REPORT

Zajęcia: Analog and digital electronic circuits

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Lab 7 and 8

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Topic:

Applying DCT for Signal Compression and Reconstruction with Thresholding

Variant 8

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Informatyka II stopień,

niestacjonarne,

1 semestr,

Gr. A

1. Problem statement:

The input signal will be processed using the Discrete Cosine Transform (DCT) to convert it into its frequency-domain representation. A threshold will be applied to the DCT coefficients to simulate compression by removing insignificant components. Finally, the signal will be reconstructed using the Inverse Discrete Cosine Transform (IDCT), and the original and reconstructed signals will be compared.

2. Input data:

f = 50
This represents the frequency of the signal in Hertz (Hz). It indicates how many cycles the signal completes in one second.

fs = 45
This is the sampling frequency.

3. Commands used (or GUI):

source code:

```
import numpy as np
from scipy.fftpack import dct, idct
import matplotlib.pyplot as plt


f = 50  # częstotliwość sygnału w Hz
fs = 45  # częstotliwość próbkowania w Hz
t = np.arange(0, 1, 1/fs)  # czas próbkowania
signal = np.sin(2 * np.pi * f * t)  # generowanie sygnału o częstotliwości f = 50
Hz

# Zastosowanie DCT
dct_coeffs = dct(signal, norm='ortho')
```

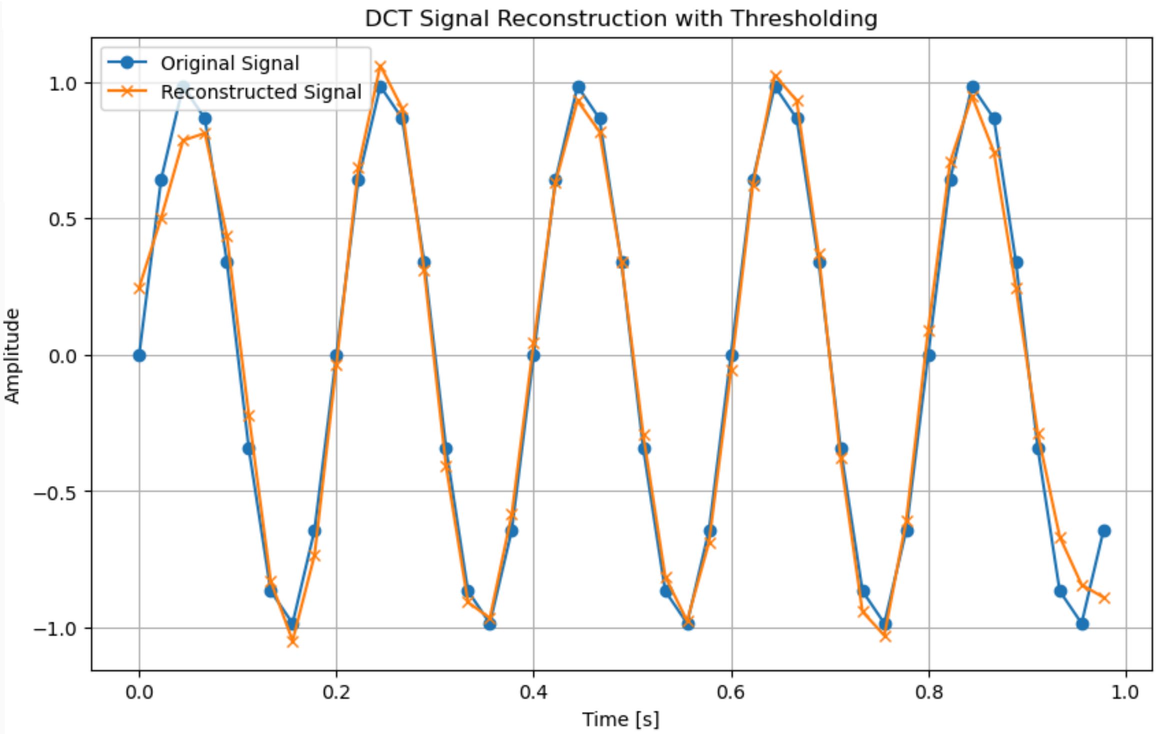
```
print("DCT Coefficients:", dct_coeffs)

# Thresholding
threshold = 0.5
dct_coeffs_thresholded = np.where(np.abs(dct_coeffs) < threshold, 0,
dct_coeffs)
print("Thresholded DCT Coefficients:", dct_coeffs_thresholded)

# Rekonstrukcja sygnału za pomocą IDCT
reconstructed_signal = idct(dct_coeffs_thresholded, norm='ortho')
print("Reconstructed Signal:", reconstructed_signal)

# Wykres oryginalnego i zrekonstruowanego sygnału
plt.figure(figsize=(10, 6))
plt.plot(t, signal, label='Original Signal', marker='o')
plt.plot(t, reconstructed_signal, label='Reconstructed Signal', marker='x')
plt.title('DCT Signal Reconstruction with Thresholding')
plt.xlabel('Time [s]')
plt.ylabel('Amplitude')
plt.legend()
plt.grid(True)
plt.show()

import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import resample

# Oryginalny sygnał ciągły
fs_continuous = 1000  # Częstotliwość próbkowania dla wizualizacji
t_continuous = np.linspace(0, 1, fs_continuous, endpoint=False)
signal_continuous = np.sin(2 * np.pi * 5 * t_continuous)  # Sygnał o
częstotliwości 5 Hz

# Próbkowanie
fs_sampled = 50  # Próbkowanie przy 50 Hz
t_sampled = np.linspace(0, 1, fs_sampled, endpoint=False)
```

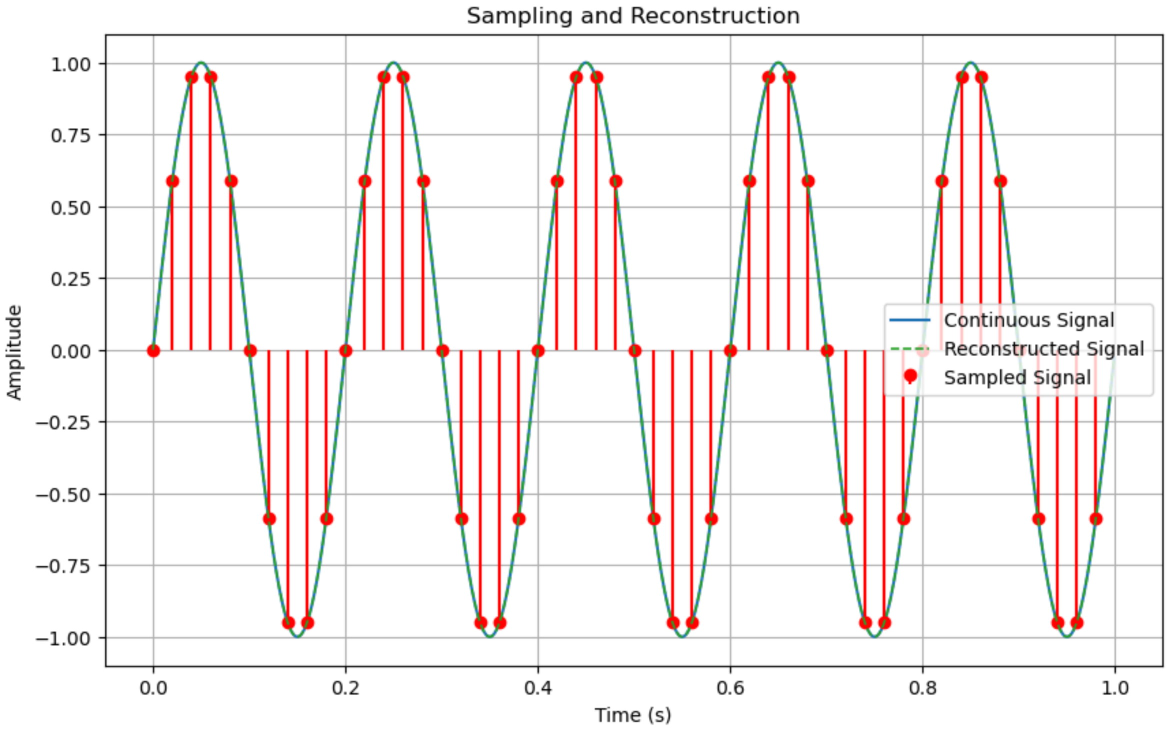
```
signal_sampled = np.sin(2 * np.pi * 5 * t_sampled)  # Próbkowanie sygnału o 5
Hz

# Rekonstrukcja
fs_reconstructed = 1000  # Rekonstrukcja do 1000 Hz
t_reconstructed = np.linspace(0, 1, fs_reconstructed, endpoint=False)
signal_reconstructed = resample(signal_sampled, len(t_reconstructed))

# Wykres
plt.figure(figsize=(10, 6))
plt.plot(t_continuous, signal_continuous, label="Continuous Signal", lw=1.5)
plt.stem(t_sampled, signal_sampled, linefmt='r-', markerfmt='ro', basefmt=" ",
label="Sampled Signal")
plt.plot(t_reconstructed, signal_reconstructed, label="Reconstructed Signal",
lw=1.2, linestyle='--')
plt.legend()
plt.xlabel("Time (s)")
plt.ylabel("Amplitude")
plt.title("Sampling and Reconstruction")
plt.grid(True)
plt.show()
```

https://github.com/HubiPX/NOD/tree/master/DSP/DSP%207%208

4. Outcomes:

```
DCT Coefficients: [ 3.70725100e-15  5.84956835e-01  1.52686024e-14  6.35394502e-01
  5.53203316e-15  7.68524057e-01  5.48693035e-15  1.12474519e+00
 -6.59888810e-15  2.99910330e+00 -1.62234399e+00 -2.69018436e+00
  1.29757316e-15 -8.09911887e-01 -2.26554886e-15 -4.41115864e-01
 -6.28316843e-15 -2.87003792e-01  3.16413562e-15 -2.03739642e-01
  5.22498711e-15 -1.52233193e-01  4.28129754e-15 -1.17529095e-01
  7.68829445e-15 -9.26953435e-02  1.06858966e-14 -7.40903487e-02
 -3.80251386e-15 -5.96224169e-02  1.45439216e-14 -4.80055956e-02
  3.16413562e-15 -3.84047252e-02  1.24344979e-14 -3.02512943e-02
 -1.06581410e-14 -2.31412004e-02 -6.99440506e-15 -1.67745892e-02
 -5.32907052e-15 -1.09184811e-02 -6.43929354e-15 -5.38218921e-03
  4.99600361e-16]
Thresholded DCT Coefficients: [ 0.          0.58495683  0.          0.6353945   0.          0.76852406
  0.          1.12474519  0.          2.9991033  -1.62234399 -2.69018436
  0.         -0.80991189  0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.        ]
Reconstructed Signal: [ 0.24671473  0.50101617  0.78581265  0.81236526  0.43708849 -0.21983528
 -0.82930948 -1.04890137 -0.73463835 -0.03540677  0.68707576  1.06051539
  0.90349185  0.31195303 -0.40754324 -0.90657527 -0.96601788 -0.58448144
  0.04411611  0.63372929  0.93207904  0.81790615  0.34202014 -0.29390089
 -0.81329669 -0.97574944 -0.68690372 -0.05830617  0.62399773  1.02535762
  0.9315485   0.37208725 -0.37948659 -0.94173304 -1.0290959  -0.60738084
  0.09185074  0.70688122  0.94809182  0.74384054  0.24695179 -0.28836
 -0.6670303  -0.84303632 -0.88950234]
```




Sampling and Reconstruction
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Amplitude
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
Continuous Signal
Reconstructed Signal
Sampled Signal

5. Conclusions:

This task demonstrated the practical application of the Discrete Cosine Transform (DCT) and thresholding in digital signal processing. By implementing these techniques in Python, the following insights were observed:

1. **Signal Representation and Reconstruction**:
   The application of DCT successfully transformed the original time-domain signal into a frequency-domain representation. Thresholding the DCT coefficients showed how compression can remove insignificant components, effectively simplifying the signal while retaining its key features. Reconstruction using the Inverse DCT (IDCT) highlighted the trade-off between signal fidelity and the level of compression, where higher thresholds can introduce more distortion.
2. **Impact of Sampling**:
   Sampling the signal at a frequency below the Nyquist rate (45 Hz for a 50 Hz signal) demonstrated the effects of aliasing. The reconstructed signal preserved some characteristics of the original but also reflected distortions due to insufficient sampling frequency. This underlines the importance of adhering to the Nyquist-Shannon theorem for accurate signal reproduction.
3. **Key Trade-offs in Thresholding**:
   While thresholding is an effective method for reducing redundancy in signal representation, it introduces a balance between compression and reconstruction quality. Lower thresholds retain more information at the cost of storage and processing, while higher thresholds improve compression at the expense of signal fidelity.

In conclusion, this task provided a foundational understanding of how DCT, thresholding, and sampling frequency affect signal compression and reconstruction. These principles are essential in real-world applications like data compression, multimedia processing, and efficient signal transmission systems.