

Einführung in die Informatik WS 2020/21

Abgabe in ILIAS bis 22.12.2020 20:00 Uhr

Übungsblatt 7

**Speicherung und Interpretation von Information , Boolesche Algebra**Aufgabe 7.1:

Welchen ganzen Zahlen entsprechen die Bitketten der Länge 3, wenn man sie als Repräsentierungen der folgenden Codes interpretiert: vorzeichenlose Ganzzahl, Exzess-4, 1-er-Komplement, 2-er-Komplement?

	vorzeichenlose Ganzzahl	Excess-4	1-er- Komplement	2-er- Komplement
<b>000</b>	0	-4	0	0
<b>001</b>	1	-3	1	1
<b>010</b>	2	-2	2	2
<b>011</b>	3	-1	3	3
<b>100</b>	4	0	-3	-4
<b>101</b>	5	1	-2	-3
<b>110</b>	6	2	-1	-2
<b>111</b>	7	3	-0	-1

Aufgabe 7.2

Berechnen Sie für die folgenden Dezimalzahlen die Gleitpunktzahlen im IEEE-754-Format in 32-Bit-Maschinenwörtern unter Nutzung der folgenden Definition (siehe auch ausführlicher in der Vorlesung):

$x = (-1)^s (1.)_m \cdot 2^e$  mit  $e = ch - 127$ , beachte "hidden bit" in normierter Darstellung

- a) -16.5    b) 86.125    c) -24.25    d) 108.025

a)  $16 : 2 = 8 \text{ R } 0$   
 $8 : 2 = 4 \text{ R } 0$   
 $4 : 2 = 2 \text{ R } 0$   
 $2 : 2 = 1 \text{ R } 0$   
 $1 : 2 = 0 \text{ R } 1$   
 $0,5 : 2 = 0,25 \text{ R } 0$   
 $0 : 2 = 0 \text{ R } 0$

$16,5_{10} = 10000,1_2$

$-16,5_{10} = -1 \cdot 0,100001_2 \cdot 2^5$   
 $= -1,00001_2 \cdot 2^5$

$ch = e4k = 4 + 127 = 131$

$131 : 2 = 65 \text{ R } 1$   
 $65 : 2 = 32 \text{ R } 1$   
 $32 : 2 = 16 \text{ R } 0$   
 $16 : 2 = 8 \text{ R } 0$   
 $8 : 2 = 4 \text{ R } 0$   
 $4 : 2 = 2 \text{ R } 0$   
 $2 : 2 = 1 \text{ R } 0$   
 $1 : 2 = 0 \text{ R } 1$

$s \quad ch \quad m$   
 $1100000110001000000000000000$

b)  $86 : 2 = 43 \text{ R } 0$   
 $43 : 2 = 21 \text{ R } 1$   
 $21 : 2 = 10 \text{ R } 1$   
 $10 : 2 = 5 \text{ R } 0$   
 $5 : 2 = 2 \text{ R } 1$   
 $2 : 2 = 1 \text{ R } 0$   
 $1 : 2 = 0 \text{ R } 1$

$0,125 : 2 = 0,0625 \text{ R } 0$   
 $0,0625 : 2 = 0,03125 \text{ R } 0$   
 $0,03125 : 2 = 0,015625 \text{ R } 0$   
 $0 : 2 = 0 \text{ R } 0$

$86,125_{10} = 1010110,001_2$   
 $= 0,1010110001_2 \cdot 2^7$   
 $= 1,010110001_2 \cdot 2^6$

$ch = e4k = 6 + 127 = 133$

$133 : 2 = 66 \text{ R } 1$   
 $66 : 2 = 33 \text{ R } 0$   
 $33 : 2 = 16 \text{ R } 1$   
 $16 : 2 = 8 \text{ R } 0$   
 $8 : 2 = 4 \text{ R } 0$   
 $4 : 2 = 2 \text{ R } 0$   
 $2 : 2 = 1 \text{ R } 0$   
 $1 : 2 = 0 \text{ R } 1$

$s \quad ch \quad m$   
 $0100010101011000100000000000$

c)  $24 : 2 = 12 \text{ R } 0$   
 $12 : 2 = 6 \text{ R } 0$   
 $6 : 2 = 3 \text{ R } 0$   
 $3 : 2 = 1 \text{ R } 1$   
 $1 : 2 = 0 \text{ R } 1$

$0,25 : 2 = 0,125 \text{ R } 0$   
 $0,125 : 2 = 0,0625 \text{ R } 0$   
 $0 : 2 = 0 \text{ R } 0$

$24,25_{10} = 11000,01_2$

$-24,25_{10} = -1 \cdot 0,1100001_2 \cdot 2^5$   
 $= -1 \cdot 1,100001_2 \cdot 2^4$

$ch = e4k = 4 + 127 = 131_{10} = 10000011$

$s \quad ch \quad m$   
 $1100001010001000000000000000$

d)  $108 : 2 = 54 \text{ R } 0$   
 $54 : 2 = 27 \text{ R } 0$   
 $27 : 2 = 13 \text{ R } 1$   
 $13 : 2 = 6 \text{ R } 1$   
 $6 : 2 = 3 \text{ R } 0$   
 $3 : 2 = 1 \text{ R } 1$   
 $1 : 2 = 0 \text{ R } 1$

$0,025 : 2 = 0,0125 \text{ R } 0$   
 $0,0125 : 2 = 0,00625 \text{ R } 0$   
 $0,00625 : 2 = 0,003125 \text{ R } 0$   
 $0,003125 : 2 = 0,0015625 \text{ R } 0$   
 $0,0015625 : 2 = 0,00078125 \text{ R } 0$   
 $0,00078125 : 2 = 0,000390625 \text{ R } 0$   
 $0,000390625 : 2 = 0,0001953125 \text{ R } 0$   
 $0,0001953125 : 2 = 0,00009765625 \text{ R } 0$   
 $0,00009765625 : 2 = 0,000048828125 \text{ R } 0$   
 $0,000048828125 : 2 = 0,0000244140625 \text{ R } 0$   
 $0,0000244140625 : 2 = 0,00001220703125 \text{ R } 0$   
 $0,00001220703125 : 2 = 0,000006103515625 \text{ R } 0$   
 $0,000006103515625 : 2 = 0,0000030517578125 \text{ R } 0$   
 $0,0000030517578125 : 2 = 0,00000152587890625 \text{ R } 0$   
 $0,00000152587890625 : 2 = 0,000000762939453125 \text{ R } 0$   
 $0,000000762939453125 : 2 = 0,0000003814697265625 \text{ R } 0$   
 $0,0000003814697265625 : 2 = 0,00000019073486328125 \text{ R } 0$   
 $0,00000019073486328125 : 2 = 0,000000095367431640625 \text{ R } 0$   
 $0,000000095367431640625 : 2 = 0,0000000476837158203125 \text{ R } 0$   
 $0,0000000476837158203125 : 2 = 0,00000002384185791015625 \text{ R } 0$   
 $0,00000002384185791015625 : 2 = 0,000000011920928955078125 \text{ R } 0$   
 $0,000000011920928955078125 : 2 = 0,0000000059604644775390625 \text{ R } 0$   
 $0,0000000059604644775390625 : 2 = 0,00000000298023223876953125 \text{ R } 0$   
 $0,00000000298023223876953125 : 2 = 0,000000001490116119384765625 \text{ R } 0$   
 $0,000000001490116119384765625 : 2 = 0,0000000007450580596923828125 \text{ R } 0$   
 $0,0000000007450580596923828125 : 2 = 0,00000000037252902984619140625 \text{ R } 0$   
 $0,00000000037252902984619140625 : 2 = 0,000000000186264514923095703125 \text{ R } 0$   
 $0,000000000186264514923095703125 : 2 = 0,0000000000931322574615478515625 \text{ R } 0$   
 $0,0000000000931322574615478515625 : 2 = 0,00000000004656612873077392578125 \text{ R } 0$   
 $0,00000000004656612873077392578125 : 2 = 0,000000000023283064365386962890625 \text{ R } 0$   
 $0,000000000023283064365386962890625 : 2 = 0,0000000000116415321826934814453125 \text{ R } 0$   
 $0,0000000000116415321826934814453125 : 2 = 0,00000000000582076609134674072265625 \text{ R } 0$   
 $0,00000000000582076609134674072265625 : 2 = 0,000000000002910383045673370361328125 \text{ R } 0$   
 $0,000000000002910383045673370361328125 : 2 = 0,0000000000014551915228366851806640625 \text{ R } 0$   
 $0,0000000000014551915228366851806640625 : 2 = 0,00000000000072759576141834259033203125 \text{ R } 0$   
 $0,00000000000072759576141834259033203125 : 2 = 0,000000000000363797880709171295166015625 \text{ R } 0$   
 $0,000000000000363797880709171295166015625 : 2 = 0,0000000000001818989403545856475830078125 \text{ R } 0$   
 $0,0000000000001818989403545856475830078125 : 2 = 0,00000000000009094947017729282379150390625 \text{ R } 0$   
 $0,00000000000009094947017729282379150390625 : 2 = 0,000000000000045474735088646411895751953125 \text{ R } 0$   
 $0,000000000000045474735088646411895751953125 : 2 = 0,0000000000000227373675443232059478759765625 \text{ R } 0$   
 $0,0000000000000227373675443232059478759765625 : 2 = 0,00000000000001136868377216160297393798828125 \text{ R } 0$   
 $0,00000000000001136868377216160297393798828125 : 2 = 0,000000000000005684341886080801486968994140625 \text{ R } 0$   
 $0,000000000000005684341886080801486968994140625 : 2 = 0,0000000000000028421709430404007434844970703125 \text{ R } 0$   
 $0,0000000000000028421709430404007434844970703125 : 2 = 0,00000000000000142108547152020037174224853515625 \text{ R } 0$   
 $0,00000000000000142108547152020037174224853515625 : 2 = 0,000000000000000710542735760100185871124267578125 \text{ R } 0$   
 $0,000000000000000710542735760100185871124267578125 : 2 = 0,0000000000000003552713678800500929355621337890625 \text{ R } 0$   
 $0,0000000000000003552713678800500929355621337890625 : 2 = 0,00000000000000017763568394002504646778106689453125 \text{ R } 0$   
 $0,00000000000000017763568394002504646778106689453125 : 2 = 0,000000000000000088817841970012523233890533447265625 \text{ R } 0$   
 $0,000000000000000088817841970012523233890533447265625 : 2 = 0,0000000000000000444089209850062616169452667236328125 \text{ R } 0$   
 $0,0000000000000000444089209850062616169452667236328125 : 2 = 0,00000000000000002220446049250313080847263336181640625 \text{ R } 0$   
 $0,00000000000000002220446049250313080847263336181640625 : 2 = 0,000000000000000011102230246251565404236316680908203125 \text{ R } 0$   
 $0,000000000000000011102230246251565404236316680908203125 : 2 = 0,0000000000000000055511151231257827021181583340461015625 \text{ R } 0$   
 $0,0000000000000000055511151231257827021181583340461015625 : 2 = 0,00000000000000000277555756156289135105907916702305078125 \text{ R } 0$   
 $0,00000000000000000277555756156289135105907916702305078125 : 2 = 0,000000000000000001387778780781445675529539583511525390625 \text{ R } 0$   
 $0,000000000000000001387778780781445675529539583511525390625 : 2 = 0,0000000000000000006938893903907228377647697917557626953125 \text{ R } 0$   
 $0,0000000000000000006938893903907228377647697917557626953125 : 2 = 0,00000000000000000034694469519536141888238489587788134765625 \text{ R } 0$   
 $0,00000000000000000034694469519536141888238489587788134765625 : 2 = 0,000000000000000000173472347597680709441192447938940673828125 \text{ R } 0$   
 $0,000000000000000000173472347597680709441192447938940673828125 : 2 = 0,0000000000000000000867361737988403547205962239694703369140625 \text{ R } 0$   
 $0,0000000000000000000867361737988403547205962239694703369140625 : 2 = 0,00000000000000000004336808689942017736029811198473516845703125 \text{ R } 0$   
 $0,00000000000000000004336808689942017736029811198473516845703125 : 2 = 0,000000000000000000021684043449710088680149055992367584228515625 \text{ R } 0$   
 $0,000000000000000000021684043449710088680149055992367584228515625 : 2 = 0,0000000000000000000108420217248550443400745279961837921142578125 \text{ R } 0$   
 $0,0000000000000000000108420217248550443400745279961837921142578125 : 2 = 0,00000000000000000000542101086242752217003726399809189605712890625 \text{ R } 0$   
 $0,00000000000000000000542101086242752217003726399809189605712890625 : 2 = 0,000000000000000000002710505431213761085001861999045948028564453125 \text{ R } 0$   
 $0,000000000000000000002710505431213761085001861999045948028564453125 : 2 = 0,0000000000000000000013552527156068805425009309995229740142822265625 \text{ R } 0$   
 $0,0000000000000000000013552527156068805425009309995229740142822265625 : 2 = 0,00000000000000000000067762635780344027125046549997648700714111328125 \text{ R } 0$   
 $0,00000000000000000000067762635780344027125046549997648700714111328125 : 2 = 0,000000000000000000000338813178901720135625232749988243503570556640625 \text{ R } 0$   
 $0,000000000000000000000338813178901720135625232749988243503570556640625 : 2 = 0,0000000000000000000001694065894508600678126163749941217517852783203125 \text{ R } 0$   
 $0,0000000000000000000001694065894508600678126163749941217517852783203125 : 2 = 0,00000000000000000000008470329472543003390630818749706087589263916015625 \text{ R } 0$   
 $0,00000000000000000000008470329472543003390630818749706087589263916015625 : 2 = 0,000000000000000000000042351647362715016953154093748530437946319580078125 \text{ R } 0$   
 $0,000000000000000000000042351647362715016953154093748530437946319580078125 : 2 = 0,0000000000000000000000211758236813575084765770468742652189731597900390625 \text{ R } 0$   
 $0,0000000000000000000000211758236813575084765770468742652189731597900390625 : 2 = 0,00000000000000000000001058791184067875423828852343713260948657989501953125 \text{ R } 0$   
 $0,00000000000000000000001058791184067875423828852343713260948657989501953125 : 2 = 0,000000000000000000000005293955920339377119144261718566304743289947509765625 \text{ R } 0$   
 $0,000000000000000000000005293955920339377119144261718566304743289947509765625 : 2 = 0,0000000000000000000000026469779601696885595721308592831523716449737548828125 \text{ R } 0$   
 $0,0000000000000000000000026469779601696885595721308592831523716449737548828125 : 2 = 0,00000000000000000000000132348898008484427978606542964157618582248687744140625 \text{ R } 0$   
 $0,00000000000000000000000132348898008484427978606542964157618582248687744140625 : 2 = 0,000000000000000000000000661744490042422139893032714820788092911243438720703125 \text{ R } 0$   
 $0,000000000000000000000000661744490042422139893032714820788092911243438720703125 : 2 = 0,0000000000000000000000003308722450212110699465163574103940464556217193603515625 \text{ R } 0$   
 $0,0000000000000000000000003308722450212110699465163574103940464556217193603515625 : 2 = 0,00000000000000000000000016543612251060553497325817870519702322781085968017578125 \text{ R } 0$   
 $0,00000000000000000000000016543612251060553497325817870519702322781085968017578125 : 2 = 0,000000000000000000000000082718061255302767486629089352598511613905429840087890625 \text{ R } 0$   
 $0,000000000000000000000000082718061255302767486629089352598511613905429840087890625 : 2 = 0,0000000000000000000000000413590306276513837433145446762992558069527149200439453125 \text{ R } 0$   
 $0,0000000000000000000000000413590306276513837433145446762992558069527149200439453125 : 2 = 0,00000000000000000000000002067951531382569187165727233814962790347635746002197265625 \text{ R } 0$   
 $0,00000000000000000000000002067951531382569187165727233814962790347635746002197265625 : 2 = 0,000000000000000000000000010339757656912845935828636169074813951738178730010986328125 \text{ R } 0$   
 $0,000000000000000000000000010339757656912845935828636169074813951738178730010986328125 : 2 = 0,0000000000000000000000000051698788284564229679143180845374069758690893650054931640625 \text{ R } 0$   
 $0,0000000000000000000000000051698788284564229679143180845374069758690893650054931640625 : 2 = 0,00000000000000000000000000258493941422821148395715904226870348793454468250274658203125 \text{ R } 0$   
 $0,00000000000000000000000000258493941422821148395715904226870348793454468250274658203125 : 2 = 0,000000000000000000000000001292469707114105741978579521134351743967272341251373291015625 \text{ R } 0$   
 $0,000000000000000000000000001292469707114105741978579521134351743967272341251373291015625 : 2 = 0,0000000000000000000000000006462348535570528709892897605671758719836361706256365955078125 \text{ R } 0$   
 $0,0000000000000000000000000006462348535570528709892897605671758719836361706256365955$

### Aufgabe 7.3

Welchen dezimalen Wert besitzen die folgenden Repräsentierungen von Gleitpunktzahlen im IEEE-754-Format in 32-Bit-Maschinenwörtern:

s	ch	m(23)
1	01111111	00000000...0
0	10000011	11010000...0
1	00000000	00001000...0
0	11111111	00000000...0

Nutzen Sie die folgenden Definitionen (siehe auch Vorlesung):

ch	m	Wert	Bemerkung
0	0	$(-1)^s * 0$	$\pm 0$
0	$\neq 0$	$(-1)^s * 2^{-k+1} * (0.m)$	nicht-normalisiert, Zahlen mit Betrag kleiner als normalisiert darstellbar
$0 < ch < 2k+1$	m beliebig	$(-1)^s * 2^{ch-k} * (1.m)$	Normalfall
$ch = 2k+1$	0	$(-1)^s * \infty$	$\pm \infty$
$ch = 2k+1$	$\neq 0$	NaN	Not a Number: unbestimmter oder unzulässiger Wert

$$2k+1 = 2 \cdot 127 + 1 = 254 + 1 = 255$$

$$255_{10} = FF_{16} = 1111\ 1111_2$$

1.  $s$   $ch$   $m$   
 $1011\ 1111\ 1000\ 0000\ 0000\ 0000\ 0000\ 0000$

$$s = 1$$

$$ch = 0111\ 1111$$

$$m = 0$$

$$\begin{aligned} & (((((1 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1 \\ &= (((3 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1 \\ &= ((7 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1 \\ &= ((15 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1 \\ &= ((31 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1 \\ &= ((63 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1 \\ &= ((127 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1 \end{aligned}$$

$$ch \neq 2k+1 \Rightarrow (-1)^7 \cdot 0 = \cancel{0} -1$$

2.  $s$   $ch$   $m$   
 $0100\ 0001\ 1110\ 1000\ 0000\ 0000\ 0000\ 0000$

$$s = 0$$

$$m = 1, 1101$$

$$ch = 1000\ 0011$$

$$\begin{aligned} & (((((1 \cdot 2 + 0) \cdot 2 + 0) \cdot 2 + 0) \cdot 2 + 0) \cdot 2 + 1) \cdot 2 + 1 \\ &= ((1 \cdot 6 + 1) \cdot 2 + 1) \cdot 2 + 1 \\ &= (6 \cdot 2 + 1) \cdot 2 + 1 \\ &= 13 \cdot 2 + 1 \\ &= 27 \end{aligned}$$

$$e = ch - k = 131 - 127 = 4$$

$$(-1)^0 \cdot 1, 1101_2 \cdot 2^4$$

$$= 1, 1101_2 \cdot 2^0$$

$$= 1, 1101_2$$

$$\begin{aligned} & (((((1 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 0) \cdot 2 + 1) \cdot 2 + 1 \\ &= (((3 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1 \\ &= ((7 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1 \\ &= ((15 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1 \\ &= ((31 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1 \\ &= ((63 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1 \\ &= ((127 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1 \end{aligned}$$

$$29$$

3.  $s$   $ch$   $m$   
 $1000\ 0000\ 0000\ 0100\ 0000\ 0000\ 0000\ 0000$

$$s = 1$$

$$ch = 0$$

$$m = 0, 00001$$

$$\Rightarrow (-1)^s \cdot 2^{k+1} \cdot 0, m$$

$$\Rightarrow (-1)^1 \cdot 2^{-126} \cdot 0, 00001_2$$

$$= -1 \cdot 2^{-131} = -2^{-131}$$

richtig aber bitte dazu schreiben das es nicht Normalisiert ist

4.  $s$   $ch$   $m$   
 $0111\ 1111\ 1000\ 0000\ 0000\ 0000\ 0000\ 0000$

$$s = 0$$

$$ch = 1111\ 1111 = 2k+1 \Rightarrow (-1)^s \cdot \infty = \underline{\underline{\infty}}$$

$$m = 0$$

#### Aufgabe 7.4:

Beweisen Sie mittels Wahrheitstabellen den Ausdruck  $a \leftrightarrow b = (\neg a + b) (a + \neg b)$

a	b	$a \leftrightarrow b$	$x = \neg a + b$	$y = a + \neg b$	$xy$
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	1	1	1	1

$$a \leftrightarrow b = (\neg a + b)(a + \neg b)$$

### Aufgabe 7.5:

Beweisen Sie mittels Wahrheitstabellen die De Morganschen Regeln!

$$(a \wedge b) = \overline{(\bar{a} \vee \bar{b})}$$

a	b	$a \wedge b$	$\neg(a \vee \neg b)$	$\bar{z}$
0	0	0	1	0
0	1	0	1	0
1	0	0	1	0
1	1	1	1	1

$$(a \vee b) = \overline{(\bar{a} \wedge \bar{b})}$$

a	b	$a \vee b$	$\neg(\bar{a} \wedge \bar{b})$	$\bar{z}$
0	0	0	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	0	1