# Week 7 - Symbolic Calculus

Using Sage we can carry out various operations from Calculus. This week we will investigate how to:

* Carry out limits in Sage;
* Carry out differentiation in Sage;
* Carry out integration in Sage.

1. Last week we saw how to define a function in Sage:

* f(x) = x ^ 3 + 3 \* x + sin(x)
* To obtain the variables of a function we can use the variables method:
* print f.variables()
* Try this with a function of more than one variable:
* f(x, y) = x \* y + x ^ 2 + y ^ 2

1. In calculus the following definition of a limit is well know:

* iff such that : .
* Let us calculate the limit of as .
* First of all let us plot :
* plot(f(x), x, .5, 10)
* The following code obtains :
* f.limit(x=1)
* We can also obtain the same result using the limit method:
* limit(f,x=1)
* Note that :
* f(1)
* This implies that is continuous at 1.

1. **TICKABLE** Plot using the default options:

* plot(f)

We see that Sage is plotting extremely high values at the discontinuity due to a root of the denominator which seems to be around . We can plot our function either side of that point and combine them. We do this by creating plot objects:

p = plot(f, x, 0.8, 10) type(p) p += plot(f, x, -10, .6) type(p) p.show()

and identify (use the solve function or the roots method, and maybe the denominator method on ) : the root of the denominator of . Obtain and . Directions of limits can be obtained using the following code:

limit(f, x=??, dir="plus") limit(f, x=??, dir="minus")

1. There are various algebraic relationships on limits:
   1. (if )

* We can verify the first identity with the following Sage code for a particular example:
* f(x) = exp(x)g(x) = sin(x)var('a')L1 = limit(f(x) + g(x), x = a)L2 = limit(f(x), x = a) + limit(g(x), x = a)bool(L1 == L2)

Note that we use the bool class to convert the symbolic equation L1==L2 to a boolean variable. Verify with some example functions the other two relationships above.

1. Basic differentiation
2. Limiting definition of a derivative
3. Plotting the limiting definition of a derivative
4. Visualising the limiting definition of a derivative
5. Differentiation rules
6. Basic integration
7. Integration by parts
8. Riemann integration
9. Numerical integration
10. Integrate polynomials in a data file