

# Computing for Mathematics: Week 1

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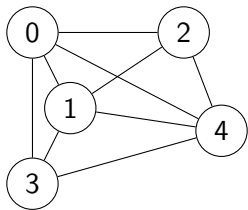
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# Programming and Mathematics

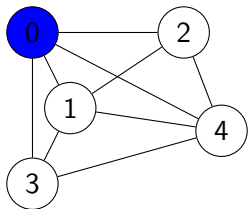
There are various areas in which computers are of major importance to Mathematicians:

- ▶ Computer assisted proofs;
- ▶ Implementation of mathematics;
- ▶ Computer generated proofs;
- ▶ Everyday mathematics.

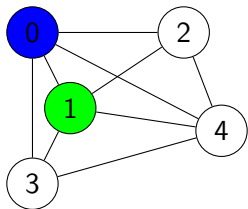
## Computer assisted proofs



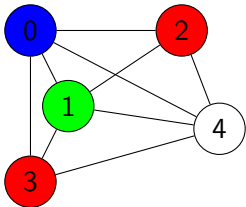
## Computer assisted proofs



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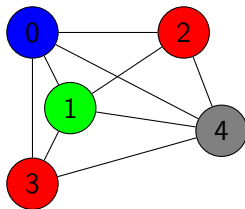


## Computer assisted proofs



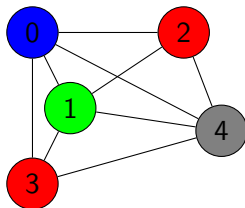


## Computer assisted proofs



- '4 colour theorem': **Any map can be coloured using 4 colours.**

## Computer assisted proofs



- ▶ '4 colour theorem': **Any map can be coloured using 4 colours.**
- ▶ Proved in 1976 by Kenneth Appel and Wolfgang Haken:

Used computers to check 1936 particular cases.

# Risk boards



# Computer assisted proofs

How to pack 3 dimensional spheres?

- ▶ In 1611 Kepler conjectured the best possible way.
- ▶ Proof in 1998 by Hales which involved a computer to minimize a function of 150 variables (100,000 times).
- ▶ **Also** involved a 100 page paper for the 'non computer assisted aspects'.

# Computer assisted proofs

How to pack 3 dimensional spheres?

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- ▶ Proof in 1998 by Hales which involved a computer to minimize a function of 150 variables (100,000 times).
- ▶ **Also** involved a 100 page paper for the 'non computer assisted aspects'.
- ▶ Referees are 99% sure.

# Implementation of mathematics

Here at Cardiff Dr Leanne Smith studied the best way to locate ambulances in Wales. This took in to account:

- ▶ Queues;
- ▶ Survival probabilities of patients;
- ▶ Time of the day...

Once the mathematics was done a computer program was built to be able to demonstrate to the Welsh Ambulance Trust.

# Computer generated proofs

Timothy Gowers

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Timothy Gowers

Theorem: Let  $X$  and  $Y$  be sets, let  $f : X \rightarrow Y$  be an injection and let  $A$  and  $B$  be subsets of  $X$ . Then  $f(A) \cap f(B) \subset f(A \cap B)$ .



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Proof: Take  $x \in f(A) \cap f(B)$ . So there is some  $y \in A$  and  $z \in B$  such that  $f(y) = f(z) = x$ . As  $f$  is injective,  $y$  and  $z$  are equal. So  $y \in A \cap B$ . So  $x = f(y) \in f(A \cap B)$ .

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The above is an example of a computer generated proof. **You do not need to know any of this!**

# Everyday mathematics

Everyday mathematicians might need to calculate an integral for a bigger project. This is some Sage code to calculate an integral:

```
1 integrate(x ^ 3, x)
```

which returns:

$$\frac{x^4}{4}$$

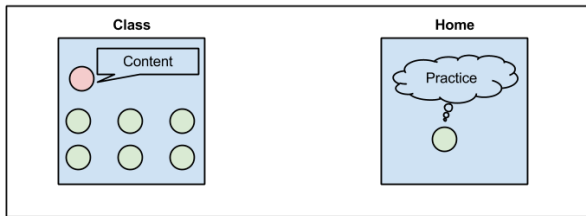
# What we will learn

- ▶ Python: general purpose programming language (Weeks 1-5).
- ▶ Sage: mathematics package (based on Python) (Weeks 5-9).
- ▶  $\text{\LaTeX}$ : a package for writing mathematics (Week 10).

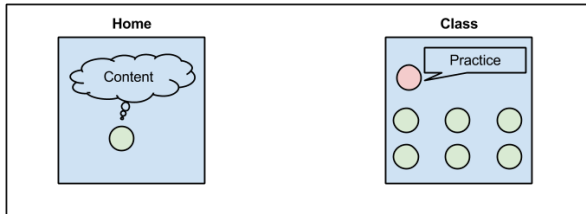
# Flipped classrooms

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**Classic  
Classroom**



**Flipped  
Classroom**



## Labs and 'Tickables'

- ▶ Every week you have 2 computer lab sessions.
- ▶ You have until the end of the second lab session to complete all exercises marked as 'TICKABLE'.
- ▶ You will need to work on these outside of the lab sessions.

## Resources

<http://www.vincent-knight.com/home/teaching/computing-for-mathematics>