Computing for Mathematics: Week 1

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$$\begin{pmatrix} (0,0) & (-1,1) & (1,-1) \\ (1,-1) & (0,0) & (-1,1) \\ (1,-1) & (-1,1) & (0,0) \end{pmatrix}$$

http://thefightclub.herokuapp.com/
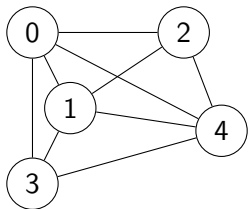
http://jayrobertvos.pythonanywhere.com/
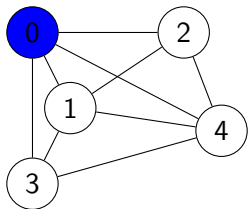
Programming and Mathematics

There are various areas in which computers are of major importance to Mathematicians:

- ▶ Computer assisted proofs;
- ▶ Implementation of mathematics;
- ▶ Computer generated proofs;
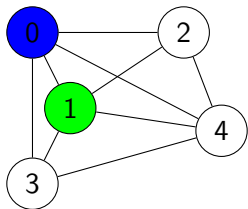- ▶ Everyday mathematics.
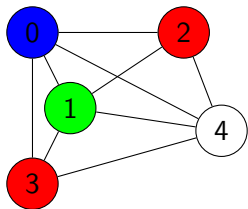
Computer assisted proofs

Computer assisted proofs
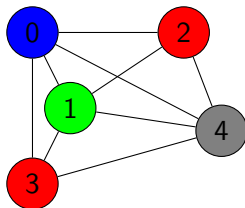
Computer assisted proofs

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- '4 colour theorem': **Any map can be coloured using 4 colours.**

Computer assisted proofs
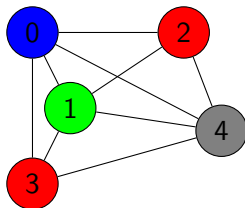
- ‘4 colour theorem’: **Any map can be coloured using 4 colours.**
- Proved in 1976 by Kenneth Appel and Wolfgang Haken:

  Used computers to check 1936 particular cases.

Risk boards

Computer assisted proofs

How to pack 3 dimensional spheres?

- ▶ In 1611 Kepler conjectured the best possible way.
- ▶ Proof in 1998 by Hales which involved a computer to minimize a function of 150 variables (100,000 times).
- ▶ **Also** involved a 100 page paper for the 'non computer assisted aspects'.

Computer assisted proofs

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- ▶ **Also** involved a 100 page paper for the 'non computer assisted aspects'.
- ▶ Referees are 99% sure.

Implementation of mathematics

Here at Cardiff Dr Leanne Smith studied the best way to locate ambulances in Wales. This took in to account:

- Queues;
- Survival probabilities of patients;
- Time of the day...

Once the mathematics was done a computer program was built to be able to demonstrate to the Welsh Ambulance Trust.

Computer generated proofs

Timothy Gowers

Computer generated proofs

Timothy Gowers

Theorem: Let X and Y be sets, let $f : X \to Y$ be an injection and let A and B be subsetsof X. Then $f(A) \cap f(B) \subset f(A \cap B)$.

Computer generated proofs

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Proof: Take $x \in f(A) \cap f(B)$. So there is some $y \in A$ and $z \in B$ such that $f(y) = f(z) = x$. As f is injective, y and z are equal. So $y \in A \cap B$. So $x = f(y) \in f(A \cap B)$.

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The above is an example of a computer generated proof. **You do not need to know any of this!**

Everyday mathematics

Everyday mathematicians might need to calculate an integral for a bigger project. This is some Sage code to calculate an integral:

```
1    integrate(x ^ 3, x)
```

which returns:

$$\frac{x^4}{4}$$
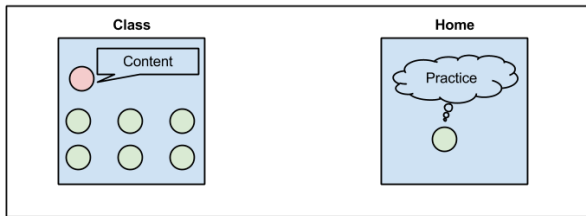
What we will learn

- Python: general purpose programming language (Weeks 1-5).
- Sage: mathematics package (based on Python) (Weeks 5-9).
- LaTeX: a package for writing mathematics (Week 10).
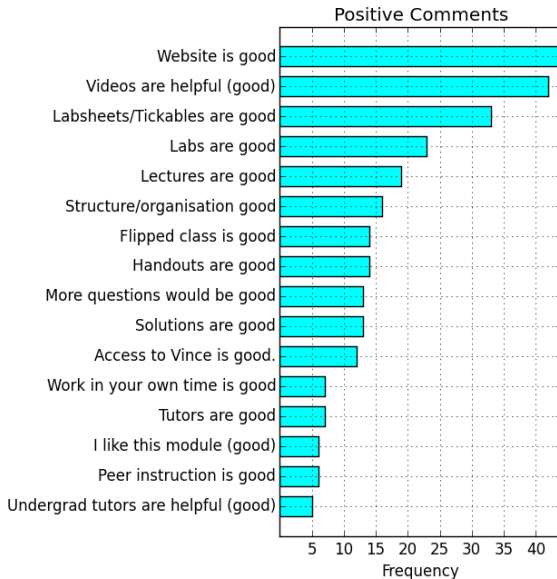
Flipped classrooms

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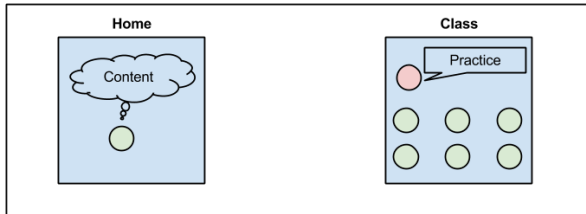
Labs and 'Tickables'

- Every week you have 2 computer lab sessions.
- You have until the end of the second lab session to complete all exercises marked as 'TICKABLE'.
- You will need to work on these outside of the lab sessions.

Resources

http://vknight.org/Computing_for_mathematics/

Feedback from last year

Positive Comments

Label	Frequency

(Horizontal bar chart, x-axis: Frequency, values 5 10 15 20 25 30 35 40)

- Website is good
- Videos are helpful (good)
- Labsheets/Tickables are good
- Labs are good
- Lectures are good
- Structure/organisation good
- Flipped class is good
- Handouts are good
- More questions would be good
- Solutions are good
- Access to Vince is good.
- Work in your own time is good
- Tutors are good
- I like this module (good)
- Peer instruction is good
- Undergrad tutors are helpful (good)

Feedback from last year

Negative Comments

	Frequency
Need a bit more help	
Not the biggest fan of some videos	
Some part of the lecture being forward looking	
Back to back tutorials are bad	
Unclear questions	
Lectures are bad	
Having a test at beginning of term is bad	
More tutors please	
Self learning is bad	

"No matter how difficult the lab sheets are, it always feels like there is sufficient support, either from Vince or from various online resources."

"Only one video tutorial is given providing only one angle"

"Would like to know about all assessment from the start, class test was only recently revealed and don't know much about the remaining 45%"

"More compulsory tickables as its easy to just not do them."

http://cardiffmathematicscodeclub.github.io/

http://pydiff.github.io/