# Week 9 - Linear Algebra

1. Use Sage to solve the following system of equations:



1. **TICKABLE** Note that the above system of equations is equivalent to the following systems of equations:

* In essense the only thing that defines the system of equations is the cofficients:
* We can of course seperate the right hand side of our equation and perhaps include those elements in a vector. Our system can now be represented as:
* Let us attempt to represent the above system in Sage.
* The following defines b as a vector:
* b = vector(0,154)
* The representation of coefficients is a well defined mathematical object called a matrix. The following code defines A as a matrix:
* A = matrix([[10, 2], [2, -1]])
* If we define a vector X as a vector of the symbolic variables:
* X = vector([x, y])
* We can **multiply** A by X:
* A \* X
* Verify that where is the solution to our system of equations (obtained in (1)).

1. **TICKABLE** In linear algebra (you will study this next semester) a matrix equation is an equation of the form:

* or
* If we define A and b as in question 2 we can solve this equation quite simply using the solve\_right or solve\_left methods. The following obtains a solution to the equation :
* A.solve\_right(b)
* Note that A\b is shorthand for A.solve\_right
* Use the above to solve the following system of equations using matrix notation:

1. For reasons that will become clear, the following definition of matrix multiplication is required:

* For matrices this is equivalent to:
* As an example create the following two matrices in Sage:
* A = matrix([[1,2],[3,4]])  
  B = matrix([[7,8],[9,10]])
* Attempt to multiply these matrices by hand and carry out their multiplication in Sage:
* A\*B
* Repeat the exercise by multiplying the following pairs of matrices:
  1. ,
  2. ,
  3. ,
  4. ,

1. **TICKABLE** The previous exercise shows that when considering matrix multiplication there exists a matrix which does not have a multiplicative affect: "the identity matrix".

* The identity matrix of size is denoted by . The following Sage code gives :
* identify\_matrix(4)
* Note also, that the previous exercise showed that we can sometimes find a matrix such that . Finding such a matrix is refered to as 'invering' and if certain properties hold (you will see this in further details next semester) this matrix is denoted .
* If we recall the matrix equation and if we assume that exists then multiplying both sides by gives:
* $$A^{-1}AX=A^{-1}b\Rightarrow \mathbb{I}\_nX=A^{-1}b=X=A^{-1}b$$
* In Sage we can obtain (if it exists) with the following code:
* A.inverse()
* Thus another approach to solving is:
* A.inverse()\*b
* Use this approach to solve the systems of equations we have considered so far.

1. Enter the following matrices in to a list. Invert all of them.
2. Plotting something?
3. Solve a large number of systems of linear equations
4. Reading in data for a big system of linear equations
5. Creating a big linear system and solve it.