Midterm Practice

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1 Short Answer Questions (45 Points)

1.1 Modigliani-Miller Theorem (5 Points)

In the following statement TRUE or FALSE:

Assuming perfect financial markets, a reduction in diversifiable risk does not increase shareholder value. Such a reduction is performed with zero extreme shareholder wealth, according to the theorem.

Explain your choice.

False

If MM & the Theorem Holds then the firm's market value, which is derived from future revenue streams, risk and overall risk, is independent of the capital structure chosen. Therefore, reducing systematic risk by changing the capital structure will not increase firm value, which means the change in shareholder's wealth

1.2 Component WaC (5 Points)

In the following statement TRUE or FALSE:

Suppose you have a portfolio with three instruments: a stock, a corporate bond, and a Treasury Bond. The Bond Market moves upward in the next week. Bill analysts forecast in the corporate bond, and the stock market. The Treasury Bond is 50% of the portfolio, the stock is 30% of the portfolio, and the corporate bond, and Treasury bond are 0.1 and 0.6, respectively. Bill's analysts forecast the stock market will increase by 2% and the corporate bond will increase by 1%. The portfolio's WaC can be calculated by adding the aforementioned component VaRs, which is equal to 30.5% million + 10.5 million = 41.25 million.

Explain your choice.

By Euler Proposition,

The portfolio total VaR is the sum of each component VaR. Thus, liquidity and position will result in profits that will equal to summing position VaR component VaR. $= 30.5 + 10.5 = 41$ mil

1.3 Parameter Estimation in the ARCH(1) Model (5 Points)

The ARCH(1) model assumes that the mean series $\{R_t\}$ is a stationary process satisfying

$$R_t = \epsilon_t,$$
$$\sigma_t^2 = \omega + \alpha \epsilon_t^2$$

where $\{\epsilon_t\}$ is standard normal and the parameters satisfy $\omega > 0$ and $0 \le \alpha < 1$. Additionally, note that the unconditional variance of R_t is $\sigma^2 = \frac{\omega}{1-\alpha}$ and $\sigma_t^2 = \mathbb{E}[R_t^2 | R_{t-1}, R_{t-2}, \ldots]$. Suppose $\omega = 0.1$ and $\alpha = 0.5$. Suppose the observed R_t equals 2 for the last two days. Compute the conditional likelihood of the joint conditional density $f(R_{t-1}, R_t | R_{t-2} = 2)$ and σ_t^2.

$\epsilon_t \sim N(0,1)$

Then $R_t = N(0,1)$

Since $\sigma_t^2 = \omega + \alpha R_t^2$

(Since the distribution is normalized)

Therefore $R_t = N(0, \frac{\omega}{1-\alpha})$

$$f(R_t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{R_t^2}{2\sigma^2}}$$

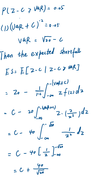
Then the likelihood is

$$L(f(R_t)) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{R_t^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi\frac{0.1}{1-0.5}}} e^{-\frac{2.2^2}{2\frac{0.1}{1-0.5}}}$$

1.4 Measuring Risk of Stock (5 Points)

The current price of a stock is \$100. Suppose that the stock has a 25% chance of being by 10% a year from now and a 75% chance of being worth \$120 a year from now. Assume the current price of the stock is \$100. Suppose the dividends and the risk-free rate is 2% per an annual period. Your investment hand-out is \$100 and you have a 1-year investment horizon. Compute the 1-year VaR. Calculate the 1-year MC VaR and the 1-year MC Expected Shortfall for your portfolio.



The cdf of the stock's distribution.

VaR $= 100 - 75 = 25$ mil

$ES = 100 - (0.0125*90 + 0.25*90/0.25)$

$= 100 - 90.5$

$= 9.75$ mil

1.5 Soft and Diversification (5 Points)

In the following statement TRUE or FALSE:

The VaR of a diversified portfolio can be much larger than the VaR of its individual components.

Explain your choice.

True. VaR does not take into account of losses to the right of the distribution. The combined distribution could exceed the original quantile, therefore various total VaR.

1.6 VaR and ES of Pareto-Distributed Random Variables (5 Points)

Let Z follow a Pareto distribution (i.e., $F(Z > z) = z^{-\alpha}$) with $\alpha = 0.5$ and $\alpha = 1$. Assume security X's return is Pareto-distributed. If Z gives some instance Z, Derive expressions for security X's 95% VaR and Expected Shortfall.

$R_X = Z \to$ where $Z \sim$ Pareto distribution

95% VaR follows:

$P(\text{VaR}) = \text{VaR} \to 1 - F(\text{VaR}) = 5\%$

$P(R_X > \text{VaR}) = 5\% = 0.05$

$P(Z > \text{VaR}) = 0.05$

$(1)(\text{VaR}.5)^{-1} = 0.05$

$\text{VaR} = 400 - 6$

Then the expected shortfall

$ES = E[Z > 6 | Z > 6 \text{ MR}]$

$= Z_0 - \frac{\int_{-\infty}^{(\text{VaR})} z f(z) dz}{\int_{-\infty}^{(\text{VaR})} f(z) dz}$

$= C - w \frac{\int_{-\infty}^{(\text{VaR})} z(\frac{1}{2}) z^{-\frac{1}{2}} dz}{\int_{-\infty}^{(\text{VaR})} f(z) dz}$

$= C - w \left[\frac{1}{2} \int_{-\infty}^{(\text{VaR})} z^{-\frac{1}{2}} dz \right]$

$= C + w \left[\frac{1}{2} \sqrt{\text{VaR}} \right]$

$= C + \frac{1}{2}\sqrt{\text{VaR}}$

1.7 Backtesting VaR (5 Points)

We want to back test our 99% daily VaR model. We have 500 observations of daily returns. Derive the probability of observing 15 or more exceptions.

$P(\text{exceptions} \ge 15) = 1 - P(\text{exceptions} \le 5)$

$= 1 - \sum_{i=0}^{5} (.01)^i (.99)^{500-i} \binom{500}{i}$

$= 1 - 0.0031 - 0.0156 - 0.0781$

$= 0.7969$

1.8 Regulatory Arbitrage (5 Points)

Consider an portfolio manager who manages small amount of funds, but one has been invested in very risky positions. The portfolio manager's compensation is based on the portfolio's Sharpe ratio. The reward series of the portfolio has an subject to some regulatory constraints to how much risk they can take. Bank regulations require that the portfolio's VaR cannot exceed a certain threshold. The portfolio manager's compensation is also subject to a bonus or VaR constraint or an expected shortfall constraint. Explain why the manager will choose to invest in very risky positions.

The manager who invests in very risky types of bonds very profile risk, or the bonds have a very high probability of incurring losses which behind risk averse and thus will only invest for high returns. On the other hand, the manager who diversifies the portfolio might be discovered by the loss distribution provided and thus an expected shortfall.

1.9 Confidence Interval of VaR (5 Points)

Suppose we are the manager of a portfolio. We estimate the 99% VaR of a portfolio from 1,000 observations and we get 10 million. We want to construct a 95% confidence interval for our estimate of the 99% VaR. Assume the portfolio's returns are normal with zero mean and standard deviation of 2%. Assume the returns are independent and identically distributed, and so forth, and so forth.

The portfolio value is

$E[R_t]_{\text{mean}} = 0$

The portfolio variance is

$\text{var}_{\text{port}} = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho_{12}$

To infer, the portfolio 99% total VaR is:

$\text{VaR} = -\text{Port}_{\text{mean}} - Z_{1-\alpha} \sigma_{\text{port}}$

$= 2.33 \sqrt{\frac{1}{1000} \sum_{i=1}^{1000} (R_i - \bar{R})^2}$

The marginal VaR for r_i is:

$D\text{VaR}_{r_i} = \frac{\partial \text{VaR}}{\partial r_i} = \frac{Z_\alpha}{\sigma_p} \left[\rho_{i,p} \sqrt{\frac{1}{1000} \sum_{i=1}^{1000} (R_i - \bar{R})^2} \right]$

1.2 Measuring the Risk of Currency Trades (15 Points)

Derrick Pitts (DP) is a German bank that manages to back in EUR. Consider 3 dollar in DM, one is long and two is short and the current exchange rate is 1.5 DM/dollar. The exchange rate per EURO is 0.5 DM/EUR and 1 dollar/EUR. The daily volatility of the DM/EUR and dollar/EUR are 0.5% and 1% respectively. The correlation between these is 0.5. Assume that the returns have means zero and are normally distributed.

1. What is the MV VaR for each dollar? (5 Points)
2. What is the MV VaR for the combined portfolio? (5 Points)
3. What are the MV Marginal and Component VaR for each dollar? (5 points)

1. Consider positions in EUR

If long 1M million USD $\to$ 1M $\times$ 0.75EE

$\to$ 1.5 1000 mil EUR

If short 2M million USD $\to$ -2M $\times$ 1.5DM

$\to$ -1.5M mil EUR.

Therefore the 99% VaR for two dollars are (assume normal)

$\text{VaR}_{\$} = (\text{Mu} + 2.33\sigma)\text{Pf}$

$= -0.0*0 \times (-1.5 - 1.5) \times [1.5]$ 99%

$= 1.33$ mil

$\text{VaR}_{\$} = (\text{Mu} + 2.33\sigma)*\text{Pf}$

$= -0.0*0 \times 1.5 \times (-1.75)$

$= 0.0175$ mil

2. The combined portfolio has return 0.

$\text{variance} \; \frac{\$}{\text{EUR}} = (1.5)^2 \sigma_{\$}^2 + (1.5)^2 \sigma_{\text{EUR}}^2 - 2(1.5)(1.5)\rho(\sigma_{\$})(\sigma_{\text{EUR}})$

$= (1.5)^2 (0.01)^2 - (1.5)^2 (1)^2 - 2(1.5)(1.5)(1)(0.5)$

$= 0.1375$ mil

Therefore, VaR for combined portfolio.

$\text{VaR}_{\$} = \sqrt{0.1375} \times (-2.33)$

3. 99% marginal VaR for dollar 0

$D\text{VaR}_{\$} = 2.33 \; \frac{(0.5)\sigma_{\$} + (1.5)(0.5)\sigma_{\$}(0.5)(1.5)\sigma_{\$}}{\sqrt{0.1375}}$

$= 0.0000$

$\text{Comp}_{\$} = 1.33 \text{ mil} * \text{comp}$

$= 0$

99% MVaR. Comp for dollar 0.

$\text{Comp}_{\$} = 1.33 \text{ mil} * \text{comp}$

$= 0.0 \text{ mil}$

2 Longer Question 1 (20 Points)

2.1 Portfolio of Bonds (10 Points)

A portfolio manager wants to invest in treasury bonds and bill for a year. The manager the portfolio manager has \$100 million to invest. The manager has a 1-year investment horizon. The manager's objective is to minimize the 1-year VaR subject to the 1-year Sharpe ratio. Assume that the 1-year Sharpe ratio. Assume that the 1-year Sharpe ratio. Assume that