Class Review and Final Preparations

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Outline

1. Empirical Methods in Finance vs. the rest of the curriculum

2. Review of main topics
   - What are main lessons?
   - What will be on the final?

Empirical Methods in Finance

vs.

Other Classes

Empirical Methods in Finance

This class is a continuation of Econometrics and Investments

Goals are

1. To provide you with the baseline econometric tools needed for subsequent MFE classes and to conduct empirical quantitative research
2. To enable you to more easily understand and implement other, potentially more advanced, time-series models for use in quantitative analysis

The class is hard, but necessarily so as we need to get you quickly up to speed in order to focus the rest of your time here on tackling a wide variety of interesting, *realistic* problems

- Relative to classes before 2017, I have reduced the amount of material (in particular, we are not covering cointegration and non-stationary time series analysis, see Ch. 18, 19, and 20 in Hamilton)
- Other topic of interest: regime switching models (see Ch. 22 in Hamilton).

The Link to Future Classes

Quantitative Asset Management

- Factor models, Fama-MacBeth regressions, portfolio sorts, 'alpha'

Fixed Income Markets

- Principal Components Analysis, forecasting regressions, VARs

Advanced Stochastic Calculus

- The APT, models of heteroscedasticity, jumps, non-normalities

Financial Risk Management

- Models of heteroscedasticity, Value-at-Risk, factor models and hedging, non-normalities

The Link to Future Classes (cont'd)

Data Analytics

- ARMA, VARs, Principal Components Analysis, cross-sectional regressions, learning and shrinkage, factor models

Behavioral Finance

- Portfolio sorts, forecasting regressions, cross-sectional regressions, factor models

Statistical Arbitrage

- 'Alpha', Principal Components Analysis, shrinkage, factor models

Credit Markets

- VARs, Principal Components Analysis, jumps, non-normalities

Class Review

Lecture Notes 1 and 2

Financial asset returns highly non-normal

- Excess kurtosis, skewness, time-varying volatility
- JB test

This questions the use of regressions

- Minimizing SSE only optimal if normal homoskedastic errors

But: OLS regression estimates still *consistent*; we do need to adjust *standard errors*

- Basics of asymptotics (Central Limit Theorem and Law of Large Numbers)
- **Stationarity** important for Central Limit Theorem, important for any moment condition that is a sample average
- White standard errors, later Hansen-Hodrick and Newey-West standard errors
- Note on Asymptotic Theory posted on CCLE

I will not ask detailed questions about aymptotic theory on the final

Lecture Notes 3 and 4

Autocorrelations and, in particular, the autocorrelation function describe the time-dependencies in a time series.

- Stock returns exhibit interesting autocorrelation patterns
  1. Short-term reversal (negative autocorrelation)
  2. Momentum (positive autocorrelation)
  3. Long-term reversal (negative autocorrelation)

Different patterns are more dominant at different frequencies

Lecture Notes 3 and 4

Know the Ljung-Box test and Variance Ratios

$$VR\left(k\right)=\frac{Var\left(r_{t+1}+r_{t+2}+...+r_{t+k}\right)}{kVar\left(r_{t+1}\right)}$$

If returns are negatively autocorrelated (the negative autocorrelation dominates at horizon k), $VR\left(k\right)<1$.

- This means that a long horizon investor, who cares about the cumulative long-run return, faces a better annualized risk-return trade-off relative to a short-horizon investor

Lecture Note 5

ARMA models

- ARMA models can capture any linear function of past returns, any autocorrelation pattern
- Optimal linear forecast (based on univarite data)
- ARMA-X: add other exogenous predictors

Know:

- Stationarity requirement
- Multi-period forecasting
- Autocorrelation functions (how to derive), partial autocorrelation function
- AIC, BIC
- Conditional and unconditional variances
- Dynamic multipliers
- Exact vs. conditional likelihood functions for AR

Lecture Note 6

Vector Autoregressions

- Multi-variate AR
- Get multi-horizon forecasts based on set of predictors
- Now how to forecast using VAR(1), dynamic multiplier

Don't worry about:

- Vec and kronecker products
- Bond pricing application

Lecture Note 7

We skipped it due to time constraints

- Long-horizon forecasting regressions

Lecture Note 8

GARCH models

Know:

- Stylized facts on stock market conditional variance
- ARCH and GARCH models
    - In particular, know ARCH(1) and GARCH(1,1)
    - What additional stylized fact can EGARCH and GJR-GARCH account for?
    - Forecasting with GARCH models
- Realized variance

Lecture Note 9

Please understand portfolio construction through factors

$$R_{Pt} = R_{ft} + w_1 R^e_{F1,t} + \ldots + w_k R^e_{Fk,t}$$

- Note: no restriction that w's need to sum to 1!

Example 1:

$$R_{Pt} = R_{ft} + 0.8 R^e_{Mkt,t}$$

- Clearly, $0.8 \neq 1$. But, not a problem.
  - Net weight in risk-free asset is $1 - 0.8 = 0.2$. Net weight in market is 0.8.

Lecture Note 9 - cont'd

Example 2:

$$R_{Pt} = R_{ft} + 0.8R^e_{Mkt,t} - 0.3R_{HML,t}$$

- Clearly, $0.8 - 0.3 \neq 1$. But, not a problem.
  - Net weight in risk-free asset is $1 - 0.8 = 0.2$. Net weight in market is ..?
  - You have an initial position in market of 0.8. But, you are then overweighting growth stocks and underweighting value stocks relative to the market portfolio since $R_{HML,t} = R_{V,t} - R_{G,t}$

Lecture Note 9 - cont'd

Example 3:

- You are evaluating a fund with historical returns R_{Pt}.

- The fund claims it follows a stock-picking strategy, but is actually a closet-indexer. In particular, it invests all it's money each period in a market index fund and goes short an amount equal to half of that in a growth stock portfolio. It then invests the proceeds of the short position in a long value portfolio. Assume the growth and value portfolios are the same as those underlying the HML factor.

- You run the regression

$$R_{Pt} - R_{ft} = \alpha + \beta_{Mkt} R^{e}_{Mkt,t} + \beta_{HML} R_{HML,t} + \varepsilon_t$$

- What are your estimated $\hat{\beta}_{Mkt}$ and $\hat{\beta}_{HML}$? $\hat{\beta}_{Mkt} = 1$, $\hat{\beta}_{HML} = 0.5$

- What is your estimate of α? $\hat{\alpha} = 0$.

Lecture Note 10

Please know factor portfolio math:

$$R_{it}^{e} = \alpha_i + \beta_i' F_t + \varepsilon_{it}, \quad \text{for all } i$$

where $E\left[\varepsilon_{it}^2\right] = \sigma_i^2$, and where $E\left[\varepsilon_{it}\varepsilon_{jt}\right] = 0$ for all $i \neq j$.

- Know how to calculate expected returns, variances, and covariances in this setup
- Please note: the factor model does not necessarily imply that $\alpha_i = 0$. Need additional assumptions for this.
  - Typically: assume ε's can be diversified away (APT)

Principal Components Analysis: natural way to find factors

- Run PCA using excess returns
- Eigenvectors are then the w's for the corresponding factor in the previous slides and give each asset's weight in the zero-investment PCA factors

Lecture Note 10

The assumption of no-arbitrage makes the α's in the factor model zero *if the factors are traded*

- In general, the factor model implies that:

  $$E\left(R_{it}^{e}\right) = \beta_i' \lambda$$

  If factor j is traded and expressed as an excess return, we have:

  $$E\left(F_{jt}\right) = \lambda^{(j)}$$

  as the factor has a beta of one with respect to itself and zero to all other factors.

- In other words, the price of risk of a traded factor is its risk premium

Lecture Note 10

Empirically, a lot of factors that drive the covariance matrix of stock returns aren't *priced*

- I.e., $\lambda^{(j)} = 0$ if factor j is not priced.
  - Thus, the risk premium of a traded factor that is priced is significantly different from zero.
- Example: industry factors are not priced, typically, while the HML factor of Fama and French is.

Thus, while PCA offers an intuitive way of getting at the most important factors (e.g., industry factors), it is an *empirical* stylized fact that priced factors in stock returns are not well-identified by PCA analysis

- PCA is useful, however, for finding factors that add variance
- We may want to hedge out such factors

Lecture Notes 11

A linear beta-pricing model (our factor models) with traded factors *prices* all assets if and only if...

- ..the factors span the mean variance efficient portfolio
- That means, the mean-variance efficient portfolio can be constructed by trading the factors only:
$$\mu' V^{-1} \mu = \lambda' \Sigma_F^{-1} \lambda$$
where the left-hand side is the maximum squared Sharpe ratio of all assets and the right-hand side is the maximum squared Sharpe ratio of the factors

Please, know mean-variance math!

Lecture Notes 11

Continuing from the previous slide.

- In general, it is true that

$$\mu' V^{-1}\mu = \lambda' \Sigma_F^{-1}\lambda + \alpha' \Sigma_\varepsilon^{-1}\alpha.$$

- Thus, under the null that a given factor model prices all assets, we have that $\alpha' \Sigma_\varepsilon^{-1}\alpha = 0$.

This is not a function of investors' preferences, mean-variance risk criteria, etc.

- It's just math. An implication of the linear factor model framework.

Another mechanical implication:

- If you uncover α's for a particular factor model, it implies that you can achieve a higher Sharpe ratio than one can using the factors alone
- Understand how to implement this

Lecture Notes 12

Cross-sectional regressions always throw students off the first time they see them...

- Another take on testing null hypothesis:

$$E\left(R_{it}^{e}\right) = \beta_i' \lambda \quad \text{for all } i$$

- Note that $\frac{1}{T}\sum_{t=1}^{T} R_{it}^{e} = \bar{R}_i^{e} = E\left(R_{it}^{e}\right) + \text{error}$

- Consider a sample of N portfolios with T time series observations. Obtain the sample means for each i and regress on betas:

$$\bar{R}_i^{e} = \lambda_0 + \lambda_1 \hat{\beta}_i + \tilde{\alpha}_i.$$

- Note that $\tilde{\alpha}_i$ is the error term in this regression. Under the null hypothesis $\lambda_0 = 0$, and $\tilde{\alpha}_i = 0$ for all i.

- But what is λ_1?

Lecture Notes 12

What is λ_1?

$$\begin{bmatrix} \lambda_0 \\ \lambda_1 \end{bmatrix} = \left(X'X\right)^{-1} X'\bar{R}^e$$

where

$$X = \begin{bmatrix} 1 & \beta_1 \\ 1 & \beta_2 \\ & \vdots \\ 1 & \beta_N \end{bmatrix}, \quad \bar{R}^e = \begin{bmatrix} \bar{R}^e_1 \\ \bar{R}^e_2 \\ \vdots \\ \bar{R}^e_N \end{bmatrix}$$

- Also, let $E^I\left[\beta_i\right]$ and $Var^I\left[\beta_i\right]$ be the cross-sectional average and variance of the β_i's, respectively.
- Then.... see next slide

Lecture Notes 12

Thus, from writing out the formula from the previous slide:

$$\begin{aligned} \lambda_1 &= \sum_{i=1}^{N} \frac{1}{N} \frac{\beta_i - E[\beta]}{Var[\beta]} \bar{R}_i^e \\ &= \sum_{i=1}^{N} w_i \bar{R}_i^e \end{aligned}$$

where $w_i = \frac{1}{N} \frac{\beta_i - E[\beta]}{Var[\beta]}$ is a portfolio weight in the excess return sense; the same as we saw in the initial slides in factor models.

- The cross-sectional regression forms a portfolio that each period is long high beta stocks and short low beta stocks.
    - The portfolio weight is linearly increasing in β_i
    - This is the factor-mimicking portfolio implied by the cross-sectional regression.
- The price of risk estimate, λ_1, is the average excess return to this portfolio

Lecture Notes 12

Fama-MacBeth runs this cross-sectional regression each period.

- Mostly useful is beta's or other firm characteristics vary over time:

$$R_{it}^{e} = \lambda_{0t} + \lambda_{1t} bm_{i,t-1} + \tilde{\alpha}_{it}$$

- Then

$$\lambda_0 = \frac{1}{T}\sum_{t=1}^{T}\lambda_{0t}, \quad \lambda_1 = \frac{1}{T}\sum_{t=1}^{T}\lambda_{1t}, \quad \tilde{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T}\tilde{\alpha}_{it}$$

- Note that this is simply a panel forecasting regression, where the regression coefficient is the same across firms.
- From this regression, we have:

$$E_{t-1}\left[R_{it}^{e}\right] = \lambda_0 + \lambda_1 bm_{i,t-1}$$

- Note, this regression does not have λ_0 as a null hypothesis ($bm_{i,t-1}$ is not a beta)
- What is λ_1 in this case?

How Should I Study for the Final?

How to Study

1. Read lecture notes and handout on factor models and asymptotics
    - The Tsay text book is a useful reference
2. Understand the homeworks, in particular the parts that were analytical.
3. You do not need to know any of the coding for the final.
4. Finals from last two years are posted
    - Remember to bring cheat sheet (letter-sized, both pages) and calculator

Extra office hours

I will hold extra office hours:

Thursday, March 14, from 2-3pm in C5.19

Good luck and thanks for being great students!!

I hope to see many of you next quarter for

Data Analytics and Machine Learning