Review of analytical instruments for EEG analysis

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Abstract: Since it was first used in 1926, EEG has been one of the most useful instruments of neuroscience. In order to start using EEG data we need not only EEG apparatus, but also some analytical tools and skills to understand what our data mean. This article describes several classical analytical tools and also new one which appeared only several years ago. We hope it will be useful for those researchers who have only started working in the field of cognitive EEG.

Competing interests: The authors have declared that no competing interests exist.

Introduction

Over the past few decades statistic have grown considerably, because new research in abstract and applied mathematics has been conducted. Outspread of computers and the creation of WWW have played a crucial role for statistical analysis. Many powerful methods such as Bayesian analysis, Monte-Carlo simulation, FFT, wavelet analysis, spline interpolation, generalized linear/additive models, machine learning can be applied because we have cheap and fast computers. Some methods such as Bayes function were discovered in the 19th or early 20th century, but they have been out of use due to hard practical implementation. Over the last decade we have seen blow up grow of machine learning methods in science and engineering, and during the past several years the most modern and impressive computer science technology is Big Data. Every day the world science community generate incredible amount of information. It is not only data but also new methods, theories, concepts and application. In order to orientate ourselves inside this "ocean of data" we need new metastructure for analysis procedures and best practices. For this purpose, we want provide an overview of some of the modern methods and show how these groups of methods are interrelated. We suggest metastructure for the data mining procedure and its application for EEG data analysis.

Layer 1: Subject area - Basic cognitive EEG concepts

Human brain produces complex electric fields patterns. In order to understand the basic EEG technique there are good books about electroactivity and neuron oscillations. The following book is good starting point for learning about biophysics that underlie the EEG signals (Nunez & Srinivasan, 2006). The authors cover small- and large-scale electrogenerators, passive volume conduction, recording strategies, spectral analysis, source localization. If you have some background knowledge in physics (electricity and magnetism), this reference book will prove useful to you. The introductory course of neurobiological mechanisms of oscillations can be found in the book by Buzsaki (Buzsáki, 2006). For general information about organization and design experiments in cognitive EEG you can read the book by Luck (Luck, 2005).
You can find comprehensive introduction in EEG analysis in the book by Cohen (Cohen, 2014). The author touches upon many practical questions, from preparing equipment for experiments to statistical inference and overview of the future of cognitive electrophysiology.

Layer 2: Preprocessing

Time-series analysis has a huge set of methods, solid mathematics background and a long history in science and engineering. We try to understand the character of our data when we see their visual representation. The investigation of graphic of the data can give keys to the nature and character of function. Even simple graphics observation (peaks, valleys, periods, linear trends) can help us in our research. There is no substitution for raw data investigation by open eye.
Before starting to use the data we often need certain preprocessing: cleaning noise, removing artifacts and malfunctioning channels, filtering low and high frequencies, applying baseline assignment. Details about data prepossessing can be found in books by Luck and Cohen (Luck, 2005), (Cohen, 2014). After preprocessing we start using all other methods of analysis.

Layer 3: Analytical decomposition. Spectral analysis and TDA

Fourier analysis: Fourier analysis is rather old and the most elaborated method of analysis. It is a mathematical technique for transforming the signal from time-based to frequency-based domain. Any function *f(t)* we can be decomposed in linear combination of *sin(x)* and *cos(x)*:

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right)$$

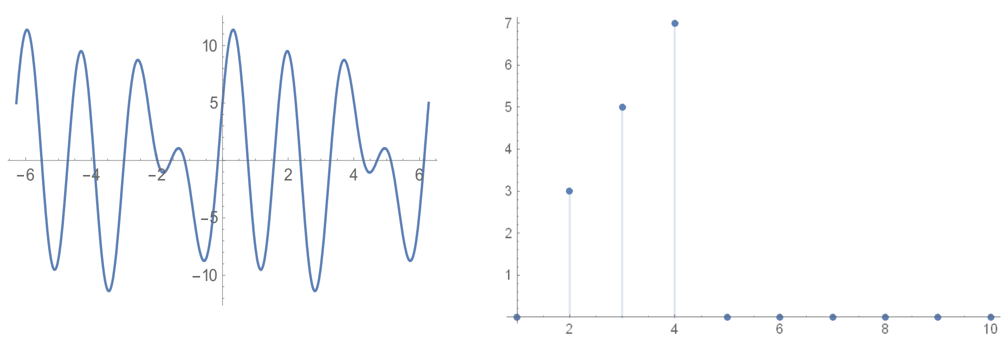


Figure 1. Function $3\sin(2t) + 5\cos(3t) + 7\sin(4t)$ and its spectrum.

Pairs (a, n), (b, n) form a spectrum of signal. Figure 1 is an example of a function and its spectral decomposition. The techniques of Fourier analysis widely known and have a long history. We can analyze spectrum by using both classical statistical methods, i.e. Bayesian and modern techniques, i.e. a topological data analysis. However, there are several limitations for Fourier transformation to be used with EEG signals. Fourier analysis is suitable for stationary and linear signals because sin waves do not have time localization, but EEG signals have non-stationary nature. Presentation of frequency dynamics in time domain is not an easy task and it is hard to understand its meaning intuitively. Due to these limitations it is better to use wavelets in EEG analysis.

Orthogonal polynomials: The next step in the process of choosing a basis function for decomposition is Chebyshev polynomials. As we know from the theory of partial differential equations, Chebyshev polynomials will be a better choice for spectrum decomposition almost in all cases unless the solution is spatially periodical. They are defined by the equation:

$$T_n(cos\theta) \equiv \cos(n\theta)$$

For non-periodic cases Legendre polynomials can also be used:

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}[(x^2\text{-}1)^n]$$

The theory of orthogonal polynomials is still being developed. And this theory supported by fast computational methods to use it in daily practice. We can try to make use of some of polynomials in EEG analysis for a spectral decomposition, because, as we know from physics, sometimes a change of basis in a system can provide a simpler picture of researched phenomena. For the overview of the modern state of the theory of orthogonal polynomials see, for example, in the lectures notes by Gautschi (Gautschi, 2006). The author provides a library of Matlab functions for computing many modern polynomials.

Wavelets: *Sin(x)* and *cos(x)* waves are not localized in time domain. By contrast, wavelets are “small” waves which have good time localization. For example, Morlet wavelet is a composition of two functions: Gauss and *sin(x)* (Figure 2).

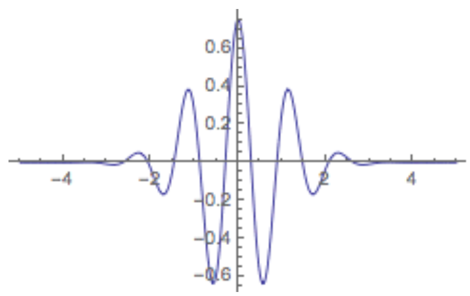
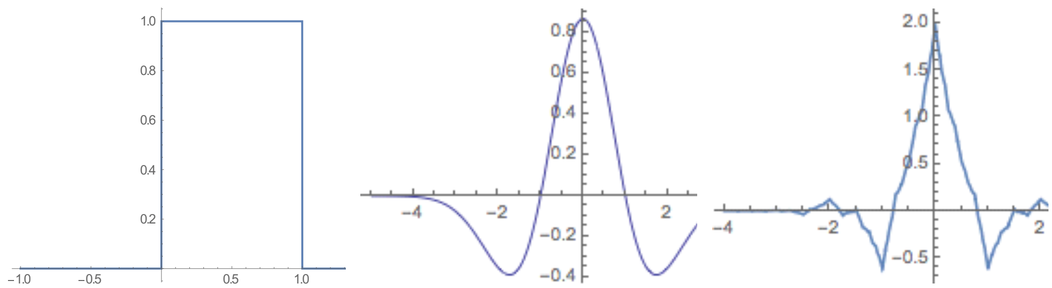


Figure 2. Example of Morlet wavelet.



Haar wavelet | Mexican Hat wavelet | Biorthogonal spline wavelet

Figure 3. Examples of wavelets.

Splines: After De Boor had introduced the *B-splines* numerical computation method (De Boor, 1972), splines spread widely in many areas of science and engineering. Fast computation and easy construction procedures for splines make them a useful tool in many numerical experiments. As a first step in the construction of a spline we divide an interval of function we wish to approximate into N subintervals. Then we allocate polynomial of degree for each subinterval n, $n = 1..N$. Thirdly, we join all the adjacent polynomials together to form a piecewise polynomial curve. The points which join polynomials are called knots. Polynomials at knots should satisfy some continuity constraint: they have to be differentiable at these points.
Splines play several roles in modern analytical methods. First, they are widely used as auxiliary entities: for approximation and interpolation of non-periodic functions, in wavelets construction (see biorthogonal spline wavelet in Figure 3). In functional data analysis (FDA) *B-splines* are used as basis functions, in Hilbert-Huang transformation they are used for the implementation of IMF. Periodic splines can be applied for Spline Harmonic Analysis which combines computation speed from FFT and approximation abilities of splines (Zheludev, 1998).

Blind source separation and ICA: For the last two decades blind source separation has applied widely in several areas as telecommunications, digital signal processing, biomedical engineering, financial data analysis, astronomical imaging. The goal of BSS acquire the several unknown sources of signals from sensor arrays. Classical problem is known as separate signals from two sources by two microphones. Independent component analysis (ICA) one of the first techniques of BSS revealing hidden parameters of observed signals. But as we know in EEG analysis extensively used only ICA and as part Hilbert – Huang transformations empirical mode decompositions.

ICA method assumes that the original sources are statistical independent and uses several good known algorithms as Infomax, maximum likelihood estimation, the maximum a posterior and Fast ICA. In EEG analysis ICA method often uses for removing artifacts from raw signals.

Modern set of BSS tools is very rich and consist of such methods as empirical mode decompositions, compressed sensing, factor analysis, dictionary learning.

We think that the modern methods of BSS paired with machine learning algorithms can give powerful tool for researchers and engineers.

Hilbert-Huang transformation: Any set of predefined functions cannot give good results in the cases when are signals nonstationary and nonlinear. In this case we need an adaptive basis for our analysis. It means that basis has to be relevant to the data or data-dependent. Hilbert - Huang transformation resolves some problems for that type of complex data (Huang et al., 1998). HHT consists of two parts – Hilbert spectral analysis and empirical mode decomposition (EMD). EMD is an adaptive signal decomposition into a sum of natural, intrinsic blocks (intrinsic mode functions IMF) which describe complex waves. EMD still lacks rigorous theoretical foundation, it is simple but "empirical" algorithm. HHT is widely used in applications in spite of some problems in theory and methodology of this method.

TDA: Topological data analysis appeared only several years ago. Its basis is algebraic topology – a branch of mathematics which is connected with dynamics and computer science. Persistent homology is the main concept of topological data analysis. Any data have a structure, and topology can give us some keys to their nature. We replace data with simplicial complexes using persistent homology, then we transform them into the parameterized form of Betty numbers called barcode. If digital EEG data a presented in time or frequency domain, we receive a point cloud. Then the 'shape' of this cloud provides us with information about the nature of underlying neuron activity. Algorithms for computing have implementations in Matlab code, and in CRAN repository of R. For introduction in some concepts of TDA you can see the book by Edelsbrunner and Harer (Edelsbrunner & Harer, 2008).

Layer 4: Synthetic and smooth methods. Statistical methods and FDA

Classical statistics: Classical statistics provide powerful tools, both basic and advanced, for any type of analysis. Simple descriptive statistics such as mean, standard deviation, quartiles can be used almost in all kind of data investigations. Hypothesis testing, p-values, confidence intervals are easy but powerful instruments of statistical inference. We use them for avoiding pitfalls in the early stages of data analysis. Analysis of variance (ANOVA), statistical independence test like chi-squared test and correlations give us opportunity to find relations in our data. To understand modern methods such independent component analysis (ICA) we need some background in information theory.

Bayesian statistics: Bayesian statistics use only one tool – the Bayes' theorem:

$$g(\theta|data) = \frac{g(\theta)\times f(data|\theta)}{\int g(\theta)\times f(data|\theta)d\theta}$$

This method has several differences from classical statistics:

- Classical or "frequentist" statistics do not mean any prior knowledge about the data which we analyze. Bayesian statistics use the prior information about the process and the information about the process is contained in data.
- Bayesian method supplies a direct tool for computation of parameters probabilities.
- Bayesian statistics can marginalize nuisance parameters effectively.

Bayesian inference can be imagined as a statistic "Occam's razor". The iterative use of Bayesian theorem eliminates all impossible hypotheses from our prior knowledge. In the end, we will receive only most probable outcomes.

Good practical modern introduction in Bayesian analysis can be found in (Kruschke, 2011).

Generalized linear/additive models: The dependence of current values on the previous values can explain the correlation between adjacent points. Time-series analysis uses such methods as autocorrelation and cross-correlation functions. Their modern modification such as ARIMA was introduced in 1970 by Box and Jenkins, and after that linear use generalized models grow significantly. Linear models are statistics models in which responses are presented as linear combinations of predictors and a sum of random error component.

Generalized linear models (GLM) weaken the linearity of linear models. They allow a expected value of response to depend on differentiable function of the predictor. Generalized additive models (GAM) are the next step from linearity in time-series analysis. They allow predictors to be sum of smooth functions, which do not have straight parametric form, and may have only limited differentiability. This evolution from linearity to non-linearity is a balance between flexibility and overfitting from one hand, and model interpretability on the other hand. For introduction in GLM/GAM with implementations in R you can read book by Wood (Wood, 2006).

FDA: Functional data analysis is a generalization of multivariate data in infinite dimension. The resulting data can be curves, surfaces or other continuous complex objects. The term functional data analysis was first introduced in the article by (Ramsay and Dalzell, 1991). This framework includes several methods, concepts and ideas from functional analysis, calculus of variations and statistics. The concept of FDA is characterized by the following criteria:

- functional data is continuous
- individual unit of data is a function
- smoothness of data is one of the key aspects of this analysis
- the derivatives of the functions often play an important role
- principal component analysis is one of central concepts

For comprehensive introduction in FDA techniques see (Ramsay and Silverman, 2005).

Layer 5: Machine learning

Over the past few decades the use machine learning methods has increased dramatically. They pull together the best from expert systems, neuron networks, Bayesian statistics and pattern recognition. The concept of machine learning is based on the four fundamental principles:

Data, Abstraction, Generalization, Evaluation.

Data: Before we start learning something we need data, which must be collected in advance. In case of EEG there must be some EEG device, input interface and data storage for collecting data. It is important to preprocess by cleaning them from any noises, artifacts and mistakes. However, gathering and storing data do not provide us with knowledge, that we should take next step.

Abstraction: At the abstraction stage we try to isolate any kinds of structures from data. They may be connected or disjoint graphs, pictures, mathematical equations, clusters, logical rules etc. During this phase we try to look at the data and identify something meaningful or obvious for us. Like ancient people who sow star constellations in the sky, we also seek any close connections in data. For example, for EEG we decompose signals in a spectrum and try to find better statistical distribution. But still it is not full knowledge and in some cases not knowledge at all.

Generalization: If we apply the gained abstracted knowledge only for one set of data, it will not be of much use. Good models can be used in many areas of science and engineering.

Evaluation: During the last stage we apply our algorithm for new dataset. It is obvious that perfect generalization of model to unseen data is very rare. In many cases failure occurs due to noise in data. If we include the noise in our model, the system may grow redundant and may start recognize of true patterns in data in worse way. This is known as "overfitting" problem.

All methods of machine learning can be divided into two groups: supervised and unsupervised. Supervised methods involve creating a predictive statistical model.

In unsupervised learning there is no model at all, but we can identify connections and structures inside the input data.

Machine learning algorithms solve several tasks: clustering, classification, prediction. Clustering is a first task in exploratory data analysis; raw data should be divided in any groups for investigation. After clusterization we can use classification for categorical data, and prediction for nominal datasets.

For introduction to machine learning with software implementation in R see (James, Witten, Hastie, and Tibshirani, 2013) and (Lantz, 2013), in Weka (Witten, Frank, and Hall, 2011), in Matlab (Theodoridis & Koutroumbas, 2008).

In EEG methods of machine learning have been widely used over the last decade, see for example (AlZoubi, Calvo, and Stevens, 2009) (Sohaib, Qureshi, Hagelbäck, Hilborn, 2013) (Höhne, Bartz, Hebart, Müller, and Blankertz, 2015).

Layer 6: Big Data

During past several years Big Data has emerged as a new keyword in data analysis. EEG data do not fit into the formal scheme of *Five V* of Big Data as it is given in (Demchenko, Grosso, De Laat, and Membrey, 2013). The *Five V* are:

- volume
- velocity
- variety
- value
- veracity

EEG data are obviously not “too” big and have structured nature.

Big Data provide several interesting tools for concurrent computations. For instance Hadoop cluster gives opportunities to create systems for processing massive data. One of the most interesting components of Hadoop environment is Apache Spark. This is an open-source, fast, general-purpose engine for large-scale data processing. Apache Spark extends the Map-Reduce models and makes it possible to combine different processing types of workloads. Spark has APIs to Python, Java, Scala, R and SQL. It consists of several integrated parts, which form the following structure:

<table>
<tr><td>SQL engine</td><td colspan="2">Spark Streaming</td><td colspan="2">Mlib, Machine learning</td><td>GraphX, graph processing</td></tr>
<tr><td colspan="6">Spark Core</td></tr>
<tr><td colspan="2">Standalone Scheduler</td><td colspan="2">YARN</td><td colspan="2">Mesos</td></tr>
</table>

The integration of SQL engine, machine learning, library, graph computing tool, and scheduler of distributed processes in one framework can offer new opportunities for EEG analysis even for relatively small sets of data.

Software tools

Matlab: Modern data analysis is almost impossible without computer programs. In neuroscience data arrays too big to handle them manually. In DSP (digital signal processing) MathWork Matlab is standard de facto. Matlab have fast linear transform procedures, so its main power in use for applications of linear algebra. Matlab have script program language and mechanism for creation and modernization of toolboxes (user application packages). For general introduction to Matlab you can read (Sizemore and Mueller, 2014).

For purposes of EEG analyzes science community created a lot of applications (toolboxes), and new projects still start every year. We list several of them: EEGLab, FieldTrip, BrainStorm. EEGlab good starting point to try. It have elaborate structure, powerful methods for time-series and independent component analyzes, see (Delorme and Makeig, 2004), (Delorme, Makeig, Debener, and Onton, 2004), (Makeig and Onton, 2011). It can read a data from many modern datafile formats. Friendly, easy tutorial you can find on *http://sccn.ucsd.edu/wiki/EEGLAB_Wiki* and video materials from workshops make it possible to start use it in practice in several days. Also EEGlab have over 20 specific plugins like ERPLab for event-related potentials (Lopez-Calderon and Luck, 2014), BCILab for brain computer interface design and analyzes, NFT – 3D-Head and source location modeling, MobiLab - Mobile brain/body imaging (MoBI) and others (Delorme et al., 2011)

R: is a free software environment for statistical computing and graphics. R open source project and freely available for several operational systems on site *https://www.r-project.org*. It is including linear, nonlinear methods, statistical tests, classical and Bayesian statistics methods, classification, clustering, machine learning libraries. Packages for R develop and maintain many groups and individual researchers. You can find them on *https://cran.r-project.org/web/packages*, each package usually contains code, documentation and some example data arrays. Several packages were developed for EEG study - eegkit, icaOcularCorrection, eegAnalysis. For modern methods of data analysis good introduction in R (James et al., 2013). Book accompanied on-line MOOC course “Statistical Learning” at Lagunita, Stanford, *https://lagunita.stanford.edu*. Authors tell about linear and polynomial regression, logistic regression and linear discriminant analysis, cross-validation and the bootstrap, model selection and regularization methods, nonlinear models, splines and generalized additive models, tree-based methods, random forests and boosting, support-vector machines and other methods. For introduction and reference in programming language of R see (Matloff, 2011), (Wickham, 2014).

Python: is a script programming language known as one of scientific research tools. It is free open-source project that has many implementations and huge community of developers. Many Linux distributives have it as installed part. Python central coordination site *https://www.python.org*, on-line tutorials are available there, it also has big library depository. Python scientific stack SciPy (*http://www.scipy.org)* you can get with distributions like Anaconda (*http://store.continuum.io/cshop/anaconda*), Enthought Canopy (*http://www.enthought.com*), Python(x,y) (*http://python-xy.github.io*), they also have reach collection of popular libraries for data analyses and complex computations. For introduction to data analysis with Python see (McKinney, 2012). It covers Ipython and NymPy libraries, Pandas data structures, data manipulations and aggregation, visualization, some beginning time-series analyzing techniques. For EEG analyze Python have libraries like MNE-Python *(http://martinos.org*). Some of its features: raw data visualization, epoching, averaging, removing artifacts with SSP projections, ICA, forward modeling, linear inverse solving, sparse inverse solvers. For example of use MNE-Python in EEG analysis area see (Gramfort et al., 2013). Another library is PyEEG (*http://pyeeg.org)* have some interesting functions: power spectrum density, petrosian fractal dimension, higuchi fractal dimension, hjorth mobility and complexity, hurst exponent, detrended fluctuation analysis and others. Example of use that packet in practice in paper (Bao, Liu, and Zhang, 2011).

Weka: is a stack of over 100 machine learning algorithms for data mining. It is a free open-source Java application developed at the University of Waikato, New Zealand (*http://www.cs.waikato.ac.nz/ml/weka)*. It can solve typical for machine learning groups of task: clustering, classification, regression, attribute selection, finding association rules and others. A companion book for the Weka software is (Witten et al., 2011), University of Waikato also has a MOOC about using Weka for practical tasks (*https://weka.waikato.ac.nz/dataminingwithweka)*. Java cross-platform object-oriented language can be powerful tool for implementation user defined algorithms. Weka have 3 different GUI which can help you design configurations for data processing, change parameters of algorithm and handle statistical tests. All its features give the Weka a low bar for starting practical data mining. For examples of use Weka for EEG analysis see (AlZoubi et al., 2009), (Sohaib, Qureshi, Hagelbäck, Hilborn, 2013).

Resume

Modern statistical methods and its computer implementations confirm ecosystems of concepts, algorithms and tools which form a several layers structure. Each level do not have strict borders and interrelate between themselves. This system always grows with new researches in mathematics, computer science and statistics. For EEG analysis we may need an orienteer to navigate in this growing and complex structure. In our paper we overview and classify modern branches of analysis. We hope it can help avoid a "analytic's paralyze" for researchers who is a new in this field of science.

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