

Computing for Mathematics: Week 1

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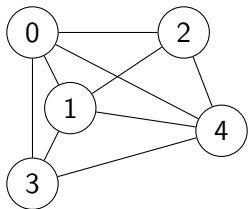
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Programming and Mathematics

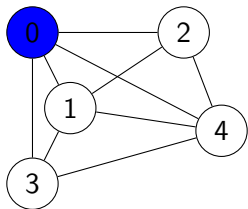
There are various areas in which computers are of major importance to Mathematicians:

- ▶ Computer assisted proofs;
- ▶ Implementation of mathematics;
- ▶ Computer generated proofs;
- ▶ Everyday mathematics.

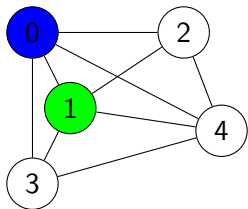
Computer assisted proofs



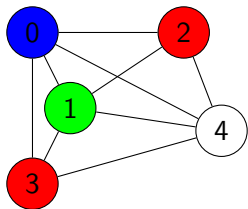
Computer assisted proofs



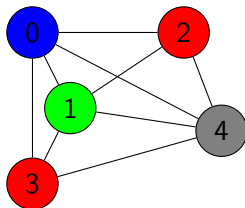
Computer assisted proofs



Computer assisted proofs

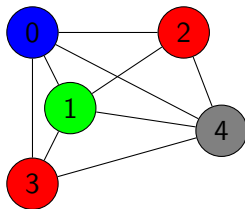


Computer assisted proofs



- '4 colour theorem': **Any map can be coloured using 4 colours.**

Computer assisted proofs



- ▶ '4 colour theorem': **Any map can be coloured using 4 colours.**
- ▶ Proved in 1976 by Kenneth Appel and Wolfgang Haken:

Used computers to check 1936 particular cases.

Risk boards



Computer assisted proofs

How to pack 3 dimensional spheres?

- ▶ In 1611 Kepler conjectured the best possible way.
- ▶ Proof in 1998 by Hales which involved a computer to minimize a function of 150 variables (100,000 times).
- ▶ **Also** involved a 100 page paper for the 'non computer assisted aspects'.

Computer assisted proofs

How to pack 3 dimensional spheres?

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- ▶ Proof in 1998 by Hales which involved a computer to minimize a function of 150 variables (100,000 times).
- ▶ **Also** involved a 100 page paper for the 'non computer assisted aspects'.
- ▶ Referees are 99% sure.

Implementation of mathematics

Here at Cardiff Dr Leanne Smith studied the best way to locate ambulances in Wales. This took in to account:

- ▶ Queues;
- ▶ Survival probabilities of patients;
- ▶ Time of the day...

Once the mathematics was done a computer program was built to be able to demonstrate to the Welsh Ambulance Trust.

Computer generated proofs

Timothy Gowers

Computer generated proofs

Timothy Gowers

Theorem: Let X and Y be sets, let $f : X \rightarrow Y$ be an injection and let A and B be subsets of X . Then $f(A) \cap f(B) \subset f(A \cap B)$.

Computer generated proofs

Timothy Gowers

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Proof: Take $x \in f(A) \cap f(B)$. So there is some $y \in A$ and $z \in B$ such that $f(y) = f(z) = x$. As f is injective, y and z are equal. So $y \in A \cap B$. So $x = f(y) \in f(A \cap B)$.

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The above is an example of a computer generated proof. **You do not need to know any of this!**

Everyday mathematics

Everyday mathematicians might need to calculate an integral for a bigger project. This is some Sage code to calculate an integral:

```
1 integrate(x ^ 3, x)
```

which returns:

$$\frac{x^4}{4}$$

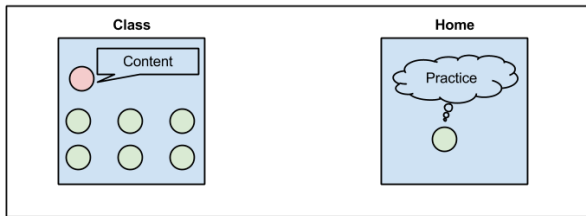
What we will learn

- ▶ Python: general purpose programming language (Weeks 1-5).
- ▶ Sage: mathematics package (based on Python) (Weeks 5-9).
- ▶ \LaTeX : a package for writing mathematics (Week 10).

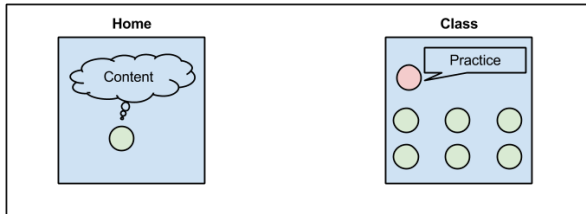
Flipped classrooms

Flipped classrooms

**Classic
Classroom**



**Flipped
Classroom**



Labs and 'Tickables'

- ▶ Every week you have 2 computer lab sessions.
- ▶ You have until the end of the second lab session to complete all exercises marked as 'TICKABLE'.
- ▶ You will need to work on these outside of the lab sessions.

Resources

<http://www.vincent-knight.com/home/teaching/computing-for-mathematics>