MEM 666 Final Project: Delta 3D Printer KHAC HIEU DINH (kkd44)

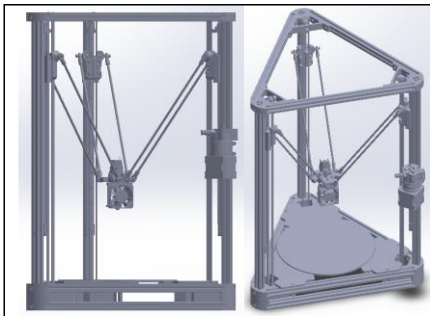


Figure 1: CAD Model of Kossel Delta 3D Printer [1].

Project Goal:

Investigate the dynamics of a generic 3D printer print head when commanded to move at various speed. Understanding the impact of layer print speed on the print head vibration (and, thus, print quality) can lead to the optimization of print speed (i.e., How far can the print speed be pushed without degrading print quality too much.)

A. Dynamics Modeling Setup:

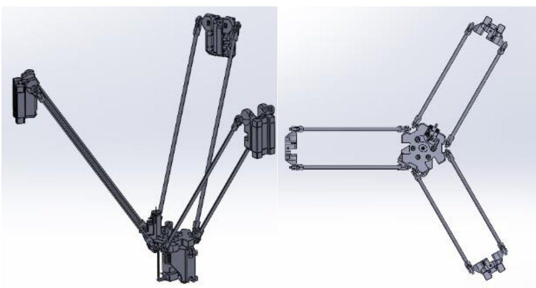


Figure 2: Interested System: Frame Removed.

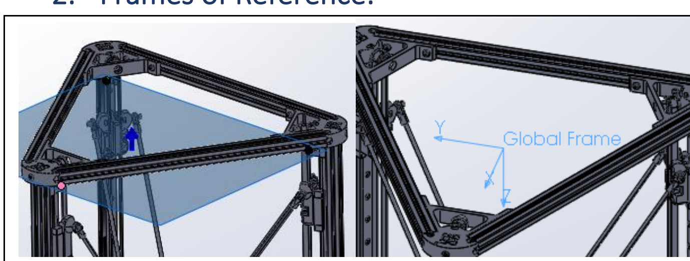
The focus of the dynamics modeling is on the print head and the 3 linkages (each with 2 rods with ball-joints, the 4-bar linkage act as a universal joint). Therefore, the frame will not be included (that is, the frame is assumed to be perfectly rigid when modeling the system.) The 3D printer technically has 3 degrees of freedom (D.O.F) corresponding to the 3 independent cartridge movements. However, this investigation treats it as a 0 D.O.F system: **the positions of all three cartridges are dictated (i.e., known in advance for all time t>0) by the printer's controller, and forces large enough to satisfy that movement will be generated by the stepper motors. This is also called position control (as opposed to force control).**

1. Generalized Coordinate Variables:

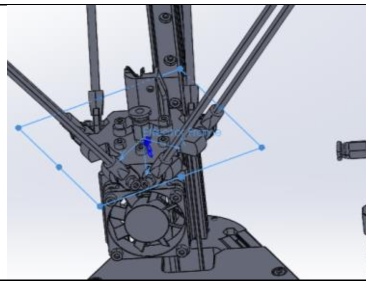
- **x_i, y_i, z_i: Description of the position of cartridge i in the base frame (i = 1,2,3).**
- **X, Y, Z: Description of the position of the end effector in the base frame.**

The carts are treated as point mass in this model (no rotation). **(X, Y, Z) and are included as independent variables even though we are not interested in the tension/compression in the linkage due to the complexity of performing the effector's kinematic analysis** (no closed form solution, must rely on numerical solve to find position → impossible to write the kinetic energy equation). Details regarding the forward kinematic analysis can be found in [2]. We also ignore the mass of the rod in the derivation of the differential equation due to: (1) The mass of the rods is small compared to the print head, (2) The added complexity (additional 9 variables, 21 variable system in total) makes this project unfeasible.

2. Frames of Reference:



Global Frame: **L**ocated on the base plane passing through all 3 home switches on top (when a cartridge is "homed", its z position is 0). The x, y, z direction is defined as shown, +z points down, +x points towards cartridge 1. The origin is the center of the equilateral triangle.

	**Effector Frame**: A frame parallel to the global frame is attached to the effector's triangle. This is possible because the effector assembly cannot rotate. The coordinate of the effector frame's origin is <X, Y, Z>

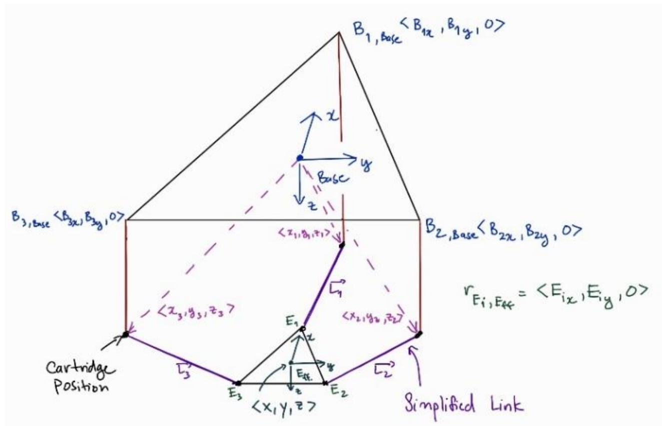


Figure 3: Simplified Sketch of the System.

B. Derivation of Differential Equation of Motion:

1. Constraint Equations – 21 in Total:

"The cartridges must move in the z axis only" – 6 Lagrange Multiples

$\boldsymbol{x_1 = x_{B_1}, \dot{x}_1 = 0\ (1)}$	$\boldsymbol{x_2 = x_{B_2}, \dot{x}_2 = 0\ (4)}$	$\boldsymbol{x_3 = x_{B_3}, \dot{x}_3 = 0\ (7)}$
$\boldsymbol{y_1 = y_{B_1}, \dot{y}_1 = 0\ (2)}$	$\boldsymbol{y_2 = y_{B_2}, \dot{y}_2 = 0\ (5)}$	$\boldsymbol{y_3 = y_{B_3}, \dot{y}_3 = 0\ (8)}$

"The movement of the cartridge in the z direction is set by the controller" – 3 Lagrange Multiples

$\boldsymbol{z_1 = z_{c1}(t)}$ → $\boldsymbol{\dot{z}_1 = \dot{z}_{c1}(t)\ (3)}$	$\boldsymbol{z_2 = z_{c2}(t)}$ → $\boldsymbol{\dot{z}_2 = \dot{z}_{c2}(t)\ (6)}$	$\boldsymbol{z_3 = z_{c3}(t)}$ → $\boldsymbol{\dot{z}_3 = \dot{z}_{c3}(t)\ (9)}$

Due to the 4-bar <u>parallelogram</u> design of the linkage, it is easily seen that **the print head must always remain parallel to the print base and cannot rotate in the z-direction** (i.e., no rotation). Each side of the effector's triangular platform on the print head must be parallel to the horizontal edge of its linked cartridge, which is always parallel to the base. The constraint equation associated with the 4 bar linkages (3 sets in total) can be found through the following vector equation:

$$r_{E_i,Eff.} = \langle E_{ix}, E_{iy}, 0\rangle = r_{E_i,Base} - r_{Eff.,Base} = \left(\langle x_i, y_i, z_i\rangle + \overrightarrow{L_\iota}\right) - \langle X, Y, Z\rangle$$

$$\overrightarrow{L_\iota} = \langle E_{ix}, E_{iy}, 0\rangle + \langle X, Y, Z\rangle - \langle x_i, y_i, z_i\rangle = \langle X - x_i + E_{ix}, Y - y_i + E_{iy}, Z - z_i\rangle$$

But $\left\|\overrightarrow{L_\iota}\right\| = l = const.$ for all 3 linkages (length of linkage is constant). Therefore:

$$l^2 = (X - x_i + E_{ix})^2 + \left(Y - y_i + E_{iy}\right)^2 + (Z - z_i)^2\ \boldsymbol{(A)}$$

Taking time derivative of both sides:

$$0 = 2(X - x_i + E_{ix})(\dot{X} - \dot{x}_i) + 2(Y - y_i + E_{iy})(\dot{Y} - \dot{y}_i) + 2(Z - z_i)(\dot{Z} - \dot{z}_i)$$

Rearranging and simplifying:

$$(X - x_i + E_{ix})\dot{X} + (Y - y_i + E_{iy})\dot{Y} + (Z - z_i)\dot{Z} + (-X + x_i - E_{ix})\dot{x}_i + (-Y + y_i - E_{iy})\dot{y}_i + (z_i - Z)\dot{z}_i = 0$$

For each cartridge (i = 1,2,3), we have a constraint equation:

$$\boldsymbol{(X - x_1 + E_{1x})\dot{X} + (Y - y_1 + E_{1y})\dot{Y} + (Z - z_1)\dot{Z} + (-X + x_1 - E_{1x})\dot{x}_1 + (-Y + y_1 - E_{1y})\dot{y}_1 + (z_1 - Z)\dot{z}_1 = 0 \; (10)}$$

$$\boldsymbol{(X - x_2 + E_{2x})\dot{X} + (Y - y_2 + E_{2y})\dot{Y} + (Z - z_2)\dot{Z} + (-X + x_2 - E_{2x})\dot{x}_2 + (-Y + y_2 - E_{2y})\dot{y}_2 + (z_2 - Z)\dot{z}_2 = 0 \; (11)}$$

$$\boldsymbol{(X - x_3 + E_{3x})\dot{X} + (Y - y_3 + E_{3y})\dot{Y} + (Z - z_3)\dot{Z} + (-X + x_3 - E_{3x})\dot{x}_3 + (-Y + y_3 - E_{3y})\dot{y}_3 + (z_3 - Z)\dot{z}_3 = 0 \; (12)}$$

2. Energy Analysis:

Cartridges: All three cartridges are identical with mass m_c:

$$T_{cartridge} = \frac{1}{2} m_c(\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2 + \dot{y}_1^2 + \dot{y}_2^2 + \dot{y}_3^2 + \dot{z}_1^2 + \dot{z}_2^2 + \dot{z}_3^2)$$

Effector: Let the mass of the effector be M:

$$T_{effector} = \frac{1}{2} M(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2)$$

Therefore, we have T:

$$T = \frac{1}{2} m_c(\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2 + \dot{y}_1^2 + \dot{y}_2^2 + \dot{y}_3^2 + \dot{z}_1^2 + \dot{z}_2^2 + \dot{z}_3^2) + \frac{1}{2} M(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2)$$

Potential Energy Function V:

$$V = -m_c g(z_1 + z_2 + z_3) - MgZ$$

3. Differential Equation of Motion:

$\boldsymbol{x_i, y_i, z_i}$ equation should be symmetrical between the 3 cartridges:

$\boldsymbol{m_c\ddot{x}_1 = \lambda_1 + \lambda_{10}(-X + x_1 - E_{1x})\ (1)}$ $\boldsymbol{m_c\ddot{x}_2 = \lambda_4 + \lambda_{11}(-X + x_2 - E_{2x})\ (4)}$ $\boldsymbol{m_c\ddot{x}_3 = \lambda_7 + \lambda_{12}(-X + x_3 - E_{3x})\ (7)}$	$\boldsymbol{m_c\ddot{y}_1 = \lambda_2 + \lambda_{10}(-Y + y_1 - E_{1y})\ (2)}$ $\boldsymbol{m_c\ddot{y}_2 = \lambda_5 + \lambda_{11}(-Y + y_2 - E_{2y})\ (5)}$ $\boldsymbol{m_c\ddot{y}_3 = \lambda_8 + \lambda_{12}(-Y + y_3 - E_{3y})\ (8)}$
$\boldsymbol{m_c\ddot{z}_1 - m_c g = \lambda_3 + \lambda_{10}(z_1 - Z)\ (3)}$ $\boldsymbol{m_c\ddot{z}_2 - m_c g = \lambda_6 + \lambda_{11}(z_2 - Z)\ (6)}$ $\boldsymbol{m_c\ddot{z}_3 - m_c g = \lambda_9 + \lambda_{12}(z_3 - Z)\ (9)}$	
$\boldsymbol{M\ddot{X} = \lambda_{10}(X - x_1 + E_{1x}) + \lambda_{11}(X - x_2 + E_{2x}) + \lambda_{12}(X - x_3 + E_{3x})\ (10)}$ $\boldsymbol{M\ddot{Y} = \lambda_{10}(Y - y_1 + E_{1y}) + \lambda_{11}(Y - y_2 + E_{2y}) + \lambda_{12}(Y - y_3 + E_{3y})\ (11)}$ $\boldsymbol{M\ddot{Z} - Mg = \lambda_{10}(Z - z_1) + \lambda_{11}(Z - z_2) + \lambda_{12}(Z - z_3)\ (12)}$	

4. Lagrange Multiple Interpretation:

It can be seen by comparing the cartridges differential equations (1-9) and that of the effector (10-12) that the terms containing $\boldsymbol{\lambda_{10}, \lambda_{11}, \lambda_{12}}$ represent interactions between the cartridges and the effector (equal and opposite pairs). This interaction come in the form of tension/compression in the pair of rods in the linkage. For example: $\boldsymbol{\lambda_{10}(-X + x_1 - E_{1x})}$ is the force on cartridge 1 in the x-direction due to the tension in the rod. $\boldsymbol{\lambda_{10}(X - x_1 + E_{1x})}$ is its counterpart in (10), representing the force at the other end of the rods. $\boldsymbol{\lambda_{1,2,4,5,7,8}}$ can be interpreted as the force on the cartridge by the railing (divided between x and y direction) and, according to Newton's 3rd Law, can be used to examine frame shake. $\boldsymbol{\lambda_{3,6,9}}$ can be interpreted as the control force required to satisfy the position control.

C. Simulation (Please Refer to the Comment in the MATLAB code for more information):

1. Setup (Constants and Initial Conditions):

The edge length of the Base Triangle is **L_big = 0.3 (m)**. The edge length of the Effector Triangle is **L_small = 0.035 (m)**. The value of $\boldsymbol{B_{1x}, B_{1y}, B_{2x}, B_{2y}, B_{3x}, B_{3y}}$ is then $\boldsymbol{\frac{L_{big}}{\sqrt{3}}, 0, \frac{-L_{big}}{2\sqrt{3}}, \frac{L_{big}}{2}, \frac{-L_{big}}{2\sqrt{3}}, \frac{-L_{big}}{2}}$ (respectively). The value of $\boldsymbol{E_{1x}, E_{1y}, E_{2x}, E_{2y}, E_{3x}, E_{3y}}$ is identical (since the 2 triangles are always parallel), but **L_big** is replaced by **L_small.** This can easily be seen in the simplified sketch (all triangles are equilateral). The other constants are mass of cartridge **mc = 0.1 (kg)**, mass of effector **M = 0.4 (kg)**, and linkage length **l = 0.25 (m).**

Fulfilling physical consistency is <u>**very important**</u> in this case, as EVERY SINGLE VARIABLE is constrained. Therefore, an inconsistent initial condition (one that does not obey the constraint) will lead to results that does not make sense. The initial condition of $\boldsymbol{x_i, y_i}$ is simply $\boldsymbol{B_{ix}, B_{iy}}$ (the rail location), as the cartridges can only travel along the rails (z-direction). Consequently, $\boldsymbol{\dot{x}_i, \dot{y}_i}$ must be initially 0.

The initial condition of z_1, z_2, z_3, X, Y, Z (both position and velocity) can be determined using inverse kinematic analysis (known X, Y, Z movement, find z_1, z_2, z_3 movement).

2. Inverse Kinematic Analysis to Determine Desired $\langle \boldsymbol{z_1, z_2, z_3} \rangle$ Control Policy:

We have the configuration constraint $(\boldsymbol{A})$. Solve for z_i (positive solution only) and substitute $x_i = B_{ix}\ ;\ y_i = B_{iy}$ (both are constants):

$$z_i = f(X, Y, Z) = Z - \sqrt{l^2 - (X - B_{ix} + E_{ix})^2 - \left(Y - B_{ix} + E_{iy}\right)^2}\ (\boldsymbol{A})$$

Taking time derivative to find the inverse velocity solution:

$$\dot{z}_i = z_i = f\left(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}\right) = \dot{Z} + \frac{(X - B_{ix} + E_{ix})\dot{X} + \left(Y - B_{ix} + E_{iy}\right)\dot{Y}}{\sqrt{l^2 - (X - B_{ix} + E_{ix})^2 - \left(Y - B_{ix} + E_{iy}\right)^2}}\ (\boldsymbol{A})$$

The inverse acceleration solution is also necessary due to the $\frac{db}{dt}$ term in the numerical simulation:

$$\{z\} = \begin{Bmatrix} \{q\} \\ \{\dot{q}\} \end{Bmatrix}, \quad \{G(z_i, t)\} = \begin{Bmatrix} \{\dot{q}\} \\ \{\ddot{q}\} \end{Bmatrix},$$

$$\begin{Bmatrix} \{\ddot{q}\} \\ \{\lambda\} \end{Bmatrix} = \begin{bmatrix} [M] & -[a]^{\mathrm{T}} \\ -[a] & [0] \end{bmatrix}^{-1} \begin{Bmatrix} \{F\} \\ \left[\frac{da}{dt}\right]\{\dot{q}\} + \left\{\frac{db}{dt}\right\} \end{Bmatrix}.$$

Figure 4: Simulation Dynamic Matrix Setup [3].

However, taking another time derivative is messy, so numerical derivative was used instead.

In this program, the desired path of the effector $\langle X, Y, Z \rangle$ is a spiral with radius **R**, pitch **pitch**, angular frequency **freq**, and starting z heigh **z_start**:

$\boldsymbol{X} = \boldsymbol{R}cos(\boldsymbol{freq} \cdot t)$ $\boldsymbol{Y} = \boldsymbol{R}sin(\boldsymbol{freq} \cdot t)$ $\boldsymbol{Z} = \boldsymbol{pitch} \cdot t + \boldsymbol{z\_start}$	$\dot{\boldsymbol{X}} = -\boldsymbol{R} \cdot \boldsymbol{freq} \cdot sin(\boldsymbol{freq} \cdot t)$ $\dot{\boldsymbol{Y}} = \boldsymbol{R} \cdot \boldsymbol{freq} \cdot cos(\boldsymbol{freq} \cdot t)$ $\dot{\boldsymbol{Z}} = \boldsymbol{pitch}$

Given a time series, the position and velocity of the effector can now be calculated. Consequently, the position, velocity, and acceleration of the cartridges can also be calculated. The procedure above is performed in **`vel_control_input.m`**

3. Results:

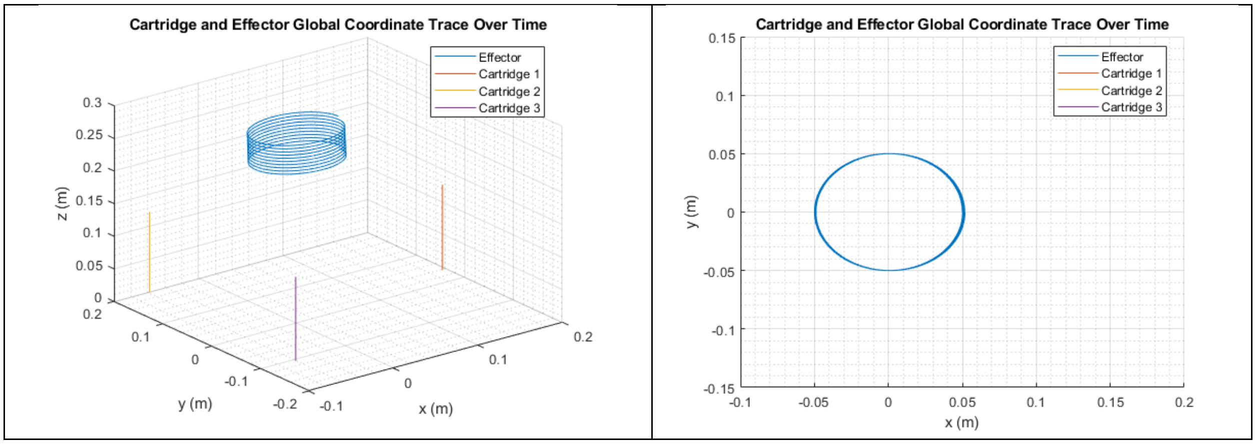


Figure 5: Effector and Cartridges Locus in 3D (Positive Z Axis Point DOWN. The Picture is Up-side-down)

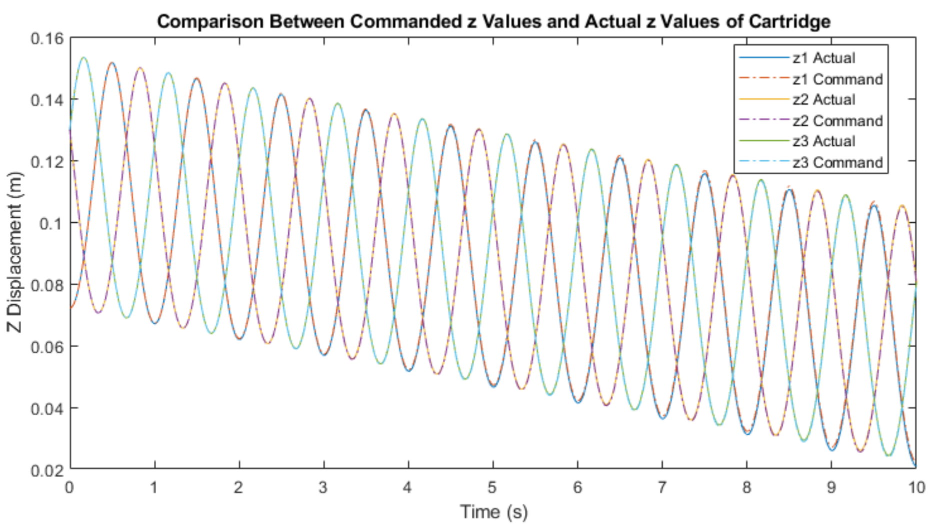


Figure 6: Commanded Position vs. Actual Position

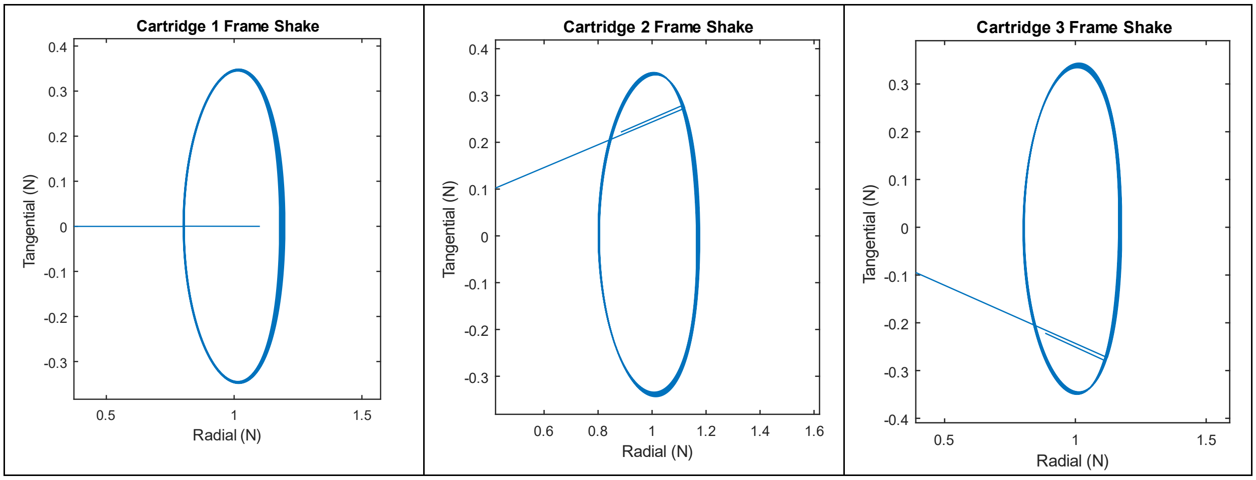


Figure 7: Visualization of Shaking Force on the Frames (the components are transformed so that x' is the radial direction, and y' is the tangential direction).

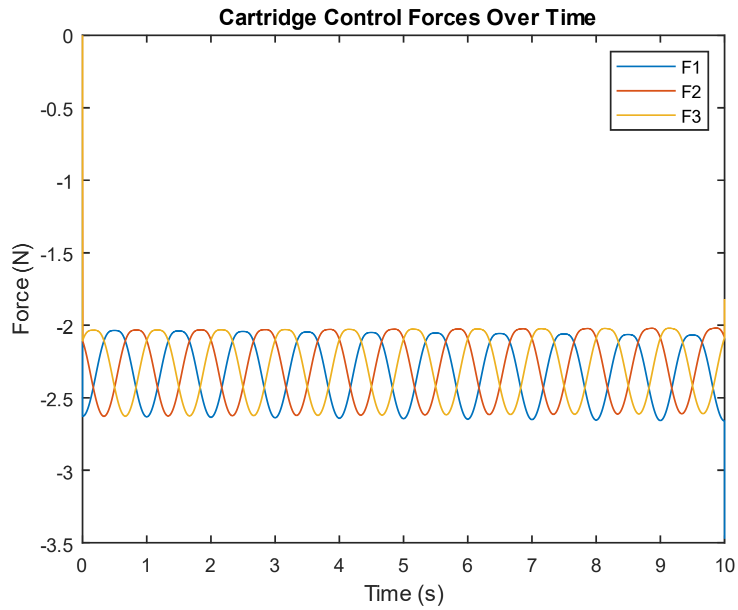


Figure 8: Control Force (Negative Force Means the Cartridge is Pulled Up – Positive Z is Down)

D. Conclusion:

It can be seen that all of the constraints are satisfied:

- The spiral motion appears as designed even though we are only manipulating the cartridges, so the linkage constraints (10-12) must have worked.
- The traces of the cartridges are all vertical, so the x/y constraints (1,2,4,5,7,8) must also have worked.
- The command and actual cartridge positions are identical, so the z constraints (3,6,9) must also have worked.

The control forces fluctuate at low levels (2.25 N), which is well below the capacity of a typical NEMA 17 stepper (holding torque of approximately 40 Ncm). Therefore, supporting the print-head is not an issue.

The magnitude of the frame shake force is low (the force range is less than 1N). However, it must be noted that the shaking is more severe in the tangential direction: the range of tangential shaking force is roughly double that of the radial shaking force, and the force changes direction (passes through 0).

Examining the phase of the tangential shaking force shows that they are out of phase across the three cartridges, which can be beneficial in avoiding resonance:

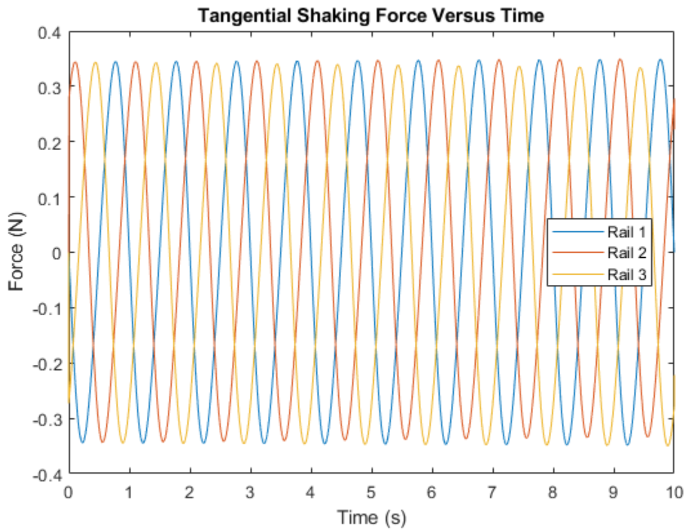


Figure 9: Phase Analysis of Tangential Shaking Forces.

References:

[1] "Free CAD Designs, Files & 3D Models | The GrabCAD Community Library." https://grabcad.com/library/kossel-ver-bettak-delta-robot-3d-printer-1 (accessed Dec. 12, 2021).

[2] Robert L. Williams II, Ph.D., "The Delta Parallel Robot: Kinematics Solutions." Ohio University, Oct. 2016. [Online]. Available: The Delta Parallel Robot: Kinematics Solutions Robert L. Williams II, Ph.D., williar4@ohio.edu Mechanical Engineering, Ohio University, October 2016

[3] J. Ginsberg, *Engineering Dynamics*, 3rd edition. Cambridge ; New York: Cambridge University Press, 2007.