UCLA Anderson School of Management
Daniel Andrei, Derivative Markets MGMTMFE 406, Winter 2018

Problem Set 4

These exercises do not need to be turned in for credit.

1 Exotic options

Consider a financial market with a riskless money market account which pays a constant interest rate r and a risky stock which pays a continuous porportional dividend at the constant rate δ. The price of the stock is reduced by the dividend payment and it follows that the stock price evolves according to

$$dS_t = S_t \left[\mu dt + \sigma dB_t\right] - \delta S_t dt \tag{1}$$

a. We call **paylater call** a European call option where the holder of the option pays no initial premium but pays an amount π at maturity only if the call ends up in the money. The payoff is therefore given by

$$S_T - K - \pi \quad \text{if} \quad S_T \geq K \tag{2}$$
$$0 \quad \text{if} \quad S_T < K \tag{3}$$

Let $c(S_0, 0; K, T)$ denote the price of a European vanilla call option at date 0 with maturity T and strike K. Compute the value of π as a function of $c(S_0, 0; K, T)$.

b. We call **power option** a European call option with payoff given by

$$\max\left[S_T^2 - K, 0\right] \tag{4}$$

Compute the dynamics of S_T^2 and then deduce a value for the power option.

2 Security design

Value the M&I stock purchase contract (pages 458-460 in McDonald 3rd edition) assuming that the 3-year interest rate is 3%, the dividend yield on the stock is 2%, and the M&I volatility is 15%. *Note: you are asked to value the* ***second*** *component of the M&I issue. Please ignore the first component (interest in the trust).*

3 An equity-linked structured note

Consider a structured investment product that is available from Credit Suisse for an initial cost of 1$ and whose payoff in 3 years is given by

$$1 + \max\left[\alpha, c\left(\frac{S_3}{S_0} - 1\right)\right] \tag{5}$$

where α, c are positive constants and S_t denotes the level of the S&P500 at time t. The riskless interest rate is 7%, the volatility of the S&P500 is equal to 20% per year and the dividend yield on the index is $\delta = 2\%$ per year.

a. What does this product do?

b. Show that the terminal payoff of the strucutred product can be written as $\beta + (c/S_0)\max(S_3 - K, 0)$ for some constants β and K to be determined.

c. Find an equation that relates the value of α to the maximum value of c that Credit Suisse can offer while being sure to not loose any money on the trade. Compute the maximal value of c when $\alpha = 2\%$.

d. Assume that Credit Suisse offers you to invest in a contract with $c = 0.8$ and $\alpha = 2\%$. What should you do? What if the volatility was equal to 35%?

4 Portfolio insurance

A fund holds a well-diversified portfolio that tracks the performance of the S&P500 and is currently worth 360,000,000. The current value of the index is 1,200 and the fund manager would like to buy insurance against a reduction of more than 5% in the index level at a six month horizon.

Assume that the riskless rate is 6%, that the dividend yield on the index is 3% and that the annualized volatility of the index is 30%. **For this exercise, assume that options are written on 1 unit of the index.**

a. If the fund manager decides to obtain his insurance by investing in put options, how much would it cost?

b. Show that if the fund manager decides to use calls instead of puts the initial cost would be the same.

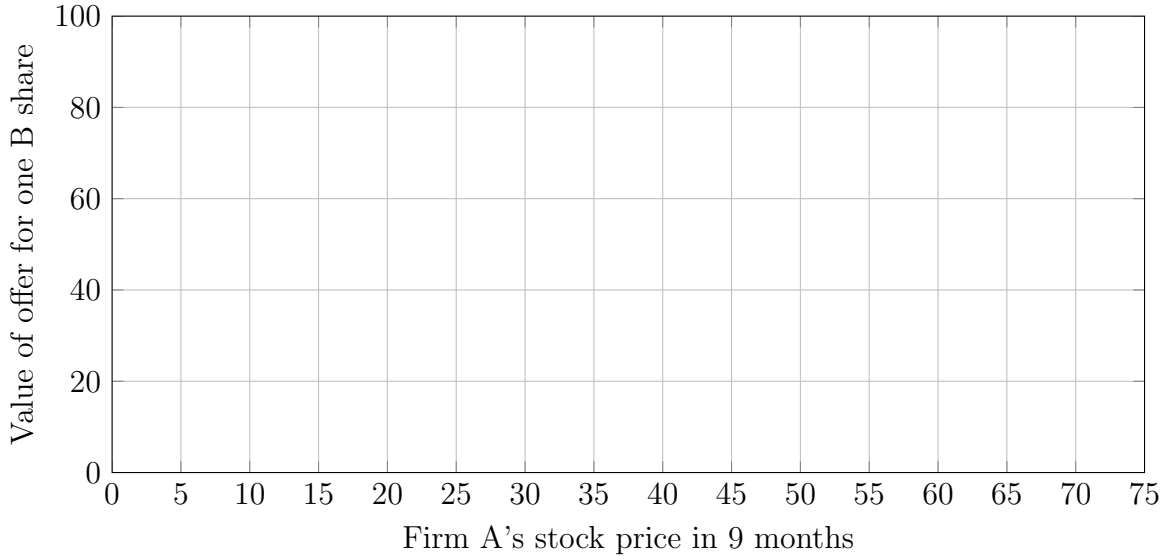
c. Assume that instead of buying options, the fund manager decides to synthetically create the option he needs by trading in the index and the riskless asset. How much should he initially invest in the riskless asset?

5 Collars in acquisitions

Firm A has a stock price of \$38 and has made an offer for firm B where A promises to pay \$60/share for B, as long as A's stock price remains between \$35 and \$45. If the price of A is below \$35, A will pay 1.714 shares, and if the price of A is above \$45, A will pay 1.333 shares. The deal is expected to close in 9 months. Assume (for A's stock): $\sigma = 40\%$, $r = 6\%$, and $\delta = 0$.

a. How are values 1.714 and 1.333 arrived at?

b. Plot the value of A's offer for one B share as a function of A's stock price in 9 months. Clearly indicate on the diagram the slope of all the lines that you draw.



c. What is the value of the offer?

d. How sensitive is the value of the offer to the volatility of A's stock? More precisely, what is the value of the offer if the volatility of A's stock goes up to 45%? Here is some information that you might find useful:

Vega of a \$35-strike call	0.1134
Vega of a \$35-strike put	0.1134
Vega of a \$45-strike call	0.1291
Vega of a \$45-strike put	0.1291