UCLA Anderson School of Management
Valentin Haddad, Financial Risk Management, Spring 2018

MFE 409 – Final Exam

June 12, 2018

Date: ______________________________

Name: ______________________________

Signature: ______________________________

- As a member of the UCLA Anderson academic community, the highest standards of academic behavior are expected of you. It is your responsibility to make yourself aware of these standards (specifically regarding plagiarism, individual work, and team work) and adhere to them. By signing the exam: (*i*) you certify your presence, and (*ii*) you state that you neither gave nor received help during the examination.
- This exam is closed book. A table of the normal distribution is given at the end of the exam. You can use a scientific calculator, and a two-page cheat sheet (i.e. one sheet of paper), **but be sure to show or explain your work.**
- You are not allowed to communicate with anyone (verbally, in writing, or electronically) during the exam period.
- You have **180 minutes** to finish the exam. The total score is **180 points**. One minute corresponds roughly to one point. Pace yourself wisely.

1 Short Answer Questions (114 Points)

1.1 Liquidity Risk in Basel III (8 Points)

What are the two ways Basel III regulates liquidity risk? Explain the rationale behind each of these two regulations.

SOLUTION: See class notes. Liquidity Coverage Ratio (High-Quality Liquid Assets/Net Cash Outflows in a 30-Day Period). Net Stable Funding Ratio (Amount of Stable Funding/Required Amount of Stable Funding). Both must be greater than 100%.

1.2 VaR for Options (8 Points)

You have a portfolio of options on Apple stocks. The stock trades at \$190 and has daily volatility of 2%. You are long 10 options with a Delta of 0.7 and a Gamma of 0.2, and short 5 options with a Delta of 0.4 and a gamma of 1. Using the delta-gamma method, what is the 1-day 99% VaR for your portfolio?

SOLUTION: Add up the delta and gamma then apply the formula from class. Value of the portfolio

$$P_t = 10P_{1,t} - 5P_{2,t}$$

So

$$\begin{aligned} P_{t+1} - P_t &= (10\Delta_1 - 5\Delta_2)(S_{t+1} - S_t) + \frac{1}{2}(10\Gamma_1 - 5\Gamma_2)(S_{t+1} - S_t)^2 \\ &= 5R_S - \frac{3}{2}R_S^2 \end{aligned}$$

whre $R_S \sim N\left(0, 0.02^2\right)$. Then

$$\begin{aligned} E\left[P_{t+1} - P_t\right] &= -\frac{3}{2}(0.02)^2 = -0.0006 \\ \cancel{var\left[P_{t+1} - P_t\right]} &\cancel{= 25(0.02)^2 + \frac{1}{2}9(0.02)^4 \approx 0.01} \end{aligned}$$

so

$$VaR_{99\%} = -0.0006 + 2.326(0.01) = 0.023$$

1.3 Back-testing a VaR Measure (8 Points)

Why is bunching of exceptions the sign of an issue with a VaR measure?

SOLUTION: VaR measure fails to capture time variation in risk or extreme tail-risk events happened. A useful VaR measure has very few exceptions and randomly distributed along the time series.

1.4 Exponential Weighting (8 Points)

When using exponential weighting to compute the VaR using the historical method, we choose typically a parameter $\lambda = 0.995$. When using exponential weighting to estimate volatility we choose typically a parameter $\lambda = 0.94$. Explain why such different values.

SOLUTION: Different λ helps to adjust the weight of historical data. Estimating 1% quantile needs a lot of data (it only uses outliers) whereas estimating volatility is easier (it uses all the data), therefore we use a smaller λ.

1.5 VaR with Trading Costs (10 Points)

You want to estimate the risk of trading an illiquid asset for a day. The mid-price of the asset today is \$10,000, and the distribution of the change in mid-price between today and tomorrow is $\mathcal{N}(0, 9)$. The bid-ask spread is 50 basis points today and the distribution of the bid ask spread tomorrow is normally distributed with mean 40 basis point and standard deviation 15 basis points. Assume the change in mid-price and the bid-ask spread are independent from each other.

What is the 99% 1-day VaR for buying one unit of the asset today and selling it tomorrow?

SOLUTION: You buy at bid today and sell at ask tomorrow.

$$
\begin{aligned}
\frac{Ask_{t+1}}{Bid_t} &= \frac{Mid_{t+1}\,(1 \cancel{+} - spread_{t+1}/2)}{Bid_t} \\
&= \frac{Mid_{t+1}\,(1 \cancel{+} - spread_{t+1}/2)}{Mid_t\,(1 \cancel{-} + spread_t/2)} \\
&\approx \Delta mid_{t+1} \cancel{+} - \frac{1}{2}\,(spread_{t+1} + spread_{t+1})
\end{aligned}
$$

Using independence, this is the sum of two normals, and you can use the standard formula.

$$\frac{Mid_{t+1}}{Mid_t} \times \frac{1 - S_{t+1}/2}{1 + S_t/2}$$

$$\left(1 + \frac{Mid_{t+1} - Mid_t}{Mid_t}\right) \times \left(1 - S_{t+1}/2 - \frac{S_t}{2}\right)$$

$$\approx 1 + \frac{Mid_{t+1} - Mid_t}{Mid_t} - \frac{S_t + S_{t+1}}{2}$$

x is small

$$\frac{1}{1-x} \approx 1 + x$$

1.6 The VIX (8 Points)

What are the advantages and limitations of the VIX as a measure of future volatility?

SOLUTION: Advantages: it is a real time and forward-looking measure. Limitation: risk-neutral (encodes variance risk premium, typically is above actual realized volatility)

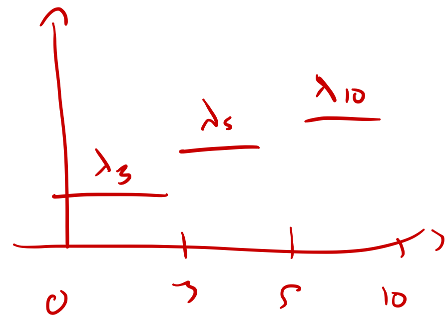
1.7 Hazard Rate (10 Points)

A bond has a probability 10% to default in the next 3 years, 20% to default in the next 5 years, and 80% to default in the next 10 years. Compute a hazard rate curve consistent with these numbers.

SOLUTION: Compute the λ.

$$\lambda_j = \frac{-\ln(V_j)}{j}$$

for each j.



$$V(0) = 1$$

$$V(3) = 0.9$$

$$V(5) = 0.8$$

$$V(10) = 0.2$$

$$\lambda_3 = \frac{\log(V(0)) - \log(V(3))}{3}$$

$$\lambda_5 = \frac{\log(V(3)) - \log(V(5))}{\underset{5-3}{\underset{\shortparallel}{2}}}$$

$$\int_{t_1}^{t_2} \lambda(t)\, dt = \log(V(t_1)) - \log(V(t_2))$$

1.8 Issue With Copulas (10 Points)

Two bonds have time to default given by the random variables T_1 and T_2. Assume that they are both exponentially distributed with parameters λ_1 and $\lambda_2 < \lambda_1$ respectively, and follow a normal copula with correlation ρ. If $\rho = 1$, what is the distribution of T_2 conditional on T_1? Comment on the result.

The c.d.f. of an exponential distribution is $1 - e^{-\lambda x}$.

SOLUTION: $\rho = 1$ means that quantiles exactly match. So $1 - \exp(-\lambda_1 T_1) = 1 - e(-\lambda_2 T_2), T_2 = \lambda_1 / lambda_2 T_1$. Issue: Defaults perfectly copula correlated means default today will exactly predict default in the future, rather than meaning defaults happen at the same time

1.9 Merton Model (12 Points)

Explain in detail how you would use market data on the equity of a company to estimate its default probability. Also provide equations for the relevant quantities.

SOLUTION: This is from class. Slides 33-339, Lecture 5. You should have the forumla of the probability of default. Explain how you would implement the estimation of the asset value of the company if debt and equity are traded and also if equity is only traded.

1.10 Credit Default Swaps (10 Points)

If there are no arbitrage opportunities, as a CDS seller, do you make more profit by selling a contract with or without the cheapest-to-deliver feature?

SOLUTION: It doesn't matter because you can charge the buyer for the cheapest-to-deliver feature. Explaining what is the cheapest-to-deliver feature is not considered an answer.

1.11 VaR for a "Straddle" (12 Points)

Assume that a stock trades at \$90 and has 10-day returns following a log-normal distribution with volatility 6%. This corresponds to the log return following a normal distribution: $\log(R_{t+10}) \sim \mathcal{N}(0, 6\%^2)$. You are short a contract that pays $|\log(P_{t+10}/90)|$ in 10 days. What is the 99% 10-day VaR of your position?

SOLUTION: The contract is really on absolute value of a normal. So you have to take the 99.5% percentile, $z = 2.58$, because both sides of the distribution give you losses (i.e., a short straddle).

1.12 MLE Estimation of Correlation (10 Points)

Assume X_1 and X_2 are multivariate normal with mean 0, variance 1 and correlation ρ. Derive an equation satisfied by the maximum-likelihood estimator of ρ. Compare it to the standard estimator.

SOLUTION:

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 1 & \rho \\ & 1 \end{bmatrix} \right)$$

Then, for any given pair of observation

$$f\left(X_1, X_2; \rho\right) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{X_1^2 + X_2^2 - 2\rho X_1 X_2}{2\left(1-\rho^2\right)} \right)$$

The joint likelihood

$$F = \Pi_{i=1}^{i=N} f\left(\left\{X_{1,i}, X_{2,i}\right\}; \rho\right) = \left(\frac{1}{2\pi\sqrt{1-\rho^2}} \right)^N \exp\left(-\frac{1}{2\left(1-\rho^2\right)} \sum_{i=1}^{N} X_{1,i}^2 + X_{2,i}^2 - 2\rho X_{1,i} X_{2,i} \right)$$

Take logs

$$-N \log\left(2\pi\sqrt{1-\rho^2}\right) - \frac{1}{2\left(1-\rho^2\right)} \sum_{i=1}^{N} X_{1,i}^2 + X_{2,i}^2 - 2\rho X_{1,i} X_{2,i}$$

$$-N \log\left(2\pi\right) - \frac{N}{2} \log\left(1-\rho^2\right) - \frac{1}{2\left(1-\rho^2\right)} \sum_{i=1}^{N} X_{1,i}^2 + X_{2,i}^2 - 2\rho X_{1,i} X_{2,i}$$

First order condition

$$\frac{-N\widehat{\rho}}{1-\widehat{\rho}^2} - \left(\frac{2\sum_{i=1}^{N} X_{1,i} X_{2,i} - 8\widehat{\rho} \sum_{i=1}^{N} X_{1,i} X_{2,i}}{4\left(1-\widehat{\rho}^2\right)^2} \right) = 0$$

Then the solution is a third order polynomial

$$-4N\widehat{\rho}^3 + \left(8\sum_{i=1}^{N} X_{1,i} X_{2,i} - 4N \right) \widehat{\rho} + 2\sum_{i=1}^{N} X_{1,i} X_{2,i} = 0$$

Where the "standard" estimator

$$\widehat{\rho} = \frac{\sum_{i=1}^{N} X_{1,i} X_{2,i}}{\sqrt{\sum_{i=1}^{N} X_{1,i}^2 \sum_{i=1}^{N} X_{2,i}^2}}$$

is not a solution

2 Long Question 1: Managing a Currency Trading Desk (30 Points)

Deutsche Bank (DB) is a German bank that manages its book in EUR. Consider 2 desks in DB, one is long 150 million USD and the other is short 50 million GBP. The exchange rates are 1 USD = 0.9163 EUR and 1 GBP = 1.3599 EUR. The daily volatilities for changes in USD/EUR and GBP/EUR are 0.40% and 0.30%, respectively and means of 1 basis point and 1.5 basis points. The correlation between them is 0.5. For risk calculations, assume that the returns have mean zero and are normally distributed.

1. What is the 99% VaR for each desk? (5 Points)
2. What is the 99% VaR for the combined portfolio? (5 Points)
3. Derive a formula for the Marginal and Component VaR for a portfolio of two normally distributed assets with positions x_1 and x_2, volatility σ_1 and σ_2, and correlation ρ. (10 Points)
4. What are the 99% Marginal and Component VaR for each desk? (5 Points)
5. How would you change the allocation of the desk? Give a quantitative argument. (5 Points)

SOLUTION:

1. Notice

$$\begin{aligned}\log\left(\frac{E_{t+1}^{USDEUR}}{E_t^{USDEUR}}\right) &\sim N\left(0.0001, 0.004\right)\\ \log\left(\frac{E_{t+1}^{GBPEUR}}{E_t^{GBPEUR}}\right) &\sim N\left(0.00015, 0.003\right)\end{aligned}$$

So, for each position in EUR

$$\begin{aligned}W^{EUR} - W_0^{EUR} &= W_0^{EUR}\left(1+\log\left(\frac{E_{t+1}}{E_t}\right)\right) - W_0^{EUR}\\ &= W_0^{EUR}\log\left(\frac{E_{t+1}}{E_t}\right)\end{aligned}$$

Thus

$$\begin{aligned}\Pr\left(W_0^{EUR}\log\left(\frac{E_{t+1}}{E_t}\right) < -VaR_{0.99}\right) &= 0.99\\ \Pr\left(\frac{\log\left(\frac{E_{t+1}}{E_t}\right)-\mu}{\sigma} < \frac{\frac{-VaR_{0.99}}{W_0^{EUR}}-\mu}{\sigma}\right) &= 0.99\\ VaR_{0.99} &= W_0^{EUR}\left[\mu - \sigma\Phi^{-1}\left(0.99\right)\right]\end{aligned}$$

The VaR, measured in EUR,of the desk with USD is

$$VaR_{0.99} = \left(-0.0001 + 2.326\left(0.004\right)\right)137.45 = 1.265m$$

The VaR, measured in EUR,of the desk with GBP is

$$VaR_{0.99} = \left(-0.00015 + 2.326\left(0.003\right)\right)67.995 = 0.4846m$$

2. The combined porfolio is bi-variate normal

$$
\begin{aligned}
\mu_P &= \frac{137.45}{205.44}(-0.0001) - \frac{67.995}{205.44}(-0.00015) \\
\sigma_P &= \sqrt{\left(\frac{137.45}{205.44}\right)^2 \sigma_1^2 + \left(\frac{67.995}{205.44}\right)^2 \sigma_2^2 - 2\left(\frac{67.995}{205.44}\right)\left(\frac{137.45}{205.44}\right)\rho} \\
VaR_{0.99} &= -\mu_P + 2.326\sigma_P
\end{aligned}
$$

3. ignore means (they are zero)

$$VaR = -\frac{1}{2}\sqrt{(x_1\sigma_1)^2 + (x_2\sigma_2)^2 + 2\rho\sigma_1\sigma_2 x_1 x_2}\Phi^{-1}(0.01)$$

   Marginal VaR

$$MVaR(x_1) = -\frac{1}{2}\frac{2x_1\sigma_1^2 + 2\rho\sigma_1\sigma_2 x_2}{\sqrt{(x_1\sigma_1)^2 + (x_2\sigma_2)^2 + 2\rho\sigma_1\sigma_2 x_1 x_2}}\Phi^{-1}(0.01)$$

   Cvar

$$CVaR(x_1) = -\frac{1}{2}\frac{2x_1^2\sigma_1^2 + 2\rho\sigma_1\sigma_2 x_2 x_1}{\sqrt{(x_1\sigma_1)^2 + (x_2\sigma_2)^2 + 2\rho\sigma_1\sigma_2 x_1 x_2}}\Phi^{-1}(0.01)$$

4. Proceed as above.

5. can compute E(r)/DVaR ratios. I suspect it will imply you should anyways be long GBP because of the positive premium

3 Long Question 2: Using CreditMetrics (36 Points)

Consider a world with only two ratings, A and B. A-rated bonds trade at \$90, B-rated bonds at \$60 and the recovery on defaulted bonds is 0. We have a portfolio worth \$1,000,000 constituted of positions in many bonds, all A rated. We want to compute the 99% 1-year VaR of the portfolio.

1. After a year, an A-rated bond can become B-rated with probability 30% and default with probability 5%. Explain how to match these transitions to a normal distribution. Give the numerical value of the thresholds. (5 Points)
2. The rating transitions follow a copula with correlation 0.05. Which fraction of bonds will default in the 1% worst-case scenario? (10 Points)
3. Comment on the relation of this result with the Vasicek model. (5 Points)
4. Which fraction of bonds will be B-rated in the 1% worst-case scenario? (11 Points)
5. What is the 99% 1-year VaR of the portfolio, using the current price as reference level? (5 Points)

SOLUTION:

1. You need to match quantiles of normal distribution. The transition to defalt is $\Phi^{-1}(0.05)$ and the transition from A to B is $\Phi^{-1}(0.35)$
2. Using $N((-1.65 + \sqrt(0.05)2.326)/\sqrt(0.95)) = 12.3$
3. Is closely related to Vasicek where you solve for the CDF to find the rate of default in the worst case.
4. $N(\Phi^{-1}(0.35) - \sqrt(0.05)2.326)/\sqrt(0.95)) = 0.5$, approx 37% of the bonds will be downgraded.
5. In the 1% worst case, 12.3% of the portfolio would go yo 0 and 37.7% of the portfolio would lose 1/3 of the value. Adding up, the total loss is 24.9% of the 1m portfolio value.

END OF EXAM

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