THE RELATIONSHIP BETWEEN

Integration and Differentiation

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Abstract

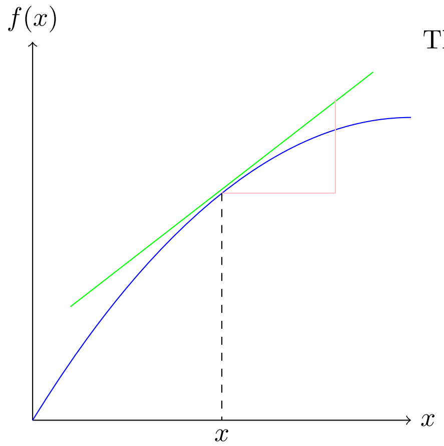
The mathematical topic Calculus is essentially the study of change, and has two major branches; integration and differentiation where integration is formally known as 'the reverse of' differentation. In this article we will explore the relationship between the two; comparing the different methods of integrating and differentiating and looking into the calculus of trigonometric functions.

1 Introduction

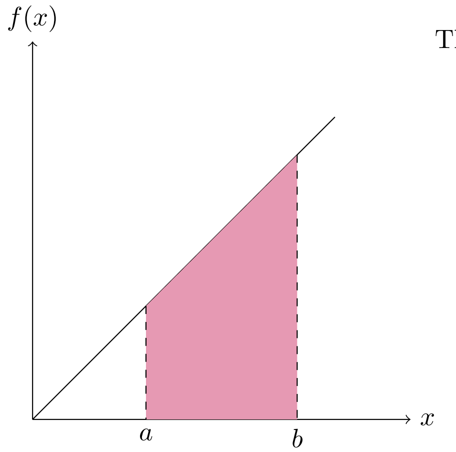
The diagrams below illustrate a geometrical interpretation of differentiation and integration.

This shows how we calculate the derivative of a function using the definition of a derivative which states that if we let $y = f(x)$ be a function then

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \tag{1}$$



This graph shows how we calculate the derivative of a function



This graph shows how we calculate the integral of a function

Thus, let us look at the function $y = x^2$ and suppose we differentiate this function, we obtain $\frac{\partial}{\partial x}x^2 = 2x$. Now, if we integrate we obtain $\int 2x\,dx = x^2 + c$. Thus, you can see from this simple example that integration is the reverse of differentiation.

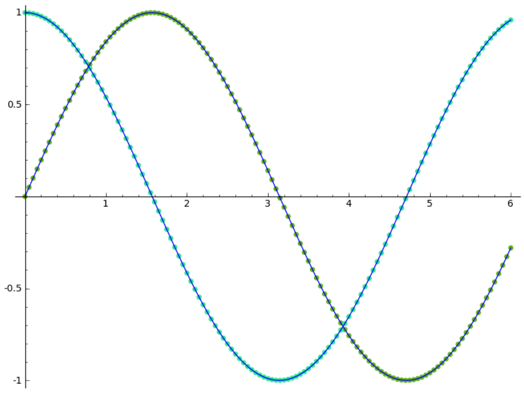
2 Trigonometric Functions

We will now look at the integration and differentiation of trigonometric functions. The table below shows a list of the most important derivatives and integrals, further showing how integration and differentiation are the reverse:

Derivative	Integral
$\frac{\partial}{\partial x} sinx = cosx$	$\int cosx\, dx = sinx$
$\frac{\partial}{\partial x} cosx = -sinx$	$\int sinx\, dx = -cosx$
$\frac{\partial}{\partial x} tanx = sec^2x$	$\int sec^2x\, dx = tanx$
$\frac{\partial}{\partial x} cotx = -cosec^2x$	$\int -cosec^2x\, dx = tanx$
$\frac{\partial}{\partial x} secx = secxtanx$	$\int secxtanx\, dx = secx$
$\frac{\partial}{\partial x} cosecx = -cosecxcotx$	$\int -cosecxcotx\, dx = cosecx$

We can use sage to work out the derivative of sin(x) and then plot the results on a graph, the following code demostrates how we do this:

```
f(x) = sin(x)
g(x) = diff(f,x)
v = [(x, f(x)) for x in [0,0.05,..,6]]
s = (points(v, rgbcolor=(0.4,0.7,0.1), pointsize=30) + plot(spline(v), 0, 6))
u = [(x, g(x)) for x in [0,0.05,..,6]]
t = (points(u, rgbcolor=(0.2,0.9,0.6), pointsize=30) + plot(spline(u), 0, 6))
show(s + t)
```

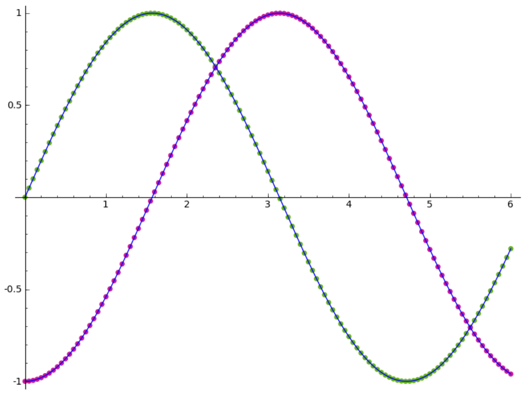


This is the graph that sage produces from the above code.

This shows the graph of $sin(x)$, which differentiates to $cos(x)$

We also can use sage to work out the integral of sin(x) and then plot the results on a graph, the following code demostrates how we do this:

```
f(x) = sin(x)
g(x) = integrate(f,x)
v = [(x, f(x)) for x in [0,0.05,..,6]]
s = (points(v, rgbcolor=(0.4,0.7,0.1), pointsize=30) + plot(spline(v), 0, 6))
u = [(x, g(x)) for x in [0,0.05,..,6]]
t = (points(u, rgbcolor=(0.8,0.1,0.6), pointsize=30) + plot(spline(u), 0, 6))
show(s + t)
```



This is the graph that sage produces from the above code.

This it the graph of $sin(x)$, which integrates to $-cos(x)$

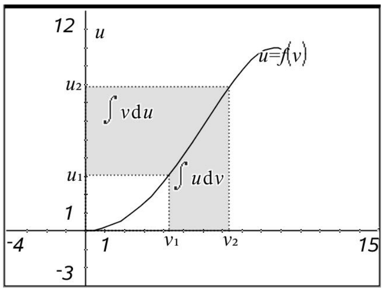


Figure 1: Visual Representation of Integration by Parts

3 Different Methods

3.1 Differentiating a Function

There are various methods you can use to differentiate a function, for example implicit differentiation, which can be approached using sage.

3.2 Ways to Integrate a Function

There are various methods you can use to integrate a function:

- Integration by parts;
- Integration by substitution.

Let us look at integration by parts, which states that:

$$\int v \frac{\partial u}{\partial x} dx = uv - \int \frac{\partial v}{\partial x} dx \tag{2}$$

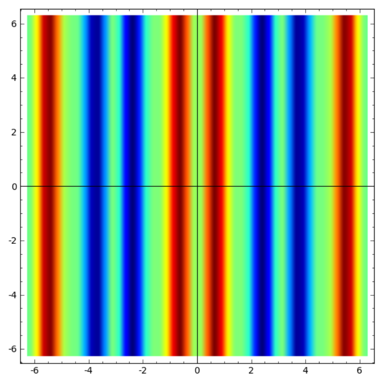
This can be represented visually in Figure 1 (taken from `http://compasstech.com.au/TNSINTRO/TI-NspireCD/mystuff/showcase.html#act11.`). There are many books with explore this method further. [1]

4 Conclusion

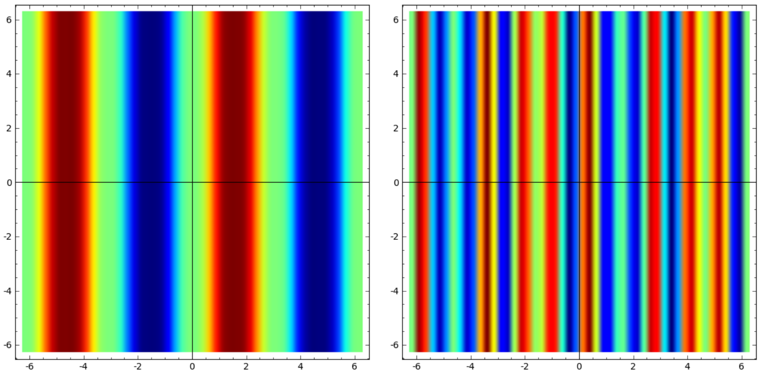
To conclude, we have briefly looked at the integration and differentiation of trigonometric functions, which have been illustrated in some simple graphs plotted on sage. However, we can also look at some more complex graphs on sage, for example, take the code:

```
density_plot(sin(x)^2 * cos(x)^3, (-2*pi, 2*pi), (-2*pi, 2*pi),
cmap='jet', plot_points=50).show(figsize=(6,6), frame=True)
```

This plots the function $sin(x)^2 cos(x)^3$ using a density plot graph, as shown below:



From this graph, we can compare the derivate and integral of the function $sin(x)^2 cos(x)^3$:



References

[1] Justin Domke. Implicit differentiation by perturbation. In *Advances in Neural Information Processing Systems*, pages 523–531, 2010.