Sec. 2.2 Examples of Linear Programming Problems

LP → single linear function $y = a\ x + b$

rate per unit of x ⇑ ⇑ #(units) ⇑ constant, fixed

Ex.1 (the diet problem): How to determine the most economical diet that satisfies the basic minimum nutritional needs for good health? Formulate this as a LP. Assume that the values of a, b, and c in the table below are given.

A

Units proposed	x_1	x_2	...	x_n	mini requir.
Foods →	F_1	F_2	...	F_n	
I_1 (ingredient) ↓	A a_{11}			a_{1n}	b_1
I_2					b_2
⋮	(nutritional contents per unit of each food)				
I_m	a_{m1}			a_{mn}	b_m
unit costs $	c_1	c_2		c_n	

Sol. Step 1. understand/organize given info (see the given data table above).

Step 2. Formulation in individual variables. let x_i = "#(units) of food i" ≥ 0, $i=1, \ldots, n$. (shown in table)

ingredient j requirement:

$$\boxed{a_{j1}x_1 + a_{j2}x_2 + \cdots + a_{jn}x_n} \geq \boxed{b_j} \qquad 1 \leq j \leq m$$

total cost: $c_1x_1 + \cdots + c_nx_n$

$j \rightarrow$

Units proposed	x_1	x_2		x_n	mini requir.
Foods →	F_1	F_2	…	F_n	
I_1 (ingredient) ↓	a_{11}	a_{12}		a_{1n}	b_1
I_2			…		b_2
…	(nutritional contents per unit of each food)				
I_m	a_{m1}	a_{m2}		a_{mn}	b_m
unit costs $	c_1	c_2		c_n	

Step 3. Compact formulation (in vector-matrix):

$$\min\ c^T x \quad \text{s.t.} \quad Ax \geq b, \quad x \geq 0$$

Step 4. std LP: ...

Step 5. Solve std LP. (not now)

Step 6. State answer (at east x^*, z^*). #

Ex.3 (shipping/transportation, e.g. automobiles): a, b, c — given.

Assembly plants Dealers

$a_1 \longrightarrow$ x_{ij} at c_{ij} b_1

… (x_{ij} at unit cost c_{ij}) …

a_m b_n

Assume(balanced system): total of a = total of b

(P): How to determine the shipping sizes that would meet the needs of dealers with the minimal total shipping cost? Formulate this as a LP.

Sol. Step 1. understand/organize given info.

Step 2. Formulation in individual variables. let x_{ij} = # (units) from source i to dest. j.

$$\min \sum_{i,j} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = a_i$$

$$\sum_{i=1}^{m} x_{ij} = b_j$$

$$x_{ij} \geq 0.$$

Ex2’. (kitchen cabinet manufacturing)

Cabinet data (per unit of cabinet):

Cabinet type (1 unit)	Wood (units)	Labor (units)	Revenue (per unit)	Proposed #(units)
Bookshelf	10	2	100	x_1
With Doors	12	4	150	x_2
With Drawers	25	8	200	x_3
Custom	20	12	400	x_4

Total (available units): 5000 1500 maximized

(P): How to allocate labor and wood capacities that would result in the maximal total revenue? Formulate this as a LP.

Sol. Introduce different numbers of units for different types of cabinets, as shown in table.

(LP) $\max z = \quad x_1 + \quad x_2 + \quad x_3 + \quad x_4$ ---- total revenue

s.t. $x_1 + \quad x_2 + \quad x_3 + \quad x_4 \leq 5000$ ----- wood limit

$x_1 + \quad x_2 + \quad x_3 + \quad x_4 \leq 1500$ ----- labor limit

$x_i \geq 0$, $i=1,2,3,4$. ----- meaningful values. #