An Investigation of the Optical Properties of a Water Droplet

Ethan Hu

Honors Research in Physics

Abstract: When a droplet of water sits on a glass surface, it acts as an imaging system. We analyzed the optical properties of the droplet by treating it as a lens. In order to do so, we developed a 2D raytracing program, which allowed us to look at the depth of focus of the droplet after fitting a function to approximate its side profile curve. We verified these properties by looking at how our droplet magnified a pixel grid.

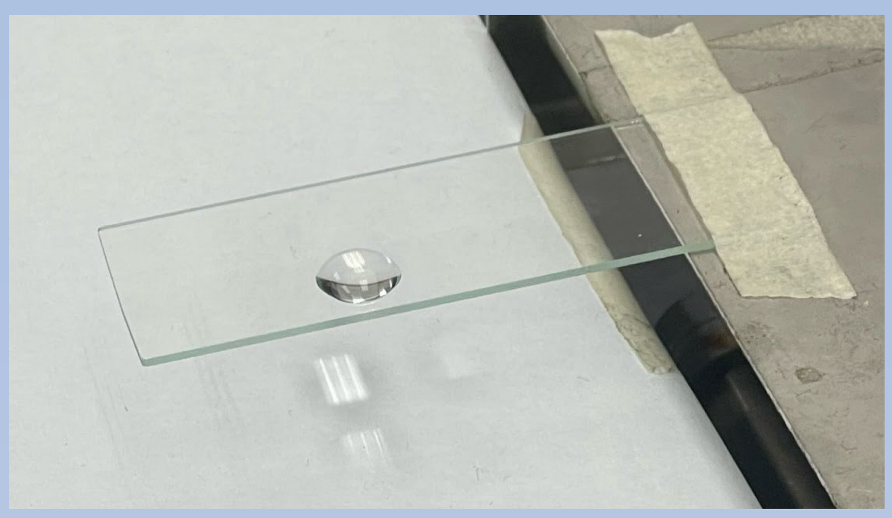


Fig 1. A water droplet on a microscope slide magnifying the light from the ceiling.

Introduction:

- When you drop water on a glass surface, it creates a curved surface. This drop of water, a type of droplet known as a sessile water drop, can be investigated as a plano convex lens.
- After determining that simple governing equations such as the lens-maker equation, or applying matrix optics were insufficient to fully grasp the magnifying properties of the system, we decided to create a raytracing program to understand the properties of this optical system.
- By assuming that each water droplet was a spherical revolution of its side profile, we were able to use Snell's Law, Equation 1, to find its depth of focus, its aperture stop, and investigate different aberrations. [1]

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Eq 1. Snell's Law of Refraction [1]

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

Eq 2. Lens Equation [1]

Experimental Data and Model:

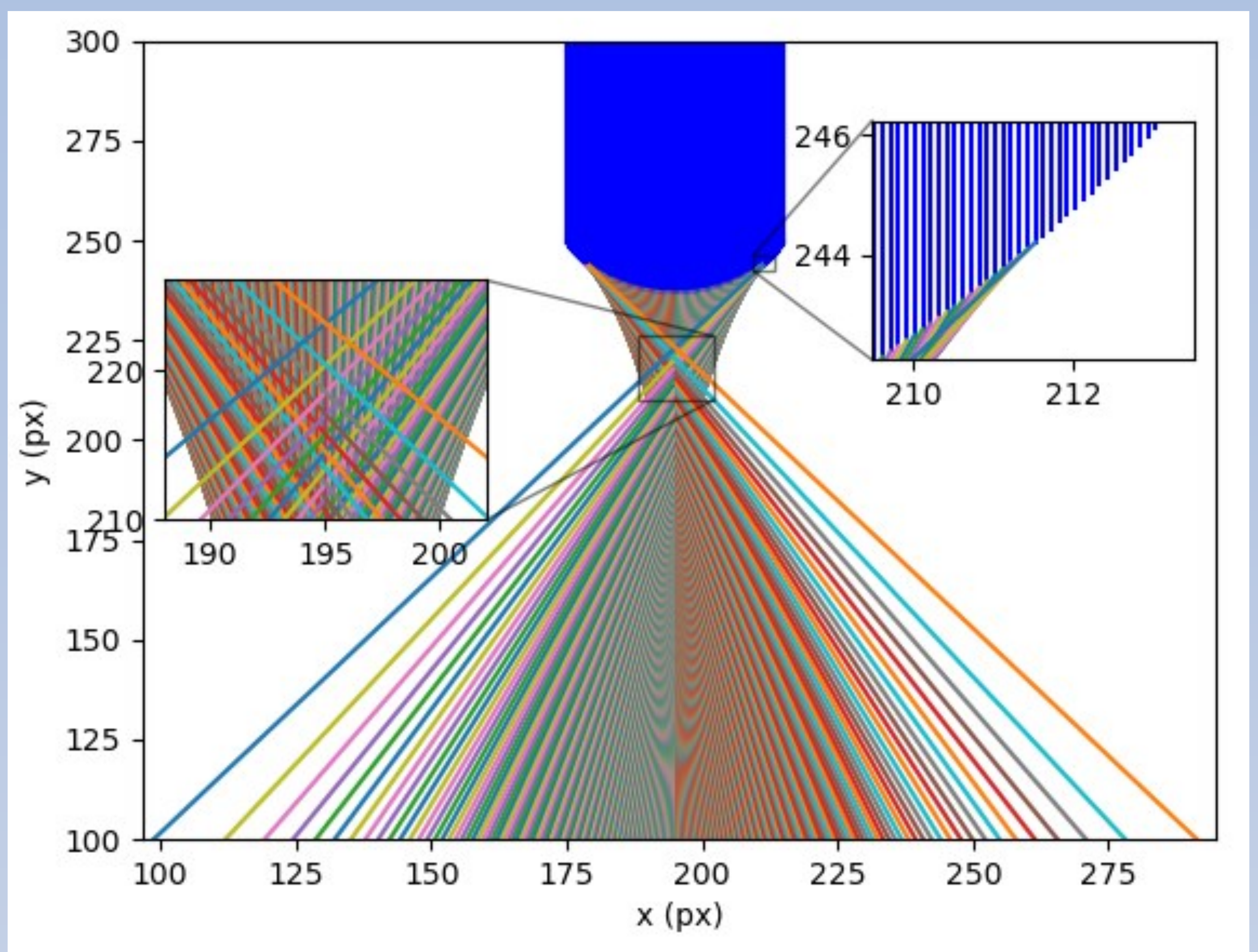


Fig 7. Generated Raytracing diagram of elliptical approximation of a droplet. Subplot on left shows how we isolated focal points, highlighting the minimum focal length. Subplot on right shows how we found the aperture size of the droplet, using the location just before total internal reflection occurs.

$$W_{\rho,\theta} = s(\rho d)^4 + c\alpha(\rho d)^3 \cos(\theta) + a\alpha^2(\rho d)^2 \cos^2(\theta) + u\alpha^2(\rho d)^2 + g\alpha^3(\rho d)\cos(\theta)$$

Eq 3. The aberration function. W is the wavefront error, which measure using the magnification at each pixel. s, c, a, u, and g are coefficients that stand for different types of aberrations(spherical, coma, astigmatism, field curvature, distortion). ρ is the relative distance from the center of focus. d is the pupil radius. Therefore, ρd is the sagittal displacement. α is the angle of the field. θ is the pupil angle. [2][3]

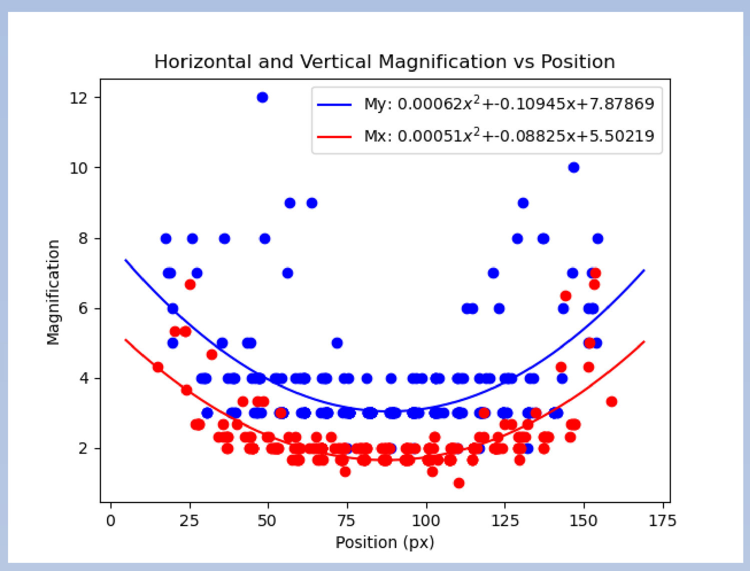
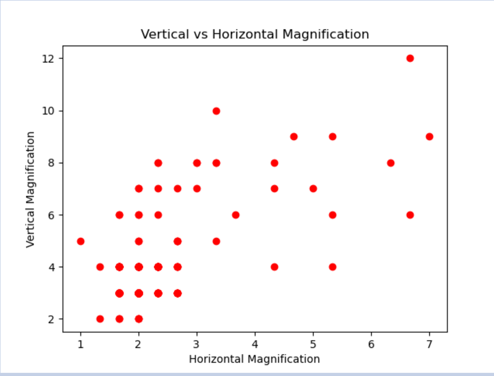


Fig 8. A plot of the horizontal and vertical magnification gotten from the Pixel Grid Setup. By taking the midpoints of the quadratic functions, we can find the center of focus.

Fig 9. A plot of the horizontal and vertical magnifications of each pixel under the droplet.



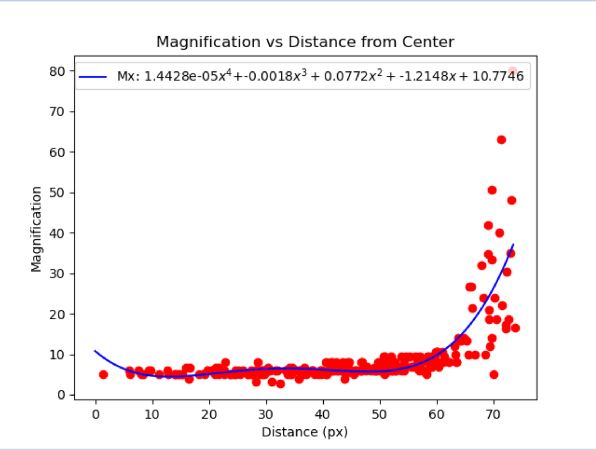


Fig 10. A plot of Area Magnification versus distance from center of focus. Used to get the Aberration Polynomial of Equation 3.

Methods

- Slides were prepared by rinsing glass microscope slides using paper napkins and tap water. We created the water droplets using droppers and syringes to control volume.
- In order to determine the shape of the water droplet, we assumed the water droplet was a segment of an circle, and fitted a function to its side view, as shown in Fig 2.
- We then used the function we fit to our edge as our approximation for the shape of the droplet, and use numerical methods to find the range of focal lengths using Snell's Law, shown in Equation 1. A graph of these results can be seen in Figure 7.
- We compared our range of focal lengths by looking at the image location For image location, we used two objects: the ceiling light, shown in Figure 3, and pixel display, shown in Figure 4. We calculated the focal length using Equation 2.
- In order to investigate the magnification, we used the pixels of a Lenovo X380 Yoga laptop, depicted in Figure 5. Using ImageJ, an image analysis software, we were able to get the dimensions of each pixel. A plot of the data determined for each pixel is shown in Figure 6.

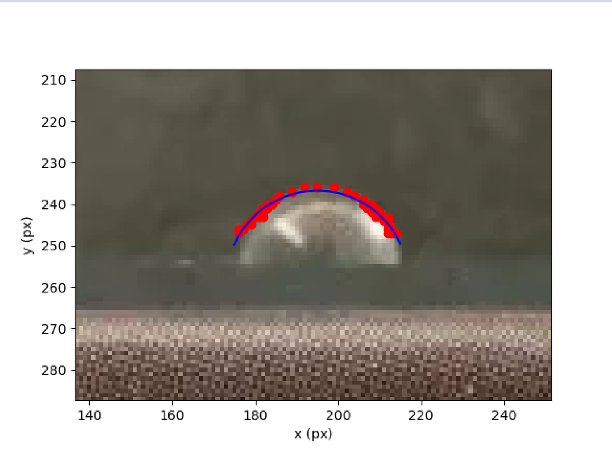


Fig 2. By applying a horizontal derivative, we were able to isolate the edge of the droplet. We then fit an ellipse equation to points detected along this edge to get an equation for our side profile.

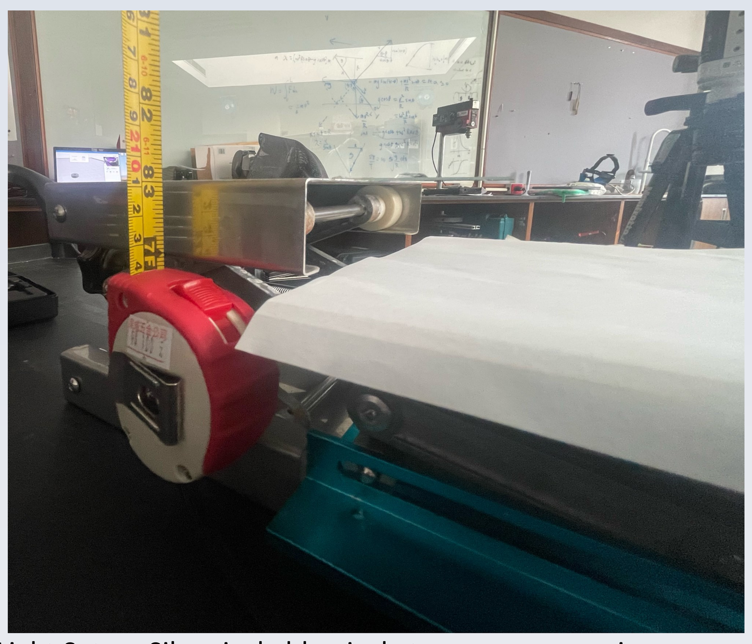


Fig 3. Ceiling Light Setup. Silver jack, blue jack, tape measure, microscope slide, water droplet.

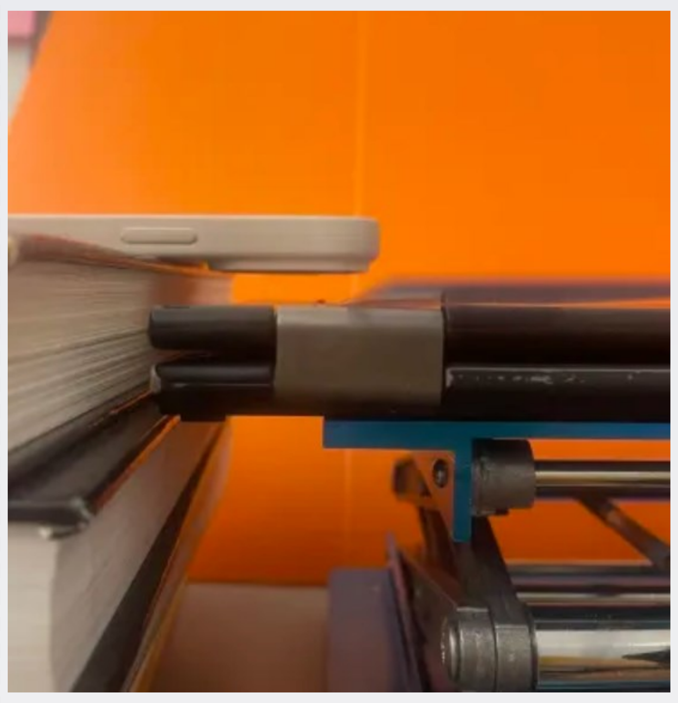


Fig 4. Pixel Grid Setup. Blue jack, laptop, microscope slide, water droplet, phone.

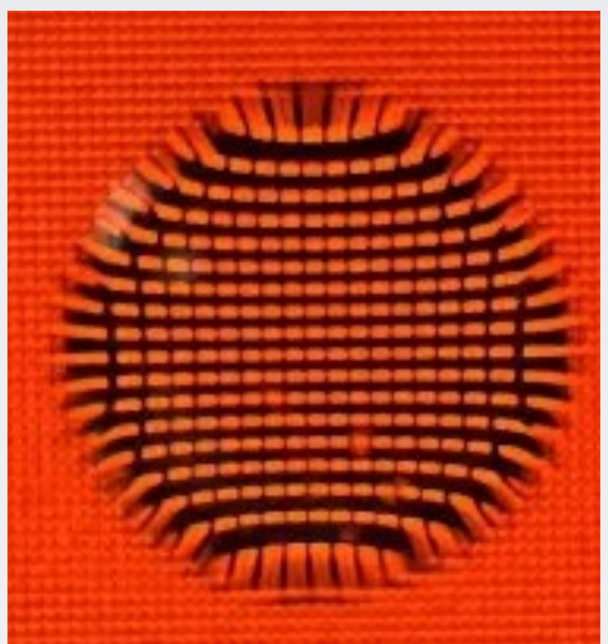


Fig 5. Image gained from Pixel Grid Setup.

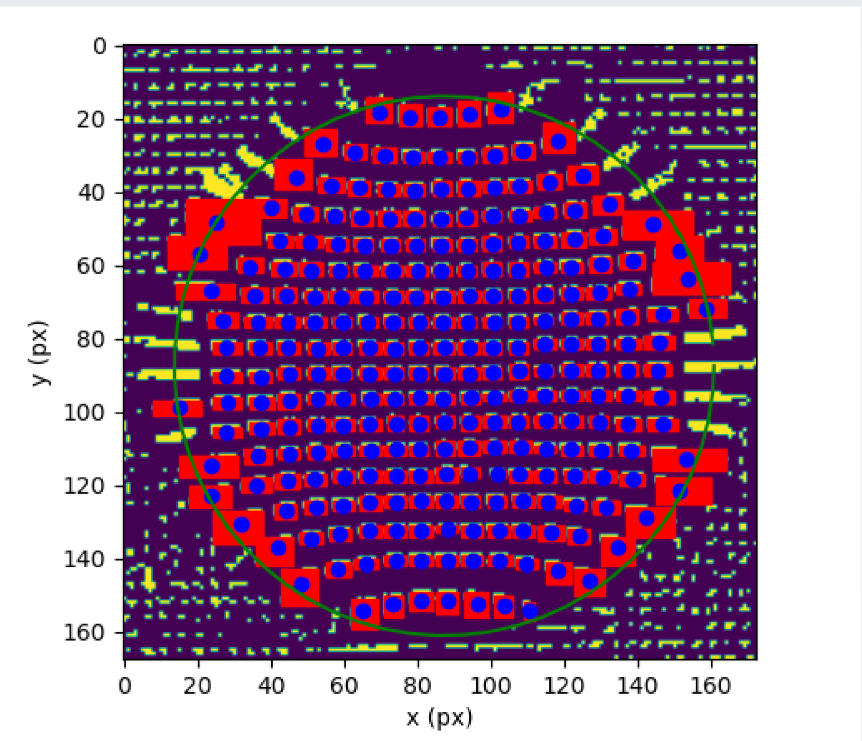


Fig 6. A plot of the data gained by ImageJ analysis of Figure 5. Each red box represents the dimensions of the respective pixel. Each blue dot represents the central location. The green circle represents the cutoff line for which pixels we analyzed, allowing us to filter out the pixels unaffected by the droplet.

Results and Discussion:

- Most experimental focal lengths fell within the depth of range predicted in Figure 6.
- Our center of focus determined in Figure 8 is different from the geometric center in Figure 4. This may have been due to the phone's center of focus.
- Our diagram in Figure 6 shows characteristics of spherical aberration. This is supported by our experimental results in Figure 10, as the quartic relationship in the wavefront error polynomial, Equation 3, indicates spherical aberration. [2]
- Horizontal and vertical magnification diverge in Figure 9, perhaps due to the effects of spherical aberration on the rectangular shape of the pixels.
- In the future, we would like to look at the resolution of the lens. The aperture in Figure 6 would tell us the diffraction limit using Rayleigh's criterion. [1] Additionally, we can generate a point spread function to get the minimum resolvable distance. [1] However, we would need to improve our side profile function to be more accurate.

References

[1] Smith, W.J. (2008) *Modern Optical Engineering: The design of Optical Systems*. New York i pozostałe: McGraw-Hill.

[2] Born, M. and Wolf, E. (1970) *Principles of optics; electromagnetic theory of propagation, interference, and diffraction of light, by Max born and Emil Wolf, with contributions by A.B. Bhatia et al.*. Oxford, NY,: Pergamon Press.

[3] Sacek, V. (2006) *3.5.1. Seidel aberrations, wavefront aberration function*, *telescopeOptics.net*. Available at: https://www.telescope-optics.net/Seidel_aberrations.htm (Accessed: 12 May 2024).

Special Thanks

- Dr. Garcia for teaching the Research in Physics Class
- Mr. Nolin, Dr. Garcia, and Mr. Laufer for supervising the Physics Research Club
- Andrew Lin for his suggestions in improving the experiments