My tea's getting cold!
An investigation into Newton's law of cooling

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1 Introduction

Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature (i.e. the temperature of its surroundings).[2]

Or:

$$\frac{dT}{dt} = -k(T - T_{\mathrm{a}}) \tag{1}$$

Using Sage to solve this differential equation, via this code:

```
typeset_mode(true)

t, A, k = var('t, A, k')    #Declaring variables

T = function('T', t)    #Declaring T as a function in terms of t

desolve(diff(T, t) == -k * (T - A), T, ivar = t)
#Solving the differential equation for T, with the independent variable t
```

$$\left(Ae^{(kt)} + C\right)e^{(-kt)}$$

(After expansion) we get this equation:

$$T = Ce^{\text{-kt}} + T_{\mathrm{a}} \tag{2}$$

Whereby:

- T = Object temperature at time t
- $\mathrm{T_a}$= Ambient temperature (room temperature)
- $\mathrm{T_i}$ = Initial temperature of the object

- k = Cooling coefficient (of tea in this case)
- t = Time (measured in seconds)
- C = Some constant

2 Assumptions

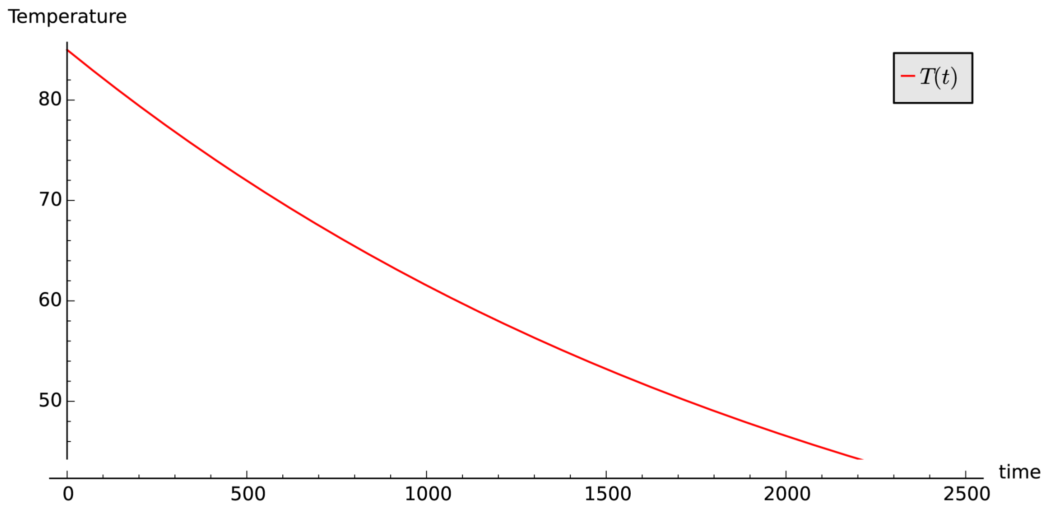
1. Room temperature is 20°C
2. Cooling coefficient of tea (k) is 0.0004476
3. Unfortunately I don't drink or interfere with my cup of tea in any way
4. The constant C has already been calculated to be 65

3 Tea

My favourite tea is the tea in standard, everyday, industrially produced teabags. This tea is black leaf tea and 85°Cis the optimum brewing temperature for taste [1]. This will be our T_i. By 45°Cthe tea is too cold to drink, we want to find the time (t) it will take to get to this temperate (T). Entering these values and the assumptions into equation 2, gives us a plot table function:

```
T(t) = 65 * e ^ (-0.0004476 * t) + 20
#Function T in terms of t with values inputed for C, k and A

plot(T, t, 0, 2500, axes_labels=["time", "Temperature"],
ymin = 45, color='red', legend_label='$T(t)$')
#Plotting the function for values 0 to 2500,
labelling axes, colouring the plot, and including a label for the plot
```



From this plot we can see that the tea will be too cold to drink in approximately 2250 seconds after being brewed. Lets try and find a more accurate time. So, using Sage to solve for t (time) we have:

```
q = T == C * e ^ (-k * t) + A

solve(q, t)

[t == log(-C/(A - T))/k]
```

Entering our values for C, A, T and k we have:

$$t = 2134.74 \text{ s}$$

or in minutes:

$$t = 35.579 \text{ minutes}$$

4 Conclusion

In conclusion, there are only approximately 35 minutes of 'good temperature' tea drinking time left; from brewing up, to your tea being consigned to the sink.

References

[1] The Rare Tea Company. How to make the perfect cup of tea.

[2] University of British Colombia. Newton's law of cooling.