Introduction

- Fundamental concept: time value of money.
- Discount factor at time zero: $D(T)$.
- Discount factor in the future: $D(t, T)$.
- Discount factor is fundamental way of defining term structure; has no compounding or daycount assumptions.
- What should $D(T)$ function look like?

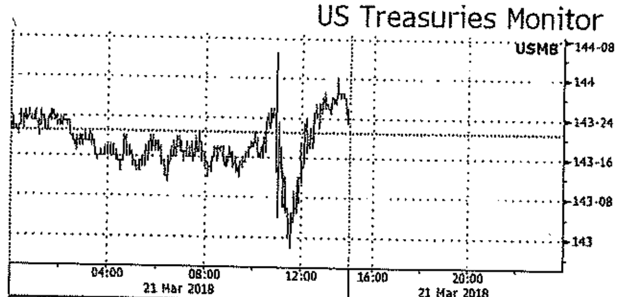
Examples of the Discount Curve

d

- Treasury bills.
- Treasury STRIPS.
  - Stripped coupon.
  - Stripped principal.
- Arbitrage case.

Security	Bid / Ask	BYld / AYld	Chg
Current Bills			
1) B 04/19/18	1.680 /1.672	1.706 /1.698	-.023
2) B 06/21/18	1.720 /1.710	1.752 /1.741	-.045
3) B 09/20/18	1.900 /1.890	1.945 /1.935	-.023
4) B 02/28/19	1.988 /1.980	2.044 /2.036	-.020
Current Notes/Bonds			
5) T 2 ¼ 02/20	99-28+ /28⅝	2.308 /2.306	+ 02⅜
6) T 2 ⅜ 03/21	99-23¾ /24	2.465 /2.462	+ 02¾
7) T 2 ⅝ 02/23	99-24+ /24¾	2.676 /2.674	+ 03¼
8) T 2 ¾ 02/25	99-19+ /20+	2.812 /2.807	+ 04+
9) T 2 ¾ 02/28	98-27 /27+	2.885 /2.883	+ 03+
10) T 3 02/15/48	97-22 /22+	3.119 /3.119	+ 07
Current/When Issued TIPS/TII			
11) TII0 ⅛ 04/22	98-12 /14	.530 /.514	+ 05+
12) TII0 ½ 01/28	97-11+ /13+	.780 /.773	+ 09
13) TII3 ⅜ 04/32	133-07 /14+	.862 /.847	+ 12¾
14) TII1 02/15/48	99-22¾ /26¾	1.011 /1.006	+ 21¾

US Treasuries Monitor



Security	Time	Last	Chg
22) USM8	d03/21	143-16 s	-0-11
23) TYM8	d03/21	119-30+ s	-0-04+
24) EDZ9	d03/21	97.065 s	-0.020
25) FVM8	d03/21	113-25+ s	-0-01¾
26) INDU Index	13:44	24682.31	-44.96
27) CRY Index	d 14:26	196.0194	+1.8651
28) CLK8	d03/21	65.49	+1.95
29) GCJ8	d03/21	1338.00	+20.40
30) BPM8	d03/21	141.84 s	+1.28
31) JYM8	d03/21	94.800 s	+0.280
32) ECM8	d03/21	1.24110 s	+0.00755

SN 648115 PDT GMT-7:00 G447-3651-0 21-Mar-2018 14:41:56

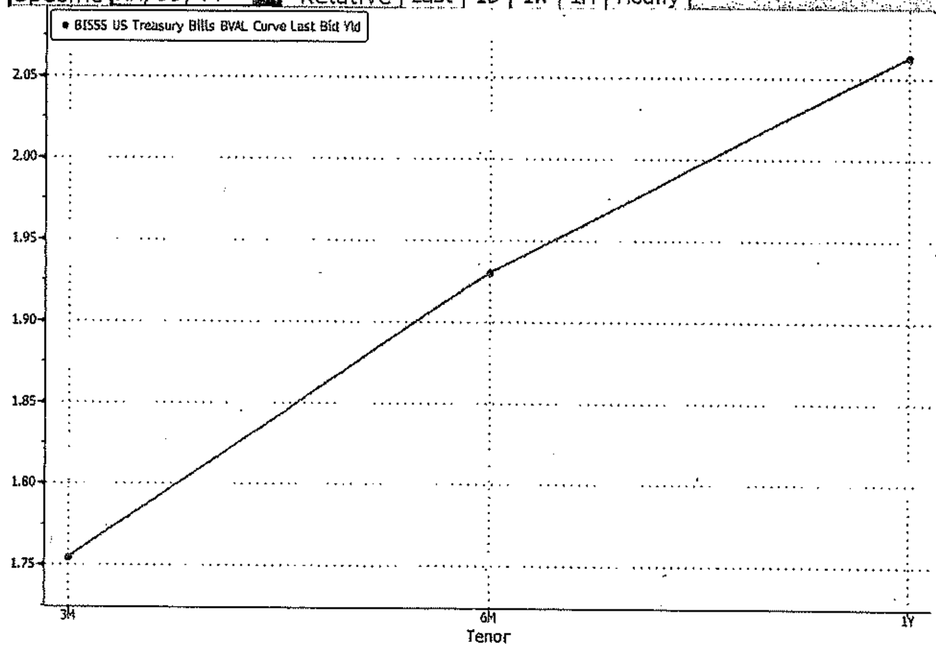
YCSW0023 1d 1.447 1w 1.711 1m 1.854 6m 2.410 1y 2.651 5y 2.833 30y 2.982

US Dollar Swaps (30/360, S/A) Curve

US Treasury Bills BVAL Cur | Graph Curves

X-Axis Y-Axis Bid Yld Currency None PCS BVAL Lower Chart Table

Specific MM/DD/YY Relative Last 1D 1W 1M Modify | Curves & Relative Value



⌄ Plot Curves

<Add Curve> Browse | CRVF »

☑ US Treasury Bills BVAL Curve

☑ Bid Yld

Base Curve

☐ Show Constituents on Base Curve

⌄ Recent Curves

› Curves related to US Dollar Swaps (3...

⌄ Bond Spread to Curve

<Add Security>

› Plot New Issues / Points

› Interpolate Curves

All Tenors ◉ Key Tenors

	Curve Id	3M	6M	1Y
11)	BI555	1.754	1.930	2.063

SN 648115 PDT GMT-7:00 G447-3651-0 21-Mar-2018 14:48:58

Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000

4

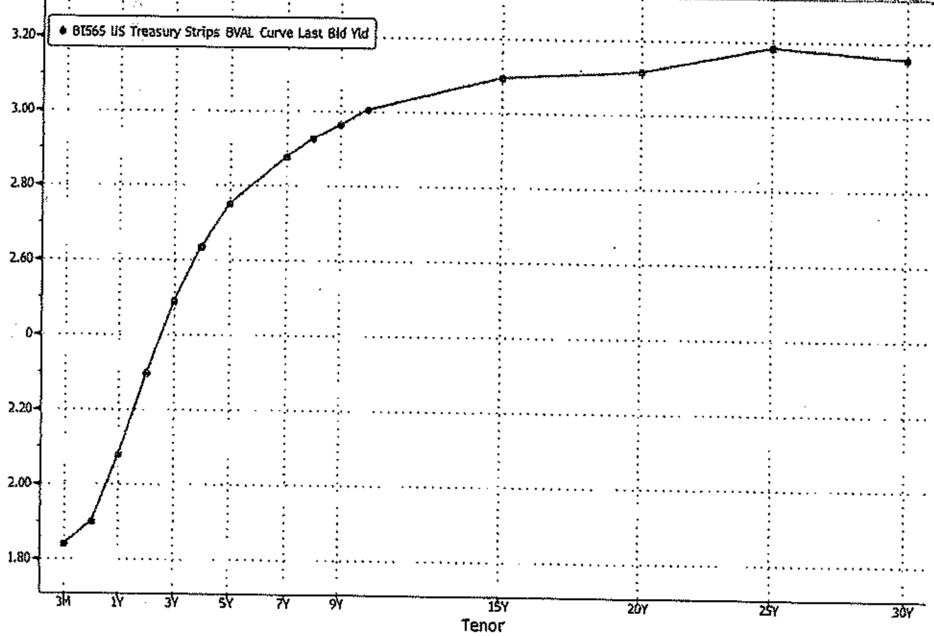
BVIS0565 1y 2.078 3y 2.489 5y 2.750 7y 2.875 10y 3.004 20y 3.116 30y 3.155
US Treasury Strips BVAL Curve

US Treasury Strips BVAL Cu

X-Axis Y-Axis Bid Yld Currency None PCS BVAL Graph Curves

Lower Chart Table

Specific MM/DD/YY Relative Last 1D 1W 1M Modify Curves & Relative Value



Plot Curves

<Add Curve> Browse | CRVF »

☑ US Treasury Strips BVAL Curve

☒ Bid Yld

Base Curve

☐ Show Constituents on Base Curve

Recent Curves

› Curves related to US Treasury Strips ...

Bond Spread to Curve

<Add Security>

› Plot New Issues / Points

› Interpolate Curves

○ All Tenors ◉ Key Tenors

	Curve Id	3M	1Y	2Y	3Y	5Y	10Y	15Y	30Y
11)	BI565	1.839	2.078	2.298	2.489	2.750	3.004	3.093	3.155

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<HELP> for explanation.

PXS STRIP PRICES

	SECURITY	BID/ASK	PRICE	RISK		SECURITY	BID/ASK	PRICE	RISK
1)S	5/31/21	1.76/74	89.51	5.66	22)S	11/15/24	2.14/12	81.23	7.91
2)S	6/30/21	1.77/76	89.31	5.73	23)S	2/15/25	2.17/15	80.60	8.05
3)S	7/31/21	1.78/77	89.10	5.79	24)S	5/15/25	2.18/16	80.06	8.19
4)S	8/15/21	1.81/79	88.93	5.81	25)S	8/15/25	2.19/18	79.52	8.33
5)S	8/31/21	1.80/78	88.90	5.84	26)S	11/15/25	2.22/20	78.85	8.45
6)S	9/30/21	1.81/79	88.70	5.90	27)S	2/15/26	2.25/23	78.19	8.58
7)S	10/31/21	1.82/81	88.49	5.96	28)S	5/15/26	2.27/25	77.59	8.70
8)S	11/15/21	1.83/81	88.42	5.99	29)S	8/15/26	2.28/26	77.03	8.83
9)S	11/30/21	1.83/82	88.28	6.02	30)S	11/15/26	2.30/28	76.43	8.95
10)S	12/31/21	1.85/83	88.07	6.08	31)S	2/15/27	2.32/30	75.83	9.07
11)S	2/15/22	1.87/85	87.78	6.17	32)S	5/15/27	2.33/31	75.31	9.19
12)S	5/15/22	1.90/88	87.20	6.34	33)S	8/15/27	2.35/33	74.74	9.30
13)S	8/15/22	1.90/88	86.78	6.53	34)S	11/15/27	2.36/34	74.20	9.42
14)S	11/15/22	1.93/91	86.15	6.69	35)S	2/15/28	2.38/36	73.59	9.52
15)S	2/15/23	1.98/96	85.43	6.85	36)S	5/15/28	2.39/37	73.04	9.63
16)S	5/15/23	2.00/98	84.85	7.01	37)S	8/15/28	2.40/38	72.52	9.74
17)S	8/15/23	2.02/00	84.29	7.17	38)S	11/15/28	2.41/39	72.02	9.85
18)S	11/15/23	2.04/02	83.69	7.32	39)S	2/15/29	2.42/40	71.48	10.0
19)S	2/15/24	2.07/05	83.06	7.48	40)S	5/15/29	2.43/41	70.97	10.1
20)S	5/15/24	2.10/08	82.43	7.62	41)S	8/15/29	2.43/41	70.50	10.2
21)S	8/15/24	2.12/10	81.85	7.77	42)S	11/15/29	2.45/43	69.92	10.3

<HELP> for explanation.

PXS STRIP PRICES

SECURITY		BID/ASK	PRICE	RISK	SECURITY		BID/ASK	PRICE	RISK
1)SP	8/15/25	2.06/04	80.64	8.46	22)SP	11/15/39	2.58/56	53.19	13.0
2)SP	2/15/26	2.13/11	79.26	8.70	23)SP	2/15/40	2.59/57	52.75	13.1
3)SP	8/15/26	2.15/13	78.24	8.97	24)SP	5/15/40	2.58/56	52.45	13.1
4)SP	11/15/26	2.17/15	77.60	9.09	25)SP	8/15/40	2.59/57	52.02	13.1
5)SP	2/15/27	2.19/16	77.07	9.22	26)SP	11/15/40	2.58/56	51.84	13.2
6)SP	8/15/27	2.21/19	76.01	9.47	27)SP	2/15/41	2.53/52	52.09	13.4
7)SP	11/15/27	2.23/21	75.45	9.58	28)SP	5/15/41	2.55/52	51.65	13.4
8)SP	8/15/28	2.26/24	73.85	9.93	29)SP	8/15/41	2.56/54	51.11	13.4
9)SP	11/15/28	2.26/24	73.44	10.1	30)SP	11/15/41	2.61/59	50.16	13.3
10)SP	2/15/29	2.28/26	72.85	10.2	31)SP	2/15/42	2.63/61	49.57	13.3
11)SP	8/15/29	2.29/27	71.98	10.4	32)SP	5/15/42	2.64/61	49.16	13.3
12)SP	5/15/30	2.31/29	70.54	10.7	33)SP	8/15/42	2.65/63	48.63	13.2
13)SP	2/15/31	2.31/29	69.28	11.0	34)SP	11/15/42	2.66/64	48.22	13.3
14)SP	2/15/36	2.31/28	61.93	12.9	35)SP	2/15/43	2.66/64	47.81	13.3
15)SP	2/15/37	2.36/34	59.79	13.1	36)SP	5/15/43	2.67/65	47.46	13.3
16)SP	5/15/37	2.35/33	59.57	13.2	37)SP	8/15/43	2.65/63	47.35	13.4
17)SP	2/15/38	2.48/47	56.79	13.0	38)SP	11/15/43	2.65/63	47.04	13.4
18)SP	5/15/38	2.49/47	56.40	13.0	39)SP	2/15/44	2.67/65	46.53	13.4
19)SP	2/15/39	2.56/54	54.43	12.9	40)SP	5/15/44	2.67/65	46.18	13.4
20)SP	5/15/39	2.57/55	54.02	13.0	41)SP	8/15/44	2.67/65	45.86	13.4
21)SP	8/15/39	2.57/55	53.61	13.0	42)SP	11/15/44	2.67/65	45.59	13.4

<HELP> for explanation, <MENU> for similar functions. DG26 **Corp** **SGY**

USE {HDF} TO MODIFY YOUR HISTORICAL DEFAULTS

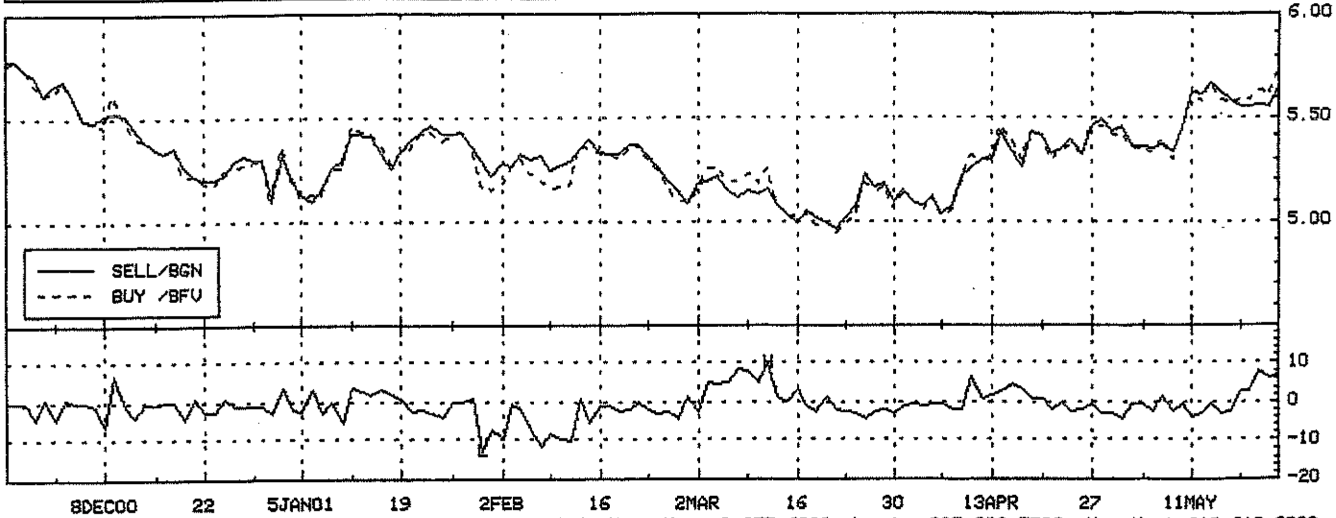
HISTORICAL YIELD SPREAD

SELL S 02/15/10	**Mid**	61.51	(5.647)	2/15/10 100
BUY SP 02/15/10	**Mid**	61.16	(5.715)	2/15/10 100

MTY, CALL, PUT

RANGE 11/24/00 TO 5/24/01 SELL BUY B/F (B=BLOOMBERG,F=FIRM)
PERIOD D (D-W-M-Q-Y) TIME FRAME N N (N=NY,F=NY 9-3,L=LONDON,T=TOKYO)
SPREAD Y P=PRICE OR Y=YIELD VALUE C C (O=OPEN,H=HIGH,L=LOW,C=CLOSE)
YIELD C CONV/SEMI-ANN/ANN MARKET M M (B=BID,A=ASK,M=MID)

HIGH 10 ON 3/13/01 **AVE** -1 **CURR** 7 **LOW** -13 ON 1/31/01



<HELP> for explanation. DG26 **Corp** **SGY**

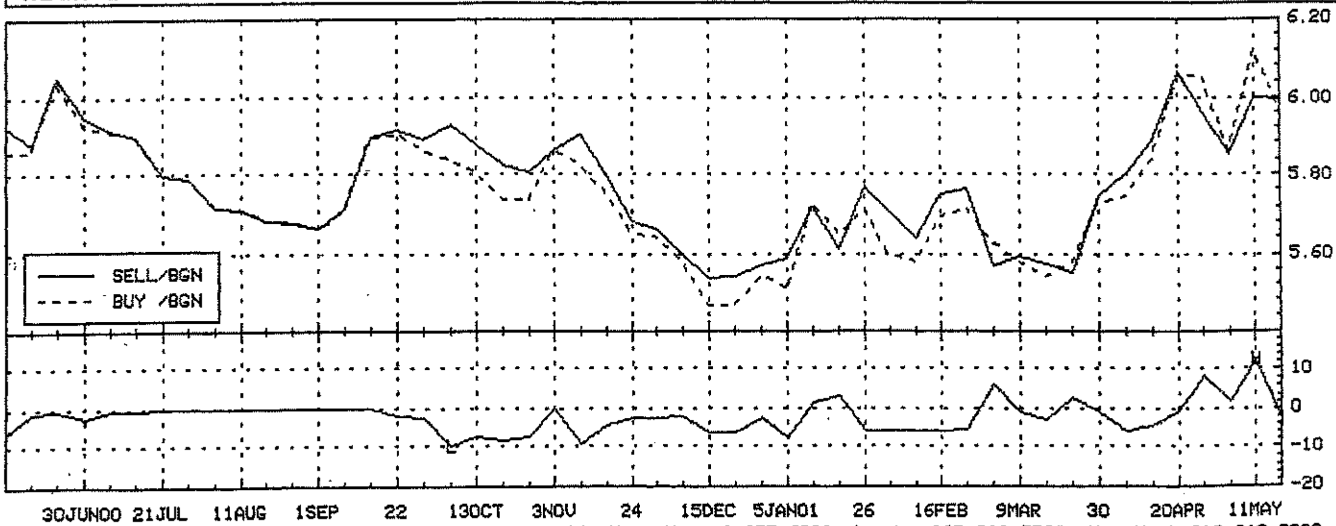
HISTORICAL YIELD SPREAD

SELL S 02/15/29	**Mid**	19.33	(6.016)	2/15/29 100
BUY SP 02/15/29	**Mid**	19.16	(6.050)	2/15/29 100

MTY, CALL, PUT

RANGE 6/ 9/00 TO 5/18/01 SELL BUY
PERIOD W (D-W-M-Q-Y) TIME FRAME N N (N=NY,F=NY 9-3,L=LONDON,T=TOKYO)
SPREAD Y P=PRICE OR Y=YIELD VALUE C C (O=OPEN,H=HIGH,L=LOW,C=CLOSE)
YIELD C CONV/SEMI-ANN/ANN MARKET M M (B=BID,A=ASK,M=MID)

HIGH 12 ON 5/11/01 **AVE** -2 **CURR** -3 **LOW** -10 ON 10/ 6/00



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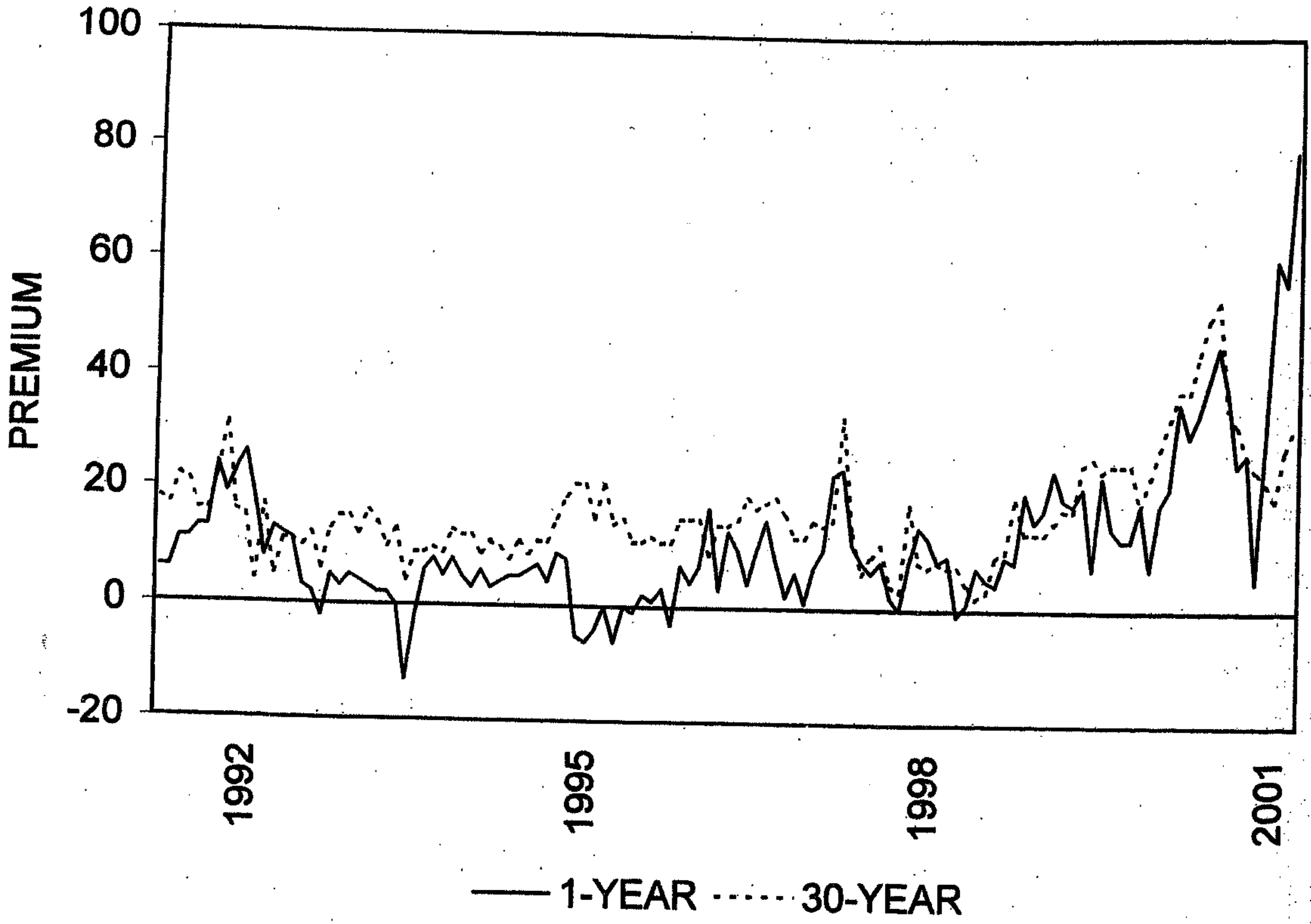

FIG. 1.—This graph shows the flight-to-liquidity premia for the 1-year and 30-year maturities. The data are monthly from April 1991 to March 2001. The flight-to-liquidity prem[illegible] are measured in basis points.

TABLE 4 **Results from Regressions of Flight-to-Liquidity Premia Explanatory Variables**

Maturity	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7	t_{β_0}	t_{β_1}	t_{β_2}	t_{β_3}	t_{β_4}	t_{β_5}	t_{β_6}	t_{β_7}	R^2
.25	.069	.469	−.342	.0015	−.0009	.524	−.394	.032	3.12*	5.68*	−2.12*	.68	−.89	.45	−1.36	2.84*	.411
.50	.046	.495	−.266	.0013	−.0009	1.702	−.275	.015	2.91*	5.98*	−2.37*	.81	−1.27	2.09*	−1.37	1.91+	.389
1.00	.018	.616	−.149	−.0005	−.0004	1.387	.162	.030	1.54	8.04*	−1.66+	−.38	−.74	2.10*	1.02	4.06*	.694
2.00	.029	.580	−.165	−.0024	−.0010	1.360	−.191	.017	2.98*	7.71*	-2.13*	−2.17*	−1.93+	2.29*	−1.37	2.72*	.655
3.00	.033	.511	−.193	−.0023	−.0006	.767	−.029	.026	3.43*	6.85*	−2.76*	−2.27*	−1.25	1.48	−.23	4.45*	.674
4.00	.031	.580	−.171	−.0028	.0001	.419	−.076	.023	3.06*	7.85*	−2.37*	−2.71*	.14	.78	−.57	3.92*	.650
5.00	.028	.622	−.205	−.0027	−.0000	.748	−.162	.015	2.84*	8.55*	-2.94*	-2.72*	−.05	1.44	−1.28	2.70*	.629
7.00	.040	.473	−.071	−.0023	.0002	1.049	.024	.019	3.25*	5.84*	−.89	−1.97*	.43	1.78+	.17	3.12*	.480
10.00	.067	.430	−.108	−.0015	.0001	.065	−.148	.025	5.68*	4.91*	−1.86+	−1.74+	.38	.15	−1.38	4.56*	.619
20.00	.049	.581	−.060	.0002	.0000	.751	−.091	.020	4.42*	7.74*	−1.16	.29	.02	1.96*	−.96	3.97*	.719
30.00	.045	.716	−.069	−.0003	−.0002	.298	−.257	.012	3.65*	9.73*	−1.06	−.36	−.45	.62	−2.08*	1.98*	.711

NOTE.—This table reports the estimated coefficients and t-statistics from the regression of the flight-to-liquidity premium on the lagged flight-to-liquidity premium and the explanatory variables described in table 3. The data are monthly from April 1991 to February 2001.

$$\text{Premium}_t = \beta_0 + \beta_1 \text{Premium}_{t-1} + \beta_2 \Delta\text{Spread}_t + \beta_3 \Delta\text{Confidence}_t + \beta_4(\Delta\text{Foreign Holdings Percent})_t$$
$$+ \beta_5(\text{MM Mutual Fund Percent})_t + \beta_6(\text{Equity Mutual Fund Percent}) + \beta_7(\text{Treasury Buyback})_t + \epsilon_t.$$

+ Significant at the 10% level.
* Significant at the 5% level.

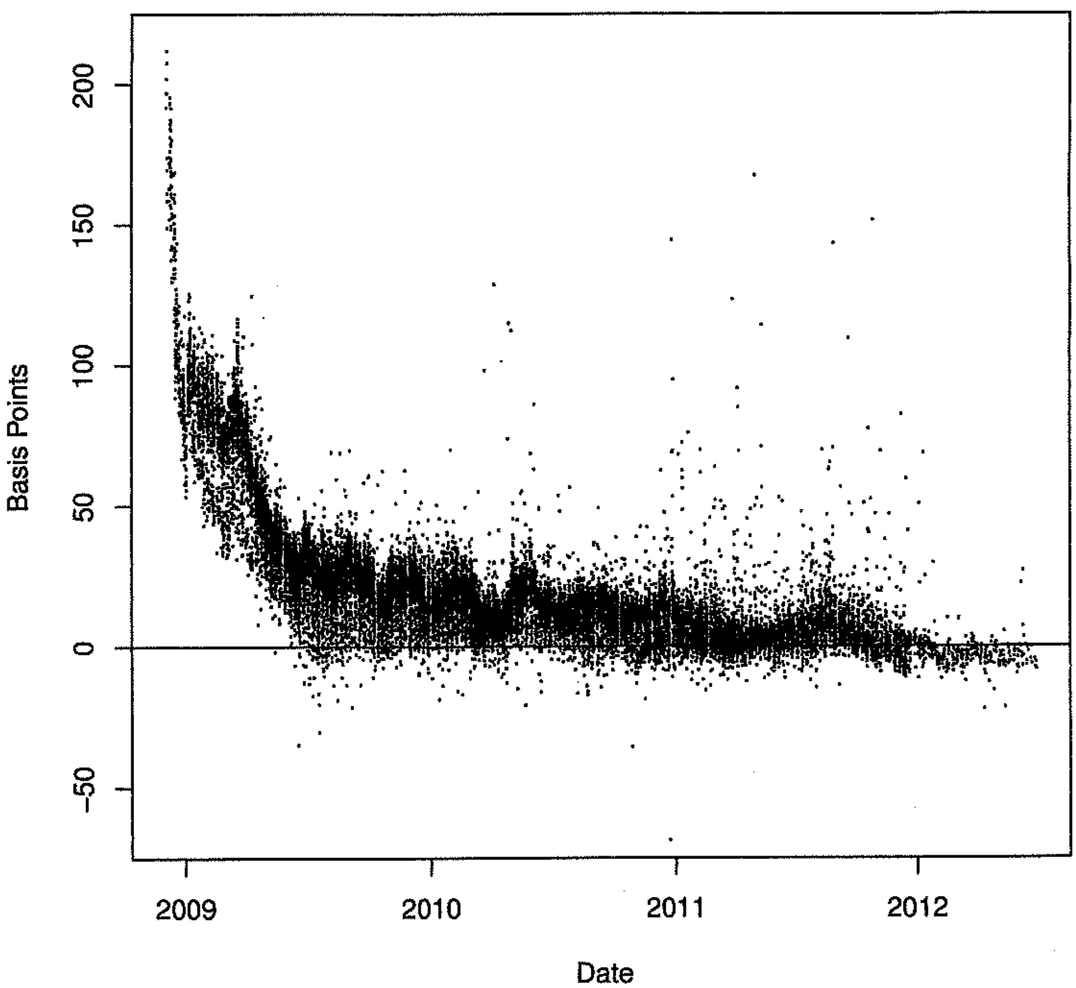

Figure 6. Mispricing of Individual Bonds. This figure plots the mispricing of the individual guaranteed bonds over time. Mispricing is measured in basis points.

Cross-Sectional Regressions of Mispricing on Explanatory Variables. This tables reports results from the cross-sectional regressions of bond-specific mispricing on the indicated variables. Time to maturity is in years. Coupon rate is expressed as a percentage. Issuer, lead underwriter, and prime dealer CDS spreads are measured in basis points. Inventory denotes the total inventory holdings of dealers as a percentage of the size of the bond issue. Institutional holdings are expressed as percentage of the size of the bond issue. Lead underwriter and prime dealer haircuts are expressed as a percentage of the value of the bond. Size of the issue and trading volume are measured in millions of dollars. The bid-ask spread is measured as a fraction of the par amount of the bond. The Amihud measure is expressed as a fraction of the par amount. Medium-term note takes value one if the bond issue is a medium-term note, and zero otherwise. Standard errors are clustered at the issuer and monthly level. The superscripts * and ** denote significance at the ten-percent and five-percent levels, respectively. The sample is monthly from December 2008 to December 2012.

Variable	Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat
Time to Maturity	10.286	9.33**	10.260	9.44**	9.639	8.10**	9.349	7.49**	9.036	7.47**
Coupon Rate	−0.001	0.00	0.129	0.23	0.274	0.59	0.465	0.84	0.047	0.11
Issuer CDS Spread			0.009	1.42	−0.002	−0.28	−0.000	−0.07	0.000	0.07
Underwriter CDS Spread					0.021	3.48**	0.014	2.27**	0.014	2.13**
Prime Dealer CDS Spread					0.010	2.02**	0.011	2.36**	0.011	2.37**
Underwriter Haircut					0.039	0.11	0.051	0.11	0.006	0.01
Prime Dealer Haircut					0.543	2.08**	0.500	2.20**	0.528	2.28**
Number of Dealers							−0.070	−1.67*	−0.061	−1.72*
Number of Investors							0.011	0.56	0.033	1.60
Inventory							−0.122	−5.90**	−0.130	−5.90**
Institutional Holdings							−0.003	−0.08	−0.032	−0.71
Log Size of Issue									−1.711	−2.62**
Log Customer Volume									0.996	2.74**
Log Interdealer Volume									−0.141	−0.79
Bid-Ask Spread									0.044	0.79
Amihud Measure									−0.498	−1.50
Medium-Term Note									0.002	0.00
Time Fixed Effects		Yes		Yes		Yes		Yes		Yes
Issuer Fixed Effects		No		No		No		No		No
Adjusted R^2		0.552		0.601		0.630		0.654		0.686
Number of Observations		1646		1646		1646		1646		1646

Valuing Bonds

3

- What is the present value of a 5-year 6 percent coupon bond?

$$= 3 \sum_{i=1}^{10} D(i/2) \; + \; 100 \; D(5)$$

$$= 3 \; PVA \; + \; 100 \; PVF$$

- Example: Treasury bonds.
- Example: Treasury triplets case.

<HELP> for explanation.

<Search> 98) Export | 161-180 of 296 results | Security Finder

30) All 31) Eqty 32) FI 33) Mtge 34) Cmdty 35) Indx/Stats 36) FX 37) Funds 38) M-Mkt

40) Corp 41) Govt 42) Loans 43) Pfd 44) CDS 45) CDS Idx 46) Muni 47) Futr 48) Optns 49) IRS 50) IRS Vols 51) Gen Govt

60) Exclude: 61) Column Settings

	R	Name	Ticker	Coupon	Maturity	Type	Ask Px	Ask Yield
			T					
1)	...	United States Tre	T	1.750	10/31/2018		101-31+	1.214
2)	...	United States Tre	T	3.750	11/15/2018		109-19+	1.182
3)	...	United States Tre	T	9.000	11/15/2018		129-17¼	1.118
4)	...	United States Tre	T	1.250	11/30/2018		100-00+	1.246
5)	...	United States Tre	T	1.375	11/30/2018		100-17¾	1.228
6)	...	United States Tre	T	1.500	12/31/2018		100-29¼	1.263
7)	...	United States Tre	T	1.375	12/31/2018		100-14¼	1.260
8)	...	United States Tre	T	1.250	01/31/2019		99-27+	1.286
9)	...	United States Tre	T	1.500	01/31/2019		100-27¼	1.284
10)	...	United States Tre	T	2.750	02/15/2019		105-29+	1.261
11)	...	United States Tre	T	8.875	02/15/2019		130-21¼	1.178
12)	...	United States Tre	T	1.500	02/28/2019		100-27¼	1.287
13)	...	United States Tre	T	1.375	02/28/2019		100-09¼	1.303
14)	...	United States Tre	T	1.625	03/31/2019		101-09	1.311
15)	...	United States Tre	T	1.500	03/31/2019		100-25	1.309
16)	...	United States Tre	T	1.625	04/30/2019		101-08	1.325
17)	...	United States Tre	T	1.250	04/30/2019		99-22+	1.321
18)	...	United States Tre	T	3.125	05/15/2019		107-17+	1.329
19)	...	United States Tre	T	1.500	05/31/2019		100-21	1.345
20)	...	United States Tre	T	1.125	05/31/2019		99-03	1.339

Zoom 100%

<HELP> for explanation.
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<Search> 98) Export 241-260 of 296 results Security Finder

30) All 31) Eqty 32) FI 33) Mtge 34) Cmdty 35) Indx/Stats 36) FX 37) Funds 38) M-Mkt
40) Corp 41) Govt 42) Loans 43) Pfd 44) CDS 45) CDS Idx 46) Muni 47) Futr 48) Optns 49) IRS 50) IRS Vols 51) Gen Govt

60) Exclude: 61) Column Settings

	R	Name	Ticker	Coupon	Maturity	Type	Ask Px	Ask Yield
			T					
1)		United States Tre	T	1.625	11/15/2022		98-18+	1.820
2)		United States Tre	T	7.625	11/15/2022		143-02+	1.726
3)		United States Tre	T	2.000	02/15/2023		101-07	1.837
4)		United States Tre	T	7.125	02/15/2023		140-02+	1.784
5)		United States Tre	T	1.750	05/15/2023		99-00	1.880
6)		United States Tre	T	2.500	08/15/2023		104-27+	1.885
7)		United States Tre	T	6.250	08/15/2023		135-02	1.823
8)		United States Tre	T	2.750	11/15/2023		106-31	1.890
9)		United States Tre	T	2.750	02/15/2024		106-31+	1.909
10)		United States Tre	T	2.500	05/15/2024		104-25	1.938
11)		United States Tre	T	2.375	08/15/2024		103-21	1.955
12)		United States Tre	T	2.250	11/15/2024		102-17+	1.964
13)		United States Tre	T	7.500	11/15/2024		150-02	1.898
14)		United States Tre	T	7.625	02/15/2025		152-04+	1.918
15)		United States Tre	T	6.875	08/15/2025		146-29	1.952
16)		United States Tre	T	6.000	02/15/2026		139-13+	2.016
17)		United States Tre	T	6.750	08/15/2026		148-16+	2.031
18)		United States Tre	T	6.500	11/15/2026		146-16+	2.054
19)		United States Tre	T	6.625	02/15/2027		148-18+	2.065
20)		United States Tre	T	6.375	08/15/2027		147-05+	2.094

Zoom 100%

<HELP> for explanation.
Screen Printed

<Search> 98) Export 281-296 of 296 results Security Finder

30) All 31) Eqty 32) FI 33) Mtge 34) Cmdty 35) Indx/Stats 36) FX 37) Funds 38) M-Mkt
40) Corp 41) Govt 42) Loans 43) Pfd 44) CDS 45) CDS Idx 46) Muni 47) Futr 48) Optns 49) IRS 50) IRS Vols 51) Gen Govt

60) Exclude: 61) Column Settings

	R	Name	Ticker	Coupon	Maturity	Type	Ask Px	Ask Yield
			T					
1)		United States Tre	T	4.750	02/15/2041		144-19+	2.431
2)		United States Tre	T	4.375	05/15/2041		137-13	2.441
3)		United States Tre	T	3.750	08/15/2041		125-03	2.459
4)		United States Tre	T	3.125	11/15/2041		112-06+	2.498
5)		United States Tre	T	3.125	02/15/2042		111-31+	2.512
6)		United States Tre	T	3.000	05/15/2042		109-14	2.520
7)		United States Tre	T	2.750	08/15/2042		104-06+	2.537
8)		United States Tre	T	2.750	11/15/2042		104-04	2.542
9)		United States Tre	T	3.125	02/15/2043		111-23	2.539
10)		United States Tre	T	2.875	05/15/2043		106-18+	2.548
11)		United States Tre	T	3.625	08/15/2043		122-10+	2.524
12)		United States Tre	T	3.750	11/15/2043		125-01	2.523
13)		United States Tre	T	3.625	02/15/2044		122-11	2.535
14)		United States Tre	T	3.375	05/15/2044		117-05	2.542
15)		United States Tre	T	3.125	08/15/2044		111-31+	2.546
16)		United States Tre	T	3.000	11/15/2044		109-12+	2.549

Zoom 100%

RATE	MATURITY MO/YR	BID	ASKED	CHG	ASK YLD

Government Bonds & Notes

RATE	MATURITY MO/YR	BID	ASKED	CHG	ASK YLD
1.625	Mar 05n	99:30	99:31	...	1.95
1.625	Apr 05n	99:27	99:28	1	2.58
6.500	May 05n	100:20	100:21	-1	2.60
6.750	May 05n	100:21	100:22	-1	2.66
12.000	May 05	101:19	101:20	-3	2.29
1.250	May 05n	99:21	99:22	...	2.63
1.125	Jun 05n	99:16	99:17	1	2.70
1.500	Jul 05n	99:14	99:15	...	2.85
6.500	Aug 05n	101:15	101:16	-1	2.92
10.750	Aug 05	103:09	103:10	-2	2.85
2.000	Aug 05n	99:17	99:18	...	2.96
1.625	Sep 05n	99:07	99:08	...	3.02
1.625	Oct 05n	99:02	99:03	...	3.09
5.750	Nov 05n	101:22	101:23	-2	3.13
5.875	Nov 05n	101:24	101:25	-2	3.14
1.875	Nov 05n	99:02	99:03	-1	3.14
1.875	Dec 05n	98:29	98:30	...	3.21
1.875	Jan 06n	98:24	98:25	...	3.27
5.625	Feb 06n	102:03	102:04	-2	3.25
9.375	Feb 06	105:16	105:17	-3	3.25
1.625	Feb 06n	98:11	98:12	-1	3.34
1.500	Mar 06n	98:02	98:03	-1	3.35
2.250	Apr 06n	98:22	98:23	-1	3.41
2.000	May 06n	98:11	98:12	-1	3.41
4.625	May 06n	101:11	101:12	-2	3.41
6.875	May 06n	103:29	103:30	-2	3.41
2.500	May 06n	98:27	98:28	-2	3.44
2.750	Jun 06n	99:02	99:03	-1	3.47
7.000	Jul 06n	104:15	104:16	-3	3.51
2.750	Jul 06n	98:30	98:31	-2	3.51
2.375	Aug 06n	98:12	98:13	-2	3.52
2.375	Aug 06n	98:09	98:10	-2	3.56
2.500	Sep 06n	98:11	98:12	-2	3.59
6.500	Oct 06n	104:13	104:14	-4	3.60
2.500	Oct 06n	98:07	98:08	-2	3.60
2.625	Nov 06n	98:11	98:12	-3	3.62
3.500	Nov 06n	99:25	99:26	-2	3.61
2.875	Nov 06n	98:22	98:23	-3	3.64
3.000	Dec 06n	98:26	98:27	-3	3.67
3.375	Jan 07i	105:00	105:01	-4	0.61
3.125	Jan 07n	98:30	98:31	-3	3.70
2.250	Feb 07n	97:10	97:11	-3	3.69
6.250	Feb 07n	104:23	104:24	-4	3.67
3.375	Feb 07n	99:10	99:11	-4	3.72
6.625	May 07n	105:29	105:30	-5	3.74
4.375	May 07n	101:08	101:09	-4	3.75
3.125	May 07n	98:21	98:22	-4	3.76
2.750	Aug 07n	97:17	97:18	-5	3.81
3.250	Aug 07n	98:22	98:23	-5	3.81
6.125	Aug 07n	105:09	105:10	-6	3.81
3.000	Nov 07n	97:24	97:25	-6	3.87
3.625	Jan 08i	107:17	107:18	-6	0.92
3.000	Feb 08n	97:14	97:15	-6	3.92
5.500	Feb 08n	104:11	104:12	-7	3.90
3.375	Feb 08n	98:15	98:16	-6	3.92
2.625	May 08n	96:00	96:01	-7	3.96
5.625	May 08n	104:28	104:29	-8	3.96
3.250	Aug 08n	97:18	97:19	-8	4.01
3.125	Sep 08n	97:01	97:02	-9	4.03
3.125	Oct 08n	97:00	97:00	-10	4.03
3.375	Nov 08n	97:22	97:23	-10	4.04
4.750	Nov 08n	102:13	102:14	-10	4.03
3.375	Dec 08n	97:19	97:20	-10	4.06
3.250	Jan 09n	97:03	97:04	-10	4.07
3.875	Jan 09i	110:07	110:08	-11	1.13
3.000	Feb 09n	96:02	96:03	-10	4.09
2.625	Mar 09n	94:20	94:21	-10	4.09
3.125	Apr 09n	96:10	96:11	-12	4.10
3.875	May 09n	99:04	99:05	-11	4.10
5.500	May 09n	105:11	105:12	-12	4.08
4.000	Jun 09n	99:16	99:17	-11	4.12
3.625	Jul 09n	97:30	97:31	-12	4.14
3.500	Aug 09n	97:14	97:15	-11	4.13
6.000	Aug 09n	107:13	107:14	-13	4.14
3.375	Sep 09n	96:24	96:25	-11	4.17
3.375	Oct 09n	96:20	96:21	-12	4.18
3.500	Nov 09n	97:04	97:05	-13	4.17
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3.625	Jan 10n	97:14	97:15	-13	4.20
4.250	Jan 10i	113:24	113:25	-16	1.30
3.500	Feb 10n	96:28	96:29	-14	4.20
6.500	Feb 10n	110:01	110:02	-15	4.21
4.000	Mar 10n	99:00	99:01	-13	4.21
0.875	Apr 10i	97:22	97:23	-13	1.34
10.000	May 10	101:05	101:06	-3	2.94
5.750	Aug 10n	107:06	107:07	-15	4.25
12.750	Nov 10	106:08	106:09	-4	3.20
3.500	Jan 11i	111:21	111:22	-18	1.41
5.000	Feb 11n	103:21	103:22	-14	4.29
13.875	May 11	111:27	111:28	-5	3.43
5.000	Aug 11n	103:20	103:21	-16	4.34
14.000	Nov 11	116:23	116:24	-5	3.58
3.375	Jan 12i	112:02	112:03	-17	1.51
4.875	Feb 12n	102:30	102:31	-18	4.37
3.000	Jul 12i	109:29	109:30	-19	1.56
4.375	Aug 12n	99:26	99:27	-18	4.40
4.000	Nov 12n	97:08	97:09	-19	4.42
10.375	Nov 12	116:08	116:09	-8	3.89
3.875	Feb 13n	96:04	96:05	-19	4.45

RATE	MATURITY MO/YR	BID	ASKED	CHG	ASK YLD
3.625	May 13n	94:18	94:19	-18	4.42
1.875	Jul 13i	101:16	101:17	-10	1.68
4.250	Aug 13n	98:08	98:09	-19	4.50
12.000	Aug 13	125:09	125:10	-11	4.01
4.250	Nov 13n	98:02	98:03	-20	4.52
2.000	Jan 14i	102:03	102:04	-13	1.74
4.000	Feb 14n	96:04	96:05	-20	4.53
4.750	May 14n	101:16	101:17	-20	4.54
13.250	May 14	134:23	134:24	-15	4.09
2.000	Jul 14i	101:28	101:29	-19	1.78
4.250	Aug 14n	97:21	97:22	-20	4.55
12.500	Aug 14	133:14	133:15	-18	4.14
11.750	Nov 14	131:28	131:29	-17	4.16
4.250	Nov 14n	97:18	97:19	-21	4.56
1.625	Jan 15i	98:08	98:09	-20	1.81
4.000	Feb 15n	95:23	95:24	-21	4.54
11.250	Feb 15	152:30	152:31	-29	4.56
10.625	Aug 15	149:20	149:21	-30	4.58
9.875	Nov 15	144:00	144:00	-29	4.61
9.250	Feb 16	139:11	139:12	-28	4.62
7.250	May 16	122:10	122:11	-27	4.66
7.500	Nov 16	125:00	125:01	-28	4.69
8.750	May 17	137:00	137:00	-32	4.72
8.875	Aug 17	138:19	138:20	-32	4.73
9.125	May 18	142:13	142:14	-34	4.75
9.000	Nov 18	142:00	142:00	-35	4.78
8.875	Feb 19	141:02	141:03	-34	4.79
8.125	Aug 19	134:01	134:02	-32	4.82
8.500	Feb 20	138:18	138:19	-33	4.84
8.750	May 20	141:21	141:22	-32	4.84
8.750	Aug 20	142:00	142:00	-34	4.85
7.875	Feb 21	133:00	133:00	-32	4.87
8.125	May 21	136:01	136:02	-33	4.87
8.125	Aug 21	136:11	136:12	-33	4.88
8.000	Nov 21	135:06	135:07	-34	4.89
7.250	Aug 22	127:06	127:07	-32	4.91
7.625	Nov 22	131:25	131:26	-33	4.91
7.125	Feb 23	126:00	126:01	-32	4.92
6.250	Aug 23	115:25	115:26	-31	4.93
7.500	Nov 24	132:06	132:07	-33	4.92
2.375	Jan 25i	106:16	106:17	-25	1.97
7.625	Feb 25	134:00	134:00	-37	4.93
6.875	Aug 25	124:27	124:28	-31	4.93
6.000	Feb 26	113:26	113:27	-31	4.93
6.750	Aug 26	123:27	123:28	-34	4.93
6.500	Nov 26	120:25	120:26	-30	4.93
6.625	Feb 27	122:20	122:21	-34	4.92
6.375	Aug 27	119:16	119:17	-34	4.93
6.125	Nov 27	116:09	116:10	-32	4.92
3.625	Apr 28i	130:10	130:11	-30	1.98
5.500	Aug 28	107:30	107:31	-33	4.92
5.250	Nov 28	104:18	104:19	-31	4.92
5.250	Feb 29	104:24	104:25	-30	4.91
3.875	Apr 29i	136:07	136:08	-45	1.98
6.125	Aug 29	117:06	117:07	-33	4.91
6.250	May 30	119:19	119:20	-34	4.89
5.375	Feb 31	108:07	108:08	-32	4.81
3.375	Apr 32i	121:06	121:07	-36	1.90

Interest Rates

Convention is to use rates rather than discount functions.

- Annual compounding

$$D(10) = \frac{1}{(1+r)^{10}} \implies r = D(10)^{-1/10} - 1$$

- Semiannual compounding

$$D(10) = \frac{1}{(1+r/2)^{20}} \implies r = 2\left(D(10)^{-1/20} - 1\right)$$

- Continuous compounding

$$D(10) = e^{-10r} \implies r = -\ln D(10)/10$$

Present Value Formulas

5

- Present value of annuity

$$PVA = \frac{1}{r}\left[1 - \frac{1}{(1+r)^N}\right]$$

$$PV\ Perpetuity = \frac{1}{r}$$

- Present value of growing annuity.

$$PVGA = \frac{1}{r-g}\left[1 - \frac{(1+g)^N}{(1+r)^N}\right]$$

$$PVG\ Perpetuity = \frac{1}{r-g}$$

<HELP> for explanation. DG26 **Corp** **PCS**
ENTER 1 <GO> TO UPDATE OR <CANCEL> TO ABORT

PRICE PROVIDER SEARCH LIST

Page 1/ 1

BRITISH GOVERNMENT AGENCY BONDS

1st WDRL UBS WARBURG
2nd EURX EUROPEAN EXCHANGE
3rd MLIL xMERRILL LYNCH INTL.
4th
5th
Prices within 7 days

Prices for all securities in the class BRITISH GOVERNMENT AGENCY BONDS are obtained using this search priority list.

Hit 1 <GO> to save this list.
Price security individually within class? N

CITY OF SWANSEA SWAN 3 ½ 01/49 35.0900/ (9.97/) **WDRL**

Pricing providers: SWAN 3 ½ 01/29/49

PCS	Provider	Pricing	PCS	Provider	Pricing
WDRL	UBS WARBURG	IntraDay			
EURX	EUROPEAN EXCHANGE	Daily			
MLIL	xMERRILL LYNCH INTL.	Some Daily			

x=NOT PRIVILEGED
Green = Monitorable source
Enter MRKT <GO> to view all pricing contributors

DES DG26 **Corp** **DES**

SECURITY DESCRIPTION

Page 1/ 1

CITY OF SWANSEA SWAN 3 ½ 01/49 NOT PRICED

ISSUER INFORMATION
Name CITY OF SWANSEA
Type Municipal-City
Market of Issue DOMESTIC

SECURITY INFORMATION
Country GB Currency GBP
Collateral Type DEBENTURES
Calc Typ(42)PERPETL PAY,EX-DIV
Maturity PERPETUAL Series
PERPETUAL
Coupon 3 ½ FIXED
S/A ACT/ACT
Announcement Dt
Int. Accrual Dt
1st Settle Date
1st Coupon Date
Iss Pr

NO PROSPECTUS

IDENTIFIERS
ISIN GB0008665472
Sedol 0866547
BB number GG7146556

RATINGS
Moody's NA
S&P NA
Fitch NA

ISSUE SIZE
Amt Issued
GBP 800.00 (M)
Amt Outstanding
GBP 459.00 (M)
Min Piece/Increment
1.00/ 1.00
Par Amount 1.00

BOOK RUNNER/EXCHANGE

LONDON

1) Additional Sec Info
2) Identifiers
3) Ratings
4) Custom Notes
5) ALLQ
6) Pricing Sources
7) Related Securities

65) Old DES
66) Send as Attachment

ORIG £600M ISS'D 9/1882. ADD'L £200M ISS'D 5/1887. £340,733 CANCELLED.

Yield to Maturity

YTM is IRR on a bond

$$P = \frac{c/2}{(1+r)} + \frac{c/2}{(1+r)^2} + \frac{c/2}{(1+r)^3} + \frac{100}{(1+r)^3}$$

- Solve for constant r.
  - Example 1.
  - Example 2.
- YTM for par bond = coupon rate.
- Excel IRR function.
- Examples of yield curves.

Case: Yield as a measure of value.

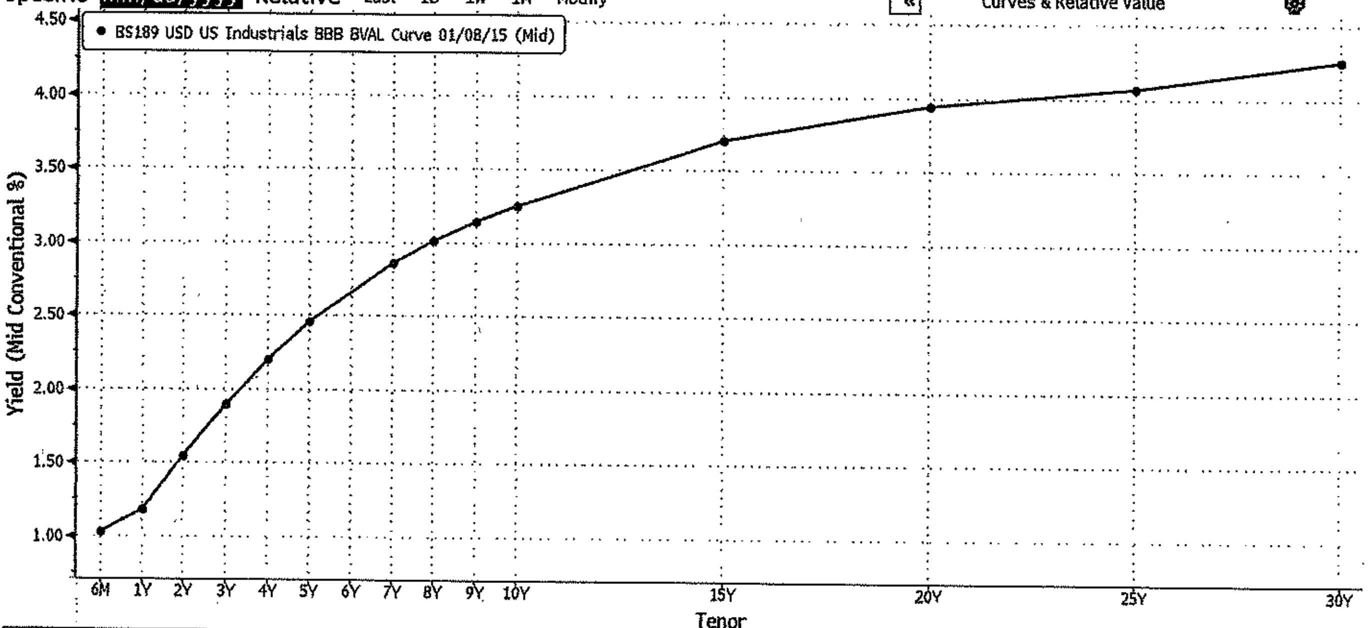
1

<Menu> to Return

USD US INDUSTRIALS BBB B | 97) Actions | 98) Table | Graph Curves

X-axis Tenor Y-axis Yield PCS BVAL Lower Chart Table

Specific mm/dd/yyyy Relative Last 1D 1W 1M Modify « Curves & Relative Value



Curve ID	1Y	2Y	3Y	5Y	7Y	10Y	15Y	30Y
BS189	1.176	1.545	1.893	2.460	2.858	3.255	3.706	4.262

3) Show All Tenors >

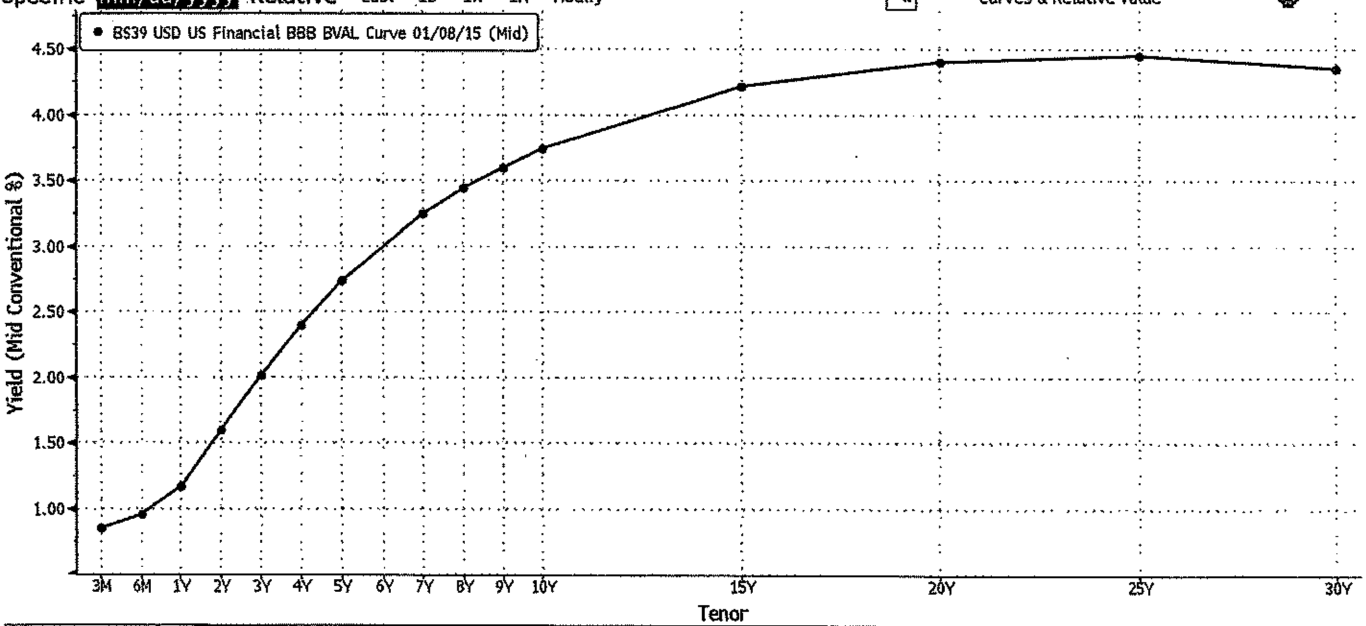
1

<Menu> to Return

USD US FINANCIAL BBB BVAL | 97) Actions | 98) Table | Graph Curves

X-axis Tenor Y-axis Yield PCS BVAL Lower Chart Table

Specific mm/dd/yyyy Relative Last 1D 1W 1M Modify « Curves & Relative Value



Curve ID	3M	1Y	2Y	3Y	5Y	7Y	10Y	15Y	30Y
BS39	0.853	1.173	1.598	2.013	2.738	3.254	3.742	4.215	4.352

3) Show All Tenors >

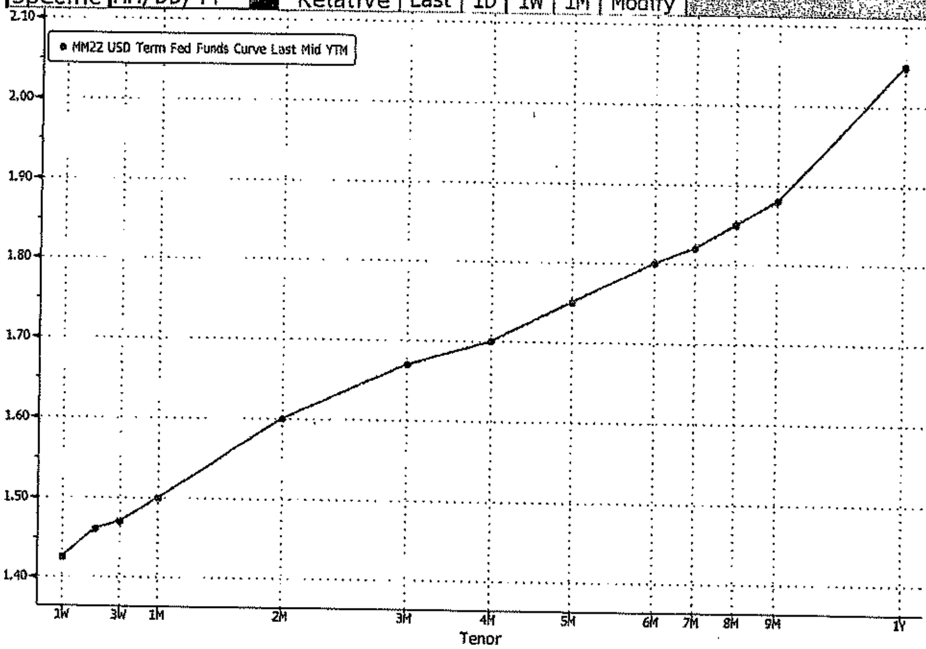
YCMM0022 1w 1.425 15d 1.460 3w 1.470 1m 1.500 2m 1.600 3m 1.670 4m 1.700
USD Term Fed Funds Curve

USD Term Fed Funds Curve

Graph Curves

X-Axis Y-Axis Mid YTM Currency None PCS BLP Lower Chart Table

Specific MM/DD/YY Relative Last 1D 1W 1M Modify » Curves & Relative Value



Plot Curves

<Add Curve> Browse | CRVF »

☑USD Term Fed Funds Curve

☑Mid YTM

Base Curve

☐ Show Constituents on Base Curve

Recent Curves

› Curves related to USD Term Fed Fund...

Bond Spread to Curve

<Add Security>

› Plot New Issues / Points

› Interpolate Curves

All Tenors Key Tenors

	Curve Id	1W	1M	2M	3M	6M	9M	1Y
11)	MM22	1.425	1.500	1.600	1.670	1.800	1.880	2.050

SN 648115 PDT GMT-7:00 G447-3651-0 21-Mar-2018 14:51:14

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Agenda

X

- Spot curve.
- Par curve.
- Forward curves.
- Forward par curve.
- Bootstrapping curves.

Spot Curve

26

- Definition: The term structure of yields on zero-coupon bonds.

$$D(T) = \frac{1}{(1 + r/2)^{2T}}$$

- Example:

$$D(1) = 0.950 \implies r_1 = 0.051956$$

$$D(2) = 0.895 \implies r_2 = 0.056242$$

$$D(3) = 0.840 \implies r_3 = 0.058970$$

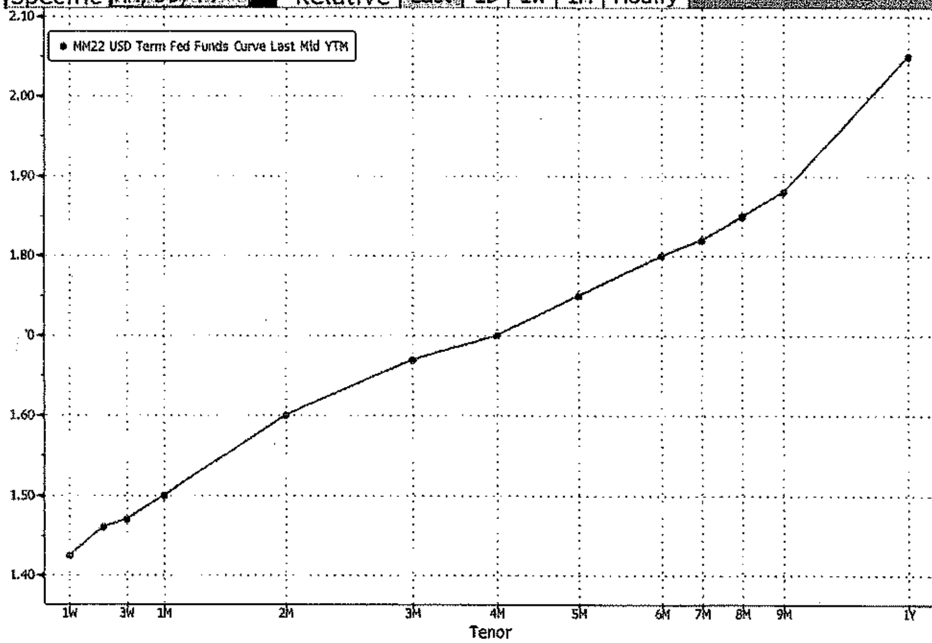
- More examples.
  - Treasury bill curve.
  - STRIPS curve.
  - Repo curve.
  - Bloomberg curves.

S

USD Term Fed Funds Curve | | Graph Curves

X-Axis | Y-Axis Mid YTM | Currency None | PCS BLP | Lower Chart Table

Specific MM/DD/YY | Relative | Last | 1D | 1W | 1M | Modify | » Curves & Relative Value



⌄ Plot Curves
<Add Curve> Browse | CRVF »
☑ USD Term Fed Funds Curve
☑ Mid YTM

Base Curve
☐ Show Constituents on Base Curve
⌄ Recent Curves
› Curves related to USD Term Fed Fund...

⌄ Bond Spread to Curve
<Add Security>

› Plot New Issues / Points

› Interpolate Curves

◎ All Tenors ◉ Key Tenors

	Curve Id	1W	1M	2M	3M	6M	9M	1Y
11)	■ MM22	1.425	1.500	1.600	1.670	1.800	1.880	2.050

SN 648115 PDT GMT-7:00 G447-3651-0 21-Mar-2018 14:51:15

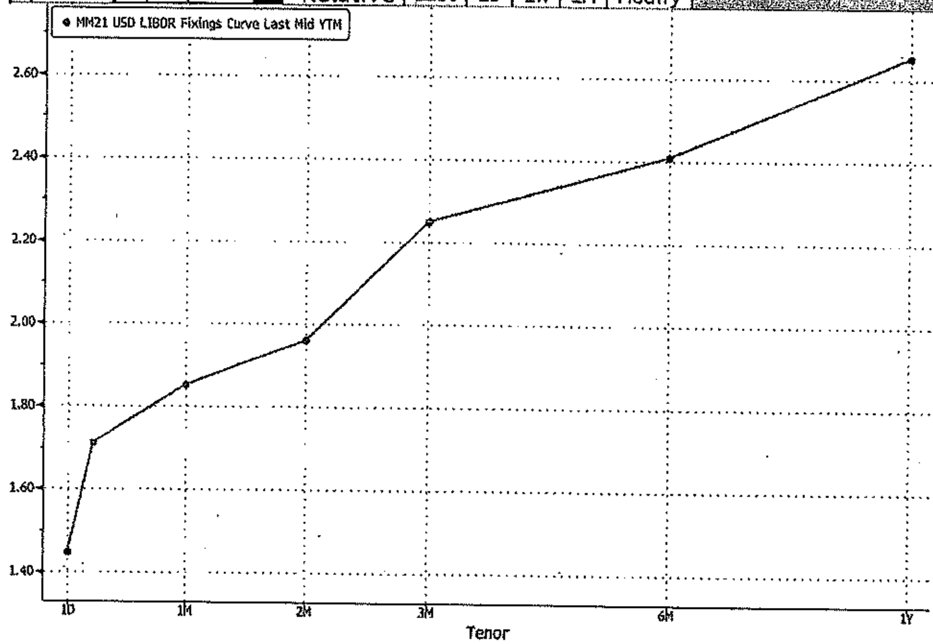
YCMM0021 1d 1.447 1w 1.711 2w 0.171 1m 1.854 2m 1.961 3m 2.248 4m 0.317
USD LIBOR Fixings Curve

USD LIBOR Fixings Curve

Graph Curves

X-Axis Y-Axis Mid YTM Currency None PCS BLP Lower Chart Table

Specific MM/DD/YY Relative Last 1D 1W 1M Modify » Curves & Relative Value



⌄ Plot Curves

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☒ USD LIBOR Fixings Curve

☒ Mid YTM

Base Curve

☐ Show Constituents on Base Curve

⌄ Recent Curves

› Curves related to USD LIBOR Fixings ...

⌄ Bond Spread to Curve

<Add Security>

› Plot New Issues / Points

› Interpolate Curves

◉ All Tenors ◉ Key Tenors

	Curve Id	1D	1W	1M	2M	3M	6M	1Y
11)	▪ MM21	1.448	1.711	1.854	1.961	2.248	2.410	2.651

SN 648115 PDT GMT-7:00 G447-3651-0 21-Mar-2018 14:51:59

28

<Menu> to Return

97) Settings 98) Output ·) Show in Launch Page 1/2 ICE Benchmark Administration

Official ICE Libor Fixings (Digital)

ICE Interest Settlement Rates

MSG Contributor 14:53:24

Zoom 100%

ICE Benchmark Administration -> Official ICE Libor Fixings (formerly known as BBA LIBOR) -> Daily Fixings -> Official ICE...

	USD		GBP		EUR	
1) O/N	1.44750	03/20/18	0.47505	03/20/18	-0.44043	03/20/18
2) 1WK	1.71088	03/20/18	0.48863	03/20/18	-0.42271	03/20/18
3) 1MO	1.85382	03/20/18	0.50706	03/20/18	-0.41086	03/20/18
4) 2MO	1.96100	03/20/18	0.55497	03/20/18	-0.39714	03/20/18
5) 3MO	2.24814	03/20/18	0.62408	03/20/18	-0.38929	03/20/18
6) 6MO	2.40988	03/20/18	0.73255	03/20/18	-0.33314	03/20/18
7) 12MO	2.65138	03/20/18	0.94388	03/20/18	-0.25271	03/20/18

Page Forward For S/N Currencies

SN 648115 PDT GMT-7:00 G447-3651-0 21-Mar-2018 14:53:24

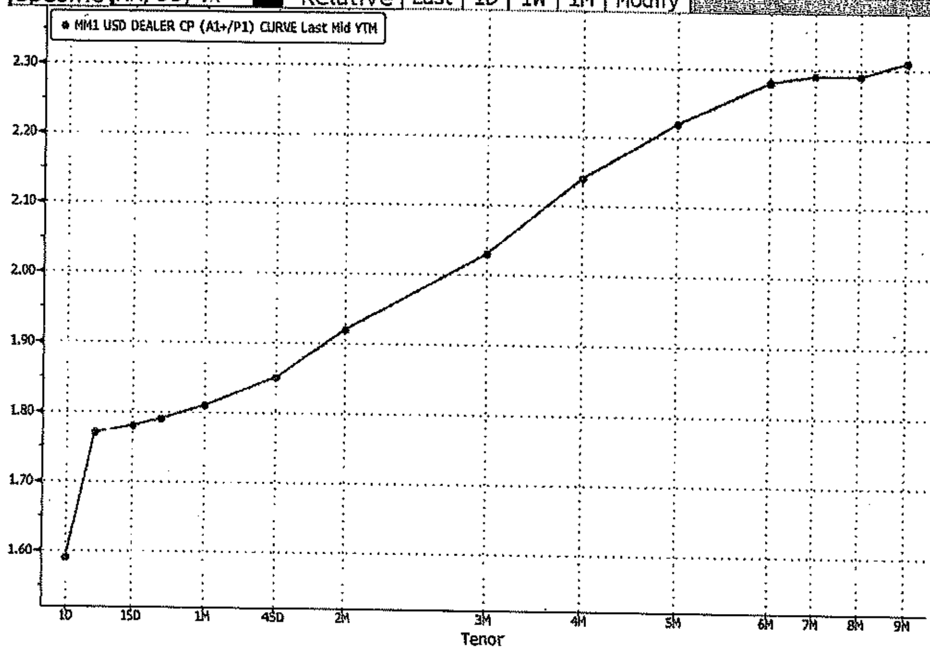
<Menu> to Return

USD DEALER CP (A1+/P1) C

Graph Curves

X-Axis Y-Axis Mid YTM Currency None PCS BLP Lower Chart Table

Specific MM/DD/YY Relative Last 1D 1W 1M Modify » Curves & Relative Value



Plot Curves

<Add Curve> Browse | CRVF »

☑ USD DEALER CP (A1+/P1) CURVE

☑ Mid YTM

Base Curve

☐ Show Constituents on Base Curve

Recent Curves

Curves related to Federal Reserve 7 ...

Bond Spread to Curve

<Add Security>

Plot New Issues / Points

Interpolate Curves

◉ All Tenors ◉ Key Tenors

	Curve Id	1D	1W	1M	2M	3M	6M	9M
11)	■ MM1	1.590	1.770	1.810	1.920	2.030	2.280	2.310

SN 648115 PDT GMT-7:00 G447-3651-0 21-Mar-2018 15:06:05

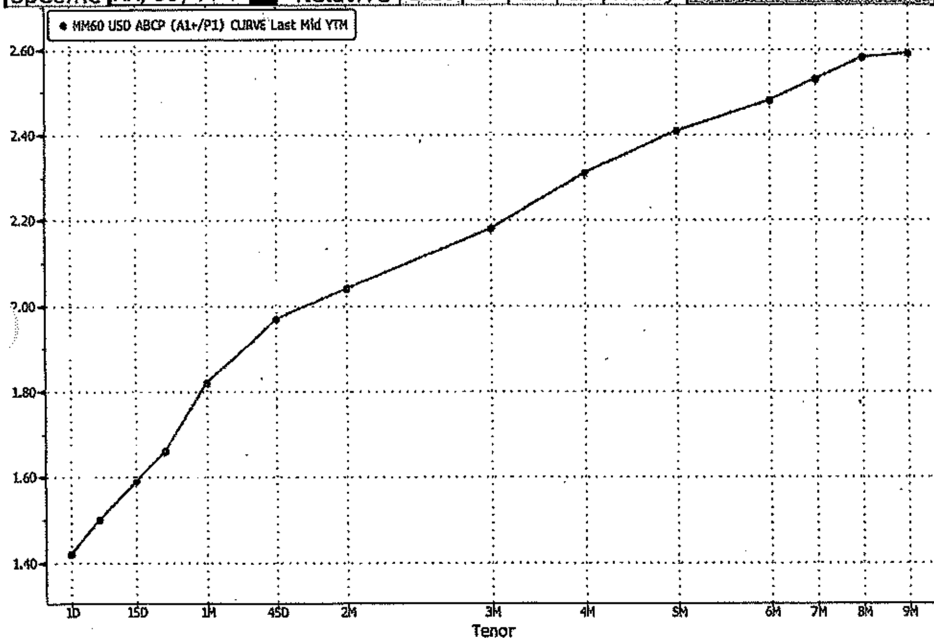
<Menu> to Return

USD ABCP (A1+/P1) CURVE

Graph Curves

X-Axis Y-Axis Mid YTM Currency None PCS BLP Lower Chart Table

Specific MM/DD/YY Relative Last 1D 1W 1M Modify » Curves & Relative Value



⌄ Plot Curves

<Add Curve> Browse | CRVF »

☑USD ABCP (A1+/P1) CURVE

☑Mid YTM

Base Curve

☐ Show Constituents on Base Curve

⌄ Recent Curves

› Curves related to Federal Reserve 7 ...

⌄ Bond Spread to Curve

<Add Security>

› Plot New Issues / Points

› Interpolate Curves

◉ All Tenors ◉ Key Tenors

	Curve Id	1D	1W	1M	2M	3M	6M	9M
11)	■ MM60	1.420	1.500	1.820	2.040	2.180	2.480	2.590

SN 648115 PDT GMT-7:00 G447-3651-0 21-Mar-2018 15:07:43

14:56 **REPURCHASE AGREEMENT RATES**

SECURITY	TIME	BID	ASK	LAST	HIGH	LOW	CLOSE
GOVERNMENT							
3) O/N	1/12	.1400	.1200	.1300	.1400	.1300	.1300
4) 1 Week	1/12	.1400	.1200	.1300	.1400	.1300	.1300
5) 2 Week	1/12	.1400	.1200	.1300	.1400	.1300	.1300
6) 3 Week	1/12	.1400	.1200	.1300	.1400	.1300	.1300
7) 1 Month	1/12	.1500	.1200	.1350	.1500	.1350	.1350
8) 2 Month	1/12	.1700	.1400	.1550	.1700	.1550	.1550
9) 3 Month	1/12	.1700	.1400	.1550	.1700	.1550	.1550
MORTGAGE							
14) O/N	1/12	.1600	.1400	.1500	.1600	.1500	.1500
15) 1 Week	1/12	.1700	.1400	.1550	.1700	.1550	.1550
16) 2 Week	1/12	.1800	.1500	.1650	.1800	.1650	.1650
17) 3 Week	1/12	.1800	.1500	.1650	.1800	.1650	.1650
18) 1 Month	1/12	.1800	.1500	.1650	.1800	.1650	.1650
19) 2 Month	1/12	.2100	.1800	.1950	.2100	.1950	.1950
20) 3 Month	1/12	.2200	.1800	.2000	.2200	.2000	.2000

Par Curve

- Definition: The term stucture of coupon rates for bonds to sell at par.
- Recall that for par bonds, coupon rate equals par rate.

$$100 = \frac{c}{2}\sum_{i=1}^{2T} D(i/2) + 100D(T)$$

$$c = 2\left[\frac{100 - 100D(T)}{\sum_{i=1}^{2T} D(i/2)}\right]$$

- Term structure data

$$D(0.5) = 0.97$$

$$D(1.0) = 0.94$$

$$D(1.5) = 0.91$$

$$D(2.0) = 0.87$$

Par Rate Examples

- Par rate for $T = 0.50$

$$= 2\left[\frac{100 - 100 \times 0.97}{0.97}\right] = 0.061855$$

Same as spot rate.

- Par rate for $T = 1.00$

$$= 2\left[\frac{100 - 100 \times 0.94}{0.97 + 0.94}\right] = 0.062827$$

Spot rate = .062842.

Par Curve Examples

5

- Constant maturity Treasury (CMT) curve.
- Constant maturity swap (CMS) curve.
- Swap spread = CMS - CMT.

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Federal Reserve Selected Interest Rates

H.15 Bloomberg Index Tickers		3/14	3/15	3/16	3/19	3/20
13) TBSM1Y	1-year	1.98	2.00	2.01	2.01	2.01
Treasury Const. Mat. Nominal						
14) H15T1M	1-month	1.71	1.70	1.71	1.70	1.76
15) H15T3M	3-month	1.76	1.77	1.78	1.80	1.81
16) H15T6M	6-month	1.94	1.95	1.96	1.99	1.97
17) H15T1Y	1-year	2.05	2.07	2.08	2.08	2.08
18) H15T2Y	2-year	2.26	2.29	2.31	2.31	2.34
19) H15T3Y	3-year	2.41	2.42	2.44	2.45	2.49
20) H15T5Y	5-year	2.61	2.62	2.65	2.65	2.69
21) H15T7Y	7-year	2.75	2.76	2.78	2.78	2.82
22) H15T10Y	10-year	2.81	2.82	2.85	2.85	2.89
23) H15T20Y	20-year	2.94	2.94	2.96	2.97	3.01
24) H15T30Y	30-year	3.05	3.05	3.08	3.09	3.12
Inflation Indexed						
25) H15X5YR	5-year	0.57	0.55	0.60	0.61	0.64
26) H15X7YR	7-year	0.69	0.66	0.71	0.72	0.74
27) H15X10YR	10-year	0.74	0.74	0.77	0.77	0.81
28) H15X20YR	20-year	0.88	0.88	0.91	0.91	0.94
29) H15X30YR	30-year	0.97	0.97	1.00	1.00	1.03

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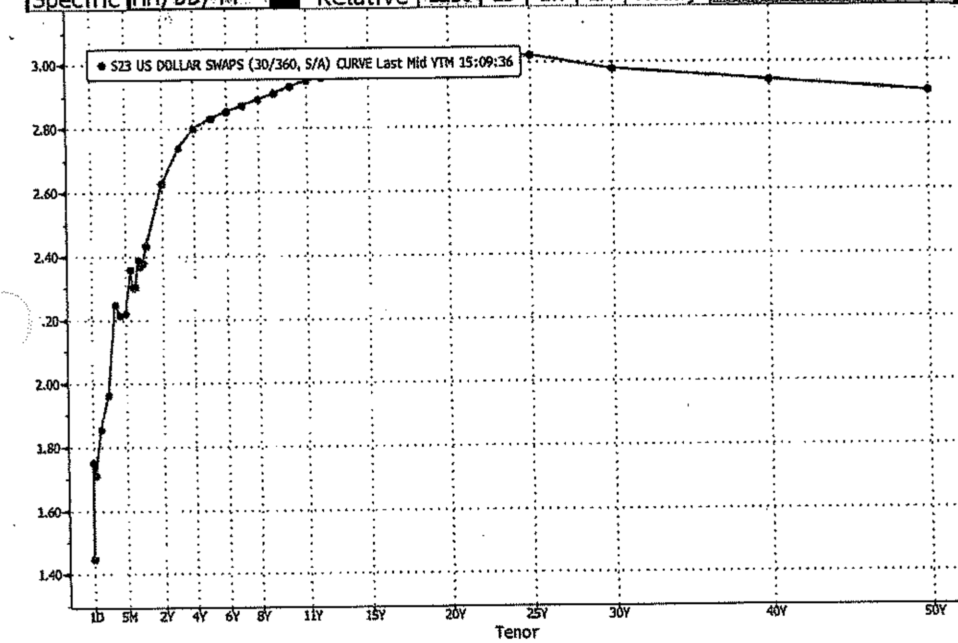
<Menu> to Return

US DOLLAR SWAPS (30/360,

Graph Curves

X-Axis Y-Axis Mid YTM Currency None PCS MULT Lower Chart Table

Specific MM/DD/YY Relative Last 1D 1W 1M Modify » Curves & Relative Value



All Tenors Key Tenors

	Curve Id	3M	1Y	2Y	3Y	5Y	10Y	15Y	30Y
11)	■ S23	2.248	2.434	2.627	2.737	2.830	2.929	2.987	2.981

Plot Curves

<Add Curve> Browse | CRVF »

US DOLLAR SWAPS (30/360, S/A...

Mid YTM

Base Curve

Show Constituents on Base Curve

Recent Curves

US EUR Sr CDS Curve

Curves related to Federal Reserve 7 ...

Bond Spread to Curve

<Add Security>

Plot New Issues / Points

Interpolate Curves

SN 648115 PDT GMT-7:00 G447-3651-0 21-Mar-2018 15:09:36

United States 98) Export 99) Settings **Interest Rate Swap Rates**

Date Range: 02/21/2018 - 03/21/2018 1 Month

40) Semi Swaps 41) Sprs to Gov. 42) Ann Swaps 43) Ann Sprs 44) OIS Swaps

USD SemiAnnual 30/360 Swap Rates Avg Now PCS CMPN

Tenor	Bid / Ask	Mid	Change	Today	#SD Δ/day	Low	Range	High	Avg	+/-BPS	#SD
1) 1 YR	2.427 / 2.429	2.428	-0.002		0.0	2.251		2.466	2.339	9.0	1.7
2) 2 YR	2.624 / 2.630	2.627	0.000		0.0	2.445		2.677	2.553	7.7	1.7
3) 3 YR	2.735 / 2.740	2.737	0.000		0.0	2.550		2.797	2.668	7.2	1.7
4) 4 YR	2.790 / 2.810	2.800	0.005		0.1	2.616		2.858	2.732	7.8	2.0
5) 5 YR	2.827 / 2.833	2.830	0.002		0.1	2.660		2.890	2.771	6.2	1.7
6) 6 YR	2.850 / 2.854	2.852	0.000		0.0	2.696		2.913	2.803	5.1	1.5
7) 7 YR	2.871 / 2.871	2.871	-0.001		0.0	2.726		2.932	2.829	4.2	1.[illegible]
8) 8 YR	2.886 / 2.894	2.890	0.000		0.0	2.753		2.949	2.854	4.0	1[illegible]
9) 9 YR	2.905 / 2.914	2.909	0.001		0.0	2.777		2.966	2.876	3.8	1.[illegible]
10) 10 YR	2.927 / 2.930	2.929	0.002		0.1	2.801		2.998	2.897	3.3	1.0
11) 15 YR	2.984 / 2.990	2.987	0.001		0.0	2.876		3.063	2.967	2.3	0.7
12) 20 YR	3.002 / 3.007	3.004	0.000		0.0	2.901		3.088	2.990	1.7	0.5
13) 25 YR	3.020 / 3.026	3.023	0.026		0.7	2.156		3.254	2.983	4.2	1.2
14) 30 YR	2.978 / 2.983	2.981	0.001		0.0	2.884		3.071	2.969	1.5	0.4

Executable quotes for Fixed Income Electronic Trading are in white tenors.

SN 648115 PDT GMT-7:00 G447-3651-0 21-Mar-2018 15:12:35

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United States 98) Export 99) Settings Interest Rate Swap Rates

Date Range: 02/21/2018 - 03/21/2018 1 Month

40) Semi Swaps | 41) Sprs to Gov | 42) Ann Swaps | 43) Ann Sprs | 44) OIS Swaps

USD Spreads to Government Avg ● Now PCS MPR

Tenor	Bid / Ask	Mid	Change	Today	#SD Δ/day	Low	Range	High	Avg	+/-BPS	#SD
1) 1 YR	38.228 / 39.260	38.744	-0.145		0.0	21.840		40.580	30.774	8.5	1.9
2) 2 YR	31.375 / 32.500	31.938	0.063		0.0	22.500		33.850	29.198	3.3	1.5
3) 3 YR	26.750 / 27.875	27.313	0.087		0.1	20.656		27.875	25.135	2.7	1.6
4) 4 YR	21.875 / 23.125	22.500	-0.006		0.0	16.015		23.526	20.144	3.0	1.6
5) 5 YR	15.000 / 15.500	15.250	-0.100		-0.1	5.500		16.450	13.137	2.4	1.2
6) 6 YR	10.250 / 11.375	10.813	0.188		0.1	4.560		11.830	8.444	2.9	1.5
7) 7 YR	5.750 / 6.500	6.125	0.125		0.1	-0.330		7.200	3.298	3.2	1.6
8) 8 YR	4.875 / 6.125	5.500	-0.125		-0.1	-0.130		6.370	3.368	2.8	1.5
9) 9 YR	4.000 / 5.250	4.625	-0.250		-0.2	-0.040		5.690	3.162	2.1	1.3
10) 10 YR	4.000 / 4.500	4.250	0.000		0.0	-0.500		5.180	2.859	1.6	1.2
11) 15 YR	4.125 / 5.375	4.750	0.500		0.4	-0.250		5.570	3.347	2.0	1.6
12) 20 YR	0.125 / 0.625	0.375	-0.063		0.0	-4.690		1.510	-0.879	1.5	1.0
13) 25 YR	-6.625 / -6.125	-6.375	-0.100		-0.1	-12.375		-5.287	-7.988	1.9	1.1
14) 30 YR	-14.375 / -13.625	-14.000	0.125		0.1	-20.938		-12.750	-16.202	2.6	1.3

Executable quotes for Fixed Income Electronic Trading are in white tenors.

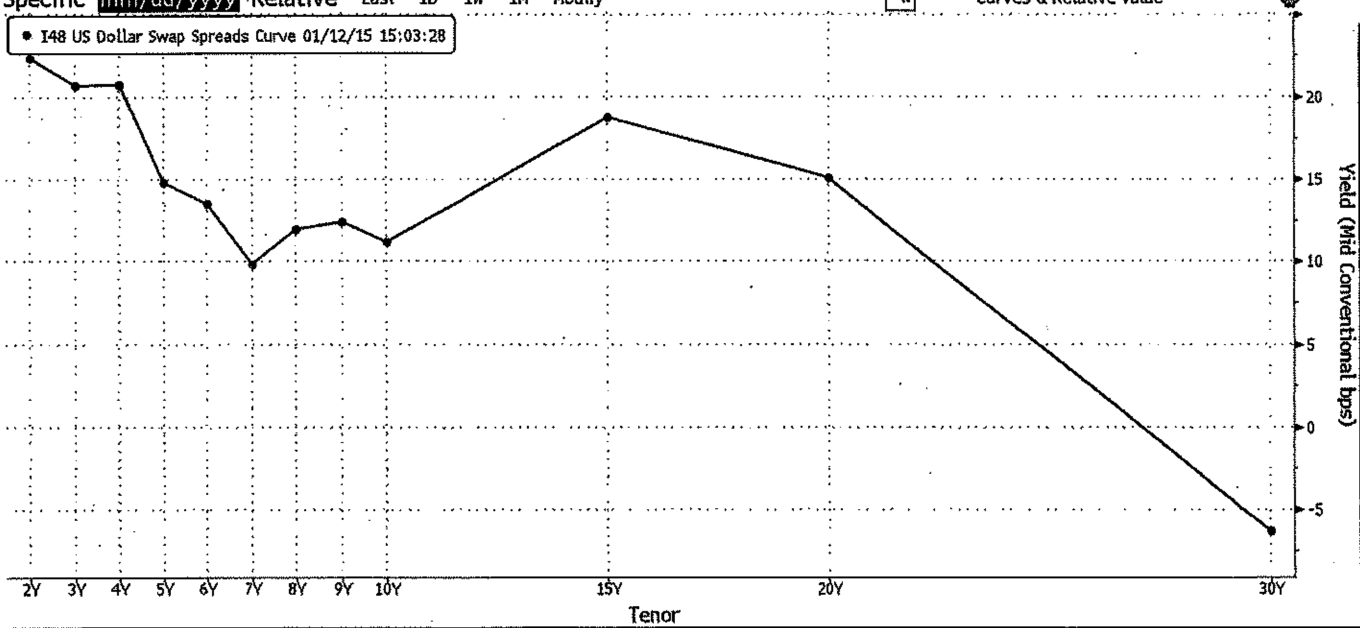
SN 648115 PDT GMT-7:00 G447-3651-0 21-Mar-2018 15:12:48

<HELP> for explanation.
<Menu> to Return

US DOLLAR SWAP SPREADS C | 97) Actions | 98) Table | Graph Curves

X-axis Tenor Y-axis Yield PCS CMPN Lower Chart Table

Specific mm/dd/yyyy Relative Last 1D 1W 1M Modify « Curves & Relative Value



Curve ID	2Y	3Y	5Y	7Y	8Y	10Y	15Y	30Y
I48	22.4	20.7	14.8	9.8	12.0	11.2	18.8	-6.8

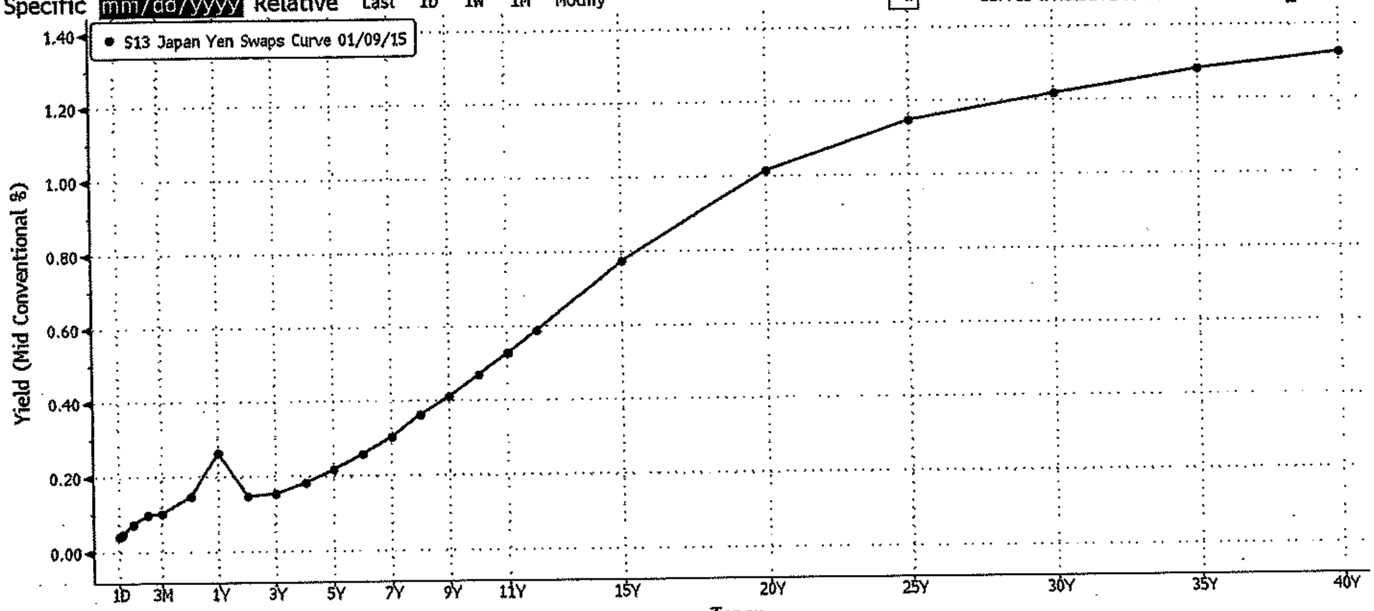
3) Show All Tenors >

<HELP> for explanation.
<Menu> to Return

JAPAN YEN SWAPS CURVE | 97) Actions | 98) Table | Graph Curves

X-axis Tenor | Y-axis Yield | PCS CMPN | Lower Chart Table

Specific mm/dd/yyyy Relative Last 1D 1W 1M Modify | « | Curves & Relative Value



Curve ID	3M	1Y	2Y	3Y	5Y	7Y	10Y	15Y	30Y
S13	0.103	0.264	0.148	0.156	0.218	0.306	0.471	0.776	1.218

3) Show All Tenors >

<HELP> for explanation.
Screen Printed

JAPAN YEN SWAPS CURVE | 97) Actions | 98) Chart | Page 1/2 | Graph Curves

X-axis Tenor Y-axis Yield PCS CMPN

Specific mm/dd/yyyy Relative Last 1D 1W 1M Modify « Curves & Relative Value

◉Values and Members ○Values ○Members ○Constituents 0) Export

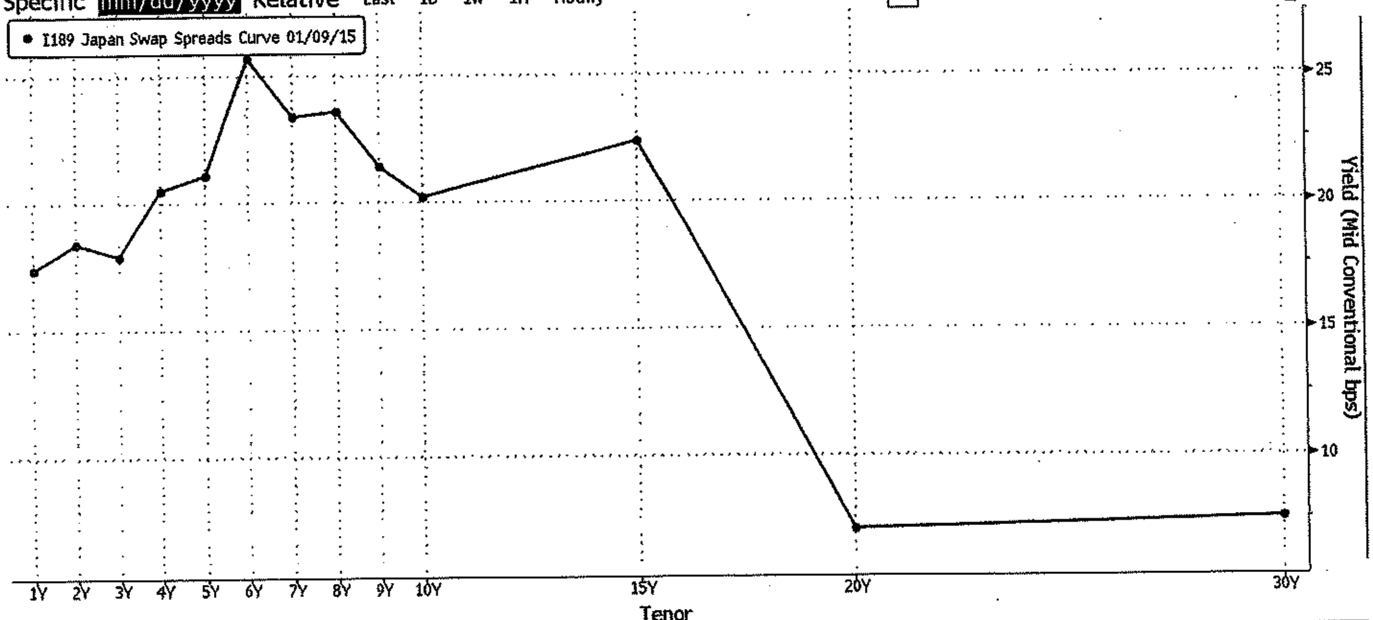
	Tenor	Description	Mid Price	Mid Yield	Source	Update
11)	1D	JY00S/N Index		0.042		01/09/2015
12)	1W	JY0001W Index		0.049		01/09/2015
13)	1M	JY0001M Index		0.074		01/09/2015
14)	2M	JY0002M Index		0.099		01/09/2015
15)	3M	JY0003M Index		0.103		01/09/2015
16)	6M	JY0006M Index		0.150		01/09/2015
17)	1Y	JY0012M Index		0.264		01/09/2015
18)	2Y	JYSWAP2 Curncy		0.148	CMPN	01/09/2015
19)	3Y	JYSWAP3 Curncy		0.156	CMPN	01/09/2015
20)	4Y	JYSWAP4 Curncy		0.183	CMPN	01/09/2015
21)	5Y	JYSWAP5 Curncy		0.218	CMPN	01/09/2015
22)	6Y	JYSWAP6 Curncy		0.260	CMPN	01/09/2015
23)	7Y	JYSWAP7 Curncy		0.306	CMPN	01/09/2015
24)	8Y	JYSWAP8 Curncy		0.363	CMPN	01/09/2015
25)	9Y	JYSWAP9 Curncy		0.414	CMPN	01/09/2015
26)	10Y	JYSWAP10 Curncy		0.471	CMPN	01/09/2015
27)	11Y	JYSW11 Curncy		0.530	CMPN	01/09/2015
28)	12Y	JYSWAP12 Curncy		0.589	CMPN	01/09/2015
29)	15Y	JYSW15 Curncy		0.776	CMPN	01/09/2015
30)	20Y	JYSW20 Curncy		1.015	CMPN	01/09/2015
31)	25Y	JYSW25 Curncy		1.149	CMPN	01/09/2015
32)	30Y	JYSW30 Curncy		1.218	CMPN	01/09/2015

<HELP> for explanation.
<Menu> to Return

JAPAN SWAP SPREADS CURVE | 97) Actions | 98) Table | Graph Curves

X-axis Tenor | Y-axis Yield | PCS CMPN | Lower Chart Table

Specific mm/dd/yyyy | Relative | Last | 1D | 1W | 1M | Modify | « | Curves & Relative Value



Curve ID	1Y	2Y	3Y	5Y	7Y	10Y	15Y	30Y
I189	17.4	18.4	17.9	21.1	23.4	20.2	22.4	7.5

3) Show All Tenors >

<HELP> for explanation.
Screen Printed

JAPAN SWAP SPREADS CURVE | 97) Actions | 98) Chart | Page 1/1 | Graph Curves

X-axis Tenor Y-axis Yield PCS CMPN

Specific mm/dd/yyyy Relative Last 1D 1W 1M Modify « Curves & Relative Value

◉Values and Members ○Values ○Members ○Constituents 0) Export

	Tenor	Description	Mid Spread	Source	Update
11)	1Y	JYSS1 Curncy	17.4	CMPN	01/09/2015
12)	2Y	JYSS2 Curncy	18.4	CMPN	01/09/2015
13)	3Y	JYSS3 Curncy	17.9	CMPN	01/09/2015
14)	4Y	JYSS4 Curncy	20.5	CMPN	01/09/2015
15)	5Y	JYSS5 Curncy	21.1	CMPN	01/09/2015
16)	6Y	JYSS6 Curncy	25.7	CMPN	01/09/2015
17)	7Y	JYSS7 Curncy	23.4	CMPN	01/09/2015
18)	8Y	JYSS8 Curncy	23.6	CMPN	01/09/2015
19)	9Y	JYSS9 Curncy	21.4	CMPN	01/09/2015
20)	10Y	JYSS10 Curncy	20.2	CMPN	01/09/2015
21)	15Y	JYSS15 Curncy	22.4	CMPN	01/09/2015
22)	20Y	JYSS20 Curncy	7.1	CMPN	01/09/2015
23)	30Y	JYSS30 Curncy	7.5	CMPN	01/09/2015

Forward Curve

- Let's compare alternative two-year-horizon investment strategies.
- Buy-and-Hold: Invest for two years in a two-year bond.

$$= (1 + r_2)^2$$

- Rollover: Invest for one year in a one-year bond, then rollover proceeds into a new one-year bond .

$$= (1 + r_1)(1 +_1 r_1)$$

- Which is larger?
- What is the breakeven rate that sets the two equal?

$$= (1 + r_1)(1 +_1 f_1) = (1 + r_2)^2$$

$$(1 +_1 f_1) = \frac{(1 + r_2)^2}{(1 + r_1)} = \frac{D(1)}{D(2)}$$

Continued.

- Example ${}_{10}r_{10}$.

$$(1 + r_{10})^{10}(1 +_{10} f_{10})^{10} = (1 + r_{20})^{20}$$

$$(1 +_{10} f_{10}) = \frac{(1 + r_{20})^{20}}{(1 + r_{10})^{10}} = \frac{D(10)}{D(20)}$$

- Special case for $m < 1$ year.

$$1 + m \times_n f_m = \frac{D(n)}{D(n+m)}$$

(ignore compounding).

Examples

- Example. We want the three-month rate five years forward, ${}_5f_{0.25}$

$$1 + 0.25 \times {}_5 f_{0.25} = \frac{D(5)}{D(5.25)}$$

$$D(5) = 0.765, D(5.25) = 0.75, \implies {}_5f_{0.25} = 0.08$$

- Bloomberg examples.

Forward Par Curves

- Imagine a bond issued not today, but N periods in the future.
- If the bond was issued at par, then its present value today would be $100D(N)$.
- What coupon rate would make the forward-starting bond worth $100D(N)$?

$$100D(N) = \frac{c}{2}\sum_{i=1}^{2M} D(N+i/2) + 100D(N+M)$$

$$c = 2\left[\frac{100D(N) - 100D(N+M)}{\sum_{i=1}^{2M} D(N+i/2)}\right]$$

<HELP> for explanation.
Run FWCV<Go> for Forward Curve Analysis

USD OIS Curve | 5) Export | 6) Graph | Forward Curve Matrix

USD OIS Curve Mid Yield Conventional « Curve List

Two Curve Spreads

Select a curve under "Curve List" for two... Bid Yield Conventional

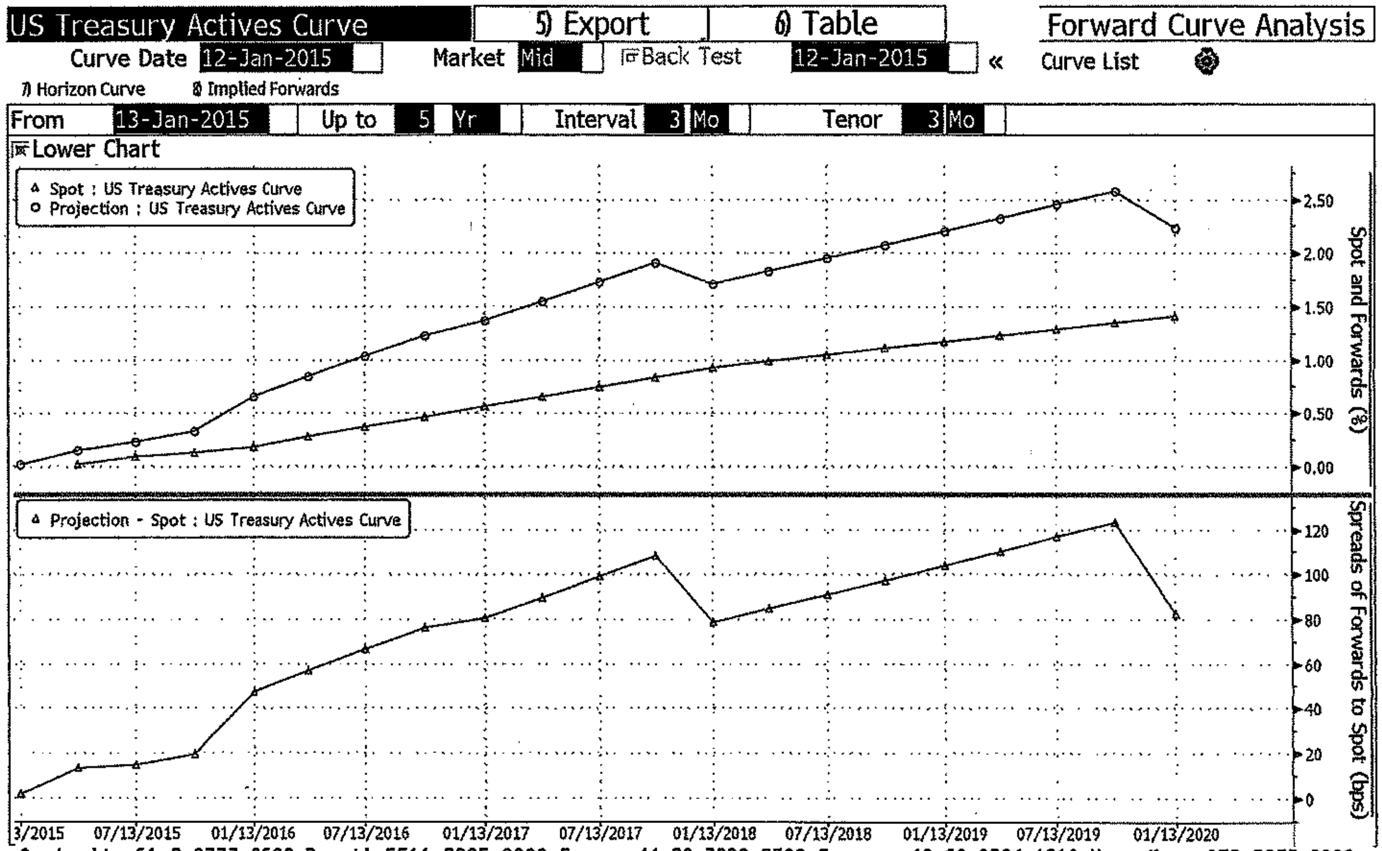
Forward Curve Date 19-Jan-2015 OIS Discounting

Spot ◉Coupon ○Zero

			Forwards									
Tenors	Coupon	1/19/2015	3Mo	6Mo	1Yr	2Yr	3Yr	4Yr	5Yr	10Yr	15Yr	30Yr
1Yr	0.2600	0.2651	0.3963	0.5659	0.9496	1.5185	1.8514	2.0143	2.0790	2.4085	2.4541	2.3269
2Yr	0.6010	0.6097	0.7485	0.9036	1.2298	1.6837	1.9323	2.0464	2.1892	2.4478	2.4682	2.3228
3Yr	0.9030	0.9093	1.0307	1.1659	1.4320	1.7920	1.9814	2.1297	2.2041	2.4504	2.4810	2.3186
4Yr	1.1360	1.1411	1.2425	1.3555	1.5734	1.8627	2.0586	2.1552	2.2444	2.4651	2.4927	2.3142
5Yr	1.3000	1.3095	1.3959	1.4906	1.6708	1.9465	2.0922	2.1964	2.2926	2.4831	2.5031	2.3096
8Yr		1.6287	1.6932	1.7630	1.8951	2.0952	2.2101	2.2910	2.3482	2.4824	2.4673	2.2952
9Yr		1.7045	1.7658	1.8317	1.9560	2.1272	2.2383	2.3076	2.3647	2.4871	2.4613	2.2901
10Yr	1.8090	1.7766	1.8300	1.8880	1.9967	2.1594	2.2578	2.3259	2.3820	2.4925	2.4566	2.2848
15Yr		1.9856	2.0241	2.0656	2.1432	2.2575	2.3299	2.3790	2.4175	2.4665	2.4303	2.2106
20Yr		2.0925	2.1228	2.1551	2.2155	2.3029	2.3563	2.3901	2.4147	2.4460	2.4053	2.1585
30Yr		2.1680	2.1901	2.2137	2.2577	2.3206	2.3579	2.3803	2.3956	2.4046	2.3396	1.9067

Grey values are extrapolated

<HELP> for explanation.



<HELP> for explanation.

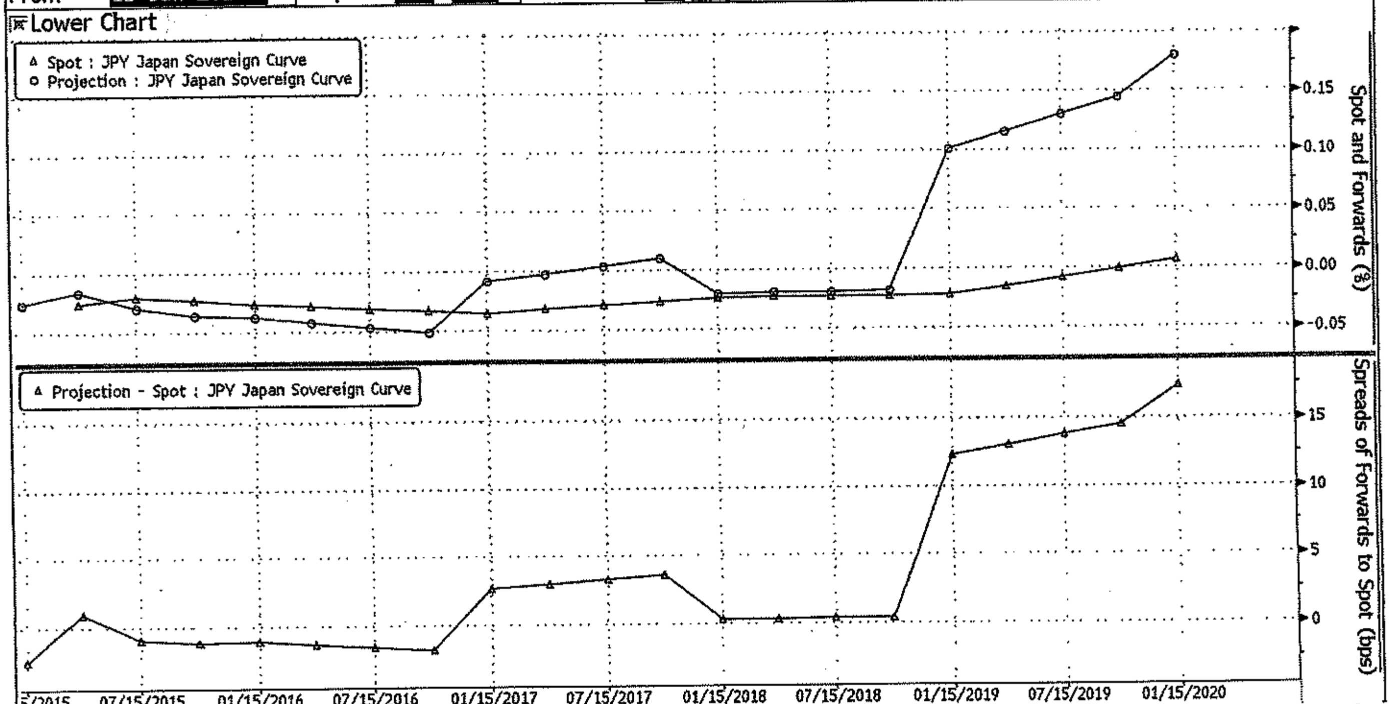
JPY Japan Sovereign Curve | 5) Export | 6) Table | Forward Curve Analysis

Curve Date 12-Jan-2015 | Market Mid | Back Test 12-Jan-2015 « | Curve List

7) Horizon Curve | 8) Implied Forwards

From 15-Jan-2015 | Up to 5 Yr | Interval 3 Mo | Tenor 3 Mo

Lower Chart



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Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000

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<HELP> for explanation.

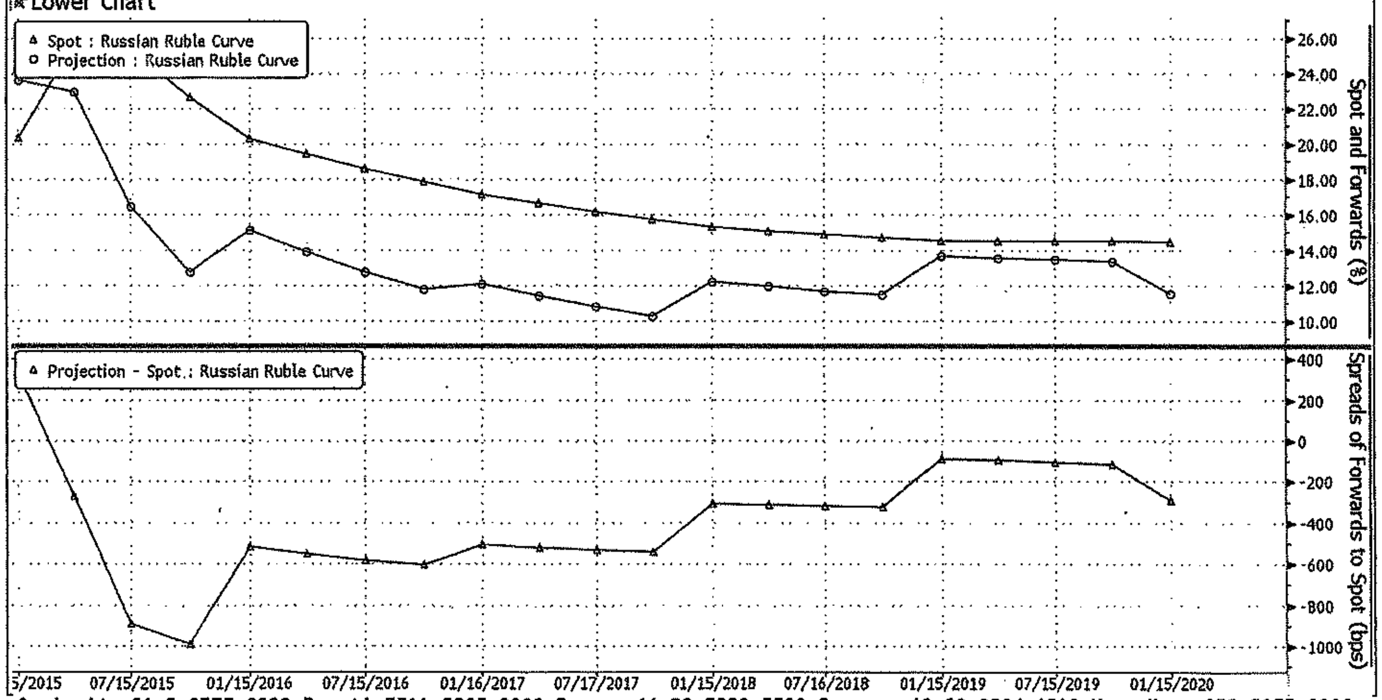
Russian Ruble Curve | 5) Export | 6) Table | Forward Curve Analysis

Curve Date 12-Jan-2015 Market Mid Back Test 12-Jan-2015 « Curve List

7) Horizon Curve 8) Implied Forwards

From 15-Jan-2015 Up to 5 Yr Interval 3 Mo Tenor 3 Mo

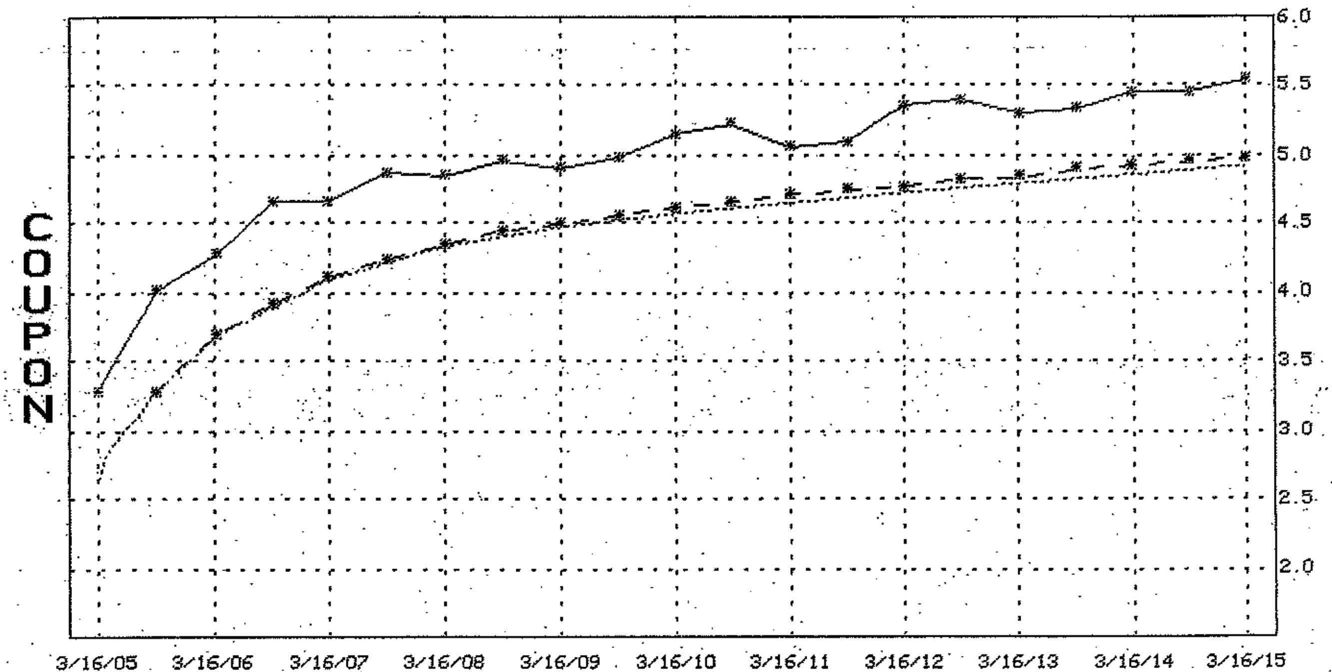
Lower Chart



<HELP> for explanation. N127 **Govt** **FWCV**

IMPLIED FORWARDS CURVE
US Dollar

6-Mo Forwards 6-Mo Intervals Date 3/16/05 Points 20 Page 1/2

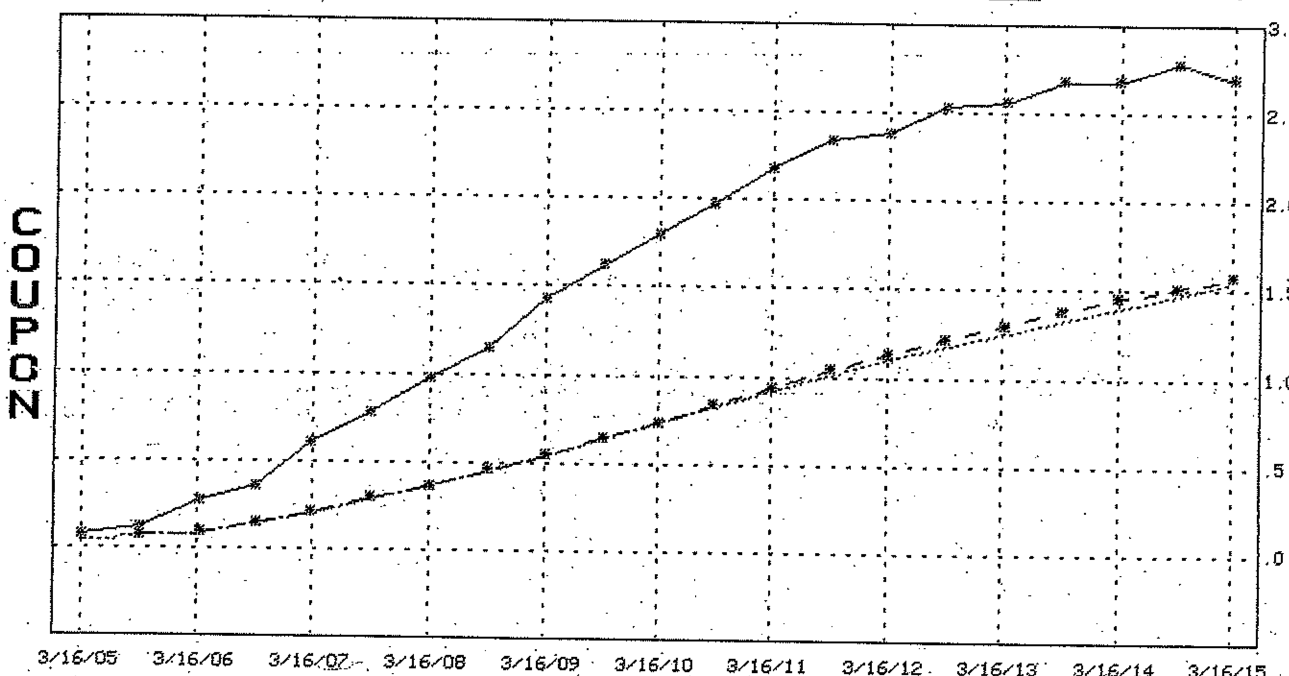


Forwards Curve Overlay Spot Curve Coupon Curve

<HELP> for explanation. N127 Govt **FWCV**

IMPLIED FORWARDS CURVE
Japanese Yen

6-Mo Forwards 6-Mo Intervals Date 3/16/05 Points 20 Page 1/2

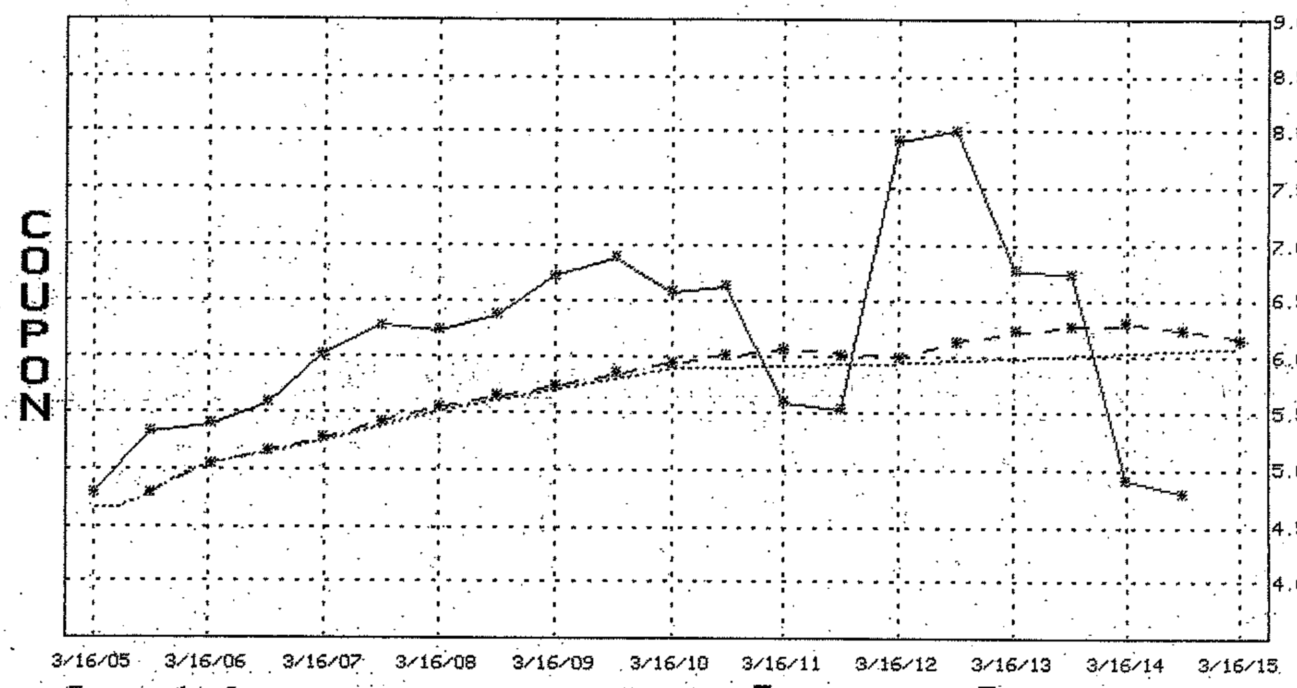


Forwards Curve Overlay Spot Curve Coupon Curve

<HELP> for explanation. N127 Govt **FWCV**

IMPLIED FORWARDS CURVE
INR MIFOR

6-Mo Forwards 6-Mo Intervals Date 3/16/05 Points 20 Page 1/2



Forwards Curve Overlay Spot Curve Coupon Curve

Bootstrapping Curves

- Often, $D(T)$ isn't observable, but coupon bond prices are.
- Can reverse engineer discount curve from coupon bond prices.

Coupon	Maturity	Price
0.00	0.50	0.974
6.25	1.0	100.27
6.50	1.5	100.41
7.00	2.0	100.79

- Example

EXHIBIT 6–4

Maturity and Yield-to-Maturity for 20 Hypothetical Treasury Securities

Maturity	Coupon Rate	Yield-to-Maturity	Price
0.50 years	0.0000	0.0800	$ 96.15
1.00	0.0000	0.0830	92.19
1.50	0.0850	0.0890	99.45
2.00	0.0900	0.0920	99.64
2.50	0.1100	0.0940	103.49
3.00	0.0950	0.0970	99.49
3.50	0.1000	0.1000	100.00
4.00	0.1000	0.1040	98.72
4.50	0.1150	0.1060	103.16
5.00	0.0875	0.1080	92.24
5.50	0.1050	0.1090	98.38
6.00	0.1100	0.1120	99.14
6.50	0.0850	0.1140	86.94
7.00	0.0825	0.1160	84.24
7.50	0.1100	0.1180	96.09
8.00	0.0650	0.1190	72.62
8.50	0.0875	0.1200	82.97
9.00	0.1300	0.1220	104.30
9.50	0.1150	0.1240	95.06
10.00	0.1250	0.1250	100.00

EXHBIT 6–5

Theoretical Spot Rates

Maturity	Yield-to-Maturity	Theoretical Spot Rate
0.50 years	0.0800	0.08000
1.00	0.0830	0.08300
1.50	0.0890	0.08930
2.00	0.0920	0.09247
2.50	0.0940	0.09468
3.00	0.0970	0.09787
3.50	0.1000	0.10129
4.00	0.1040	0.10592
4.50	0.1060	0.10850
5.00	0.1080	0.11021
5.50	0.1090	0.11175
6.00	0.1120	0.11584
6.50	0.1140	0.11744
7.00	0.1160	0.11991
7.50	0.1180	0.12405
8.00	0.1190	0.12278
8.50	0.1200	0.12546
9.00	0.1220	0.13152
9.50	0.1240	0.13377
10.00	0.1250	0.13623

Smoothing Curves

5

- Often bootstrapped curves are very jagged.
- Examples.
- Another approach is to linearly interpolate curve.
- Tuckman example.
- Regression approach–homework.

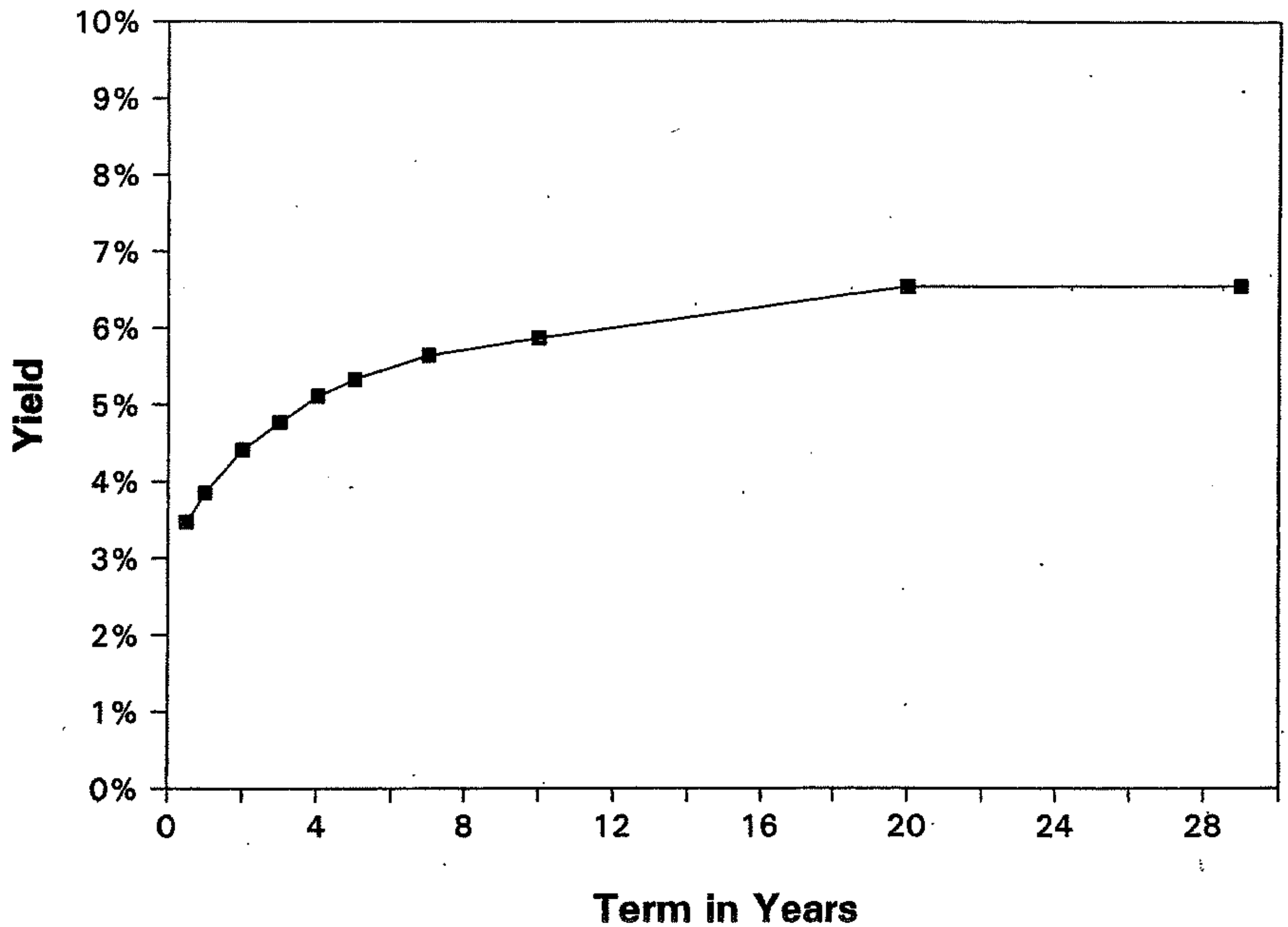


FIGURE 4.4 Linear yield interpolation on February 15, 1994.

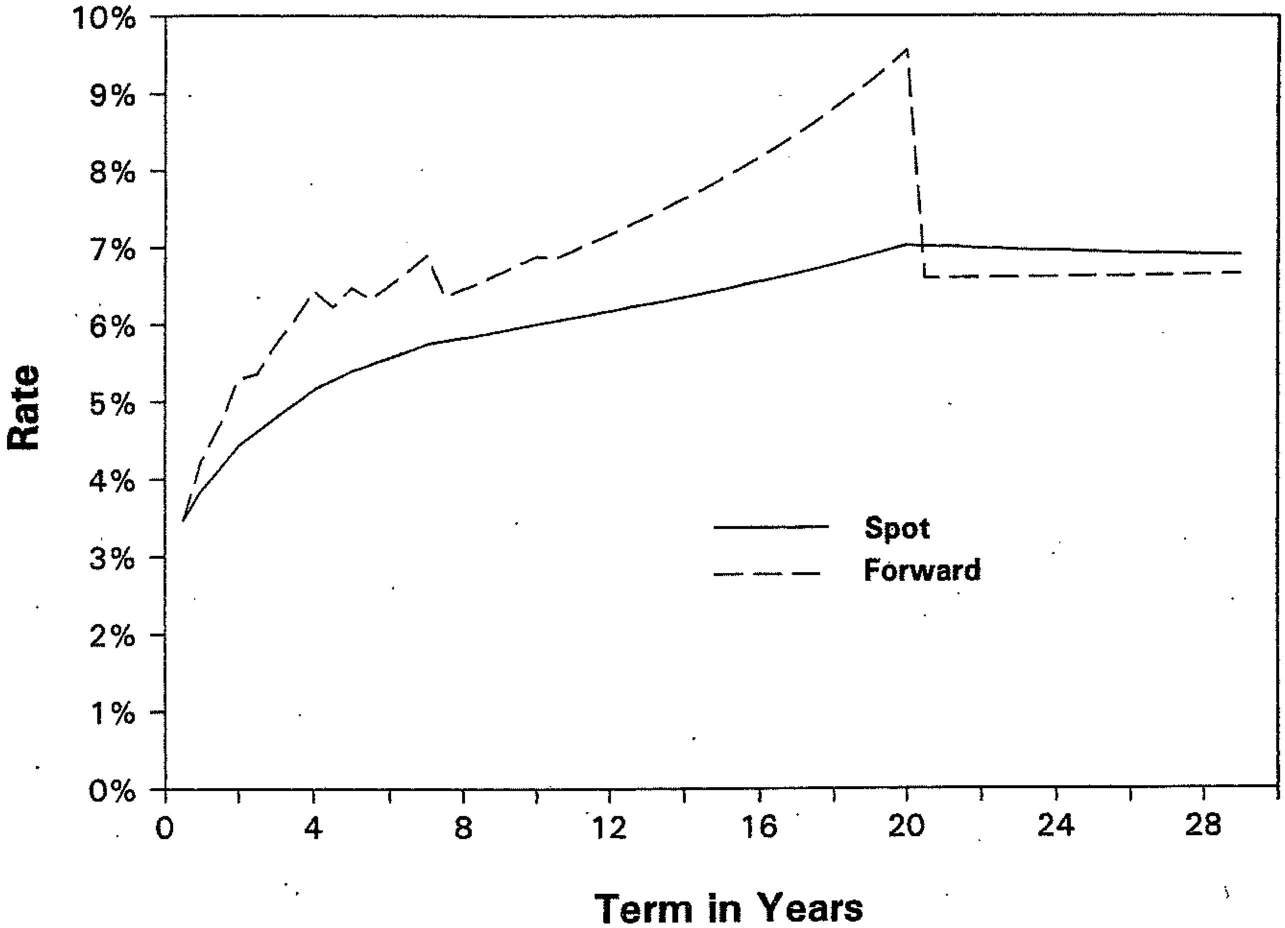


FIGURE 4.5 Spot and forward rates as of February 15, 1994: yield interpolation.

Agenda

9

- Price/Yield Relations.
- DV01.
- Duration.
- Convexity.
- Bond Returns.
- Applications of Convexity.

Price/Yield Relations

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- Bond prices and yields are inversely related.
- Price/yield relation isn't the same for all bonds.
- Price/yield relation is approximately symmetric for small changes in yield.
- Price/yield relation isn't symmetric for large changes in yields.

EXHIBIT 11-1

Price/Yield Relationship for 12 Hypothetical Bonds

		Yield Level						
Coupon (%)	Term (years)	10.00%	10.01%	10.10%	10.50%	11.00%	12.00%	13.00%
.00%	5	$ 61.39	$ 61.36	$ 61.10	$ 59.95	$ 58.54	$ 55.84	$ 53.27
.00	15	23.14	23.10	22.81	21.54	20.06	17.41	15.12
.00	30	5.35	5.34	5.20	4.64	4.03	3.03	2.29
8.00	5	92.28	92.24	91.91	90.46	88.69	85.28	82.03
8.00	15	84.63	84.56	83.95	81.32	78.20	72.47	67.35
8.00	30	81.07	80.99	80.29	77.30	73.83	67.68	62.42
10.00	5	100.00	99.96	99.61	98.09	96.23	92.64	89.22
10.00	15	100.00	99.92	99.24	96.26	92.73	86.24	80.41
10.00	30	100.00	99.91	99.06	95.46	91.28	83.84	77.45
14.00	5	115.44	115.40	115.02	113.35	111.31	107.36	103.59
14.00	15	130.74	130.65	129.81	126.15	121.80	113.76	106.53
14.00	30	137.86	137.73	136.60	131.79	126.17	116.16	107.52

EXHIBIT 11-1

Concluded

		Yield Level						
Coupon (%)	Term (years)	10.00%	9.99%	9.90%	9.50%	9.00%	8.00%	7.00%
.00%	5	$ 61.39	$ 61.42	$ 61.68	$ 62.87	$ 64.39	$ 67.56	$ 70.89
.00	15	23.14	23.17	23.47	24.85	26.70	30.83	35.63
.00	30	5.35	5.37	5.51	6.18	7.13	9.51	12.69
8.00	5	92.28	92.32	92.65	94.14	96.04	100.00	104.16
8.00	15	84.63	84.70	85.31	88.13	91.86	100.00	109.20
8.00	30	81.07	81.15	81.87	85.19	89.68	100.00	112.47
10.00	5	100.00	100.04	100.39	101.95	103.96	108.11	112.47
10.00	15	100.00	100.08	100.77	103.96	108.14	117.29	127.57
10.00	30	100.00	100.09	100.95	104.94	110.32	122.62	137.42
14.00	5	115.44	115.49	115.87	117.59	119.78	124.33	129.11
14.00	15	130.74	130.84	131.69	135.60	140.72	151.88	164.37
14.00	30	137.86	137.99	139.13	144.44	151.60	167.87	187.31

EXHIBIT 11–3

Dollar Price Change per $100 of Par Value as Yield Changes for 12 Hypothetical Bonds

		Change in Basis Points from 10%					
		1	*10*	*50*	*100*	*200*	*300*
		New Yield Level					
Coupon (%)	*Term (years)*	*10.01%*	*10.10%*	*10.50%*	*11.00%*	*12.00%*	*13.00%*
.00%	5	$ –.03	$ –.26	$ –1.15	$ –1.41	$ –2.70	$ –2.57
.00	15	–.03	–.33	–1.59	–3.07	–5.73	–8.02
.00	30	–.02	–.15	–.71	–1.33	–2.32	–3.07
8.00	5	–.04	–.37	–1.81	–3.58	–7.00	–10.25
8.00	15	–.07	–.68	–3.31	–6.43	–12.16	–17.27
8.00	30	–.08	–.78	–3.78	–7.25	–13.39	–18.65
10.00	5	–.04	–.39	–1.91	–3.77	–7.36	–10.78
10.00	15	–.08	–.76	–3.74	–7.27	–13.76	–19.59
10.00	30	–.09	–.94	–4.54	–8.72	–16.16	–22.55
14.00	5	–.04	–.42	–2.09	–4.14	–8.08	–11.85
14.00	15	–.09	–.94	–4.59	–8.94	–16.98	–24.22
14.00	30	–.13	–1.25	–6.07	–11.68	–21.70	–30.34

EXHIBIT 11–3

Concluded

		Change in Basis Points from 10%					
		–1	*–10*	*–50*	*–100*	*–200*	*–300*
		New Yield Level					
Coupon (%)	*Term (years)*	*9.99%*	*9.90%*	*9.50%*	*9.00%*	*8.00%*	*7.00%*
.00%	5	$ .03	$ .29	$ 1.48	$ 3.00	$ 6.17	$ 9.50
.00	15	.03	.33	1.72	3.56	7.69	12.49
.00	30	.02	.16	.82	1.78	4.15	7.34
8.00	5	.04	.37	1.86	3.77	7.72	11.88
8.00	15	.07	.69	3.51	7.23	15.37	24.57
8.00	30	.08	.79	4.12	8.61	18.93	31.40
10.00	5	.04	.39	1.95	3.96	8.11	12.47
10.00	15	.08	.77	3.96	8.14	17.29	27.59
10.00	30	.09	.95	4.94	10.32	22.62	37.42
14.00	5	.04	.42	2.14	4.34	8.89	13.66
14.00	15	.09	.95	4.85	9.98	21.13	33.63
14.00	30	.13	1.27	6.58	13.74	30.01	49.45

EXHIBIT 11–4

Percentage Price Change as Yield Changes for 12 Hypothetical Bonds

		Change in Basis Points from 10%					
		1	10	50	100	200	300
Coupon (%)	Term (years)	10.01%	10.10%	10.50%	11.00%	12.00%	13.00%
		New Yield Level					
.00%	5	–.05%	–.47%	–2.35%	–4.64%	–9.04%	–13.22%
.00	15	–.14	–1.42	–6.89	–13.28	–24.75	–34.66
.00	30	–.29	–2.82	–13.30	–24.80	–43.38	–57.30
8.00	5	–.04	–.40	–1.97	–3.88	–7.58	–11.11
8.00	15	–.08	–.80	–3.91	–7.60	–14.37	–20.41
8.00	30	–.10	–.96	–4.66	–8.94	–16.52	–23.01
10.00	5	–.04	–.39	–1.91	–3.77	–7.36	–10.78
10.00	15	–.08	–.76	–3.74	–7.27	–13.76	–19.59
10.00	30	–.09	–.94	–4.54	–8.72	–16.16	–22.55
14.00	5	–.04	–.37	–1.81	–3.58	–7.00	–10.26
14.00	15	–.07	–.72	–3.51	–6.84	–12.99	–18.52
14.00	30	–.09	–.91	–4.40	–8.48	–15.74	–22.01

EXHIBIT 11–4

Concluded

		Change in Basis Points from 10%					
		–1	–10	–50	–100	–200	–300
		New Yield Level					
Coupon (%)	Term (years)	9.99%	9.90%	9.50%	9.00%	8.00%	7.00%
.00%	5	.05%	.48%	2.41%	4.89%	10.04%	15.48%
.00	15	.14	1.44	7.41	15.40	33.25	53.98
.00	30	.29	2.90	15.38	33.16	77.57	137.10
8.00	5	.04	.40	2.02	4.08	8.37	12.87
8.00	15	.08	.81	4.14	8.54	18.16	29.03
8.00	30	.10	.98	5.08	10.62	23.35	38.73
10.00	5	.04	.39	1.95	3.96	8.11	12.47
10.00	15	.08	.77	3.96	8.14	17.29	27.59
10.00	30	.09	.95	4.94	10.32	22.62	37.42
14.00	5	.04	.37	1.86	3.76	7.70	11.84
14.00	15	.07	.73	3.71	7.63	16.16	25.72
14.00	30	.09	.92	4.78	9.96	21.77	35.87

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EXHIBIT 11–7

Price Change for a 100-Basis-Point Change in Yield for 10%, 15-Year Bonds Trading at Different Yield Levels

Yield Level (%)	Initial Price ($)	New Price ($)*	Price Decline ($)	Percent Decline (%)
7%	$127.57	$117.29	$10.28	8.1%
8	117.29	108.14	9.15	7.8
9	108.14	100.00	8.14	7.5
10	100.00	92.73	7.27	7.3
11	92.73	86.24	6.49	7.0
12	86.24	80.41	5.83	6.8
13	80.41	75.18	5.23	6.5

* As a result of a 100-basis-point increase in yield.

DV01

- Definition: The price change resulting from a one-basis point change in yield.
- Examples.
- DV01 hedging case.

EXHIBIT 12–1

Computation of the Price Value of a Basis Point

Bond Coupon (%)	Term (years)	Price ($) 10.00%	Price ($) 10.01%	Price Value of a Basis Point ($)*
.00%	5	$ 61.3913	$ 61.3621	$.0292
.00	15	23.1377	23.1047	.0330
.00	30	5.3536	5.3383	.0153
8.00	5	92.2783	92.2415	.0367
8.00	15	84.6275	84.5595	.0681
8.00	30	81.0707	80.9920	.0787
10.00	5	100.0000	99.9614	.0386
10.00	15	100.0000	99.9232	.0768
10.00	30	100.0000	99.9054	.0946
14.00	5	115.4435	115.4011	.0423
14.00	15	130.7449	130.6506	.0943
14.00	30	137.8586	137.7323	.1263

Bond Coupon (%)	Term (years)	Price ($) 10.00%	Price ($) 9.99%	Price Value of a Basis Point ($)
.00%	5	$ 61.3913	$ 61.4206	$.0292
.00	15	23.1377	23.1708	.0331
.00	30	5.3536	5.3689	.0153
8.00	5	92.2783	92.3150	.0367
8.00	15	84.6275	84.6957	.0681
8.00	30	81.0707	81.1496	.0788
10.00	5	100.0000	100.0386	.0386
10.00	15	100.0000	100.0769	.0769
10.00	30	100.0000	100.0947	.0947
14.00	5	115.4435	115.4858	.0424
14.00	15	130.7449	130.8393	.0944
14.00	30	137.8586	137.9851	.1265

*Absolute value per $100 of par value.

MacCauley Duration

- What is the "true" maturity of a bond?
- Graphical intuition.
- Definition:

$$= \frac{\sum_{i=1}^{N} iCF(i)D(i)}{\sum_{i=1}^{N} CF(i)D(i)}$$

$$= \frac{\sum_{i=1}^{N} iCF(i)D(i)}{Price}$$

- Examples.

EXHIBIT 13–1

Calculation of Macaulay Duration and Modified Duration for a 10%, 5-year Bond Selling to Yield 10%

Coupon rate = 10.00%;
Maturity (years) = 5;
Initial yield = 10.00%.

Period (t)	*Cash Flow ($)**	*Present Value of $1 at 5%*	*Present Value of Cash Flow (PVCF; ($))*	*t× PVCF ($)*
1	$ 5.00	0.952380	$ 4.761905	$ 4.761905
2	5.00	0.907029	4.535147	9.070295
3	5.00	0.863837	4.319188	12.957563
4	5.00	0.822702	4.113512	16.454049
5	5.00	0.783526	3.917631	19.588154
6	5.00	0.746215	3.731077	22.386461
7	5.00	0.710681	3.553407	24.873846
8	5.00	0.676839	3.384197	27.073574
9	5.00	0.644608	3.223045	29.007401
10	105.00	0.613913	64.460892	644.608920
Total			$100.000000	$810.782168

*Cash flow per $100 of par value.

$$\text{Macaulay duration (in half years)} = \frac{810.782168}{100.000000} = 8.11.$$

$$\text{Macaulay duration (in years)} = \frac{8.11}{2} = 4.05.$$

$$\text{Modified duration (in years)} = \frac{4.05}{1.0500} = 3.86.$$

EXHIBIT 13–2

Calculation of Macaulay Duration and Modified Duration for a 14%, 5-year Bond Selling to Yield 10%

Coupon rate = 14.00%;
Maturity (years) = 5;
Initial yield = 10.00%.

Period (t)	*Cash Flow ($)**	*Present Value of $1 at 5%*	*Present Value of Cash Flow (PVCF; $)*	*t× PVCF ($)*
1	$ 7.00	0.952380	$ 6.666667	$ 6.666667
2	7.00	0.907029	6.349206	12.698412
3	7.00	0.863837	6.046863	18.140589
4	7.00	0.822702	5.758917	23.035669
5	7.00	0.783526	5.484683	27.423415
6	7.00	0.746215	5.223508	31.341046
7	7.00	0.710681	4.974769	34.823385
8	7.00	0.676839	4.737876	37.903004
9	7.00	0.644608	4.512262	40.610361
10	107.00	0.613913	65.688718	656.887180
Total			$115.443470	$889.529728

*Cash flow per $100 of par value.

$$\begin{matrix}\text{Macaulay duration}\\ \text{(in half years)}\end{matrix} = \frac{889.529728}{115.443470} = 7.71.$$

$$\begin{matrix}\text{Macaulay duration}\\ \text{(in years)}\end{matrix} = \frac{7.71}{2} = 3.85.$$

$$\begin{matrix}\text{Modified duration}\\ \text{(in years)}\end{matrix} = \frac{3.85}{1.0500} = 3.67.$$

Shorter Formula

- The formula

$$= 1 + \frac{1}{y} + \left[100(N-1) - \frac{1}{y}(100 + CN)\right] \frac{PV}{P}$$

- Examples

Price Sensitivity

- Approximate percentage change in prices

$$\frac{\Delta P}{P} = -\frac{1}{1+y} \times MC \times \Delta y \times 100$$

$$\frac{\Delta P}{P} = -MD \times \Delta y \times 100$$

- Dollar price change

$$\Delta P = -MD \times P \times \Delta y$$

$$DV01 = MD \times P \times 0.0001$$

- DV01 is the slope of the price/yield relation.

Immunization Case

- The investment objective.
- The constraints.
- Plan A.
- Plan B.
- Plan C.
- Implications for portfolio management.
- Implications for risk management.

Accumulated Value and Total Return:
5.5-Year, 12.5% Coupon Bond Selling to Yield 12.5%

Investment horizon (years) = 5.5;
Coupon rate = 12.5%;
Maturity (years) = 5.50;
Yield to maturity = 12.5%;
Price = 100.00000;
Par value purchased = $8,820,262;
Purchase price = $8,820,262;
Target accumulated value = $17,183,033.

	Over 5.5 Years				
New Yield (%)	*Coupon ($)*	*Interest on Interest ($)*	*Price of Bond ($)*	*Accumulated Value ($)*	*Total Return (%)*
16.0%	$6,063,930	$3,112,167	$8,820,262	$17,996,360	13.40%
15.5	6,063,930	2,990,716	8,820,262	17,874,908	13.26
14.5	6,063,930	2,753,177	8,820,262	17,637,369	13.00
14.0	6,063,930	2,637,037	8,820,262	17,521,230	12.88
13.5	6,063,930	2,522,618	8,820,262	17,406,810	12.75
13.0%	$6,063,930	$2,409,894	$8,820,262	$17,294,086	12.62%
12.5	6,063,930	2,298,840	8,820,262	17,183,033	12.50
12.0	6,063,930	2,189,433	8,820,262	17,073,625	12.38
11.5	6,063,930	2,081,648	8,820,262	16,965,840	12.25
11.0	6,063,930	1,975,462	8,820,262	16,859,654	12.13
10.5	6,063,930	1,870,852	8,820,262	16,755,044	12.01
10.0	6,063,930	1,767,794	8,820,262	16,651,986	11.89
9.5	6,063,930	1,666,266	8,820,262	16,550,458	11.78
9.0	6,063,930	1,566,246	8,820,262	16,450,438	11.66
8.5	6,063,930	1,476,712	8,820,262	16,351,904	11.54
8.0	6,063,930	1,370,642	8,820,262	16,254,834	11.43
7.5	6,063,930	1,275,014	8,820,262	16,159,206	11.32
7.0	6,063,930	1,180,808	8,820,262	16,065,000	11.20
6.5	6,063,930	1,088,003	8,820,262	15,972,195	11.09
6.0	6,063,930	996,577	8,820,262	15,880,769	10.98
5.5	6,063,930	906,511	8,820,262	15,790,703	10.87
5.0	6,063,930	817,785	8,820,262	15,701,977	10.77

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Accumulated Value and Total Return:
15-Year, 12.5% Coupon Bond Selling to Yield 12.5%

Investment horizon (years) = 5.5;
Coupon rate = 12.5%;
Maturity (years) = 15;
Yield to maturity = 12.5%;
Price = $100.00000;
Par value purchased = $8,820,262;
Purchase price = $8,820,262;
Target accumulated value = $17,183,033.

	Over 5.5 Years				
New Yield (%)	Coupon ($)	Interest on Interest ($)	Price of Bond ($)	Accumulated Value ($)	Total Return (%)
16.0%	$6,063,930	$3,112,167	$7,337,902	$16,514,000	11.73%
15.5	6,063,930	2,990,716	7,526,488	16,581,134	11.81
14.5	6,063,930	2,753,177	7,925,481	16,742,587	12.00
14.0	6,063,930	2,637,037	8,136,542	16,837,510	12.11
13.5	6,063,930	2,522,618	8,355,777	16,942,325	12.23
13.0%	$6,063,930	$2,409,894	$8,583,555	$17,057,379	12.36%
12.5	6,063,930	2,298,840	8,820,262	17,183,033	12.50
12.0	6,063,930	2,189,433	9,066,306	17,319,669	12.65
11.5	6,063,930	2,081,648	9,322,113	17,467,691	12.82
11.0	6,063,930	1,975,462	9,588,131	17,627,523	12.99
10.5	6,063,930	1,870,852	9,864,831	17,799,613	13.18
10.0	6,063,930	1,767,794	10,152,708	17,984,432	13.38
9.5	6,063,930	1,666,266	10,452,281	18,182,477	13.59
9.0	6,063,930	1,566,246	10,764,095	18,394,271	13.82
8.5	6,063,930	1,467,712	11,088,723	18,620,366	14.06
8.0	6,063,930	1,370,642	11,426,770	18,861,342	14.31
7.5	6,063,930	1,275,014	11,778,867	19,117,812	14.57
7.0	6,063,930	1,180,808	12,145,682	19,390,420	14.85
6.5	6,063,930	1,086,003	12,527,914	19,679,847	15.14
6.0	6,063,930	996,577	12,926,301	19,986,808	15.44
5.5	6,063,930	906,511	13,341,617	20,312,058	15.76
5.0	6,063,930	817,785	13,774,677	20,656,393	16.09

EXHIBIT 13–7

Change in Interest on Interest and Price Due to Interest Rate Change:
15-Year, 12.5% Coupon Bond Selling to Yield 12.5%

New Yield (%)	*Change in Interest on Interest ($)*	*Change in Price ($)*	*Total Change in Accumulated Value ($)*
16.0%	$813,327	–$1,482,360	–$669,033
15.5	691,875	–1,293,774	–601,898
14.5	454,336	–894,781	–440,445
14.0	338,197	–683,720	–345,523
13.5	223,778	–464,485	–240,707
13.0	111,054	–236,707	–125,654
12.5	0	0	0
12.0	–109,407	246,044	136,636
11.5	–217,192	501,851	284,659
11.0	–323,378	767,869	444,491
10.5	–427,989	1,044,569	616,581
10.0	–531,046	1,332,446	801,400
9.5	–632,574	1,632,019	999,445
9.0	–732,594	1,943,833	1,211,239
8.5	–831,128	2,268,461	1,437,333
8.0	–928,198	2,606,508	1,678,309
7.5	–1,023,826	2,958,605	1,934,779
7.0	–1,118,032	3,325,420	2,207,388
6.5	–1,210,838	3,707,652	2,496,814
6.0	–1,302,263	4,106,039	2,803,776
5.5	–1,392,329	4,521,355	3,129,026
5.0	–1,481,055	4,954,415	3,473,360

EXHIBIT 13–9

Change in Interest on Interest and Price Due to Interest-Rate Change:
8-Year, 10.125% Coupon Bond Selling to Yield 12.5%

New Yield (%)	*Change in Interest on Interest ($)*	*Change in Price ($)*	*Total Change in Accumulated Value ($)*
16.0%	$746,911	–$676,024	$70,887
15.5	635,377	–583,314	52,063
14.5	417,235	–394,112	23,123
14.0	310,580	–297,579	13,001
13.5	205,504	–199,730	5,774
13.0	101,985	–100,544	1,441
12.5	0	0	0
12.0	–100,473	101,925	1,452
11.5	–199,456	205,254	5,798
11.0	–296,971	310,010	13,038
10.5	–393,039	416,215	23,176
10.0	–487,681	523,894	36,212
9.5	–580,918	633,071	52,153
9.0	–672,770	743,771	71,000
8.5	–763,258	856,019	92,760
8.0	–852,402	969,841	117,439
7.5	–940,221	1,085,263	145,042
7.0	–1,026,734	1,202,311	175,578
6.5	–1,111,961	1,321,014	209,053
6.0	–1,195,921	1,441,399	245,478
5.5	–1,278,632	1,563,494	284,862
5.0	–1,360,112	1,687,328	327,216

Accumulated Value and Total Return:
8-Year, 10.125% Coupon Bond Selling to Yield 12.5%

Investment horizon (years) = 5.5;
Coupon rate = 10.125%;
Maturity (years) = 8;
Yield to maturity = 12.5%;
Price = $88.20262;
Par value purchased = $10,000,000;
Purchase price = $8,820,262;
Target accumulated value = $17,183,033.

	Over 5.5 Years				
New Yield (%)	*Coupon ($)*	*Interest on Interest ($)*	*Price of Bond ($)*	*Accumulated Value ($)*	*Total Return (%)*
16.0%	$5,568,750	$2,858,028	$8,827,141	$17,253,919	12.58%
15.5	5,568,750	2,746,494	8,919,852	17,235,096	12.56
14.5	5,568,750	2,528,352	9,109,054	17,206,156	12.53
14.0	5,568,750	2,421,697	9,205,587	17,196,034	12.51
13.5	5,568,750	2,316,621	9,303,435	17,188,807	12.51
13.0%	$5,568,750	$2,213,102	$9,402,621	$17,184,473	12.50%
12.5	5,568,750	2,111,117	9,503,166	17,183,033	12.50
12.0	5,568,750	2,010,644	9,605,091	17,184,485	12.50
11.5	5,568,750	1,911,661	9,708,420	17,188,831	12.51
11.0	5,568,750	1,814,146	9,813,175	17,196,071	12.51
10.5	5,568,750	1,718,078	9,919,380	17,206,208	12.53
10.0	5,568,750	1,623,436	10,027,059	17,219,245	12.54
9.5	5,568,750	1,530,199	10,136,236	17,235,185	12.56
9.0	5,568,750	1,438,347	10,246,936	17,254,033	12.58
8.5	5,568,750	1,347,859	10,359,184	17,275,793	12.60
8.0	5,568,750	1,258,715	10,473,006	17,300,472	12.63
7.5	5,568,750	1,170,897	10,588,428	17,328,075	12.66
7.0	5,568,750	1,084,383	10,705,477	17,358,610	12.70
6.5	5,568,750	999,156	10,824,180	17,392,086	12.73
6.0	5,568,750	915,197	10,944,565	17,428,511	12.77
5.5	5,568,750	832,486	11,066,660	17,467,895	12.82
5.0	5,568,750	751,005	11,190,494	17,510,248	12.86

DV01 \ Duration

1. $$P = c\sum_{i=1}^{N}\frac{1}{(1+y)^i} + \frac{100}{(1+y)^N}$$

- c is semiannual coupon
- y is semiannual rate
- N number of semiannual periods

$$\frac{\partial P}{\partial y} = \frac{-c}{(1+y)}\sum_{i=1}^{N}\frac{i}{(1+y)^i} - \frac{100N}{(1+y)}\frac{1}{(1+y)^N}$$

$$= \frac{-1}{(1+y)}\left[c\sum_{i=1}^{N}\frac{i}{(1+y)^i} + \frac{100\cdot N}{(1+y)^N}\right]$$

2. $$P = \frac{c}{y}\left[1 - \frac{1}{(1+y)^N}\right] + \frac{100}{(1+y)^N}$$

$$\frac{\partial P}{\partial y} = \frac{-c}{y^2}\,PVA + \frac{cN}{y(1+y)}\,PV - \frac{100N}{(1+y)}\,PV$$

$$= \frac{-c}{y}PVA - \frac{N}{(1+y)}\left[100 - c/y\right]PV$$

$$= -\frac{c}{y}\left[\frac{P - 100PV}{c}\right] - \frac{N}{(1+y)}\left[100 - c/y\right]PV$$

$$= -\frac{P}{y} + \frac{100PV}{y} - \frac{N}{(1+y)}\left[100 - c/y\right]PV$$

- Duration $= -\frac{dP/dy}{P} \times (1+y)$
- DV01 $= \frac{dP}{dy} \times .00005$

$\Rightarrow$

$$\frac{(1+y)}{y} - \frac{100(1+y)}{y}R + N\left[100 - c/y\right]R$$

$$= \frac{(1+y)}{y} + \left[100N - \frac{1}{y}\left(100 + 100y + cN\right)\right]R$$

$$= 1 + \frac{1}{y} + \left[100(N-1) - \frac{1}{y}(100 + cN)\right]R$$

Key Rate Durations

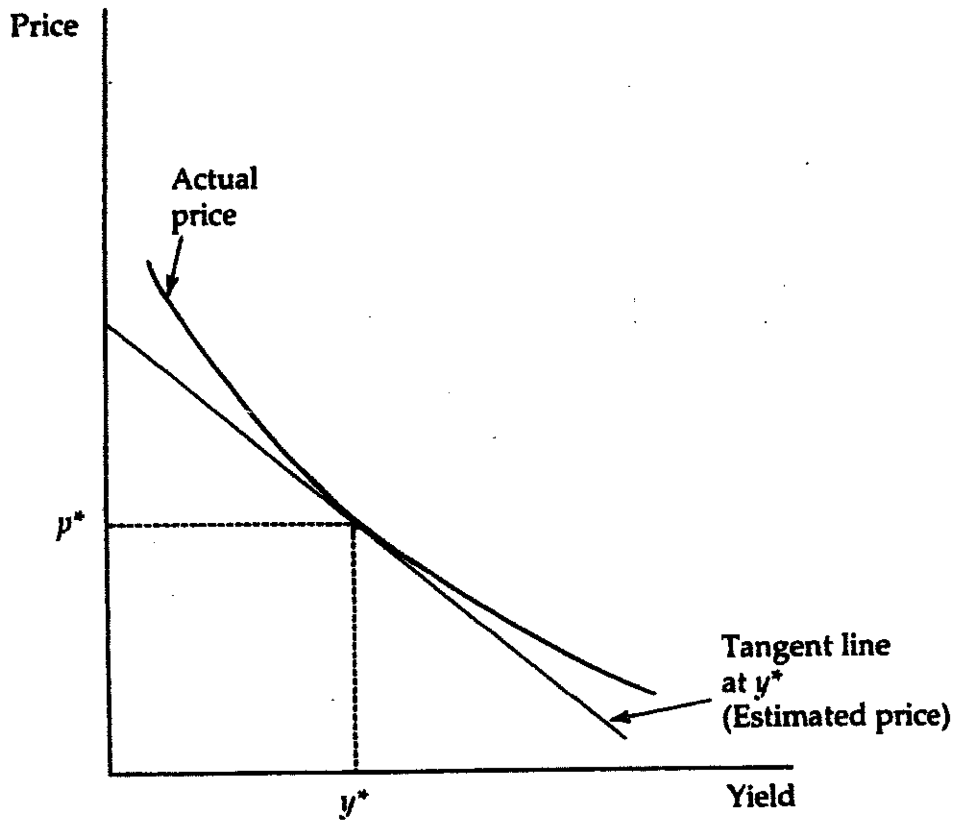
- Duration helps measure the change in the bond price as the yield to measure changes.
- An alternative approach is to measure the change in the bond price as each individual spot rate changes, say from 0.50 to 10 years in steps of 0.50 years.
- These sensitivities are referred to as key rate durations (or KRDs).
- Changing each spot rate along the curve is the same as a parallel shift. Thus, the sum of all KRDs is the same as the effect of a parallel shift (which is the effect of ordinary duration).

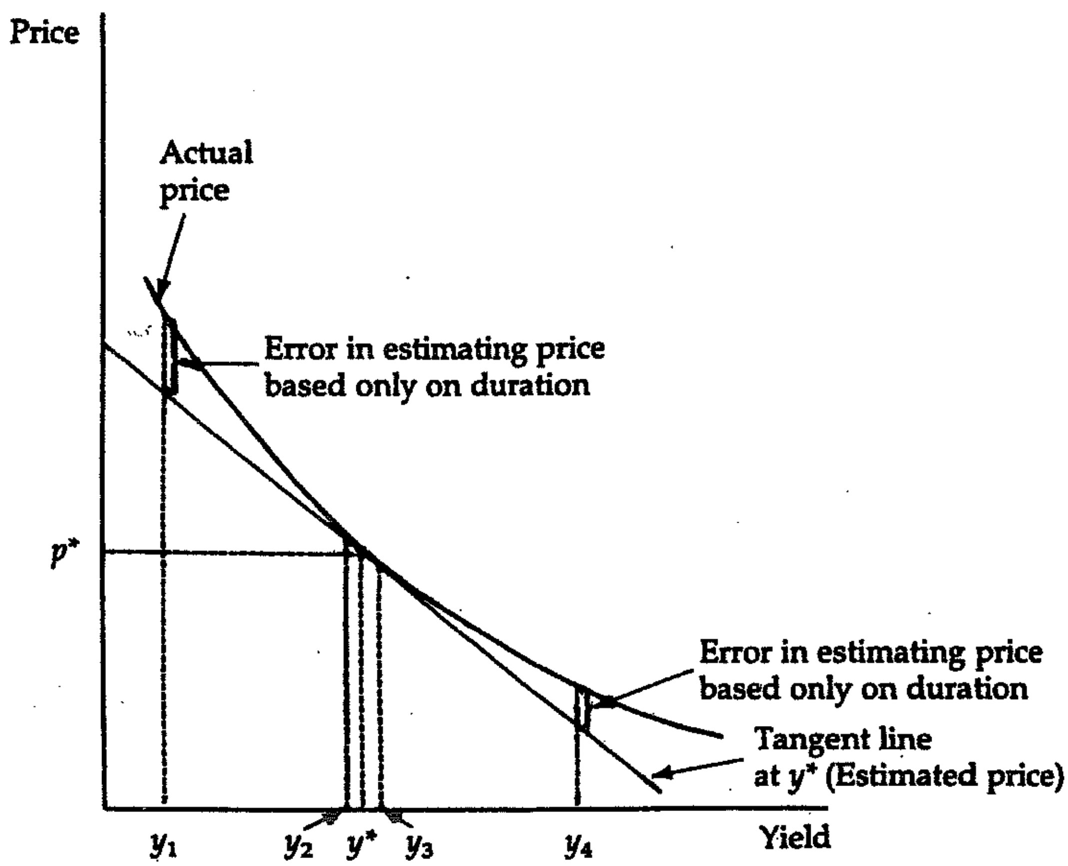
Convexity

- Convexity is the curvature of the bond price.
- Slope of the slope.
- The convexity paradox.

EXHIBIT 14–1

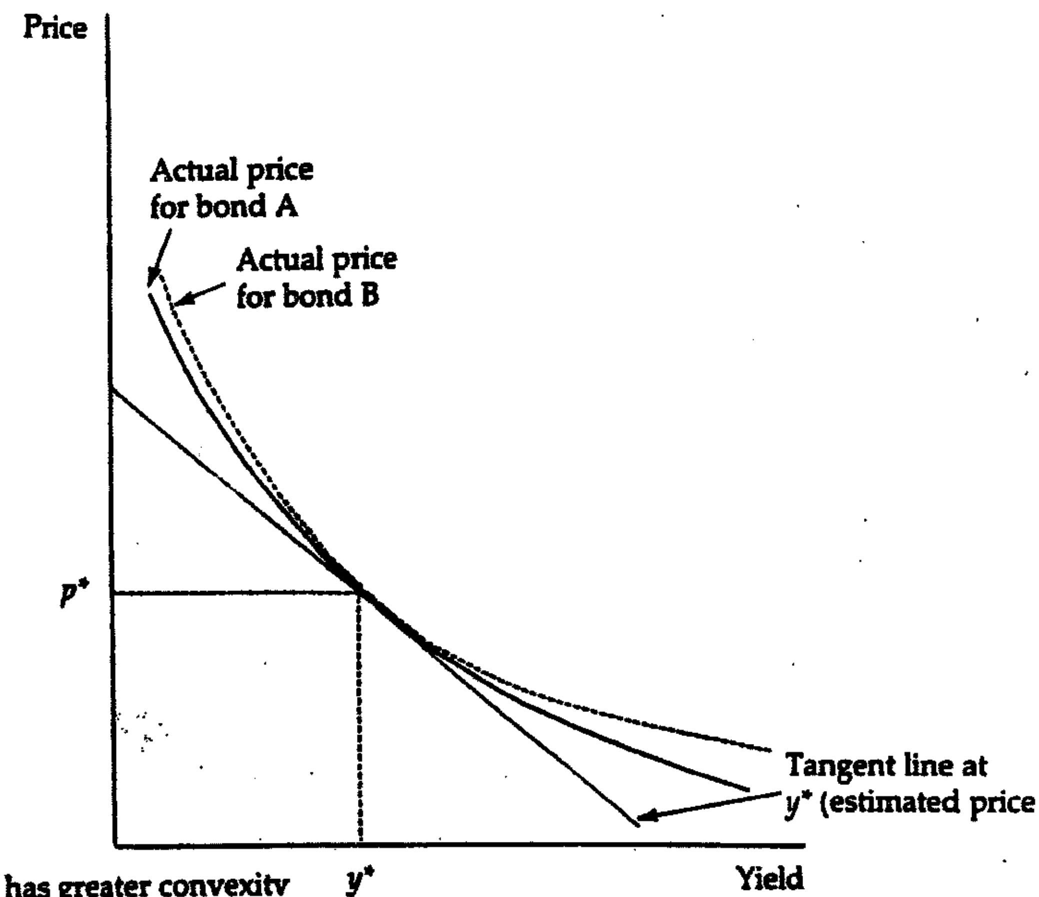
Tangent to Price/Yield Relationship





Price
Actual
price
Error in estimating price
based only on duration
p^*
Error in estimating price
based only on duration
Tangent line
at y^* (Estimated price)
y_1
y_2
y^*
y_3
y_4
Yield

Comparison of Two Bonds with Different Convexities but the Same Duration



ond B has greater convexity than bond A

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Measuring Convexity

- Definition:

$$= \frac{\sum_{i=1}^{N} i \times (i+1) CF(i) D(i)}{(1+y/k)^2 \times k^2 \times Price}$$

  where k is the number of periods in a year.
- Numerical examples.
- Example of zero-coupon bond.

$$= \frac{N \times (N+1)}{(1+y)^2}$$

EXHIBIT 13–4

Percentage Price Change Not Explained by Modified Duration

			Yield Change (in basis points) from 10%					
			1	10	50	100	200	300
			New Yield Level					
Coupon (%)	Term (years)	Modified Duration (years)	10.01%	10.10%	10.50%	11.00%	12.00%	13.00%
.00%	5	4.76	.00%	.00	.03%	.12%	.48%	1.06%
.00	15	14.29	.00	.01	.26	1.01	3.83	8.21
.00	30	28.57	.00	.04	.99	3.77	13.76	28.41
8.00	5	3.98	.00	.00	.02	.10	.38	.83
8.00	15	8.05	.00	.00	.12	.45	1.73	3.74
8.00	30	9.72	.00	.01	.20	.78	2.92	6.15
10.00	5	3.86	.00	.00	.02	.09	.36	.80
10.00	15	7.69	.00	.00	.11	.42	1.62	3.48
10.00	30	9.46	.00	.01	.19	.74	2.76	5.83
14.00	5	3.67	.00	.00	.02	.09	.34	.75
14.00	15	7.22	.00	.00	.10	.38	1.45	3.14
14.00	30	9.17	.00	.01	.18	.69	2.60	5.50

EXHIBIT 14–4

Worksheet for Computation of Convexity for a 5-Year, 8% Coupon Bond Selling to Yield 10%

Coupon rate = 8%;
Term (years) = 5;
Initial yield = 10%;
Price = 92.27826.

Period (t)	*Cash Flow*	*PVCF ($)*	*t(t + 1)*	*PVCF × t(t + 1) ($)*
1	$ 4.00	$3.8095	2	$ 7.6190
2	4.00	3.6281	6	22.7687
3	4.00	3.4554	12	41.4642
4	4.00	3.2908	20	65.8162
5	4.00	3.1341	30	94.0231
6	4.00	2.9849	42	125.3642
7	4.00	2.8427	56	159.1926
8	4.00	2.7074	72	194.9297
9	4.00	2.5784	90	232.0592
10	104.00	63.8470	110	7,023.1676
			Total	$7,965.4046

$$\text{Convexity (in half years)} = \frac{7{,}965.4046}{(1.05)^2\ 92.27826} = 78.2942.$$

$$\text{Convexity (in years)} = \frac{78.2942}{2^2} = 19.58.$$

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EXHIBIT 14–5

Worksheet for Computation of Convexity for a 5-Year, 14% Coupon Bond Selling to Yield 10%

Coupon rate = 14%;
Term (years) = 5;
Initial yield = 10%;
Price = 115.4434.

Period (t)	*Cash Flow ($)*	*PVCF ($)*	*t(t + 1)*	*PVCF × t(t + 1) ($)*
1	$ 7.00	$6.6667	2	$ 13.333
2	7.00	6.3492	6	38.0952
3	7.00	6.0469	12	72.5624
4	7.00	5.7589	20	115.1783
5	7.00	5.4847	30	164.5405
6	7.00	5.2235	42	219.3873
7	7.00	4.9748	56	278.5871
8	7.00	4.7379	72	341.1270
9	7.00	4.5123	90	406.1036
10	107.00	65.6887	110	7,225.7590
			Total	$8,874.6738

$$\text{Convexity (in half years)} = \frac{8{,}874.6738}{(1.05)^2\ 115.4434} = 69.7276.$$

$$\text{Convexity (in years)} = \frac{69.7276}{2^2} = 17.44.$$

Percentage Change in Bond Prices

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- With only duration

$$\frac{\Delta P}{P} = -MD \times \Delta y \times 100$$

- With duration and convexity

$$\frac{\Delta P}{P} = -MD \times \Delta y \times 100 + \frac{1}{2} \times Convexity \times \Delta y^2 \times 100$$

- Sign of the convexity effect.
- Examples of its magnitude.
- Convexity always adds value to the portfolio.

EXHIBIT 14–6

Percentage Price Change Due to Convexity

			Change (in basis points)					
Coupon (%)	Term (years)	Convexity	1	10	50	100	200	300
0%	5	24.94	.00%	.00%	.03%	.12%	.50%	1.12%
0	15	210.88	.00	.01	.26	1.05	4.22	9.49
0	30	829.94	.00	.04	1.04	4.15	16.60	37.35
8	5	19.58	.00	.00	.02	.10	.39	.88
8	15	94.36	.00	.00	.12	.47	1.89	4.25
8	30	167.56	.00	.01	.21	.84	3.35	7.54
10	5	18.74	.00	.00	.02	.09	.37	.84
10	15	87.62	.00	.00	.11	.44	1.75	3.94
10	30	158.70	.00	.01	.20	.79	3.17	7.14
14	5	17.44	.00	.00	.02	.09	.35	.78
14	15	78.90	.00	.00	.10	.39	1.58	3.55
14	30	148.28	.00	.01	.19	.74	2.97	6.67

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EXHIBIT 14-7

Estimated Percentage Price Change Using Duration and Convexity

		Yield Change (in basis points) from 10%					
		1	*10*	*50*	*100*	*200*	*300*
		New Yield Level					
Coupon (%)	*Term (years)*	*10.01%*	*10.10%*	*10.50%*	*11.00%*	*12.00%*	*13.00%*
0%	5	–.05%	–.47%	–2.35%	–4.64%	–9.02%	–13.16%
0	15	–.14	–1.42	–6.88	–13.24	–24.36	–33.38
0	30	–.29	–2.82	–13.25	–24.42	–40.54	–48.36
8	5	–.04	–.40	–1.97	–3.88	–7.57	–11.06
8	15	–.08	–.80	–3.91	–7.58	–14.21	–19.90
8	30	–.10	–.96	–4.65	–8.88	–16.09	–21.62
10	5	–.04	–.39	–1.91	–3.77	–7.35	–10.74
10	15	–.08	–.76	–3.74	–7.25	–13.63	–19.13
10	30	–.09	–.94	–4.53	–8.67	–15.75	–21.24
14	5	–.04	–.37	–1.81	–3.58	–6.99	–10.23
14	15	–.07	–.72	–3.51	–6.83	–12.86	–18.11
14	30	–.09	–.91	–4.40	–8.43	–15.37	–20.84

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EXHIBIT 14–8

Estimated Percentage Price Change Not Explained by Using Both Duration and Convexity

		Yield Change (in basis points) from 10%					
		1	10	50	100	200	300
		New Yield Level					
Coupon (%)	Term (years)	10.01%	10.10%	10.50%	11.00%	12.00%	13.00%
0%	5	.00%	.00%	.00%	.00%	–.02%	–.07%
0	15	.00	.00	.00	–.05	–.39	–1.28
0	30	.00	.00	–.05	–.38	–2.83	–8.94
8	5	.00	.00	.00	.00	–.02	–.05
8	15	.00	.00	.00	–.02	–.15	–.51
8	30	.00	.00	–.01	–.06	–.43	–1.39
10	5	.00	.00	.00	.00	–.01	–.05
10	15	.00	.00	.00	–.01	–.14	–.46
10	30	.00	.00	–.01	–.06	–.42	–1.31
14	5	.00	.00	.00	.00	–.01	–.04
14	15	.00	.00	.00	–.02	–.13	–.41
14	30	.00	.00	–.01	–.05	–.36	–1.17

Convexity Intuition

- The curl benefit of large changes in yields.
- The volatility effect on expected returns.

The Effect of Carry

- The price/yield approximation assumes instantaneous change in yield.
- In reality, need to consider the effect of the passage of time as well.
- General expression

$$\frac{\Delta P}{P} = \text{"}YTM\text{"} - MD \times \Delta y \times 100 + \frac{1}{2} \times Convexity \times \Delta y^2 \times 100$$

- Styles of investment management.
- Prepackaged duration and convexity.

Expected Returns

- Taking expectation of the previous formula

$$E\left[\frac{\Delta P}{P}\right] = \text{"}YTM\text{"} - MD \times E[\Delta y] + \frac{1}{2} \times Convexity \times Var[\Delta y]$$

- Provides a benchmark for performance evaluation.
  - Yields.
  - Duration.
  - Convexity.

The Bullet-Barbell Case

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- Two ways to invest with 6.434 year duration.
- Bullet or single bond.
- Barbell or combination of long and short bonds.
- Decision made on the basis of yield.
- Decision made when taking convexity into account.
- The yield/convexity tradeoff.

EXHIBIT 7–15
Dumbbell-Bullet Analysis

Three bonds used in analysis

Bond	Coupon	Maturity (Years)	Price Plus Accrued	Yield	Dollar Duration	Dollar Convexity
A	8.50	5	100	8.50%	4.00544	19.8164
B	9.50	20	100	9.50	8.88151	124.1702
C	9.25	10	100	9.25	6.43409	55.4506

Bullet: Bond C

Dumbbell: Bonds A and B

Composition of dumbbell: 50.2% of bond A; 49.8% of bond B

Dollar duration of dumbbell =
$.502 \times 4.00544 + .498 \times 8.88151 = 6.434$

Average yield of dumbbell =
$.502 \times 8.50 + .498 \times 9.5 = 8.998$

Strategy: Sell the dumbbell and buy the bullet

Analysis based on average yield

Yield pickup = Yield on bullet − Average yield of dumbbell
= 9.25 − 8.998 = .252, or 25.2 basis points

Analysis based on duration, convexity, and average yield

Dollar convexity of dumbbell =
$.502 \times 19.8164 + .498 \times 124.1702 = 71.7846$

Yield pickup = Yield on bullet − Average yield of dumbbell
= 9.25 − 8.998 = .252, or 25.2 basis points

Convexity giveup = Convexity of dumbbell − Convexity of bullet
= 71.7846 − 55.4506 = 16.334

Analysis based on duration, convexity, and cash-flow yield

Cash-flow yield of dumbbell* =

$$\frac{(8.5 \times .502 \times 4.00544) + (9.5 \times .498 \times 8.88151)}{6.434} = 9.187$$

Yield pickup = Yield on bullet − Cash-flow yield
= 9.25 − 9.187 = .063, or 6.3 basis points

Convexity giveup = Convexity of dumbbell − Convexity of bullet
= 71.7846 − 55.4506 = 16.334

* The calculation shown is actually a dollar-duration-weighted yield, a very close approximation to cash flow yield.

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EXHIBIT 7–16
Dumbbell-Bullet Analysis Based on Horizon Analysis: Bullet Minus Dumbbell

	Parallel Shift		Nonparallel Shift*		Nonparallel Shift†	
Yield	Dollar Return	Horizon Return	Dollar Return	Horizon Return	Dollar Return	Horizon Return
−5.000	−3.59613	−7.19	−5.34489	−10.69	−1.94264	−3.89
−4.750	−3.13782	−6.28	−4.80478	−9.61	−1.56223	−3.12
−4.500	−2.72030	−5.44	−4.30917	−8.62	−1.21906	−2.44
−4.250	−2.34103	−4.68	−3.85538	−7.71	−0.91076	−1.82
−4.000	−1.99764	−4.00	−3.44084	−6.88	−0.63511	−1.27
−3.750	−1.68787	−3.38	−3.06316	−6.13	−0.39002	−0.78
−3.500	−1.40960	−2.82	−2.72005	−5.44	−0.17349	−0.35
−3.250	−1.16081	−2.32	−2.40937	−4.82	0.01635	0.03
−3.000	−0.93962	−1.88	−2.12906	−4.26	0.18126	0.36
−2.750	−0.74421	−1.49	−1.87721	−3.75	0.32293	0.65
−2.500	−0.57291	−1.15	−1.65201	−3.30	0.44291	0.89
−2.250	−0.42410	−0.85	−1.45173	−2.90	0.54271	1.09
−2.000	−0.29628	−0.59	−1.27475	−2.55	0.62373	1.25
−1.750	−0.18802	−0.38	−1.11954	−2.24	0.68729	1.37
−1.500	−0.09798	−0.20	−0.98465	−1.97	0.73464	1.47
−1.250	−0.02489	−0.05	−0.86872	−1.74	0.76695	1.53
−1.000	0.03245	0.06	−0.77046	−1.54	0.78534	1.57
−0.750	0.07518	0.15	−0.68864	−1.38	0.79086	1.58
−0.500	0.10434	0.21	−0.62213	−1.24	0.78448	1.57
−0.250	0.12095	0.24	−0.56984	−1.14	0.76714	1.53
0.000	0.12596	0.25	−0.53074	−1.06	0.73970	1.48
0.250	0.12025	0.24	−0.50387	−1.01	0.70300	1.41
0.500	0.10466	0.21	−0.48834	−0.98	0.65780	1.32
0.750	0.07999	0.16	−0.48327	−0.97	0.60484	1.21
1.000	0.04698	0.09	−0.48786	−0.98	0.54479	1.09
1.250	0.00632	0.01	−0.50136	−1.00	0.47830	0.96
1.500	−0.04132	−0.08	−0.52305	−1.05	0.40598	0.81
1.750	−0.09533	−0.19	−0.55225	−1.10	0.32839	0.66
2.000	−0.15512	−0.31	−0.58834	−1.18	0.24606	0.49
2.250	−0.22015	0.44	−0.63071	−1.26	0.15949	0.32
2.500	−0.28991	−0.58	−0.67881	−1.36	0.06916	0.14
2.750	−0.36391	−0.73	−0.73212	−1.46	−0.02450	−0.05
3.000	−0.44169	−0.88	−0.79012	−1.58	−0.12109	−0.24
3.250	−0.52285	−1.05	−0.85237	−1.70	−0.22020	−0.44
3.500	−0.60698	−1.21	−0.91843	−1.84	−0.32149	−0.64
3.750	−0.69370	−1.39	−0.98788	−1.98	−0.42462	−0.85
4.000	−0.78268	−1.57	−1.06035	−2.12	−0.52927	−1.06
4.250	−0.87358	−1.75	−1.13548	−2.27	−0.63515	−1.27
4.500	−0.96611	−1.93	−1.21292	−2.43	−0.74198	−1.48
4.750	−1.05997	−2.12	−1.29237	−2.58	−0.84952	−1.70
5.000	−1.15491	−2.31	−1.37352	−2.75	−0.95752	−1.92

* Change in yield for bond C. Nonparallel shift as follows:

Yield change bond A = Yield change bond C + 25 basis points

Yield change bond B = Yield change bond C − 25 basis points

† Change in yield for bond C. Nonparallel shift as follows:

Yield change bond A = Yield change bond C − 25 basis points

Yield change bond B = Yield change bond C + 25 basis points

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Salomon Brothers

Antti Ilmanen

Convexity Bias and the Yield Curve

Understanding the Yield Curve: Part 5

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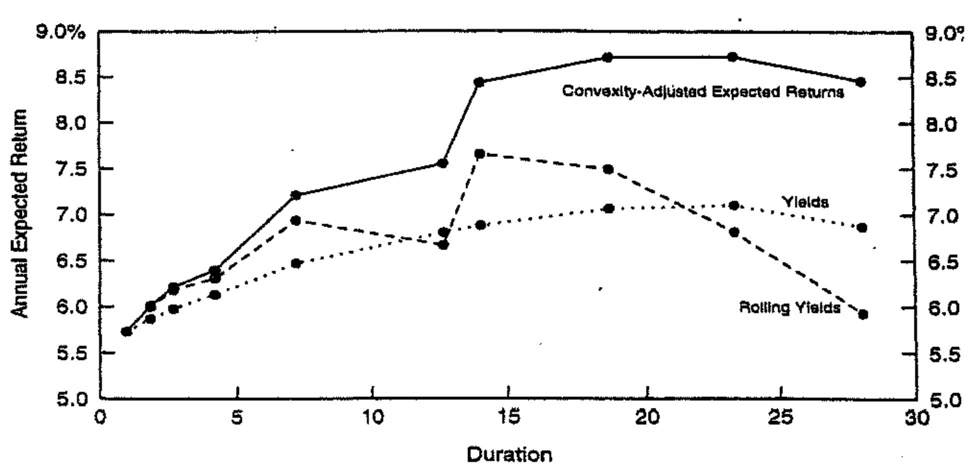
INTRODUCTION

Few fixed-income assets' values are linearly related to interest rate ' ls most bonds' price-yield curves exhibit positive or negative convexit Market participants have long known that positive convexity can enhance bond portfolio's performance. Therefore, convexity differentials across bonds have a significant effect on the yield curve's shape and on bond returns. This report describes these effects and presents empirical evidenc of their importance in the U.S. Treasury market.

For a given level of expected returns, many investors are willing to acce lower yields for more convex bond positions. Long-term bonds are much more convex than short-term bonds because convexity increases very quickly as a function of duration. Because of the value of convexity, long-term bonds can have lower yields than short-term bonds and yet, of the same near-term expected returns. **Thus, the convexity differentials across bonds tend to make the Treasury yield curve inverted or "humped."** We refer to the impact of such convexity differentials on the yield curve shape as the convexity bias. Our historical analysis shows th the bias is small at the front end of the curve, but it can be quite large a the long end.

Convexity bias can also be viewed from another perspective — the valu of convexity as a part of the expected bond return. Widely used relative value tools in the Treasury market, such as yield to maturity and rolling yield, assign no value to convexity. **In this report, we show how yield-based expected return measures can be adjusted to include the value of convexity.** The value of convexity depends crucially on the yie volatility level; the larger the yield shift, the more beneficial positive convexity is. In contrast, the rolling yield is a bond's expected holdi period return given *one* scenario, an unchanged yield curve. Thus, t rolling yield implicitly assumes zero volatility and ignores the value convexity, making it a downward-biased measure of near-term expected bond return. To counteract this problem, we can simply add up the two sources of expected return. **A bond's convexity-adjusted expected retu is equal to the sum of its rolling yield and the value of convexity.** Figure 1 shows that, at long durations, the convexity-adjusted expected returns can be substantially different from the yield-based expected retu (We describe the construction of this figure further in the report.)

Figure 1. Three Alternative Expected Return Curves, as of 1 Sep 95



Note: Each curve is constructed by connecting ten individual bonds' yields, rolling yields or convexity-adju xp returns. The first six points on each curve represent par bonds of 1- to 30-year maturities and the last four point represent zero-coupon bonds of 15- to 30-year maturities, estimated from the Salomon Brothers Treasury Model c

In the section "Basics of Convexity," we define convexity, describe how it varies across bonds and discuss the relation between volatility and the value of convexity. We then examine convexity's impact on the yield curve shape and on expected returns and explain why we advocate the use of convexity-adjusted expected returns in the evaluation of duration-neutral barbell-bullet trades. Finally, we present historical evidence about convexity's impact on realized long-term bond returns and on the performance of a barbell-bullet trade.

While this report focuses on convexity's impact on the yield curve (and on bond returns), we stress that the convexity bias is not the only determinant of the yield curve shape. Positive bond risk premia tend to offset the negative impact of convexity, making the yield curve slope upward, at least at short durations. Moreover, the market's expectations about future rate changes can make the yield curve take any shape. This report is the fifth part of a series titled *Understanding the Yield Curve;* earlier reports in this series describe how the market's rate expectations and the required bond risk premia influence the curve shape.

BASICS OF CONVEXITY[1]

What Is Convexity and How Does It Vary Across Treasury Bonds?

Convexity refers to the curvature (nonlinearity) in a bond's price-yield curve. All noncallable bonds exhibit varying degrees of positive convexity. When a price-yield curve is positively convex, a bond's price rises more for a given yield decline than it falls for a similar yield increase. It is often stated that positive convexity can only improve a bond portfolio's performance. Figure 2, which shows the price-yield curve of a 30-year zero, illustrates in what sense this statement is true: A linear approximation of a positively convex curve always lies below the curve. That is, a duration-based approximation of a bond's price change for a given yield change will always understate the bond price. The error is small for small yield changes but large for large yield changes. We can approximate the true price-yield curve much better by adding a quadratic (convexity) term to the linear approximation. Thus, a bond's percentage price change ($100 * \Delta P/P$) for a given yield change is:[2]

$$100 * \Delta P/P \approx -\text{ duration} * \Delta y + 0.5 * \text{convexity} * (\Delta y)^2 \qquad (1)$$

where duration $= -(100/P) * (dP/dy)$, convexity $= (100/P) * (dP^2/dy^2)$, Δy is the yield change, and yields are expressed in percentage terms.

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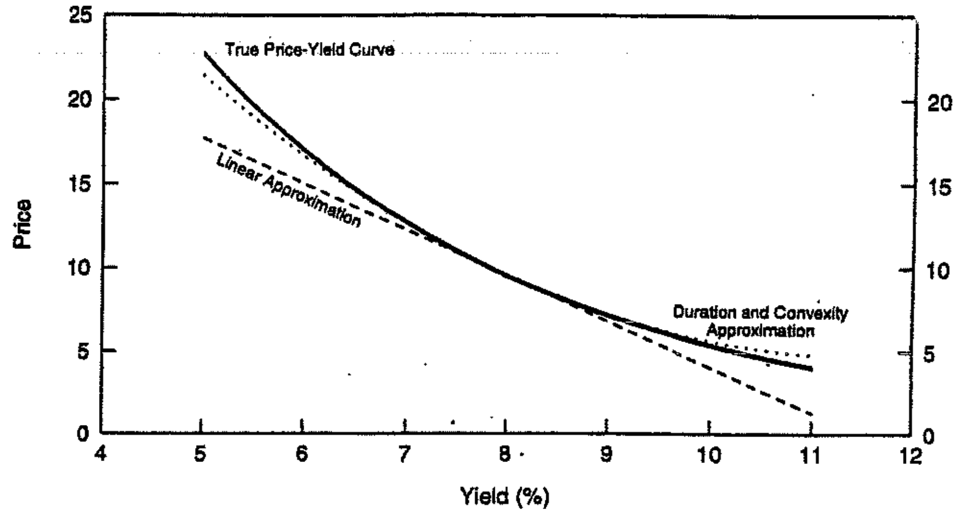
[1] This section provides a brief overview of convexity. Readers who are not familiar with this concept may want to read first a text with a more extensive discussion, such as Klotz (1985) or Tuckman (1995).

[2] Equation (1) is based on a two-term Taylor series expansion of a bond's price as a function of its yield, divided by the price. The Taylor series can be used to approximate the bond price with any desired level of accuracy. A duration-based approximation is based on a one-term Taylor series expansion; it only uses the first derivative of the price function (dP/dy). The two-term Taylor series expansion also uses the second derivative (d2P/dy2) but ignores higher-order terms. In Equation (1), the word "convexity" is used narrowly for the difference between the two-term approximation and the linear approximation, but the word is sometimes used more broadly for the whole difference between the true price-yield curve and the linear approximation. Given the price-yield curves of Treasury bonds and typical yield volatilities in the Treasury market, the two-term approximation in Equation (1) is quite accurate. As an "eyeball test," we note that Figure 2 shows the most nonlinear price-yield curve among noncallable Treasury bonds and yet, the two-term approximation is visually indistinguishable from the true price-yield curve within a 300-basis-point yield range.

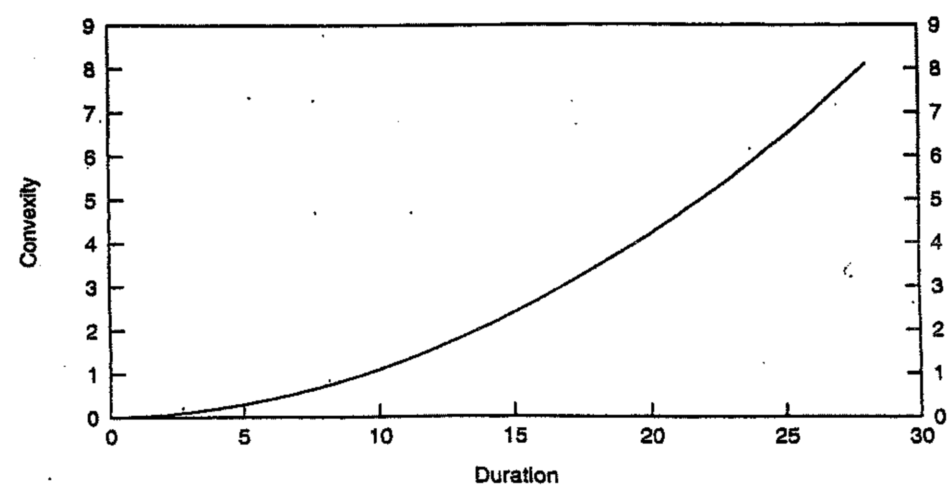
Figure 2. Price-Yield Curve of a 30-Year Zero



In general, **the most important determinants of bond convexity are t[illegible] option features** attached to bonds. Bonds with embedded short options often exhibit negative convexity. The negative convexity arises because borrower's call or prepayment option effectively caps the bond's price appreciation potential when yields decline. However, this report does no analyze bonds with option features. **For noncallable bonds, convexity depends on duration and on the dispersion of cash flows** (see Appen[illegible] A for details).

Figure 3 shows the convexity of zero-coupon bonds as a function of (modified) duration. Convexity not only increases with duration, but it increases at a rising speed. For zeros, a good rule of thumb is that

Figure 3. Convexity of Zeros as a Function of Duration



convexity equals the square of duration (divided by 100).[3] **Convexity also increases with the dispersion of cash flows.** A barbell portfolio of a short-term zero and a long-term zero has more dispersed cash flows than a duration-matched bullet intermediate-term zero. **Of all bonds with the same duration, a zero has the smallest convexity because it has no cash flow dispersion.** As discussed in Appendix A, a coupon bond's or a portfolio's convexity can be viewed as the sum of a duration-matched zero's convexity and the additional convexity caused by cash flow dispersion.

Volatility and the Value of Convexity

Convexity is valuable because of a basic characteristic of positively convex price-yield curves that we alluded to earlier: A given yield decline raises the bond price more than a yield increase of equal magnitude reduces it. Even if investors know nothing about the direction of rates, they can expect gains to be larger than losses because of the nonlinearity of the price-yield curve. Figure 2 illustrated that convexity has little impact on the bond price if the yield shift is small, but a big impact if the yield shift is large. The more convex the bond and the larger the absolute magnitude of the yield shift, the greater the realized value of convexity is. We do not know in advance how large the realized yield shift will be, but we can measure its expected magnitude with a volatility forecast.[4] **If we expect high near-term yield volatility, we expect a high value of convexity.**

The value of convexity is a nebulous concept; it may be hard for investors to see how higher volatility can increase expected returns. We try to make the concept more concrete and intuitive with the following example. Figure 4 compares the expected value of a 30-year zero in a world of certainty and in a world of uncertainty. In a world of certainty, investors know that a bond's yield will remain unchanged at 8%; thus, there is no volatility and convexity has no value. In the second case, we introduce uncertainty in the simplest possible way: The bond's yield either moves to 10% or to 6% immediately, with equal probability. That is, investors do not know in which direction the rates are moving (on average, they expect no change), but they do know that the rates will shift up or down by 200 basis points. Note that the two possible final bond prices (y = 10%, P = \$5.40 and y = 6%, P = \$17.00) are higher than those implied by a linear approximation. The expected bond price is an average of the two possible final prices: E(P) = 0.5 * \$5.40 + 0.5 * \$17.00 = \$11.20. This expected price is higher than the price given no yield change (y = 8%, P = \$9.50). The \$1.70 price difference reflects the expected value of convexity; the bond's expected price is \$1.70 higher if volatility is 200 basis points than if volatility is 0 basis points. Thus, higher volatility enhances the (expected) performance of positively convex positions.[5]

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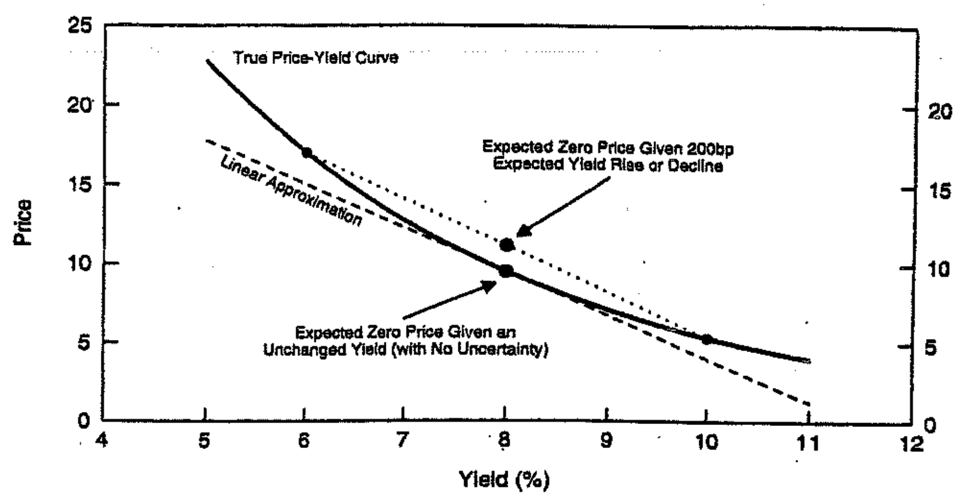
3 The convexity of a given security can be quoted in many ways, depending, in part, on the way that yields are quoted. If yields are *expressed in percent* (200 basis points = 2%), as in Equation (1), the convexity of a long zero with a duration of 15 is quoted as roughly 2.25 (= 15^2 / 100). However, if yields are *expressed in decimals* (200 basis points = 0.02), the same bond's convexity is quoted as 225 (= 15^2). We decided to use the former method of expressing yields and quoting convexity because it is more common in practice. (For careful readers, we point out that in Appendix A of *Overview of Forward Rate Analysis*, titled "Notation and Definitions Used in the Series Understanding the Yield Curve," we expressed yields in decimals and, thus, used the other quotation method for duration and convexity.) Fortunately, the quotation method does not influence convexity's impact on bond returns. The convexity impact of a 200-basis-point yield change on the long zero's return is approximately $0.5 * \text{convexity} * (\Delta y_{percent})^2 = 0.5 * 2.25 * 2^2 = 4.5\%$. We get the same result if the yield change is expressed in percent and convexity is scaled correctly: $0.5 * (100 * \text{convexity}) * (\Delta y_{decimal})^2 = 0.5 * 225 * 0.02^2 = 0.045$ or 4.5%.

4 Equation (1) shows that the impact of convexity on percentage price changes can be approximated by 0:5 * convexity * $(\Delta y)^2$. The expected value of convexity is, therefore, 0.5 * convexity * $E(\Delta y)^2$. Appendix B shows that $E(\Delta y)^2$ is roughly equal to the squared volatility of basis-point yield changes, $(Vol(\Delta y))^2$.

5 This example suggests that scenario analysis is one way to incorporate the value of convexity to expected returns. If we compare the average expected bond price from two rate scenarios (+/-2%) to the expected price given one scenario, the difference will be positive for positively convex bonds (if the scenarios are not biased). In reality, more than two possible rate scenarios exist, but the same intuition holds; the expected value of convexity depends on volatility (also if this is computed from 500 yield curve scenarios instead of two).

Figure 4. Value of Convexity in the Price-Yield Curve of a 30-Year Zero



bp Basis points.

The impact of volatility is very clear in the spread behavior between positively and negatively convex bonds (noncallable government bonds versus callable bonds or mortgage-backed securities). It is more subtle ir the spread behavior within the government bond market where all bonds exhibit positive convexity. When volatility is high, the yield curve tends be more humped and is more likely to be inverted at the long end, widening the spreads between duration-matched barbells and bullets and between duration-matched coupon bonds and zeros.

CONVEXITY, YIELD CURVE AND EXPECTED RETURNS

Convexity Bias: The Impact of Convexity on the Curve Shape

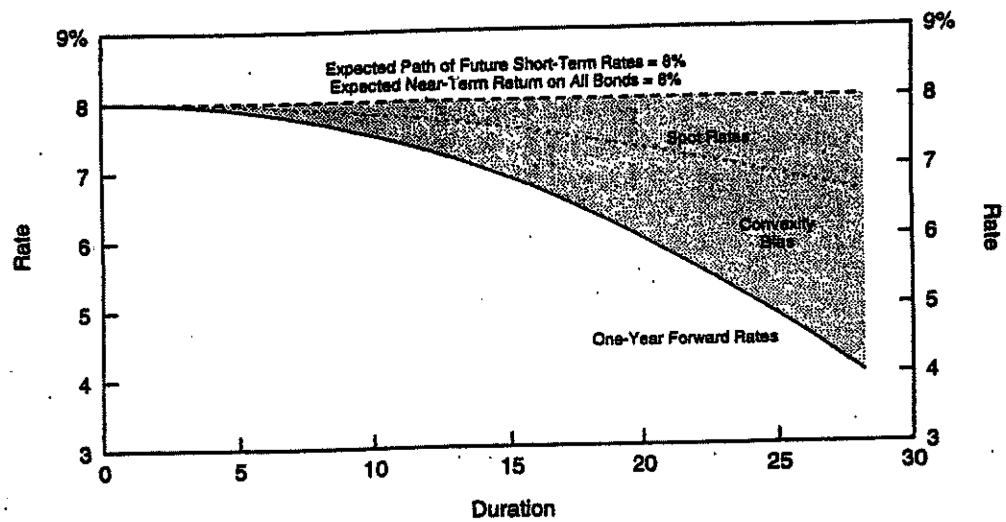
We have demonstrated that positive convexity is a valuable property for fixed-income asset and that different-maturity bonds exhibit large conve: differences. Now we will show that these convexity differences give rise offsetting yield differences across maturities. Investors tend to demand l yield for more convex positions because they have the prospect of enhancing their returns as a result of convexity. In particular, Figure 3 showed that long-term bonds exhibit very high convexity. Because of th high convexity, these bonds can offer lower yields than a short-term boı and still offer the same near-term expected returns.

We isolate the impact of convexity on the yield curve shape, or the convexity bias[6], by presenting a hypothetical situation where the other influences on the curve shape are neutral. Specifically, we assume that a bonds have the same expected return (8%) and that the market expects t short-term rates to remain at the current (8%) level, and we examine the behavior of the spot curve and the curve of one-year forward rates. Witl no bond risk premia and no expected rate changes, one might expect the curves to be horizontal at 8%. Instead, Figure 5 shows that **they slope**

[6] Our use of the term "convexity bias" is slightly different from its use in a recent article "A Question of Burghardt and Hoskins, *Risk*, March 1995. In that article, convexity bias refers to the difference between t price and the futures price in the Eurodollar market. This bias also reflects varying degrees of curvature in the price-yield curves of different fixed-income assets; the mark-to-market system makes the future's price-yield curve linear, while the forward price is a convex function of yield.

down at an increasing pace because lower yields are needed to offset the convexity advantage of long-duration bonds (and thus to equate the near-term expected returns across bonds). Note the symmetry between the curve shapes in Figures 3 and 5.

Figure 5. Pure Impact of Convexity on the Yield Curve Shape



Note: Convexity bias is the difference between the curve of one-year forward rates and the expected return curve. Formally, Convexity bias ≈ -0.5 * convexity * $(\mathrm{Vol}(\Delta y))^2$, adjusted for the fact that the bond price changes do not occur instantaneously but at the end of a one-year horizon. The assumed yield volatility is 100 basis points per annum for all bonds; that is $\mathrm{Vol}(\Delta y) = 1\%$.

Where did the numbers in Figure 5 come from? Unlike the real world, where the spot rates are the easiest to observe, in this example, we take the expected returns as given and work our way back to forward rates and then to spot rates. Given our assumption that the market has no directional views about the yield curve, each zero earns the near-term expected return from the rolling yield[7] and from convexity:[8]

Convexity-adjusted expected return = rolling yield + value of convexity, (2)

where value of convexity ≈ 0.5 * convexity * $(\mathrm{Vol}(\Delta y))^2$.

Using our assumption that all bonds have convexity-adjusted expected return of 8% and using some volatility assumption (which determines the value of convexity), we can back out the rolling yields for various-maturity zeros from Equation (2). Our volatility assumption of 100 basis points means roughly that we expect all rates to move 100 basis points (up or down) from their current level over the next year. For example, if the convexity of a long zero is 2.25 (see footnote 3), the value of convexity is

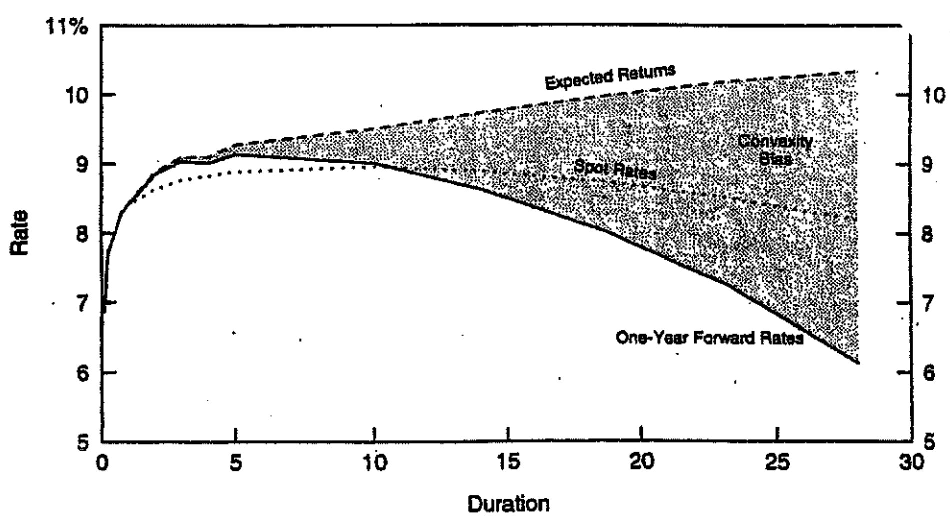
[7] The rolling yield is a bond's holding-period return given an unchanged yield curve. If a downward-sloping yield curve remains unchanged, long-term bonds earn their initial yields and negative rolldown returns (because they "roll up the curve" as their maturities shorten). An n-year zero-coupon bond's rolling yield over the next year is equal to the one-year forward rate between n-1 and n. For details, see *Market's Rate Expectations and Forward Rates*, Salomon Brothers Inc, June 1995.

[8] Here is an intuitive "proof." A bond's expected holding-period return can be split into *a part that reflects an unchanged yield curve* (the rolling yield) and *a part that reflects expected changes in the yield curve*. The second part can be approximated by taking expectations of Equation (1). If we expect the yield curve to remain unchanged, as a base case, but allow for positive volatility, the duration impact will be zero, leaving only the value of convexity. (Some modifications are needed because Equation (1) holds instantaneously for constant-maturity rates, while the actual bond price changes occur over a horizon.)

approximately 0.5 * 2.25 * 1^2 = 1.125%. The zero's rolling yield is 6.875% but its annualized near-term expected return is 8%, by assumpti For coupon bonds, which have smaller convexities, the value of cc is much smaller. The final step in constructing Figure 5 is to compu spot curve from the curve of one-year forward rates (the rolling yield curve).

Convexity bias is simply the inverse of the value of convexity, or -0.5 * convexity * $(\text{Vol}(\Delta y))^2$. Figure 5 shows that the convexity bias, by itsel tends to make the yield curve inverted, especially at long durations. However, actual yield curves rarely invert as they do in this hypothetica example, in which we assumed, in particular, that all bonds across the curve have the same near-term expected return and the same basis-poin yield volatility. We now relax each of these two assumptions, one at a time. First, convexity is not the only influence on the curve shape. The typical historical yield curve shape is upward sloping, probably reflectir positive bond risk premia (the fact that investors require higher expecte returns for long-term bonds than for short-term bonds). At the front enc the curve, the convexity bias is so small that it does not offset the imp of positive bond risk premia. At the long end, the convexity bias can b large that the yield curve becomes inverted in spite of positive risk pre Figure 6 shows that **in the presence of positive risk premia, convexit bias tends to make the yield curve humped rather than inverted.** In figure, we use historical average returns of various maturity subsectors proxy for expected returns.

Figure 6. Impact of Convexity with Positive Bond Risk Premia

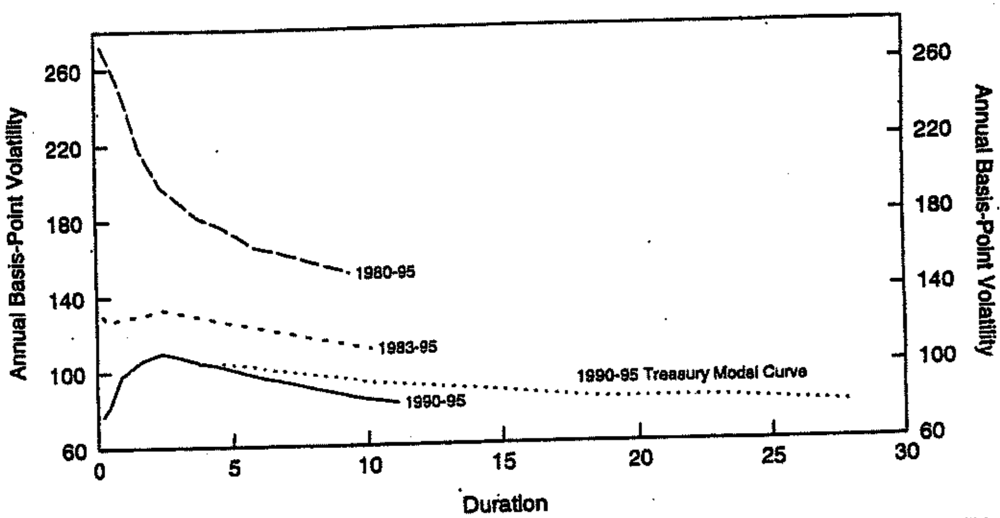


Note: The figure is constructed in the same way as Figure 5 except that all bonds' expected returns are not 8% are based on the (arithmetic) mean realized returns of Treasury bond maturity-subsectors between 1970 and 199 curve is extrapolated between ten- and 30-year durations because of a lack of data. The curve of one-year forwa rates is computed by adding the convexity bias from Figure 5 to the expected return curve. The spot curve is computed from the curve of one-year forward rates.

As explained earlier, the value of convexity increases with yield volatil **Thus far we have assumed that yield volatility is equally high acros the curve. Figure 7 shows that historically, the term structure of volatility has often been inverted** — long-term rates have been l volatile than short-term rates. Therefore, the value of convexity dc o increase quite as a square of duration even though convexity itself does

However, **the value of convexity does increase quite quickly with duration even when the volatility term structure is taken into account;** its inversion only dampens the rate of increase (see Figure 8).

Figure 7. Historical Term Structure of (Basis-Point) Yield Volatility

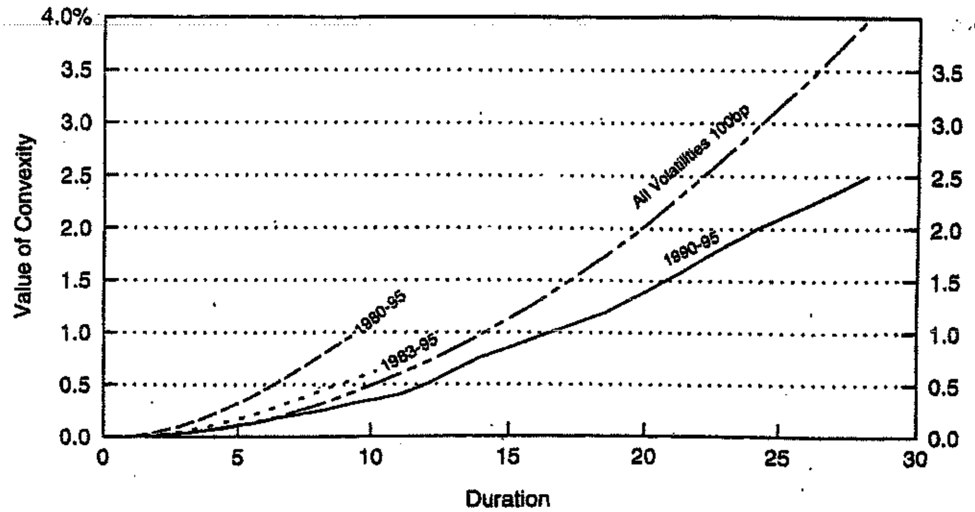


Note: Historical yield volatility is the annualized standard deviation of weekly basis-point yield changes. Yield volatilities are computed for several on-the-run Treasury bill and bond series (and plotted on their average durations) over three sample periods. In addition, yield volatilities are computed for the five-, ten-, 15-, 20-, 25-, and 30-year points on the Salomon Brothers Treasury Model spot curve over the January 1990-August 1995 period. Yields are compounded annually and the yield volatilities are expressed in basis points.

The levels and shapes of the volatility term structures are very different in Figure 7, depending on the sample period. In the 1980s — and especially at the beginning of the decade — yield volatilities were very high and the term structure of volatility was inverted. In the 1990s, volatilities have been lower and the term structure of volatility has been flat or humped. It is difficult to choose the appropriate sample period for computing the yield volatility and Figure 8 shows that this choice will have a significant impact on the estimated value of convexity. Our view is that the relevant choice is between the 1983-95 and the 1990-95 sample periods because we do not expect to see again the volatility levels experienced in 1979-82 — at least not without clear warning signs. This period coincided with a different monetary policy regime in which the Federal Reserve targeted the money supply and tolerated much higher yield volatility than after October 1982.[9]

[9] Whenever the period 1979-82 is included in a historical sample, the estimated volatilities will be much higher, the term structures of volatility will be more inverted and basis-point yield volatilities will appear to be more "level-dependent" than if the sample period begins after 1982. In many countries outside the United States, the inversion and the level-dependency also have been apparent features of the volatility structure recently. These features seem to become stronger if the central bank subordinates the short-term rates to be tools for some other monetary policy goal, such as money supply (United States 1979-82) or currency stability (for example, countries in the European Monetary System). Figure 7 also illustrates interesting findings about the term structure of volatility in the 1990s. The shape is humped, not inverted, because the intermediate-term yields have been more volatile than either the short-term or long-term yields. Moreover, yield volatility is not just a function of duration; it also depends on a bond's cash flow distribution. *For a given duration*, zeros have exhibited greater yield volatility than coupon bonds. This pattern probably reflects the coupon bonds' diversification benefits (unlike zeros, these bonds have cash flows in many parts of the yield curve that are imperfectly correlated) as well as the humped shape of the volatility structure.

Figure 8. Value of Convexity Given Various Volatility Structures



bp Basis points.
Note: Value of Convexity = 0.5 * convexity * $(Vol(\Delta y))^2$, expressed in percent per annum, adjusted for the fact th bond price changes do not occur instantaneously but at the end of a one-year horizon. Yield volatilities are based Figure 7. Because we only have spot rate data for the 1990s, we cannot compute long zeros' value of convexity f two longer samples.

Instead of sample-specific historical volatilities, we could use implied volatilities from current option prices (based on the cap-curve, options o various futures contracts, OTC options on individual on-the-run bonds) t compute the (expected) value of convexity. The main reason that we not done this is that such implied volatilities are not available for al maturities. In addition, it is not clear from empirical evidence that impli volatilities predict future yield volatilities any better than historical volatilities do.

In Appendix B, we describe the various volatility measures used in this report and discuss the relations between them. In particular, we emphasi that the option prices are typically quoted in relative yield volatilities ($Vol(\Delta y/y)$) rather than in the basis-point volatilities ($Vol(\Delta y)$) that we u For example, a 13% implied volatility quote has to be multiplied by the yield level, say 7%, to get the basis-point volatility (91 basis points = 0.91% = 13% * 7%).

The Impact of Convexity on Expected Bond Returns

Figure 8 shows that positive convexity can be quite valuable, especially a high-volatility environment. However, yield-based measures of expecte bond return assign no value to convexity. For example, the rolling yield a bond's holding-period return given *one* scenario (an unchanged yield curve), essentially assuming no rate uncertainty. **Because volatility can only be positive, the rolling yield is a downward-biased measure of expected return** for bonds with positive convexity.[10] Fortunately, it is possible to add the impact of rate uncertainty (the expected value of convexity) to rolling yields. Equation (2) showed that if the base case expectation is an unchanged yield curve, **a bond's near-term expected**

[10] This point is most easily seen by considering a horizontal yield curve. All bonds have same yields and rolling yields, but their expected returns are not the same. Long-term bonds are more convex than short-term bonds; thus, have higher near-term expected returns.

return is simply the sum of the rolling yield and the value of convexity.[11] This relation holds approximately for coupon bonds as well as for zeros.

In Figure 9, we calculate three expected return measures (yield, rolling yield, convexity-adjusted expected return) and the value of convexity on September 1, 1995 for six Treasury par bonds and four long-duration zeros (estimated from the Salomon Brothers Treasury Model curve which represents off-the-run bonds). In addition, we describe two barbell positions that can be compared with duration-matched bullets. Figure 1 showed graphically the three alternative expected return curves as a function of duration.

Figure 9. Expected One-Year Returns on Various Bonds as of 1 Sep 95

	(Modified) Duration	Convexity	Historical Vol(Δy)	(Annual) Yield	Rolling Yield	Value of Convexity	Conv.-Adjusted Expected Return
Par Bonds							
1 Year	0.95	0.02	0.98%	5.73%	5.73%	0.00%	5.73%
2 Year	1.84	0.05	1.06	5.87	6.00	0.01	6.01
3 Year	2.67	0.10	1.10	5.98	6.18	0.03	6.21
5 Year	4.20	0.23	1.04	6.13	6.31	0.09	6.40
10 Year	7.20	0.67	0.95	6.47	6.94	0.27	7.21
30 Year	12.66	2.57	0.82	6.81	6.67	0.88	7.55
Long Zeros							
15 Year	14.03	2.10	0.89%	6.88%	7.66%	0.78%	8.44%
20 Year	18.68	3.66	0.83	7.07	7.49	1.22	8.71
25 Year	23.34	5.67	0.83	7.11	6.81	1.91	8.72
30 Year	28.07	8.14	0.79	6.88	5.93	2.53	8.46
Par Barbells							
1 Year and 10 Year	4.19	0.36	0.95%	6.11%	6.35%	0.14%	6.50%
1 Year and 30 Year	7.18	1.38	0.82	6.30	6.23	0.47	6.70

Note: Convexity-adjusted expected return = rolling yield + value of convexity, where rolling yield = yield + rolldown return and where value of convexity = .5 * convexity * $(\text{Vol}(\Delta y))^2$, adjusted for the fact that the bond price changes do not occur instantaneously but at the end of the one-year horizon. Historical volatilities are the annualized standard deviations of weekly basis-point yield changes between January 1990 and August 1995. All measures use annually compounded yields and are expressed in percentage terms. The first (second) barbell is a combination of the one-year par bond and the ten-year (30-year) par bond, duration-matched to the end-of-horizon duration of the five-year (ten-year) par bond; thus, the current durations are not exactly matched. All other measures for the barbells are market value-weighted averages, but the barbell's yield volatility is market value * duration-weighted.

We use maturity-specific historical volatilities from the 1990-95 period to proxy for expected volatility, and we use a one-year horizon. These choices give one illustration of the ideas developed in this report; we stress that it is possible to use other volatility measures or other horizon. In particular, Figure 7 shows that the volatility estimates would be much higher if we extended our sample period to the 1980s. (The par bonds' yield volatilities are similar to those of the on-the-run bonds in Figure 7.) For a given yield curve, these higher volatility estimates could more than double the estimated value of convexity and, thus, increase the convexity-adjusted expected returns. Using a one-year horizon makes the notation easier because the value of convexity is expressed in annualized terms as are yields and volatilities. If we used a three-month horizon, all three expected return measures and the value of convexity would be roughly one fourth of the numbers in Figure 9. For example, if a 30-year par bond's convexity is 2.57 and the annual volatility is 82 basis points, the quarterly volatility is approximately 41 basis points ($82/\sqrt{4}$), and the quarterly value of convexity is $0.5 * 2.57 * 0.41^2 = 0.22\%$ ($\approx 0.88\%/4$), or 22 basis points.

[11] Our empirical analysis in *Market's Rate Expectations and Forward Rates* indicates that it is reasonable to take today's yield curve as the base forecast for the future yield curve. Therefore, the rolling yield can proxy for a bond's near-term expected return (assuming zero volatility). Other hypotheses about the yield curve behavior would lead to other expected return proxies than the rolling yield, but the value of convexity could be added to any such proxy. For example, if the implied forward curve were the best forecast for the future yield curve, the near-term expected return of each bond would be the sum of the near-term riskless rate and the (bond-specific) value of convexity. Or, if investors have strong subjective expectations about curve-reshaping, the impact of such expectations can be easily added to the convexity-adjusted expected returns — as a third term on the right-hand side of Equation (2).

Figures 1 and 9 show that **the convexity adjustment has little impact a short durations because short-term bonds exhibit little convexity**. Eve for the longest coupon bond, the annual impact is 88 basis points. **In contrast, for the longest zeros, the value of convexity is very larg** as an absolute number (253 basis points) and as a proportion of their expected return (30% = 2.53/8.46). More generally, the value of convexi can partly explain the rolling yield curve's typical concave (humped) shape, but even the convexity-adjusted expected return curve inverts afte 25 years. The longest-maturity zeros appear to have genuinely low expected returns, perhaps reflecting their liquidity advantage and financi advantage.

One advantage of this analysis is that it gives an **improved view of the overall reward-risk trade-off in the government bond market.** Until 1970s, fixed-income investors evaluated this reward - risk trade-off by plotting bond yields on their maturities. Eventually investors learned tha the rolling yield measures near-term expected return better than yield an that duration measures risk better than maturity.[12] In the mid-1980s, investors became familiar with the concept of convexity (see Literature Guide), although few have incorporated it formally into their expected return measures. However, convexity-adjusted expected returns are even better expected return measures than rolling yields — and the adjustmen reasonably simple. To move all the way to mean-variance analysis, as advocated by the modern portfolio theory, we should adjust bond durati by their yield volatilities; then, Figure 1 would plot bonds' expected returns on their return volatilities. Of course, convexity-adjusted expecte returns are not perfect; for example, if investors can predict yield curve reshapings consistently, they can construct even better expected return measures.

In addition, our analysis helps investors to interpret varying yield curve shapes, and more directly, it gives them tools to evaluate relative value trades between duration-matched barbells and bullets and between duration-matched coupon bonds and zeros. This is the topic of the next subsection.

Applications to Barbell-Bullet Analysis

A barbell-bullet trade involves the sale of an intermediate bullet bond a the purchase of a barbell portfolio of a short-term bond and a long-term bond. Often the trade is weighted so that it is cash-neutral and duration-neutral; that is, one unit of the intermediate bond is sold, a duration-weighted amount of the long bond is bought and the remainin proceeds from the sale are put into "cash" (a short-term bond that matu at the end of horizon). For simplicity, we will only study such barbells this report. In Appendix A, we explain that a barbell portfolio has a convexity advantage over a duration-matched bullet because the barbell duration varies more (inversely) with the yield level. Figure 3 provides another illustration of the convexity difference between barbells and bullets. If we draw a straight line between any two points on the zeros' convexity-duration curve, each point on this line corresponds to a barb portfolio (with varying weights of the long-term and the short-term zer The convexity of this barbell is the market-value-weighted average of t component bonds' convexities. Because the connecting straight line alw

[12] *Total Return Management*, Martin L. Leibowitz, Salomon Brothers Inc, 1979, and *Understanding Duration a Volatility*, Salomon Brothers Inc, September 1985, among other papers, made the concepts of rolling yield and d widely known among bond investors.

lies above the zeros' convexity-duration curve, the barbell's convexity is always higher than that of a duration-matched bullet. Furthermore, the maximum convexity pick-up for any duration occurs when we connect the shortest and longest zeros.

In a similar way, we can connect any two points in Figure 5 and find that the rolling yield of any barbell is below the rolling yield of a duration-matched bullet. More generally, the rolling yield curve (as well as the yield curve) almost always has a concave shape as a function of duration; that is, the curve increases at a decreasing rate or decreases at an increasing rate. Therefore, a rolling yield disadvantage tends to offset the convexity advantage of a barbell-bullet trade. **If an investor wants to evaluate the relative cheapness of a barbell-bullet trade, he needs to compare two numbers, the rolling yield give-up and the convexity pick-up. The advantage of the convexity-adjusted expected return is that it provides a single number to measure the attractiveness of these trades.** For example, the ones-30s barbell in Figure 9 has a 71-basis-point rolling yield give-up relative to the ten-year bullet (= 6.23%-6.94%), but how does this give-up compare with the convexity pick-up (1.38 versus 0.67)? The numbers in the last column show that the barbell still has a 51-basis-point give-up (= 6.70%-7.21%) when measured in terms of convexity-adjusted expected returns and given our volatility forecasts. Incidentally, the shorter barbell in Figure 9 even picks up rolling yield over the duration-matched five-year bullet; this exceptional situation reflects the convex shape in parts of the rolling yield curve in Figure 1.

The performance of a duration-neutral barbell-bullet trade depends on curve reshaping, on parallel curve shifts and on the initial yields: (1) The trade profits from curve flattening and loses from curve steepening (between the two longer bonds); (2) the trade is constructed to be neutral to small parallel curve shifts, but the barbell profits from large shifts in either direction because of its convexity advantage; and (3) the initial rolling yield give-up is greater the more curved (concave) the yield curve is. Such a shape may be caused by the market's expectations of curve flattening or of high volatility, either of which would generate capital gains for the trade in the future.

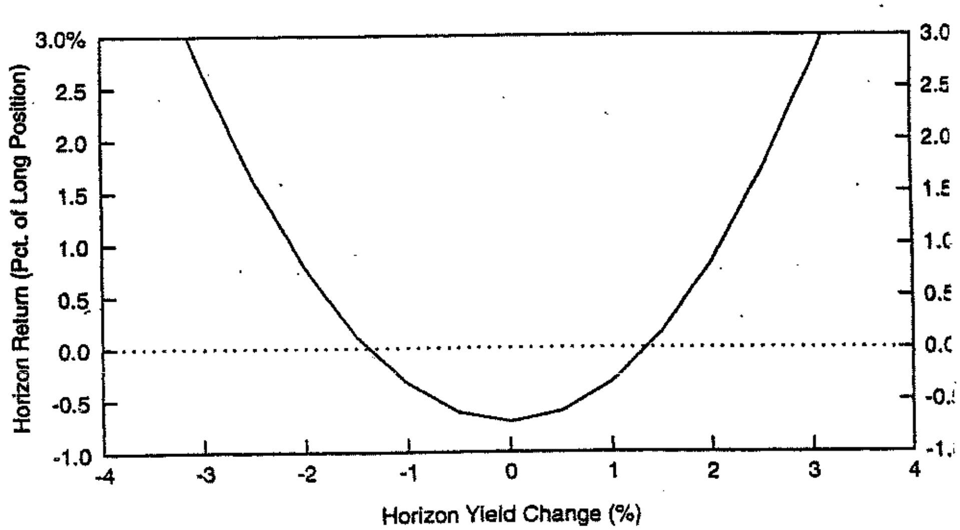
Typical barbell-bullet trades are more curve flattening trades than convexity trades. The following break-even analysis illustrates this point. Consider the long barbell-bullet trade in Figure 9. It consists of selling a ten-year par bond (rolling yield 6.94%) and buying a barbell of the 30-year par bond (rolling yield 6.67%) and the one-year bond (rolling yield 5.73%), with a one-year investment horizon. Thus, at the end of horizon, the components will be a nine-year bond, a 29-year bond and cash. The constraints that the trade is duration-neutral and cash-neutral require weights 0.53 and 0.47 for the long bond and the short bond. Given the duration-neutral weighting of the barbell, the rolling yield give-up is 71 basis points (= 0.53 * 6.67% + 0.47 * 5.73% - 6.94%). We isolate the flattening and convexity effects in the trade by asking two questions:

- How much would the yield spread between tens and 30s (or more exactly, between nines and 29s at the end of horizon) have to narrow to offset this give-up, if no parallel shifts occur?
- How large must the parallel shifts be to make the convexity advantage offset this give-up, if no curve reshaping occurs?

A little math shows that the necessary break-even changes are an 11-basis-point spread narrowing (curve flattening) and a 138-basis-point parallel shift. Historical experience suggests that the former event i[illegible] plausible than the latter: Over the past 15 years, the tens-30s spread narrowed by at least 11 basis points in a year 30% of the time, while th ten-year yield level shifted by more than 138 basis points in a year only 17% of the time. Thus, **it is more likely that a given rolling yield disadvantage is offset via curve flattening than via the barbell's convexity advantage.** However, the relative roles of curve-reshaping an convexity vary across different barbell-bullet trades. The reshaping effec are clearly more important at shorter durations (between most coupon bonds), while convexity can be more important at longer durations (between very long zeros). It follows that the time-variation in the rollir yield spread between barbell and bullet coupon bonds— or in the yield curve curvature below the ten-year duration — depends more on the market's changing expectations about future curve flattening/steepening than on its changing volatility expectations.

The convexity aspect of the previous example illustrates **the similarity between a barbell-bullet trade and a purchase of a long option stra** (a purchase of a call and a put with the same strike price and exercise date). Figure 10 shows the almost U-shaped pattern that is familiar fror option analysis. The rolling-yield disadvantage corresponds to the long and put positions' initial cost (premium), which large market movemen either direction would offset. The trade would only be profitable if the yield level increased or declined by at least 138 basis points, assuming

Figure 10. The Payoff Profile of a Barbell-Bullet Trade, Assuming Parallel Yield Shifts



parallel yield shifts. If the yield curve does not move at all from the initial level, the maximum loss (71 basis points) occurs. Of course, Figure 10 ignores the substantial curve-reshaping risk in this trade.[13]

Another way to measure the cheapness of the barbell-bullet trade is to compute its implied yield volatility and compare it with the implied volatility in option markets. We can back out an implied volatility number for each barbell-bullet trade based on the observable rolling yield spread and convexity difference, if we assume that the duration-matched barbell and bullet earn the same expected returns and that the rolling yield spread reflects only the value of convexity — and no curve-flattening expectations.[14] In that case, high curvature (concavity) in the yield curve and high bullet-barbell rolling yield spreads indicate high implied volatility. In contrast, if the yield curve is a convex function of duration, barbells pick up yield *and* convexity and the implied volatility is negative — typically an indication of the market's strong expectations about near-term curve steepening.

HISTORICAL EVIDENCE ABOUT CONVEXITY AND BOND RETURNS

The intuition behind convexity-adjusted expected returns is that if investors care about expected return rather than yield, they will rationally accept lower yields and rolling yields from more convex bonds. In this sense, convexity is priced: It influences bond yields. However, **a more subtle question is whether convexity also influences expected returns** that are not directly observable. It is possible that the rolling yield disadvantage exactly offsets convexity advantage so that two bond positions with the same duration but different convexities have the same near-term expected return. It is also possible that convexity is such a desirable characteristic — because of the insurance-type payoff pattern — that the market (investors in the aggregate) accepts lower expected returns for more convex bonds. Finally, it is possible that current-income seekers dominate the marketplace, leading to a price premium (lower expected returns) for higher-yielding, less convex bonds. The jury is still out on this question. The evidence from historical bond returns that we present below suggests that more convex positions earn somewhat lower returns in the long run than less convex positions.

13 The barbell-bullet trade that we analyze over a static one-year horizon is comparable to a strategy of buying and holding a straddle. Readers familiar with options know that the profitability of this strategy depends solely on the starting and ending yield levels and not on the yield path during the horizon. Option traders may use this strategy if they expect yields to end up far away from the current levels. It is useful to contrast this option strategy with another option strategy: buying a delta-hedged straddle and rebalancing the position dynamically throughout the horizon. The profitability of this strategy depends on the level of volatility (yield path) during the horizon and not on the ending yield level. Option traders initiate this strategy (and "go long volatility") when they think that the current implied volatility is "too low." If the realized volatility turns out to be higher than the initial implied volatility, the trade makes money from profitable rebalancing trades — even if the ending yield is the same as the starting yield. These two option positions are analogous to two types of barbell-bullet strategies. In the first type (our example), the barbell and the bullet are duration-matched to horizon and no rebalancing occurs. In the second type, the trade is duration-matched instantaneously and the match is rebalanced frequently. The appropriate strategy in a given situation depends on several factors, including the following: (1) whether the investor has a particular view about the likely horizon yields (for example, "far away from the current level") or about the implied volatility during the horizon; (2) whether the investor tolerates some duration drift (because in the first case, the duration would drift during the year) or has a strict duration target; and (3) whether the investor expects rates to be mean-reverting (in which case he may want to rebalance and lock in the convexity gains after significant rate movements).

14 The assumption of no curve-flattening expectations is realistic when describing the long-run average behavior of the yield curve, but may be unrealistic at times, especially if the Fed has recently begun easing or tightening. Because the performance of the barbell-bullet trade depends more on the curve reshaping than on convexity effects, curvature (the rolling yield spread between a barbell and a bullet) provides very noisy implied volatility estimates. Thus, it might be more useful to try to extract the market's curve-flattening expectations from the curvature by subtracting the value of convexity (based on, say, the implied volatility from option prices) from the rolling yield spread.

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In this final section, we examine the historical performance of a long-term bond position and of a wide barbell-bullet position between January 1980 and December 1994, focusing on the impact of convexity on realized returns. The first strategy involves always investing in the on-the-run 30-year Treasury bond; this strategy is long convexity by holding a long-duration bond. The second strategy involves rolling over a fives-thirties flattening trade each month. Specifically, we sell short the on-the-run five-year Treasury bond each month and buy a barbell of the 30-year bond and one-month bill. The trade is duration-matched to horizo: that is, the weight of the 30-year bond in the barbell is such that the barbell and the bullet have the same expected duration at the end of the month. A little algebra shows that the weight is the ratio of the five-year bond's duration to the 30-year bond's duration (at horizon). Although the trade is cash-neutral and duration-neutral, it is long convexity because a barbell is more convex than a bullet.

We first show some summary statistics of various bond positions in Figu 11 but focus on the last two columns. **The bullet has roughly a 100-basis-point higher average return and average yield than the duration-matched barbell.**[15] Thus, the barbell's convexity pick-up (0.69 versus 0.19) and the impact of yield curve reshaping do not offset its initial yield give-up. However, the barbell does have clearly lower return volatility than the bullet, reflecting the lower yield volatility of the 30-ye bond than the five-year bond.

Figure 11. Description of Various On-the-Run Bond Positions, 1980-94

	1-Month	30-Year	5-Year "Bullet"	"Barbell"
Average Return	7.21%	10.34%	9.75%	
Volatility of Return	0.91	12.84	7.02	
Average Yield	7.34	9.61	9.20	[illegible]
Volatility of Yield Change	3.46	1.44	1.88	1.4
Average Duration	0.08	9.87	3.91	3.91
Average Convexity	0.00	1.79	0.19	0.69

Note: Average returns are simply annualized by *12 and volatilities by * $\sqrt{12}$. The barbell is a combination of the one-month bill and the 30-year bond, duration-matched each month to the end-of-month duration of the five-year t All other measures for the barbell are market value-weighted averages, but the barbell's yield volatility is market va * duration-weighted.

We can decompose any bond's holding-period return into four parts the yield impact; the duration impact; the convexity impact; and a residual term. Recall from Equation (1) that duration and convexity effects can approximate a bond's instantaneous return well. Over time, a bond also earns some income from coupons or from price accrual; we estimate this income from a bond's yield. Thus, we approximate a bond' holding-period return by Equation (3).[16] The difference between the actu return and its three-term approximation is the residual term; if the approximation is good, the residual should be relatively small. We split t

[15] The bullet's outperformance is consistent with the finding in Part 3 of this series, *Does Duration Extension Enh Long-Term Expected Returns?*, that historical average returns do not increase linearly with duration. Instead, the average return curve is concave, indicating that the intermediate-term bonds earn higher average returns than duration-matched pairs of short-term bonds and long-term bonds.

[16] Why is the first term on the right-hand side of Equation (3) yield and not rolling yield? Equation (3) is n way to approximate a bond's holding-period return when we study actual bond-specific yield changes (whi t viewed as the sum of the rolldown yield changes and the changes in constant-maturity rates). In this case, the rolld return is a part of the duration and convexity impact. Alternatively, if we studied in Equation (3) the changes in constant-maturity rates (which do not include the rolldown yield change), we should include the rolldown return in the first term on the right-hand side; it would be rolling yield instead of yield.

30-year bond's monthly returns to four components and describe the average behavior and volatility of each component in the top panel of Figure 12.[17]

$$\text{Return} \approx \text{yield impact} - \text{duration} * \Delta y + 0.5 * \text{convexity} * (\Delta y)^2. \quad (3)$$

The return volatility numbers in the top panel of Figure 12 show that in any given month, the duration impact largely drives the long bond's return — it is the source behind 99% of the monthly return fluctuations. However, yield increases and decreases tend to offset each other over time, having little impact on long-term average returns.[18] Over our 15-year sample period, the long bond's average return reflects more the average yield (91%) and less the convexity (14%) and duration (-5%) effects. The residual term has a small mean and volatility, indicating that the approximation in Equation (3) works well. Subperiod analysis shows that over three-year horizons, the duration effect can still have a significant positive or negative impact — the 1983-85 and 1989-91 subperiods were clearly bull markets and the three other subperiods were bear markets. In contrast, the yield and convexity effects are always positive (by construction). The convexity impact was largest in the early 1980s when yield volatility was very high. **During the whole sample, the annualized convexity impact was 148 basis points. In the 1990s, it was about half of that.**

Similarly, we can split the five-year bullet's and the duration-matched barbell's monthly returns into four components based on Equation (3). The lower panel of Figure 12 describes the average behavior and volatility of their difference, which can be viewed as a duration-matched and cash-neutral barbell-bullet trade. Again, the volatility numbers show that most of the monthly fluctuations (99%) come from the duration impact. The trade is duration-neutral; thus, the duration impact refers to the capital gains or losses caused by curve reshaping. That is, although $Dur_{Barbell} = Dur_{Bullet}$, the duration impacts of the barbell and the bullet differ unless the yield changes are parallel ($-Dur_{Barbell} * \Delta y_{Barbell} \neq -Dur_{Bullet} * \Delta y_{Bullet}$). Over the whole sample, these effects tend to cancel out, and the average return depends largely (90%) on initial yields. The barbell has a 105-basis-point lower average annual return than the bullet, mainly because of its yield disadvantage (-95 basis points) and partly due to losses caused by the curve steepening (-36 basis points); these are only partly offset by the barbell's convexity advantage (30 basis points). **In four out of five subperiods, the bullet outperformed the barbell, suggesting that a barbell's convexity advantage is rarely sufficient to offset the negative**

[17] The percentage contributions of average returns in Figure 12 add up to 100% because we use an approximate method of annualizing monthly returns (multiplying by 12). In contrast, the percentage contributions of volatilities do not add up to 100% because volatilities are not additive (whether annualized or not).

[18] A careful reader may find it puzzling that the average duration impact on bond returns is negative over a sample period when the bond yields declined, on average. There are two explanations. First, the duration impact is a product of duration and yield changes, and it turns out that yield declines (from high yield levels) tended to coincide with relatively short durations, while yield increases (from low yield levels) tended to coincide with long durations. Thus, yield increases are "weighted" more heavily than yield declines. Second, historical yield changes that are based on a time series of on-the-run yield levels can be misleading because they ignore the impact of changing on-the-run bonds. For example, if a new bond is issued on August 15, the on-the-run yield change from July 31 to August 31 compares the yields of different bonds, the old one and the new one. Typically, the old bond loses some of its liquidity premium; thus, its end-of-month yield tends to be higher than that of the new bond — a pattern hidden in the on-the-run yield level series. For the analysis in Figure 12, we create a clean series of yield changes that always compares the beginning- and end-of-month yields of one bond. The average monthly yield change in the clean series is one basis point higher than in the unadjusted series.

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Figure 12. Decomposing Returns to Yield, Duration and Convexity Effects

1980-94	Total Return	Yield Impact	Duration Impact	Convexity Impact	R
30-Year Bond's Monthly Returns					
Average Return	10.34%	9.41%	-0.49%	**1.48%**	-0.06%
Volatility of Return	12.84	0.61	12.66	0.61	0.11
Pct. of Average Return	100	**91**	-5	14	-1
Pct. of Volatility of Return	100	5	**99**	5	1
Subperiod Average Returns					
1980-82	11.19%	12.21%	-3.70%	**2.83%**	-0.15%
1983-85	14.83	11.22	2.24	**1.44**	-0.07
1986-88	8.16	8.27	-1.73	**1.65**	-0.03
1989-91	13.57	8.26	4.56	**0.80**	-0.04
1992-94	3.94	7.10	-3.81	**0.67**	-0.02
Barbell-Bullet Trade's Monthly Returns					
Average Return	-1.05%	-0.95%	**-0.36%**	**0.30%**	**-0.04%**
Volatility of Return	2.96	0.19	2.93	0.18	0.09
Pct. of Average Return	100	**90**	34	-28	4
Pct. of Volatility of Return	100	6	**99**	6	3
Subperiod Average Returns					
1980-82	**-1.44%**	-0.66%	-1.10%	**0.42%**	-0.09%
1983-85	**-1.86**	-1.21	-0.94	**0.39**	-0.10
1986-88	**0.58**	-0.97	1.12	**0.42**	0.01
1989-91	**-2.39**	-0.65	-1.90	**0.17**	-0.01
1992-94	**-0.14**	-1.25	1.02	**0.10**	0.00

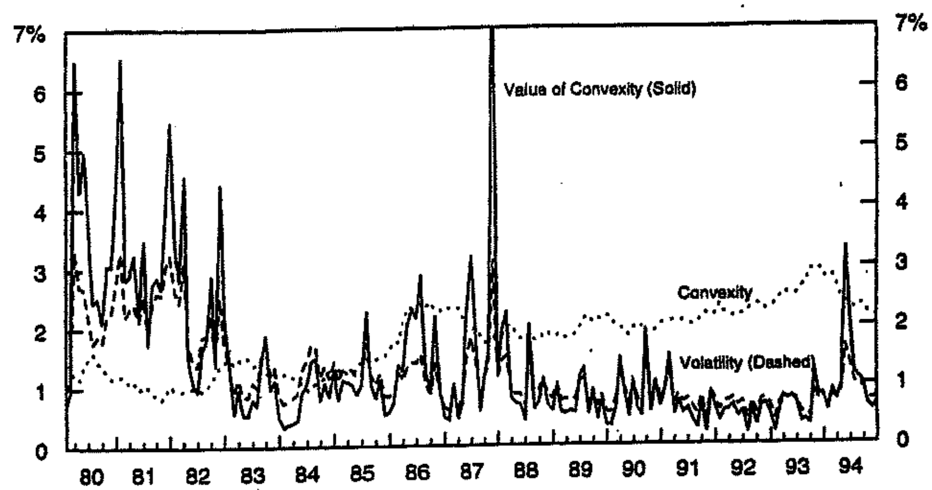
Note: For the bond, the returns and their components are raw returns. For the barbell-bullet, these figures are return differences between the barbell and duration-matched five-year bullet. Averages of total returns and their components are simply annualized by *12 and expressed in percent; volatilities are annualized by * $\sqrt{12}$. Yield impact is the return from yield (where the barbell's yield is market value-weighted). Duration impact is -Duration-at-horizon * where the yield change for the barbell is market value * duration-weighted. Convexity impact is 0.5 * convexity-at-horizon * $(\Delta y)^2$. Residual is the differe between the total return and the three components (yield impact, duration impact and convexity impact).

carry over a multiyear period.[19] In addition, the impact of curve-reshaping is larger, in absolute magnitude, than the convexity impa in each subperiod. Again, the residual has a small mean and volatilit thus, the approximation in Equation (3) appears to work well.

Figure 12 describes the impact of convexity, and two other effects, on realized bond returns. While characterization of past returns is sometimes useful, most investors are more interested in the future impact of convexity. If volatility and convexity were constant, we could use the historical average convexity impact to proxy for the expected value of convexity. However, volatility and convexity vary over time. Figure 13 shows the behavior of convexity, the rolling 20-day historical volatility a the (expected) value of convexity of the 30-year bond between 1980 and 1994. (Recent historical volatility is often used as an estimate for near-te future volatility.) Convexity has increased as yields declined, but the volatility level has declined even more except for spikes after the 1987 stock market crash and after the Fed's tightening in spring 1994. **In the early 1980s, convexity was worth several hundred basis points for th 30-year bond — while more recently, the value of convexity has rare exceeded 100 basis points.** Such variation implies that any estimates of the value of convexity are as good as the underlying estimates of future volatility. Therefore, when computing convexity-adjusted expected return investors should use the information in the current yield curve combined with their best forecasts of the near-term yield volatility.

19 One should not generalize these findings about wide barbells to narrower barbells. The yield curve exhi less curvature in the intermediate sector than between the extreme front end and long end. For example, a barbell-bulle trade from fives to twos and tens tends to have a much smaller yield give-up than the trade from fives to cash and thirties — and a smaller convexity pick-up.

Figure 13. Convexity and Volatility of the 30-Year Bond Over Time



Note: Volatility (Vol (Δy)) is the annualized 20-day historical volatility of the 30-year on-the-run bond's basis-point-yield changes. Convexity is the same bond's convexity. Value of convexity = 0.5 * convexity * $(\mathrm{Vol}(\Delta y))^2$.

APPENDIX A. HOW DOES CONVEXITY VARY ACROSS NONCALLABLE TREASURY BONDS?

For bonds with known cash flows, **convexity depends on the bond's duration and on the dispersion of the bond's cash flows.** The longer the duration, the higher the convexity (for a given cash flow dispersion), and the more dispersed the cash flows, the higher the convexity (for a given duration). In this subsection, we discuss the algebra and the intuition behind these relations. We begin by analyzing zero-coupon bonds.

The price of an n-year zero is

$$P = \frac{100}{(1 + y/100)^n} \tag{4}$$

where P is the bond's price, y is its annually compounded yield, expressed in percent, and n is its maturity. Taking the derivative of price with respect to yield reveals that

$$\frac{dP}{dy} = \frac{-n}{(1 + y/100)^{n+1}} = \frac{-n * (P/100)}{1 + y/100} \tag{5}$$

The second equality holds because $1/(1 + y/100)^n = P/100$, based on Equation (4). Multiplying both sides of Equation (5) by (-100/P) gives the definition of (modified) duration:

$$\text{Dur} = \frac{100}{P} * \frac{dP}{dy} = \frac{n}{1 + y/100}.$$

For zeros, maturity (n) equals Macaulay duration (T). Thus, Equation (6) confirms the familiar relation between modified duration and Macaulay duration: Dur = T/(1 + y/100), given annual compounding.

Taking the second derivative of price with respect to yield reveals that

$$\frac{d^2P}{dy^2} = \frac{-n * (-n-1)}{100 * (1 + y/100)^{n+2}} = \frac{(n^2 + n) * (P/100)}{100 * (1 + y/100)^2}.$$

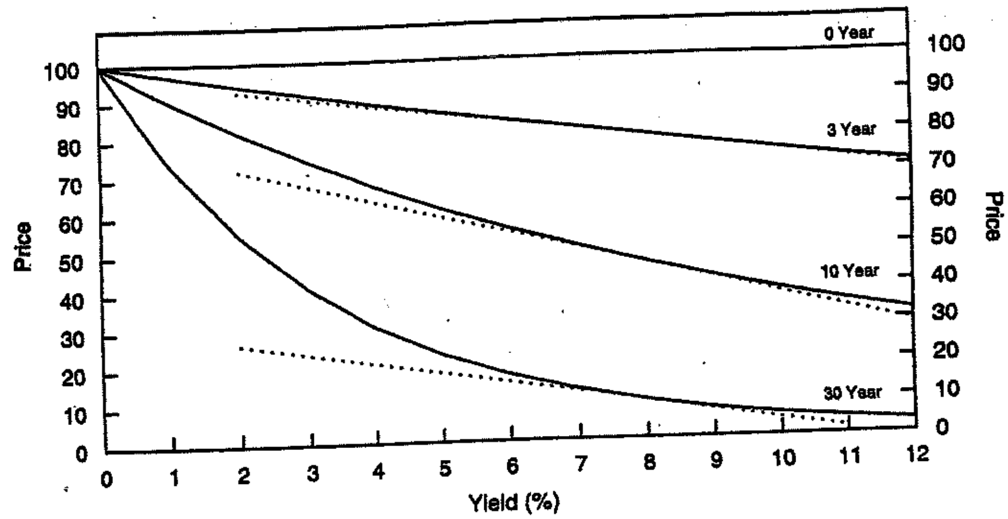
Multiplying both sides by (100/P) gives the definition of convexity (Cx)

$$Cx = \frac{100}{P} * \frac{d^2P}{dy^2} = \frac{n^2 + n}{100 * (1+ y/100)^2}.$$

Expressed in terms of Macaulay duration or modified duration, a ze convexity is $(T^2 + T)/[100 * (1 + y/100)^2] = [\text{Dur}^2 + \text{Dur}/(1+y/100)$ For long-term bonds, the square of duration is much larger than duratioı — thus, the **rule of thumb that the convexity of zeros increases as a square of duration** divided by 100. For example, for a zero with modif duration of 20 and yield of 8%, convexity is approximately 4.0 (= 20^2/: ≈ $(20^2 + 20/1.08)/100 = 4.18$).

The relation between the convexity and duration of zeros, illustrated in Figure 3, is simply a mathematical fact. With Figure 14 we try to offer some intuition as to *why* long-term bonds have much *more nonlinear* (convex) price-yield curves than short-term bonds. This figure shows pr as a function of yield for various-maturity zeros. All curves are downw sloping but not linear. However large the discounting term $(1 + y/100)^n$ prices cannot become negative as long as y > 0. Intuitively, high conve (that is, a large change in the slope of the price-yield curve) is needed t keep bond prices positive if the price-yield curve is initially very steep. Otherwise the linear approximation of the long bond's price-yield curve would hit zero very fast (at a yield of 11% for a 30-year zero in Figure versus at a yield of 43% for a three-year zero).

Figure 14. Price-Yield Curves of Zeros with Various Maturities and Their Linear Approximations



For a given duration, convexity increases with the dispersion of cash flows. A barbell portfolio of a short-term zero and a long-term zero has more dispersed cash flows than a duration-matched bullet intermediate-term zero. The bullet, in fact, has no cash flow dispersion. **The barbell exhibits more convexity because of the inverse relation between yield level and portfolio duration.** A given yield rise reduces the present value of the longer cash flow more than it reduces that of the shorter cash flow, and the decline in the longer cash flow's relative weight shortens the barbell's duration, limiting losses if yields rise further. (Recall that the Macaulay duration of a portfolio is the *present-value-weighted* average duration of its constituent cash flows.) Of all bonds with the same duration, a zero has the smallest convexity because it has no cash flow dispersion. Thus, its Macaulay duration does not vary with the yield level.

In fact, **a coupon bond's or a portfolio's convexity can be viewed as a sum of a duration-matched zero's convexity and additional convexity caused by cash flow dispersion.** That is, the convexity of a bond portfolio with a Macaulay duration T is:

$$Cx = \frac{T^2 + T}{100 * (1 + y/100)^2} + \frac{\text{Dispersion}}{100 * (1 + y/100)^2} \qquad (9)$$

where the first term on the right-hand side equals a duration-matched zero's convexity (see Equation (8)) and "dispersion" is the standard deviation of the maturities of the portfolio's cash flows about their present-value-weighted average (the Macaulay duration).[20]

[20] Stan Kogelman derived Equation (9) in "Dispersion: An Important Component of Convexity and Performance," an unpublished research piece, Salomon Brothers Inc, 1986.

Implications of Convexity

- Zero-coupon bonds.
- Long-duration bonds.

DURA
ENTER ALL VALUES AND HIT <GO>.

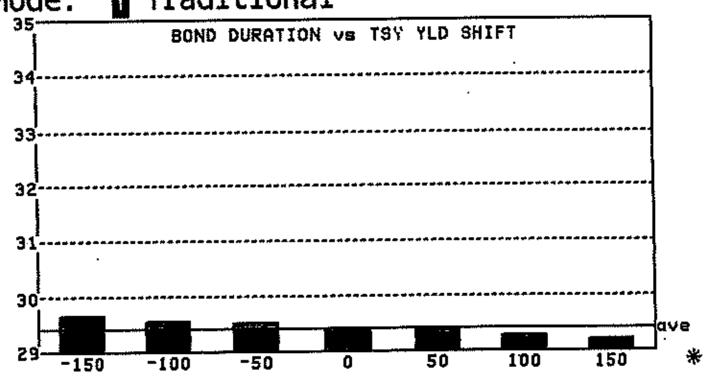
Duration Analysis for SP 0 11/15/44

Settlement 1/23/15 Price 47.234945 Yield 2.532001 to 11/15/44 @ 100

YLD SHFT	S/A Reinv	Pricing at Traded to			1/26/15 SPRD*	HORIZON Yield	Price	Mod Duration Bond	30YR	%PROB
-150	1.03	MTY	11/15/44	100	+ 6.7	1.032	73.583	29.65	21.81	
-100	1.53	MTY	11/15/44	100	+ 6.7	1.532	63.457	29.57	21.26	
-50	2.03	MTY	11/15/44	100	+ 6.7	2.032	54.744	29.50	20.70	
0	2.53	MTY	11/15/44	100	+ 6.7	2.532	47.245	29.43	20.12	100.0
50	3.03	MTY	11/15/44	100	+ 6.7	3.032	40.788	29.36	19.54	
100	3.53	MTY	11/15/44	100	+ 6.7	3.532	35.226	29.28	18.94	
150	4.03	MTY	11/15/44	100	+ 6.7	4.032	30.433	29.21	18.34	
ExVal	2.53				6.7	2.532	47.245	29.43	20.12	

Mode: T Traditional

Fixed Yld Convention? Y



BMK TSY YLD 16:15	
30YR	2.465
10YR	1.870

Probabilities V
C-Custom
V-Yld Std Dev at
24 bp/year Log? Y
10.0
% Yld Volat.

View D
Duration

* SPRDS done to interpolated BMRK Curve

130

CVXA

ENTER ALL VALUES AND HIT <GO>.

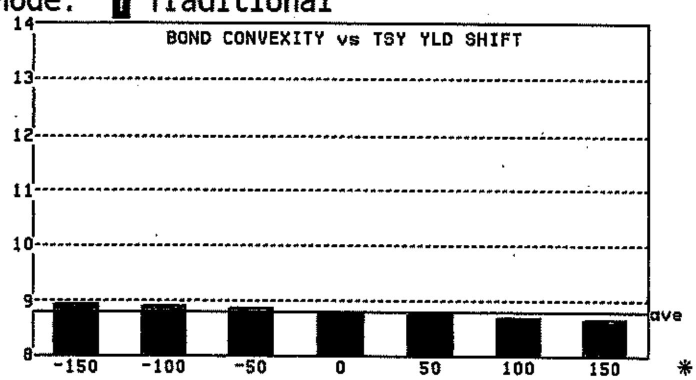
Convexity Analysis for SP 0 11/15/44

Settlement 1/23/15 Price 47.234945 Yield 2.532001 to 11/15/44 @ 100

YLD SHFT	S/A Reinv	Traded to				SPRD*	Yield	Price	Convexity Bond	Convexity 30YR	%PROB
		Pricing at 1/26/15 HORIZON									
-150	1.03	MTY	11/15/44	100	+	6.7	1.032	73.583	8.94	5.83	
-100	1.53	MTY	11/15/44	100	+	6.7	1.532	63.457	8.89	5.63	
-50	2.03	MTY	11/15/44	100	+	6.7	2.032	54.744	8.85	5.42	
0	2.53	MTY	11/15/44	100	+	6.7	2.532	47.245	8.81	5.21	100.0
50	3.03	MTY	11/15/44	100	+	6.7	3.032	40.788	8.76	5.00	
100	3.53	MTY	11/15/44	100	+	6.7	3.532	35.226	8.72	4.79	
150	4.03	MTY	11/15/44	100	+	6.7	4.032	30.433	8.68	4.57	
ExVal	2.53					6.7	2.532	47.245	8.81	5.21	

Mode: T Traditional

Fixed Yld Convention? Y



BMK TSY YLD 16:16	
30YR	2.465
10YR	1.870

Probabilities V

C-Custom

V-Yld Std Dev at

24 bp/year Log? Y

10.0

% Yld Volat.

View C

Convexity

* SPRDS done to interpolated BMRK Curve

DES

WALT DISNEY CO   DIS7.55 07/93-23   123.052/123.052   (4.537/4.537) TRAC @ 1/08

DIS 7.55 07/15/93 Corp

Page 1/11 Description: Bond

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35) CACS Corp Action
36) CF Prospectus
37) CN Sec News
38) HDS Holders
39) VPR Underly Info

66) Send Bond

Issuer Information

Name	WALT DISNEY COMPANY/THE
Industry	Entertainment Content

Security Information

Mkt Iss	Domestic MTN		
Country	US	Currency	USD
Rank	Sr Unsecured	Series	MTN
Coupon	7.55	Type	Fixed
Cpn Freq	S/A		
Day Cnt	30/360	Iss Price	100.00000
Maturity	07/15/2093		
CALL 07/15/23@103.02			
Iss Sprd	95.00bp vs T 7 $\frac{1}{8}$ 02/15/23		

Calc Type	(1)STREET CONVENTION
Announcement Date	07/21/1993
Interest Accrual Date	07/15/1993
1st Settle Date	07/28/1993
1st Coupon Date	01/15/1994

Identifiers

CUSIP	254687AH9
ISIN	US254687AH95
ID Number	DD5300973

Bond Ratings

Moody's	A1
S&P	A
Fitch	A
Composite	A

Issuance & Trading

Amt Issued/Outstanding

USD	300,000.00 (M) /
USD	201,169.00 (M)

Min Piece/Increment

25,000.00 / 1,000.00

Par Amount	1,000.00
Book Runner	MLPFS,MS
Reporting	TRACE

SETTLEMENT: NEW YORK FUNDS. CO ACQ'D CAPITAL CITIES/ ABC INC EFF 2/12/96.

132

DURA
ENTER ALL VALUES AND HIT <GO>.

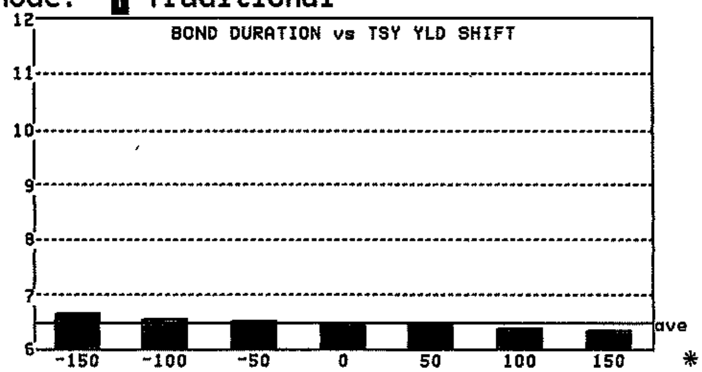
Duration Analysis for DIS 7.55 07/93

Settlement 1/27/15 Price 123.052 Yield 4.536791 to 7/15/23 @ 103.02

YLD SHFT	S/A Reinv	Pricing at 1/28/15 HORIZON Traded to			SPRD*	Yield	Price	Mod Duration Bond	10YR	%PROB
-150	3.04	CALL	7/15/23	103.02	+281.1	3.037	135.8	6.63	8.89	
-100	3.54	CALL	7/15/23	103.02	+281.1	3.537	131.38	6.59	8.84	
-50	4.04	CALL	7/15/23	103.02	+281.1	4.037	127.13	6.54	8.79	
0	4.54	CALL	7/15/23	103.02	+281.1	4.537	123.05	6.49	8.75	100.0
50	5.04	CALL	7/15/23	103.02	+281.1	5.037	119.13	6.44	8.70	
100	5.54	CALL	7/15/23	103.02	+281.1	5.537	115.36	6.39	8.65	
150	6.04	CALL	7/15/23	103.02	+281.1	6.037	111.74	6.34	8.60	
ExVal	4.54				281.1	4.537	123.05	6.49	8.75	

Mode: T Traditional

Fixed Yld Convention? Y



BMK TSY YLD 16:17	
10YR	1.870
5 YR	1.349

Probabilities V
C-Custom
V-Yld Std Dev at
17 bp/year Log? Y
10.0
% Yld Volat.

View D
Duration

* SPRDS done to interpolated BMRK Curve

CVXA
ENTER ALL VALUES AND HIT <GO>.

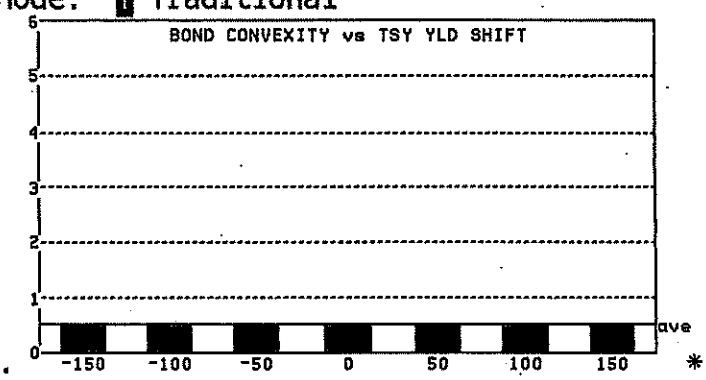
Convexity Analysis for DIS 7.55 07/93

Settlement 1/27/15 Price 123.052 Yield 4.536791 to 7/15/23 @ 103.02

YLD SHFT	S/A Reinv	Pricing at Traded to			1/28/15 SPRD*	HORIZON Yield	Price	Convexity Bond	10YR	%PROB
-150	3.04	CALL	7/15/23	103.02	+281.1	3.037	135.8	0.54	0.88	
-100	3.54	CALL	7/15/23	103.02	+281.1	3.537	131.38	0.53	0.88	
-50	4.04	CALL	7/15/23	103.02	+281.1	4.037	127.13	0.53	0.87	
0	4.54	CALL	7/15/23	103.02	+281.1	4.537	123.05	0.52	0.86	100.0
50	5.04	CALL	7/15/23	103.02	+281.1	5.037	119.13	0.51	0.85	
100	5.54	CALL	7/15/23	103.02	+281.1	5.537	115.36	0.51	0.85	
150	6.04	CALL	7/15/23	103.02	+281.1	6.037	111.74	0.50	0.84	
ExVal	4.54				281.1	4.537	123.05	0.52	0.86	

Mode: T Traditional

Fixed Yld Convention? Y



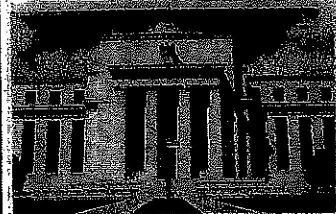
BMK TSY YLD 16:17	
10YR	1.870
5 YR	1.349

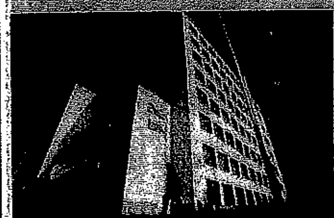
Probabilities V
C-Custom
V-Yld Std Dev at
17 bp/year Log? Y
10.0

View C
Convexity

* SPRDS done to interpolated BMRK Curve

The World's Safest Bonds May Be Wild Risks


Federal Reserve in Washington


Bundesbank in Frankfurt


Bank of Japan in Tokyo


Bank of France in Paris

Notes: European bond data as of May 18, 2016, London close. Calculations for Japanese securities use a simple yield and ignore leap years. All others are computed with a yield to maturity and count actual days.

Sources: WSJ Market Data Group (U.S., Japan); Tradeweb (Germany and France)
Photos from top: ZUMAPRESS, Associated Press, EPA, AFP/Getty Images

THE WALL STREET JOURNAL.

Desperate for yield, investors are buying government bonds that come due further and further in the future. If you lend your money to the government, you expect to get it back. What could possibly go wrong?

Unfortunately, a lot. A small move in the yield on these increasingly popular 40-, 50- and sometimes even 100-year bonds can have a crippling effect on their capital value.

Anyone who might be thinking about selling those bonds before they mature (hint: that's everyone alive today for the longest-dated bonds) should consider how they'd feel if their supposedly safe bond lost a quarter of its value in two months. It happened to the rock-solid German 30-year bund just last year.

The operative word is duration: the sensitivity of bonds to changes in interest rates. For U.S. Treasurys, duration rose to its highest level last week in 15 years, according to Barclays PLC data. The risks of an interest-rate surprise were underscored Wednesday by the minutes of the Federal Open Market Committee's meeting in April, which showed markets have underestimated the possibility of a rate hike by the Fed.

The chart shows how moves of one or more percentage points would affect the value of government bonds around the world.

—Charles Forelle

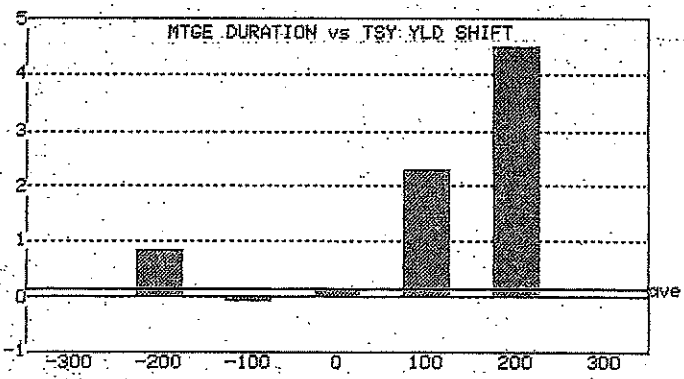
135

DURA   N270 **Mtge** **DURA**

Duration Analysis for GNSF 7

Settle 4/20/05 Px 105-27 4.665 469 PSA WAM 26y 6m WAC 7.50 AL 2.91

TSY YLD SHIFTS	Pricing @ 0 mth Horizon w/ 40 % yld curve adjust.					PPM Duration		
		WAL	TsySprd	BEY	PRICE	MTGE	3YR	PROB%
-300	873 PSA	1.4	+101/AL	1.474	107.058	n.a.	n.a.	
-200	865 PSA	1.4	+100/AL	2.477	105.736	.83	2.68	0.0
-100	741 PSA	1.7	+92/AL	3.533	105.295	-.05	2.66	0.0
0bp	469 PSA	2.9	+72/AL	4.665	105-27	.12	2.65	100
100	255 PSA	5.3	+54/AL	5.784	105.045	2.28	2.63	0.0
200	177 PSA	7.1	+47/AL	6.832	101.041	4.47	2.62	0.0
300	151 PSA	8.0	+44/AL	7.853	95.977	n.a.	n.a.	
ExpdVal	469	2.9	+72/AL	4.665	105-27	.12	2.65	



(0=CUSTOM)
PREPAY MODEL
1 D.TABLE

Hit <HELP> for Details

BASE PREPAY
MED B.Median

Treasury Curve	
3mo	2.787
6mo	3.089
2yr	3.734
3yr	3.946
5yr	4.221
10yr	4.542
30yr	4.825

Probabilities V
C-Custom
V-YLD Std Dev at
39 bp/year Log? Y
10.0% Yld Volat.

View D T-TotRet,
C-CVX, D-DUR, A-AvLife

136

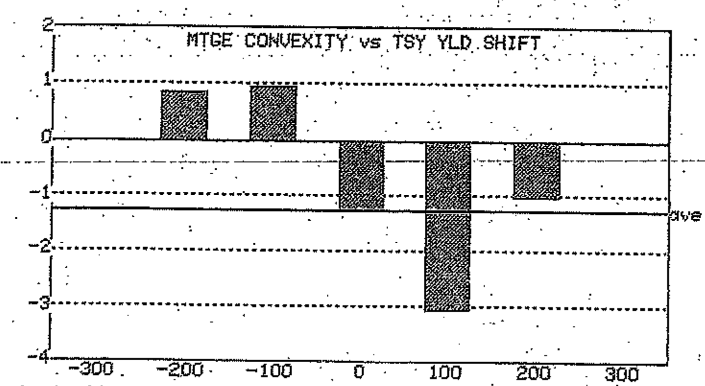
CVXA

N270 **Mtge** **CVXA**

Convexity Analysis for GNSF 7

Settle 4/20/05 Px 105-27 4.665 469 PSA WAM 26 y 6 m WAC 7.50 AL 2.91

TSY YLD SHIFTS	Pricing @ 0 mth Horizon w/	40 % yld curve adjust.				PPM Convexity		
		WAL	TsySprd T	BEY	PRICE	MTGE	3YR	PROB%
-300	873 PSA	1.4	+101/AL	1.475	107.056	n.a.	n.a.	
-200	865 PSA	1.4	+100/AL	2.478	105.734	.83	.09	0.0
-100	741 PSA	1.7	+92/AL	3.534	105.294	.94	.09	0.0
0bp	469 PSA	2.9	+72/AL	4.665	105-27	-1.27	.09	100
+100	255 PSA	5.3	+54/AL	5.784	105.045	-3.04	.08	0.0
200	177 PSA	7.1	+47/AL	6.832	101.041	-1.04	.08	0.0
300	151 PSA	8.0	+44/AL	7.853	95.977	n.a.	n.a.	
ExpdVal	469	2.9	+72/AL	4.665	105-27	-1.27	.09	



<0=CUSTOM>
PREPAY MODEL 3 D.TABLE

Hit <HELP> for Details

BASE PREPAY 16 B.Median

Treasury Curve	
3mo	2.798
6mo	3.099
2yr	3.734
3yr	3.946
5yr	4.221
10yr	4.542
30yr	4.825

Probabilities V
C-Custom
V-YLD Std Dev at
39 bp/year Log? Y
10.0 % Yld Volat.

View C T-TotRet,
C-CVX, D-DUR, A-AvLife

137

<HELP> for explanation, <MENU> for similar functions. N270 **Mtge** **VALL**

Bloomberg DEALER PREPAYMENT FORECASTS Pg. 1-2

For: GNSF 7.00 [illegible] TBA GNMA Single Famil 7.00% as of: 3/15/05

Firm	OBP PSA	WAM Yr	WAM Mo	WAC	-300	-200	-100	-50	+0	+50	+100	+200	+300
FBC	335	26	4	7.00	776	690	648	534	335	221	172	131	113
DB	295	25	9	7.50	467	455	379	340	295	236	183	145	133
UBS	910	29	10	7.50	973	1088	1315	1177	910	604	319	163	132
BS	535	26	7	7.50	767	728	702	664	535	394	287	195	151
ML	472	27	0	7.50	1057	941	745	613	472	349	263	202	170
LB	325	26	4	7.50	1005	1004	741	442	325	284	240	187	155
SAL	419	26	4	7.50	996	925	657	498	419	323	234	152	132
GCM				n/a for this security									
MS	480	26	9	7.50	1155	1066	817	665	480	333	255	177	138
JPS	689	27	4	7.50	857	865	850	799	689	527	375	219	165
BOA	469	27	10	7.50	783	759	759	608	469	363	267	194	154
GS	382	26	4	7.50	873	821	633	505	382	289	230	176	158
Avg	483				883	849	750	622	483	357	257	176	146
MED	469			Bloomberg MEDIAN PREPAYMENTS	873	865	741	608	469	333	255	177	151

(Column headings -300 to +300: basis point shift)

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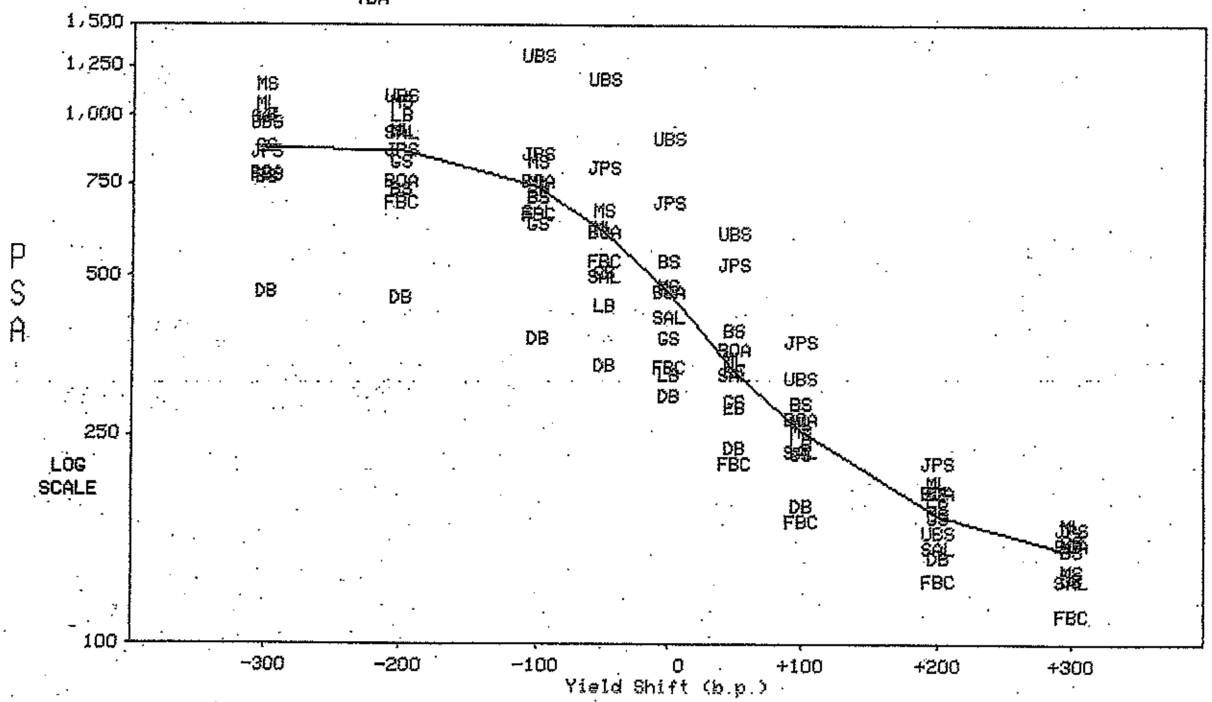
<HELP> for explanation. N270 **Mtge** **VALL**

Bloomberg CMO DEALER PREPAYMENT FORECASTS Pg 2-2

For: GNSF 7.00 T GNMA Single Famil 7.00% as of: 3/15/05

TBA



Median	
-300	873
-200	865
-100	741
-50	608
0	469
+50	333
+100	255
+200	177
+300	151

Format 1-3
Line? Y
Log? Y

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Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000
G447-63-0 15-Mar-05 16:09:06

Bloomberg TERMINAL

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