

$$\begin{aligned}
 1. \quad m(a+bY) &= a + bm(Y) \\
 &= \frac{1}{N} \sum_{i=1}^N (a + by_i) \\
 &= \frac{1}{N} \sum_{i=1}^N a + b \frac{1}{N} \sum_{i=1}^N y_i \\
 &= \frac{1}{N} Na + b \frac{1}{N} \sum_{i=1}^N y_i = a + bm(Y) = m(a+bY)
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \text{cov}(X, a+bY) &= \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(a + by_i - m(a+bY)) \\
 &= \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(a + by_i - a - bm(Y)) \\
 &= \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(by_i - bm(Y)) \\
 &= b \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(y_i - m(Y)) = b \text{cov}(X, Y)
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \text{cov}(a+bX, a+bX) &= b^2 \text{cov}(X, X), \quad \text{cov}(X, X) = s^2 \\
 &= \frac{1}{N} \sum_{i=1}^N (a + bx_i - m(a+bX))(a + bx_i - m(a+bX)) \\
 &= \frac{1}{N} \sum_{i=1}^N (bx_i - bm(X))(bx_i - bm(X)) \\
 &= b^2 \sum_{i=1}^N (x_i - m(X))(x_i - m(X)) = b^2 \text{cov}(X, X) \\
 \sum_{i=1}^N (x_i - m(X))(x_i + m(X)) &= \sum_{i=1}^N (x_i - m(X))^2 = s^2
 \end{aligned}$$

4. The median of transformed variable that is non decreasing can shift the median. This also applies to quantiles, IQR, and range. A rescale will shift the distribution of values, which can change median.

5. This is not always true, as both the symmetry and median can be affected unless it is a linear transformation.