1. Ito's lemma: $dM = \frac{\partial M}{\partial t}dt + \frac{\partial M}{\partial S_1}dS_1 + \frac{\partial M}{\partial S_2}dS_2 + \frac{1}{2}\frac{\partial^2 M}{\partial S_1^2}(dS_1)^2 + \frac{1}{2}\frac{\partial^2 M}{\partial S_2^2}(dS_2)^2 + \frac{1}{2}\frac{\partial^2 M}{\partial S_1 \partial S_2}dS_1 dS_2$

∵ only consider delta approach, we assume $\Gamma = \frac{\partial^2 M}{\partial S^2} = 0$. and assume $dt = 1$

$\therefore dM = \Delta_1 dS_1 + \Delta_2 dS_2 = \Delta_1 u_1 S_1 dt + \Delta_1 \sigma_1 S_1 dW_1 + \Delta_2 u_2 S_2 dt + \Delta_2 \sigma_2 S_2 dW_2$

$= (\Delta_1 u_1 S_1 + \Delta_2 u_2 S_2)dt + \Delta_1 \sigma_1 S_1 dW_1 + \Delta_2 \sigma_2 S_2 dW_2$

$E(dM) = \Delta_1 u_1 S_1 + \Delta_2 u_2 S_2$ $\quad Var(dM) = \Delta_1^2 \sigma_1^2 S_1^2 dt + \Delta_2^2 \sigma_2^2 S_2^2 dt + 2\Delta_1 \Delta_2 \sigma_1 \sigma_2 S_1 S_2 \rho dt$

$= \Delta_1^2 \sigma_1^2 S_1^2 + \Delta_2^2 \sigma_2^2 S_2^2 + 2\Delta_1 \Delta_2 \sigma_1 \sigma_2 S_1 S_2 \rho$

$P(M < M_0 - VaR) = 0.01$.

$P(dM < -VaR) = 0.01$

$u - 2.326\sigma = -VaR$

$VaR = 2.326\sigma - u =$

$2.326\sqrt{(\Delta_1^2 \sigma_1^2 S_1^2 + \Delta_2^2 \sigma_2^2 S_2^2 + 2\Delta_1 \Delta_2 \sigma_1 \sigma_2 S_1 S_2 \rho)} - \Delta_1 u_1 S_1 - \Delta_2 u_2 S_2$

2. $dM = \Delta_1 dS_1 + \Delta_2 dS_2 + \frac{1}{2}\Gamma_1 (dS_1)^2 + \frac{1}{2}\Gamma_2 (dS_2)^2 + \Gamma_{12} dS_1 dS_2$

$= \Delta_1 u_1 S_1 dt + \Delta_1 \sigma_1 S_1 dW_1 + \Delta_2 u_2 S_2 dt + \Delta_2 \sigma_2 S_2 dW_2 + \frac{1}{2}\Gamma_1 \sigma_1^2 S_1^2 dt + \frac{1}{2}\Gamma_2 \sigma_2^2 S_2^2 dt + \Gamma_{12}\sigma_1 \sigma_2 S_1 S_2 \rho dt$

$E(dM) = \Delta_1 u_1 S_1 + \Delta_2 u_2 S_2 + \frac{1}{2}\Gamma_1 \sigma_1^2 S_1^2 + \frac{1}{2}\Gamma_2 \sigma_2^2 S_2^2 + \Gamma_{12}\sigma_1 \sigma_2 S_1 S_2 \rho$

$Var(dM) = \Delta_1^2 \sigma_1^2 S_1^2 + \Delta_2^2 \sigma_2^2 S_2^2 + 2\Delta_1 \Delta_2 \sigma_1 \sigma_2 S_1 S_2 \rho$

$VaR = 2.326\sigma - u$

$= 2.326\sqrt{(\Delta_1^2 \sigma_1^2 S_1^2 + \Delta_2^2 \sigma_2^2 S_2^2 + 2\rho \Delta_1 \Delta_2 \sigma_1 \sigma_2 S_1 S_2)}$

$- \Delta_1 u_1 S_1 - \Delta_2 u_2 S_2 - \frac{1}{2}\Gamma_1 \sigma_1^2 S_1^2 - \frac{1}{2}\Gamma_2 \sigma_2^2 S_2^2 - \Gamma_{12}\sigma_1 \sigma_2 S_1 S_2 \rho$.