# Probability of the Golden State Warriors losing consecutive games during the 2016-2017 season

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# 1 Problem

1) After the Golden State Warriors acquired former MVP Kevin Durant in 2016, some NBA fans speculated that the Warriors would not lose consecutive games at any point of the season given they have an 80% chance of winning each game.

## 1.1 Solution

Consider two variables $s_n$ and $f_n$ where:

$s_n$ represents the probability of having no consecutive losses during the first $n$ games and;
$f_n$ represents the probability of consecutive losses occuring during the first $n$ games.

Clearly,

$$s_n + f_n = 1 \tag{1}$$

Consider an arbitrary $n > 0$. Assume that we know $s_i$ and $f_i$ for all $0 < i < n$. There are 2 contributing components to $f_n$:

1. Runs that already failed before game $n$
2. Runs that fail at game $n$

The first component is equal to $f_{n-1}$ which we assume is known. The second component is slightly more complicated. Failing on game $n$ requires that the previous game, $n-1$, be a loss as well. But, for this run not to be included as part of $f_{i-1}$, the team must win game $n-2$. Since winning a game will not cause the run to fail, we simply look at all the runs that haven't failed before game $n-3$ which equals $s_{n-3}$. The overall probability of the second part of $f_n$ therefore must equal:

$$s_{n-3} \cdot \mathrm{P}(\textit{Win game n-2}) \cdot \mathrm{P}(\textit{Lose game n-1}) \cdot \mathrm{P}(\textit{Lose game } n) \tag{2}$$

And the overall equation for $f_n$ including (2) must be:

$$f_n = f_{n-1} + s_{n-3} \cdot \mathrm{P}(\textit{Win game n-2}) \cdot \mathrm{P}(\textit{Lose game n-1}) \cdot \mathrm{P}(\textit{Lose game } n) \tag{3}$$

Let $\alpha$ equal $\mathrm{P}(\textit{Win game n-2}) \cdot \mathrm{P}(\textit{Lose game n-1}) \cdot \mathrm{P}(\textit{Lose game } n)$. From (1), we know that $s_i + f_i = 1$. Therefore, rearranging (3) gives:

$$\begin{aligned} 1 - s_n &= 1 - s_{n-1} + s_{n-3} \cdot \alpha \\ 0 &= s_{n-3} - s_{n-1} + \alpha \cdot s_{n-3} \end{aligned} \tag{4}$$

Consider a variable $r$that is the ratio between consecutive terms such that for $i \geq 3$

$$s_i = r \cdot s_{i-1} = r^2 \cdot s_{i-2} = r^3 \cdot s_{i-3} \tag{5}$$

Using (5) we can rewrite (4) and solve for r:

$$\begin{aligned} 0 &= r^3 \cdot s_{n-3} - r \cdot s_{n-3} + \alpha \cdot s_{n-3} \\ 0 &= s_{n-3} \cdot (r^3 \cdot -r + \alpha) \\ 0 &= r^3 - r + \alpha \end{aligned} \tag{6}$$

Since P(*Win game n*)=0.8, we know intuitively that P(*Lose game n*)=0.2 and

$$\alpha = 0.8 \cdot 0.2 \cdot 0.2 = 0.032 = \frac{4}{125} \tag{7}$$

Substituting (7) into (6), factoring the obvious root of x=1/5 and using the quadratic formula on the second degree polynomial gives:

$$\begin{aligned} 0 &= r^3 - r + \frac{4}{125} \\ 0 &= \frac{r^3}{1} - \frac{5 \cdot r}{5} + \frac{4}{125} \\ 0 &= (r - \frac{1}{5}) \cdot (r^2 - \frac{4}{5} \cdot r - \frac{-4}{25}) \\ 0 &= (r - \frac{1}{5}) \cdot (r - \frac{2 - 2 \cdot \sqrt{2}}{5}) \cdot (r - \frac{2 + 2 \cdot \sqrt{2}}{5}) \end{aligned} \tag{8}$$

Leaving three possible solutions:

$$r = 1/5 \qquad r = \frac{2 - 2 \cdot \sqrt{2}}{5} \qquad r = \frac{2 + 2 \cdot \sqrt{2}}{5} \tag{9}$$

The first two roots are clearly incorrect, since 1/5 is too small and the second root is negative which is impossible since $s_i \geq 0$ for all $i \geq 3$. Therefore we know that:

$$r = \frac{2 + 2 \cdot \sqrt{2}}{5} \tag{10}$$

Since (3) requires $s_{n-3}$ to be defined, we need to define $s_1$, $s_2$, and $s_3$.
$s_1$ clearly equals 1 since it is impossible to lose two games when you only play one.
After 2 games, the only possible unsuccesful run is each game is lost. Therefore:

$$\begin{aligned} s_2 &= 1 - P(\textit{lose game 1}) \cdot P(\textit{lose game 2}) \\ s_2 &= 1 - 0.2 \cdot 0.2 \\ s_2 &= 0.96 \end{aligned} \tag{11}$$

We now must find $s_3$.Let W represent a win and L represent a loss. A three game run will only be unsuccessfull if the games play out in the following manner: WLL, LLW, or LLL. Therefore:

$$\begin{aligned} s_3 &= 1 - P(WLL) - P(LLL) - P(LLW) \\ s_3 &= 1 - 0.8 \cdot 0.2 \cdot 0.2 - 0.2 \cdot 0.2 \cdot 0.8 - 0.2 \cdot 0.2 \cdot 0.2 \\ s_3 &= 0.928 \end{aligned} \tag{12}$$

Using our constant r defined in (5) and solved for in (10), the probability of a successful season after $n \geq 3$ is equal to:

$$\begin{aligned} s_n &= s_3 \cdot r \\ s_n &= 0.928 \cdot (\frac{2 + 2 \cdot \sqrt{2}}{5})^{n-3} \end{aligned} \tag{13}$$

Therefore the probability of a successful season after n games equals:

$$s_{n=1} = 1, \qquad s_{n=2} = 0.96, \qquad s_{n=i} = 0.928 \cdot (\frac{2 + 2 \cdot \sqrt{2}}{5})^{n-3} \, for \, i \geq 3 \tag{14}$$

And the probability of a successful season after 82 games is equal to

$$\begin{aligned} s_{82} &= 0.928 \cdot (\frac{2 + 2 \cdot \sqrt{2}}{5})^{82-3} \, for \, i \geq 3 \\ s_{82} &= 0.928 \cdot (\frac{2 + 2 \cdot \sqrt{2}}{5})^{79} \, for \, i \geq 3 \\ s_{82} &\approx 0.0588 \\ s_{82} &\approx 5.88\% \end{aligned} \tag{15}$$