MFE 409 Lecture 3
Risk for Options

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Lecture Objectives

Risk management for option trading

- What are the risks of option strategies?
- How to quantify these risks?

Trading Derivatives and Risk Management

- Two broad levels of risk management inside financial institutions
  - Trader level: (hard) risk limits
  - Institution level: aggregate positions and construct broad measures of risk

Trading Derivatives and Risk Management

- Two broad levels of risk management inside financial institutions
  - Trader level: (hard) risk limits
    - ⋆ Often expressed in terms of Greeks
  - Institution level: aggregate positions and construct broad measures of risk
    - ⋆ Often around VaR

Delta

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- Buying $-\Delta$ of the underlying protects the portfolio against local changes in underlying price

- Can also hedge with another option

Linear Products

- If the value of the portfolio is linear in the price of the underlying, delta-hedging eliminates all risk
- Examples: forwards, futures, fixed promises in foreign currency, ...
- Static hedging works perfectly: "hedge and forget"

Example

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VaR for Options: Delta Approach

- Portfolio:
  - Long EUR10m, EUR/USD $= 1.436$, volatility of EUR/USD 0.65%)
  - Short 10m in puts to sell euros in 6m, $\Delta = -0.5044$
- 1% VaR?

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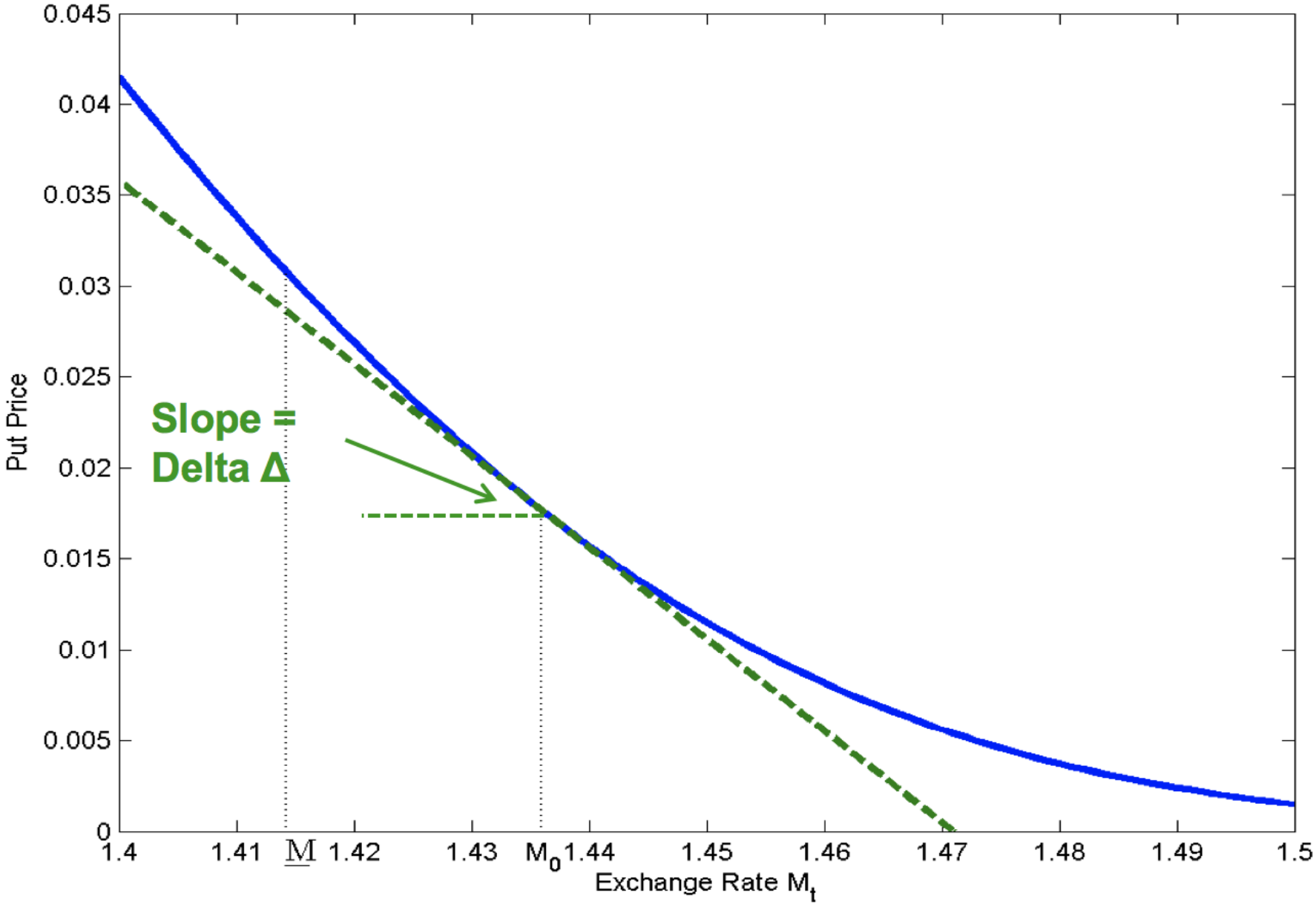
Put Price: Delta Approximation



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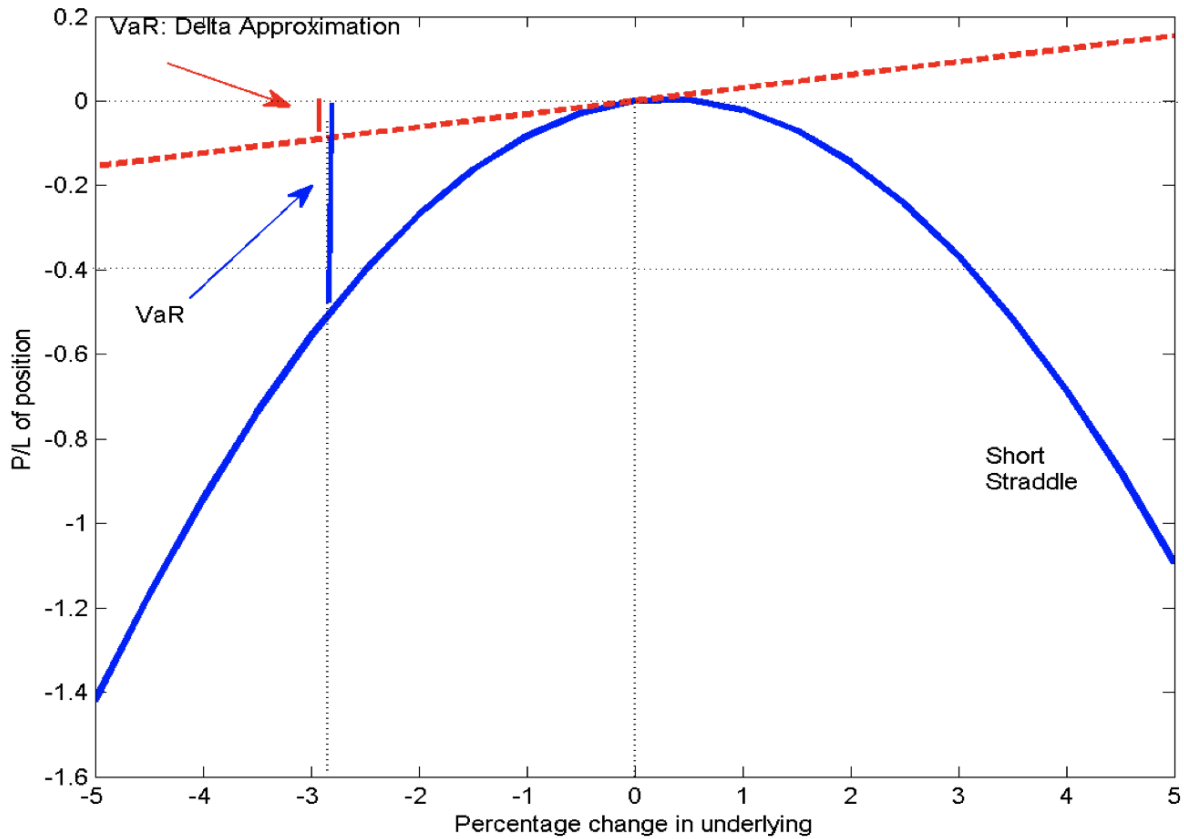

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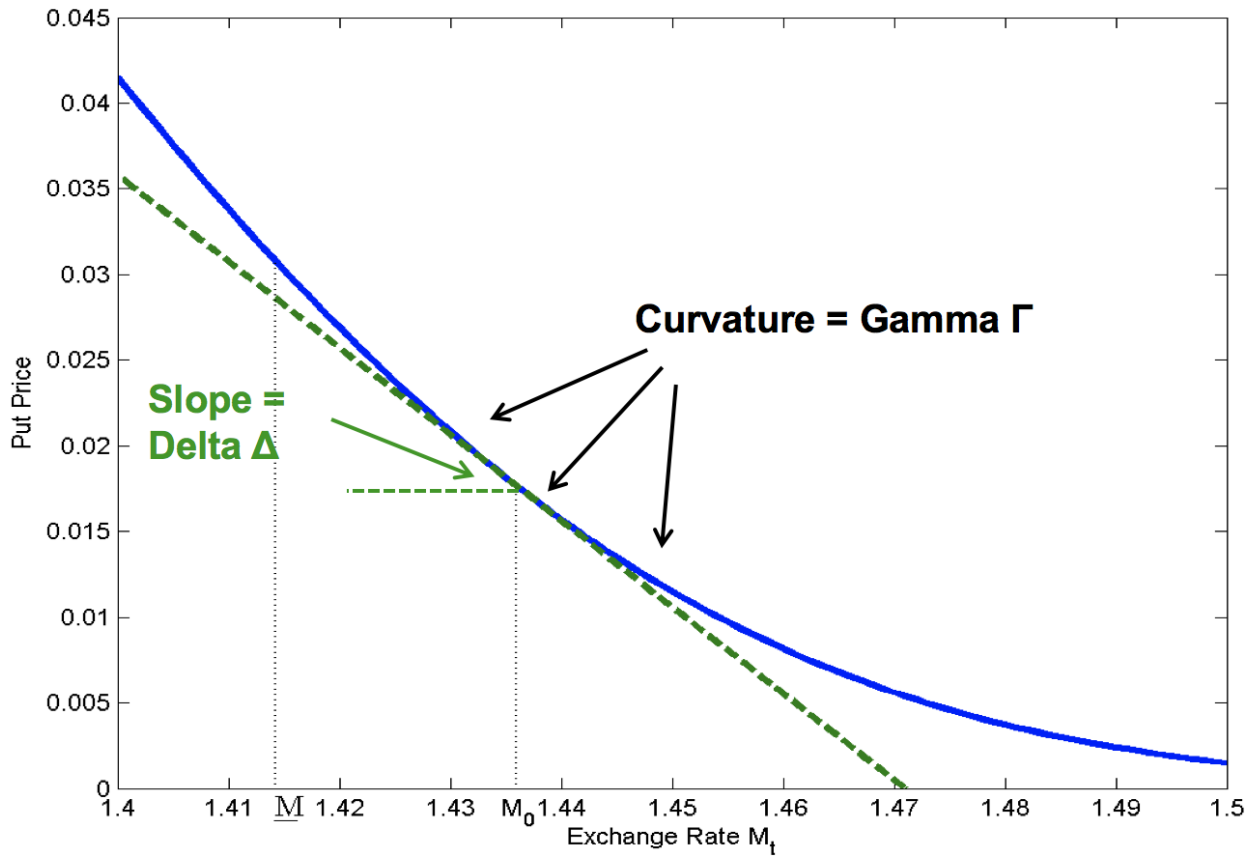
Put Price: Delta Gamma Approximation

$$P(S) \approx P(S_0) + \underbrace{P'(S_0)}_{\Delta}(S - S_0) + \frac{1}{2}\underbrace{P''(S_0)}_{\Gamma}(S - S_0)^2$$



VaR for Options: Delta-Gamma Approach

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- How to incorporate into VaR calculation?
- **Cornish-Fisher expansion**: asymptotic expansion for the quantile of a distribution
  - Skewness: $\xi_P = \mathbb{E}[(R_P - \mu_P)^3]/\sigma_P^3$
  - Quantile $1 - c$:

$$\mu_P + \left(z(1-c) + \frac{1}{6}\left(z(1-c)^2 - 1\right)\xi_P \right)$$

Vega

- Vega (ν): derivative of option value with respect to the volatility of the underlying asset

$$\nu = \frac{\partial P}{\partial \sigma}$$

- Under the assumptions of Black-Scholes, there is no risk of change in volatility ... but in practice volatility can move

- We can add changes in volatility to our previous calculations:

Other Greeks

- Theta (Θ): change of the value of the portfolio due to passage of time:

$$\Theta = \frac{\partial P}{\partial t}$$

- Rho: change of the value of the portfolio due to a parallel shift in all interest rates in a particular country

$$\text{Rho} = \frac{\partial P}{\partial r}$$

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- Traders must keep Gamma and Vega within limits set by risk management
  - Adjust whenever the opportunity arises
- Delta can be adjusted by trading the underlying
- Gamma and Vega need trading of other options

Takeaways

- When trading options, identify the key risks and hedge them
- Think one step ahead and about potential large shocks: gamma-hedging
- For risk management: crucial to take into account the non-linearity of option contracts