Derivative Markets MGMTMFE 406

Futures, Futures Options, and Swaps (week 10)

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SCHOOL of MANAGEMENT

Winter 2019

Outline

Outline

Currency Futures

Currency Futures

Japanese Yen (CME)-¥12,500,000; $ per 100¥						
Dec	1.2747	1.2783	1.2685	**1.2717**	-.0031	132,480
March'13	1.2786	1.2786	1.2711	**1.2731**	-.0031	339
Canadian Dollar (CME)-CAD 100,000; $ per CAD						
Dec	1.0181	1.0256	1.0177	**1.0198**	.0020	186,994
March'13	1.0160	1.0230	1.0154	**1.0175**	.0019	2,523
British Pound (CME)-£62,500; $ per £						
Dec	1.6190	1.6213	1.6119	**1.6138**	-.0043	177,894
March'13	1.6181	1.6195	1.6113	**1.6134**	-.0042	168
Swiss Franc (CME)-CHF 125,000; $ per CHF						
Dec	1.0761	1.0793	1.0738	**1.0771**	.0012	36,271
March'13	1.0782	1.0791	1.0782	**1.0789**	.0012	10
Australian Dollar (CME)-AUD 100,000; $ per AUD						
Dec	1.0186	1.0213	1.0092	**1.0108**	-.0073	157,624
March'13	1.0111	1.0129	1.0026	**1.0037**	-.0073	249
June	1.0035	1.0035	1.0035	**.9973**	-.0072	7
Mexican Peso (CME)-MXN 500,000; $ per 10MXN						
Dec	.07790	.07843 ▲	.07745	**.07763**	-.00025	210,254
March'13	.07753	.07763 ▲	.07753	**.07693**	-.00028	427
Euro (CME)-€125,000; $ per €						
Dec	1.3025	1.3080	1.3002	**1.3042**	.0014	220,184
March'13	1.3036	1.3085	1.3022	**1.3055**	.0014	690
Euro/Japanese Yen (ICE-US)-€125,000; ¥ per €						
Dec	101.275	101.275 ▲	101.275	**102.5600**	.3600	4,365
Euro/British Pound (ICE-US)-€125,000; £ per €						
Dec	...	...	...	**0.8082**	.0030	2,590
Euro/Swiss Franc (ICE-US)-€125,000; CHF per €						
Dec	...	...	...	**1.2108**	-.0001	1,187

Figure 1: Listing of various currency futures contracts from the *Wall Street Journal*, October 6-7, 2012.

Currency Contracts

- Currency forwards and futures are widely used to manage foreign exchange risk
- Suppose that T years from today you want to buy ¥1 with dollars. Denote the yen-denominated interest rate by r_y, the dollar-denominated interest rate by r, and the exchange rate today (\$/¥) by x_0
- The forward price for a yen is

$$F_{0,T} = x_0 e^{(r-r_y)T} \tag{1}$$

- The forward currency rate will exceed the current exchange rate when the domestic risk-free rate is higher than the foreign risk-free rate
- Notice that equation (1) is just like the formula for stock index futures, with the foreign interest rate equal to the dividend yield.

Currency Contracts (cont'd)

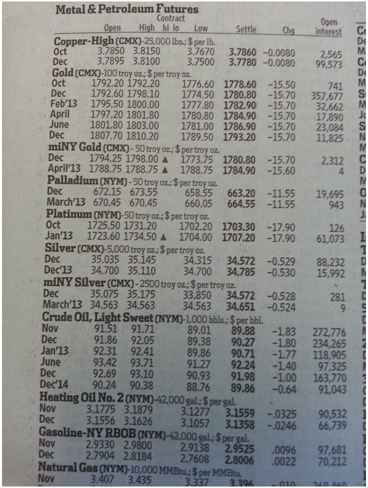
- We can prove equation (1) by absence of arbitrage and by building a synthetic currency forward: borrowing in one currency and lending in another, which creates the same cash-flow as the forward contract:

	Cash Flows			
	Year 0		Year T	
Transaction	**\$**	**¥**	**\$**	**¥**
Borrow $x_0 e^{-r_y T}$ dollar at r	$+x_0 e^{-r_y T}$	—	$-x_0 e^{(r-r_y)T}$	—
Convert to yen @ x_0	$-x_0 e^{-r_y T}$	$+e^{-r_y T}$	—	—
Invest in yen-denominated bill at r_y	—	$-e^{-r_y T}$	—	1
Total	0	0	$-x_0 e^{(r-r_y)T}$	1

- Example: Suppose that the yen-denominated interest rate is $r_y = 2\%$ and the dollar-denominated rate is $r = 6\%$. The current exchange rate rate is $x_0 = 0.009$ dollars per yen. The 1-year forward rate is

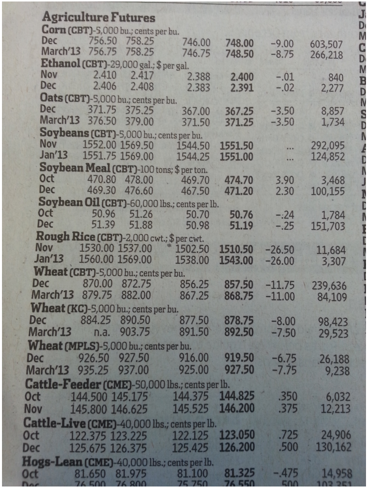
$$0.009e^{0.06-0.02} = 0.009367$$

Commodity Forwards and Futures



Metal & Petroleum Futures

	Open	High	hi lo	Low	Settle	Chg	Open interest
Copper-High (CMX)-25,000 lbs.; $ per lb.							
Oct	3.7850	3.8150		3.7670	**3.7860**	-0.0080	2,565
Dec	3.7895	3.8100		3.7500	**3.7780**	-0.0080	99,573
Gold (CMX)-100 troy oz.; $ per troy oz.							
Oct	1792.20	1792.20		1776.60	**1778.60**	-15.50	741
Dec	1792.60	1798.10		1774.50	**1780.80**	-15.70	357,677
Feb'13	1795.50	1800.00		1777.80	**1782.90**	-15.70	32,662
April	1797.20	1801.80		1780.80	**1784.90**	-15.70	17,890
June	1801.80	1803.00		1781.00	**1786.90**	-15.70	23,084
Dec	1807.70	1810.20		1789.50	**1793.20**	-15.70	11,825
miNY Gold (CMX) - 50 troy oz.; $ per troy oz.							
Dec	1794.25	1798.00	▲	1773.75	**1780.80**	-15.70	2,312
April'13	1788.75	1788.75	▲	1788.75	**1784.90**	-15.60	4
Palladium (NYM) - 50 troy oz.; $ per troy oz.							
Dec	672.15	673.55		658.55	**663.20**	-11.55	19,695
March'13	670.45	670.45		660.05	**664.55**	-11.55	943
Platinum (NYM)-50 troy oz.; $ per troy oz.							
Oct	1725.50	1731.20		1702.20	**1703.30**	-17.90	126
Jan'13	1723.60	1734.50	▲	1704.00	**1707.20**	-17.90	61,073
Silver (CMX)-5,000 troy oz.; $ per troy oz.							
Dec	35.035	35.145		34.315	**34.572**	-0.529	88,232
Dec'13	34.700	35.110		34.700	**34.785**	-0.530	15,992
miNY Silver (CMX) - 2500 troy oz.; $ per troy oz.							
Dec	35.075	35.175		33.850	**34.572**	-0.528	281
March'13	34.563	34.563		34.563	**34.651**	-0.524	9
Crude Oil, Light Sweet (NYM)-1,000 bbls.; $ per bbl.							
Nov	91.51	91.71		89.01	**89.88**	-1.83	272,776
Dec	91.86	92.05		89.38	**90.27**	-1.80	234,265
Jan'13	92.31	92.41		89.86	**90.71**	-1.77	118,905
June	93.42	93.71		91.27	**92.24**	-1.40	97,325
Dec	92.69	93.10		90.93	**91.98**	-1.00	163,770
Dec'14	90.24	90.38		88.76	**89.86**	-0.64	91,043
Heating Oil No. 2 (NYM)-42,000 gal.; $ per gal.							
Nov	3.1775	3.1879		3.1277	**3.1559**	-.0325	90,532
Dec	3.1556	3.1626		3.1057	**3.1358**	-.0246	66,739
Gasoline-NY RBOB (NYM)-42,000 gal.; $ per gal.							
Nov	2.9330	2.9800		2.9138	**2.9525**	.0096	97,681
Dec	2.7904	2.8184		2.7608	**2.8006**	.0022	70,212
Natural Gas (NYM)-10,000 MMBtu.; $ per MMBtu.							
Nov	3.407	3.435		3.337	**3.396**	-.010	269,460



Agriculture Futures

Corn (CBT)-5,000 bu.; cents per bu.						
Dec	756.50	758.25	746.00	**748.00**	-9.00	603,507
March'13	756.75	758.25	746.75	**748.50**	-8.75	266,218
Ethanol (CBT)-29,000 gal.; $ per gal.						
Nov	2.410	2.417	2.388	**2.400**	-.01	840
Dec	2.406	2.408	2.383	**2.391**	-.02	2,277
Oats (CBT)-5,000 bu.; cents per bu.						
Dec	371.75	375.25	367.00	**367.25**	-3.50	8,857
March'13	376.50	379.00	371.50	**371.25**	-3.50	1,734
Soybeans (CBT)-5,000 bu.; cents per bu.						
Nov	1552.00	1569.50	1544.50	**1551.50**	...	292,095
Jan'13	1551.75	1569.00	1544.25	**1551.00**	...	124,852
Soybean Meal (CBT)-100 tons; $ per ton.						
Oct	470.80	478.00	469.70	**474.70**	3.90	3,468
Dec	469.30	476.60	467.50	**471.20**	2.30	100,155
Soybean Oil (CBT)-60,000 lbs.; cents per lb.						
Oct	50.96	51.26	50.70	**50.76**	-.24	1,784
Dec	51.39	51.88	50.98	**51.19**	-.25	151,703
Rough Rice (CBT)-2,000 cwt.; $ per cwt.						
Nov	1530.00	1537.00	1502.50	**1510.50**	-26.50	11,684
Jan'13	1560.00	1569.00	1538.00	**1543.00**	-26.00	3,307
Wheat (CBT)-5,000 bu.; cents per bu.						
Dec	870.00	872.75	856.25	**857.50**	-11.75	239,636
March'13	879.75	882.00	867.25	**868.75**	-11.00	84,109
Wheat (KC)-5,000 bu.; cents per bu.						
Dec	884.25	890.50	877.50	**878.75**	-8.00	98,423
March'13	n.a.	903.75	891.50	**892.50**	-7.50	29,523
Wheat (MPLS)-5,000 bu.; cents per bu.						
Dec	926.50	927.50	916.00	**919.50**	-6.75	26,188
March'13	935.25	937.00	925.00	**927.50**	-7.75	9,238
Cattle-Feeder (CME)-50,000 lbs.; cents per lb.						
Oct	144.500	145.175	144.375	**144.825**	.350	6,032
Nov	145.800	146.625	145.525	**146.200**	.375	12,213
Cattle-Live (CME)-40,000 lbs.; cents per lb.						
Oct	122.375	123.225	122.125	**123.050**	.725	24,906
Dec	125.675	126.375	125.425	**126.200**	.500	130,162
Hogs-Lean (CME)-40,000 lbs.; cents per lb.						
Oct	81.650	81.975	81.100	**81.325**	-.475	14,958
Dec	76.500	76.800	75.750	**76.550**	.500	103,351

Figure 2: Listing of various currency futures contracts from the *Wall Street Journal*, October 6-7, 2012.

Introduction to Commodity Forwards and Futures

- Financial forward prices are described by the general formula

$$F_{0,T} = S_0 e^{(r-\delta)T}$$

- At a general level, commodity forward prices can be described by the same formula. There are, however, important differences:
  - For financial assets δ is the dividend yield, whereas for commodities δ is the **commodity lease rate**
  - While the dividend yield for a financial asset can typically be observed directly, the lease rate for a commodity can be estimated **only by observing the forward price**
- The formula for a commodity forward price is

$$F_{0,T} = S_0 e^{(r-\delta_l)T} \tag{2}$$

Forward Prices and The Lease Rate

- When we observe the forward price, we can infer the lease rate. Specifically, if the forward price is $F_{0,T}$, the annualized lease rate is

$$\delta_l = r - \frac{1}{T} \ln\left(\frac{F_{0,T}}{S_0}\right) \tag{3}$$

- If instead we use an effective annual interest rate, the effective annual lease rate is

$$\delta_l = \frac{1+r}{\left(F_{0,T}/S_0\right)^{1/T}} - 1 \tag{4}$$

Suppose that on June 6, 2001, the gold spot price is \$265.7 and the gold future price with maturity in December is \$269. The June to December interest rate (annualized, effective) is 3.9917%. What is the annualized 6-month gold lease rate?

- Using equation (4), the annualized 6-month lease rate is

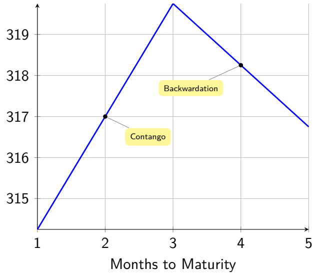
$$\text{6-month lease rate } = \frac{1+0.039917}{(269/265.7)^{1/0.5}} - 1 = 1.456\%$$

The Forward Curve

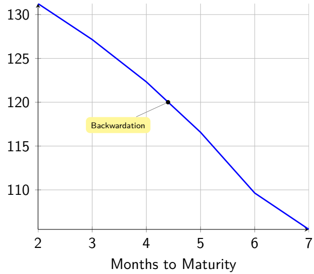
- Commodities are complex because every commodity market differs in the details. For example:
  - Storage is not possible for electricity
  - Gold is durable and relatively inexpensive to store (compared to its value)
  - Some commodities feature seasonality in production (for example, corn in the United States is harvested primarily in the fall)
- One way to observe and understand this heterogeneity is to build the **forward curve** (or the **forward strip**), i.e., the set of prices for different expiration dates for a given commodity
  - If on a given date the forward curve is upward-sloping, then the market is in **contango**.
  - If the forward curve is downward sloping, the market is in **backwardation**
  - Forward curves can have portions in backwardation and portions in contango

Forward Curves for Various Commodities, May 5, 2004

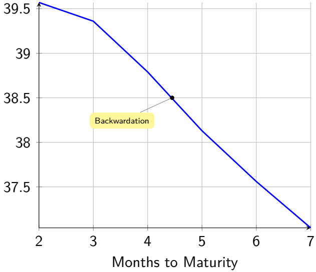
Corn Futures Price (cents per bushel)



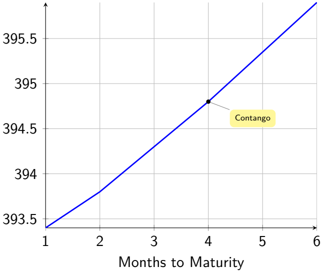
Gasoline Futures Price (cents per gallon)



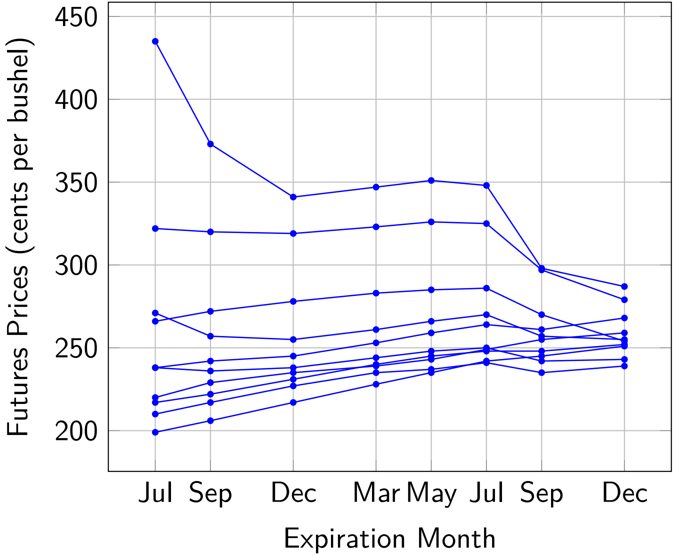
Crude Oil Futures Price ($ per barrel)



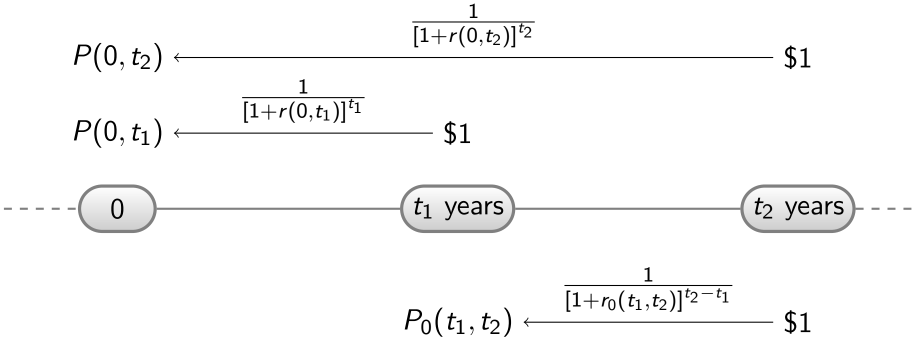
Gold Futures Price ($ per ounce)



Futures Prices for Corn for the first Wednesday in June, 1995-2004



Interest Rate Forwards and Futures: Bond Basics



- Notation:
  - $P(0,t_1)$ and $P(0,t_2)$: zero coupon bond
  - $r(0,t_1)$ and $r(0,t_2)$: effective annual interest rate (yield to maturity)
  - $P_0(t_1,t_2)$: implied forward zero-coupon bond price (quoted at time 0)
  - $r_0(t_1,t_2)$: implied forward rate (prevailing at time 0)

Interest Rate Forwards and Futures: Bond Basics

- Suppose that today is 0 and we plan to borrow \$1 at t_1 and pay back at t_2
- We can synthetically create this by trading zero-coupon bonds

	Cash Flows		
Transaction	**Time 0**	**Time t_1**	**Time t_2**
Sell $P(0,t_1)/P(0,t_2)$ z-c bonds maturing @ t_2	$+P(0,t_1)$	—	$-\frac{P(0,t_1)}{P(0,t_2)}$
Buy 1 z-c bond maturing @ t_1	$-P(0,t_1)$	\$1	—
Total	0	\$1	$-\frac{P(0,t_1)}{P(0,t_2)}$

- It follows that the implied forward zero-coupon bond price must be consistent with the implied forward interest rate and the zero-coupon bond prices with maturities t_1 and t_2:

$$P_0(t_1,t_2) = \frac{1}{[1+r_0(t_1,t_2)]^{t_2-t_1}} = \frac{P(0,t_2)}{P(0,t_1)}$$

Forward Rate Agreements

- Consider a firm expecting to borrow \$100m for 91 days (3 months), beginning 120 days from today, in June (using our previous notation, $t_1 = 120/360$ is the borrowing date and $t_2 = 211/360$ is the loan repayment date).
- In June, the effective quarterly interest rate can be either 1.5% or 2%. The implied June 91-day forward rate (the rate from June to September) is 1.8%.
- Here is the risk faced by the borrower, assuming no hedging:

Transaction	**June** ($t_1 = 120/360$)	**September** ($t_2 = 211/360$)	
		$r_{quarterly} = 1.5\%$	$r_{quarterly} = 2\%$
Borrow \$100m	$+\$100m$	-\$101.5m	-\$102.0m

Forward Rate Agreements (cont'd)

- A **forward rate agreement** (FRA) is an over-the-counter contract that guarantees a borrowing or lending rate on a given notional principal amount.
- FRAs can be settled at maturity (**in arrears**) or at the initiation of the borrowing or lending transaction
  - FRA settlement in arrears:

$$\text{Payment to the borrower} = \left(r_{quarterly} - r_{FRA}\right) \times \text{notional principal}$$

  - FRA settlement at the borrowing date:

$$\text{Payment to the borrower} = \frac{r_{quarterly} - r_{FRA}}{1 + r_{quarterly}} \times \text{notional principal}$$

Forward Rate Agreements (cont'd)

- FRA settlement in arrears:

Transaction	**June** ($t_1 = 120/360$)	**September** ($t_2 = 211/360$)	
		$r_{quarterly} = 1.5\%$	$r_{quarterly} = 2\%$
Borrow \$100m	$+\$100m$	-\$101.5m	-\$102.0m
FRA payment if $r_{quarterly} = 1.5\%$	—	-\$0.3m	—
FRA payment if $r_{quarterly} = 2\%$	—	—	+\$0.2m
Total	+\$100m	-\$101.8m	-\$101.8m

- FRA settlement at the time of borrowing:

Transaction	**June** ($t_1 = 120/360$)	**September** ($t_2 = 211/360$)	
		$r_{quarterly} = 1.5\%$	$r_{quarterly} = 2\%$
Borrow \$100m	$+\$100m$	-\$101.5m	-\$102.0m
FRA payment if $r_{quarterly} = 1.5\%$	$-\$295,566.50$	-\$0.3m	
FRA payment if $r_{quarterly} = 2\%$	$+\$196,078.43$		+\$0.2m
Total	+\$100m	-\$101.8m	-\$101.8m

Eurodollar Futures

Specifications for the Eurodollar futures contract

- Where traded: Chicago Mercantile Exchange
- Size: 3-month Eurodollar time deposit, $1 million principal
- Months: Mar, Jun, Sep, Dec, out 10 years, plus 2 serial months and spot month
- Trading ends: 5 A.M. (11 A.M. London) on the second London bank business day immediately preceding the third Wednesday of the current month
- Delivery: Cash settlement
- Settlement: 100 - British Banker's Association Futures Interest Settlement Rate for 3-Month Eurodollar Interbank Time Deposits. (This is a 3-month rate annualized by multiplying by 360/90.)

Eurodollar Futures (cont'd)

- Since the notional principal is \$1 million, a change in annualized LIBOR of 1 basis point would change the borrowing cost by

$$\frac{0.0001}{4} \times \$1\text{million} = \$25$$

- The payoff at expiration of a long Eurodollar futures contract is

$$[(100 - r_{LIBOR}) - \text{Futures Price}] \times \boxed{100} \times \boxed{\$25}$$

Convert the change in the futures price to basis points

Change in borrowing cost per basis point

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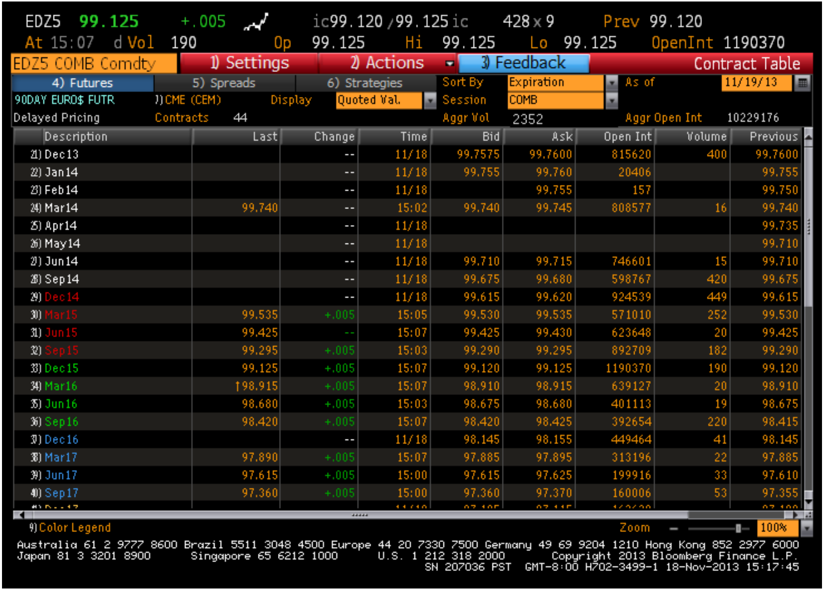

EDZ5 99.125 +.005 ic99.120 / 99.125 ic 428 x 9 Prev 99.120
At 15:07 d Vol 190 Op 99.125 Hi 99.125 Lo 99.125 OpenInt 1190370
EDZ5 COMB Comdty 1) Settings 2) Actions 3) Feedback Contract Table
4) Futures 5) Spreads 6) Strategies Sort By Expiration As of 11/19/13
90DAY EURO$ FUTR 7) CME (CEM) Display Quoted Val. Session COMB
Delayed Pricing Contracts 44 Aggr Vol 2352 Aggr Open Int 10229176

Description	Last	Change	Time	Bid	Ask	Open Int	Volume	Previous
21) Dec13		--	11/18	99.7575	99.7600	815620	400	99.7600
22) Jan14		--	11/18	99.755	99.760	20406		99.755
23) Feb14		--	11/18		99.755	157		99.750
24) Mar14	99.740	--	15:02	99.740	99.745	808577	16	99.740
25) Apr14		--	11/18					99.735
26) May14		--	11/18					99.710
27) Jun14		--	11/18	99.710	99.715	746601	15	99.710
28) Sep14		--	11/18	99.675	99.680	598767	420	99.675
29) Dec14		--	11/18	99.615	99.620	924539	449	99.615
30) Mar15	99.535	+.005	15:05	99.530	99.535	571010	252	99.530
31) Jun15	99.425	--	15:07	99.425	99.430	623648	20	99.425
32) Sep15	99.295	+.005	15:03	99.290	99.295	892709	182	99.290
33) Dec15	99.125	+.005	15:07	99.120	99.125	1190370	190	99.120
34) Mar16	↑98.915	+.005	15:07	98.910	98.915	639127	20	98.910
35) Jun16	98.680	+.005	15:03	98.675	98.680	401113	19	98.675
36) Sep16	98.420	+.005	15:07	98.420	98.425	392654	220	98.415
37) Dec16		--	11/18	98.145	98.155	449464	41	98.145
38) Mar17	97.890	+.005	15:07	97.885	97.895	313196	22	97.885
39) Jun17	97.615	+.005	15:00	97.615	97.625	199916	33	97.610
40) Sep17	97.360	+.005	15:00	97.360	97.370	160006	53	97.355

9) Color Legend Zoom 100%

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2013 Bloomberg Finance L.P.
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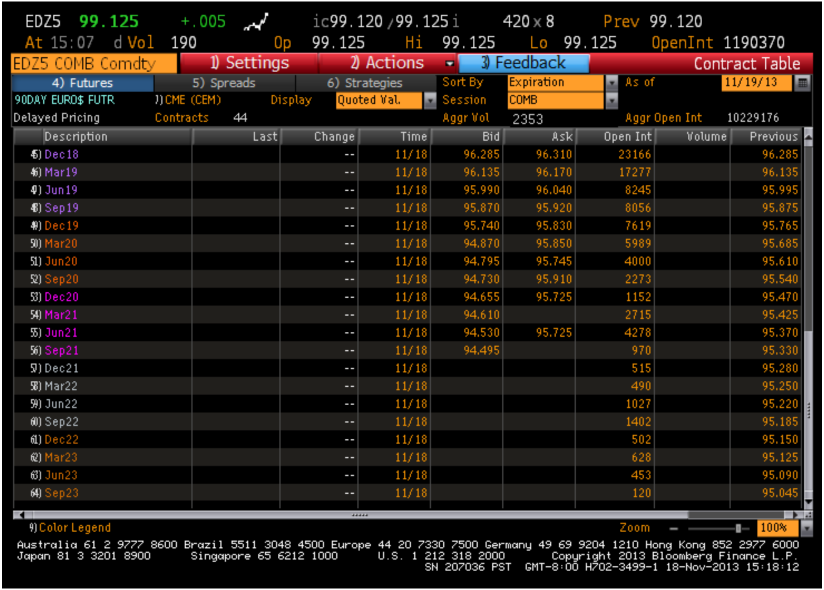

EDZ5 99.125 +.005 ic99.120 / 99.125 i 420 x 8 Prev 99.120

At 15:07 d Vol 190 Op 99.125 Hi 99.125 Lo 99.125 OpenInt 1190370

EDZ5 COMB Comdty 1) Settings 2) Actions 3) Feedback Contract Table

4) Futures 5) Spreads 6) Strategies Sort By Expiration As of 11/19/13

90DAY EURO$ FUTR 7) CME (CEM) Display Quoted Val. Session COMB

Delayed Pricing Contracts 44 Aggr Vol 2353 Aggr Open Int 10229176

Description	Last	Change	Time	Bid	Ask	Open Int	Volume	Previous
45) Dec18		--	11/18	96.285	96.310	23166		96.285
46) Mar19		--	11/18	96.135	96.170	17277		96.135
47) Jun19		--	11/18	95.990	96.040	8245		95.995
48) Sep19		--	11/18	95.870	95.920	8056		95.875
49) Dec19		--	11/18	95.740	95.830	7619		95.765
50) Mar20		--	11/18	94.870	95.850	5989		95.685
51) Jun20		--	11/18	94.795	95.745	4000		95.610
52) Sep20		--	11/18	94.730	95.910	2273		95.540
53) Dec20		--	11/18	94.655	95.725	1152		95.470
54) Mar21		--	11/18	94.610		2715		95.425
55) Jun21		--	11/18	94.530	95.725	4278		95.370
56) Sep21		--	11/18	94.495		970		95.330
57) Dec21		--	11/18			515		95.280
58) Mar22		--	11/18			490		95.250
59) Jun22		--	11/18			1027		95.220
60) Sep22		--	11/18			1402		95.185
61) Dec22		--	11/18			502		95.150
62) Mar23		--	11/18			628		95.125
63) Jun23		--	11/18			453		95.090
64) Sep23		--	11/18			120		95.045

9) Color Legend Zoom 100%

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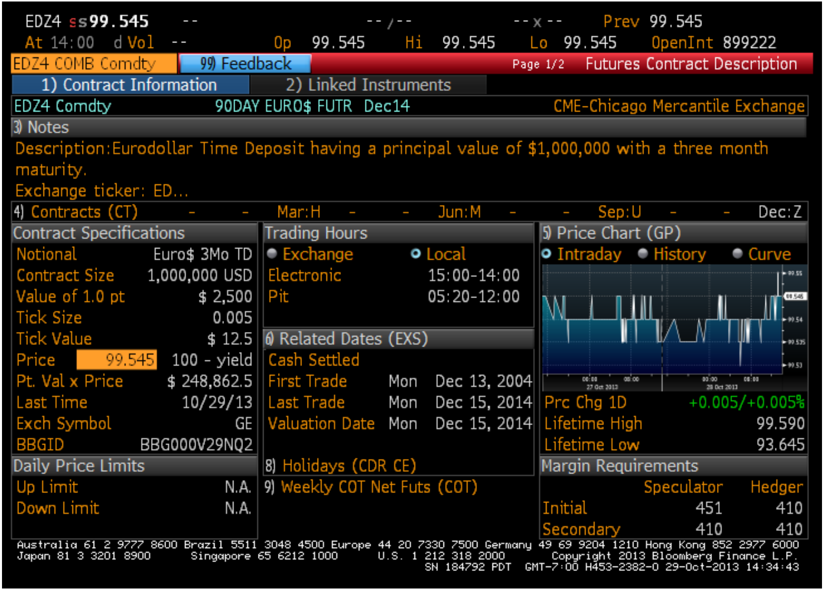


Hedging with Eurodollar Futures

- Let's consider again the example in which we wish to guarantee a borrowing rate for a \$100m loan from June to September
- Suppose the June Eurodollar futures price is 92.8. Implied 3-month LIBOR is

$$\frac{100-92.8}{4} = 1.8\% \text{ over 3 months}$$

- As before, consider 2 possible 3-month borrowing rates in June:
  - 1.5%, which correspond to a Eurodollar futures price in June of 94
  - 2%, which correspond to a Eurodollar futures price in June of 92

Hedging with Eurodollar Futures (cont'd)

- We expect to borrow \$100m for 91 days, beginning 120 days from today. Thus, we are short the interest rate.
- We wish to hedge this exposure $\Rightarrow$ we short 100 Eurodollar futures contracts
- The futures contract settles in June:

$[(92.8 - 94) \times 100 \times \$25] \times 100$

Transaction	June ($t_1 = 120/360$)	September ($t_2 = 211/360$) $r_{quarterly} = 1.5\%$	September ($t_2 = 211/360$) $r_{quarterly} = 2\%$
Borrow \$100m	$+\$100m$	-\$101.5m	-\$102.0m
Eurodollar payment if $r_{quarterly} = 1.5\%$	$-\$300,000$		
Eurodollar payment if $r_{quarterly} = 2\%$	$+\$200,000$		

$[(92.8 - 92) \times 100 \times \$25] \times 100$

Hedging with Eurodollar Futures (cont'd)

- We are almost done, with one exception: the futures contract settles in June, but our interest expense is not paid until September.
- We need to tail the position in order to earn or pay interest on our Eurodollar gain or loss before we actually have to make the interest payment:

$$\text{Number of Eurodolar contracts} = -\frac{100}{1+0.018} = -98.2318$$

Transaction	**June** $(t_1 = 120/360)$	**September** $(t_2 = 211/360)$	
		$r_{quarterly} = 1.5\%$	$r_{quarterly} = 2\%$
Borrow \$100m	$+\$100m$	-\$101.5m	-\$102.0m
Eurodollar payment if $r_{quarterly} = 1.5\%$	$-\$294,695$	-\$299,116	
Eurodollar payment if $r_{quarterly} = 2\%$	$+\$196,464$		$+\$200,393$
Total	$+\$100m$	-\$101.799m	-\$101.799m

Outline

Futures Options

- The options we have considered so far, **spot options**, provide the holder with the right to buy or sell a certain asset by a certain date for a certain price
- For **futures options**, the exercise of the option gives the holder a position in the futures contract.
- Examples of futures options:
  - Crude Oil Futures Options
  - S&P 500 Futures Options
  - Eurodollar Futures Options
  - Treasury Bond Futures Options

Why trade option on futures rather than options on the underlying asset?

- Futures contracts are, in many circumstances, more liquid than the underlying asset
- A futures price is known immediately from trading on the futures exchange, whereas the spot price of the underlying may not be so readily available. For instance, a Treasury bond futures price is known immediately from exchange trading, whereas the current market price of a bond can be obtained only by contacting one or more dealers
- Futures on commodities are often easier to trade than the commodities themselves (much easier to make or take delivery of a live-cattle futures contract than of the cattle themselves)
- Futures and options are traded side by side in the same exchange. This facilitates hedging, arbitrage, and speculation
- Futures options entail lower transactions costs than spot options in many situations

Futures Options: Mechanics

- Futures options are referred to by the delivery month of the underlying futures contract—not by the expiration month of the option
- The expiration date of a futures option contract is usually on, or a few days before, the earliest delivery date of the underlying futures contract
- Most futures options are American
- Call Futures Options
  - When a call futures option is exercised, the holder acquires
    1. A long position in the futures
    2. A cash amount equal to the excess of the most recent settlement futures price over the strike price
- Put Futures Options
  - When a put futures option is exercised, the holder acquires
    1. A short position in the futures
    2. A cash amount equal to the excess of the strike price over the most recent settlement futures price

Valuation of Futures Options Using Binomial Trees

- If we compare how the spot and futures price evolve on the tree, we observe a different solution for u and d in the case of future prices
- The solution is exactly what we would get for an option on a stock index if δ, the dividend yield, were equal to the risk-free rate:

$$
\begin{array}{ccc}
 & & \begin{array}{c} S_{t+h} = S_t e^{(r-\delta)h+\sigma\sqrt{h}} \\ F_{t+h,T} = F_{t,T} e^{\sigma\sqrt{h}} \end{array} \\
 & \nearrow & \\
\begin{array}{r} S_t \\ F_{t,T} = S_t e^{(r-\delta)nh} \end{array} & & \\
 & \searrow & \\
 & & \begin{array}{c} S_{t+h} = S_t e^{(r-\delta)h-\sigma\sqrt{h}} \\ F_{t+h,T} = F_{t,T} e^{-\sigma\sqrt{h}} \end{array}
\end{array}
$$

- Thus, the nodes are constructed as

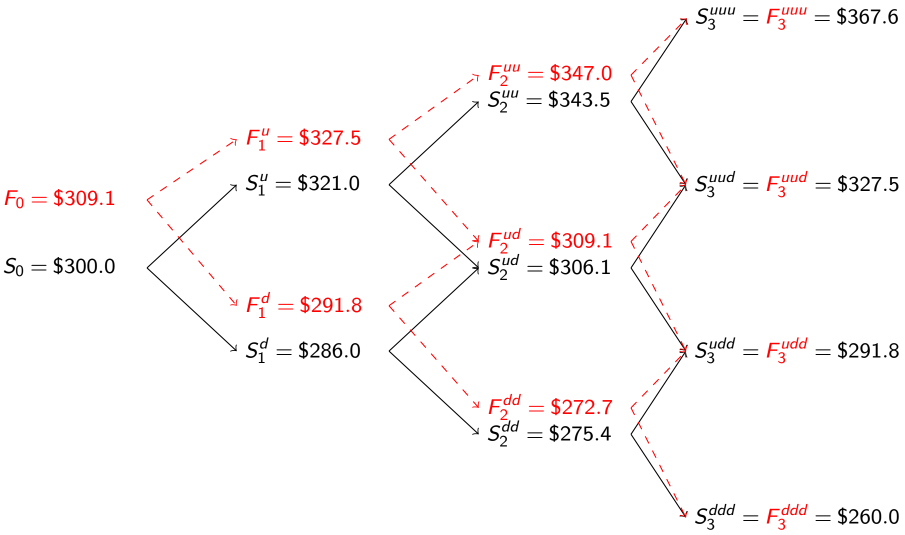
$$u = e^{\sigma\sqrt{h}}$$

$$d = e^{-\sigma\sqrt{h}}$$

Binomial Tree Example

- An option has a gold futures contract as the underlying asset. Assume $S = \$300$, $K = \$300$, $\sigma = 0.1$, $r = 0.05$, $T = 1$ year, $\delta = 0.02$, and $h = 1/3$.
- The following figure shows the tree for pricing an American call option on a gold futures contract.

Binomial Tree Example (cont'd)



Binomial Tree Example (cont'd)

- Early exercise is optimal when the futures price is \$347.0 (which corresponds to a \$343.5 spot price for the gold).

$F_0 = \$309.1$
$C_0 = \$17.1$
$C_{0,NO} = \$17.1$
$C_{0,EX} = \$9.1$

$F_1^u = \$327.5$
$C_1^u = \$29.1$
$C_{1,NO}^u = \$29.1$
$C_{1,EX}^u = \$27.5$

$F_1^d = \$291.8$
$C_1^d = \$6.3$
$C_{1,NO}^d = \$6.3$
$C_{1,EX}^d = \$0$

$F_2^{uu} = \$347.0$
$\boxed{C_2^{uu} = \$47.0}$
$C_{2,NO}^{uu} = \$46.2$
$C_{2,EX}^{uu} = \$47.0$

$F_2^{ud} = \$309.1$
$C_2^{ud} = \$13.1$
$C_{2,NO}^{ud} = \$13.1$
$C_{2,EX}^{ud} = \$9.1$

$F_2^{dd} = \$275.4$
$C_2^{dd} = \$0$
$C_{2,NO}^{dd} = \$0$
$C_{2,EX}^{dd} = \$0$

$F_3^{uuu} = \$367.6$
$C_3^{uuu} = \$67.6$

$F_3^{uud} = \$327.5$
$C_3^{uud} = \$27.5$

$F_3^{udd} = \$291.8$
$C_3^{udd} = \$0$

$F_3^{ddd} = \$260.0$
$C_3^{ddd} = \$0$

$C_{\cdot,NO}$ = value of call if not exercised
$C_{\cdot,EX}$ = value of call if exercised

American Futures Options vs. American Spot Options

- It is not generally true that an American futures option is worth the same as the corresponding American spot option **when the futures and the option contracts have the same maturity**.
  - Suppose that the market is **contango**:

$$C_0^{\text{Futures}} > C_0^{\text{Spot}} \tag{5}$$

$$P_0^{\text{Futures}} < P_0^{\text{Spot}} \tag{6}$$

  - Suppose that the market is **backwardation**:

$$C_0^{\text{Futures}} < C_0^{\text{Spot}} \tag{7}$$

$$P_0^{\text{Futures}} > P_0^{\text{Spot}} \tag{8}$$

- These differences are even greater **when the futures contract expires later than the option contract**.

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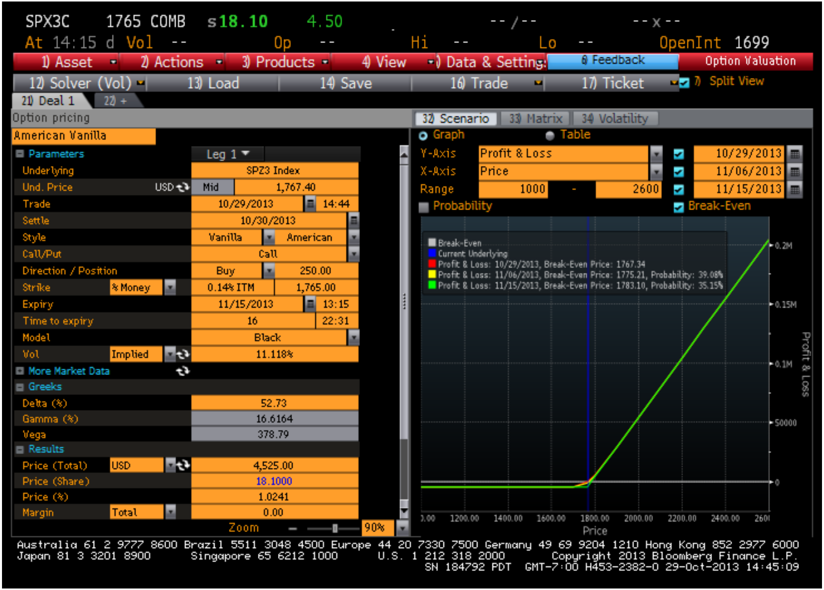

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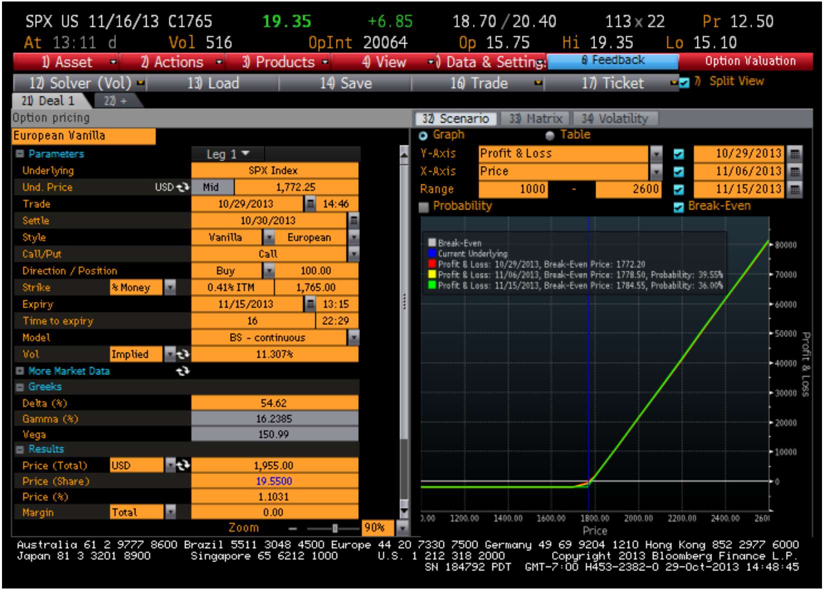


Black's Model for Valuing Futures Options

- European futures options can be valued by extending the Black-Scholes result (Black, 1976)
- The European call price C_0^f and the European put price P_0^f for futures options are (replace S_0 with F_0 and δ with r in the Black-Scholes formula):

$$C_0^f = e^{-rT}\left[F_0 N(d_1) - K N(d_2)\right] \tag{9}$$

$$P_0^f = e^{-rT}\left[K N(-d_2) - F_0 N(-d_1)\right] \tag{10}$$

where

$$d1 = \frac{\ln(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}} \tag{11}$$

$$d2 = \frac{\ln(F_0/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{12}$$

Black's Model for Valuing Futures Options (cont'd)

- Observe that the maturity of the futures contract does not enter in the valuation formula (except implicitly into the futures price today, F_0)
- If the futures contract matures at the same time as the option, then $F_T = S_T$ and the two options are equivalent
- When the lease rate is a function only of time, it can be shown that the volatility of the futures price is the same as the volatility of the underlying asset
- For European futures options, the put-call parity relationship is

$$C_0^f - P_0^f = F_0 e^{-rT} - K e^{-rT} \tag{13}$$

- See also Chapter 17 in Hull, 8th edition.

Outline

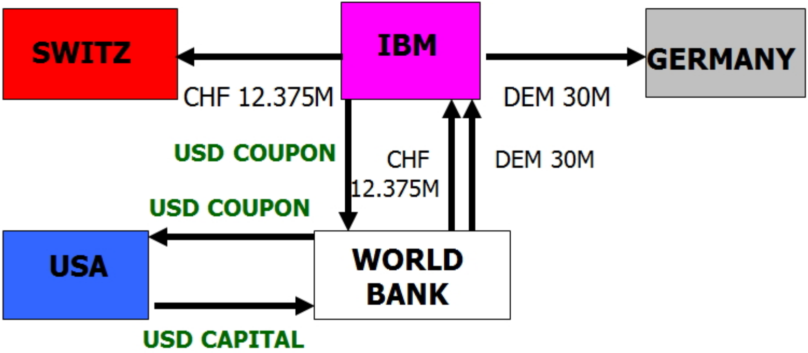
Introduction to Swaps

- A **swap** is a contract calling for an exchange of payments, on one or more dates, determined by the difference in two prices
- A swap provides a means to hedge a **stream** of risky payments
- A single-payment swap is the same thing as a cash-settled forward contract

The First Swap (1981): IBM and the World Bank

- The World Bank borrows funds internationally and loans those funds to developing countries
- In 1981, the World Bank had borrowed its allowed limit in Swiss Francs (CHF) and Deutsche Marks (DEM)
- IBM had large debt payments to pay in CHF and DEM
- IBM and the World Bank worked out an arrangement in which the World Bank borrowed dollars in the U.S. market and **swapped** the dollar payment obligation to IBM in exchange for taking over IBM's CHF and DM obligations.
- This **currency swap** was proposed by Solomon Brothers.

The First Swap (1981): IBM and the World Bank



Documentation

The swap contract's documentation must describe:

1. the currencies of the cash flows
2. the interest rates
3. the principal amount
4. whether the principal is exchanged
5. whether the counterparties make future cash payments in their entirety or through a net payment
6. the tenor (or maturity) of the swap
7. the frequency of cash payments (every six months for a majority of simple swaps)
8. how to terminate a swap early
9. what to do if one side defaults

An Example of a Commodity Swap

- An industrial producer, IP Inc., needs to buy 100,000 barrels of oil 1 year from today and 2 years from today
- The forward prices for delivery in 1 year and 2 years are \$20 and \$21/barrel
- The 1- and 2-year zero-coupon bond yields are 6% and 6.5% (annualized, effective)

An Example of a Commodity Swap (cont'd)

- IP can guarantee the cost of buying oil for the next 2 years by entering into long forward contracts for 100,000 barrels in each of the next 2 years. The present value of this cost per barrel is

$$\frac{\$20}{1.06} + \frac{\$21}{1.065^2} = \$37.383$$

- Thus, IP could pay an oil supplier any payment stream with a present value of \$37.383. Typically, a swap will call for equal payments in each year
- For example, the payment per year per barrel, x, will have to satisfy the following equation

$$\frac{x}{1.06} + \frac{x}{1.065^2} = \$37.383$$

- We find $x = \$20.483$ and we say that the 2-year swap price is \$20.483

Computing the Commodity Swap Rate

- Suppose there are n swap settlements, occurring on dates t_i, $i = 1, ..., n$
- The price of a zero-coupon bond maturing on date t_i is $P(0, t_i)$
- The fixed payment on a commodity swap is

$$\bar{F} = \frac{\sum_{i=1}^{n} P(0, t_i) F_{0,t_i}}{\sum_{i=1}^{n} P(0, t_i)} \tag{14}$$

  where F_{0,t_i} is the forward price with maturity t_i
- The commodity swap price is a weighted average of commodity forward prices, where zero-coupon bond prices are used to determine the weights

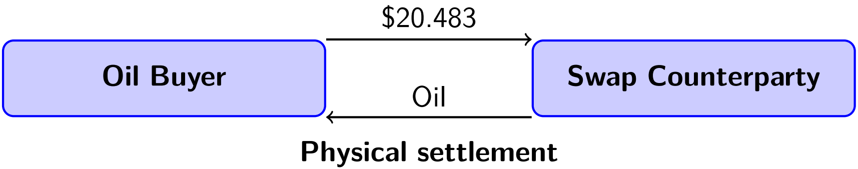
Computing the Commodity Swap Rate (cont'd)

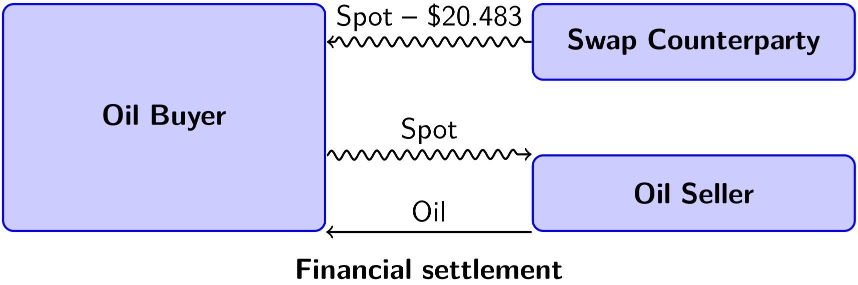
- A buyer with seasonally varying demand (e.g., someone buying gas for heating) might enter into a swap in which **quantities** vary over time
- The swap price with seasonally-varying quantities is

$$\bar{F} = \frac{\sum_{i=1}^{n} Q_{t_i} P(0, t_i) F_{0,t_i}}{\sum_{i=1}^{n} Q_{t_i} P(0, t_i)} \tag{15}$$

  where Q_{t_i} is the quantity of gas purchased at time t_i
- when $Q_t = 1$, the formula is the same as equation (14), when the quantity is not varying
- It is also possible for **prices** to be time-varying (e.g., a gas buyer who needs gas for heating can enter into a swap in which the summer price is fixed at a low value, and then winter price is then determined by the zero present value condition)

Physical Versus Financial Settlement





Swaps are nothing more than forward contracts coupled with borrowing and lending money:

- Consider the swap price of \$20.483/barrel. Relative to the forward curve price of \$20 in 1 year and \$21 in 2 years, we are overpaying by \$0.483 in the first year, and we are underpaying by \$0.517 in the second year
- Thus, by entering into the swap, we are lending the counterparty money for 1 year. The interest rate on this loan is

$$\frac{0.517}{0.483} - 1 = 7\%$$

- Given 1- and 2- year zero-coupon bond yields of 6% and 6.5%, the 1-year implied forward yield from year 1 to year 2 is

$$r_0(1,2) = \frac{P(0,1)}{P(0,2)} - 1 = 7\%$$

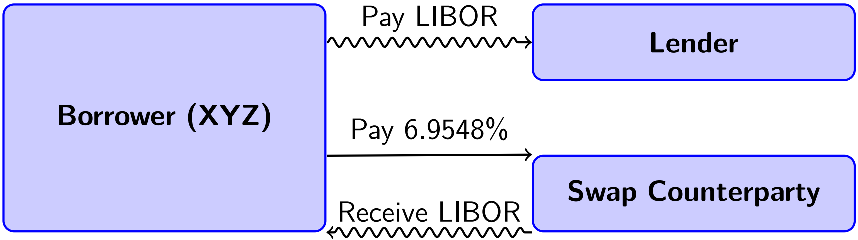
Interest Rate Swaps

- The **notional principal** of the swap is the amount on which the interest payments are based
- The life of the swap is the **swap term** or **swap tenor**
- If swap payments are made at the end of the period (when interest is due), the swap is said to be settled **in arrears**

An Example of an Interest Rate Swap

- XYZ Corp. has \$200M of floating-rate debt at LIBOR, i.e., every year it pays that year's current LIBOR
- XYZ would prefer to have fixed-rate debt with 3 years to maturity
- XYZ could enter a swap, in which they receive a floating rate and pay the fixed rate, which is 6.9548%

An Example of an Interest Rate Swap (cont'd)



- On net, XYZ pays 6.9548%

$$\text{XYZ net payment} = -\text{LIBOR} + \text{LIBOR} - 6.9548\% = -6.9548\%$$

Computing the Swap Rate

- Suppose there are n swap settlements, occurring on dates t_i, $i = 1, ..., n$
- The implied forward interest rate from date t_{i-1} to date t_i, known at date 0, is $r_0(t_{i-1}, t_i)$
- The price of a zero-coupon bond maturing on date t_i is $P(0, t_i)$
- The fixed swap rate, R, is

$$R = \frac{\sum_{i=1}^{n} P(0, t_i) r_0(t_{i-1}, t_i)}{\sum_{i=1}^{n} P(0, t_i)} \tag{16}$$

where $\sum_{i=1}^{n} P(0, t_i) r_0(t_{i-1}, t_i)$ is the present value of interest payments implied by the strip of forward rates, and $\sum_{i=1}^{n} P(0, t_i)$ is the present value of a \$1 annuity when interest rates vary over time

Computing the Swap Rate (cont'd)

- We can rewrite equation (16) to make it easier to interpret

$$R = \sum_{i=1}^{n} \left[\frac{P(0, t_i)}{\sum_{j=1}^{n} P(0, t_j)} \right] r_0(t_{i-1}, t_i) \tag{17}$$

- Thus, the fixed swap rate is a weighted average of the implied forward rates, where zero-coupon bond prices are used to determine the weights