UCLA Anderson School of Management

Daniel Andrei, Option Markets 237D, Winter 2016

MFE – Midterm

February 2016

Date: ______________________________

Your Name: ______________________________

Your Signature:[1] ______________________________

- This exam is open book, open notes. You can use a calculator or a computer, **but be sure to show or explain your work**.
- You are not allowed to communicate with anyone (verbally, in writing, or electronically), except for me, during the exam period.
- You may present calculations in non-reduced form (e.g., as "$e^{0.095} - 1$").
- **If you are stuck on something, make an assumption, tell me what it is, and do the best you can.** I give partial credit if you provide enough correct information.

TIME LIMIT: 2 hours

TOTAL POINTS: 100

[1] As a member of the UCLA Anderson academic community, the highest standards of academic behavior are expected of you. It is your responsibility to make yourself aware of these standards (specifically regarding plagiarism, individual work, and team work) and adhere to them.

By signing the exam: (*i*) you certify your presence, and (*ii*) you state that you neither gave nor received help during the examination.

1 (20 points) Answer the following questions.

a. (5 points) The effective annual interest rate is 12%. What is the equivalent continuously compounded interest rate?

Interest rate

b. (5 points) A stock has an annual volatility of 30%. What is the 1-week volatility?

1-week volatility

c. (10 points) Let C_1, C_2, and C_3 be the prices of European call options with strikes X_1, X_2, and X_3, respectively. Suppose that

$$X_1 < X_2 < X_3$$
$$X_3 - X_2 = X_2 - X_1.$$

All options are on the same stock and have the same maturity. Suppose further that

$$C_2 > \frac{C_1 + C_3}{2}.$$

Is there an arbitrage available? If yes, describe the arbitrage strategy.

1 Solution

a. $\ln(1.12) = 0.1133$. You can check this by verifying that $e^{0.1133} = 1.12$.

b. The 1-week volatility is $0.30 \times \sqrt{1/52} = 0.0416$.

c. The arbitrage strategy and cash flows are shown in the table below.

Action Today	Cash Flow Today	Cash Flows at Expiry			
		$S_T < X_1$	$X_1 \leq S_T < X_2$	$X_2 \leq S_t < X_3$	$X_3 \leq S_T$
Buy C_1	$-C_1$	0	$S_T - X_1$	$S_T - X_1$	$S_T - X_1$
Buy C_3	$-C_3$	0	0	0	$S_T - X_3$
Short $2 \times C_2$	$+2C_2$	0	0	$-2(S_T - X_2)$	$-2(S_T - X_2)$
Net	$+2C_2 - C_1 - C_3$	0	$S_T - X_1 \geq 0$	$2X_2 - S_T - X_1$ $= X_3 - S_T \geq 0$	0

2 (5 points) Consider a clothing manufacturer who is having financial trouble. Their stock does not pay any dividends, and the price is currently \$100. Assume that the stock price will either be \$120 or \$20 at the end of one year. In an effort to raise cash today, the company is running the following promotion: with the purchase of every suit, a customer will receive the coice of

- \$8 cash today, or
- a contract that specifies that if the company's stock price at the end of one year is above its current level of \$100, the customer will receive \$20 one year from today. If, however, the stock price is below \$100, the customer will receive nothing.

Assume that the company's stock is actively traded and that the risk-free rate is 8%. Assuming you buy a suit from the company, which would you choose (i.e., which financial asset do you choose, not which suit)?

2 Solution

We find that $u = 1.2$, $d = 0.2$, and since $r = 0.08$, we therefore have

$$p = \frac{e^{0.08} - 0.2}{1} = 0.8833 \tag{1}$$

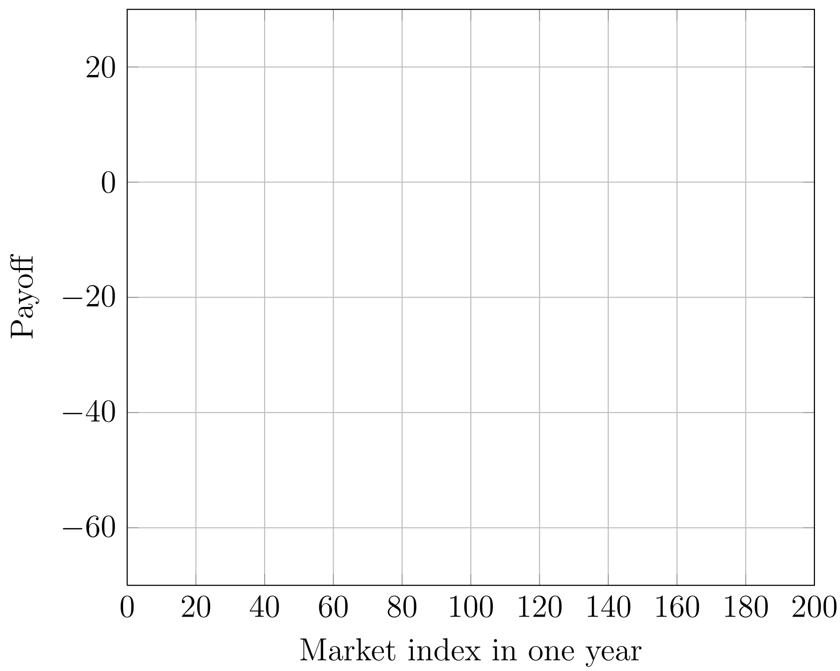
The value of the derivative contract is therefore

$$e^{-0.08}\left(0.8833 \times 20 = 0.1167 \times 0\right) = \$16.31 \tag{2}$$

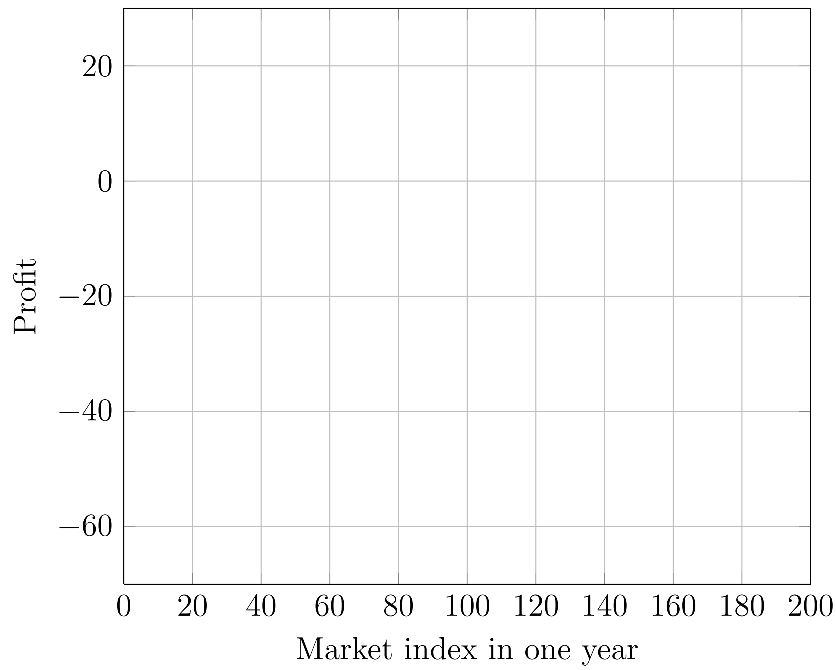
Therefore, you should choose the second option.

3 (15 points) The market index today is \$100. Assume you buy one European at-the-money put option (with strike \$100) and sell two European out-of-the-money put options (with strike \$80). Both options expire in one year. You have just built a **put ratio spread**.

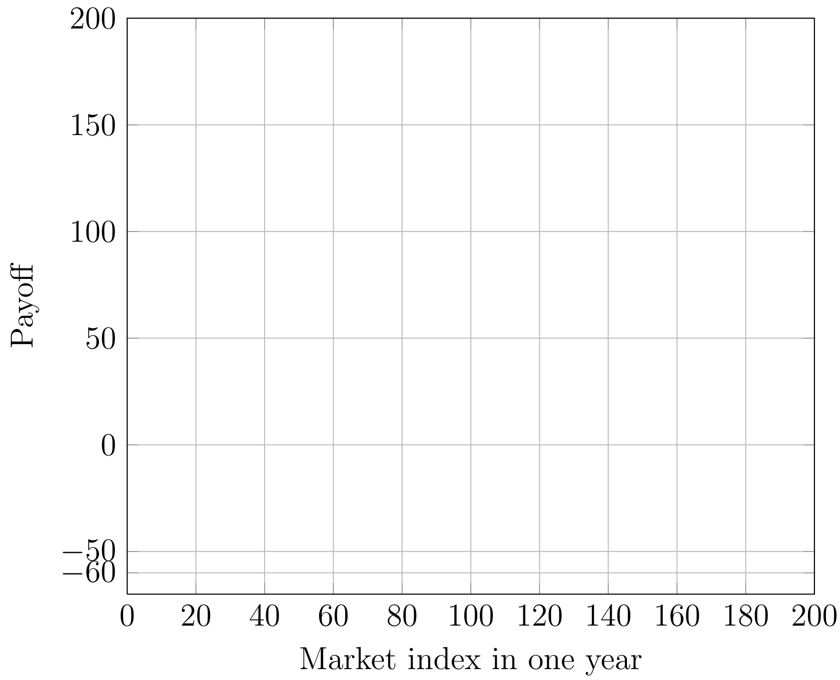
a. (5 points) Draw the **payoff** diagram for the put-ratio spread.



b. (5 points) The price of one at-the-money put (with strike \$100) is \$10. The price of one out-of-the-money put (with strike \$80) is \$5. Draw the **profit** diagram for the put-ratio spread.

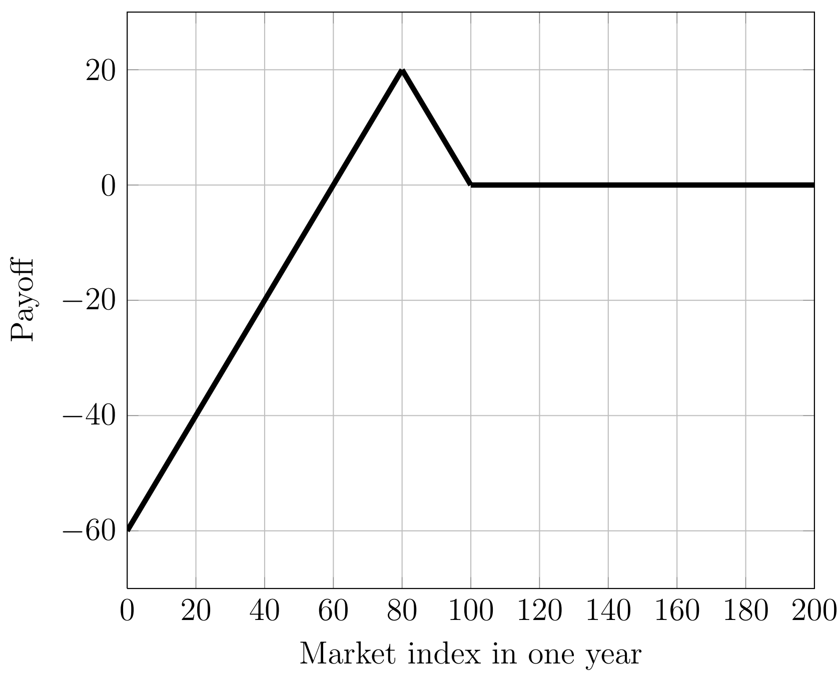


c. (5 points) Assume now that you also enter the market and buy one unit of the index. Draw the **payoff** diagram for your new position. Describe the risks and benefits of this trading strategy.



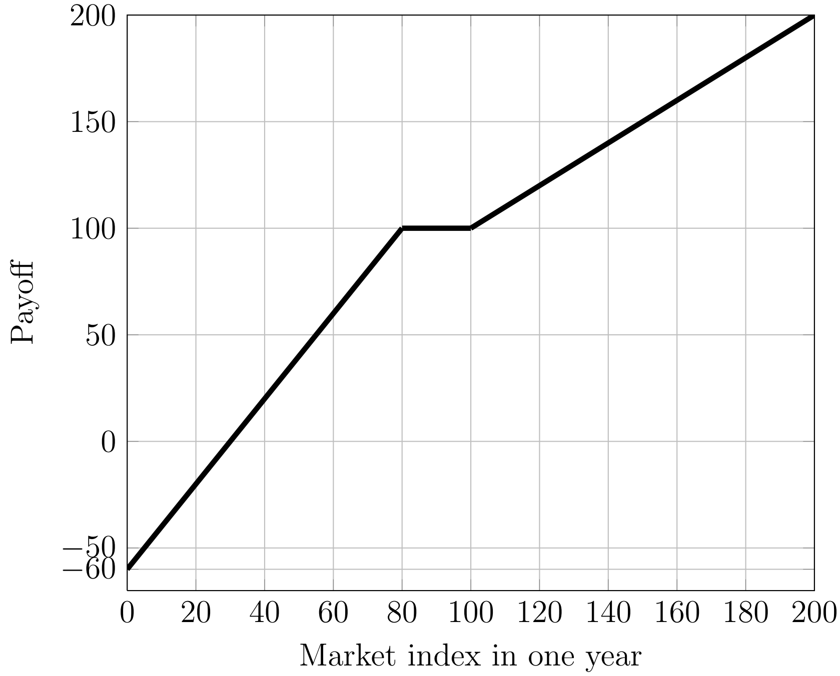
3 Solution

a. The payoff of the put ratio spread is



b. The cost of the put ratio spread is zero, and thus the profit diagram is exactly the same as the payoff diagram.

c. The payoff diagram for the new position is



This strategy can be looked as cheap insurance. There is a large downside price risk, should the market drop extremely sharply. If the index ends up between \$80 and \$100, you are *protected at zero cost*. However, below \$80, the insurance becomes a burden.

4 (15 points) Assume the following:

a. The stock price is \$100.

b. The **effective** annual risk-free rate is 10%. This means that if you invest \$1, after one year you will have \$1.1.

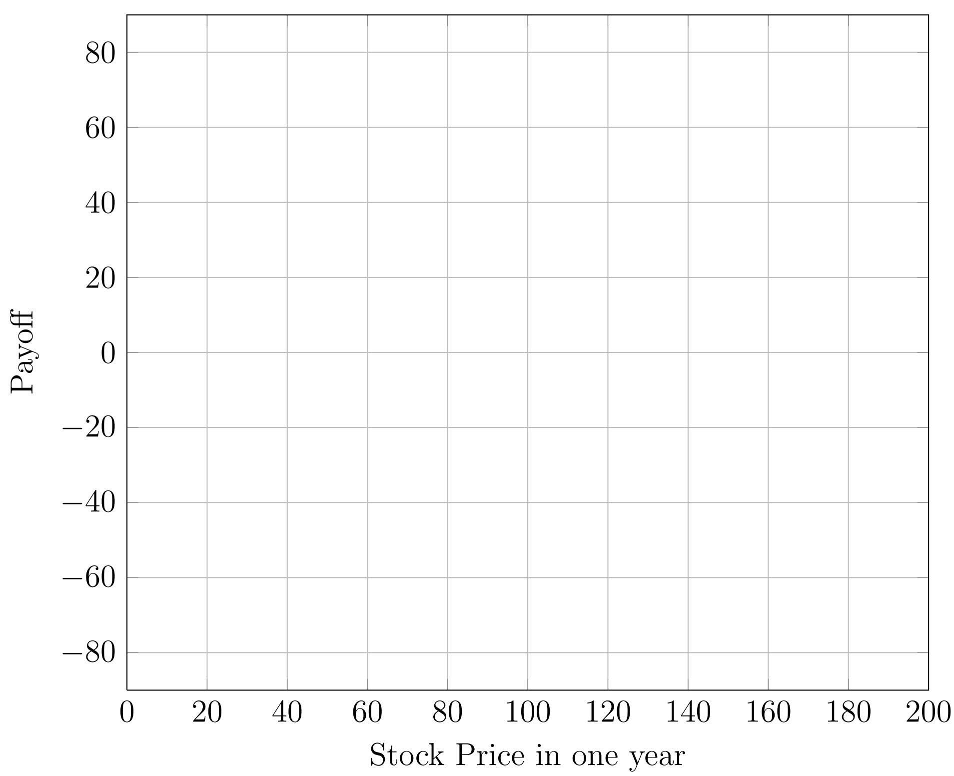
c. Here are option prices for you to use as necessary (these are Black-Scholes prices for options with one year to maturity):

Strike	80	90	100	110	120
Calls	29.15	22.24	16.49	11.92	8.44
Puts	1.88	4.06	7.40	11.92	17.53

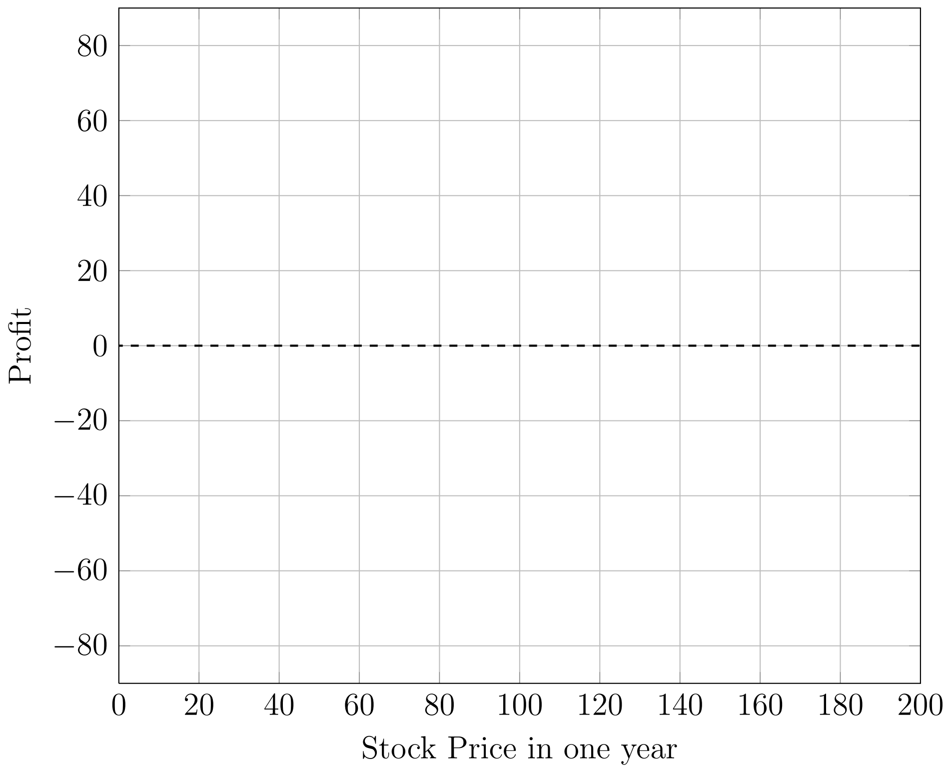
Consider the following position, built today and held for one year:

- sell one 120-strike call;
- buy one 80-strike put;

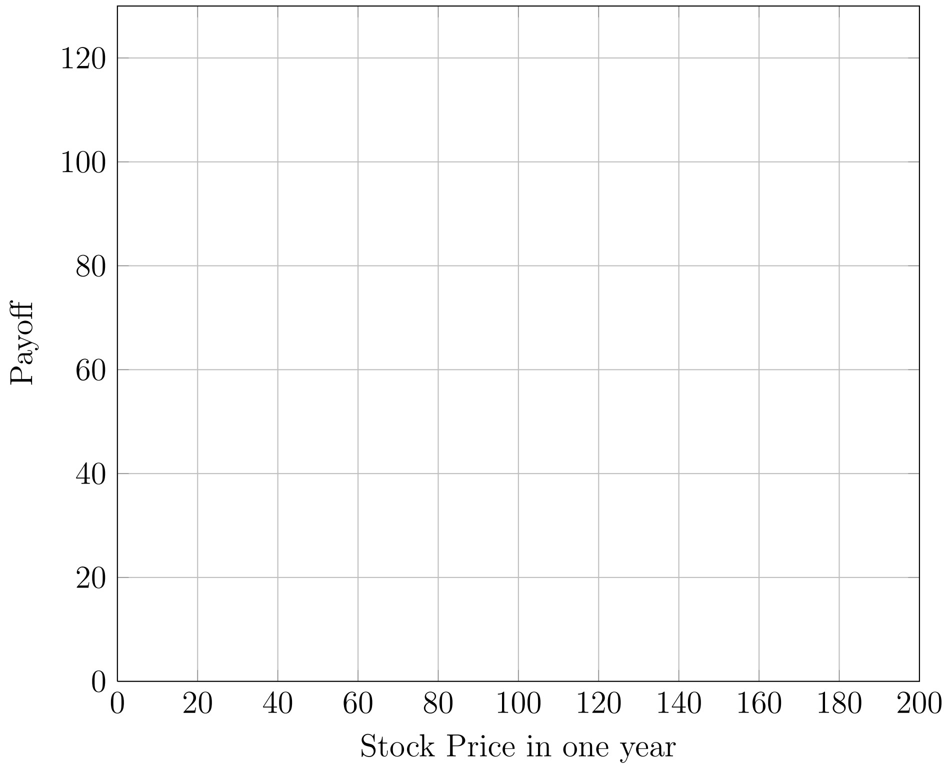
a. (5 points) Draw a **payoff** diagram for this position. What is the height of the plot at \$140? Clearly indicate this point on the diagram.



b. (5 points) Draw a **profit** diagram for this position after one year. At what point(s) does the graph cross the X-axis? Clearly indicate this point on the diagram.



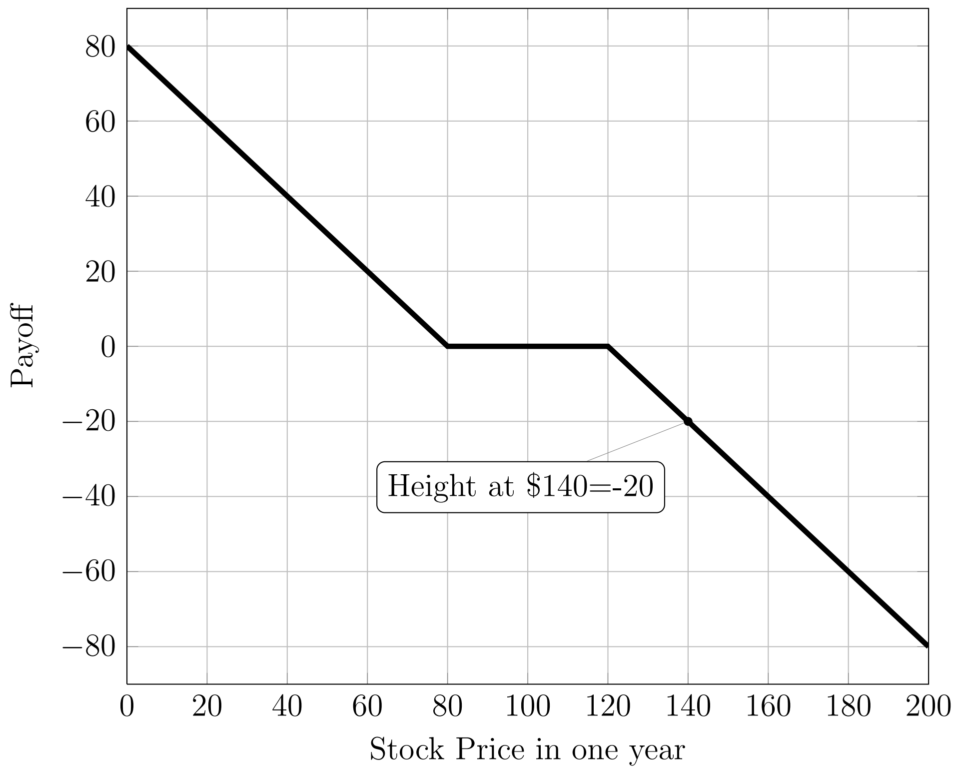
c. (5 points) Assume now that you also buy one unit of the stock. Draw the **payoff** diagram for your new position. Describe the risks and benefits of this trading strategy.



4 Solution

a. The payoff is below. The height at \$140 is given by

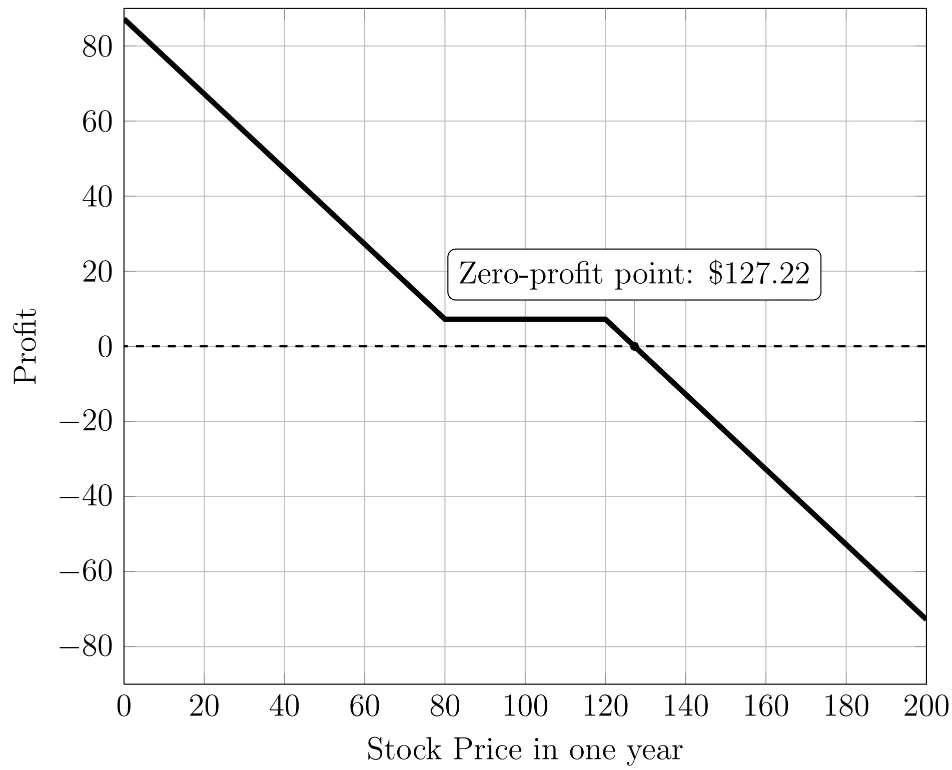
$$-\max(0, S_1 - 120) + \max(0, 80 - S_1) = -20 + 0 = -\$20$$



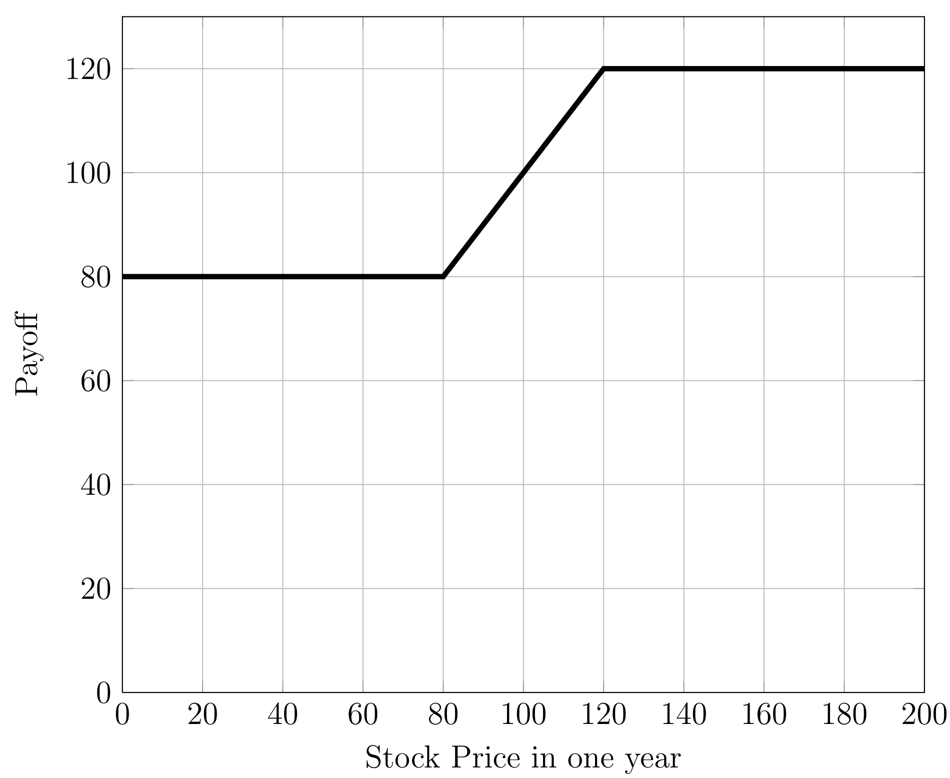
b. The profit diagram is below. The income from building the position is

$$\$8.44 - 1.88 = \$6.56$$

The future value of this income is $\$6.56 \times 1.1 = \$7.22$. The X-axis intercept is at $S_1 = 127.22$.



c. The payoff of the new position is



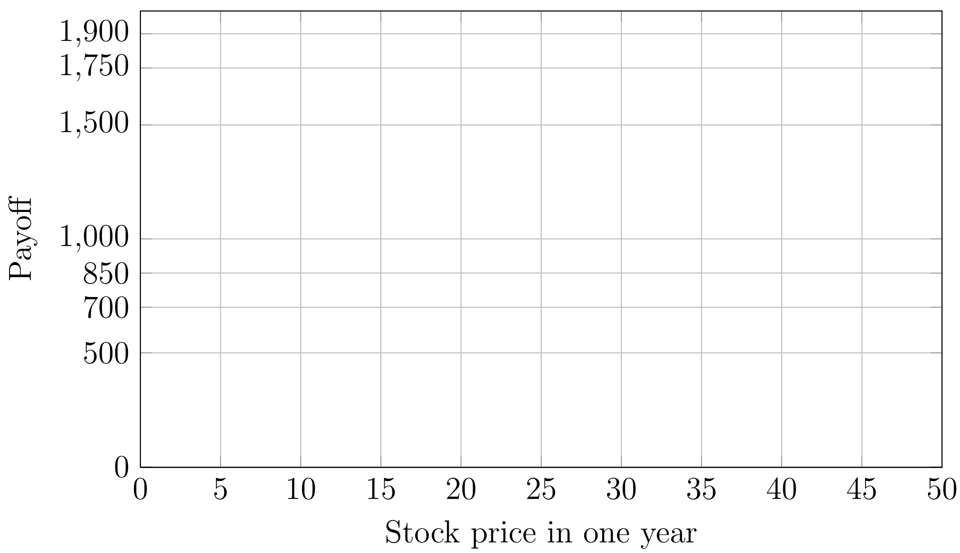
This is a **collared stock**. The long put option serves as insurance, whereas the short call option reduces the cost of the insurance. The investment preserves some of the upside potential while hedging the downside risk of holding the stock (i.e., it is capped both upside and downside). The strategy is also called a **bull spread**.

5 (25 points) An option whose payoff is based on the price of an underlying asset raised to a power is called a **power option**. Such an option has a nonlinear payoff at maturity. For this exercise, we will consider a power call option. Its payoff is:

$$\max\left[S_T^2 - 750, 0\right] \tag{3}$$

where the maturity T is 1 year. That is, the option pays the **square** of the stock price less the strike price of \$750 if the owner of the option chooses to exercise it. For example, if the stock price is \$30, the claim when exercised pays $\$900 - \$750 = \$150$.

a. (5 points) Draw the payoff diagram of the power call option. What is the height of the plot at \$40? Clearly indicate this point on the diagram.



For the rest of this exercise, assume that the annual continuously compounded interest rate is $r = 15\%$, the annual dividend yield on the stock is $\delta = 10\%$, and there are 6 months between binomial nodes. Use the tree on the following page to answer the following questions. **Pay attention to the extra information provided in the tree.**

b. (5 points) What is the annualized volatility used to construct the tree?

Volatility:

c. (5 points) What is the risk-neutral probability of an up move?

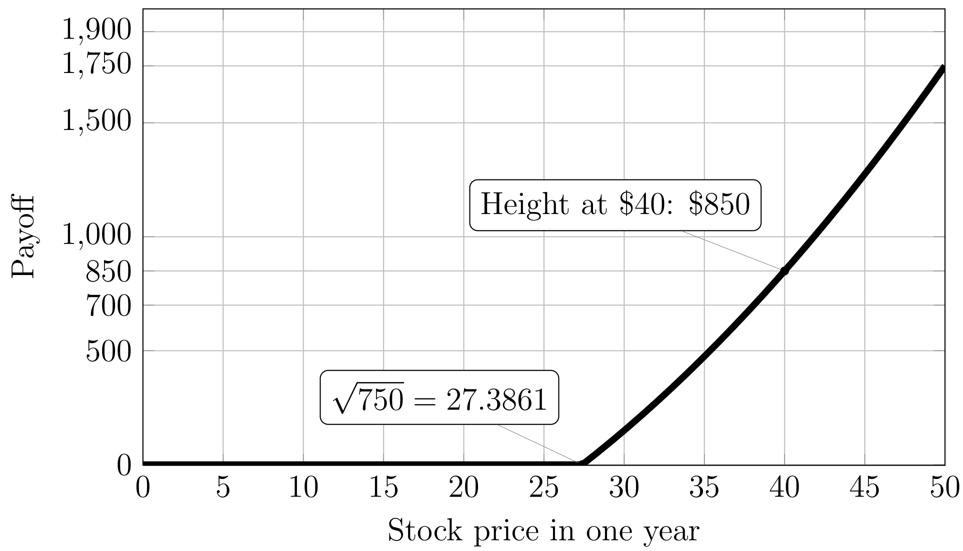
Risk-neutral Probability:

d. (10 points) At each node in the tree, fill in the prices for the American and European versions of this option (when you exercise the American version at time t, you receive $\max[S_t^2 - \$750, 0]$, as at expiration). **Put an asterisk at each intermediary node where the American option is exercised.**

$$
\begin{array}{ccccc}
 & & & & S_2^{uu} = 45 \\
 & & & & Eur = \\
 & & & & Amer = \\
 & & S_1^u = 30 & & \\
 & & Eur = & & \\
 & & Amer = & & \\
S_0 = 20 & & & & S_2^{ud} = 22.5 \\
Eur = & & & & Eur = \\
Amer = & & & & Amer = \\
 & & S_1^d = 15 & & \\
 & & Eur = & & \\
 & & Amer = & & \\
 & & & & S_2^{dd} = 11.25 \\
 & & & & Eur = \\
 & & & & Amer =
\end{array}
$$

5 Solution

a. Payoff:



b. From the tree, we observe that $u = 1.5$ and $d = 0.75$. The annualized volatility used to construct the tree is then:

$$\sigma = \frac{1}{2\sqrt{h}} \ln\left(\frac{u}{d}\right) = \frac{1}{2\sqrt{6/12}} \ln\left(\frac{1.5}{0.75}\right) = 0.4901 \tag{4}$$

c. The risk-neutral probability of an up move is

$$p^* = \frac{e^{(0.15-0.1)\times 0.5} - 0.75}{1.5 - 0.75} = 0.3671$$

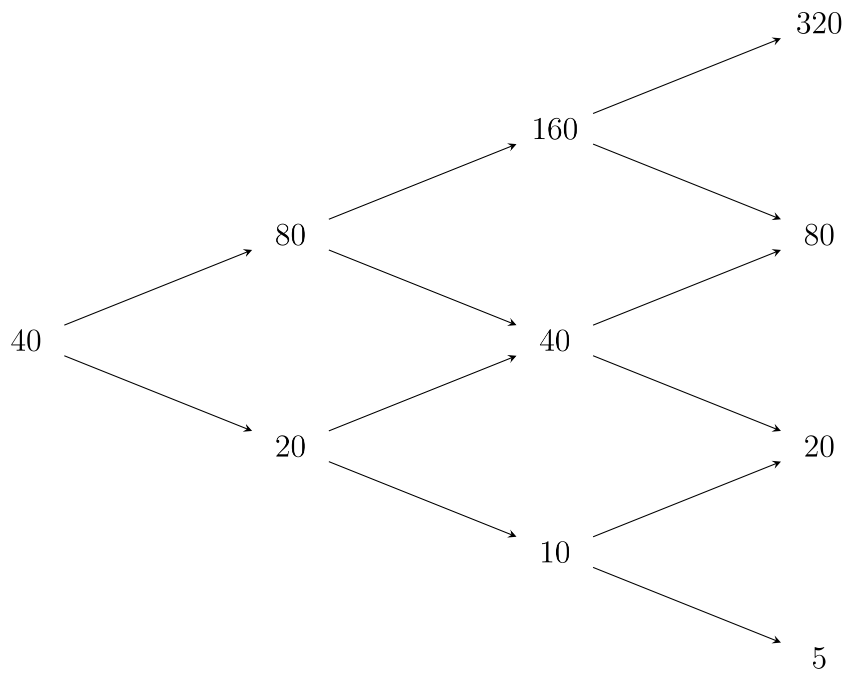
d. This claim is valued exactly like any other option. The expiration payoff is $\max\left[S_T^2 - \$750, 0\right]$. Then you work backward on the tree. For example, when the stock price is \$30, the price of the European claim is

$$e^{-.15\times 0.5}\left[0.3671 \times 1275 + (1 - 0.3671) \times 0\right] = 434.2171$$

The value of the option at each node is given in the tree below. The American option is never exercised and thus has the same price as the European option.

$$
\begin{tikzcd}[column sep=large]
 & & \begin{array}{c} S_2^{uu} = 45 \\ Eur = 1275 \\ Amer = 1275 \end{array} \\
 & \begin{array}{c} S_1^{u} = 30 \\ Eur = 434.2171 \\ Amer = 434.2171 \end{array} \arrow[ur] \arrow[dr] & \\
\begin{array}{c} S_0 = 20 \\ Eur = 147.8780 \\ Amer = 147.8780 \end{array} \arrow[ur] \arrow[dr] & & \begin{array}{c} S_2^{ud} = 22.5 \\ Eur = 0 \\ Amer = 0 \end{array} \\
 & \begin{array}{c} S_1^{d} = 15 \\ Eur = 0 \\ Amer = 0 \end{array} \arrow[ur] \arrow[dr] & \\
 & & \begin{array}{c} S_2^{dd} = 11.25 \\ Eur = 0 \\ Amer = 0 \end{array}
\end{tikzcd}
$$

6 (20 points) Consider a stock that pays no dividends and whose current price is \$40. The binomial tree below describes the possible stock prices over the next several periods. Assume that the risk-free rate is 10% and that the length of each period is 1 year.



a. (5 points) Consider a European **lookback** put option on the stock with maturity 3 years. The payoff at maturity of the option is equal to $\max(S_{max} - S_3, 0)$, where S_{max} is the maximum stock price reached during the life of the option, and S_3 is the final stock price. What is the value of the European lookback put option?

b. (15 points) Now consider an American **lookback** put option on the stock with maturity 3 years. If exercised at any time t prior to maturity, the payoff is equal to $\max(S_{max} - S_t, 0)$, where S_{max} is the maximum stock price that occurred between time 0 and time t. What is the value of the American lookback put option?

6 Solution

a. The risk neutral probability is $p^* = 0.4035$. There are 8 possible states at the end:

State	Probability
0	0.066
80	0.097
0	0.097
60	0.144
0	0.097
20	0.144
20	0.144
35	0.212

The value of the European option is thus:

$$P_0^E = e^{-0.1\times 3}\mathbb{E}\left[P_3\right] = \$21.896. \tag{5}$$

b. The value of the American option is $24.505.