$\mathbb{ITYM}$ 2021 - Problem 7: Proper Numberings of Graphs

Presented by Achille De Ridder
France

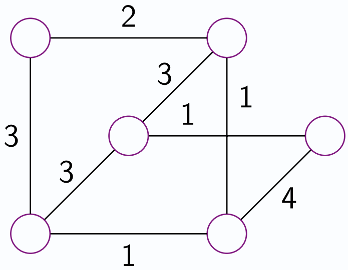
Exposition of the problem

- $G = (V, E, \lambda)$
- $|\nu(u) - \nu(v)| \geq \lambda(e)$
- $s(v) = \sum_{(u,v) \in E} \lambda(e)$
- $S(G) = \max_{v \in V} s(v)$

Greedy Algorithm

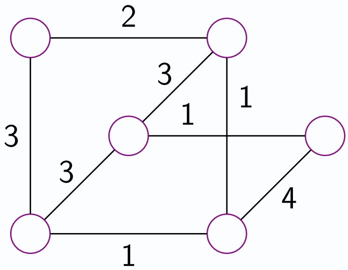
In a arbitrary order, give to each vertex the least possible number for which the required condition for a proper k-numbering will not be violated.

Greedy Algorithm



Graph labelled with the Greedy Algorithm

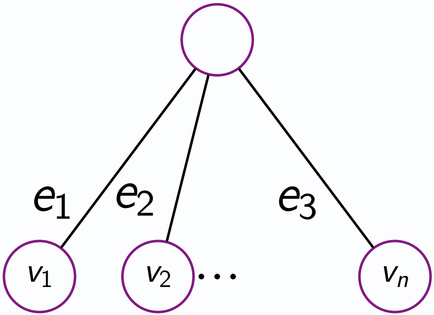
Greedy Algorithm



Graph labelled with the Greedy Algorithm

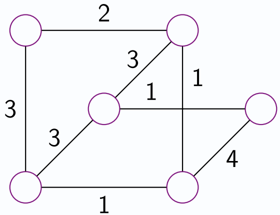
Greedy algorithm

For a vertex v of a graph G with at least 2 edges, there is an integer between 1 and $2S(G) - 1$ which can be used to label v without violating the required condition for a proper vertex k-numbering.



Proper vertex $S(G) + 1$-numbering

All graphs have a proper vertex $S(G) + 1$-numbering



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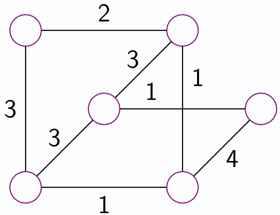

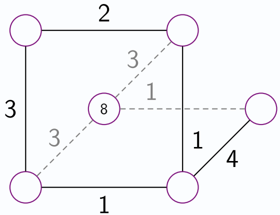

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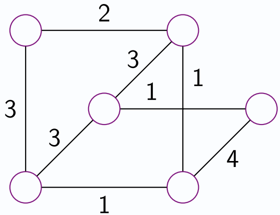

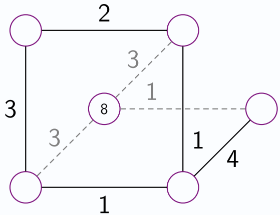

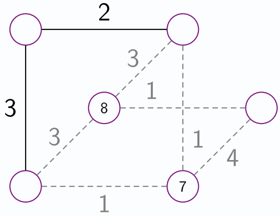

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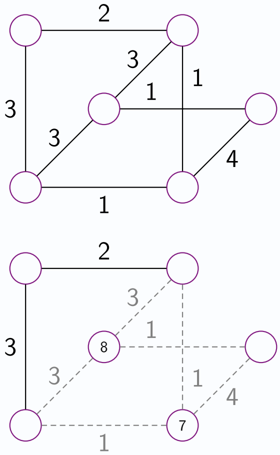
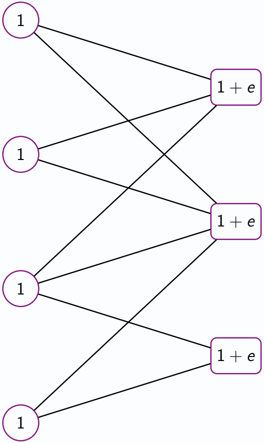


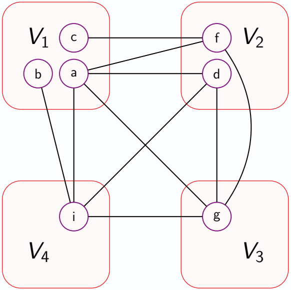
Colouring graphs

The smaller integer k for which G has a proper vertex k-numbering is equal to $\lambda(e) + 1$.

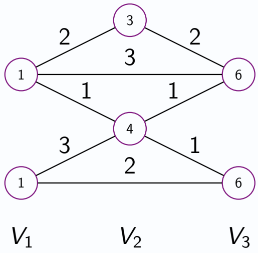


Graphs which are c-colorable

If G is c-colorable, then it has a proper vertex $(1 + \sum_{i=1}^{c-1} \lambda(e_i))$-numbering.



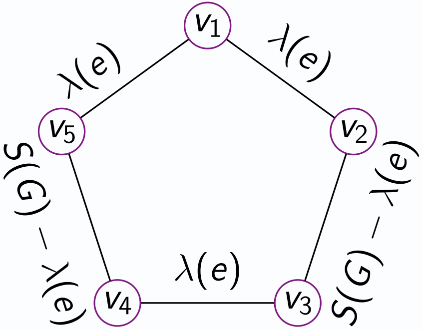
3-Colorable graph



For $\mathbf{max}(\lambda(e)) < M$, if for each $v \in V_2$, $w(v, 1) + w(v, 3)c < M$ then there exists a proper vertex M-numbering of G.

Odd cycle

If there exists an edge e such that $\lambda(e) \neq \frac{S(G)}{2}$, then G has a proper vertex $S(G)$-numbering.



Reciprocal of coloration

Proposition 10 :

If a graph G has a proper vertex k-numbering, then it is k-colorable.

Graphs which are labelled with the same integers

Proposition 11 :

A graph G which is labelled with 1 has a proper vertex $S(G)$-numbering if and only if it is not a complete graph or an odd cycle.

Thanks for listening !