# Lecture 4

# Momentum and Long-Term Reversal: Violations of the Random Walk Hypothesis

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# Overview of Lecture 4

Autocorrelation in financial asset returns

1. Momentum revisited

2. Testing the Random Walk Hypothesis
   - Variance ratios
   - Are stocks less risky in the long run?
   - Long-term reversal

# Momentum Anomaly in Stocks

- Can we exploit violations of weak market efficiency?

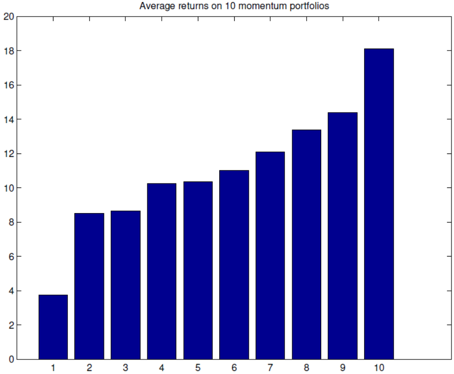
- there is a small amount of positive autocorrelation in individual monthly stock returns
  - At shorter horizons of less than 12 months, stock returns tend to be weakly positively autocorrelated.

- (in theory) this can be exploited to construct profitable trading strategies

# Momentum Anomaly in Stocks

- Ken French posts momentum portfolios on his web site
  - The portfolios at $t$ are constructed monthly using NYSE prior ($t-2$ to $t-12$) return decile breakpoints.
- this is called (cross-sectional) **momentum trading**
  - first discovered by Werner de Bondt, a Belgian economist now at DePaul University in Chicago, and Richard Thaler, of the University of Chicago Booth School of Business. See De Bondt and Thaler (1985).

# Momentum in US Stock Returns




Average returns on momentum portfolios. Source: data from Kenneth French's website. The portfolios are constructed monthly using NYSE prior (2-12) return decile breakpoints. Sample: 1927-2013.

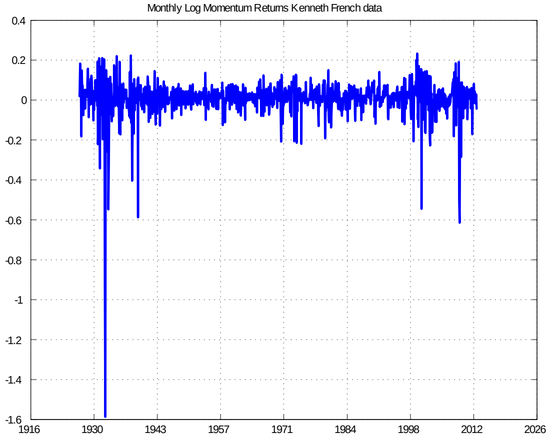
# Momentum Sorting

- **Cross-sectional momentum**, by sorting stocks into portfolios based on past performance, basically exploits (small) positive autocorrelation at short horizons between 1 and 12 months.
  - To learn more about the time-series origins of cross-sectional momentum, see Moskowitz, Ooi, and Pederson (2012).

# Momentum Factor Structure

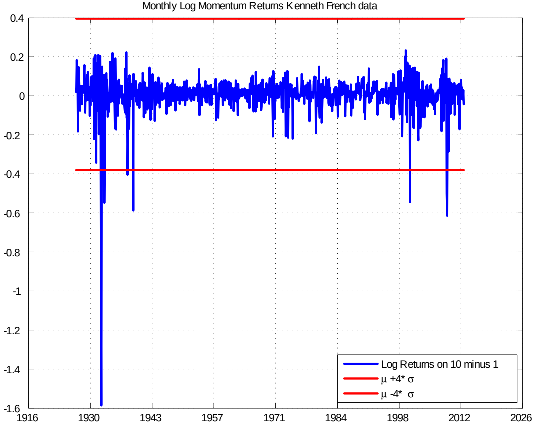
- momentum stocks have a factor structure:
  - high momentum stocks co-move
  - low momentum stocks co-move
- this risk cannot be diversified away
- some of this may be tail risk: Daniel, Jagannathan, and Kim (2012).
- not a free lunch
- maybe momentum returns compensate for tail risk

# Momentum Risk




Log Returns on Portfolio 10 minus 1. data from Kenneth French's website. The portfolios are constructed monthly using NYSE prior (2-12) return decile breakpoints. Sample: 1927-2013.

# Tail Risk in Momentum




Log Returns on Portfolio 10 minus 1. data from Kenneth French's website. The portfolios are constructed monthly using NYSE prior (2-12) return decile breakpoints. Sample: 1927-2013. $\sigma$ is 0.0971. $\mu$ is 0.0079. The skewness is –6.39. The kurtosis is 86.66.

# Momentum Factor Structure

- This momentum strategy works well across several asset classes.
- Asness, Moskowitz, and Pedersen (2013) document the pervasiveness of momentum effects in
    - currencies,
    - commodities,
    - bonds.
- We've seen autocorrelation patterns in returns. Next, we consider formal tests of the Random Walk Hypothesis and further implications of its rejection

# The Random Walk

## Definition

The logarithm of prices follows a **random walk with drift** and has normally distributed increments when $p_t$ is given by:

$$p_t = \mu + p_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

or equivalently

$$r_t = \Delta p_t = \mu + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

- (demeaned) changes in $p_t$ (demeaned $r_t$) are white noise
- drift $\mu$
- variance $\sigma_\varepsilon^2$
- random walk sets drift to zero $\mu = 0$

# The Random Walk Hypothesis

- if stock prices are a random walk, log-returns should have no autocorrelation.

- estimate the sample auto-covariance:

$$\widehat{\gamma}_k = \frac{1}{T} \sum_{t=1}^{T-k} (r_t - \overline{r})(r_{t+k} - \overline{r})$$

- estimate the sample auto-correlation:

$$\widehat{\rho}_k = \frac{\widehat{\gamma}_k}{\widehat{\gamma}_0}$$

- we could use the Ljung-Box statistic to test the null that all autocorrelations are zero

# The Random Walk Hypothesis

## Definition

The **Ljung-Box** test statistic tests the null that $H_0 : \rho_1 = \ldots = \rho_m = 0$:

$$Q(m) = T(T+2)\sum_{j=1}^{m} \frac{\widehat{\rho}_j^2}{T-j}$$

$Q(m)$ is asymptotically $\chi^2$ with $m$ degrees of freedom.

# The Random Walk Hypothesis

- if prices are a random walk, then they never revert back to a long-run level.
- is there mean reversion in returns?
  - do negative returns tend to be followed by positive returns?
- does the risk-return trade-off improve over longer holding periods?
- are stocks less risky in the long-run?

# Holding Periods

- the log return over holding period $k$ is just the sum of the one-period holding returns

$$r_t(k) = r_t + r_{t+1} + \ldots + r_{t+k}$$

- we will look mostly at log excess returns:

$$r_t^e(k) = [r_t + r_{t+1} + \ldots + r_{t+k}] - [r_t^f + r_{t+1}^f + \ldots + r_{t+k}^f]$$

- to check for mean reversion, we examine the properties of log returns over longer holding periods

- we will examine the variance ratio (over horizon $q$) of log returns:

$$VR(q) = \frac{V\left[r_t\left(q\right)\right]}{q \times V\left[r_t\right]}$$

# Holding Periods

- recall that the variance of longer holding period log returns is a linear function of time in case of a random walk
- if the log return $r_t = p_t - p_{t-1}$ is *IID*
  - the variance of the sum of log returns satisfies:

$$V[r_t + r_{t+1}] = V[r_t] + V[r_{t+1}] = 2 \times V[r_t]$$

  - **independence**: covariance terms are zero
  - **identically dist**: same variance each period
  - similarly for $k$ holding period log returns:

$$V[r_t + r_{t+1} + \ldots + r_{t+k}] \; = \; V[r_t] + V[r_{t+1}] + \ldots + V[r_{t+k}] \; = \; k \times V[r_t]$$

# Variance Ratios and Autocorrelations

What happens if returns are autocorrelated but with the same variance?

the variance of the sum of log returns satisfies:

$$V\left[r_t + r_{t+1}\right] = V\left[r_t\right] + V\left[r_{t+1}\right] + 2\text{Cov}\left[r_t, r_{t+1}\right]$$

- **dependent**: covariance term is not zero
- **identically dist**: same variance each period

Divide both sides by $2V\left[r_t\right]$

$$\frac{V\left[r_t + r_{t+1}\right]}{2V\left[r_t\right]} = 1 + \rho_1$$

# Variance Ratios and Autocorrelations

In general, the variance ratios can be stated as follows:

$$VR(q) = \frac{V\left[r_t\left(q\right)\right]}{qV\left[r_t\right]} = 1 + 2\sum_{k=1}^{q-1}\left(1 - \frac{k}{q}\right)\rho_k.$$

the VR is a linear combination of the autocorrelation coefficients with linearly declining weights

- negative autocorrelation pushes the VR below one (mean reversion)
- positive autocorrelation pushes the VR above one (mean aversion)

# Variance Ratios and Autocorrelations

- consider the 2-period holding return

- if log prices follow a random walk (*RW1*), then the variance ratios :

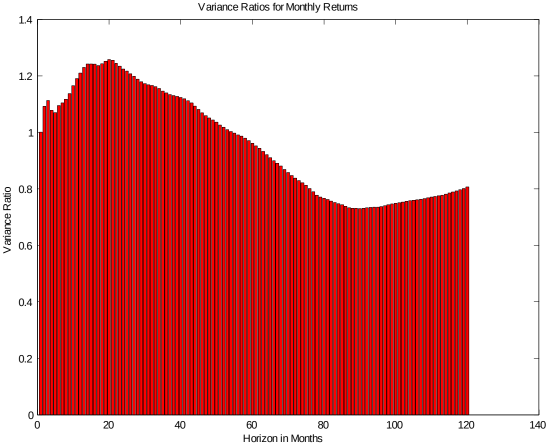
$$VR(2) = \frac{var[r_t(2)]}{2var[r_t]} = \frac{var[r_t + r_{t+1}]}{2var[r_t]} = 1 + \rho_1$$

  are equal to one because independence implies $\rho_1 = 0$.

- this provides a testable implication

# Variance Ratios of Monthly Log Excess Returns

- Some evidence of mean-reversion in log returns




VR(q) for Monthly log excess returns on VW-CRSP Index. 1926-2012. Monthly data.

# Variance Ratios of Monthly Log Excess Returns

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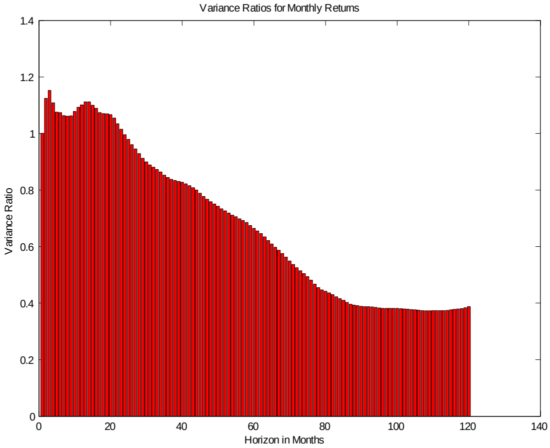
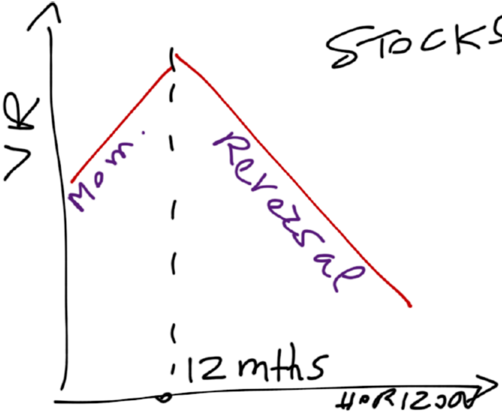



VR(q) for Monthly log excess returns on EW-CRSP Index. 1926-2012. Monthly data.

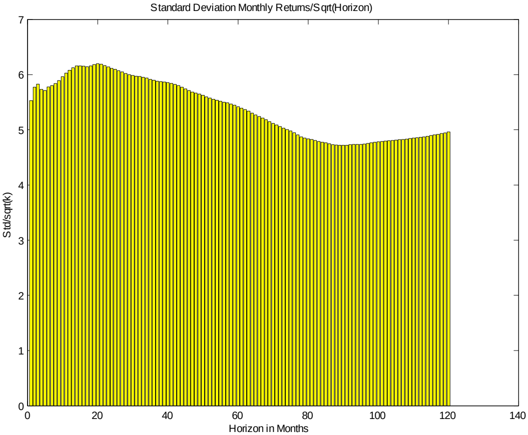
# Variance Ratios as a function of horizon

- VR levels off after +/- 12 months



# Standard Deviation of Monthly Log Excess Returns

- standard deviation does not grow as fast as the square root of the horizon




$\sigma\left(r_t\left(k\right)\right)/\sqrt{k}$ (in percentage points) for Monthly log excess returns on VW-CRSP Index. 1926-2012. Monthly data.

# Standard Deviation of Monthly Log Excess Returns

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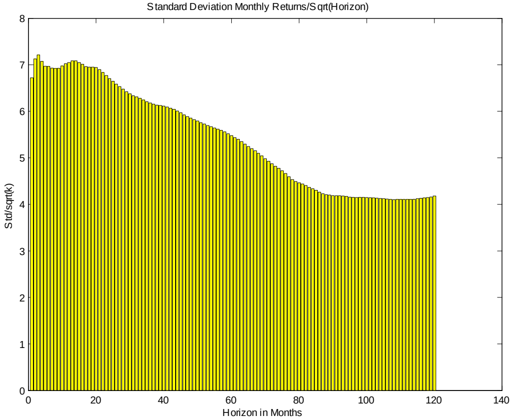



$\sigma\left(r_t\left(k\right)\right)/\sqrt{k}$ (in percentage points) for Monthly log excess returns on EW-CRSP Index. 1926-2012. Monthly data.

# Sharpe Ratios of longer holding periods

- a popular measure of the risk-return trade-off in financial markets is the Sharpe ratio:

$$SR_t = \frac{E[r_t^e]}{\sigma[r_t^e]}$$

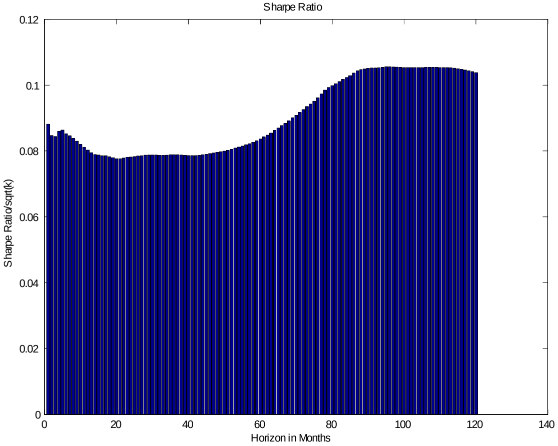
- we can also examine the SR over longer holding periods:

$$SR(k) = \frac{E[r_{t+1}^e + r_{t+2}^e + ... + r_{t+k}^e]}{\sigma[r_{t+1}^e + r_{t+2}^e + ... + r_{t+k}^e]}$$

- **If returns $r_t$ are i.i.d., then Sharpe ratios $SR(k)$ grow at a rate $\sqrt{k}$**

# Sharpe Ratio of Monthly Log Excess Returns

- SR's grow faster than the square root of the horizon




$SR\left(r_{t+1}+\ldots+r_{t+k}\right)/\sqrt{k}$ (in percentage points) for Monthly log excess returns on VW-CRSP Index. 1926-2012. Monthly data.

# Sharpe Ratio of Monthly Log Excess Returns

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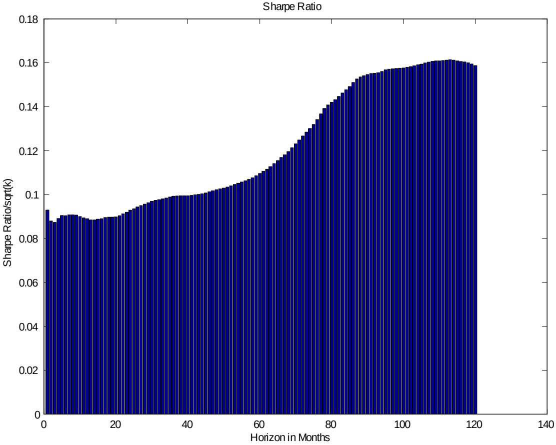



$SR\left(r_{t+1}+...+r_{t+k}\right)/\sqrt{k}$ (in percentage points) for Monthly log excess returns on EW-CRSP Index. 1926-2012. Monthly data.

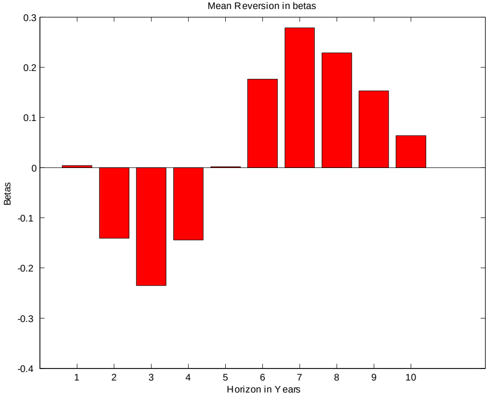
# Mean-Reversion Betas of Monthly Log Returns

Define: $r_{t+1}(k) \equiv r_{t+1} + r_{t+2} + ... + r_{t+k}$

- compute $\beta_k$ in
$$r_t(k) = \alpha_k + \beta_k r_{t-k}(k) + \varepsilon_t$$
- negative $\beta_k$ means mean reversion
- positive $\beta_k$ means mean aversion

# Mean-Reversion Betas of Monthly Log Returns

- high returns followed by low returns up to 5 years




This figure plots $\beta_k$ for Monthly log excess returns on VW-CRSP Index in $r_t(k) = \alpha_k + \beta_k r_{t-k}(k) + \varepsilon_t(k)$. 1926-2012. Monthly data.

# Mean-Reversion Betas of Monthly Log Returns

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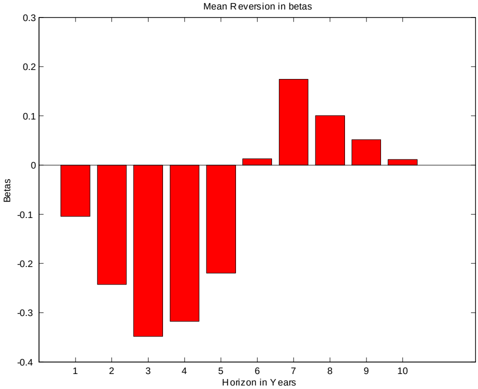


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# Less Risky for the Long-Run

- Some evidence of mean reversion in stock returns at investment horizons that exceed one year.
  - Fama and French (1988) documented evidence of mean-reversion in stock returns using variance ratios (also see Poterba and Summers (1988)).
  - Cochrane (1999) summarizes the evidence on long-run mean-reversion in returns on stocks (pp. 63-64).
  - Pastor and Stambaugh (2012) point out that there is a lot of statistical uncertainty about the mean reversion in stock returns.
- Mean reversion implies that stocks are less risky for long-run investors.

# Portfolio Implications

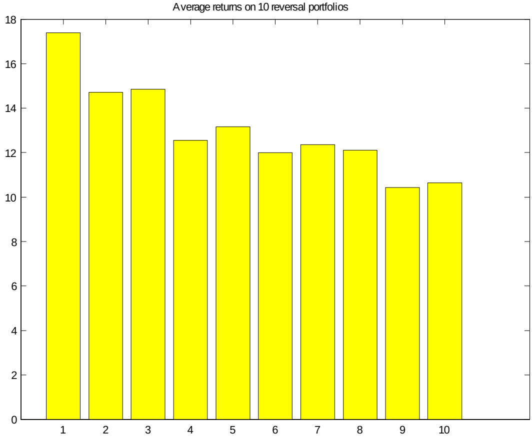
- Big picture: why does all this matter?
- Mean reversion implies that stocks are less risky for long-run investors.
- If stock returns are i.i.d. over time, then a mean-variance investor will choose a **constant stocks/bond allocation** that does not depend on his investment horizons
  - as a result, there is no role for the investor's horizon in the benchmark model!
- mean-reversion will induce investors with a longer investment horizon (e.g, younger investors, university endowments etc.) to allocate (on average) a larger share of their portfolio to the risky asset.

# Long-Term Reversal

# Long-Term Reversals in Stocks

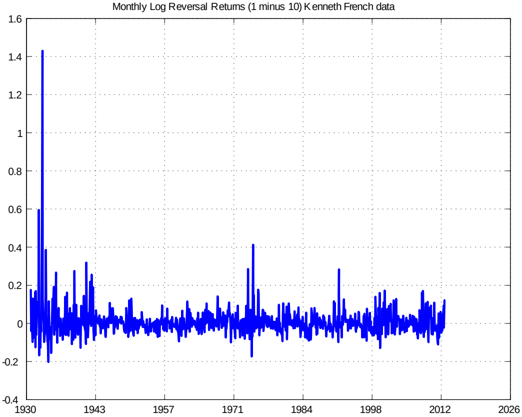
- cross-sectional trading strategy that exploits individual stock returns reversals
- there is a small amount of negative autocorrelation in individual monthly stock returns at longer horizons
  - At horizons in excess of 12 months, stock returns tend to be weakly negatively autocorrelated.
- this can be exploited to construct profitable trading strategies
- French posts LT reversal portfolios on his web site
  - The portfolios at $t$ are constructed monthly using NYSE prior ($t-13$ to $t-60$) return decile breakpoints.
- this is called (cross-sectional) **LT reversal trading**

# Long-Term Reversals in Stocks



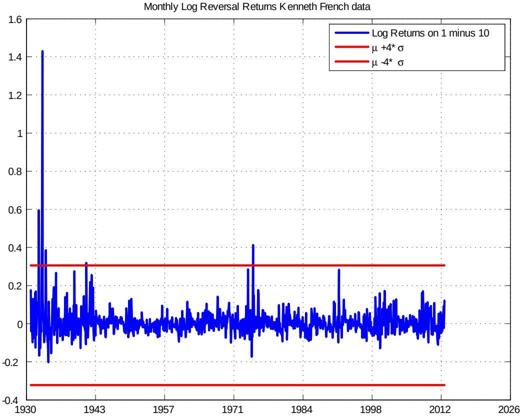

Average returns on reversal portfolios. Source: data from Kenneth French's website. The portfolios are constructed monthly using NYSE prior (13-60) return decile breakpoints. Sample: 1931-2013.

# Reversal Risk




Source: Log Returns on Portfolio 1 minus 10. data from Kenneth French's website. The portfolios are constructed monthly using NYSE prior (13-60) return decile breakpoints. Sample: 1927-2013.

# Little Tail Risk in LT Reversals

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# Long-Term Reversals and Value

- LT reversal is closely related to 'value'
  - returns on portfolios sorted by B/M ratios are correlated with returns on portfolios sorted by returns over past 5 years
- This LT reversal/value strategy works well across several asset classes.
- Asness, Moskowitz, and Pedersen (2013) document the pervasiveness of LT reversal/value effects in
  - currencies,
  - commodities,
  - bonds.
- LT reversal returns are negatively correlated with momentum returns!
- adding momentum and LT reversals increases the efficiency of the portfolio

# Over- and Underreaction

- behavioral interpretation (Barberis, Shleifer, and Vishny (1998))
  - **under-reaction** of investors to news is responsible for positive autocorrelations at horizons up to 12 months: news is slowly incorporated into prices
  - **over-reaction** of investors is responsible for negative autocorrelations at horizons after 12 months: securities that have experienced good news become overpriced.

# Conclusion

- stock returns have:
  1. positive autocorrelation at horizons of less than one year
     - exploited by momentum trading strategies
  2. negative autocorrelation at horizons of more than one year
     - exploited by reversal trading strategies
- because of (2), stocks are less risky for long run investors

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