MFE 409 Lecture 3
Risk for Options

Valentin Haddad

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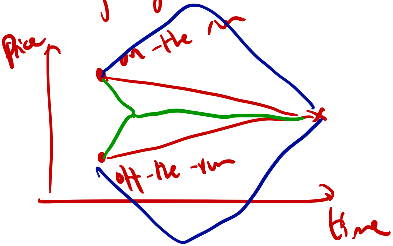
UCLA Anderson
SCHOOL of MANAGEMENT

LTCM

- Leverage
- Very big



Limits to arbitrage

Lecture Objectives

Risk management for option trading

- What are the risks of option strategies?
- How to quantify these risks?

Trading Derivatives and Risk Management

- Two broad levels of risk management inside financial institutions
  - Trader level: (hard) risk limits
  - Institution level: aggregate positions and construct broad measures of risk

Trading Derivatives and Risk Management

- Two broad levels of risk management inside financial institutions
  - Trader level: (hard) risk limits
    - Often expressed in terms of Greeks
  - Institution level: aggregate positions and construct broad measures of risk
    - Often around VaR

Delta

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- Delta (Δ) of a portfolio: change in portfolio price in response to a change in underlying price

$$\Delta = \frac{\partial P}{\partial S}$$

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- Buying $-\Delta$ of the underlying protects the portfolio against local changes in underlying price

- Can also hedge with another option

Linear Products

- If the value of the portfolio is linear in the price of the underlying, delta-hedging eliminates all risk
- Examples: forwards, futures, fixed promises in foreign currency, ...
- Static hedging works perfectly: "hedge and forget"

Example

- A U.S. company has a receivable of EUR 10mil in one year.
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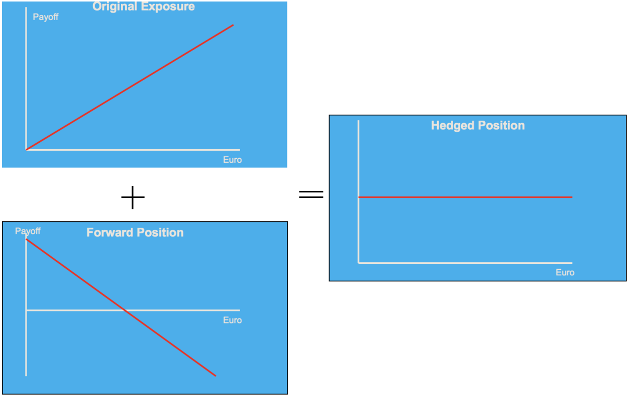

Nonlinear Products

- If portfolio payoff nonlinear, static delta-hedging does not protect against larger shocks
- But ...

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- But ...

  - Continuous delta-hedging eliminates all risk
  - By no arbitrage, can be used to find option prices

VaR for Options: Delta Approach

Volatility = s.d. of returns

- Portfolio:
  - Long EUR10m, EUR/USD $= 1.436$, volatility of EUR/USD 0.65%
  - Short 10m in puts to sell euros in 6m mth, $\Delta = -0.5044$
- 99 1% VaR?

$$R_p = 10m \times (M_{t+1} - M_t) - 10m\,(\underbrace{P_{t+1} - P_t}_{\approx \Delta (M_{t+1} - M_t)})$$

$$= 10m \times (1 - \Delta) \times (M_{t+1} - M_t)$$

$$= 10m \times M_t \times (1 - \Delta)\left(\frac{M_{t+1} - M_t}{M_t}\right)$$

$$= 14.36m \times (1 - \Delta)\, R_{M,t+1}$$

$$P(M_{t+1}) \approx P(M_t) + \underbrace{\frac{\partial P}{\partial M}}_{= \Delta}(M_{t+1} - M_t)$$

$$VaR = 2.32 \times 14.36 \times (1 - \Delta)\,\sigma$$
$$= \$325{,}000$$

VaR for Options: Delta Approach

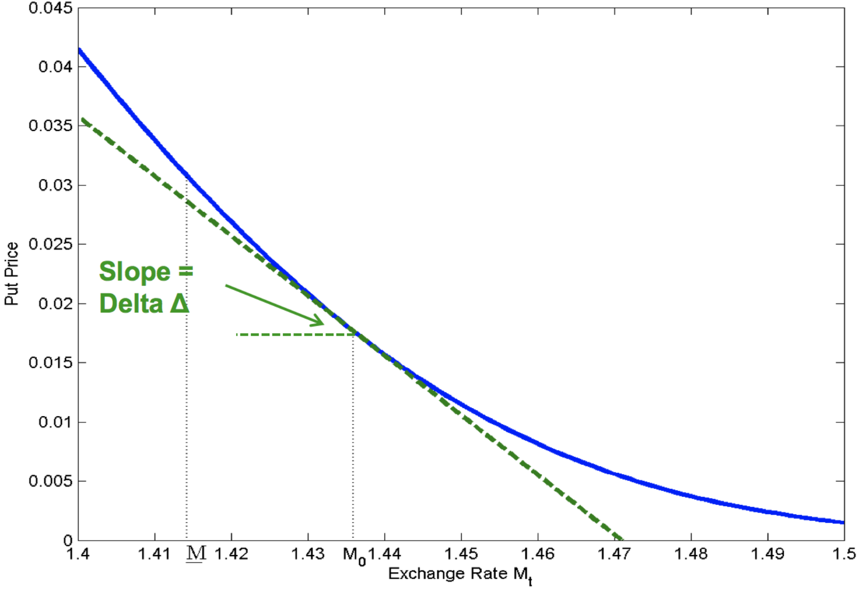
- Portfolio:
  - Long EUR10m, EUR/USD $= 1.436$, volatility of EUR/USD 0.65%)
  - Short 10m in puts to sell euros in 6m, $\Delta = -0.5044$
- 1% VaR?
  - Put price $p_t = G(M_t)$, where
  - Approximately:

    $p_{t+1} - p_t \approx G'(M_t) \times (M_{t+1} - M_t) = \Delta \times (M_{t+1} - M_t)$
  - Portfolio gain:

$$\begin{aligned} V_{t+1} - V_t &= 10\text{m} \times (M_{t+1} - M_t) + 10\text{m} \times (p_{t+1} - p_t) \\ &\approx 10\text{m} \times (1 + \Delta) \times (M_{t+1} - M_t) \\ &\approx \$14.36\text{m} \times (1 + \Delta) \times R_{M,t} \end{aligned}$$

  - ~~99% 1-day VaR~~ $\cancel{= 0.4956 \times \$217,204 = \$107,604}$

Put Price: Delta Approximation



When The Delta Approach Goes Wrong

- Nick Leeson

When The Delta Approach Goes Wrong

- Nick Leeson, 1995 Barings Bank 

- Short puts and Calls with the same strike price on Nikkei Index

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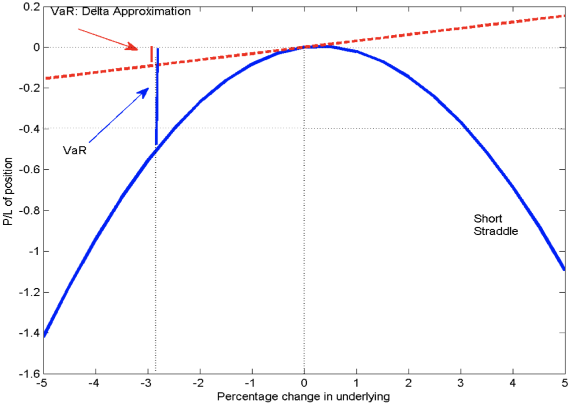

$$f(x) \approx f(x_0) + \underbrace{f'(x_0)}_{\Delta}(x-x_0) + \frac{1}{2}\underbrace{f''(x_0)}_{\Gamma}(x-x_0)^2$$

Gamma

- Gamma (Γ): second derivative of portfolio price with respect to the price of the underlying asset

$$\Gamma = \frac{\partial^2 P}{\partial S^2}$$

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- Also rate of change of Delta with respect to the price of the underlying asset:

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- Also rate of change of Delta with respect to the price of the underlying asset:

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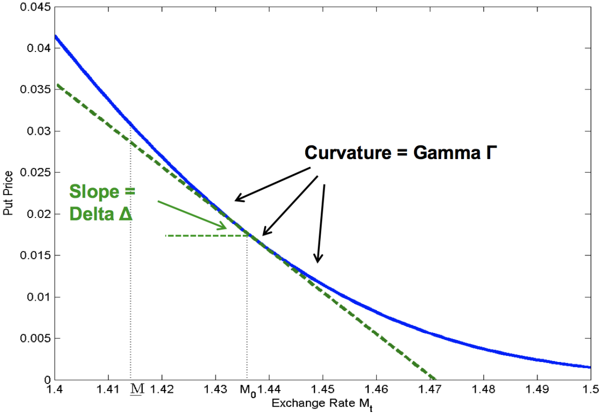
  - Delta-Gamma hedging does better with less frequent readjustments

Put Price: Delta Gamma Approximation

$$P(S) \approx P(S_0) + \underbrace{P'(S_0)}_{\Delta}(S - S_0) + \frac{1}{2}\underbrace{P''(S_0)}_{\Gamma}(S - S_0)^2$$



VaR for Options: Delta-Gamma Approach

- Assume change in underlying price $S_{t+1} - S_t \sim \mathcal{N}(\mu_S, \sigma_S^2)$
- Change in portfolio value:

$$P_{t+1} - P_t \approx \Delta \times (S_{t+1} - S_t) + \frac{1}{2}\Gamma \times (S_{t+1} - S_t)^2$$

$P_{t+1} - P_t \sim \mathcal{N}(\mu_P, \sigma_P^2)$

$VaR = -\mu_P + 2.32\,\sigma_P$

$E(P_{t+1} - P_t) = \Delta \mu_S + \frac{1}{2}\Gamma(\mu_S^2 + \sigma_S^2)$

$\sigma_P^2 =$

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- Can compute moments of $R_S = S_{t+1} - S_t$:

$$\mathbb{E}[R_s] =$$

$$\mathbb{E}[R_s^2] =$$

$$\mathbb{E}[R_s^3] =$$

$$\mathbb{E}[R_s^4] =$$

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$$\mathbb{E}[R_s] = \mu_S$$
$$\mathbb{E}[R_s^2] = \sigma_S^2 + \mu_S^2$$
$$\mathbb{E}[R_s^3] = \mu_S^3 + 3\mu_S\sigma_S^2$$
$$\mathbb{E}[R_s^4] = \mu_S^4 + 6\mu_S^2\sigma_S^2 + 3\sigma^4$$

VaR for Options: Delta-Gamma Approach

- Obtain mean and variance of $P_{t+1} - P_t$:

$$\mathbb{E}[P_{t+1} - P_t] =$$

$$\mathrm{var}[P_{t+1} - P_t] =$$

VaR for Options: Delta-Gamma Approach

- Obtain mean and variance of $P_{t+1} - P_t$:

$$\begin{aligned}
\mathbb{E}[P_{t+1} - P_t] &= \Delta\mathbb{E}[R_s] + \frac{1}{2}\Gamma\mathbb{E}[R_s^2] \\
&= \Delta\mu_S + \frac{1}{2}\Gamma(\mu_S^2 + \sigma_S^2) \\
\mathsf{var}[P_{t+1} - P_t] &= \Delta^2\mathsf{var}[R_s] + \frac{1}{4}\Gamma^2\mathsf{var}[R_s^2] + \frac{1}{2}\Delta\Gamma\mathsf{cov}[R_s, R_s^2] \\
&= \Delta^2\sigma_S^2 + \frac{1}{2}\Gamma^2\sigma_S^2(2\mu_S^2 + \sigma_S^2) + \Delta\Gamma\mu_S\sigma_S^2
\end{aligned}$$

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\end{aligned}$$

- Plug in the estimator for the normal distribution:

$$\mathsf{VaR}(c) = -\mathbb{E}[P_{t+1} - P_t] + z(c)\mathsf{var}[P_{t+1} - P_t]$$

VaR for Options: Delta-Gamma Approach

- Obtain mean and variance of $P_{t+1} - P_t$:

$$\begin{aligned}\mathbb{E}[P_{t+1} - P_t] &= \Delta \mathbb{E}[R_s] + \frac{1}{2}\Gamma \mathbb{E}[R_s^2] \\ &= \Delta \mu_S + \frac{1}{2}\Gamma(\mu_S^2 + \sigma_S^2) \\ \mathsf{var}[P_{t+1} - P_t] &= \Delta^2 \mathsf{var}[R_s] + \frac{1}{4}\Gamma^2 \mathsf{var}[R_s^2] + \frac{1}{2}\Delta\Gamma \mathsf{cov}[R_s, R_s^2] \\ &= \Delta^2 \sigma_S^2 + \frac{1}{2}\Gamma^2 \sigma_S^2 (2\mu_S^2 + \sigma_S^2) + \Delta\Gamma \mu_S \sigma_S^2\end{aligned}$$

- Plug in the estimator for the normal distribution:

$$\mathsf{VaR}(c) = -\mathbb{E}[P_{t+1} - P_t] + z(c)\mathsf{var}[P_{t+1} - P_t]$$

- Can also deal with portfolio of options with more than one risk

Cornish-Fisher Expansion

- With this approach, we could also compute any moments of the portfolio: skewness, kurtosis, ...
- How to incorporate into VaR calculation?

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- How to incorporate into VaR calculation?

- **Cornish-Fisher expansion**: asymptotic expansion for the quantile of a distribution
  - Skewness: $\xi_P = \mathbb{E}[(R_P - \mu_P)^3]/\sigma_P^3$
  - Quantile $1 - c$:

$$\mu_P + \left(\overbrace{z(1-c)}^{-2.32} + \frac{1}{6}\left(z(1-c)^2 - 1\right)\xi_P \right) \sigma_P$$

$\Rightarrow VaR = -\mu_P - \sigma_P \left(z(1-c) + \frac{1}{6}\left(z(1-c)^2 - 1\right) \xi_P \right)$

  - Can also include kurtosis and higher moments

Vega

- Vega (ν): derivative of option value with respect to the volatility of the underlying asset

$$\nu = \frac{\partial P}{\partial \sigma}$$

- Under the assumptions of Black-Scholes, there is no risk of change in volatility ... but in practice volatility can move

- We can add changes in volatility to our previous calculations:

$$P_{t+1} - P_t = \Delta \times (S_{t+1} - S_t) + \frac{1}{2}\Gamma \times (S_{t+1} - S_t)^2 + \nu(\sigma_{t+1} - \sigma_t) + ...$$

Other Greeks

- Theta (Θ): change of the value of the portfolio due to passage of time:

$$\Theta = \frac{\partial P}{\partial t}$$

  - Often ignored for risk management (same as means)
- Rho: change of the value of the portfolio due to a parallel shift in all interest rates in a particular country

$$\text{Rho} = \frac{\partial P}{\partial r}$$

  - Particularly relevant for interest rate and exchange rate products

In practice

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- Traders must keep Gamma and Vega within limits set by risk management
  - Adjust whenever the opportunity arises

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- Traders must be delta neutral at least once a day
- Traders must keep Gamma and Vega within limits set by risk management
  - Adjust whenever the opportunity arises
- Delta can be adjusted by trading the underlying
- Gamma and Vega need trading of other options

Takeaways

- When trading options, identify the key risks and hedge them
- Think one step ahead and about potential large shocks: gamma-hedging
- For risk management: crucial to take into account the non-linearity of option contracts