

Sec. 2.2 Examples of Linear Programming Problems

LP \rightarrow single linear function $y = a x + b$

rate per unit of x $\uparrow \uparrow$ \uparrow constant, fixed
#(units)

Ex.1 (the diet problem): How to determine the most economical diet that satisfies the basic minimum nutritional needs for good health? Formulate this as a LP. Assume that the values of a , b , and c in the table below are given.

units proposed	x_1	x_2	...	x_n	mini requir.
Foods \rightarrow	F_1	F_2	...	F_n	
I_1 (ingredient) \downarrow	a_{11}			a_{1n}	b_1
I_2					b_2
\vdots	(nutritional contents per unit of each food)				
I_m	a_{m1}			a_{mn}	b_m
unit costs \$	c_1	c_2		c_n	

Sol. Step 1. understand/organize given info (see the given data table above).

Step 2. Formulation in individual variables. let $x_i = \text{"#(units) of food i"} \geq 0, i=1, \dots, n.$ (shown in table)

ingredient j requirement:

$$a_{j1}x_1 + a_{j2}x_2 + \dots + a_{jn}x_n$$

total cost: $c_1x_1 + \dots + c_nx_n$

$$\geq b_j$$

$$(j \leq m)$$

Step 3. Compact formulation (in vector-matrix):

$$\min c^T x \text{ s.t. } Ax \geq b, x \geq 0$$

Step 4. std LP: ...

Step 5. Solve std LP. (not now)

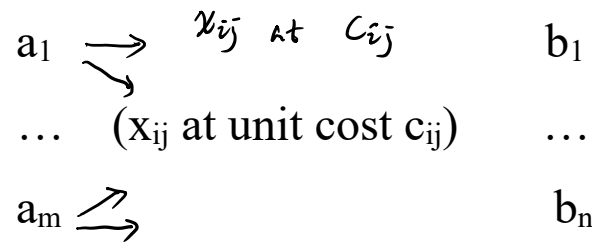
Step 6. State answer (at east x^*, z^*). #

Units proposed	x_1	x_2		x_n	mini requir.
Foods \rightarrow	F_1	F_2	...	F_n	
I_1 (ingredient) \downarrow	a_{11}	a_{12}		a_{1n}	b_1
I_2			...		b_2
...	(nutritional contents per unit of each food)				
I_m	a_{m1}	a_{m2}		a_{mn}	b_m
unit costs \$	c_1	c_2		c_n	

Ex.3 (shipping/transportation, e.g. automobiles): a, b, c — given.

Assembly plants

Dealers



Assume(balanced system): total of a = total of b

(P): How to determine the shipping sizes that would meet the needs of dealers with the minimal total shipping cost? Formulate this as a LP.

Sol. Step 1. understand/organize given info.

Step 2. Formulation in individual variables. let x_{ij} = # (units) from source i to dest. j .

$$\begin{aligned}
 \min \quad & \sum_{i,j} c_{ij} x_{ij} \\
 \sum_{j=1}^n x_{ij} &= a_i \\
 \sum_{i=1}^m x_{ij} &= b_j \\
 x_{ij} &\geq 0
 \end{aligned}$$

Ex2'. (kitchen cabinet manufacturing)

Cabinet data (per unit of cabinet):

Cabinet type (1 unit)	Wood (units)	Labor (units)	Revenue (per unit)	Proposed #(units)
Bookshelf	10	2	100	x_1
With Doors	12	4	150	x_2
With Drawers	25	8	200	x_3
Custom	20	12	400	x_4

Total (available units): 5000 1500 maximized

(P): How to allocate labor and wood capacities that would result in the maximal total revenue? Formulate this as a LP.

Sol. Introduce different numbers of units for different types of cabinets, as shown in table.

(LP) $\max z = \quad x_1 + \quad x_2 + \quad x_3 + \quad x_4 \quad \text{---- total revenue}$

 s.t. $x_1 + \quad x_2 + \quad x_3 + \quad x_4 \leq 5000 \quad \text{----- wood limit}$

$x_1 + \quad x_2 + \quad x_3 + \quad x_4 \leq 1500 \quad \text{----- labor limit}$

$x_i \geq 0, i=1,2,3,4. \quad \text{----- meaningful values.} \quad \#$