Linear Algebra - Matrices

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In this Mathematical Report, I shall be discussing various areas of Matrices. Coding with Sage and other Mathematical tools. This includes:

- Basic Matrices
- Inverse Matrices
- Matrices Transformations

1 Basic Matrices

Sage can be used to Define and Multipy Matrices, even though the Mathematical concepts behind Matrices are simple. The larger and more complex Matrices are the easier it is to use Sage to solve them.

- Defining Matrix
  The following code is used to define a Matrix, within sage:

  ```
  A = matrix([[13, 5], [2, -1]])
  A
  ```

  This code will then print the Matrix in Sage, and be show in this form:

  $$\begin{pmatrix} 13 & 5 \\ 2 & -1 \end{pmatrix}$$

  By defining Matrices this allows us to now solve and multiply them.

- Multiplying Matrices
  Now, by using the defining a Matrix Code we can now use sage to multiply matrices, and this code couldn't be more simply. We want to multiply matrix A and B, which are coded as:

  ```
  A = matrix([[13, 5], [2, -1]])  .
  B = matrix([[3, -1], [-1, -0]])
  ```

  To Multiply we use the code:

  ```
  A*B
  ```

  Sage will then print the answer:

  $$\begin{pmatrix} 34 & -13 \\ 7 & -2 \end{pmatrix}$$

  Even though multiplying matrices by hand is straight forward, the bigger the matrices get the easier it is to use Sage, to solve the question, or simply to check your answer.

2 Inverse Matrices

The inverse of a Matrix is defined by this formula:

$$\mathbf{A}^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

It is simple to show the inverse of a matrix in Sage, by using the following code:

For example, if we wanted to find the inverse of this Matrix A;

```
A = matrix([[4, 12], [32, -13]])
A.inverse()
```

Then Sage would compute the inverse, which is:

$$\begin{pmatrix} 13/436 & 3/109 \\ 8/109 & -1/109 \end{pmatrix}$$

3 Matrices Transformations

There are many different Transformation Matrices including; reflections in x and y axis, various rotations, enlargements and more. You can use Sage to show and draw the transformations that the matrices create.
I am going to show how to draw Matrix transformation with the following transformation:

$$\begin{pmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{pmatrix}$$

This is a rotation transformation, through an angle (which has to be in radians), and will rotate the shape anticlockwise.

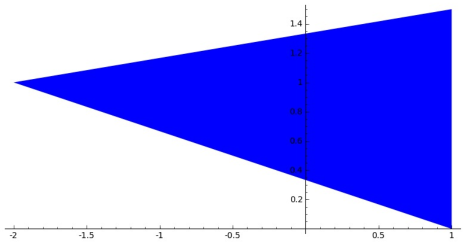
Firstly, we have to draw our shape we shall be applying the matrix to using the code:

```
p1 = vector([1,1.5])
p2 = vector([1,0])
p3 = vector([-2,1])
```

This code then plots the shape:

```
mypoints = list([p1,p2,p3])
P = polygon(mypoints)
P.set_aspect_ratio(1)
Print(P)
```

Sage then prints this graph:



Now, this code multiplies our orginal set of points by the transformation matrices:

```
def matrix_transformation(mypoints, theta):
   A = matrix([[cos(theta),-(sin(theta))],[sin(theta),cos(theta)]])
   mypoints2 = []
   for mypoint in mypoints:
      mypoint2 = A* mypoint
      mypoints2.append(mypoint2)
```

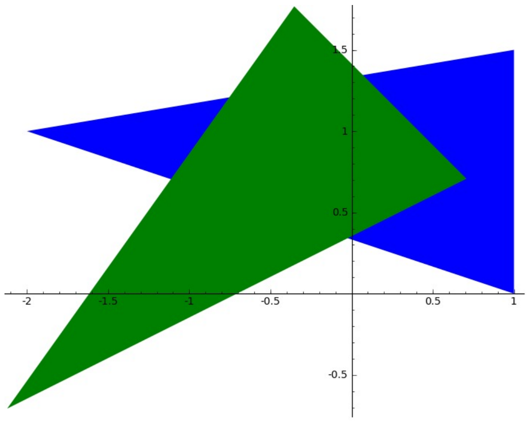
Next, this sets the value of angle, to be any radian, and in this case 45 degrees:

```
mypoints3 = matrix_transformation(mypoints, pi/4)
```

Finally, this prints the transformed shape and the orginal shape so you can spot the transformation:

```
P = polygon(mypoints) + polygon(mypoints3,color="green")
P.set_aspect_ratio(1)
show(P)
```

Sage then prints this graph:



There are many more Matrices Transformations all of which can be coded, in order to show the starting shape and followed by the Matrix Transformation.

4 Conclusion

To conclude, Sage can be used in many different ways to help solve and understand Matrices. The code to use in Sage is straightforward and easy to understand. The code I used in order to write this report is shown in this Sage worksheet `https://sage.maths.cf.ac.uk/home/pub/30/`..

References

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[5] http://a-little-book-of-sage.readthedocs.org/en/latest/src/matrixtransformations.html Cited: 06/12/13