Solutions to Final Exam 2018, MGMTMFE 407 – Empirical Methods in Finance

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Multiple Choice (80 points) 2 points per question. **All numbers are rounded to two decimal points. Some questions continue on the following page, make sure you see all answer options before answering.**

1. Consider the sample average of daily stock market returns, using data from 1926 until 2017, as the estimate of the unconditional mean of daily stock market returns. Which of the below statements best describes the statistical properties of this sample mean?

   (a) The sample mean displays negative excess kurtosis and positive skewness

   (b) The sample mean displays negative excess kurtosis and negative skewness

   (c) The sample mean displays positive excess kurtosis and positive skewness

   (d) The sample mean displays positive excess kurtosis and negative skewness

   (e) **None of the above**

2. The Jarque-Bera test statistics is JB = (S(r))^2 / (6/T) + (K(r) - 3)^2 / (24/T). What is the null hypothesis? Choose the answer that is the most accurate

   (a) That the underlying series has conditional skewness equal to zero and conditional kurtosis equal to three

   (b) That the underlying series is lognormally distributed

   (c) **That the underlying series has unconditional skewness equal to zero and unconditional kurtosis equal to three**

   (d) None of the above

3. Assume you have 64 observations of annual stock returns. The sample standard deviation of excess returns is 16%. The sample mean excess return is 6%. What is the standard error of the estimate of the unconditional mean based on this data?

   (a) 0.09375%

(b) 0.25%
(c) 0.75%
(d) **2.00%**
(e) None of the above

4. Assume the log price follows a Random Walk with drift. Assume the drift term is 5% and the volatility of the shocks to price is 10%. What is the unconditional variance of this process?

   (a) 5%
   (b) 10%
   (c) $(5\%)^2$
   (d) $(10\%)^2$
   (e) **None of the above**

5. Newey-West standard errors for linear regression estimates account for (choose the best answer):

   (a) Heteroskedasticity and autocorrelation in the dependent variable
   (b) Heteroskedasticity and autocorrelation in the independent variable
   (c) **Heteroskedasticity and autocorrelation in the residuals**
   (d) Autocorrelation in the residuals
   (e) None of the above

6. Assume the single factor Market Model with uncorrelated residuals. Assume the standard deviation of market returns is 20%. Firm A returns has beta of 0.9 and a residual standard deviation of 5%. Firm B returns has a beta of 1.1 and a standard deviation of 3%. What is the covariance of the returns to firm A and firm B?

   (a) **3.96%**
   (b) 4.3%
   (c) 19.8%
   (d) 27.8%
   (e) None of the above

7. Which of the below is **not** a reason to sort into portfolios when testing an expected return model?

   (a) It decreases the standard error of estimated factor betas

   (b) **It creates a greater spread in true factor betas**

   (c) It makes it easier to deal with firm exit and entry

   (d) It makes it easier to estimate the variance-covariance matrix of residuals

   (e) None of the above

8. Which of the below is **not** true about the HML factor in the Fama-French 3 factor?

   (a) A regression of the HML factor on the market factor yields a positive alpha

   (b) The market beta of the HML factor is close to zero

   (c) **High book-to-market firms tend to have negative loadings on the HML factor**

   (d) If a mutual fund loads positively on the HML factor, it's alpha will be smaller under the Fama-French model than under the CAPM (all else equal)

   (e) None of the above

9. The first eigenvector (associated with the largest eigenvalue) gives

   (a) Expected returns to all stocks if the CAPM holds

   (b) The variance of each stock

   (c) **The factor loading (beta) of each stock with respect to the first principal component**

   (d) The R^2 of each stock in a regression of the stock on the first principal component

   (e) None of the above

10. The following is true about the APT:

    (a) The factors need to be uncorrelated

    (b) The expected return on a stock equals the sum of the stock's factor betas

    (c) **The residual risk is diversifiable**

    (d) The factors cannot be autocorrelated

(e) None of the above

11. In a time-series CAPM regression of excess returns to stock i on excess returns of the market, a negative alpha (ignoring statistical uncertainty) means that

    (a) The expected return on the stock must be lower than the expected return on the market portfolio
    (b) Investors should avoid the stock altogether
    (c) **The maximal Sharpe ratio available is higher than the market Sharpe ratio**
    (d) The market has the highest Sharpe ratio, but the stock can be a good hedge for certain investors
    (e) None of the above

12. Consider the Fama-French three factor model. The prices of risk are estimated to be: 6% (Mkt), 5% (HML), 3% (SMB). The risk-free rate is 2%. A firm with betas 1.0 (Mkt), 1.2 (HML), -0.2 (SMB) has expected return equal to:

    (a) 11.4%
    (b) **13.4%**
    (c) 15.4%
    (d) 17.4%
    (e) None of the above

13. The portfolio weights in the mean-variance efficient portfolio are

    (a) proportional to the product of the expected return vector and the variance-covariance matrix of returns
    (b) proportional to the product of the expected excess return vector and the variance-covariance matrix of returns
    (c) proportional to the product of the expected return vector and inverse of the variance-covariance matrix of returns
    (d) **proportional to the product of the expected excess return vector and inverse of the variance-covariance matrix of returns**
    (e) None of the above

14. If market returns are positively autocorrelated

    (a) **The variance ratio is increasing with the horizon**
    (b) The variance ratio is decreasing with the horizon
    (c) The variance ratio is constant
    (d) None of the above

15. The GRS statistic in a test of the CAPM is a measure of the distance between the

    (a) Expected return on the market and the expected return on the mean-variance efficient portfolio
    (b) **Sharpe ratio of the market and the Sharpe ratio of the mean-variance efficient portfolio**
    (c) Average alpha of the test assets and the average excess returns to the market
    (d) Average R2 in the firms' time series regressions on the market, and zero

16. Assume you have found the mean-variance efficient portfolio. Which of the following is true:

    (a) All stocks have $\alpha = 0$ and $\beta = 1$ with respect to this portfolio
    (b) All investors should hold this portfolio
    (c) This portfolio is not tradeable
    (d) **This portfolio has the highest expected return of all possible portfolios with the same variance**
    (e) None of the above

17. For a covariance-stationary process, the unconditional covariance between r_t and r_{t+j}

    (a) Must be positive
    (b) Must be negative
    (c) Depends on the current value of r_t
    (d) Is a function of time t
    (e) **None of the above**

18. The Ljung-Box test is a test of whether the m first autocorrelations of a series is zero. What is its asymptotic distribution?

    (a) F with (m, T) degrees of freedom
    (b) Standard Normal
    (c) **Chi-square with m degrees of freedom**
    (d) Chi-square with $T - m$ degrees of freedom
    (e) None of the above

19. A long-horizon investor should invest less in stocks than a short-horizon investor if

    (a) **the variance ratio is increasing over time**
    (b) the risk premium is positive
    (c) the standard deviation of returns is mean-reverting
    (d) none of the above

20. Assume x_t follows an AR(1) with unconditional mean equal to 0.1 and an autocorrelation coefficient of 0.5. If the current value of x_t equals 0.1, what is the two periods ahead predicted value, $E_t[x_{t+2}]$?

    (a) 0.025
    (b) 0.05
    (c) 0.075
    (d) **0.10**
    (e) 0.125
    (f) None of the above

21. When we use the I-GARCH(1,1) process with zero intercept in the vol specification for forecasting the variance at long horizons,

    (a) the forecast converges to zero
    (b) **the forecast is an exponentially weighted moving average of past squared residuals**
    (c) the forecast converges to the unconditional variance

(d) the forecast is an exponentially weighted moving average of past absolute value of the residuals

(e) None of the above

22. The first-order autocorrelation of the MA(1), $x_{t+1} = -\theta_1 \varepsilon_t + \varepsilon_{t+1}$, with $\theta_1 = 0.5$, is:

    (a) -0.5

    (b) $\mathbf{-0.4}$

    (c) 0.4

    (d) 0.5

    (e) none of the above

23. To check the stationarity of an MA(q) process with $q < \infty$ and $|\theta_j| < \infty$ for all $j = 1, ..., q$, we need to look at

    (a) the roots of the characteristic equation of the MA-component

    (b) Ensure the sum $\Sigma_{j=1}^{q} |\theta_j| < 1$

    (c) Ensure the sum $\Sigma_{j=1}^{q} |\theta_j^2| < 1$

    (d) **None of the above**

24. Assume you ran a principal components analysis on stock returns and use the 5 biggest factors as you pricing factors in an expected-return beta pricing model (a la the 5-factor Fama and French model). Which of the following is **not** true. Choose the best answer.

    (a) The factors are the 5 most important factors in terms of explaining the stocks' variance-covariance matrix

    (b) The factors are uncorrelated

    (c) The factors have different variance

    (d) **The factors will, by virtue of being the top 5 principal components, do well in terms of explaining the cross-section of expected stock returns**

    (e) None of the above

25. Returns to a mutual fund exhibit positive alpha with respect to the CAPM (the market), but negative alpha with respect to the Fama-French 3-factor model. Which of the following sentences is likely to be the most accurate:

    (a) The fund has a higher average return than the market, but a negative loading on HML

    (b) The fund has a positive loading on SMB and a negative loading on HML

    (c) The market beta of the fund is positive, but less than one

    (d) **The fund has a positive loading on HML and SMB**

    (e) The fund likely trades momentum

26. The log-linearization of the return equation around the mean log price/dividend ratio delivers the following expression for log returns:

$$r_{t+1} = \Delta d_{t+1} + \rho pd_{t+1} + k - pd_t,$$

with a linearization coefficient ρ that depends on the mean of the log price/dividend ratio pd:$\rho = \frac{e^{pd}}{e^{pd}+1} < 1$. Assume that $\rho = 0.96$, that the volatility of annualized dividend growth and returns are 12% and 16%, respectively. Given this information, which of the following statements is the most correct?

    (a) Returns and dividend growth are i.i.d.

    (b) Returns and dividend growth must both be positively correlated

    (c) **If dividend growth is i.i.d., returns must be predictable**

    (d) The price-dividend ratio is constant

    (e) None of the above

27. For an AR(1) process $y_t = \phi_0 + \phi_1 y_{t-1} + \varepsilon_t$, the half-life of a shock h is given by

    (a) $h = 0.5/\log \phi_1$ periods.

    (b) $h = 0.5/\phi_1$ periods.

    (c) $h = \phi_1/0.5$ periods.

    (d) $\mathbf{h} = \log \mathbf{0.5}/\log |\boldsymbol{\phi}_1|$ **periods.**

    (e) None of the above.

28. Assume the following ARMA(1,1) process: $y_t = 0.9y_{t-1} - 0.8\varepsilon_{t-1} + \varepsilon_t$. If $y_t = 0.3$ and $\varepsilon_t = -0.3$, what is $E_t[y_{t+2}]$?

    (a) 0.00
    (b) **0.459**
    (c) 0.51
    (d) 0.558
    (e) None of the above.

29. Choose the statement that is the most correct from the below. The Fama-French five-factor model predicts that (choose the most accurate statement):

    (a) Firms with high investment and high book-to-market have low discount rates
    (b) **Firms with high investment and low book-to-market have low discount rates**
    (c) Firms with low investment and high book-to-market have low discount rates
    (d) Firms with low investment and low book-to-market have low discount rates

30. The notion that high momentum stocks deliver high subsequent returns is consistent with

    (a) i.i.d. stock returns.
    (b) mean reversion in stock returns.
    (c) heteroskedasticity in stock returns
    (d) **underreaction to news**
    (e) None of the above.

31. Ross' arbitrage pricing theory (APT) says that, in a well-defined asset pricing model,

    (a) only traded assets are valid risk factors with a well-defined risk price
    (b) only non-traded assets are valid risk factors with a well-defined risk price
    (c) the return on the market must be a risk factor
    (d) **None of the above**

32. Assume a forecasting regression of annual excess stock market returns on a one-year lagged predictive variable yields an R^2 of 4%. If stock return volatility is 20%, what is the volatility of the conditional annual risk premium, as estimated by this regression?

   (a) 0.0%

   (b) 0.2%

   (c) 1.0%

   (d) **4.0%**

   (e) None of the above

33. When we use the ARCH(1) model $\varepsilon_t = \sigma_t \eta_t;\ \ \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$ for forecasting, the 2-period ahead forecast is given by:

   (a) $E_t\left[\sigma_{t+2}^2\right] = \frac{\alpha_0}{1-\alpha_1}$

   (b) $E_t\left[\sigma_{t+2}^2\right] = \alpha_0 + \alpha_1^2 \varepsilon_t^2 + \alpha_1 \sigma_t^2$

   (c) $\mathbf{E}_t\left[\sigma_{t+2}^2\right] = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1^2 \boldsymbol{\varepsilon}_t^2$

   (d) None of the above

34. The unconditional variance of a GARCH(1,1) variable $\varepsilon_t = \sigma_t \eta_t;\ \ \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

   (a) $\alpha_0/(1-(\alpha_1+\beta_1)^2)$

   (b) $\boldsymbol{\alpha}_0/(\mathbf{1} - \boldsymbol{\alpha}_1 - \boldsymbol{\beta}_1)$

   (c) α_0

   (d) σ_t^2

   (e) None of the above

35. The $q+1$-period ahead forecast of an MA(q) is

   (a) zero

   (b) the conditional mean

   (c) **the unconditional mean**

   (d) None of the above

36. The Value-at-Risk for horizon k, using the RiskMetrics methodology and the 2.5% quantile is:

    (a) VaR(k)= amount of position $\times 1.65 k \sigma_{t+1}^2$
    (b) VaR(k)= amount of position $\times 1.65\sqrt{k}\sigma_{t+1}$
    (c) VaR(k)= amount of position $\times 1.96 k \sigma_{t+1}$
    (d) **VaR(k)= amount of position** $\times 1.96\sqrt{k}\sigma_{t+1}$
    (e) None of the above

37. To test a multi-factor asset pricing model, we can test whether the intercepts in the time-series regressions of excess returns on the factors are equal to zero:

    (a) if the factors have a mean of zero
    (b) if the factors are not traded returns
    (c) **if the factors are traded returns**
    (d) in none of these cases

38. Your analyst has run forecasting regressions using overlapping returns, but wonders how many lags he/she should use when applying Hansen-Hodrick standard errors. The data is quarterly, the forecasting horizon is annual. How many lags is the most appropriate in this case?

    (a) 0
    (b) 1
    (c) **3**
    (d) More than 3 but less than 9
    (e) You should always use as many lags as possible (i.e., 9 or more)

39. The EGARCH and GJR-GARCH models can explain the following stylized fact of stock market returns that the GARCH model cannot explain

    (a) Excess kurtosis
    (b) Clustering
    (c) Time-varying expected returns
    (d) **Skewness**

(e) None of the above

40. Assume a market where the autocorrelation function of realized variance drops off after 1 lag, while the partial autocorrelation functiondecays very slowly. Assume you will use 1 year of daily data to estimate your model. Given this information, which of the following models do you think will likely give the best out-of-sample forecasts of stock market variance?

    (a) An AR(1) model on realized variance
    (b) ARCH(1)
    (c) ARCH(20)
    (d) **GARCH(1,1)**
    (e) GARCH(20,20)

1. **Market return forecasting (25 pts)**: You are tasked with providing a expected market log returns for the next 20 years to be used by a pension fund in their long-term asset allocation strategy. You have T annual time series observations of r_t (log market return) and the aggregate (market) log price-dividend ratio, pd_t. You run the following regression:

$$r_{t+1} = \phi_{10} + \phi_{11} r_t + \phi_{12} pd_t + \phi_{13} pd_{t-1} + \varepsilon_{r,t+1}, \tag{1}$$

and find that all coefficients are strongly significant.

   (a) (**10 points**) You decide to use a VAR (Vector Autoregression) model for your analysis as it can both capture the findings in Equation (1) and be used to find long-run (e.g., 20-year) expected returns. Write down the VAR model you will use and give the assumptions you need to make with respect to the residuals. Make sure to clearly define all variables, including all elements in each vector and matrix, and take care to use appropriate time subscripts so the forecasting aspect of your model is clear.
   Solution:
   The question asks for a VAR(1) specification:

$$\mathbf{z}_{t+1} - \boldsymbol{\mu} = \boldsymbol{\phi}\left(\mathbf{z}_t - \boldsymbol{\mu}\right) + \boldsymbol{\varepsilon}_{t+1}$$

   where

$$\mathbf{z}_t = \begin{bmatrix} r_t \\ pd_t \\ pd_{t-1} \end{bmatrix}$$

   and $\boldsymbol{\mu} = [\mu_1 \;\; \mu_2 \;\; \mu_2]'$ is a 3×1 vector and $\boldsymbol{\phi}$ is a 3×3 matrix with elements:

$$\boldsymbol{\phi} = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ 0 & 1 & 0 \end{bmatrix}.$$

   The residual vector $\boldsymbol{\varepsilon}_t = [\varepsilon_{r,t} \;\; \varepsilon_{pd,t} \;\; 0]$ has a 2×2 covariance matrix Σ. We could assume the residuals are jointly Normal and i.i.d., but this is not needed for consistency. You do need your residuals to be stationary, however. This could be implied from the assumptions you state. Your residual assumption is important for the discussion of the estimation below.

(b) (**5 points**) How would you estimate your model and obtain standard errors for the relevant estimated parameters? Clearly state, e.g., any regressions you would run, the maximum likelihood specification, or similar.

Solution:

Estimate the model via OLS. That is, run the following two regressions

$$\begin{aligned} r_{t+1} &= \phi_{01} + \phi_{11} r_t + \phi_{12} pd_t + \phi_{13} pd_{t-1} + \varepsilon_{r,t+1} \\ pd_{t+1} &= \phi_{02} + \phi_{21} r_t + \phi_{22} pd_t + \phi_{23} pd_{t-1} + \varepsilon_{pd,t+1} \end{aligned}$$

OLS is consistent even if the errors are non-normal and/or not i.i.d. In the case of heteroskedasticity and autocorrelation, the standard errors of the regression coefficients must be estimated accordingly using, e.g., Newey-West standard errors. The number of lags is unclear here and an empirical question. One could also estimate with maximum likelihood given a particular assumption on the residual distribution.

(c) (**5 points**) Give the conditions for stationarity of the VAR. No explicit calculations are needed, just the general condition.

Solution: The eigenvalues of the $\boldsymbol{\phi}$ matrix must all be less than 1 in modulus.

(d) (**5 points**) Assuming you have successfully estimated the parameters of your model, and that the VAR is indeed stationary, write down the equations, based on your VAR, that give the current (time T) estimate of the cumulative expected log returns to the market. That is, what is $E_t\left[\Sigma_{j=1}^{20} r_{t+j}\right]$?

Solution: The eigenvalues of $\boldsymbol{\phi}$ need to all be less than one.

Define the vector $e_r = [1\ 0\ 0]$. Thus, $e_r \mathbf{z}_{t+k} = r_{t+k}$. Then the k-period ahead forecast at time t is:

$$E_t\left[e_r \mathbf{z}_{t+k}\right] = e_r\left(\boldsymbol{\mu} + \boldsymbol{\phi}^k\left(\mathbf{z}_t - \boldsymbol{\mu}\right)\right).$$

So, the cumulative 20-year forecast is:

$$E_t\left[\Sigma_{j=1}^{20} r_{t+j}\right] = e_r \sum_{k=1}^{20}\left(\boldsymbol{\mu} + \boldsymbol{\phi}^k\left(\mathbf{z}_t - \boldsymbol{\mu}\right)\right).$$

This is sufficient for full credit. But, it looks perhaps a little nicer if you define

$$\boldsymbol{\phi}^{(20)} \equiv \sum_{k=1}^{20} \boldsymbol{\phi}^k = (I_3 - \boldsymbol{\phi})^{-1} \left(I_3 - \boldsymbol{\phi}^{21}\right)$$

Then:

$$E_t\left[\Sigma_{j=1}^{20} r_{t+j}\right] = e_r \left\{ \left(I_3 - \boldsymbol{\phi}^{(20)}\right) \boldsymbol{\mu} + \boldsymbol{\phi}^{(20)} \mathbf{z}_t \right\}.$$

2. **Cross-Sectional Factor Models (25 pts)**: Assume you have a stock-specific signal at each time t, $z_{i,t}$, that you believe is a useful predictor of cross-sectional differences in expected returns (just like $bm_{i,t}$ is, as discussed in class).

   (a) (**8 points**) You decide to run Fama-MacBeth regressions to see if your signal is indeed informative about stock's conditional expected excess return. Assume you have access to T returns for N stocks, as well as the corresponding T signals, $z_{i,t}$, for each stock $i = 1, ..., N$. Give the regressions you will run and the test you will use to evaluate if the signal is useful.

   Solutions:

   For each time t, run:

$$R_{i,t}^e = \lambda_{0,t} + \lambda_{1,t} z_{i,t-1} + \varepsilon_{i,t}.$$

   Collect $\{\lambda_{1,t}\}_{t=1}^T$ and test whether

$$\lambda_1 = \frac{1}{T} \sum_{t=1}^T \lambda_{1,t}$$

   equals zero, by noting that given a relatively large sample and uncorrelated $\lambda_{1,t}$ over time, we have that:

$$\frac{\lambda_1}{\sigma(\lambda_{1,t}) / \sqrt{T}} \sim N(0,1).$$

   (b) (**5 points**) Explain concisely how, in the Fama-MacBeth regressions, the intercept and slope coefficients at each time t are portfolio returns. As a part of this explanation, give the analytical expression for the slope coefficient at an arbitrary time t.

   Solutions:

Note that:

$$\begin{bmatrix} \lambda_{0,t} \\ \lambda_{1,t} \end{bmatrix} = \left(Z'_{t-1}Z_{t-1}\right)^{-1} Z'_{t-1}R^e_t,$$

where

$$Z_{t-1} = \begin{bmatrix} 1 & z_{1,t-1} \\ 1 & z_{2,t-1} \\ \vdots & \vdots \\ 1 & z_{N,t-1} \end{bmatrix}, \quad R^e_t = \begin{bmatrix} R^e_{1,t} \\ R^e_{2,t} \\ \vdots \\ R^e_{N,t} \end{bmatrix}.$$

We can think of the $2 \times N$ vector

$$\mathbf{w}_{t-1} = \left(Z'_{t-1}Z_{t-1}\right)^{-1} Z'_{t-1}$$

as 2 sets of portfolio weights (really, leverage multipliers) for each stock in the two portfolios underlying the excess portfolio returns $\lambda_{0,t}$ and $\lambda_{1,t}$. Note that the weights are known at time $t-1$, while the returns are realized from end of time $t-1$ to end of time t.

For the slope coefficient, in particular, we have:

$$\lambda_{1,t} = \textstyle\sum_{i=1}^{N} w_{i,t-1}R^e_{i,t},$$

where

$$w_{i,t-1} = \frac{z_{i,t-1} - E^N_{t-1}\left[z_{t-1}\right]}{N \times Var^N_{t-1}\left[z_{t-1}\right]}.$$

(c) (**5 points**) Assume you have constructed a portfolio that trades on your signal. Denote the associated portfolio excess returns as $R^e_{p,t}$. You will trade this portfolio as well as the market portfolio, which has excess returns $R^e_{m,t}$.

You run the following regression:

$$R^e_{p,t} = \alpha_p + \beta_p R^e_{m,t} + \varepsilon_{p,t},$$

where $\alpha_p = 4\%$, $\beta_p = 0.5$, and $\sigma\left(\varepsilon_{p,t}\right) = 10\%$. Let the sample average return and standard deviation of $R^e_{m,t}$ be 5% and 15%, respectively. What is the sample average return and return standard deviation of $R^e_{p,t}$ given this information and the above regression?

Solutions:

$$\begin{aligned}
\bar{R}_p^e &= 4\% + 0.5 \times 5\% = 6.5\% \\
\sigma_T\left(R_{p,t}^e\right) &= \sqrt{0.5^2 0.15^2 + 0.1^2} = 12.5\%
\end{aligned}$$

(d) (**7 points**) What is the maximal Sharpe ratio you can obtain by trading a combination of $R_{p,t}^e$ and $R_{m,t}^e$? Give this number along with the portfolio weights (leverage loadings) in the two portfolios for the maximal Sharpe ratio combination that delivers a portfolio return volatility of 15%. Make sure to show the math behind your calculations.

Solutions:

$$\begin{aligned}
IR &= \sqrt{4\% \times (10\%)^{-2} \times 4\%} = 0.4, \\
SR_m &= 1/3.
\end{aligned}$$

Thus, the max Sharpe ratio is:

$$\max SR = \sqrt{0.4^2 + 1/9} = 0.52.$$

Consider the two assets:

$$\begin{aligned}
R_t^\alpha &= R_{p,t}^e - 0.5 R_{m,t}^e = 4\% + 0.1\varepsilon_{\alpha,t}, \\
R_{m,t}^e &= 5\% + 0.15\varepsilon_{m,t}
\end{aligned}$$

where $\varepsilon_{\alpha,t}$ and $\varepsilon_{m,t}$ are uncorrelated with unit variance. Thus:

$$\begin{aligned}
w^{MVE} &= k \begin{bmatrix} 0.1^2 & 0 \\ 0 & 0.15^2 \end{bmatrix}^{-1} \begin{bmatrix} 4\% \\ 5\% \end{bmatrix} \\
&= k \begin{bmatrix} 208.27 \\ 115.71 \end{bmatrix}.
\end{aligned}$$

The standard deviation of this portfolio should be 15%:

$$\begin{aligned}
15\% &= k\sqrt{\begin{bmatrix} 208.27 \\ 115.71 \end{bmatrix}' \begin{bmatrix} 0.1^2 & 0 \\ 0 & 0.15^2 \end{bmatrix} \begin{bmatrix} 208.27 \\ 115.71 \end{bmatrix}} \\
&= 27.111k.
\end{aligned}$$

Thus:

$$k = 5.53 \times 10^{-3}$$

The MVE weights are thus:

$$w^{MVE} = \begin{bmatrix} 1.152 \\ 0.640 \end{bmatrix}$$

Finally, we have to remember that the α-asset is not the fundamental asset, so we have to adjust the market weight by subtracting off 0.5×1.152 (beta times above MVE portfolio weight in alpha asset) of the market. Thus:

$$\begin{aligned} \text{Portfolio weight in } R_p^e &= 1.152 \\ \text{Portfolio weight in } R_m^e &= 0.064 \end{aligned}$$