Lab 1

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6.008 Fall 2018

September 14, 2018

1 Part A

$$v_i = \prod_{t=0}^{N-1} 1 + f(w_t) \tag{1}$$

The value of the bond after N days does *not* depend on the order of the weather sequence, since the ordering of the product above does not change its result.

2 Part B

For $N = 25$ the investment allocation I calculated was

[0.28851674, 0.21591271, 0.29925704, 0.19631352]

3 Part C

Determine values α and β that satisfy

$$\alpha < \sum_{m=0}^{M-1} e^{c_m - C} \leq \beta$$

Because C is defined as the maximum value of all c_i, the exponent will never be greater than zero. In other words,

$$c_m - C \leq 0 \implies e^{c_m - C} \leq 1 \implies \beta = M$$

The smallest value of the sum is obtained when all c_i except for one are much smaller than the maximum, C. For $c_i = C$, the expression in the sum will be 1, and for all other c_i it will be vanishingly small.

$$\implies \alpha = 1$$

We can use these bounds to show that operating on values in the logarithmic domain will stay between $(1, m]$.

$$\frac{e^{c_i}}{\sum_{m=0}^{M-1} e^{c_m}} \tag{2}$$

$$= \frac{e^{c_i}}{\sum_{m=0}^{M-1} e^{c_m}} \frac{e^{-C}}{e^{-C}} \tag{3}$$

$$= \frac{e^{c_i - C}}{\sum_{m=0}^{M-1} e^{c_m - C}} \tag{4}$$

Using the bounds derived above, we know that

$$0 < e^{c_i - C} \leq 1 \tag{5}$$

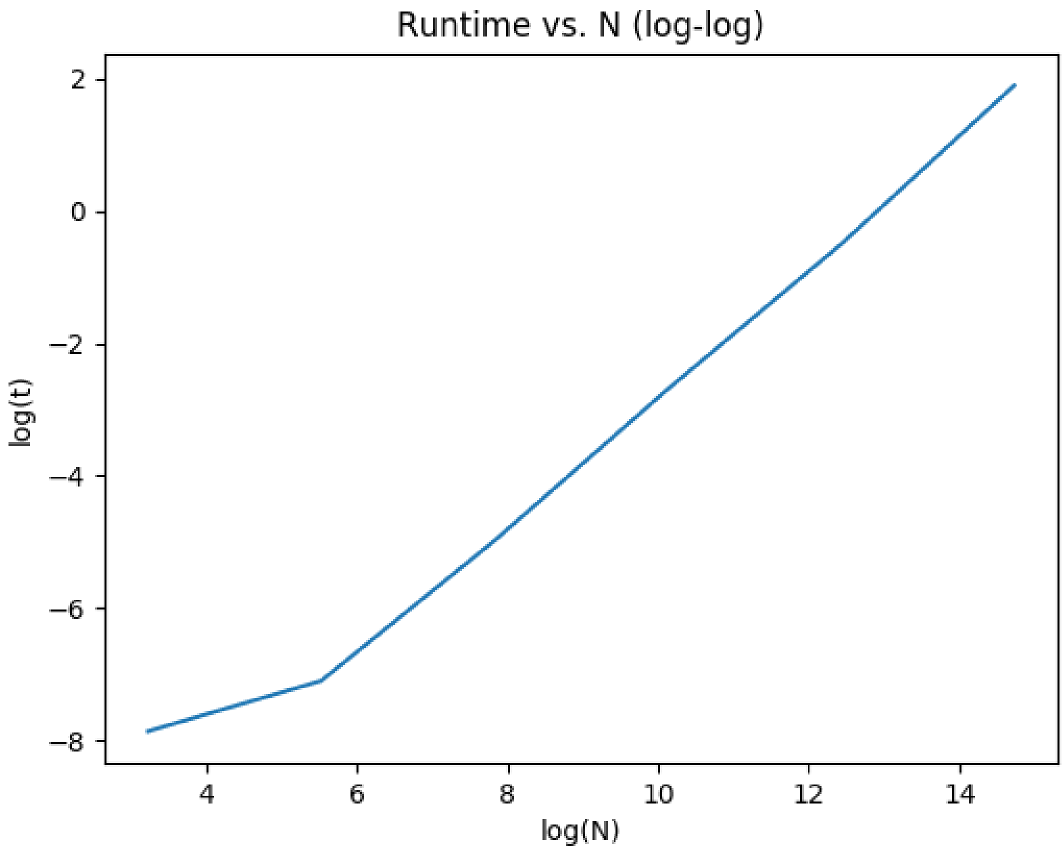
$$1 < \sum_{m=0}^{M-1} e^{c_m - C} \leq M \tag{6}$$

which proves that these values will not overflow or underflow, assuming M does not overflow.

4 Part D

In the table below are the allocations for each value of N.

N	p_1	p_2	p_3	p_4
25	0.289	0.216	0.299	0.196
250	0.215	0.126	0.391	0.268
2500	9.595e-04	5.021e-05	0.486	0.5131
25000	0.000	0.000	0.0123	0.9876
250000	0.000	0.000	0.000	1.000
2500000	0.000	0.000	0.000	1.000


Runtime vs. N (log-log)
log(t)
log(N)
2
0
−2
−4
−6
−8
4
6
8
10
12
14