

Binary and Hexadecimal Data Representation

Samia Hilal

Marianopolis College

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Number Systems

Decimal	4-Bit Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Powers of 2

Power of 2	Decimal Value
2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64
2^7	128
2^8	256
:	:

Example 1

Convert 11110001 to decimal:

$$\rightarrow 1*2^7 + 1*2^6 + 1*2^5 + 1*2^4 + 0*2^3 + 0*2^2 + 0*2^1 + 1*2^0$$

$$\rightarrow 128 + 64 + 32 + 16 + 0 + 0 + 0 + 1$$

$$\rightarrow 241$$

Example 2

Convert $2AB9_{16}$ to decimal

$$\begin{aligned} 2AB9_{16} &= 2*16^3 + A*16^2 + B*16^1 + 9*16^0 \\ &= 2*16^3 + 10*16^2 + 11*16^1 + 9*16^0 \\ &= 10937 \end{aligned}$$

Converting To Binary

Subtraction Remainder Method

Suppose we want to convert 190 to binary:

- The largest power of 2 that can be subtracted is $2^7=128$.
- Subtract 128 from 190, giving 62.
- The next power of 2, $2^6 = 64$ is too large, so we assign a placeholder of zero.
- The next power of 2, $2^5 = 32$. We'll need one of these, so subtract 32 and write down the result.
- The next power of 2, $2^4 = 16$. We can subtract one of these too.
- The next power of 2, $2^3 = 8$.
- Continue, until all powers of 2 are represented including place holders.

$$190 = 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$\begin{array}{r} 190 \\ -128 = 1 \times 2^7 \\ \hline 62 \\ -0 = 0 \times 2^6 \\ \hline 62 \\ -32 = 1 \times 2^5 \\ \hline 30 \\ -16 = 1 \times 2^4 \\ \hline 14 \\ -8 = 1 \times 2^3 \\ \hline 6 \\ -4 = 1 \times 2^2 \\ \hline 2 \\ -2 = 1 \times 2^1 \\ \hline 0 \\ -0 = 0 \times 2^0 \\ \hline 0 \end{array}$$

Convert 190 to Binary - Continued

$$190 = 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

Note: The bit corresponding to exponent 0 is on the right (***least significant bit***) and the bit corresponding to largest exponent is the ***most significant bit*** (*on the left*).

$$190 = 10111110_2$$

Converting To Binary

Division Method

Converting 190 to binary...

- First we take the number that we wish to convert and carry out subsequent divisions by 2.
- At each step, record the quotient and the remainder.
- Read remainders from **bottom to top** and record them **from most significant to least significant bit**.
- $190_{10} = 10111110_2$

$$\begin{array}{r|l} 2 & 190 \\ \hline 2 & 95 & 0 \\ 2 & 47 & 1 \\ 2 & 23 & 1 \\ 2 & 11 & 1 \\ & 5 & 1 \\ & 2 & 1 \\ & 1 & 0 \\ & 0 & 1 \end{array}$$

Example 3

Convert 147_{10} to binary using subtraction method:

$$\begin{array}{rcl} 147 & & \\ -128 & 2^7 & \\ \hline 19 & & \\ -16 & 2^4 & \\ \hline 3 & & \\ -2 & 2^1 & \\ \hline 1 & & \\ -1 & 2^0 & \\ \hline 0 & & \end{array}$$

$$\begin{aligned} 147_{10} &= 1*2^7 + 0*2^6 + 0*2^5 + 1*2^4 + 0*2^3 + 0*2^2 + 1*2^1 + 1*2^0 \\ &= 10010011 \text{ (binary)} \\ &= 93_{16} \text{ (hex)} \end{aligned}$$

Example 4

Convert 147_{10} to binary.

2	147	1	2 divides 147 73 times with a remainder of 1
2	73	1	2 divides 73 36 times with a remainder of 1
2	36	0	2 divides 36 18 times with a remainder of 0
2	18	0	2 divides 18 9 times with a remainder of 0
2	9	1	2 divides 9 4 times with a remainder of 1
2	4	0	2 divides 4 2 times with a remainder of 0
2	2	0	2 divides 2 1 time with a remainder of 0
2	1	1	2 divides 1 0 times with a remainder of 1
		0	

Reading the remainders from bottom to top, we have: $147_{10} = 10010011_2$.

Example 5

Convert 10111100011110001 to hexadecimal:

10111100011110001

= 178F₁₆ (hex)

Example 6

Convert 2AB9₁₆ to binary

2AB9₁₆ = 0010101010111001

= 10101010111001

Note: 0 digits to the left of Binary or hex numbers have no value. Same as decimal.

Example 7

Convert 299_{10} to hex

16|299 **B** divides 18 times ($16 \times 18 = 288$) with remainder of 11 (B)

16|18 **2** divides 1 times with a remainder of 2

16|1 **1** divides 0 times with a remainder of 1

0

Read remainders from ***bottom to top***

Answer: $12B_{16}$