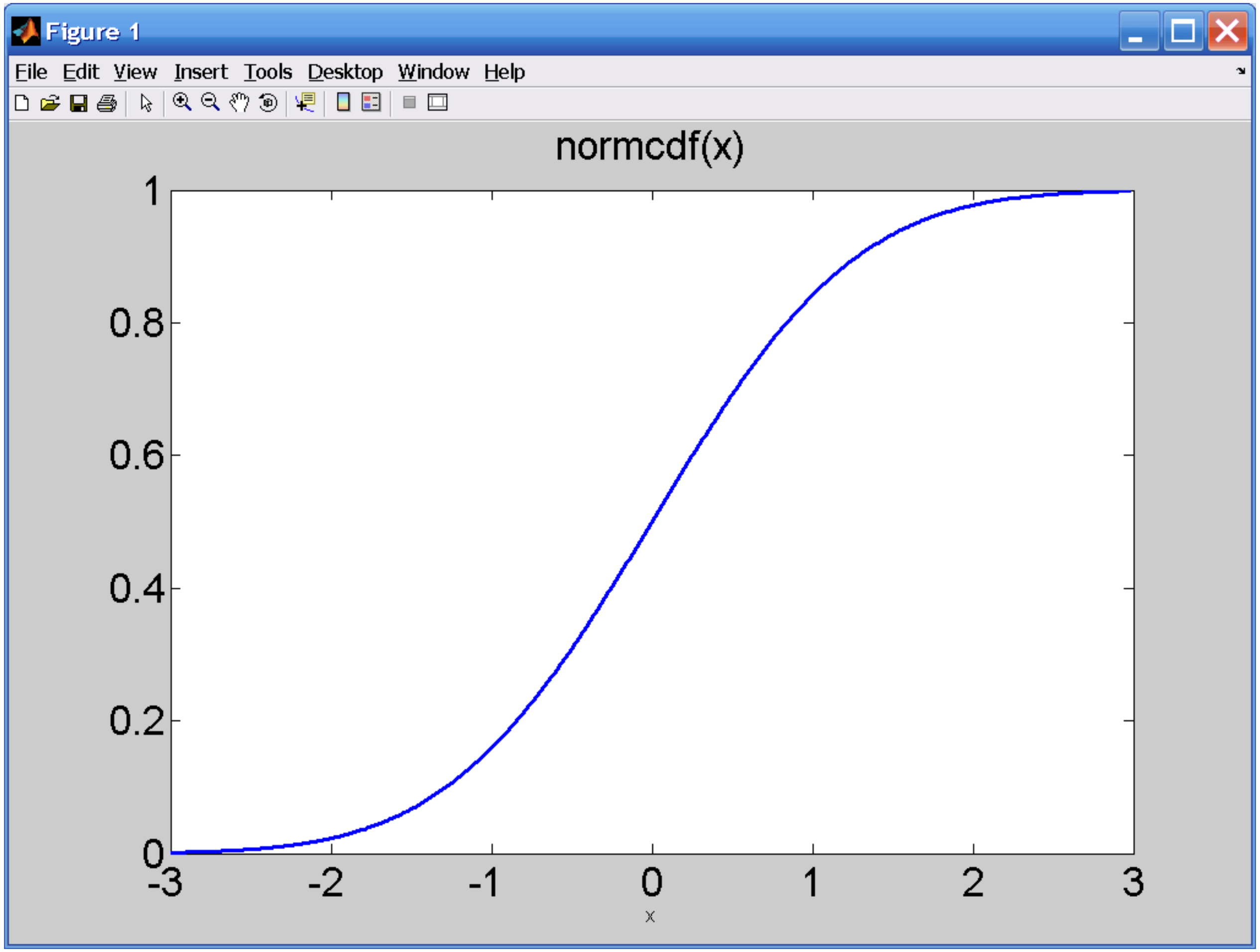
Modulating the Reversion Alpha

- Volume and Autocovariances in Short-Horizon Individual Security Returns
- Journal of Finance (1994)
- Jennifer Conrad, Allaudeen Hameed and Cathy Niden
- High-volume stocks experience price reversals, while the returns of low-volume securities are positively autocovarying

General Approach

- Suppose you have some variable θ_{ti} which tells you when the signal works and when it doesn't work
- Cross-sectionally demean, standardize and windsorized
- Modulated alpha: $\alpha_{ti} \times \text{normcdf}(\theta_{ti})$

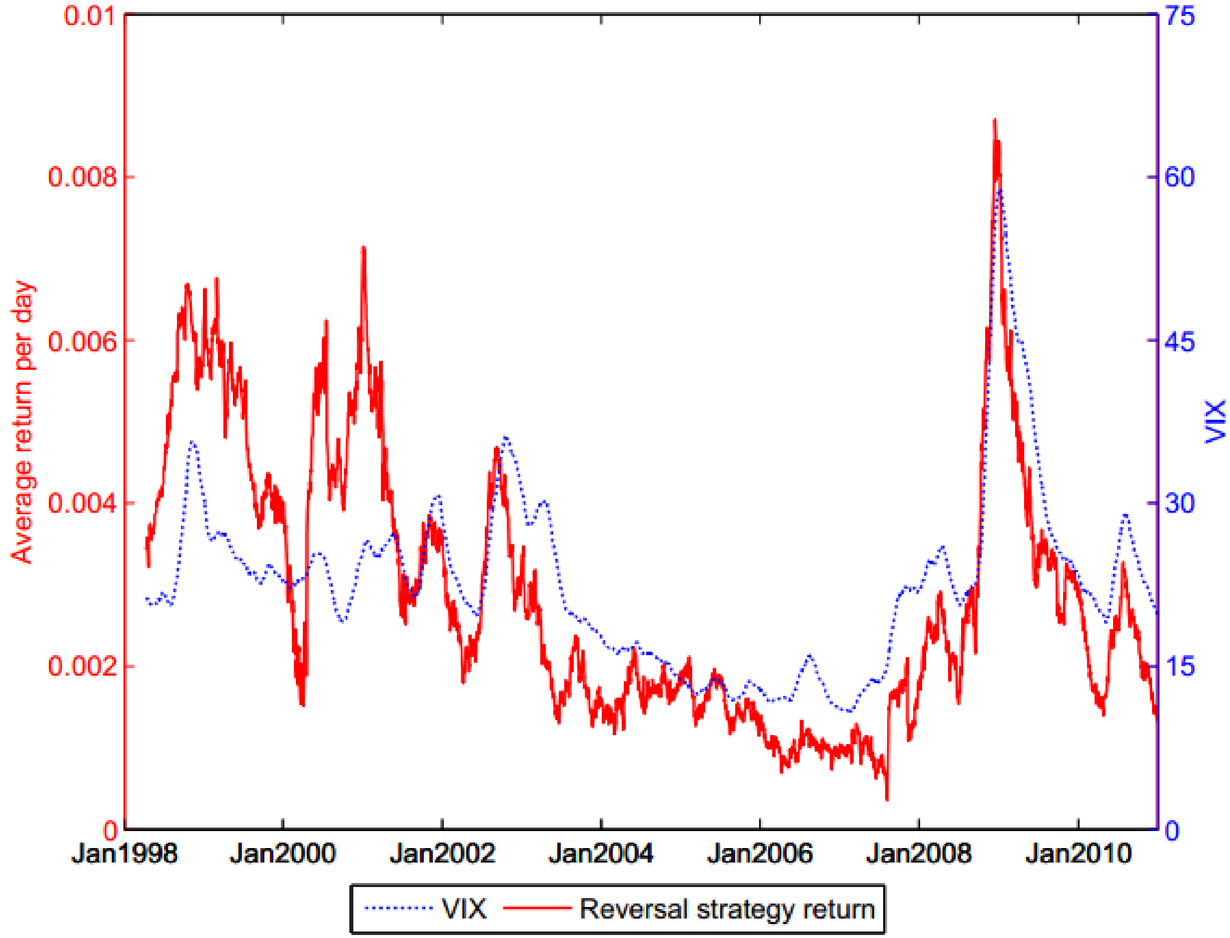
Normal Cumulative Distribution Function



When Reversion Works Well

- Evaporating Liquidity, Stefan Nagel (Stanford)
- Review of Financial Studies (July 2012)
- Expected return from reversal strategies is strongly time-varying and highly predictable with the VIX index
- Reversion works better when there is more volatility
- Reversion profit = reward for providing liquidity

5-Day Reversal



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Recommendation Revision Alpha

- Analyzing the Analysts: When Do Recommendations Add Value?
- Narasimhan Jegadeesh, Joonghyuk Kim, Susan D. Krische and Charles M. C. Lee
- Journal of Finance (2004)
- Recommendation levels have no predictive power
- Recommendation changes do!

6-month return post revision

Panel C: Market-adjusted Returns by Consensus Recommendation Change Quintile			
Quintile	Coded as	Mean	Median
Best = Increase	1.00	−0.004	−0.025
	0.75	−0.007	−0.015
	0.50	−0.022	−0.044
	0.25	−0.004	−0.023
Worst = Decrease	0.00	−0.031	−0.051
Increase – Decrease		+0.027***	+0.031***

Mathematical Definition

- x_{ti} = number of upgrades – number of downgrades on day t for stock i
- $\alpha_{ti} = (x_{ti} + x_{t-1,i} + \ldots + x_{t-44,i}) / 45$
- Cross-sectionally demean, standardize and windsorize every day

Triangular Decay

- x_{ti} = number of upgrades – number of downgrades on day t for stock i
- $w_j = (1/23) - (1/1035) \cdot j$ for j=0,...,45

$$\alpha_{ti} = w_0 \cdot x_{ti} + w_1 \cdot x_{t-1,i} + \dots + w_{44} \cdot x_{t-44,i}$$

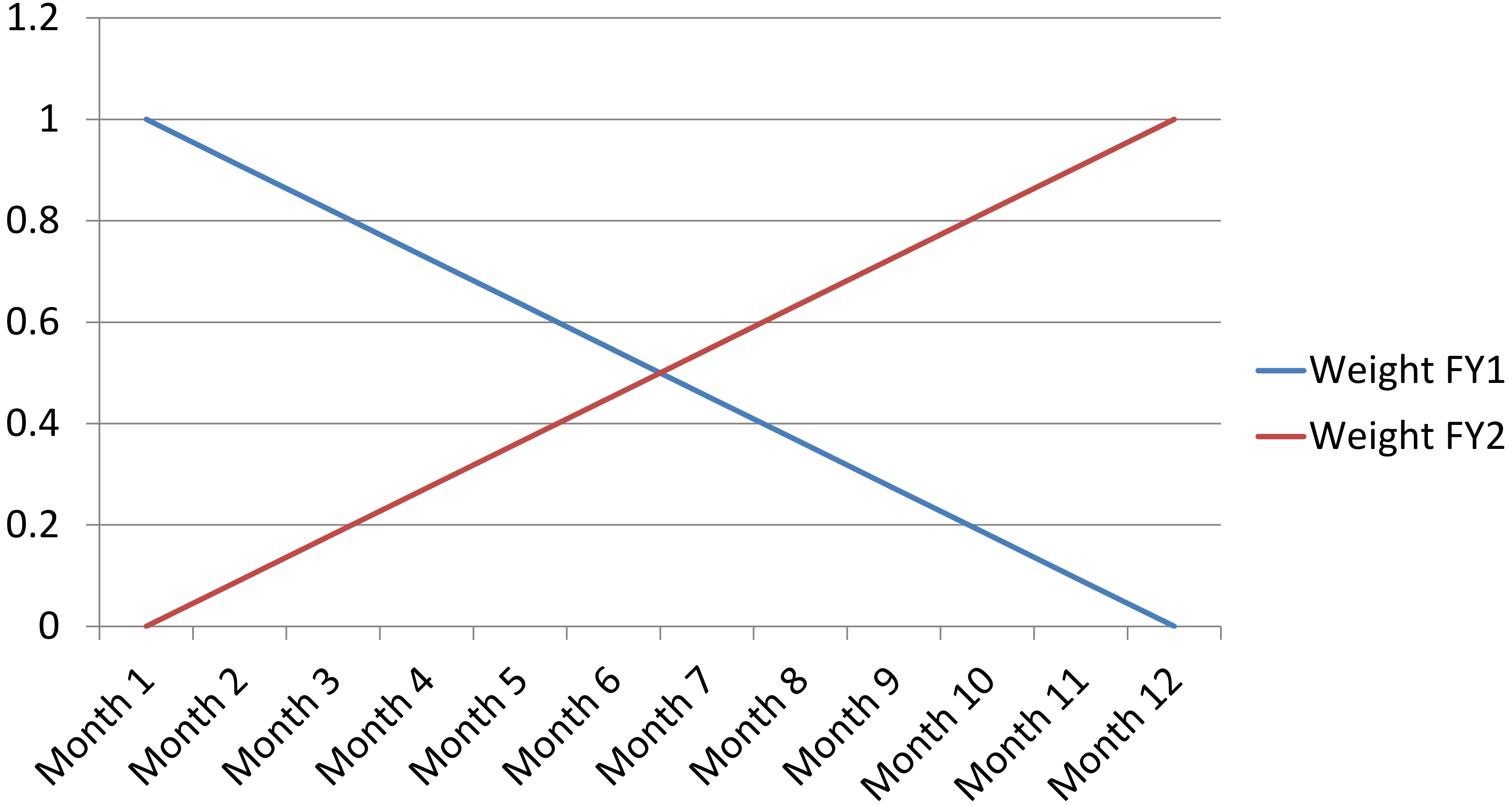
- Cross-sectionally demean, standardize and windsorize every day

Variations on the Theme

- Analyst price target revisions

- Analyst earnings forecast revisions
- Analyst sales forecast revisions
- Analyst dividends forecast revisions
- Analyst cash flow forecast revisions

Combine Next 2 Fiscal Years



Modulating Factors

Analyst Forecast Revisions and Market Price Discovery (2003), Cristi Gleason & Charles Lee

- Whether the revision moves towards the consensus or away from it
- Whether the analyst is a celebrity or not
- Number of analysts covering the stock

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Pre-Trade

- Output the trade to an Excel spreadsheet
- Display summary statistics :
  - buys ($ value, number of stocks, average size)
  - sells ($ value, number of stocks, average size)
  - long position after the trade ($ value, number of stocks, average size)
  - short position after the trade ($ value, number of stocks, average size)
- Sort trade with biggest buys/sells on top

Visual Inspection

- Do you see anything out of the ordinary?
- This should be quick (<30 seconds if nothing is out of the ordinary)
- Then press button to submit Excel spreadsheet to electronic broker

Intraday Trade Impact Graph

- Use snapshot price as reference
- If you could execute all your trade at snapshot price then price impact would be zero
- By showing order to market, you push prices away from you:
  - on average, buys go up
  - on average, sells go down
- Plot second-by-second graph showing impact

Fills/Positions

- Compare trade execution report received from broker with order you sent out
- Should be nearly 100% filled
- Should not be any overfills

- Compare back-office position with what you think your position is
- Reconcile any differences (stock splits, etc)

P&L

- Intraday P&L Graph updated every second
- Compare P&L number generated by back office to what you think your P&L is
- Reconcile any differences (dividends, etc)

P&L Convergence

- Plot realized cumulative P&L over past 250 days with backtest cumulative P&L over the same period
- If you upgraded your model, realized should be below backtest
- Difference should stabilize over recent days

Inventory Convergence

- As a function of time, plot L^1 distance between positions held in reality, and positions the backtest thought you should have held
- Do it for the past 250 days
- You should see that difference converge to zero over the recent past
- Means you’re trading what you think you’re trading

Trade Convergence

- Same concept, but for trade instead of inventory
- Compute L^1 distance between trade really executed on a given day and trade that your (current) backtest says you should have done
- Should converge towards zero in the recent past
- Converges less quickly than inventory

Transaction Cost

- Plot cumulation of the difference between t-cost your model says you should have paid, vs. what you actually paid
- Should be pretty flat overall
- Also do it broken down by country
- And by liquidity buckets (illiquid/medium/very liquid)

Do you have market exposure?

- At the end of the day, plot your P&L over each 10-minute interval vs. the market return
- Is there a pattern: e.g., are you making money when the market is going up?
- Compute your intraday beta (very noisy!)
- Plot the cumulative sum of all your intraday betas over the past 250 days
- Should be random walk, neither up nor down

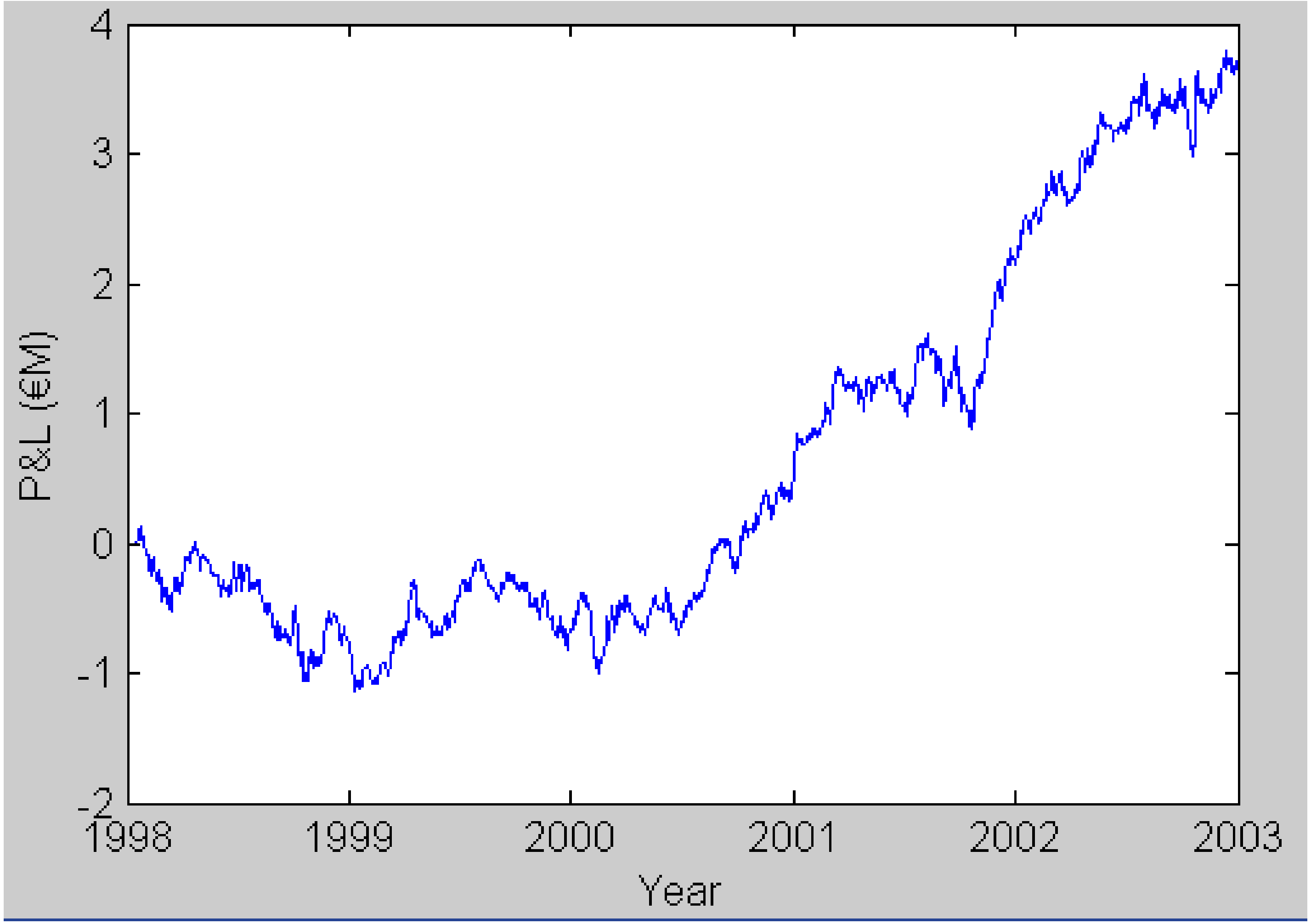
How could this happen?

- If your alphas are correlated with the estimation error in your betas
- Remedy: instead of having constraint in the optimizer that portfolio beta is zero, shoot for an offsetting target beta

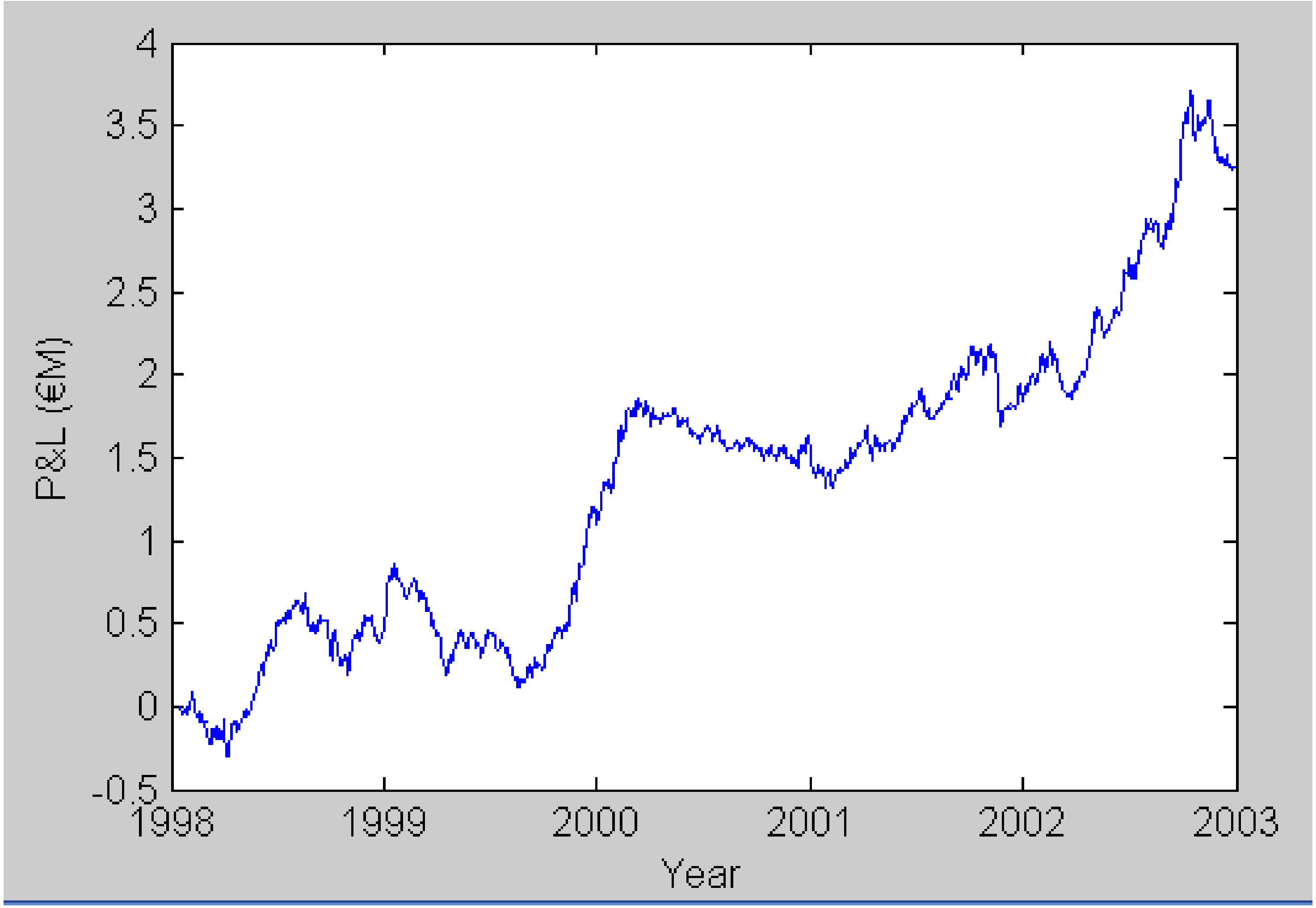
Marginal Contributions

- Run backtest with all alphas
- Then run it again with one alpha removed
- Plot the difference
- It should go up if the alpha is good

Marginal Contribution of Value



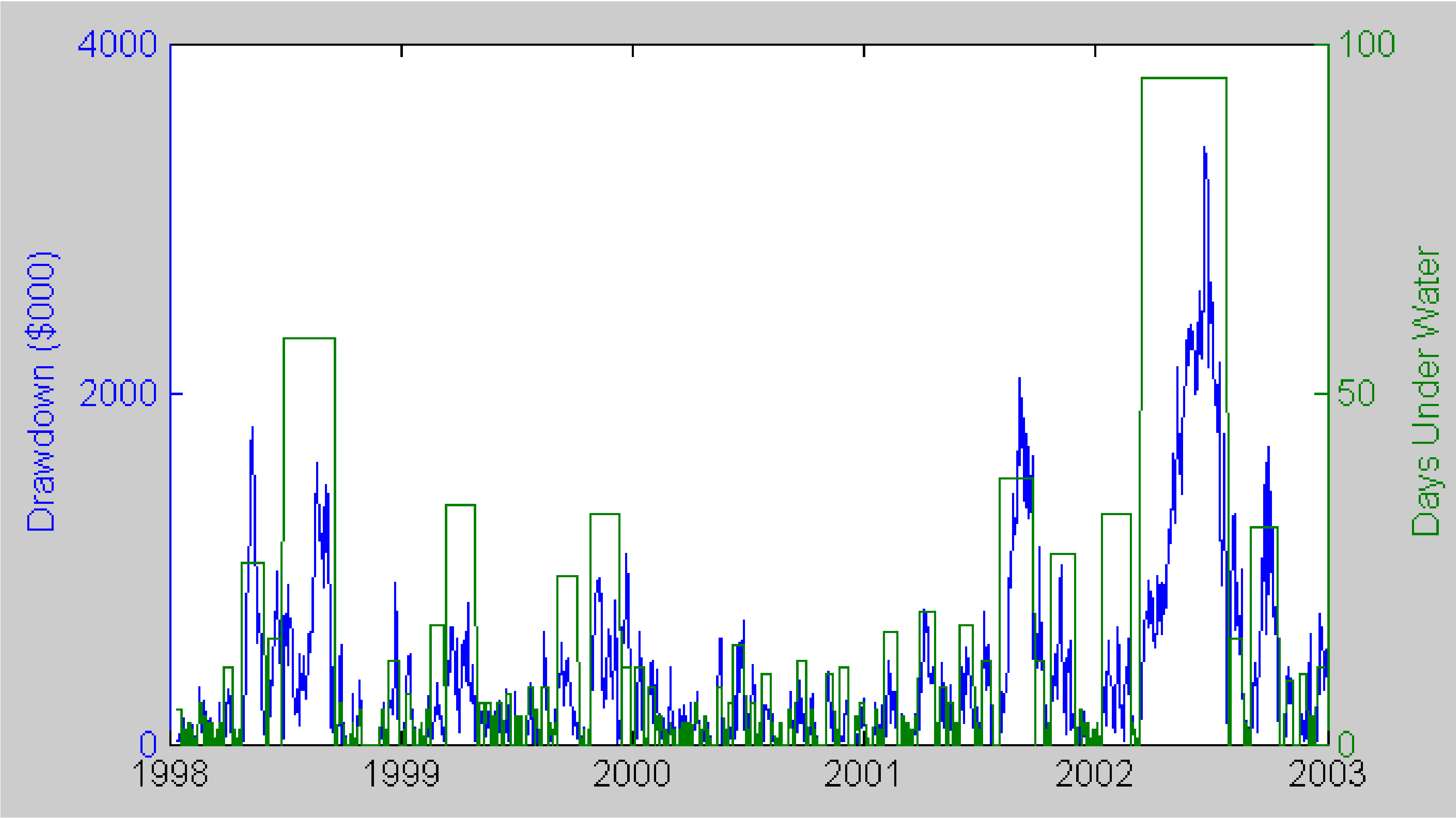
Contribution of Momentum Alpha



Drawdown

- High-watermark = maximum cumulative P&L reached on or at date t
- If P&L at date t < high-watermark then you have entered a drawdown
- Maximum drawdown duration?
- Maximum loss in a drawdown relative to previous high-watermark

Drawdown Graph



Worry Index

- Is the past week’s performance worrisome?
- Is the past month’s performance worrisome?

- For k = 1 to 25, compare the P&L over the past k days to all k-days P&Ls in the backtest
- Find out what percentile it ranks at
- Report the worst percentile
- <u>Example:</u> past 8 days performance was worse than 76% of all 8-days performances

Adjusted Worry Index

- In this example, 76% is the raw worry index
- Problem: biased upwards (largest out of 25)
- Worry too often $\Rightarrow$ cry wolf!
- Solution: compute raw worry index for every day in the past 5 years
- What percentile is today's raw worry index relative to distribution of raw worry index?
- Adjusted worry index: 50% = neutral