Recall(Sec.2.1): Example. LP1: min $z = c^T x$, s.t. $Ax \leq b$, $x \geq 0$ ←→ std LP?

LP1 ←→ LP2: min $z = c^T x$, s.t. $Ax + s = b$, $x \geq 0$, slack variables $s \geq 0$.

←→ Std LP3: min $z' = c'^T x'$, s.t. $A'x' = b'$, $x' \geq 0$, where $x' = \begin{bmatrix} x \\ s \end{bmatrix}$, $A' = [A\ I]$, $b' = b$, $c' = \begin{bmatrix} c \\ 0 \end{bmatrix}$

Equivalence of LP1 and LP3: same x^* part, $s^* = b - Ax^*$, same z^*.

Proof. (1). x^*, z^* opt. to LP1 → $\begin{bmatrix} x^* \\ b - Ax^* \end{bmatrix} = x'^*$, $z'^* = z^*$ opt to LP3.

(a). $x'^* = \begin{bmatrix} x^* \\ b - Ax^* \end{bmatrix}$ is feasible to LP3. ✓

$Ax^* \leq b$, $x^* \geq 0$ $\Rightarrow$ $s^* = b - Ax^* \geq 0$, $x^* \geq 0$ $\Rightarrow$ $x'^* \geq 0$, $A'x'^* \overset{?}{=} b'$ ✓ $\Leftrightarrow$ $Ax^* + s^* = b$ ✓

(b). $x'^* = \begin{bmatrix} x^* \\ b - Ax^* \end{bmatrix}$, z^* opt to LP3.

For any x' feasible to LP3,

↓

$x \geq 0$, $s \geq 0$, $Ax + s = b$

↓

$x \geq 0$, $Ax \leq b$ $\Longrightarrow$ x is feasible to LP1 → $c^T x^* \leq c^T x$

$w(x'^*) \leq w(x')$ ✓

(2). $x'^* = \begin{bmatrix} x^* \\ s^* \end{bmatrix}$, z'^* opt. to LP3 $\Rightarrow$ x^* opt to LP1 with $z^* = z'^*$.

(a). x^* is feasible to LP1.

(x^*, s^*) is feasible to LP_3 $\rightarrow$ $Ax^* + s^* = b$, $x^* \geq 0$ $\rightarrow$ $Ax^* \leq b$, $x^* \geq 0$.

Thus x^* is feasible to LP1.

(b). x^* is opt to LP1 with $z^* = z'^*$.

For any x feasible to LP_1, $Ax \leq b$, $x \geq 0$. Let $s = b - Ax$. Then $Ax + s = b$, $x \geq 0$ $s \geq 0$. Thus $x' = (x, s)$ is feasible to LP_3 with $w = c^T x + 0$. Since $x'^* = (x^*, s^*)$ is opt to LP_3, we must have $w^* = c^T x^* \leq c^T x = w$. Hence, $z^* = c^T x^* \leq c^T x = z$. So x^* is opt to LP_1. #