# Week 8 - Linear Algebra

1. Use Sage to solve the following system of equations:



1. **TICKABLE** Note that the above system of equations is equivalent to the following systems of equations:

* In essense the only thing that defines the system of equations is the cofficients:
* We can of course seperate the right hand side of our equation and perhaps include those elements in a vector. Our system can now be represented as:
* Let us attempt to represent the above system in Sage.
* The following defines b as a vector:
* b = vector(0,154)
* The representation of coefficients is a well defined mathematical object called a matrix. The following code defines A as a matrix:
* A = matrix([[10, 2], [2, -1]])
* If we define a vector X as a vector of the symbolic variables:
* X = vector([x, y])
* We can **multiply** A by X:
* A \* X
* Verify that where is the solution to our system of equations (obtained in (1)).

1. **TICKABLE** In linear algebra (you will study this next semester) a matrix equation is an equation of the form:

* or
* If we define A and b as in question 2 we can solve this equation quite simply using the solve\_right or solve\_left methods. The following obtains a solution to the equation :
* A.solve\_right(b)
* Note that A\b is shorthand for A.solve\_right
* Use the above to solve the following system of equations using matrix notation:

1. For reasons that will become clear, the following definition of matrix multiplication is required:

* For matrices this is equivalent to:
* As an example create the following two matrices in Sage:
* A = matrix([[1,2],[3,4]])  
  B = matrix([[7,8],[9,10]])
* Attempt to multiply these matrices by hand and carry out their multiplication in Sage:
* A\*B
* Repeat the exercise by multiplying the following pairs of matrices:
  1. ,
  2. ,
  3. ,
  4. ,

1. **TICKABLE** The previous exercise shows that when considering matrix multiplication there exists a matrix which does not have a multiplicative affect: "the identity matrix".

* The identity matrix of size is denoted by . The following Sage code gives :
* identify\_matrix(4)
* Note also, that the previous exercise showed that we can sometimes find a matrix such that . Finding such a matrix is refered to as 'invering' and if certain properties hold (you will see this in further details next semester) this matrix is denoted .
* If we recall the matrix equation and if we assume that exists then multiplying both sides by gives:
* In Sage we can obtain (if it exists) with the following code:
* A.inverse()
* Thus another approach to solving is:
* A.inverse()\*b
* Use this approach to solve the systems of equations we have considered so far.

1. **TICKABLE** Recalling your basic python knowledge. Lists can be used to hold any sort of object. Obtain a list of the inverses of the following matrices (when the inverse exists, you might need to look up information on try and except):

* For every matrix in this list and the original list obtain the result of the det method. This gives the **determinant** of the matrices. It is a very important quantity that will be explained next semester.

7 **TICKABLE** The random\_matrix command can be used to obtain a random matrix:

random\_matrix(ZZ, 5) # Gives a random square matrix of size 5 in $\mathbb{Z}$  
 random\_matrix(QQ, 5) # Gives a random square matrix of size 5 in $\mathbb{Z}$  
  
Using this attempt to conjecture a connection between the determinant of a matrix and it's inverse (and the determinant of it's inverse).

1. **TICKABLE** The file [W08\_D01.txt](./Data/W08_D01.txt) contains 4 columns of data:

* a, b, c, d, f, g
* For each row of data, obtain the solution to the system of equations:
* Write to file a new data set containing the following columns:
* A, B, C, D
* Where is the number of the original data set, and are the solutions to the system of equation in question: . is the 'norm of the solution vector': . Write the data to file in such a way as it is sorted by (attempt to do this without the inbuilt sort method).
* If there is no solution to the system of equations set B=C=D=False. The data set is a randomly sampled set of problems, how often does a solution exist? What is the mean value of ?

1. The file [W08\_D02.txt](./Data/W08_D02.txt) contains 20 rows and 20 columns of numbers. This file represents a matrix. Invert the matrix, what is it's determinant? What is the determinant of it's inverse?