## Unit 5:- Application of partial Differentiation.

\* Question Bank \*.

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92.7 (1	
(g·1)	TF x=uv, y= u+v then find d(x,y)(u,v)
Sain	Given, $x = uv$ , $y = u + v$
Loi	$\frac{u-v}{u-v}$
	consider, $T = d(x,y) =  xu   xv $
- 12	d(21,v) yu yv
	$\chi_{u=v}$ , $\chi_{v=u}$ .
3.3	$yu = (u-v)(1) - (u+v)(1) = u-v-u-v = -2v$ $(u-v)^{2} \cdot (u-v)^{2} \cdot (u-v)^{2}$
	$(u-v)^2$ $(u-v)^2$
	$yv = (u-v)(1) - (v+v)(-1) = u-v+u+v = 2u$ $(v-v)^{2} - (u-v)^{2} - (u-v)^{2}$
	$(U-V)^2 \qquad (U-V)^2$
	T = V
	-2V 2U
	$(u-v)^2$ $(u-v)^2$
Mary	$J = \frac{2uv}{(u-v)^2} + \frac{2uv}{(u-v)^2} = \frac{4uv}{(u-v)^2}$
	$(u-v)^2  (u-v)^2  .$
	But $J' = J(u,v) = 1 = (u-v)^2$
	$\partial(\alpha,y)$ J 4uv.
1 6	
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$ \gamma = f_{XX} = 2 $ $ s = f_{Xy} = f_{yx} = 0 $ $ t = f_{yy} = 2 $ $ \therefore rt - s^2 = (2)(2) - 0 = 4 > 0 $ $ \gamma > 0 $ $ \uparrow \text{ is minimum at } (-3,6) $ $ f_{min} = (-3)^2 + (0) + 6(-3) + 12 $ $ = 9 - 18 + 12 $		www.zealeducation.c
Soin Given: F(x,y) = x²+y²+6x+12  Fox = 0 → 2x+6=0  2x=-6	(2.2)	Discuss maxima and minima of
Soln Given: $F(x,y) = x^2 + y^2 + 6x + 12$ $F(x) = 0 \rightarrow 2x + 6 = 0$ $2x = -6$ $x = -3$ $f(y) = 0 \rightarrow 2y = 0$ $f(y) = 0$ Stationary points is $(x,y) = (-3,0) \rightarrow 2y = 0$ $f(y) = 0 \rightarrow 2y = 0$ $f(y) = $	204	$F(x,y) = x^2 + y^2 + 6x + 12$ .
$7x=-6  \boxed{x=-3}$ $\exists y=0 \rightarrow 2y=0  \boxed{y=0}$ $\text{Stationary points is } (x,y)=(-3,0) \rightarrow 2\underline{(}$ $x=\exists xx=2$ $s=\exists xy=\exists yx=0$ $t=\exists yy=2$ $\therefore xt-s^2=(2)(2)-0=4>0$ $\boxed{x>0}$ $\exists x>0$ $\boxed{x>0}$ $\exists x=2>0$ $\boxed{x>0}$ $\exists x=2>0$ $\boxed{x>0}$ $\boxed{x>0}$ $\exists x=2>0$ $\boxed{x>0}$ $\boxed{x>0}$ $\exists x=2>0$ $\boxed{x>0}$ $\boxed{x>0}$ $\boxed{x>0}$	Soin	Given: $F(x,y) = x^2 + y^2 + 6x + 12$
$\exists y = 0 \to 2y = 0  y = 0$ $\text{Stationary points is } (x,y) = (-3,0) \to 2\underline{(}$ $x = \exists xx = 2$ $s = \exists xy = \exists yx = 0$ $t = \exists yy = 2$ $x = 2 \times 2 \times 3 \times 4 \times 4 \times 5 \times 6 \times 6$	- 5	$FRC = 0 \rightarrow 2x + 6 = 0$
Stationary points is $(x,y) = (-3,0) \rightarrow 2\underline{(}$ $x = fxx = 2$ $s = fxy = fyx = 0$ $t = fyy = 2$ $\therefore xt - s^2 = (2)(2) - 0 = 4 > 0$ This minimum at $(-3,6)$ This minimum at $(-3,6)$ $fx = (-3)^2 + (0) + 6(-3) + 12$ $fx = (-3)^2 + (0) + 6(-3) + 12$		2x = -6  x = -3
		$fy=0 \rightarrow 2y=0$ $y=0$
$S = fxy = Jyx = 0$ $t = fyy = 2.$ $r + -S^2 = (2)(2) - 0 = 4 > 0$ $r + 2 > 0$ $r + 3 > 0$		Stationary points is (x,y)= (-3,0) , 2m
		S = fxy = Jyx = 0
		t = fyy = 2.
7 = 2 > 0 $7 > 0$ $7 = 3 > 0$ $7 = 3 > 0$ $7 = 3 = 3 = 0$ $7 = 3 = 3 = 0$ $7 = 3 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 = 0$ $7 = 3 =$		$\therefore \Upsilon t - S^2 = (2)(2) - 0 = 4 > 0$
		$\gamma = 2 > 0$
		$[\gamma > 0]$
= 9-18+12		7 is minimum at (-3,0)
		$4min = (-3)^2 + (0) + 6(-3) + 12$
fmin=3		= 9-18+12
	· ·	Imin=3
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Q.3)	In calculating the volume of a right circular cone
	emors of 2% and 1% are made in measuring
	the height and radius of base respectively &
Soin	
	volume of Right (v) = 1 TTr2h
	volume of Right (v) = 1 Tir2h  Circular cone
- 1	
	take log on both side
	$\log v = \log \left( \frac{1}{3} \pi r^2 h \right)$
:194	. (3
= -	$\log V = \log \left(\frac{\pi}{3}\right) + \log \left(r^2\right) + \log h \qquad \rightarrow \underline{Im}$
at (	$\log V = \log \left( \frac{\pi}{3} \right) + 2 \log r + \log h$
	on differention.
	$\frac{1}{V} \frac{dv}{dv} = 0 + 2 \frac{1}{V} \frac{dr}{dr} + \frac{1}{V} \frac{dh}{dh}$
	100 dv = 2 (100 dr) + 100 dh -> 1m
	V n
	$\frac{1}{1} = 2(1) + 2$
-	/·∨ = 4 → 1m
15	
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Examine whether u=x+y, v= tan'x+tan'y: If
1-xy
dependent, if so, find, the relation between
them.
u = x+y, v = tantx + tanty } ariven.
1-xy
consider, d(u,v) = Ux Uy
O(x,y) Vx Vy
The protect of the second
$Ux = (1-xy)(1) - (x+y)(-y) = 1+y^2$ $(1-xy)^2 - (1-xy)^2$
$(1-xy)^2$
Uy=(1-xy)(1)-(x+y)(-x)=1+x2
$(1-xy)^2 \qquad (1-xy)^2$
Now,
1+x2 1+y2
consider,
$\partial(u,v) =  u_x v_y =  1+y^2 $ 1+ x <sup>2</sup>
$\frac{\partial(x,y)}{\partial(x,y)} = \sqrt{x} \sqrt{y} \sqrt{(1-xy)^2} \sqrt{(1-xy)^2}$
1+x2 1+y2
$\therefore \partial(u,v) = 0 \qquad \qquad 2 M$
$\partial(x,y)$
: u & v are functionally dependent.
4 <u>1</u> m
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	Relation: - V = tan'x + tan'y
	$V = 4an^{-1} \left( x + y \right)$
	$\left(\frac{1-xy}{1-xy}\right)$
	$V = +an^{-1}(u)$
	[V-landul mall
	V= tan-u or tanv=u -> 2M
<b>Q</b> 5)	Find the Extreme values of x2+y2+2+2
2010	Given: - F(x,y) = x2+y2+2+2 - 1
	$\frac{\partial F}{\partial x} = 0 \rightarrow 2x + 0 + 2\left(-\frac{1}{x^2}\right) = 0$
	02
	2(-1) = 0
	$\chi^2$ $[a^3-b^3=(a-b)]$
	$\chi^3 - 1 = 0 \qquad (a^2 + ab + b^2)$
3 7 6	$(x-1)(x^2+x+1)=0$
	x = 1
	$\partial F = 0 \rightarrow 24 + 2/-1 = 0$
	$\overline{dy}$ $\overline{y^2}$
	$y^3 - 1 = 0$
6	
	Stationary point in $(x,y) = (1,1) \rightarrow 2m$ Zeal Education Institutes

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	(II) Find r,s,t	
	$\Upsilon = F_{XX} = 2 + 4$	
	S = Fxy = Fyx = 0	
TO ST	t = Fyy = 2+4 y3	
	a+ (1,1) +	
	$\gamma = 6$ , $S = 0$ , $t = 6$	-> LM
	[III] $rt-s^2 = 6(6)-0=36>0$	
	8 γ=6>0	
3	$rightarrow F$ is minimum at (1,1) is $f_{min} = (1)^2 + (1)^2 + 2 + 2$	-> 1 <u>m</u>
	$f_{min} = (1)^2 + (1)^2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 +$	-> IM

If u= x+y2, v=y+x2, w= z+x2, using Jacoblans
Find dx.
dy
10+
$f = x + y^2 - u = 0$
$CI = 11 + 7^2 - V = 0$
$g = y + z^2 - v = 0$ $h = z + x^2 - w = 0$
$\therefore \partial x = -\partial (F_1g_1h)_{-}$
$\frac{\partial u}{\partial (u,y,z)} = -N - 0 \rightarrow 10$
$\frac{\partial (F,q,h)}{\partial (F,q,h)}$
$\frac{\partial(x,y,z)}{\partial(x,y,z)}$
OCA, G.Z.
N=0(F,q,h) =   Fu Fy Fz   -1 24 0
$\frac{\partial(u,y,z)}{\partial(u,u,z)} = \frac{\partial u}{\partial u} = \frac{\partial u}{\partial z}$
hu hu ha 0 0 1
Thu ny
N = -1(1) - 2y(0) + 0
14 10 -900
N = -17 - 6
$D = \partial (F, q, h) =  Fx  Fx$
$D = \partial(F,g,h) = Fx Fy Fz$ $\partial(x,y,z) = gx gy gz$
ha hu hz
The try
D= 1 211 0 1
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
2x 0 1

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ser fass	= 1+8xyz - 3 -> 2m
	put in eqn(1) values of eqn(2) & (3)
	$\frac{-N = -(-1)}{D} = \frac{1}{1+8xyz} \rightarrow \underline{m}$
Q.7	A power dissipited in a resistor is given by $p = E^2$ . If Emors of 3% and 2% are four
	in & & Respectively, find the percentage emor in P.
Soli	$p = \frac{\varepsilon^2}{p} - given.$
	take log on both sides $log p = log \left(\frac{\epsilon^2}{R}\right)$
	logp = log 82-log R
	logp=2loge-log R — ① → 2M on differentiation
	$\frac{1}{p} \frac{dp}{e} = \frac{2 \cdot 1}{e} \frac{de}{R}$
•	$\frac{100  dP}{P} = 2 \left( \frac{100  dR}{E} \right) - \left( \frac{100  dR}{R} \right) \qquad $
6	$\frac{100 \text{ dp}}{p} = 2(3) - 2 = 4 \longrightarrow \underline{m}$
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- 6	
(8.8)	Using Lagrange's method find extreme
72.	Using Lagrange's method find extreme value of xyz if x + y + z = a.
Som	
i	1010+4=xyz-0
	① 10+ $u = xyz - ①$ $\phi = x+y+z = -a = 0 - ②$
M	(2) consider
	$F = u + \lambda \phi$
	F=242+2(2+2-a)-3
	$\frac{\partial F}{\partial x} = 0 \rightarrow yz + \lambda(1) = 0$
	$-\lambda = yz - (4)$
1.	$\frac{\partial F}{\partial r} = 0 \rightarrow \chi z + \chi(1) = 0$
	Oy C
	$-\lambda = \chi \chi \qquad (5)$
	$\frac{\partial F}{\partial r} = 0 \longrightarrow xy + \lambda(1) = 0$
	dz
	$\left[-\lambda = xy\right] - C$
-	$\Theta$ $A$
-	TT From eq n (4),(5) & (6)
	$yz = \chi z = \chi y$
	divide by xyx
	airide og kg k
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	47 = 27 = 24
	247 247
	1 = 1 = 1  (say)
	x y x x
	$\rightarrow \chi = k$ , $y = k$ , $\chi = k$ $\rightarrow \underline{IM}$
	Egn(2) becomes
	$\frac{\mathcal{E}_{q} \cap (2) \text{ becomes}}{\chi + y + z = a}$ $\frac{\mathcal{E}_{q} \cap (2) \text{ becomes}}{\chi + k + k = a}$
	k+k+k=a
	3k = q
	[k=q]
	3
	$\therefore x = a$ , $y = a$ , $z = a$ $\rightarrow Lm$
	3 3
	Stationary points are
	(x,y,z) = (a,a,a)
17/5	$(x,y,z) = (\frac{a}{3}, \frac{a}{3}, \frac{a}{3})$
	: Extreme value and Uex = a . a . a = a <sup>3</sup> 3 3 3 27
	3 3 3 27
ĺ	
3	
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(0.9)	IF x = u+v2, y=v+w2, Z = w+v2, using Jacobi
1	find d(u,v,w)
	d(x,y,z).
Som	Given: x = u+v2, y= v+w2, x = w+u2
	consider,
11 7	J=0(x,y,z) = xy xw
	d(u,v,w) yy yw
	Zy Zv Zw
Ε.	T = 1 2 V O
1 34	0 1 2w = 1(1) - 2v(-4uw) + 0
-	24 0 1
5	
	T=1+8UVW ->4M
1.5	Now,
	$\mathcal{J}' = \mathcal{O}(x, y, w)$
	d(x,y,z)
	1 = 1 m
111	J 1+8uvw

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IF x = u+v, y=v2+w2, Z= u3+w3 +hen
Find dy
dx.
1et:- 7 = 4+v-x, g= v2+w2-y, h= u3+w3-z
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
$\frac{\partial x}{\partial x} = \frac{\partial (x,y,y)}{\partial (x,y,y)} = -N - 0$
O(F,g,h)
d (u,v,w)
$N = \partial(F,g,h) =  Fx  Fv Fw  -1  0$ $\partial(x,v,w) =  gx  qv qw =  0  2v 2w$
2(x,v,w) gx gv = 0 2 V 200
ha hy hw 0 0 3 w <sup>2</sup>
$N = 6VW^2 \longrightarrow I^{N}$
After solving determinant we get
$D = 6 w^2 v + 6 u^2 w. \longrightarrow 2 M$
·. dy = - (-6vw2)
0x 6vw2+6u2w
- du = 1/10
dy 1/12 + 112
VW 14
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(010)	IF x= u+v, y=v2+w2, Z= u3+w3 +hen
	Find du
	dx.
Soin	1et:- = = u+v-x, g= v2+w2-g, h= u3+w3-z
	$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}$
	$\partial x \qquad \partial (x, y, w) = -N - 0 \rightarrow Iw$
	O(F,g,h) D
	d (u,v,w)
	$N = \partial(F, g, h) = Fx Fv Fw = -1 1 0$ $\partial(x, v, w) = gx gv gw = 0 2v 2w$
	d(x,v,w) gx gv = 0 2 v 2 w
	ha hy hw 0 0 3 w2
	$N = 6 V W^2$ $\rightarrow 1 M$
71.	After solving determinant we get
	$D = 6 w^2 v + 6 u^2 w. \longrightarrow 2 m$
	·. dy = - (-6vw2)
	0x 6vw2+6u2i
71	
	-1.0u = vw
	$\partial x  vw + u^2$
c	

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(4)	
911.	Discuss maxima and minima OFF (x,y)
327	$= x^2 + y^2 + xy + x - 4y + 5$ .
2017	Gilven:-
-	$(T) F(x,y) = x^2 + y^2 + xy + x - 4y + 5 - 1$
	dF = 2x + 0 + y + 1 - 0 + 0
	dx
	$\partial F = 2x + 4 + 1 = 0 - (2)$
	dn
	$\partial F = 2y + x - 4 = 0 - (3)$
	dy
	After solving (2) & (3) we get
	9c=-2 8y=3
	: Stationary point is (x,y) = (-2,3),7
22	(I) Find r,s,t
	$\gamma = f \chi \chi = 2$
	S = Fxy = Fyx = 1
	t = Fyy = 2
	$\widehat{m}$ $\gamma t - s^2 = (2)(2) - 1 = 3 > 0$
	$\chi \sim 0$
	-) f is minimum at (-2,3)+ 11
	: - Imin at (-2,3)
	4min = 4+9-6-2-12+15 = -2
	$Fmin=-2$ $\rightarrow 10$
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(0.12)	Examine whether u=x-y, v=tantx-tanty
	1+xy
	dependent, if so, Find the relation between
	them.
Soln	Given: - u = x-y, v = +antx - +anty.
	1+24
	consider
	0(4,v)= Un Uy
	3(x,y) Vx Vy
	:. Ux = 1+xy(1) - (x-y)(y) = 1+y2
	$\frac{1}{10000000000000000000000000000000000$
	$Uy = (1+xy)(-1) - (x-y)(x) = -1-x^2.$
	$0y = (1+xy)(-1) - (x-y)(x) = -1-x^{2}$ $(1+xy)^{2}$ $(1+xy)^{2}$
1	
	$Vx = 1 \qquad Vy = -1$
	$1+x^2 \qquad 1+y^2.$
	d(u,v) = Ux Uy = 1+y2 -1-x2
177	O(x,y) Vx Vy (1+xy)2 (1+xy)2
	1 -1
	1+22 1+42
-	· d(u,v) = 0 u & v are functionally -, 30
Y 6	d(x,y) dependent
	· Relation: - V=tan-1x-tan-1y
0	$v = +an^{-1}(x-y) \left[v = +an^{-1}u\right] or$
11 -	(1+xy/ = tanv= u)
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- 1	