Derivative Markets MGMTMFE 406

Black-Scholes: Formula, Greeks, Practical Uses, Extensions (weeks 7, 8 and 9)

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Outline

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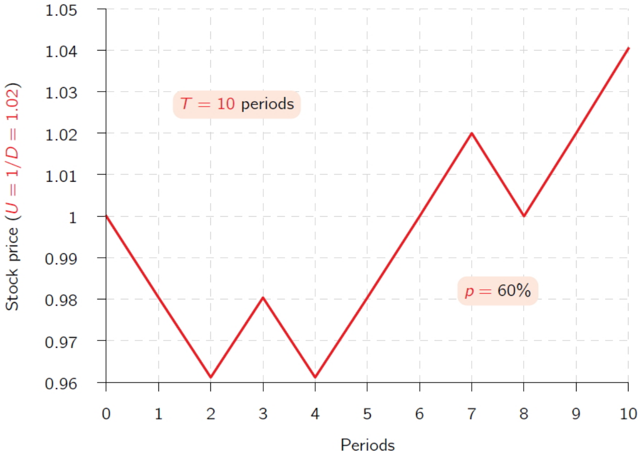
Discrete Time vs Continuous Time

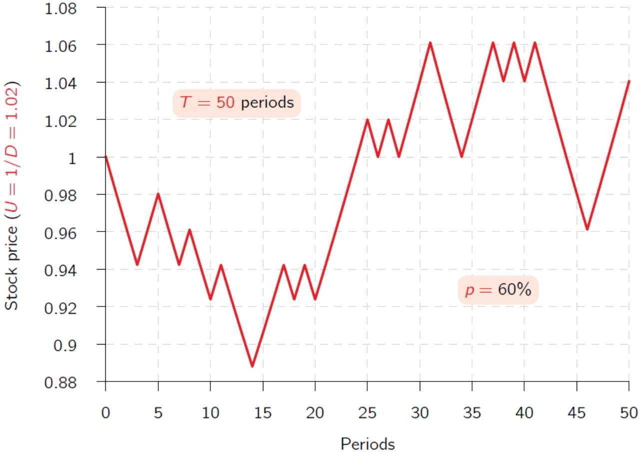
- We refer to the structure of the binomial model as **discrete time**, which means that time moves in distinct increments
- This is much like looking at a calendar and observing only months, weeks, or days
- We know that time moves forward at a rate faster than one day at a time: hours, minutes, seconds, fractions of seconds, and fractions of fractions of seconds
- When we talk about time moving in the tiniest increments, we are talking about **continuous time**.

Discrete time world	**Continuous time world**
Binomial model	Black-Scholes model

Discrete Time vs Continuous Time



Discrete Time vs Continuous Time



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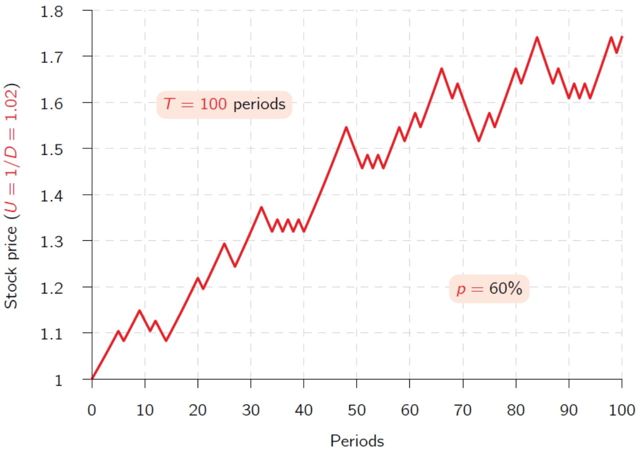

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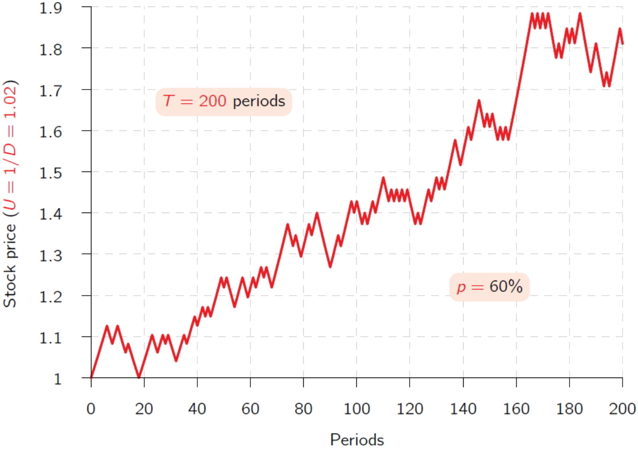

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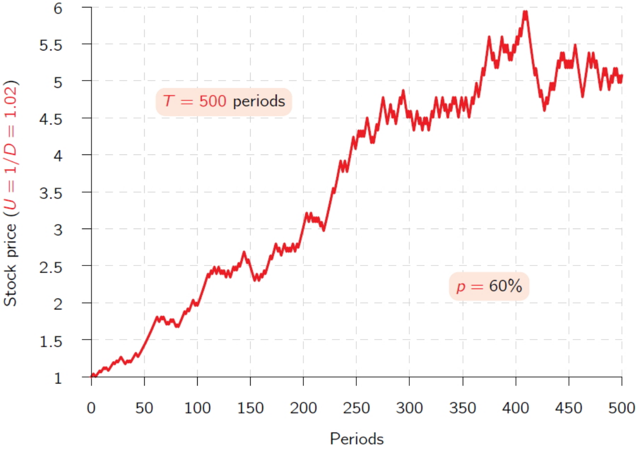

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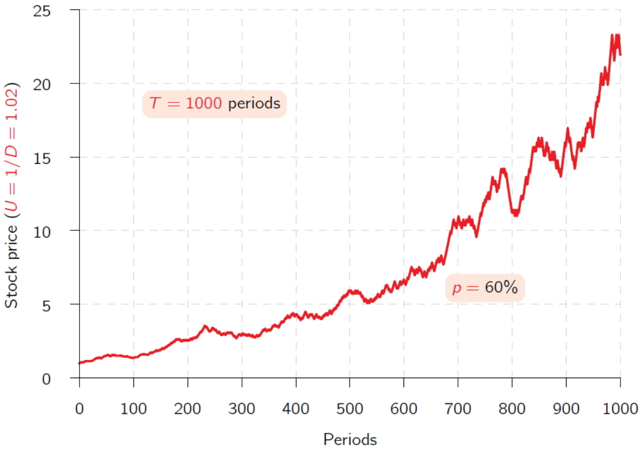


The Limiting Case of the Binomial Formula

- An obvious objection to the binomial calculations thus far is that the stock can only have a few different values at expiration. **It seems unlikely that the option price calculation will be accurate**.
- The solution to this problem is to divide the time to expiration into more periods, generating a more realistic tree.
- To illustrate how to do this, we will re-examine the 1-year European call option, which has a $40 strike and initial stock price $41.
- Let there be 3 binomial periods. Since it is a 1-year call, this means that the length of a period is $h = 1/3$. We will assume that other inputs stay the same, so $r = 0.08$ and $\sigma = 0.3$.

The Limiting Case of the Binomial Formula (cont'd)

- Since the length of the binomial period is shorter, u and d are closer to 1 than before (1.2212 and 0.8637 as opposed to 1.4623 and 0.8025 with $h = 1$).
- The risk-neutral probability of the stock price going up in a period is

$$p^* = \frac{e^{(0.08-0)\times 1/3} - 0.8637}{1.2212 - 0.8637} = 0.4568$$

- The binomial model implicitly assigns probabilities to the various nodes. The risk-neutral probability for each final period node, together with the call value, is:

Call Price in 1 Year (3 periods)	**Probability**
\$34.678	$p^{*3} = 0.0953$
\$12.814	$3p^{*2}(1-p^*) = 0.34$
\$0	$3p^*(1-p^*)^2 = 0.4044$
\$0	$(1-p^*)^3 = 0.1603$

The Limiting Case of the Binomial Formula (cont'd)

- Thus, the price of the European call option is given by

$$\begin{aligned} C_0 &= e^{0.08\times 1}(0.0953 \times \$34.678 + 0.34 \times \$12.814) \\ &= \$7.0739 \end{aligned}$$

- We can vary the number of binomial steps, holding fixed the time to expiration, T. The general formula is

$$C_0 = e^{-rT}\sum_{k=0}^{n}\frac{n!}{k!\,(n-k)!}p^{*k}(1-p^*)^{n-k}\max\left[S_0 u^k d^{n-k} - K, 0\right] \quad (1)$$

(we also need to modify u, d, and p^* at each time).

The Limiting Case of the Binomial Formula (cont'd)

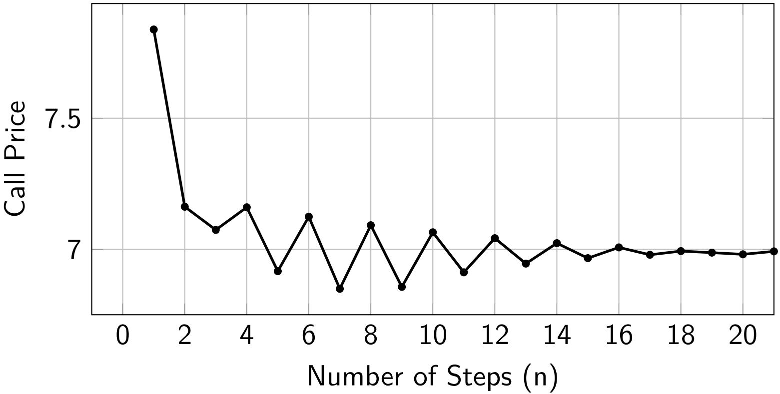
- The following table computes binomial call option prices, using the same inputs as before.

Number of steps (n)	Binomial Call Price ($)
1	$7.839
2	$7.162
3	$7.074
4	$7.160
10	$7.065
50	$6.969
100	$6.966
500	$6.960
∞	$6.961

- Changing the number of steps changes the option price, but once the number of steps becomes great enough we appear to approach a limiting value for the price, given by the **Black-Scholes formula**.

The Limiting Case of the Binomial Formula (cont'd)

- This can be seen graphically below:



Problem 12.2: Using the *BinomCall* function, compute the binomial approximations for the following call option: $S_0 = \$41$, $K = \$40$, $\sigma = 0.3$, $r = 0.08$, $T = 0.25$ (3 months), and $\delta = 0$. Be sure to compute prices for $n = 8, 9, 10, 11$, and 12. What do you observe about the behavior of the binomial approximation?

Problem 12.2: Using the *BinomCall* function, compute the binomial approximations for the following call option: $S_0 = \$41$, $K = \$40$, $\sigma = 0.3$, $r = 0.08$, $T = 0.25$ (3 months), and $\delta = 0$. Be sure to compute prices for $n = 8, 9, 10, 11$, and 12. What do you observe about the behavior of the binomial approximation?

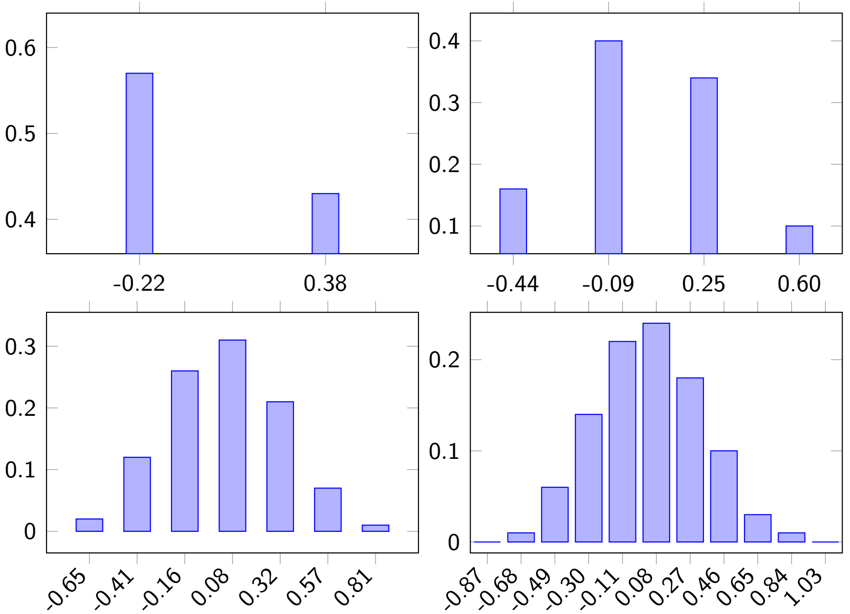
N	Call Price
8	3.464
9	3.361
10	3.454
11	3.348
12	3.446

The observed values are slowly converging towards the Black-Scholes value (3.399). Please note that the binomial solution oscillates as it approaches the Black-Scholes value.

Lognormality and the Binomial Model

- The lognormal distribution is the probability distribution that arises from the assumption that continuously **compounded returns on the stock are normally distributed**.
- In a binomial tree, as we increase the number of periods until expiration, continuously compounded returns approach a normal distribution.
- We can plot probabilities of outcomes (of stock returns) from the binomial tree for different values of n (2, 3, 6, and 10), as shown in the following figure.
- The 10-period binomial tree approaches fairly well the log-normal distribution.

Lognormality and the Binomial Model (cont'd)



Problem 11.13: Let $S = \$100$, $\sigma = 0.30$, $r = 0.08$, $t = 1$, and $\delta = 0$. Use equation (1) to compute the probability of reaching a terminal node and the price at that node and plot the risk-neutral distribution of year-1 stock prices for $n = 50$. What is the risk-neutral probability that $S_1 < \$80$? $S_1 > \$120$?

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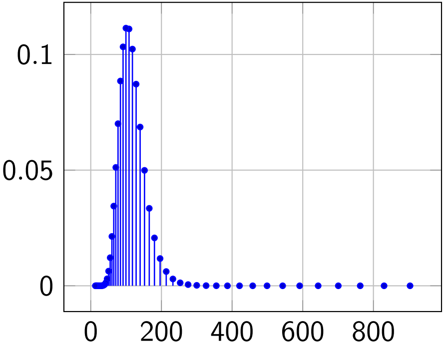
For $n = 50$, we have $u = 1.0450$, $d = 0.9600$, and $p^* = 0.4894$. We get the following diagram:
To obtain the required probabilities we sum all probabilities for which the final stock price is below 80 or above 120 respectively. We obtain

$$\Pr(S < 80) = 0.2006$$
$$\Pr(S > 120) = 0.2829$$



Black-Scholes Assumptions

- Assumptions about stock return distribution
  - Continously compounded returns on the stocks are normally distributed and independent over time
  - The volatility of continuously compounded returns is known and constant
  - Future dividends are known, either as dollar amount or as a fixed dividend yield
- Assumptions about the economic environment
  - The risk-free rate is known and constant
  - There are no transaction costs or taxes
  - It is possible to short-sell costlessly and to borrow at the risk-free rate

Inputs in the Binomial Model and in Black-Scholes

- There are seven inputs to the binomial model and six inputs to the Black-Scholes model:

Inputs	**Binomial Model**	**Black-Scholes**
Current price of the stock, S_0	✓	✓
Strike price of the option, K	✓	✓
Volatility of the stock, σ	✓	✓
Continuously compounded risk-free interest rate, r	✓	✓
Time to expiration, T	✓	✓
Dividend yield on the stock, δ	✓	✓
Number of binomial periods, n	✓	

Convergence from binomial tree to Black-Scholes

- See Appendix of Chapter 12 in Hull (8^{th} edition)
- The binomial price is

$$
\begin{aligned}
C_0 &= e^{-rT}\sum_{k=0}^{n}\frac{n!}{(n-k)!k!}p^{*k}(1-p^*)^{n-k}\max\left[S_0u^kd^{n-k}-K,0\right] && (2)\\
&= e^{-rT}\sum_{k>\alpha}\frac{n!}{(n-k)!k!}p^{*k}(1-p^*)^{n-k}\left(S_0u^kd^{n-k}-K\right) && (3)\\
&= e^{-rT}\left(S_0U_1-KU_2\right) && (4)
\end{aligned}
$$

where

$$
\alpha = \frac{n}{2}-\frac{\ln(S_0/K)+(r-\delta)T}{2\sigma\sqrt{T/n}} \qquad (5)
$$

Convergence from binomial tree to Black-Scholes (cont'd)

- The term

$$U_2 = \sum_{k>\alpha} \frac{n!}{(n-k)!k!} p^{*k}(1-p^*)^{n-k} \tag{6}$$

represents the probability of the option being in the money at maturity. When $n \to \infty$, this probability tends to

$$U_2 = N\left(\frac{np^* - \alpha}{\sqrt{np^*(1-p^*)}}\right) \tag{7}$$

- Replace α, p^*, u, and d in Equation (7) to obtain:

$$U_2 = N\left(\underbrace{\frac{\ln(S_0/K) + (r - \delta - \sigma^2/2)\,T}{\sigma\sqrt{T}}}_{d_2}\right) \tag{8}$$

Convergence from binomial tree to Black-Scholes (cont'd)

- Take now the term

$$U_1 = \sum_{k>\alpha} \frac{n!}{(n-k)!k!} (p^* u)^k [(1-p^*)d]^{n-k} \tag{9}$$

and define

$$q \equiv \frac{p^* u}{p^* u + (1-p^*)d} = \frac{p^* u}{e^{(r-\delta)T/n}} \tag{10}$$

It then follows that

$$1 - q = \frac{(1-p^*)d}{e^{(r-\delta)T/n}} \tag{11}$$

and thus

$$U_1 = e^{(r-\delta)T} \sum_{k>\alpha} \frac{n!}{(n-k)!k!} q^k (1-q)^{n-k} \tag{12}$$

Convergence from binomial tree to Black-Scholes (cont'd)

- This looks like (6), except that we have q instead of p^*. When $n \to \infty$, this probability tends to

$$U_1 = e^{(r-\delta)T} N\left(\frac{nq - \alpha}{\sqrt{nq(1-q)}}\right) \tag{13}$$

- Replace q, α, p^*, u, and d in Equation (13) to obtain:

$$U_1 = e^{(r-\delta)T} N\left(\underbrace{\frac{\ln(S_0/K) + (r - \delta + \sigma^2/2)\,T}{\sigma\sqrt{T}}}_{d_1}\right) \tag{14}$$

- We obtain the Black-Scholes formula:

$$\begin{aligned} C_0 &= e^{-rT}(S_0 U_1 - K U_2) && (15) \\ &= S_0 e^{-\delta T} N(d_1) - K e^{-rT} N(d_2) && (16) \end{aligned}$$

Outline

Black-Scholes Formula for a European Call Option

- The Black-Scholes formula for a European call option on a stock that pays dividends at the continuous rate δ is

$$C_0(S_0, K, \sigma, r, T, \delta) = S_0 e^{-\delta T} N(d_1) - K e^{-rT} N(d_2) \tag{17}$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \delta + \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}} \tag{18}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{19}$$

- $N(x)$ is the cumulative normal distribution function, which is the probability that a number randomly drawn from a standard normal distribution will be less than x.

Black-Scholes Formula for a European Call Option (cont'd)

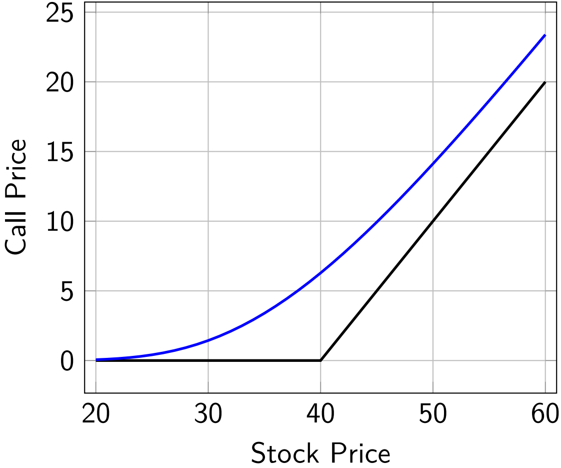
- Let $S_0 = \$41$, $K = \$40$, $\sigma = 0.3$, $r = 0.08$, $T = 1$ year, and $\delta = 0$. Computing the Black-Scholes call price, we obtain

$$\begin{aligned} C_0 &= \$41 \times e^{-0} \times N(0.49898) - \$40 \times e^{-0.08} \times N(0.19898) \\ &= \$41 \times e^{-0} \times 0.6911 - \$40 \times e^{-0.08} \times 0.5789 \\ &= \$6.961 \end{aligned}$$

- Note that this result corresponds to the limit obtained from the binomial model.

Black-Scholes Formula for a European Call Option (cont'd)

- The following figure plots Black-Scholes call option prices (today and at expiration) for stock prices ranging from \$20 to \$60.



A Remarkable Result

- In the binomial model, we have not specified the probabilities of the stock going up and down (which would give us the expected return of the stock).
- The expected return on the stock does not appear in the Black-Scholes formula either.
- This raises a question: Consider a stock with a higher beta (and hence having a higher expected return). A call option on this stock would have a higher probability of settlement in-the-money, hence a higher option price. **Why is this not the case?**

A Remarkable Result (cont'd)

- The Black-Scholes formula (17) provides the answer. This formula shows the composition of the replicating portfolio, which in the binomial case was

$$C_0 = \Delta S_0 + B$$

- We can easily identify Δ (the position in the risky asset) and B (the dollar amount of borrowing or lending) in the Black-Scholes formula:

$$\Delta = e^{-\delta T} N(d_1) \tag{20}$$

$$B = -Ke^{-rT} N(d_2) \tag{21}$$

A Remarkable Result (cont'd)

- If β_S is the stock beta, then the option beta is

$$\beta_{Option} = \frac{\Delta S_0}{\Delta S_0 + B}\beta_S \tag{22}$$

- A high stock beta implies a high option beta, so the discount rate for the expected payoff of the option is correspondingly greater.
- The net result—one of the key insights from the Black-Scholes analysis—is that beta is irrelevant: **The larger average payoff to options on high beta stocks is exactly offset by the larger discount rate.**

Black-Scholes Formula for a European Put Option

- The Black-Scholes formula for a European put option is

$$P_0(S_0, K, \sigma, r, T, \delta) = -S_0 e^{-\delta T} N(-d_1) + K e^{-rT} N(-d_2) \tag{23}$$

where d_1 and d_2 are given by equations (18) and (19).

- Put-call parity must hold:

$$P_0(S_0, K, \sigma, r, T, \delta) = C_0(S_0, K, \sigma, r, T, \delta) + K e^{-rT} - S e^{-\delta T} \tag{24}$$

- This follows from the formulas (17) and (23), together with the fact that for any x, $N(-x) = 1 - N(x)$.

Black-Scholes Formula for a European Put Option (cont'd)

- Let $S_0 = \$41$, $K = \$40$, $\sigma = 0.3$, $r = 0.08$, $T = 1$ year, and $\delta = 0$. Computing the Black-Scholes put price, we obtain

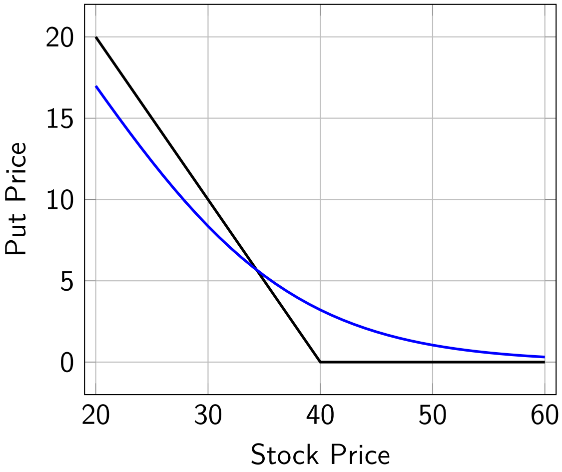
$$\begin{aligned} P_0 &= -\$41 \times e^{-0} \times N(-0.49898) + \$40 \times e^{-0.08} \times N(-0.19898) \\ &= -\$41 \times e^{-0} \times 0.3089 + \$40 \times e^{-0.08} \times 0.4211 \\ &= \$2.886 \end{aligned}$$

- In the binomial model, if we fix the number of periods to $n = 500$, we obtain a price of \$2.885.
- Computing the price using put-call parity (equation 24) yields

$$\begin{aligned} P_0 &= \$6.961 + \$40e^{-0.08} - \$41 \\ &= \$2.886 \end{aligned}$$

Black-Scholes Formula for a European Put Option (cont'd)

- The following figure plots Black-Scholes put option prices (today and at expiration) for stock prices ranging from \$20 to \$60.



Problem 12.20: Let $S = \$100$, $K = \$90$, $\sigma = 30\%$, $r = 8\%$, $\delta = 5\%$, and $T = 1$.

a. What is the Black-Scholes call price?
b. Now price a put where $S = \$90$, $K = \$100$, $\sigma = 30\%$, $r = 5\%$, $\delta = 8\%$, and $T = 1$.

Problem 12.20: Let $S = \$100$, $K = \$90$, $\sigma = 30\%$, $r = 8\%$, $\delta = 5\%$, and $T = 1$.

a. What is the Black-Scholes call price?

	A	B	C	D
1				
2		S0	100	
3		K	90	
4		sigma (v)	0.3	
5		r	0.08	
6		Maturity (T)	1	
7		delta (d)	0.05	
8				
9		=BSCall(C2,C3,C4,C5,C6,C7)		

The Black-Scholes call price is \$17.70

b. Now price a put where $S = \$90$, $K = \$100$, $\sigma = 30\%$, $r = 5\%$, $\delta = 8\%$, and $T = 1$.

	A	B	C	D
1				
2		S0	90	
3		K	100	
4		sigma (v)	0.3	
5		r	0.05	
6		Maturity (T)	1	
7		delta (d)	0.08	
8				
9		=bsput(C2,C3,C4,C5,C6,C7)		

The Black-Scholes put price is \$17.70

Outline

Volatility is Crucial in Finance

- for forecasting return
- for the pricing of derivatives
- for asset allocation (trade-off between return and risk)
- for risk management (evaluation of the risk of a portfolio)

The major problem with volatility is that it is not directly observable from returns

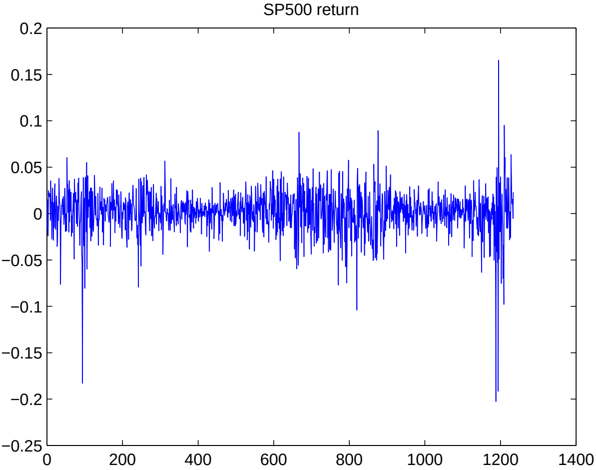
- **Unconditional volatility** is estimated as the sample standard deviation

$$\sigma = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_t - \mu)^2} \tag{25}$$

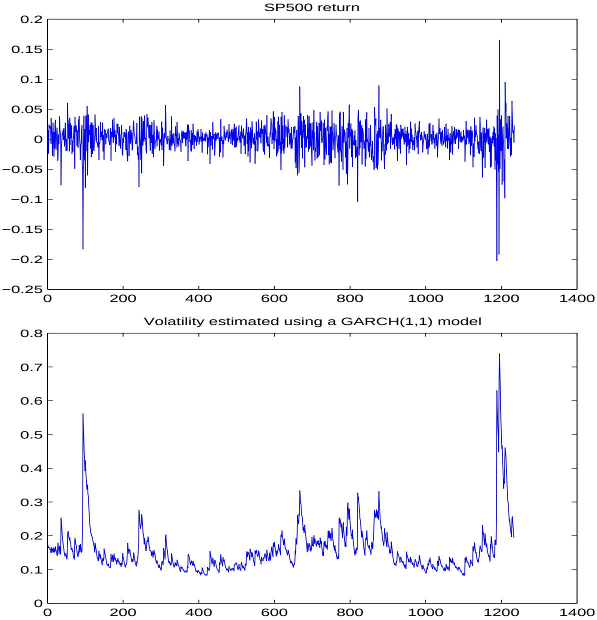
  where r_t is the log return on period t and μ is the sample mean over T periods

- However, volatility is actually not constant through time. Therefore, **conditional volatility** σ_t is a more relevant measure of risk at time t.

Volatility is not constant through time **(volatility clustering)**



GARCH(1,1) volatility for the S&P 500 (weekly data)



Implied Volatility

- Volatility is unobservable
- Choosing a volatility to use in pricing an option is difficult but important
- Using history of returns is not the best approach, because history is not a reliable guide to the future.
- We can invert Black-Scholes formula to obtain implied volatility
- We cannot use implied volatility to assess whether an option price is correct, but implied volatility does tell us the market's assessment of volatility

Implied Volatility (cont'd)

- Example: Let $S = \$100$, $K = \$90$, $r = 8\%$, $\delta = 5\%$, and $T = 1$. The market option price for a call option is \$18.25. What is the volatility that gives this option price?
- We must invert the following formula

$$\$18.25 = \text{BSCall}(100, 90, \sigma, 0.08, 1, 0.05)$$

- We use the implied volatility function:

	A	B	C	D
1				
2		S0	100	
3		K	90	
4				
5		r	0.08	
6		Maturity (T)	1	
7		delta (d)	0.05	
8				
9		Call Price	18.25	
10				
11		=bscallimpvol(C2,C3,C4,C5,C6,C7,C9)		

We find that setting $\sigma = 31.73\%$ gives us a call price of \$18.25

Implied Volatility (cont'd)

- There is a systematic pattern of implied volatility across strike prices, called **volatility skew**
- The volatility skew is not related to whether an option is a put or a call, but rather to differences in the strike price and time to expiration
- Explaining these patterns is a challenge for option pricing theory

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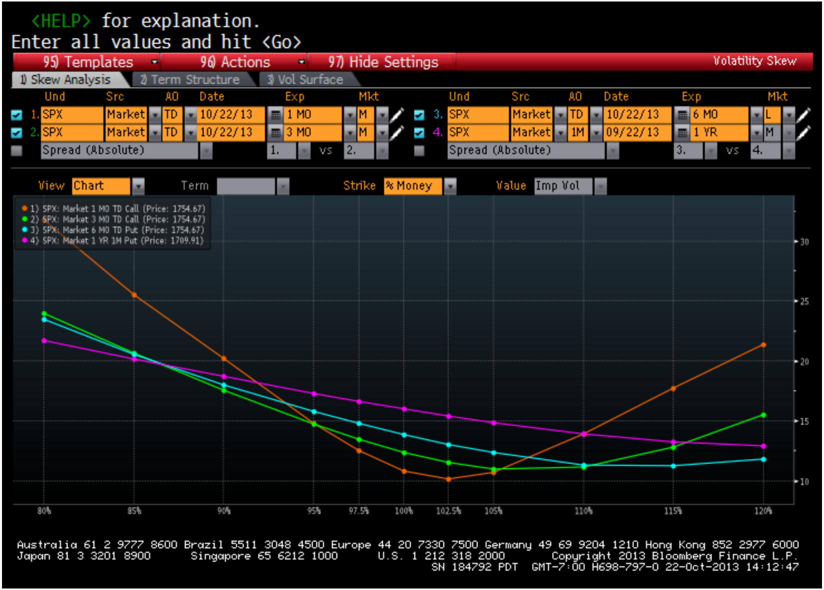

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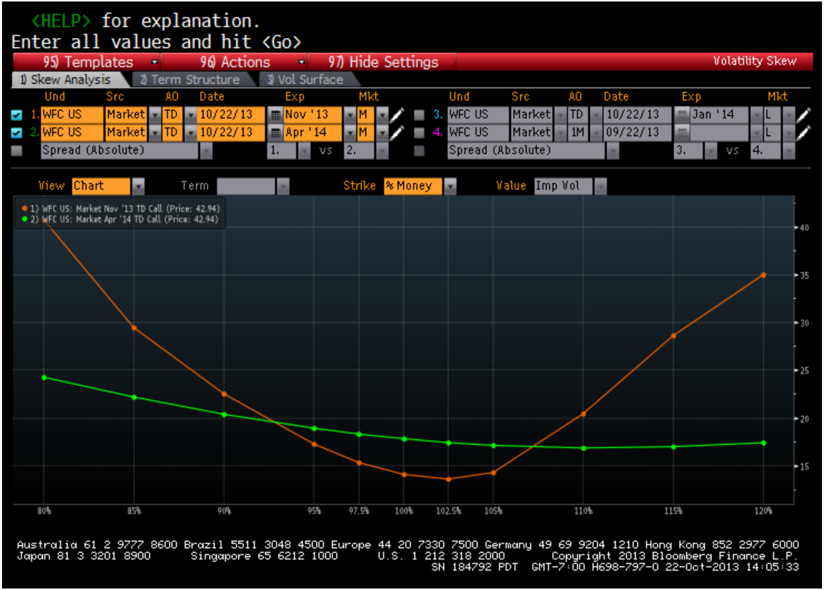


Using Implied Volatility

- Implied volatility is important for a number of reasons
  - If you need to price an option for which you **cannot** observe a market price, you can use implied volatility to generate a price consistent with the price of traded options
  - Implied volatility is often used as a quick way to describe the level of option prices on a given underlying asset. Option prices are quoted sometimes in terms of volatility, rather than as a dollar price
  - Volatility skew provides a measure of how well option pricing models work
- Just as stock markets provide information about stock prices and permit trading stocks, option markets provide information about volatility, and, in effect, permit the trading of volatility.

Volatility Trading

- Just as stock investors think they know something about the direction of the stock market, or bond investors think they can foresee the probable direction of interest rates, so you may think you have insight into the level of future volatility
- What do you do if you simply want exposure to a stock's volatility?
- Stock options are impure: they provide exposure to both the direction of the stock price and its volatility

Volatility Trading: The Traditional Way

- Buy/sell straddles or strangles. Easy to implement, but has drawbacks:
  - Straddles and strangles yield a non-null delta once the stock price moves away from the initial ATM strike price
  - Prices need to move sharply in the case of a buy-and-hold strategy
- Option delta-hedging (avoiding sensitivity to asset price). Drawbacks:
  - The P&L generated by delta hedging an option is a function of numerous sources of risk: variance risk, volatility path dependency risk, model risk, liquidity risk, dividends risk.
  - Variance risk sometimes accounts only for 50% of the total P&L.

Advantages of Variance and Volatility Swaps

- The easy way to trade volatility is to use **volatility swaps**, sometimes called **realized volatility forward contracts**
- These products provide pure exposure to volatility (and only to volatility)
- No need to delta hedge $\Rightarrow$ allows for buy-and-hold variance strategy
- OTC products but standardized contracts with maturity similar to listed options (April 14, June 14, ...)
- Strong liquidity thanks to several investment banks providing live prices
- Either long or short positions
- On indices as well as single stocks

Volatility Swaps

- A stock volatility swap is a forward contract on annualized volatility. Its payoff at expiration is equal to

$$(\sigma_R - K_{vol}) \times N \tag{26}$$

  where
  - σ_R is the realized stock volatility (quoted in annual terms) over the life of the contract
  - K_{vol} is the annualized volatility delivery price, typically quoted as a volatility, for example 30%
  - N is the notional amount of the swap in dollars per annualized volatility point, for example $N = \$250,000/(\text{volatility point})$

Volatility Swaps (cont'd)

- The holder of a volatility swap at expiration receives N dollars for every point by which the stock's realized volatility σ_R has exceeded the volatility delivery price K_{vol}
- He or she is **swapping** a fixed volatility K_{vol} for the actual (floating) future volatility σ_R
- The procedure for calculating the realized volatility should be clearly specified with respect to the following aspects:
  - How frequently the return is measured
  - Whether returns are continuously compounded or arithmetic
  - Whether the variance is measured by subtracting the mean or by simply squaring the returns
  - The period of time over which variance is measured
  - How to handle days on which trading does not occur

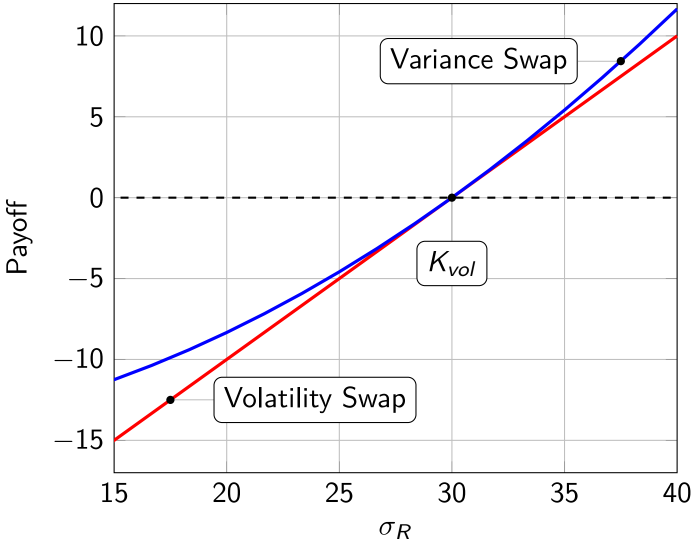
Variance Swaps

- A variance swap is a forward contract on annualized variance, the square of the realized volatility. Its payoff at expiration is equal to

$$\left(\sigma_R^2 - K_{var}\right) \times N \tag{27}$$

  where
  - σ_R^2 is the realized stock variance (quoted in annual terms) over the life of the contract
  - K_{var} is the delivery price for variance, for example $(30\%)^2$
  - N is the notional amount of the swap in dollars per annualized volatility point squared, for example $N = \$100,000/(\text{volatility point})^2$

Variance vs. Volatility Contracts



Replicating Variance Swaps

- If you own a portfolio of options of all strikes, weighted in inverse proportion to the square of the strike level, you will obtain an exposure to variance that is independent of stock price, just what is needed to trade variance.
- There is no simple replication strategy for synthesizing a volatility swap.
- We can approximate a volatility swap by statically holding a suitably chosen variance contract.

The CBOE Volatility Index (VIX)

- In 1993, the Chicago Board Options Exchange (CBOE) introduced the CBOE Volatility Index, VIX, which was originally designed to measure the market's expectation of 30-day volatility implied by at-the-money S&P 100 Index option prices
- Ten years later in 2003, CBOE and Goldman Sachs updated the VIX to reflect a new way to measure expected volatility. The new VIX is based on the S&P 500 Index, and estimates expected volatility by averaging the weighted prices of puts and calls over a wide range of strike prices.

The CBOE Volatility Index (VIX)

- The CBOE utilizes a wide variety of strike prices for SPX puts and calls to calculate the VIX
- VIX provides important information about investor sentiment. Since volatility often signifies financial turmoil, the VIX is often referrred to as the **investor fear gauge**.
- Investors can use VIX options and VIX futures to hedge their portfolios. The VIX is a good hedging tool because it has a strong negative correlation to the S&P 500

The CBOE Volatility Index (VIX)

Bloomberg: VIX <INDEX> DES <GO>

VIX ↑ 13.59 +.92

At 12:07 d O 12.99 H 13.79 L 12.93 Prev 12.67

VIX Index | 99) Feedback | Description

CBOE SPX VOLATILITY INDX

The Chicago Board Options Exchange Volatility Index reflects a market estimate of future volatility, based on the weighted average of the implied volatilities for a wide range of strikes. 1st & 2nd month expirations are used until 8 days from expiration, then the 2nd and 3rd are used.
[BBGID BBG000JW9B77]

3) Price Chart (GP)

Prices

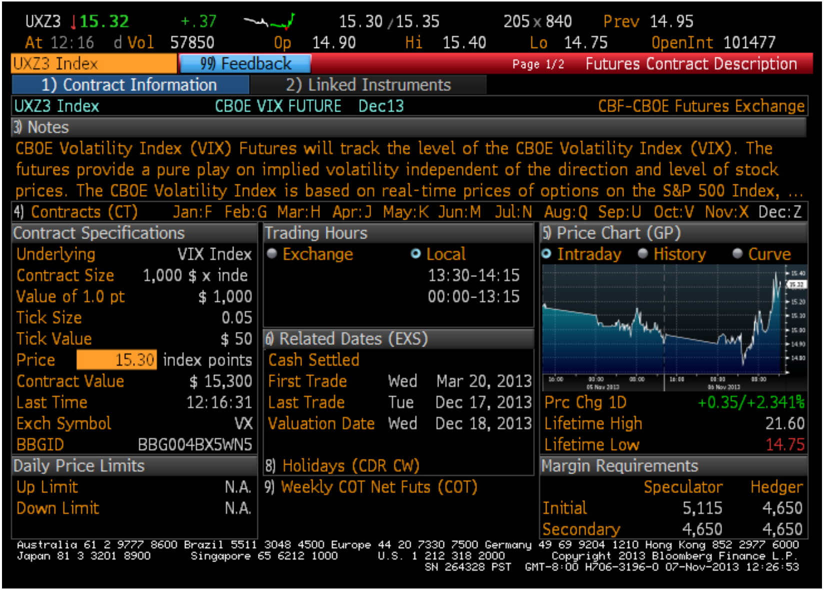
5) Intraday Chart (GIP)	Last	13.58 (12:06:15)
6) Bar Chart (GPO)	52 Wk High	22.72 (12/28/12)
	52 Wk Low	11.05 (03/14/13)

Index Information

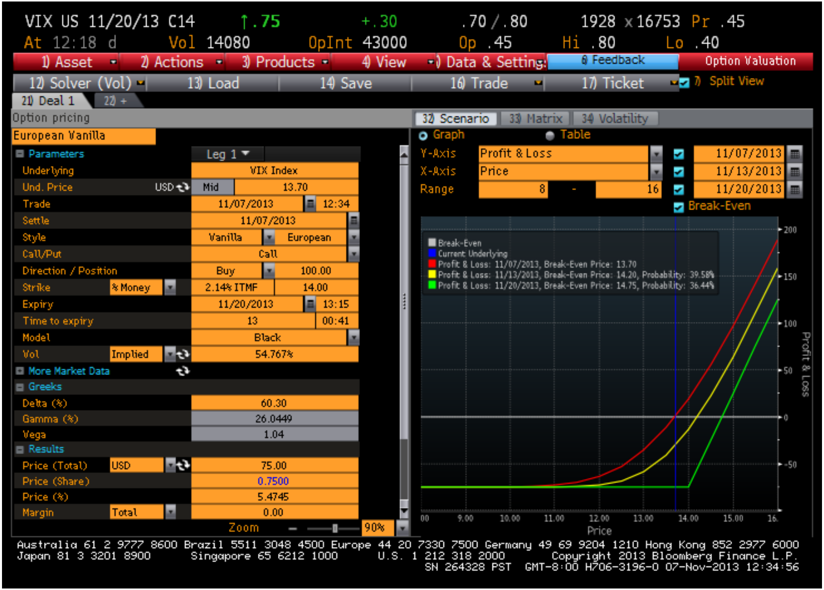
Trading Hours	06:30-13:15
Currency	USD
Volume	N.A.

4) Return Analysis (TRA)		% Chg	Annual
1 Day	12.67	+7.18	+Lge
5 Days	13.75	-1.24	-47.73
MTD	13.75	-1.24	-47.73
QTD	16.60	-18.19	-85.47
YTD	18.02	-24.64	-28.25
1 Month	19.41	-30.04	-98.51
3 Months	12.98	+4.62	+19.64
6 Months	12.83	+5.85	+11.93
1 Year	19.08	-28.83	-28.83
2 Years	29.85	-54.51	-32.51
5 Years	56.10	-75.79	-24.69
Qtr 3:12	17.08	-7.90	-28.39
Qtr 4:12	15.73	+14.56	+73.53
Qtr 1:13	18.02	-29.52	-75.80
Qtr 2:13	12.70	+32.76	+215.54
Qtr 3:13	16.86	-1.54	-6.11

Bloomberg: UXZ3 <INDEX> DES <GO>



Bloomberg: VIX UX 11/20/13 C14 <INDEX> OV <GO>



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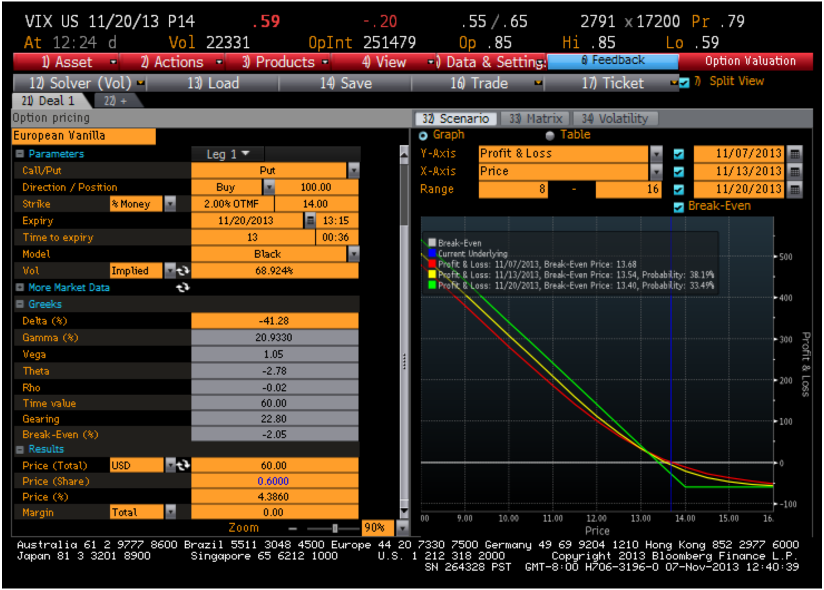
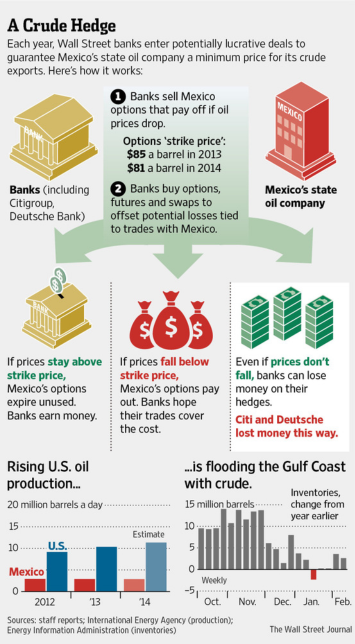


Outline

WSJ, Feb 2014: Big Banks Take Hits On Trusty Oil Hedge



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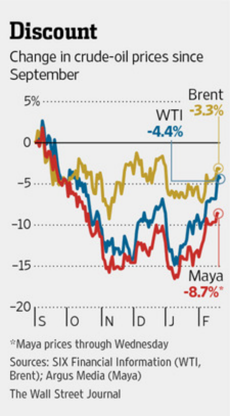

Reuters, Oct 15, 2014: Banks rush to hedge oil option deltas, accelerating rout

- Wall Street banks have scrambled to **neutralize their exposure** to big oil options trades.
- Banks have written protection to companies and they sell futures contracts to offset option deals that are **unexpectedly** in the money.
- Oil producers seek to **hedge their production** by buying put options with strikes \$75 to \$85.
- As futures prices approach the these strike levels, big banks that have sold put options (or similar hedges) to oil producers are forced to protect themselves through **delta hedging**: they sell futures to remain **market neutral**.
- The delta hedging selling was cited by several traders as a factor behind Tuesday's **rapid swoon in prices**.

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Market-Maker Risk

- A **market-maker** stands ready to sell to buyers and to buy from sellers.
- Without hedging, an active market-maker will have an arbitrary position generated by fulfilling customer orders. This arbitrary portfolio has uncontrolled risk.
- Consequently, market-makers attempt to hedge the risk of their position.
- We will se here how they do so.

Market-Maker Risk (cont'd)

- Suppose a customer wishes to buy 100 European call options with maturity of 91 days. The market-maker fills this order by selling 100 call options. To be specific, suppose that $S = \$41$, $K = \$40$, $\sigma = 0.3$, $r = 0.08$ (continuously compounded), and $\delta = 0$. We will let T denote the expiration time of the option and t the present, so time to expiration is $T - t$. Thus, $T - t = 91/365 = 0.249$.
- Suppose that the market-maker does not hedge the written option (**naked position**) and the stock has the following evolution over the next 5 days:

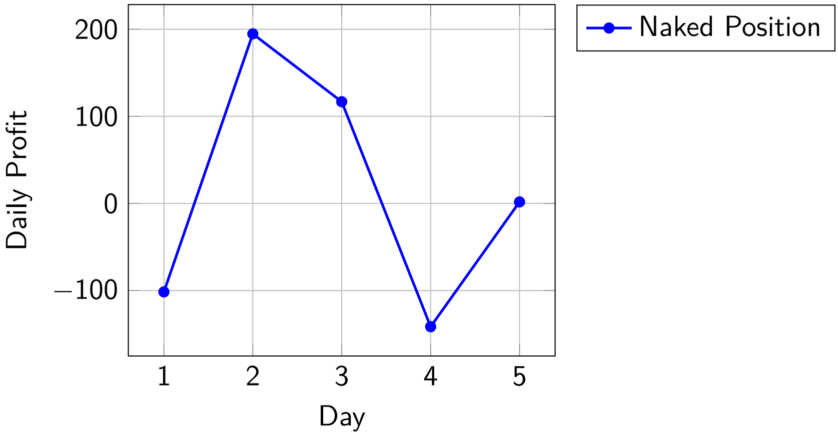
Day	Stock ($)
0	41
1	42.5
2	39.5
3	37
4	40
5	40

Market-Maker Risk (cont'd)

- We can measure the profit of the market-maker by **marking-to-market** the position: **if we liquidated the position today, what would be the gain or loss?**

Day	Stock ($)	Call Position ($)	Daily Profit ($)
0	41	-339.47	
1	42.5	-441.04	-101.57
2	39.5	-246.31	194.73
3	37	-129.49	116.82
4	40	-271.04	-141.55
5	40	-269.27	1.77

Market-Maker Risk (cont'd)



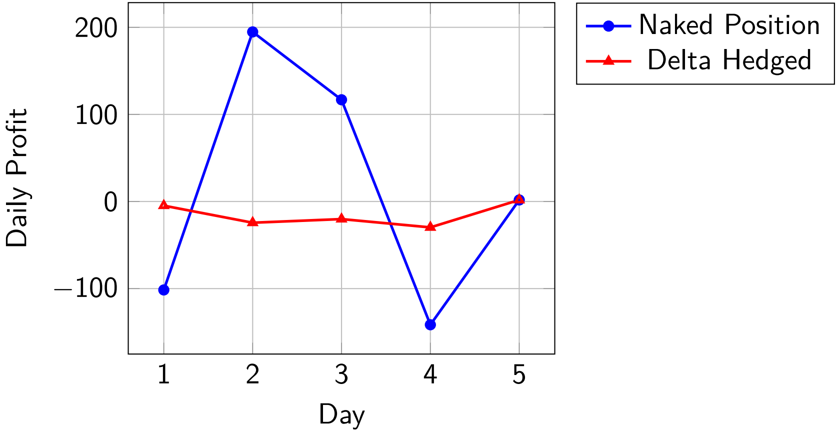
Delta-Hedging

- Suppose the market-maker hedges the position with shares. At time 0, the delta of a call at a stock price of \$41 is 0.645.
- This suggests that a \$1 increase in the stock price should increase the value of the option by approximately \$0.645.
- The market-maker takes an offsetting position in shares, position that hedges the fluctuations in the option price. We say that such a position is **delta-hedged**.
- Then, the market-maker rebalances the portfolio each day, by computing the new delta of the call.
- The following table summarizes delta, the number of purchased shares, the net investment, and profit for each day for 5 days (interest expenses are ignored for simplicity).

Delta-Hedging (cont'd)

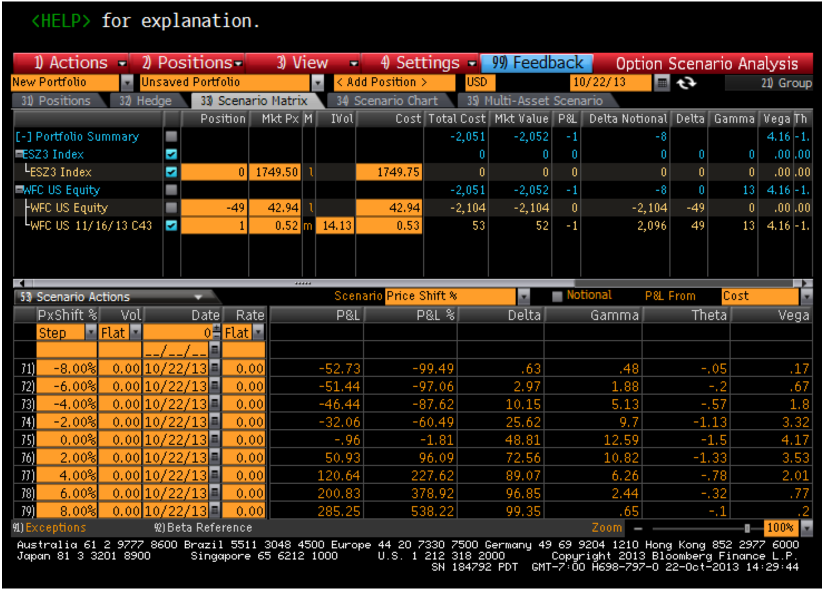
Day	Stock ($)	Option Delta	Stock Position (# shares)	Daily Profit (Call)	Daily profit (Shares)	Daily profit (Total)
0	41	0.645	64.54			
1	42.5	0.730	73.03	-101.57	96.81	-4.77
2	39.5	0.548	54.81	194.73	-219.10	-24.38
3	37	0.373	37.27	116.82	-137.02	-20.20
4	40	0.581	58.06	-141.55	111.82	-29.73
5	40	0.580	58.01	1.77	0.00	1.77

Delta-Hedging (cont'd)



- Delta hedging prevents the position from reacting to small changes in the underlying stock. **For large changes, we need to take into account the fluctuations in delta.**

WFC US <EQUITY> OSA <GO> (not delta-hedged)



<HELP> for explanation.

1) Actions | 2) Positions | 3) View | 4) Settings | 99) Feedback | Option Scenario Analysis

New Portfolio | Unsaved Portfolio | < Add Position > | USD | 10/22/13 | 21) Group

31) Positions | 32) Hedge | 33) Scenario Matrix | 34) Scenario Chart | 35) Multi-Asset Scenario

	Position	Mkt Px	M	IVol	Cost	Total Cost	Mkt Value	P&L	Delta Notional	Delta	Gamma	Vega	Th
[-] Portfolio Summary						-2,051	-2,052	-1	-8			4.16	-1.
ESZ3 Index						0	0	0	0	0	0	.00	.00
ESZ3 Index	0	1749.50	l		1749.75	0	0	0	0	0	0	.00	.00
WFC US Equity						-2,051	-2,052	-1	-8	0	13	4.16	-1.
WFC US Equity	-49	42.94	l		42.94	-2,104	-2,104	0	-2,104	-49	0	.00	.00
WFC US 11/16/13 C43	1	0.52	m	14.13	0.53	53	52	-1	2,096	49	13	4.16	-1.

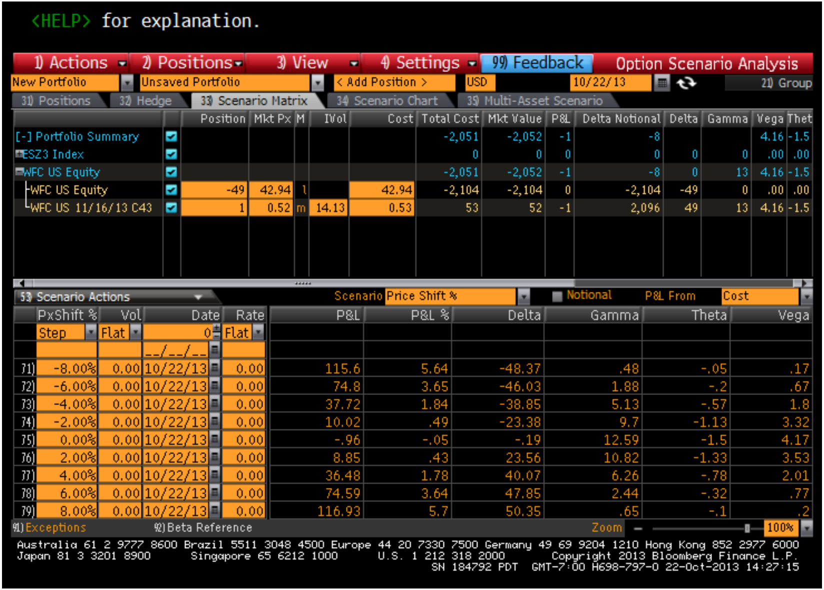
53) Scenario Actions | Scenario Price Shift % | Notional | P&L From Cost

	PxShift %	Vol	Date	Rate	P&L	P&L %	Delta	Gamma	Theta	Vega
	Step	Flat	0	Flat						
			__/__/__							
71)	-8.00%	0.00	10/22/13	0.00	-52.73	-99.49	.63	.48	-.05	.17
72)	-6.00%	0.00	10/22/13	0.00	-51.44	-97.06	2.97	1.88	-.2	.67
73)	-4.00%	0.00	10/22/13	0.00	-46.44	-87.62	10.15	5.13	-.57	1.8
74)	-2.00%	0.00	10/22/13	0.00	-32.06	-60.49	25.62	9.7	-1.13	3.32
75)	0.00%	0.00	10/22/13	0.00	-.96	-1.81	48.81	12.59	-1.5	4.17
76)	2.00%	0.00	10/22/13	0.00	50.93	96.09	72.56	10.82	-1.33	3.53
77)	4.00%	0.00	10/22/13	0.00	120.64	227.62	89.07	6.26	-.78	2.01
78)	6.00%	0.00	10/22/13	0.00	200.83	378.92	96.85	2.44	-.32	.77
79)	8.00%	0.00	10/22/13	0.00	285.25	538.22	99.35	.65	-.1	.2

91) Exceptions | 92) Beta Reference | Zoom 100%

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2013 Bloomberg Finance L.P.
SN 184792 PDT GMT-7:00 H698-797-0 22-Oct-2013 14:29:44

WFC US <EQUITY> OSA <GO> (delta-hedged)



<HELP> for explanation.

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[-] Portfolio Summary						-2,051	-2,052	-1	-8			4.16	-1.5
ESZ3 Index						0	0	0	0	0	0	.00	.00
WFC US Equity						-2,051	-2,052	-1	-8	0	13	4.16	-1.5
WFC US Equity	-49	42.94	l		42.94	-2,104	-2,104	0	-2,104	-49	0	.00	.00
WFC US 11/16/13 C43	1	0.52	m	14.13	0.53	53	52	-1	2,096	49	13	4.16	-1.5

53) Scenario Actions | Scenario Price Shift % | Notional | P&L From Cost

	PxShift %	Vol	Date	Rate	P&L	P&L %	Delta	Gamma	Theta	Vega
	Step	Flat	0	Flat						
			__/__/__							
71)	-8.00%	0.00	10/22/13	0.00	115.6	5.64	-48.37	.48	-.05	.17
72)	-6.00%	0.00	10/22/13	0.00	74.8	3.65	-46.03	1.88	-.2	.67
73)	-4.00%	0.00	10/22/13	0.00	37.72	1.84	-38.85	5.13	-.57	1.8
74)	-2.00%	0.00	10/22/13	0.00	10.02	.49	-23.38	9.7	-1.13	3.32
75)	0.00%	0.00	10/22/13	0.00	-.96	-.05	-.19	12.59	-1.5	4.17
76)	2.00%	0.00	10/22/13	0.00	8.85	.43	23.56	10.82	-1.33	3.53
77)	4.00%	0.00	10/22/13	0.00	36.48	1.78	40.07	6.26	-.78	2.01
78)	6.00%	0.00	10/22/13	0.00	74.59	3.64	47.85	2.44	-.32	.77
79)	8.00%	0.00	10/22/13	0.00	116.93	5.7	50.35	.65	-.1	.2

91) Exceptions | 92) Beta Reference | Zoom 100%

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2013 Bloomberg Finance L.P.
SN 184792 PDT GMT-7:00 H698-797-0 22-Oct-2013 14:27:15

Outline

Option Greeks

- Option Greeks are formulas that express the change in the option price when an input to the formula changes, **taking as fixed all other inputs**.
- They are used to assess risk exposures. For example:
  - A market-making bank with a portfolio of options would want to understand its exposure to stock price changes, interest rates, volatility, maturity, etc.
  - A portfolio manager wants to know what happens to the value of a portfolio of stock index options if there is a change in the level of the stock index.
  - An options investor would like to know how interest rate changes and volatility changes affect profit and loss.

Option Greeks (cont'd)

- Before providing detailed definition of the Greeks, let's have some intuition on how changes in inputs affect option prices:

Change in input	**Change in call price**	**Change in put price**
$S_t \uparrow$	$C_t \uparrow$	$P_t \downarrow$
$\sigma \uparrow$	$C_t \uparrow$	$P_t \uparrow$
$T - t \downarrow (t \uparrow)$	C_t generally $\downarrow$	P_t ambiguous
$r \uparrow$	$C_t \uparrow$	$P_t \downarrow$
$\delta \uparrow$	$C_t \downarrow$	$P_t \uparrow$

Table 1 : Changes in Black-Scholes inputs and their effect on option prices.

- An increase in the stock price (S_t) raises the chance that the call will be exercised, thus raises the call option price. Conversely, it lowers the put option price.

Option Greeks (cont'd)

- An increase in volatility raises the price of a call or put option, because it increases the expected value if the option is exercised.
- Options generally—but not always—become less valuable as time to expiration decreases, i.e., there is a **time decay**. There are exceptions, for example deep-in-the-money call options on an asset with high dividend yield and deep-in-the-money puts.
- A higher interest rate reduces the present value of the strike (to be paid by a call option holder), and thus increases the call price. The put option entitles the owner to receive the strike, whose present value is lower with a higher interest rate. Thus, a higher interest rate decreases the put price.
- A call entitle the holder to receive stock, but without dividends prior to expiration. Thus, the greater the dividend yield, the lower the call price. Conversely, a put option is more valuable when the dividend yield is greater.

Option Greeks (cont'd)

- The Greeks are tools that let us to quantify these relationships:

Input	Greek	Definition	Mnemonic
S_t	Δ (Delta)	Measures the option price change when the stock price increases by \$1	
S_t	Γ (Gamma)	Measures the change in Δ when the stock price increases by \$1	
S_t	Ω (Elasticity)	Measures the percentage change in the option price when the stock price increases by 1%	
σ	Vega	Measures the option price change when there is an increase in volatility of 1%	vega $\leftrightarrow$ volatility
t	θ (theta)	Measures the option price change when there is a decrease in the time to maturity (increase in calendar time) of 1 day	theta $\leftrightarrow$ time
r	ρ (rho)	Measures the option price change when there is an increase in the interest rate of 1% (100 basis points)	rho $\leftrightarrow$ r
δ	Ψ (Psi)	Measures the option price change when there is an increase in the continuous dividend yield of 1% (100 basis points)	

Option Greeks (cont'd)

- Let us come back to Table 1 and complete it with the proper signs of the Greeks:

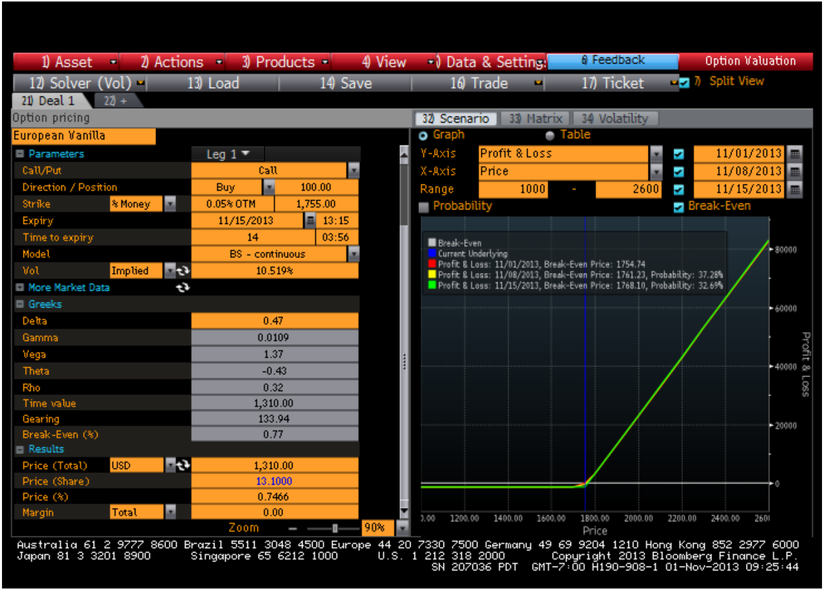
Input	Call Option		Put Option	
$S_t \uparrow$	$C_t \uparrow$	$\Delta_{Call} > 0$	$P_t \downarrow$	$\Delta_{Put} < 0$
	$\Delta_{Call} \uparrow$	$\Gamma_{Call} > 0$	$\Delta_{Put} \uparrow$	$\Gamma_{Put} > 0$
	$\Omega_{Call} \geq 1$		$\Omega_{Put} \leq 0$	
$\sigma \uparrow$	$C_t \uparrow$	$Vega_{Call} > 0$	$P_t \uparrow$	$Vega_{Put} > 0$
$t \uparrow$	C_t gen. $\downarrow$	θ_{Call} gen. < 0	P_t ambig.	θ_{Put} any
$r \uparrow$	$C_t \uparrow$	$\rho_{Call} > 0$	$P_t \downarrow$	$\rho_{Put} < 0$
$\delta \uparrow$	$C_t \downarrow$	$\Psi_{Call} < 0$	$P_t \uparrow$	$\Psi_{Put} > 0$

Option Greeks: Example

- The Greeks are mathematical derivatives of the option price formula with respect to the inputs.
- Suppose that the stock price is $S_t = \$41$, the strike price is $K = \$40$, volatility is $\sigma = 0.3$, the risk-free rate is $r = 0.08$, the time to expiration is $T - t = 1$, and the dividend yield is $\delta = 0$. The values for the Greeks are

Input	**Call Option**		**Put Option**	
S_t	$\Delta_{Call} = 0.691$	> 0	$Delta_{Put} = -0.309$	< 0
	$\Gamma_{Call} = 0.029$	> 0	$\Gamma_{Put} = 0.029$	> 0
	$\Omega_{Call} = 4.071$	≥ 1	$\Omega_{Put} = -4.389$	≤ 0
σ	$Vega_{Call} = 0.144$	> 0	$Vega_{Put} = 0.144$	> 0
t	$\theta_{Call} = -0.011$	gen. < 0	$\theta_{Put} = -0.003$	any
r	$\rho_{Call} = 0.214$	> 0	$\rho_{Put} = -0.156$	< 0
$\delta \uparrow$	$\Psi_{Call} = -0.283$	< 0	$\Psi_{Put} = 0.127$	> 0

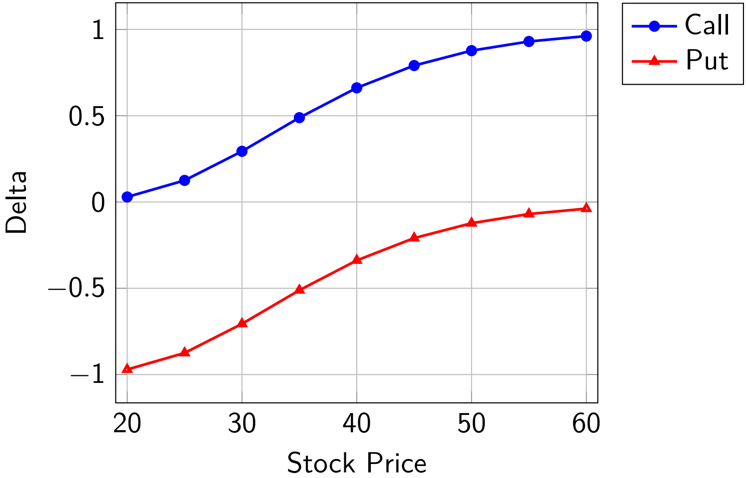
Bloomberg: SPX 11/16/13 C1755 <INDEX> OV <GO>



Bloomberg: Greeks Convention

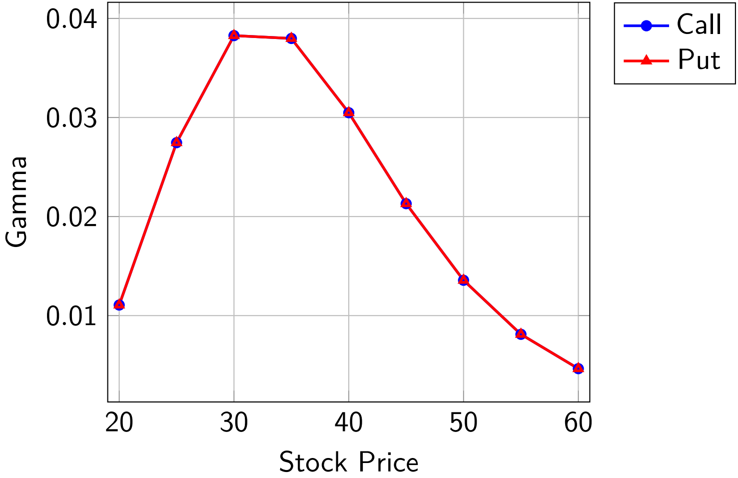
Delta (Δ)

Measures the change in the option price for a \$1 change in the stock price:



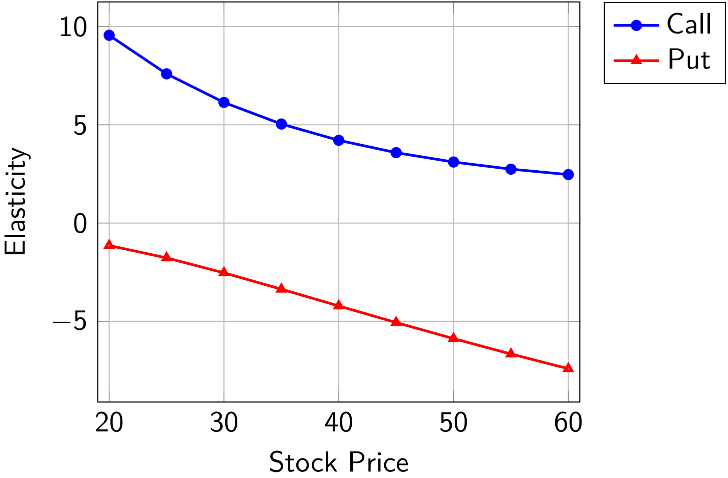
Gamma (Γ)

Measures the change in delta when the stock price changes:



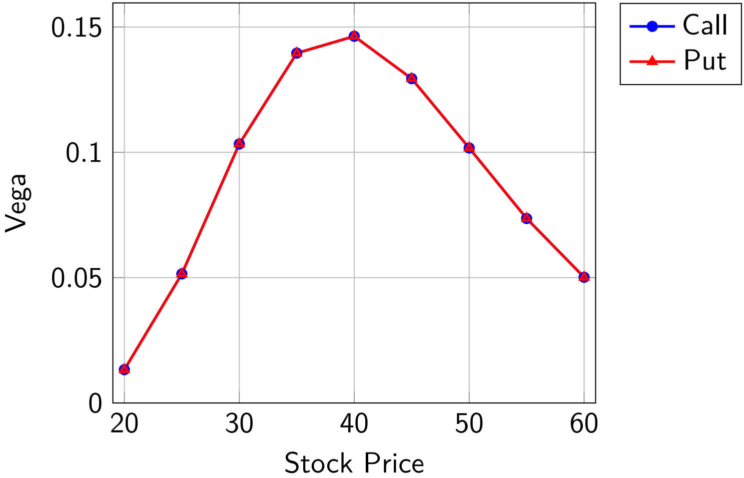
Elasticity (Ω)

Measures the percentage change in the option price relative to the percentage change in the stock price:



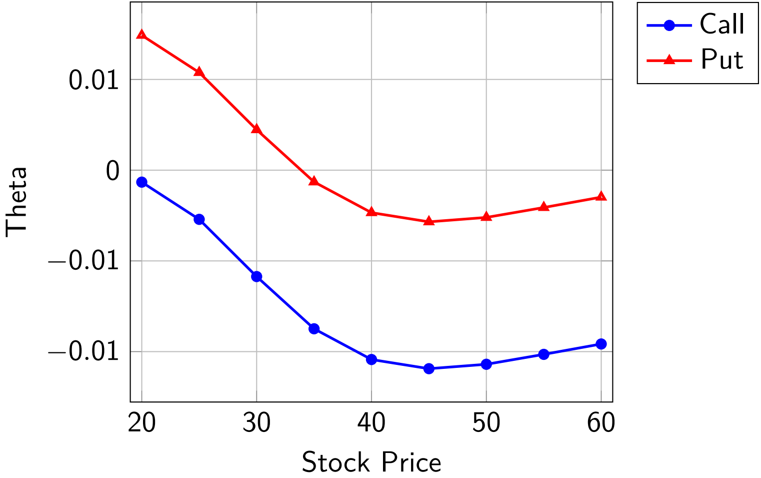
Vega

Measures the change in the option price when volatility changes (divide by 100 for a change per percentage point):



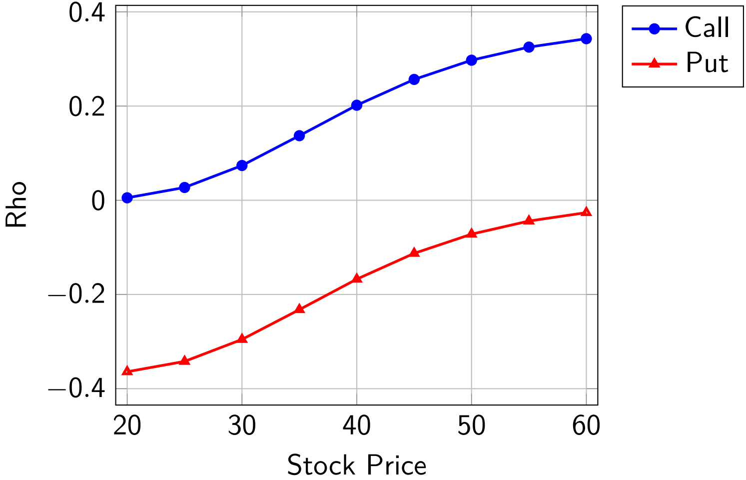
Theta (θ)

Measures the change in the option price with respect to calendar time, t, holding fixed the maturity date T. To obtain per-day theta, divide by 365.



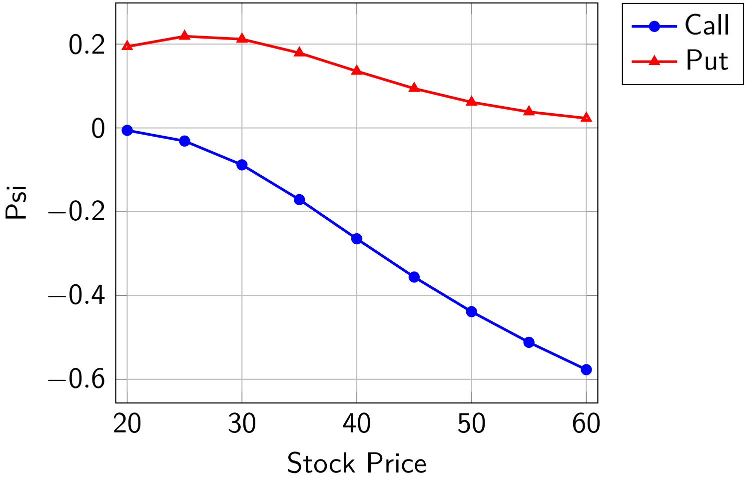
Rho (ρ)

Measures the change in the option price when the interest rate changes (divide by 100 for a change per percentage point, or by 10,000 for a change per basis point):



Psi (Ψ)

Measures the change in the option price when the continuous dividend yield changes (divide by 100 for a change per percentage point):



Outline

Gamma-Neutrality

- Gamma hedging is the construction of options positions that are hedged such that the total gamma of the position is zero.
- We cannot do this using just the stock, because the gamma of the stock is zero (the delta of a stock is constant and equal to 1).
- Hence, we must acquire another option in an amount that offsets the gamma of the written call.
- In addition to the 91-day call from the previous example, consider a 45-strike 120-day call.
- The ratio of the gamma of the two options is

$$\frac{\Gamma_{K=40,T-t=91}}{\Gamma_{K=45,T-t=120}} = \frac{0.0606}{0.0540} = 1.1213 \tag{28}$$

- Thus, we need to buy 1.1213 of the 45-strike options for every 40-strike option we have sold.

Gamma-Neutrality (cont'd)

- The Greeks resulting form this position are in the last column of the following table:

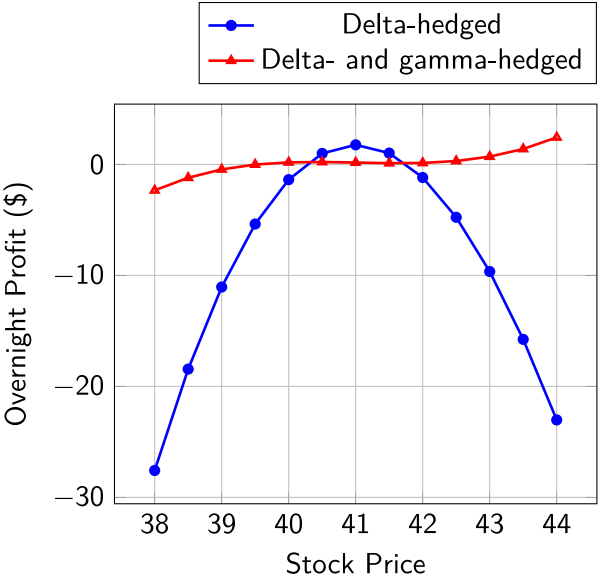
	40-Strike Call	**45-Strike Call**	**Total Position**
Price (\$)	3.395	1.707	-1.481
Delta (Δ)	0.645	0.381	-0.218
Gamma (Γ)	0.061	0.054	0
Theta (θ)	-0.018	-0.014	0.002

- Since delta is -0.218, we need to buy 21.8 shares of stock to be both delta- and gamma-hedged.

Gamma-Neutrality (cont'd)

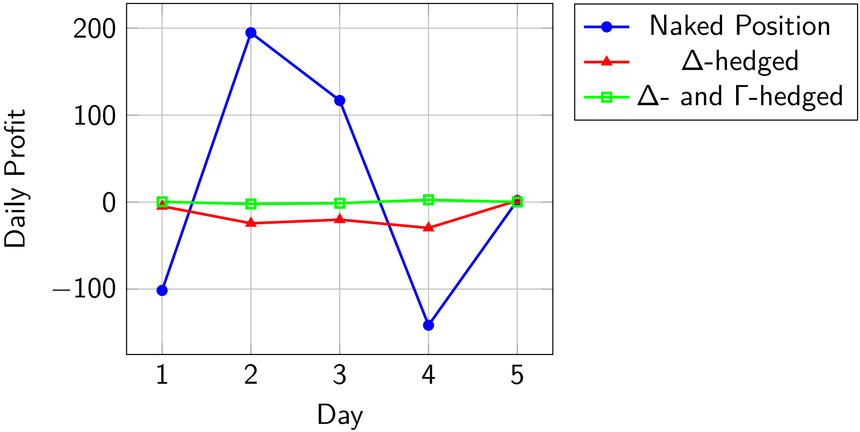
- We can compare the delta-hedged position with the delta- and gamma-hedged position. The delta-hedged position has the problem that large moves always cause losses.
- The delta- and gamma-hedged position loses less if there is a large move down, and can make money if the stock price increases.
- This can be seen from the following figure. It compares the 1-day holding period profit for delta-hedged position described earlier and delta- and gamma hedged position.

Gamma-Neutrality (cont'd)

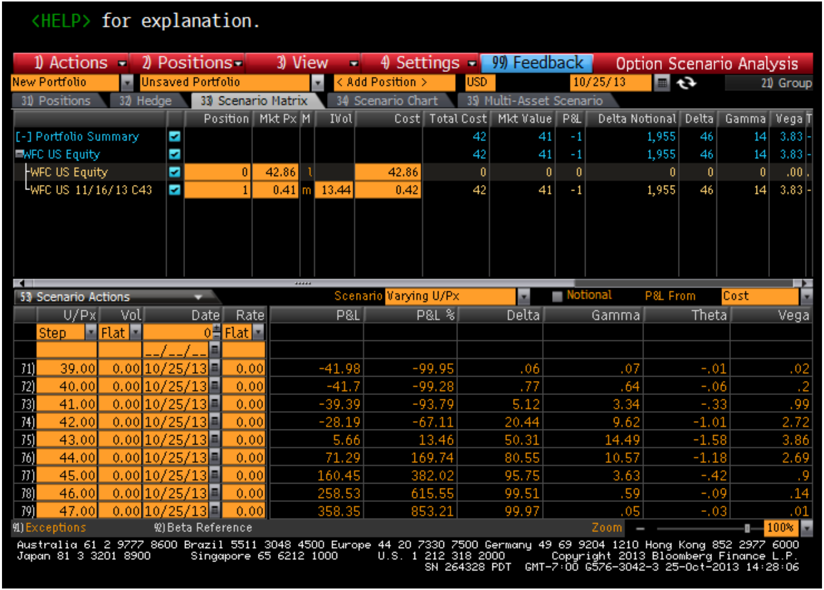


Gamma-Neutrality (cont'd)

- Delta-gamma hedging prevents the position from reacting to large changes in the underlying stock:



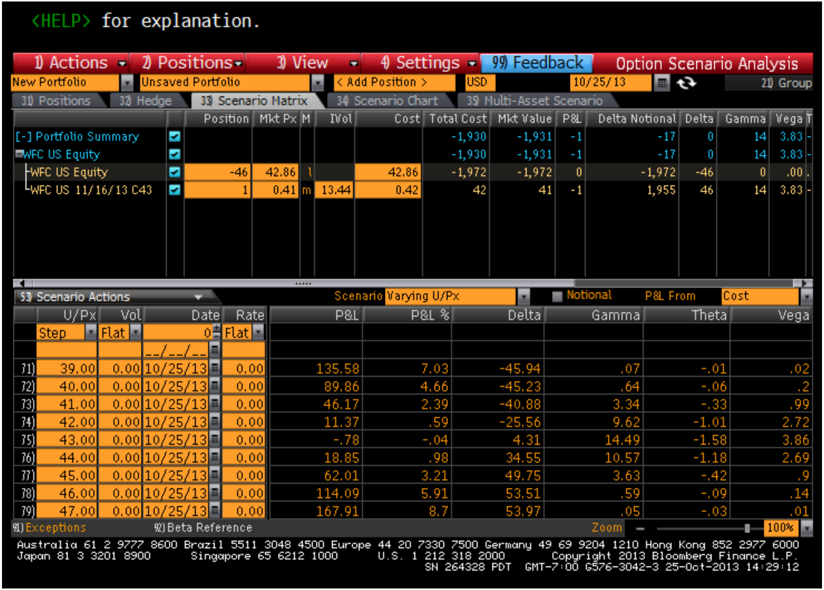
No Hedge



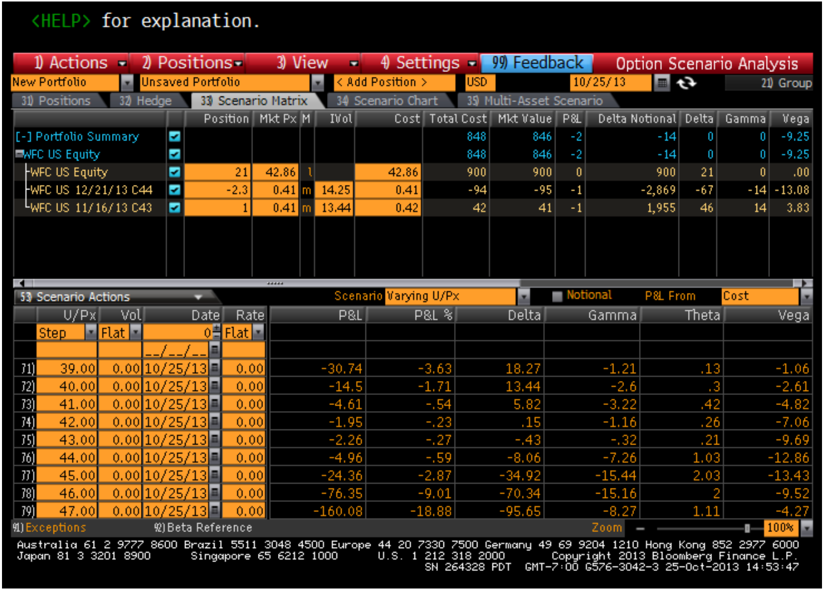
		Position	Mkt Px	M	IVol	Cost	Total Cost	Mkt Value	P&L	Delta Notional	Delta	Gamma	Vega	T
[-] Portfolio Summary							42	41	-1	1,955	46	14	3.83	-
WFC US Equity							42	41	-1	1,955	46	14	3.83	-
WFC US Equity		0	42.86	l		42.86	0	0	0	0	0	0	.00	.
WFC US 11/16/13 C43		1	0.41	m	13.44	0.42	42	41	-1	1,955	46	14	3.83	-

	U/Px	Vol	Date	Rate	P&L	P&L %	Delta	Gamma	Theta	Vega
	Step	Flat	0	Flat						
			__/__/__							
71)	39.00	0.00	10/25/13	0.00	-41.98	-99.95	.06	.07	-.01	.02
72)	40.00	0.00	10/25/13	0.00	-41.7	-99.28	.77	.64	-.06	.2
73)	41.00	0.00	10/25/13	0.00	-39.39	-93.79	5.12	3.34	-.33	.99
74)	42.00	0.00	10/25/13	0.00	-28.19	-67.11	20.44	9.62	-1.01	2.72
75)	43.00	0.00	10/25/13	0.00	5.66	13.46	50.31	14.49	-1.58	3.86
76)	44.00	0.00	10/25/13	0.00	71.29	169.74	80.55	10.57	-1.18	2.69
77)	45.00	0.00	10/25/13	0.00	160.45	382.02	95.75	3.63	-.42	.9
78)	46.00	0.00	10/25/13	0.00	258.53	615.55	99.51	.59	-.09	.14
79)	47.00	0.00	10/25/13	0.00	358.35	853.21	99.97	.05	-.03	.01

Delta-Hedged



Delta-Gamma-Hedged



<HELP> for explanation.

1) Actions | 2) Positions | 3) View | 4) Settings | 99) Feedback | Option Scenario Analysis

New Portfolio | Unsaved Portfolio | < Add Position > | USD | 10/25/13 | 21) Group

31) Positions | 32) Hedge | 33) Scenario Matrix | 34) Scenario Chart | 35) Multi-Asset Scenario

	Position	Mkt Px	M	IVol	Cost	Total Cost	Mkt Value	P&L	Delta Notional	Delta	Gamma	Vega
[-] Portfolio Summary						848	846	-2	-14	0	0	-9.25
WFC US Equity						848	846	-2	-14	0	0	-9.25
WFC US Equity	21	42.86	l		42.86	900	900	0	900	21	0	.00
WFC US 12/21/13 C44	-2.3	0.41	m	14.25	0.41	-94	-95	-1	-2,869	-67	-14	-13.08
WFC US 11/16/13 C43	1	0.41	m	13.44	0.42	42	41	-1	1,955	46	14	3.83

53) Scenario Actions | Scenario Varying U/Px | Notional | P&L From Cost

	U/Px	Vol	Date	Rate	P&L	P&L %	Delta	Gamma	Theta	Vega
	Step	Flat	0	Flat						
			__/__/__							
71)	39.00	0.00	10/25/13	0.00	-30.74	-3.63	18.27	-1.21	.13	-1.06
72)	40.00	0.00	10/25/13	0.00	-14.5	-1.71	13.44	-2.6	.3	-2.61
73)	41.00	0.00	10/25/13	0.00	-4.61	-.54	5.82	-3.22	.42	-4.82
74)	42.00	0.00	10/25/13	0.00	-1.95	-.23	.15	-1.16	.26	-7.06
75)	43.00	0.00	10/25/13	0.00	-2.26	-.27	-.43	-.32	.21	-9.69
76)	44.00	0.00	10/25/13	0.00	-4.96	-.59	-8.06	-7.26	1.03	-12.86
77)	45.00	0.00	10/25/13	0.00	-24.36	-2.87	-34.92	-15.44	2.03	-13.43
78)	46.00	0.00	10/25/13	0.00	-76.35	-9.01	-70.34	-15.16	2	-9.52
79)	47.00	0.00	10/25/13	0.00	-160.08	-18.88	-95.65	-8.27	1.11	-4.27

91) Exceptions | 92) Beta Reference | Zoom 100%

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Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2013 Bloomberg Finance L.P.
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Outline

Calendar Spreads

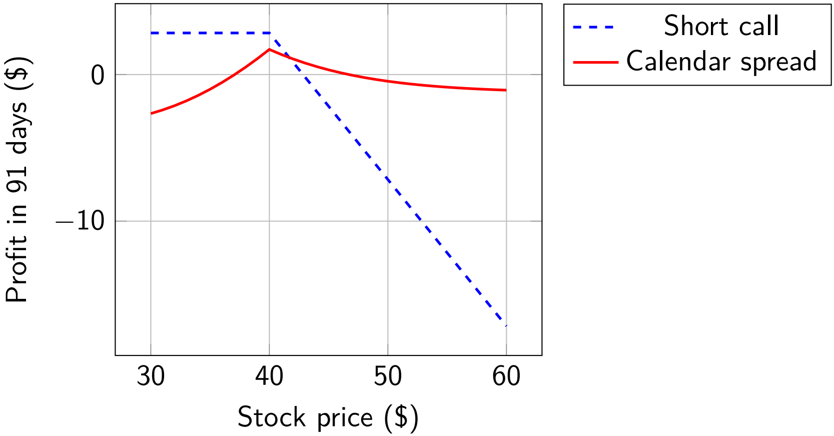
- To protect against a stock price increase when selling a call, you can simultaneously buy a call option with the same strike and greater time to expiration.
- This purchased calendar spread exploits the fact that the written near-to-expiration option exhibits greater time decay than the purchased far-to-expiration option, and therefore is profitable if the stock price does not move.

Calendar Spreads (cont'd)

- Suppose you sell a 40-strike call with 91 days to expiration and buy a 40-strike call with 1 year to expiration. Assume a stock price of \$40, $r = 8\%$, $\sigma = 30\%$, and $\delta = 0$.
- The premiums are \$2.78 for the 91-day call and \$6.28 for the 1-year call.
- Theta is more negative for the 91-day call (-0.0173) than for the 1-year call (-0.0104). Thus, if the stock price does not change over the course of 1 day, the position will make money since the written option loses more value than the purchased option.

Calendar Spreads (cont'd)

- The profit diagram for this position for a holding period of 91 days is displayed below



Outline

Real Options Revisited: Evaluating Projects with an Infinite Investment Horizon

- Consider the investment under uncertainty problem from the binomial option pricing section.
- A project requires an initial investment of \$100. Thus, $K = 100$.
- The project is expected to generate a perpetual cash flow stream, with a first cash flow \$18 in one year, expected to grow at 3% annually. Assume a discount rate of 15%.

$$\Rightarrow \begin{cases} \text{Perpetual growing annuity} \Rightarrow PV = \frac{\$18}{0.15-0.03} = \$150 \\ \text{Static NPV} = \$150 - \$100 = \$50 \\ \text{Cont. compounded div. yield } \delta = \ln\left(\frac{\$18}{\$150} + 1\right) = 0.1133 \end{cases} \tag{29}$$

- The cont. compounded risk-free rate is $r = 6.766\%$. The cash flows of the project are normally distributed with a volatility of $\sigma = 50\%$.

Real Options Revisited: Evaluating Projects with an Infinite Investment Horizon (cont'd)

- The above example assumes that we must start the project by year 2.
- Suppose instead that the project can be started at any time and then will live forever.
- The project is then a **perpetual call option**

Valuing Perpetual Options

- Calls and puts that never expire ar known as **perpetual options**.
- Perpetual American options always have the same time to expiration, namely infinity.
- Because time to expiration is constant, the option exercise problem will look the same today, tomorrow, and forever.
- Thus, the price at which it is optimal to exercise the option is constant.
- The optimal exercise strategy entails picking the exercise barrier that maximizes the value of the option, and then exercising the option the first time the stock price reaches that barrier.

Valuing Perpetual Options (cont'd)

- First, define h_1 and h_2:

$$h_1 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \tag{30}$$

$$h_2 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} - \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \tag{31}$$

- Perpetual American Call:

$$C_{perpetual} = (H_c - K)\left(\frac{S}{H_c}\right)^{h_1}, \text{ where } H_c = K\frac{h_1}{h_1 - 1} \tag{32}$$

- Perpetual American Put:

$$P_{perpetual} = (K - H_p)\left(\frac{S}{H_p}\right)^{h_2}, \text{ where } H_p = K\frac{h_2}{h_2 - 1} \tag{33}$$

Real Options Revisited: Evaluating Projects with an Infinite Investment Horizon (cont'd)

- Using continuously compounded imputs, we compute:

$$\text{CallPerpetual}(\$150, \$100, 0.5, 0.06766, 0.1133) = \{\$63.4, \$245.7\} \tag{34}$$

- When the project value is \$150, the option value is \$63.4 and the optimal investment trigger is \$245.7.
- In other words, we invest when the project is worth \$245.7, more than twice the investment cost.
- If we invest immediately, the project is worth \$50.
- The ability to wait increases that value by \$13.4.

The Option to Abandon

- Firms worry that new projects will not pay off
- Having the option to abandon a project that does not pay off can be valuable
- The option to abandon takes on the characteristics of a put option:
  - V is the remaining value on a project if it continues to the end of its life
  - L is the liquidation (abandonment) value
  - Payoff from owning an abandonment option:

$$\text{Payoff } = \begin{cases} 0 & \text{if } V > L \\ L - V & \text{if } V \leq L \end{cases}$$

- Having a option to abandon a project can make otherwise unacceptable projects acceptable

Collars in Acquisitions: Valuing an Offer

The Northrop Grumman—TRW merger

In July 2002, Northrop Grummann and TRW agreed that Northrop would pay \$7.8 billion for TRW. The number of Northrop Grumman shares to be exchanged for each TRW share is

$$\begin{aligned} 0.5357 \text{ shares} \quad & \text{if } S_{NG} \leq \$112 \\ \$60/S_{NG} \text{ shares} \quad & \text{if } \$112 < S_{NG} < \$138 \\ 0.4348 \text{ shares} \quad & \text{if } S_{NG} \geq \$138 \end{aligned}$$

where S_{NG} is the average Northrop Grumman price over the 5 days preceding the close of the merger.

Suppose that TRW shareholders were certain the merger would occur at time $T = 5/12$. Assume a risk-free rate of 1.5%, and the volatility Northrop Grumman shares is 36%. Northrop Grumman pays no dividends. The closing price of Northrop Grumman is \$120.

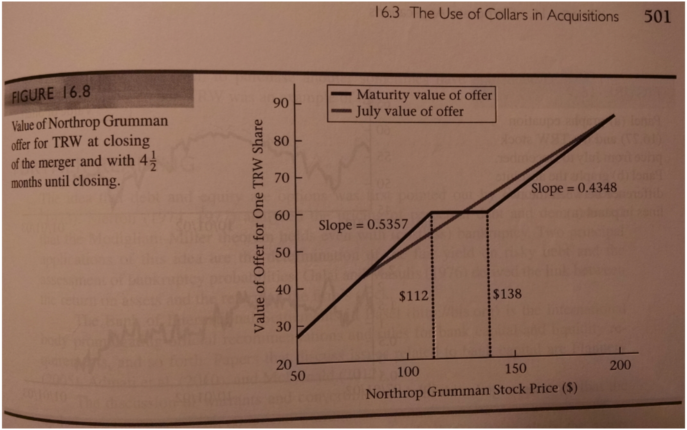
How would TRW shareholders value the Northrop offer?

Collars in Acquisitions: Valuing an Offer

- The offer is equivalent to:
  1. Buying 0.5357 shares of Northrop Grumman
  2. Selling 0.5357 112-strike calls (the Black-Scholes price is \$15.6/call)
  3. Buying 0.4348 138-strike calls (the Black-Scholes price is \$5.22/call)
- To understand this, plot the value of the Northrop Grumman offer for one TRW share, as a function of the Northrop Grumman share price (you should obtain a floating collar offer—see page 501 in McDonald and next slide)
- The value of TRW shares would then be

$$0.5357 \times 120 - 0.5357 \times 15.6 + 0.4348 \times 5.22 = \$58.2 \tag{35}$$

Collars in Acquisitions: Valuing an Offer



Collars in Acquisitions: Valuing an Offer

- The theoretical value of a TRW share under the terms of the offer is greater than the market price of a TRW share
- This is what we would expect to see, since in order to induce the target company to accept an offer, the acquirer generally has to offer a price greater than the perceived value of the target as a stand-alone company
- The difference between the two values declines toward zero as the merger is likely to take place or diverges if the merger is cancelled for some reason
- Risk arbitrageurs take positions in the two stocks in order to speculate on the success or failure of the merger
- Mitchell and Pulvino (*Characteristics of Risk and Return in Risk Arbitrage*, Journal of Finance, 2001) examine the historical returns earned by risk arbitrageurs

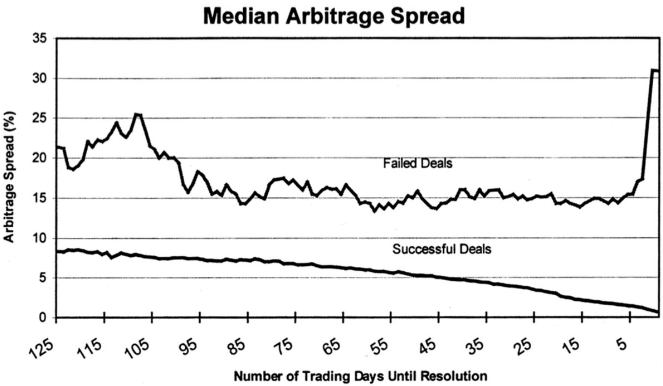


Figure 1. This figure plots the median arbitrage spread versus time until deal resolution. The arbitrage spread is defined to be the offer price minus the target price divided by the target price. For failed deals, the deal resolution date is defined as the date of the merger termination announcement. For successful deals, the resolution date is the consummation date.

Portfolio Insurance

- A portfolio manager is often interested in acquiring a put option on his or her portfolio
- The option can be created synthetically
- This involves maintaining a position in the underlying asset so that the delta of the position is equal to the delta of the required option
- There are two reasons why it might be more attractive to create the required put option synthetically than to buy it in the market:
  - Option markets do not always have the liquidity to absorb the trades required by managers of large funds
  - Fund managers often require strike prices and exercise dates that are different from those available in exchange-traded option markets

Portfolio Insurance

Example

A portfolio is worth \$90 million. To protect against market downturns the managers of the portfolio require a 6-month European put option on the portfolio with a strike of \$87 million.

The risk-free rate is 9% per annum, the dividend yield is 3% per annum, and the volatility of the portfolio is 25% per annum. The S&P 500 stands at 900.

The portfolio is considered to mimic the S&P 500 fairly closely.

Create the required option synthetically.

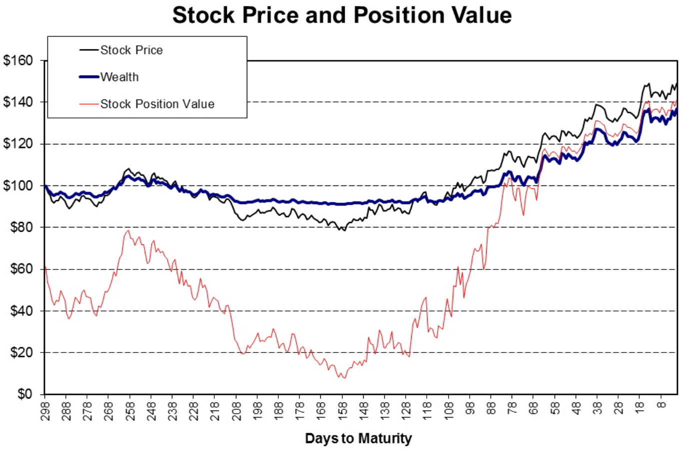
Portfolio Insurance

- In this case, $S_0 = 90$ million, $K = 87$ million, $r = 0.09$, $\delta = 0.03$, $\sigma = 0.25$, and $T = 0.5$
- The delta of the put option is

$$-e^{-\delta T} N(-d_1) = -0.3215 \tag{36}$$

- This shows that 32.15% of the portfolio (\$28.94 million) should be sold initially and invested in risk-free assets to match the delta of the required option.
- The amount of the portfolio should be monitored frequently:
  - If the portfolio reduces to \$88 million after 1 day, the delta of the required option changes to -0.3681 and further 4.66% of the original portfolio (\$3.46 million) should be sold and invested in the risk-free asset
  - If the portfolio increases to \$92 million, delta changes to -0.2787 and 4.29% (\$3.3 million) should be repurchased
- Calculations are more involved if the portfolio's beta (β) is not 1.

Example: Option-Based Portfolio Insurance (OBPI) strategy over 298 trading days, for an initial investment of \$100, with a floor of \$95.



Outline

1. Option Pricing When the Stock Price Can Jump

- Stock prices sometimes move more than would be expected from a lognormal distribution:
  - If market volatility is 20% and the expected return is 15%, a one-day 5% drop in the market occurs about once every 2.5 million days
  - A 20% drop (as in October 1987) is virtually impossible if prices are lognormally distributed
- Jumps have important implications for hedging strategies
- Merton (1976) used the Poisson distribution to count the number of price jumps that occur over a period of time.
- The Poisson distribution is summarized by the parameter λ:
  - λh is the probability that one event occurs over the short interval h.
  - Thus, λ is like an annualized probability of the event occurring over a short interval.

1. Option Pricing When the Stock Price Can Jump (cont'd)

- Let the jump magnitude be Y. That is, if S is the pre-jump price, $Y \times S$ is the post-jump price
- Let us assume that Y is lognormally distributed (hence, this is called the **Poisson-lognormal** model):

$$\ln(Y) \sim N\left(\alpha_J, \sigma_J^2\right)$$

- You can interpret $\ln(Y)$ as the continuously compounded stock return if there is a jump; $\ln(Y)$ can go from $-\infty$ to $+\infty$
- You can interpret $Y - 1$ as the effective stock return if there is a jump; $Y - 1$ can go from -1 to ∞, or Y can go from 0 to ∞
- Thus, the Merton formula for pricing with jumps requires 3 extra parameters:
  1. The jump probability, λ
  2. The expected size of a jump when one does occur, α_J
  3. The volatility of the jump magnitude, σ_J

1. Option Pricing When the Stock Price Can Jump (cont'd)

- The resulting process for the stock is

$$\frac{dS_t}{S_t} = (\alpha - \lambda k)\,dt + \sigma dZ + \begin{cases} 0, & \text{if there is no jump} \\ Y - 1, & \text{if there is a jump} \end{cases} \tag{37}$$

- Merton (1976) shows that with the stock following equation (37), and with jumps diversifiable, the price of an European call is

$$\sum_{i=0}^{\infty} \frac{e^{-\lambda' T}(\lambda' T)^i}{i!} \text{BSCall}\left(S, K, \sqrt{\sigma^2 - \frac{i\sigma_J^2}{T}}, r - \lambda k + \frac{i\alpha_J}{T}, T, \delta\right) \tag{38}$$

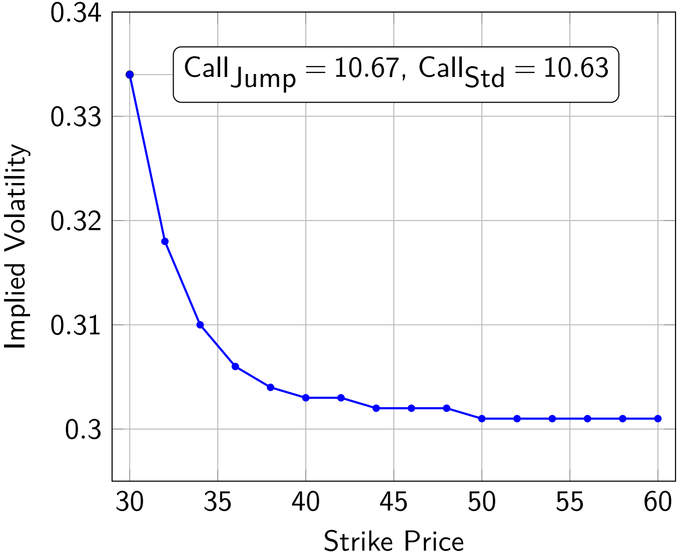
Jump Risk and Implied Volatility

- Suppose that the stock can jump to zero (i.e., $Y = 0$) with $\lambda = 0.5\%$ probability per year
- In this case, the value of a European call becomes

$$\text{BSCall}\,(S, K, \sigma, r + \lambda, T, \delta)$$

- Suppose that other parameters are: $S = \$40$, $\sigma = 30\%$, $r = 8\%$, $T = 0.25$, and $\delta = 0$. The jump parameters are: $\lambda = 0.005$, $\alpha_J = \ln(0) = -\infty$, and $\sigma_J = 0$
- We do the following experiment: generate **correct** option prices—i.e., prices properly accounting for the jump—for a variety of strikes
- We then ask what implied volatility we would compute for these options using the ordinary Black-Scholes formula

Jump Risk and Implied Volatility (cont'd)



2. The Heston Stochastic Volatility Model

- The Heston model is based on the following stock price and variance dynamics

$$\frac{dS_t}{S_t} = \mu dt + \sqrt{v_t} dZ_{1,t} \tag{39}$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t} dZ_{2,t} \tag{40}$$

  where $\kappa, \theta, \sigma > 0$ are constant parameters. The two Brownian motions, $Z_{1,t}$ and $Z_{2,t}$, are correlated, i.e., $\mathrm{Corr}[dZ_{1,t}, dZ_{2,t}] = \rho dt$.
- The dynamics of the stock price in (39) is a geometric Brownian motion with time varying volatility.
- The variance v_t in (40) follows a square root process (also known as the Feller process or the Cox-Ingersoll-Ross process).
- The parameter θ corresponds to the long-run average of v_t, and κ controls the speed by which v_t returns to its long-run mean.

2. The Heston Stochastic Volatility Model (cont'd)

- Set $x_t = \log(S_t)$ and $\tau = T - t$. The call price is

$$C(S_t, v_t, K, T, t) = S_t \mathbb{P}_1 - K e^{-r\tau} \mathbb{P}_2 \tag{41}$$

$$\mathbb{P}_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \mathrm{Re}\left[\frac{e^{-iu\log(K)} \phi_{j;x_t,v_t,t}(u)}{iu}\right] du \tag{42}$$

$$\phi_{j;x_t,v_t,t}(u) = e^{A_j(\tau,u) + B_j(\tau,u) v_t + iux_t} \tag{43}$$

$$A_j(\tau, u) = rui\tau + \frac{\kappa\theta}{\sigma^2}\left[(b_j - \rho\sigma ui + d_j)\tau - 2\log\left(\frac{1 - g_j e^{d_j\tau}}{1 - g_j}\right)\right] \tag{44}$$

$$B_j(\tau, u) = \frac{b_j - \rho\sigma ui + d_j}{\sigma^2} \times \frac{1 - e^{d_j\tau}}{1 - g_j e^{d_j\tau}} \tag{45}$$

2. The Heston Stochastic Volatility Model (cont'd)

with

$$g_j = \frac{b_j - \rho\sigma ui + d_j}{b_j - \rho\sigma ui - d_j} \tag{46}$$

$$d_j = \sqrt{(\rho\sigma ui - b_j)^2 - \sigma^2(2u_j ui - u^2)} \tag{47}$$

$$u_1 = 1/2, \text{ and } u_2 = -1/2 \tag{48}$$

$$b_1 = \kappa + \lambda - \rho\sigma, \text{ and } b_2 = \kappa + \lambda \tag{49}$$

- Even though these formulae look complicated, it is relatively easy to implement them numerically.
- The parameter λv_t can be interpreted as the volatility risk premium (Heston assumes that the volatility risk premium is a linear function of v_t).

Implied Volatility in the Heston Model

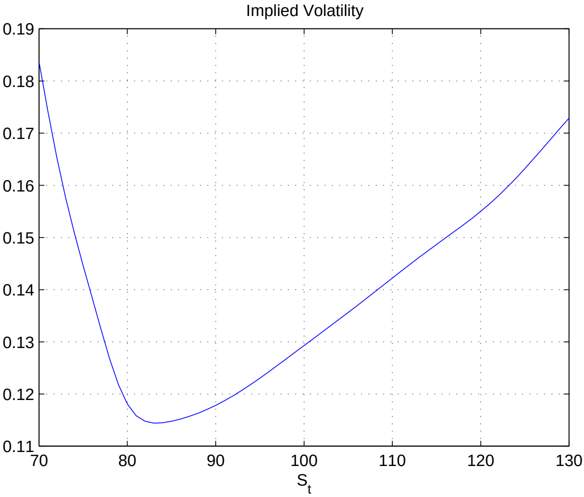


Figure 1 : The parameters are: $K = 100$, $r = 0.04$, $T = 0.5$, $t = 0$, $\sigma = 0.3$, $\rho = -0.5$, $v_t = 0.01$, $\kappa = 6$, $\theta = 0.02$, and $\lambda = 0$.

Outline

Appendix: Formulas for Option Greeks

- Delta (Δ) measures the change in the option price for a \$1 change in the stock price:

$$\Delta_{Call} = \frac{\partial C}{\partial S} = e^{-\delta(T-t)} N(d_1) \tag{50}$$

$$\Delta_{Put} = \frac{\partial P}{\partial S} = -e^{-\delta(T-t)} N(-d_1) \tag{51}$$

- Gamma (Γ) measures the change in delta when the stock price changes:

$$\Gamma_{Call} = \frac{\partial^2 C}{\partial S^2} = \frac{e^{-\delta(T-t)} N'(d_1)}{S\sigma\sqrt{T-t}} \tag{52}$$

$$\Gamma_{Put} = \frac{\partial^2 P}{\partial S^2} = \Gamma_{Call} \tag{53}$$

Appendix: Formulas for Option Greeks (cont'd)

- Elasticity (Ω) measures the percentage change in the option price relative to the percentage change in the stock price:

$$\Omega_{Call} = \frac{S_t \Delta_{Call}}{C_t} \tag{54}$$

$$\Omega_{Put} = \frac{S_t \Delta_{Put}}{P_t} \tag{55}$$

- Vega measures the change in the option price when volatility changes (divide by 100 for a change per percentage point):

$$\text{Vega}_{Call} = \frac{\partial C}{\partial \sigma} = Se^{-\delta(T-t)} N'(d_1)\sqrt{T-t} \tag{56}$$

$$\text{Vega}_{Put} = \frac{\partial P}{\partial \sigma} = \text{Vega}_{Call} \tag{57}$$

Appendix: Formulas for Option Greeks (cont'd)

- Theta (θ) measures the change in the option price with respect to calendar time, t, holding fixed the maturity date T. To obtain per-day theta, divide by 365.

$$\theta_{Call} = \frac{\partial C}{\partial t} = \delta S e^{-\delta(T-t)} N(d_1) - rKe^{-r(T-t)} N(d_2) - \frac{Ke^{-r(T-t)} N'(d_2)\sigma}{2\sqrt{T-t}} \tag{58}$$

$$\theta_{Put} = \frac{\partial P}{\partial t} = \theta_{Call} + rKe^{-r(T-t)} - \delta S e^{-\delta(T-t)} \tag{59}$$

- Rho (ρ) measures the change in the option price when the interest rate changes (divide by 100 for a change per percentage point, or by 10,000 for a change per basis point):

$$\rho_{Call} = \frac{\partial C}{\partial r} = (T-t) Ke^{-r(T-t)} N(d_2) \tag{60}$$

$$\rho_{Put} = \frac{\partial P}{\partial r} = -(T-t) Ke^{-r(T-t)} N(-d_2) \tag{61}$$

Appendix: Formulas for Option Greeks (cont'd)

- Psi (Ψ) Measures the change in the option price when the continuous dividend yield changes (divide by 100 for a change per percentage point):

$$\Psi_{Call} = \frac{\partial C}{\partial \delta} = -(T-t)Se^{-\delta(T-t)}N(d_1) \tag{62}$$

$$\Psi_{Put} = \frac{\partial P}{\partial \delta} = (T-t)Se^{-\delta(T-t)}N(-d_1) \tag{63}$$

Four Common Strategies in Unsettled Markets (*Wall Street Journal*, October 31, 2012

1. Hedge an Existing Portfolio

You have a sizeable allocation in the U.S. stock market but are worried that it could tank if the economy weakens. To hedge, you take out an insurance policy by purchasing an out-of-the-money put option on a stock-index futures contract, such as the E-mini 500 that trades at the CME. An out-of-the-money put is one whose strike price is below the current market price of the underlying instrument — in this case, the stock-index futures contract. If the market price later falls below the strike price, you'll probably have lost money on your stock portfolio. But you'll have earned money on your put, cushioning the blow. If the market price doesn't fall below the strike price, you'll be out the cost of the put, but at least your stock portfolio won't have tanked.

Four Common Strategies in Unsettled Markets (*Wall Street Journal*, October 31, 2012 (cont'd)

2. Put a Collar Around Your Portfolio

If you don't like the idea of paying for the cost of the "insurance" described in the previous example — but don't like the idea of going without a hedge, either — you could pair your put with the sale of an out-of-the-money call on the same stock-index futures contract, creating what traders refer to as a collar. Here, the strike price on the call is above the current market value of the underlying stock index futures contract. As such, the call functions as a cap on any potential gains in your stock portfolio, explains Mark Sebastian, chief operating officer and director of education with Option Pit. His company provides online option mentoring programs for traders.

If the market value of the futures contract rises above the call's strike price, you'll be forced to settle your contract at that price, no matter how much higher it goes — foregoing some of the gain you might otherwise have enjoyed. On the other hand, if the market price never reaches the strike price, your call option will expire worthless. Either way, the premium you earned for selling the call will have offset some, if not all, of the cost of the premium you paid to buy your put.

Four Common Strategies in Unsettled Markets (*Wall Street Journal*, October 31, 2012 (cont'd)

3. Get Your Feet Wet in the Stock Market — Again

Maybe you've been out of the market since 2009, have seen the S&P 500 roughly double in value since then and now want to get back into the market, but aren't willing to risk the entire amount you have to invest. One alternative, says Larry Shover, chief investment officer for Solutions Funds Group, which manages a futures mutual fund, would be to buy an out-of-the-money stock-index call option good for, say, nine to 12 months. If the stock index goes up, your call will increase in value. You won't make any money if the index goes down, but you'll only be out the small fraction of the index's value that you paid for the call.

Four Common Strategies in Unsettled Markets (*Wall Street Journal*, October 31, 2012 (cont'd)

4. Hedge a Bond Portfolio

The Federal Reserve plans to keep short-term interest rates low at least into 2015, and that is helping to keep a lid on interest rates in general. If interest rates should start to rise, however, it would ding the value of bond portfolios. Bondholders concerned about that possibility, says professional trader Yra Harris of Praxis Trading, could hedge some of their risk by purchasing put options on Treasury bonds or Treasury bond futures. Those options would increase in value if bond prices started to fall.