Transformations Using Matricies

Catherine Askill

December 11, 2013

Introduction

Points on a graph can be represented in matrix form, so a shape on the cartesian plane can be repesented by the coordinates of the vertices of that shape.

$$P = \begin{pmatrix} x_1 & x_2 & x_3 & ... \\ y_1 & y_2 & y_3 & ... \end{pmatrix}$$

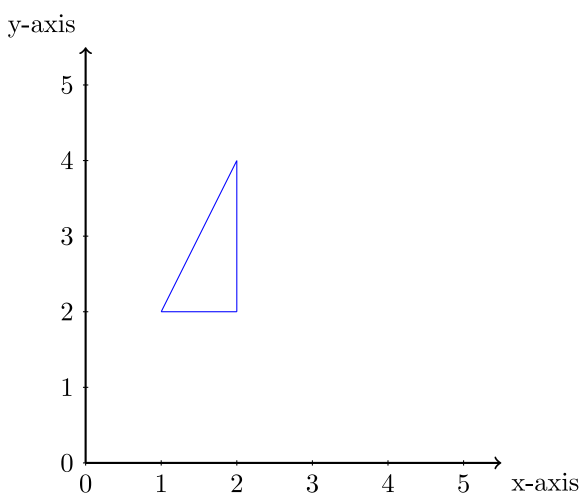

Figure 1: the plot of a= $\left(\begin{smallmatrix} 1 & 2 & 2 \\ 2 & 2 & 4 \end{smallmatrix}\right)$.

The shape on the graph can therefore be transformed in the ways:

- Enlargement
- Reflection
- Rotaion

This can be done by multiplying the point matrix representing your shape by the respective transformation matrix. When doing this the transformation matrix must lead the multiplication so that the multiplication works, this is because when multiplying matrices the number of rows in the first matrix must equal the number of columns in the second matrix.

1 Enlargements

A polygon can be enlarged uniformly by scale factor n, centre (0,0) by multiplying the point matrix by n. The polygon could also be scaled by factor a parallel to the x-axis and factor b parallel to the y-axis. This can be acheived by multiplying the point matrix by the transformation matrix T.

$$T = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

The follow sage code calculates the matrix of the image after it has been scaled by factor a=3 and b=2

```
A=matrix([[1, 2, 2], [2, 2, 4]])
B=matrix([[3,0], [0,2]])
B*A
```

Giving the result:

$$I = \begin{pmatrix} 3 & 6 & 6 \\ 4 & 4 & 8 \end{pmatrix}$$

2 Reflections

Our polygon can easily be reflected in the x-axis, y-axis, the line y=x and the line y=-x by multiplying the point matrix by on of the following:

For a reflection in the x-axis	$T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
For a reflection in the y-axis	$T = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
For a reflection in the line y=x	$T = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
For a reflection in the line y=-x	$T = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

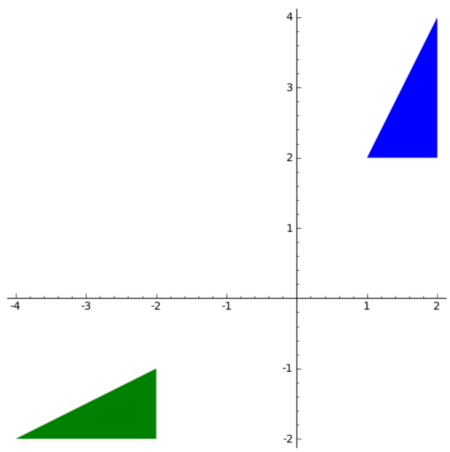

Figure 2: The plot of the original polygon represented by a= $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 2 & 4 \end{pmatrix}$ (blue) and the image after reflection in the line y=-x (green) using sage.

3 Rotations

A polygon on the cartesian plane can be rotated by θradians anti-clockwise about the centre (0,0) using the matrix

$$T = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$$

The most common rotations are when θ= 90°= $\frac{\pi}{2}$, 180°= π and 270°= $\frac{3\pi}{2}$. These rotations are represented by the matrices:

$(a)$when $\theta = 90°$ $\quad T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\qquad (b)$when $\theta = 180°$ $\quad T = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ $\qquad (c)$when $\theta = 270°$ $\quad T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Sage can be used to plot the rotaion of any degree using the following code from [1]:

```
def rotate_polygon(mypoints, theta): # Defining the rotation of the polygon.
A = matrix([[cos(theta),-(sin(theta))],[sin(theta),cos(theta)]]) # Defining the Matrix for rotation.
   mypoints2 = []
   for mypoint in mypoints:
      mypoint2 = A* mypoint
      mypoints2.append(mypoint2)
   return(mypoints2)
mypoints5 = rotate_polygon(mypoints, pi/3) #Setting theta as pi/3.
P = polygon(mypoints) + polygon(mypoints5,color="lime")
P.set_aspect_ratio(1)
show(P)
```

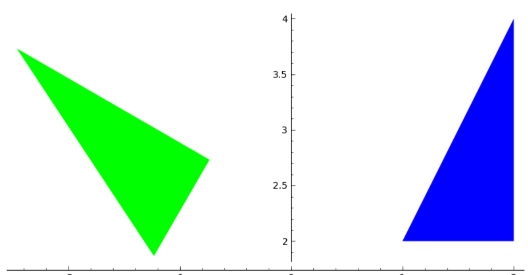

Figure 3: The plot of the oringinal polygon and the image after rotation of $\frac{\pi}{3}$ radians.

In conclusion any polygon can easily be tranformed in the above three ways by multiplying the correct transformation matrix by the point matrix representing your polygon. For more information look at Edexcel Further Pure 1 book.

References

[1] http://a-little-book-of-sage.readthedocs.org/en/latest/src/matrixtransformations.html.