MGMT MFE 431-3
Statistical Arbitrage
Lecture 06: More Alphas

Professor Olivier Ledoit

University of California Los Angeles
Anderson School of Management
Master of Financial Engineering

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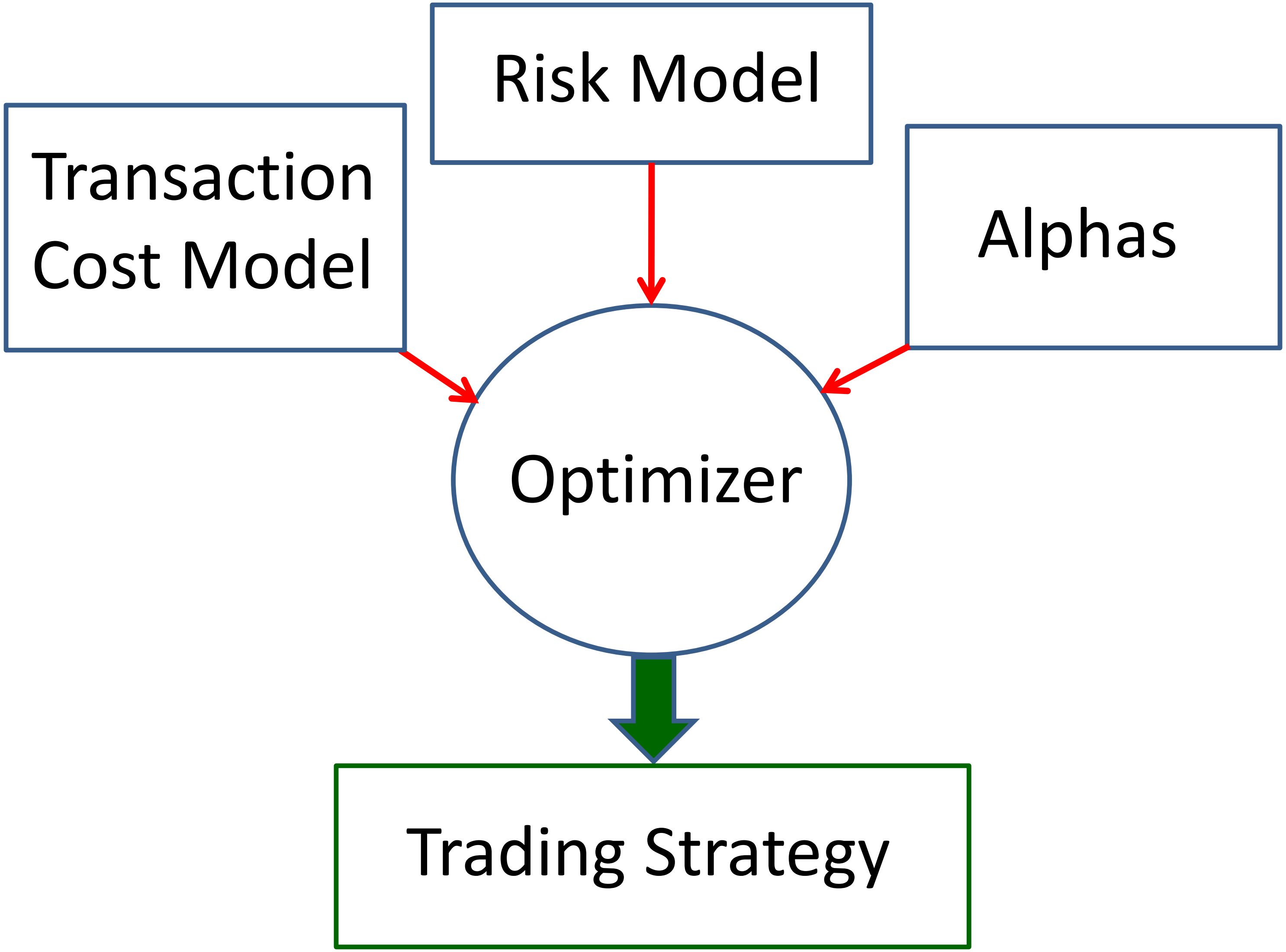
Overall Structure



What We've Seen So Far

- Risk Model: shrinkage estimator of the covariance matrix of stock returns

- Transaction Cost Model: 1bp + ½ bid-ask spread

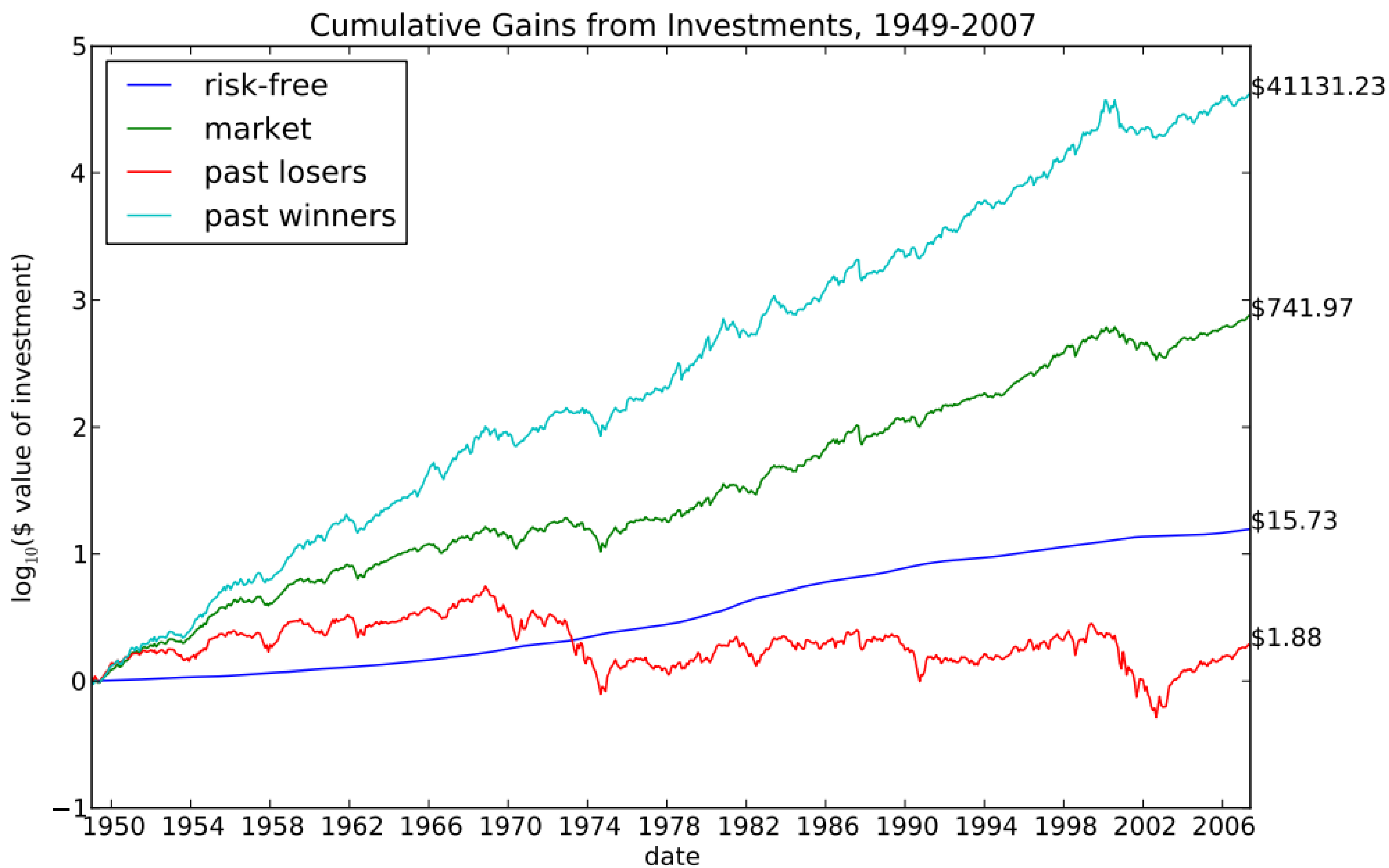
- Alpha: Weighted blend of various standardized, windsorized alphas

Follow-up on Momentum

- Returns to momentum strategies experience infrequent but strong and persistent strings of losses
- These momentum “crashes” are forecastable: they occur:
  - following market declines
  - when market volatility is high
  - and contemporaneous with market “rebounds”

Momentum Crashes - Daniel (2011)

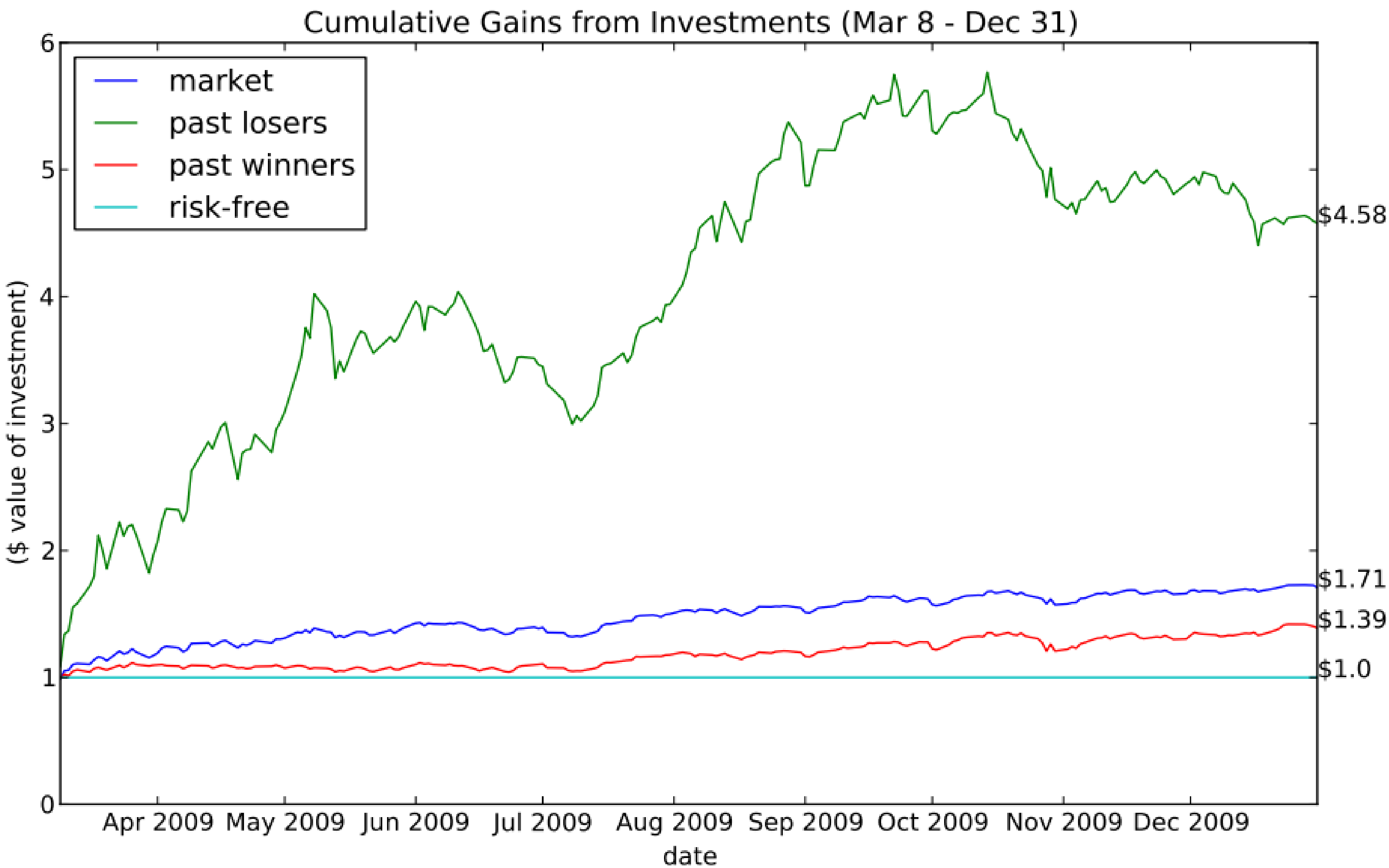
Figure 2: Momentum Components, 1949-2007



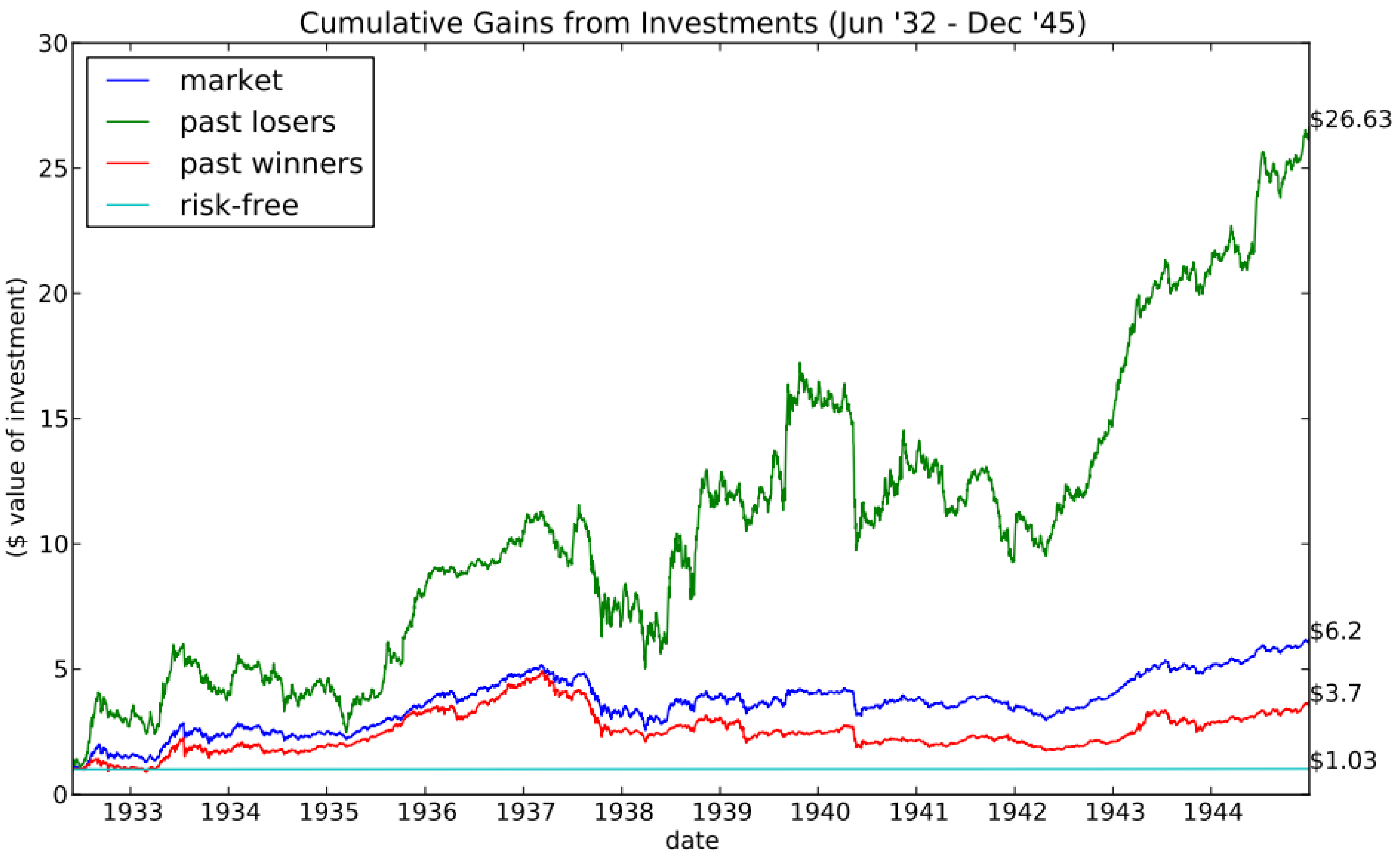
Worst Monthly Momentum Returns

Rank	Month	WML_t	Mkt-2y	Mkt_t
1	1932-08	-0.7896	-0.6767	0.3660
2	1932-07	-0.6011	-0.7487	0.3375
3	2009-04	-0.4599	-0.4136	0.1106
4	1939-09	-0.4394	-0.2140	0.1596
5	1933-04	-0.4233	-0.5904	0.3837
6	2001-01	-0.4218	0.1139	0.0395
7	2009-03	-0.3962	-0.4539	0.0877
8	1938-06	-0.3314	-0.2744	0.2361
9	1931-06	-0.3009	-0.4775	0.1380
10	1933-05	-0.2839	-0.3714	0.2119
11	2009-08	-0.2484	-0.2719	0.0319

2009 Momentum Performance



Momentum in the Great Depression



Explanation

- Poor performance due to short side:
  the short side of the portfolio (the losers) are crashing up rather than down.
- Loser portfolio optionality:
  loser up-market beta >> loser down-market beta

- *Dangerous to be short losers after bear market when volatility is high... because they might rebound!*

Alphas Everywhere

Johnson and Magee.pdf

Page: 1 / 24 Find:

Journal of Accounting and Economics 7 (1985) 151–174. North-Holland

AN ANALYSIS OF THE STOCK PRICE REACTION TO SUDDEN EXECUTIVE DEATHS

Implications for the Managerial Labor Market*

W. Bruce JOHNSON and Robert P. MAGEE

Northwestern University, Evanston, IL 60201, USA

Nandu J. NAGARAJAN

University of Pittsburgh, Pittsburgh, PA 15260, USA

Harry A. NEWMAN

University of Michigan, Ann Arbor, MI 48109, USA

Haugen & Baker (2010) Case Closed

Most important factors for predicting the cross-section of US stock returns:

1) Residual Return = last month's residual stock return unexplained by the market
2) Cash Flow-to-Price = 12-month trailing cash flow-per-share divided by current price
3) Earnings-to-Price = 12-month trailing earnings-per-share divided by current price

Factors 4-6

4) Return On Assets = 12-month trailing total income divided by most recently reported total assets
5) Residual Risk = 24-month trailing variance of residual stock return unexplained by market return
6) 12-month Return = total return for the stock over trailing twelve months

Factors 7-9

7) Return on Equity = 12-month trailing earnings-per-share divided by most recently reported book value-per-share
8) Variance = 24-month trailing variance of total stock return
9) Book-to-Price = most recently reported book value of equity divided by current market price

Factors 10-12

10) Profit Margin = 12-month trailing earnings before interest divided by 12-month trailing sales

11) 3-month Return = total return for the stock over trailing 3 months

12) Sales-to-Price = 12-month trailing sales-per-share divided by market price

Lessons

- Value, momentum, reversion: we knew
- Some accounting performance ratios:
  - Return on Book Value of Total Assets
  - Return on Book Value of Equity
  - Profit Margin: Earnings divided by Sales
- Low residual risk, low variance: Haugen & Baker's contribution
- Missing: analyst revisions, earnings announcements

Optimizer

- Inputs:
  - position as of close of business on day $t\text{-}1$
  - alphas using data observed up to day $t\text{-}1$
  - t-costs using data observed up to day $t\text{-}1$
  - risk model using data observed up to day $t\text{-}1$
  - constraints using data observed up to day $t\text{-}1$
- Output: trade to be executed on day t

final position($t+1$) = final position(t) + trade (t)

Timeline

- Day *t-1*: most recent available data
- Day *t*: trade gets executed
- Day *t+1*: returns start to be earnt

Backtest Code

- Load all necessary data into memory
- Create the alphas
- Start from portfolio with zero dollar invested
- Loop over all days in backtest period
  - Every day: call optimizer to find optimal rebalancing trade given initial position
  - End-of-day position becomes initial position of next day
- Compute P&L

Notation

- x: (n × 1) vector of desired portfolio weights
- w: (n × 1) vector of initial portfolio weights
- Σ: (n × n) covariance matrix of stock returns
- α: (n × 1) vector of aggregate alphas
- β: (n × 1) vector of historical betas
- τ: (n × 1) vector of transaction costs

Objectives and Constraints

- Minimize risk: $x' \Sigma x$
- Maximize exposure to alpha: $\alpha ' x$
- Neutralize exposure to beta: $\beta ' x = 0$
- Minimize transaction costs: $\tau ' |x-w|$
- Other constraints:
  - maximum trade size
  - maximum position size
  - maximum industry and country exposure

Optimization Problem

$$\max_x \ \alpha' x - \lambda \cdot \tau' |x-w| - \mu \cdot x' \Sigma x$$

subject to: $\beta' x = 0$

and other constraints:

- maximum trade size
- maximum position size
- maximum industry exposure
- maximum country exposure

Objective Function

controls turnover

controls book size

$$\max_x \; \alpha' x - \lambda \cdot \tau' |x-w| - \mu \cdot x'\Sigma x$$

maximize alpha exposure

minimize transaction cost

minimize portfolio variance

Maximum Trade Size

- 1% of Average Daily Volume (ADV)
  Can go to 2% if necessary (big book)

- Capped so liquid stocks do not dominate
- Example: cap at $150K
  Can go higher if necessary (big book)

Maximum Position Size

- Multiple of maximum trade size
- I want to be able to liquidate every position in how many days?
- 10 days $\Rightarrow$ 10 × max trade size
- Can be big relative to book size
- Keep balance between liquid and illiquid stocks

min(10 × max trade size, 2.5% of long side of book)

Merge the 2 Constraints

- Max trade size for i^th^ stock: θ_i

$$\Rightarrow \quad w_i - \theta_i \le x_i \le w_i + \theta_i$$

- Max position size for i^th^ stock: π_i

$$\Rightarrow \quad -\pi_i \le x_i \le \pi_i$$

- Enforce both constraints simultaneously:

$$\underbrace{\max(w_i - \theta_i, -\pi_i)}_{\gamma_i} \le x_i \le \underbrace{\min(w_i + \theta_i, \pi_i)}_{\delta_i}$$

$$\gamma_i \le x_i \le \delta_i$$

Industry Constraints

- Sectors are a factor of risk
- Difficult to time sector performance
- Constrain industry exposure
- But not to zero (too much transaction cost)
- For \$50 × 50M book size: r* = \$300,000 limit

Industry Dummy

- ρ industries
- Boolean matrix R of dimension $(n \times \rho)$
- $R(i,j) = 1$ if i^{th} stock belongs to j^{th} industry
- $R(i,j) = 0$ if i^{th} stock is outside j^{th} industry
- Every row of matrix R has exactly one entry equal to 1; all other entries are equal to 0
- Constraint: $-r^* \cdot \mathbf{1} \le R'x \le r^* \cdot \mathbf{1}$
  where $\mathbf{1}$ = vector of ones of the right dimension

Country Constraints

- Countries are a factor of risk
- Difficult to time country performance
- Constrain country exposure
- But not to zero (too much transaction cost)
- For $50 × 50M book size: f* = $100,000 limit
- Tighter than industry exposure

Country Dummy

- φ countries
- Boolean matrix F of dimension $(n \times \varphi)$
- $F(i,j) = 1$ if i^{th} stock belongs to j^{th} country
- $F(i,j) = 0$ if i^{th} stock does not belong to j^{th} country
- Every row of matrix F has exactly one entry equal to 1; all other entries are equal to 0
- <u>Constraint:</u> $-f^* \cdot \mathbf{1} \leq F'x \leq f^* \cdot \mathbf{1}$

Overall Problem

$$\max_x \; \alpha' x - \lambda \cdot \tau' |x-w| - \mu \cdot x' \Sigma x$$

<u>Subject to:</u>

- beta neutrality: $\beta' x = 0$
- max trade and position: $\gamma \le x \le \delta$
- industry constraint: $- r^* \cdot \mathbf{1} \le R'x \le r^* \cdot \mathbf{1}$
- country constraint: $- f^* \cdot \mathbf{1} \le F'x \le f^* \cdot \mathbf{1}$

Is this standard Quadratic Programming?

Quadratic Programming

- Quadratic programming (QP) is fast, efficient and guaranteed to converge
- Excellent off-the-shelf software
- Matlab optimization toolbox

- Problem: the absolute value in the transaction cost term is not *standard* quadratic programming: $\tau'|x-w|$

Split Variables

- Classic solution: split each variable into 2
- Drawback: twice as many variables
- Advantage: no need to use nonlinear programming
- Define:
  - $y = \max(x-w,0)$
  - $z = \max(w-x,0)$
- Then $y \geq 0$, $z \geq 0$, $x = w + y - z$ and $|x-w| = y+z$

Indeterminacy?

- Initial problem strictly convex
  $\Rightarrow$ unique solution in x
- Twice as many variables:
  solution still unique in y and z?
- Replace y by y+1 and z by z+1
  $\Rightarrow$ x = w + y – z remains unchanged!

- Still OK because |x-w| = y+z penalized

New Formulation

$\max_{y,z}\ \alpha'(w+y-z) - \lambda \cdot \tau'(y+z) - \mu \cdot (w+y-z)'\Sigma(w+y-z)$

<u>Subject to:</u>

- beta neutrality: $\beta\,'\,(w+y-z) = 0$
- max trade and position: $\gamma \le w+y-z \le \delta$
- industry constraint: $-\,r^* \cdot \mathbf{1} \le R'\,(w+y-z) \le r^* \cdot \mathbf{1}$
- country constraint: $-\,f^* \cdot \mathbf{1} \le F'\,(w+y-z) \le f^* \cdot \mathbf{1}$

Very close to standard Quadratic Programming

Standard Quadratic Programming

$$\min_u \; 0.5\, u'\, H\, u + g'\, u$$

Subject to:

- $A\, u \leq b$
- $C\, u = d$
- $LB \leq u \leq UB$

Rewrite Optimization Problem

$$\min_{y,z} - \alpha'(y\text{-}z) + \lambda \cdot \tau'(y+z) + 2\mu \cdot w'\Sigma(y\text{-}z)$$
$$+ \mu \cdot (y\text{-}z)'\Sigma(y\text{-}z) + \text{constant}$$

Subject to:

- beta neutrality: $\beta'(y\text{-}z) = -\beta' w$
- max trade and position: $\gamma - w \leq y\text{-}z \leq \delta - w$
- industries: $-r^* \cdot \mathbf{1} - R'w \leq R'(y\text{-}z) \leq r^* \cdot \mathbf{1} - R'w$
- countries: $-f^* \cdot \mathbf{1} - F'w \leq F'(y\text{-}z) \leq f^* \cdot \mathbf{1} - F'w$

Maps into standard Quadratic Programming

Mapping Objective Function

- $u = \begin{bmatrix} y \\ z \end{bmatrix}$

- $H = 2\,\mu \begin{pmatrix} \Sigma & -\Sigma \\ -\Sigma & \Sigma \end{pmatrix}$

- $g = \begin{pmatrix} 2\mu\,\Sigma w - \alpha + \lambda\,\tau \\ -\,2\mu\,\Sigma w + \alpha + \lambda\,\tau \end{pmatrix}$

Mapping Inequality Constraints

- $A = \begin{pmatrix} R' & -R' \\ -R' & R' \\ F' & -F' \\ -F' & F' \end{pmatrix}$
- $b = \begin{pmatrix} r^* \cdot \mathbf{1} - R'w \\ r^* \cdot \mathbf{1} + R'w \\ f^* \cdot \mathbf{1} - F'w \\ f^* \cdot \mathbf{1} + F'w \end{pmatrix}$

Mapping Equality Constraints

- C $= \begin{pmatrix} \beta' & -\beta' \end{pmatrix}$
- d $= -\beta' w$

Bounds on Optimization Variables

- Lower bound:
  LB = vector of zeros of dimension $(2n \times 1)$

- Upper bound:
  $$UB = \begin{pmatrix} \max(0,\min(\theta,\pi-w)) \\ \max(0,\min(\theta,\pi+w)) \end{pmatrix}$$

Matlab Quadratic Optimizer

- quadprog.m
- No starting point needed
- options = optimset('Algorithm','interior-point-convex')
- options = optimset(options,'Display','iter')

- [u,fval,exitflag,output] = quadprog(H,g,A,b,C,d,LB,UB,[],options)

Other Good Optimizers

- IBM: CPLEX
- FICO: Xpress
- Sunset: XA
- Stanford: QPOPT, SQOPT and MINOS
- Roger Fletcher: BQPD
- KNITRO

- Not cheap!

Required Readings for Next Lecture

1. Cristi A. Gleason and Charles M. C. Lee. Analyst forecast revisions and market price discovery. The Accounting Review, 78(1):pp. 193–225, 2003.
2. Narasimhan Jegadeesh, Joonghyuk Kim, Susan D. Krische, and Charles M. C. Lee. Analyzing the analysts: When do recommendations add value? The Journal of Finance, 59(3):1083–1124, 2004.