

Week 7 - Symbolic Calculus

Using Sage we can carry out various operations from Calculus. This week we will investigate how to:

- Carry out limits in Sage;
- Carry out differentiation in Sage;
- Carry out integration in Sage.

1. Last week we saw how to define a function in Sage:

$$f(x) = x^3 + 3x + \sin(x)$$

To obtain the variables of a function we can use the `variables` method:

```
print f.variables()
```

Try this with a function of more than one variable:

$$f(x, y) = x^2y + x^2 + y^2$$

2. In calculus the following definition of a limit is well known:

$$\lim_{x \rightarrow a} f(x) = L \text{ iff } \forall \epsilon > 0 \exists \delta \text{ such that } \forall x: |x - a| < \delta \Rightarrow |f(x) - L| \leq \epsilon.$$

Let us calculate the limit of $f(x) = \frac{3x^2}{x^3+x-1}$ as $x \rightarrow 1$.

First of all let us plot $f(x)$:

```
plot(f(x), x, .5, 10)
```

The following code obtains $\lim_{x \rightarrow 1} f$:

```
f.limit(x=1)
```

We can also obtain the same result using the `limit` method:

```
limit(f,x=1)
```

Note that $f(1) = \lim_{x \rightarrow 1} f(x)$:

```
f(1)
```

This implies that f is continuous at 1.

3. **TICKABLE** Plot $f(x) = \frac{3x^2}{x^3+x-1}$ using the default options:

```
plot(f)
```

We see that Sage is plotting extremely high values at the discontinuity due to a root of the denominator which seems to be around $x = .7$. We can plot our function either side of that point and combine them. We do this by creating plot objects:

```
p = plot(f, x, 0.8, 10)
type(p)
p += plot(f, x, -10, .6)
type(p)
p.show()
```

and identify (use the `solve` function) α : the root of the denominator of f . Obtain $\lim_{x \rightarrow \alpha+} f(x)$ and $\lim_{x \rightarrow \alpha-} f(x)$. Directions of limits can be obtained using the following code:

```
limit(f, x=??, dir="plus")
limit(f, x=??, dir="minus")
```

4. Algebra of limits
5. $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$
6. e^x
7. Basic differentiation
8. Limiting definition of a derivative
9. Plotting the limiting definition of a derivative
10. Visualising the limiting definition of a derivative
11. Differentiation rules
12. Basic integration
13. Integration by parts
14. Riemann integration
15. Numerical integration
16. Integrate polynomials in a data file