Computing for Mathematics: Week 1

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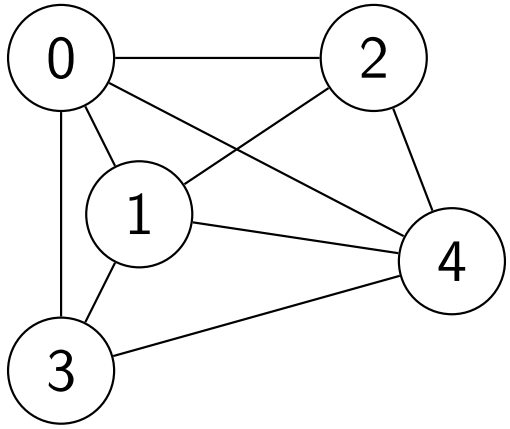
$$\begin{pmatrix} (0,0) & (-1,1) & (1,-1) \\ (1,-1) & (0,0) & (-1,1) \\ (1,-1) & (-1,1) & (0,0) \end{pmatrix}$$

Programming and Mathematics

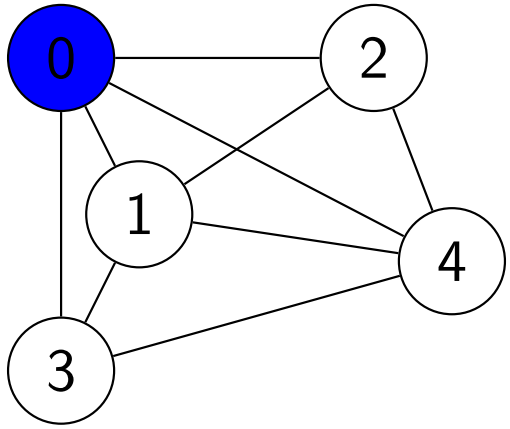
There are various areas in which computers are of major importance to Mathematicians:

- Computer assisted proofs;
- Implementation of mathematics;
- Computer generated proofs;
- Everyday mathematics.

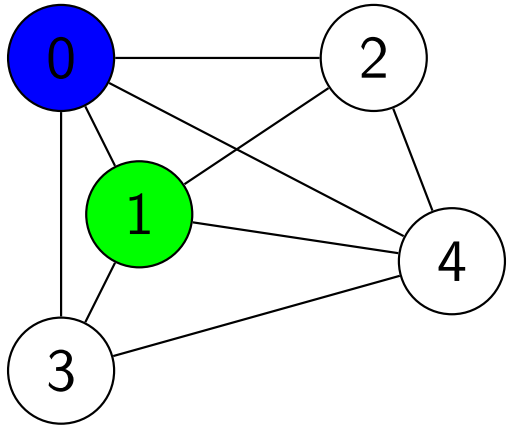
Computer assisted proofs



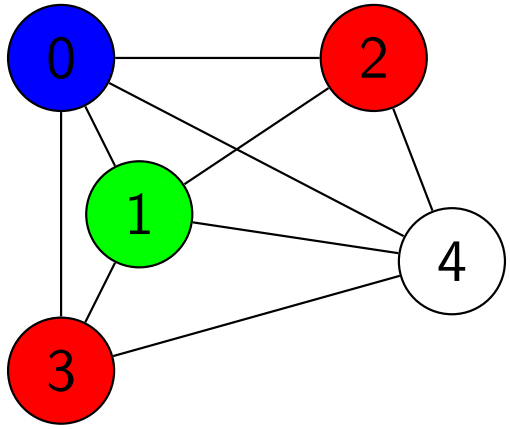
Computer assisted proofs



Computer assisted proofs



Computer assisted proofs



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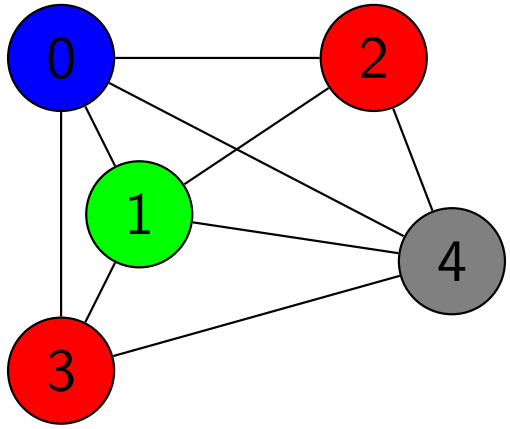
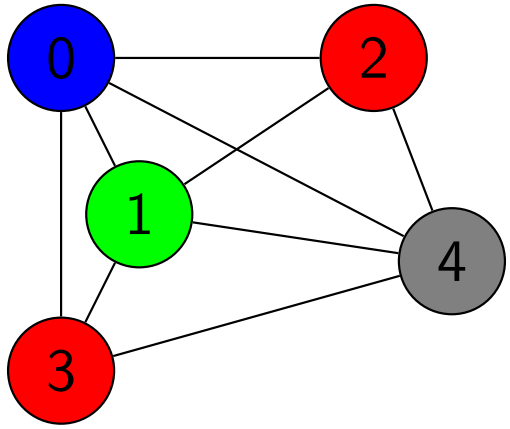


- '4 colour theorem': **Any map can be coloured using 4 colours.**

Computer assisted proofs



- '4 colour theorem': **Any map can be coloured using 4 colours.**
- Proved in 1976 by Kenneth Appel and Wolfgang Haken:

  Used computers to check 1936 particular cases.

Risk boards


Risk
TRANSFORMERS
AUTOBASE
THE DECAGON
TYREST
THE TAGON HEIGHTS
THE NEUTRAL TERRITORIES
PLASMA ENERGY CHAMBER
ORBITAL BETA PLATFORM
CHAMBER OF THE ANCIENTS
CENTRAL CITY
HIGH COUNCIL PAVILIONS
NOVA CRONUM
TYGER PAX
STELLA GALLERIES
THE PRAETORUS WHARF
TARN
VOS
LAUNCH PAD
UNICRON
IACON
THE ORACLE
URAYA
PROTIHEX
PRIMUS
HEXIMA STATE
THE ARENA
THE RAD ZONE
MACCADAMS
TRI-TORUS LOOP
KOLKULAR
THE SONIC CANYONS
KAON
ALTIHEX
HYDRAX PLATEAU
MOON ALPHA
SMELTING POOL
POLYHEX
PERIHEX
THE BORDER REGIONS
KALIS
THE SPACE BRIDGE
DAYS
1
2
3
4
5
6
LAUNCH
ZONES ▶ ROBOTS
SECTOR BONUS
7 5 5 3 2 2

Computer assisted proofs

How to pack 3 dimensional spheres?

- In 1611 Kepler conjectured the best possible way.
- Proof in 1998 by Hales which involved a computer to minimize a function of 150 variables (100,000 times).
- **Also** involved a 100 page paper for the 'non computer assisted aspects'.

Computer assisted proofs

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- **Also** involved a 100 page paper for the 'non computer assisted aspects'.
- Referees are 99% sure.

Implementation of mathematics

Here at Cardiff Dr Leanne Smith studied the best way to locate ambulances in Wales. This took in to account:

- Queues;
- Survival probabilities of patients;
- Time of the day...

Once the mathematics was done a computer program was built to be able to demonstrate to the Welsh Ambulance Trust.

Computer generated proofs

Timothy Gowers

Computer generated proofs

Timothy Gowers

Theorem: Let X and Y be sets, let $f : X \to Y$ be an injection and let A and B be subsetsof X. Then $f(A) \cap f(B) \subset f(A \cap B)$.

Computer generated proofs

Timothy Gowers

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Proof: Take $x \in f(A) \cap f(B)$. So there is some $y \in A$ and $z \in B$ such that $f(y) = f(z) = x$. As f is injective, y and z are equal. So $y \in A \cap B$. So $x = f(y) \in f(A \cap B)$.

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The above is an example of a computer generated proof. **You do not need to know any of this!**

Everyday mathematics

Everyday mathematicians might need to calculate an integral for a bigger project. This is some Sage code to calculate an integral:

```
1 integrate(x ^ 3, x)
```

which returns:

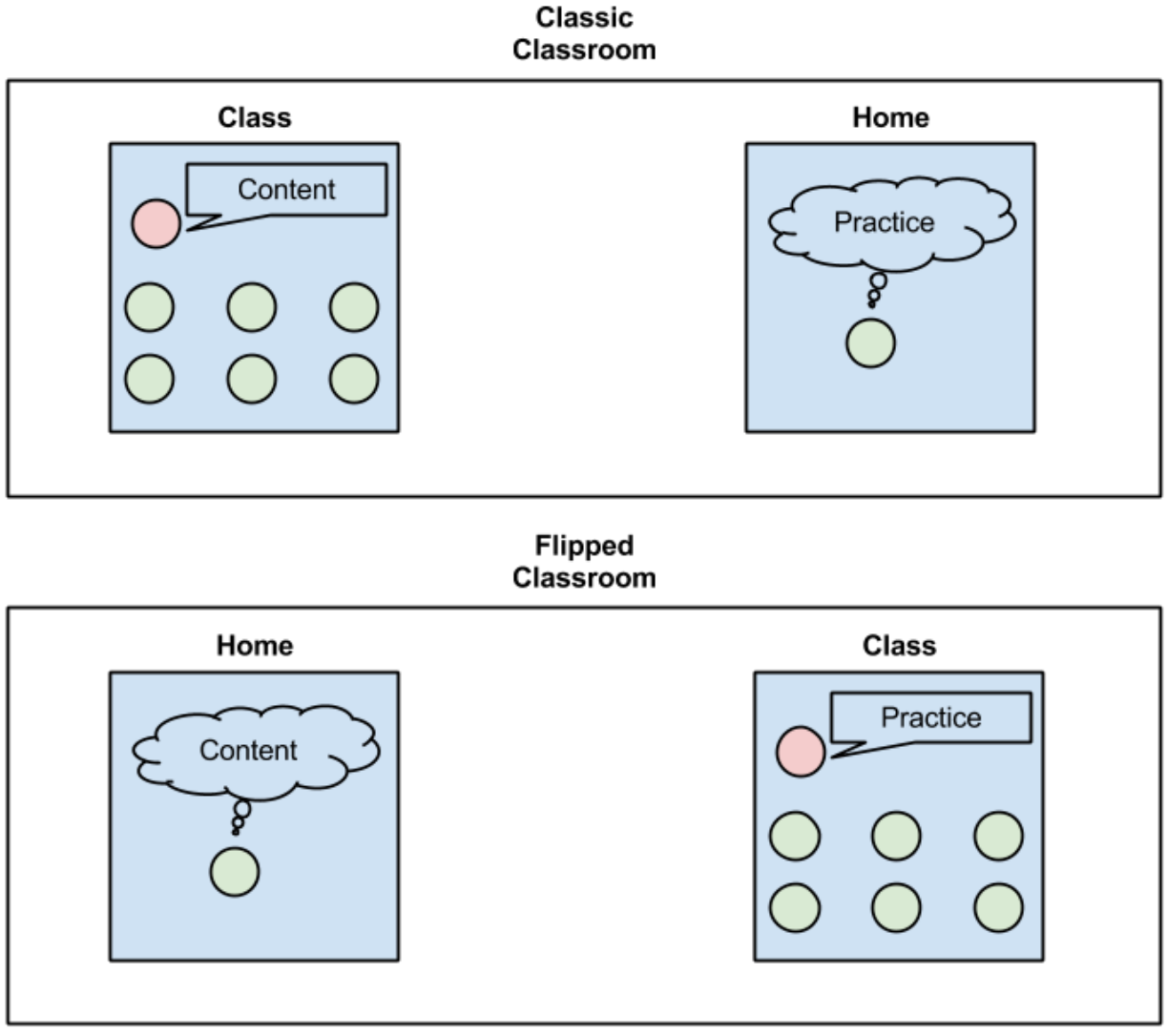
$$\frac{x^4}{4}$$

What we will learn

- Python: general purpose programming language (Weeks 1-5).
- Sage: mathematics package (based on Python) (Weeks 5-9).
- LaTeX: a package for writing mathematics (Week 10).

Flipped classrooms

Flipped classrooms



Labs and ‘Tickables’

- Every week you have 2 computer lab sessions.
- You have until the end of the second lab session to complete all exercises marked as ‘TICKABLE’.
- You will need to work on these outside of the lab sessions.

Resources

http://www.vincent-knight.com/home/teaching/computing-for-mathematics