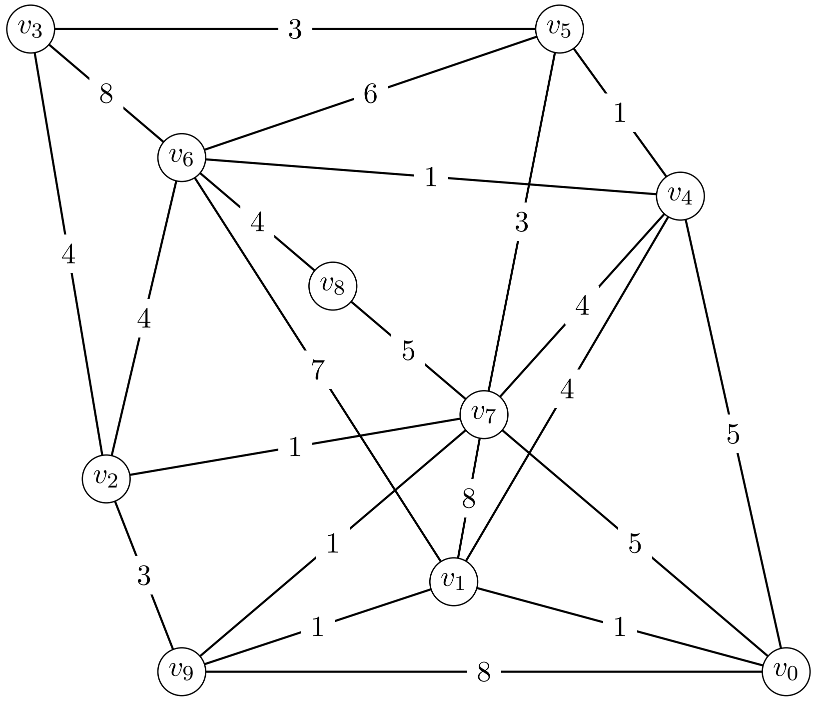
3rd EXERCISE
104.272 Discrete Mathematics

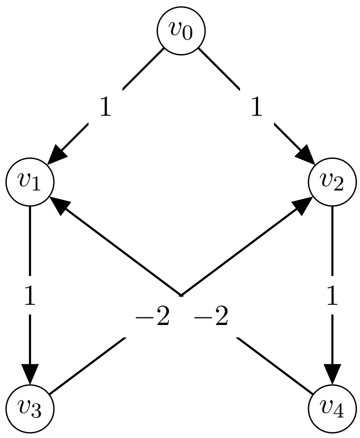
(21) Compute a minimum and a maximum spanning tree of the graph below using Kruskal's algorithm.



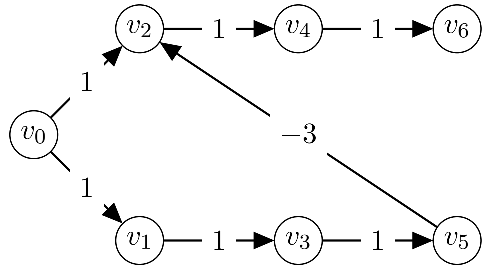
(22) Compute the distances from vertex v_0 in the graph above using Dijkstra's algorithm, highlighting the distance tree computed by Dijkstra's algorithm with extra colour. Show, using a very small counterexample, that the distance tree (as produced for example by Dijkstra's algorithm) is, in general, not a minimum spanning tree, and that the minimum spanning tree is in general not the tree with minimal distances.

(23) Give an example of a weighted graph G for which Dijkstra's algorithm fails.

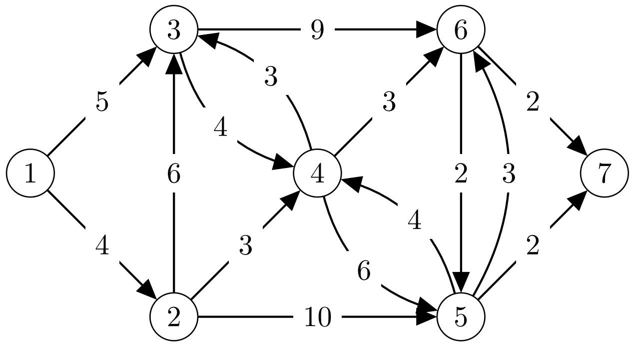
(24) Compute the distances from vertex v_0 in the graph below using the algorithm by Moore (aka Bellman-Ford). Explain why this implies that we cannot define a 'distance tree' for graphs with negative edge weights.



(25) Compute the distances from vertex v_0 in the graph below using the algorithm by Moore, illustrating why it computes the distances correctly.

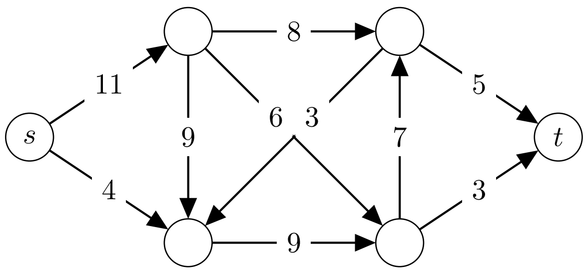


(26) Compute the minimal distances between all pairs of vertices in the following graph using the Floyd-Warshall algorithm:



(27) Let ϕ_1 and ϕ_2 be flows on a weighted digraph G. Let γ_1 and γ_2 be non-negative reals with $\gamma_1 + \gamma_2 \leq 1$. Show that $\gamma_1\phi_1 + \gamma_2\phi_2$ is a flow on G.

(28) Use the algorithm of Ford and Fulkerson to compute a maximal flow in the following network:



(29) Assume that we have given the graph above and that its vertices have weights (=bounded capacity), too. Is it still possible to apply the algorithm of Ford and Fulkerson to determine a maximal flow?

(30) The Edmonds-Karp algorithm looks for augmenting paths from s to t for a given flow ϕ by using breadth-first search, i.e., it selects an augmenting path with the least number of edges possible. Find a maximal flow, using the Edmonds-Karp algorithm, for a (small) graph of your choice, where at least once at least one backward edge must be used.

Hint: one possibility is to adapt the graph below.