

★ 2.2.1

Thm 9.3.1) 증명

(4)  $y = \cot^{-1} x$  의 도함수

Sol)

$$x = \cot y \Rightarrow x = \frac{1}{\tan y} \quad \frac{dx}{dy} = \frac{-\sec^2 y}{\tan^2 y} \quad \frac{dy}{dx} = \frac{\tan^2 y}{-\sec^2 y}$$

$$\tan^2 y + 1 = \sec^2 y$$

$$-\sec^2 y = -(\tan^2 y + 1) \quad \tan^2 y = \frac{1}{x^2}$$

$$= -\left(\frac{1}{x^2} + 1\right)$$

$$\frac{dy}{dx} = \frac{\frac{1}{x^2}}{-\frac{1}{x^2} - 1} = \frac{\frac{1}{x^2}}{\frac{-1-x^2}{x^2}} = \frac{1}{-x^2-1}$$

$$\boxed{\text{답}}: y' = \frac{1}{x^2-1} = -\frac{1}{1-x^2}$$

(4)  $y = \csc^{-1} x$  의 도함수

Sol)

$$x = \csc y = \frac{1}{\sin y} \quad \frac{dx}{dy} = \frac{-\cos y}{\sin^2 y} \quad \frac{dy}{dx} = \frac{\sin^2 y}{-\cos y}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\cos y = \sqrt{1 - \sin^2 y}$$

$$= \sqrt{1 - \frac{1}{x^2}}$$

$$\frac{dy}{dx} = \frac{\frac{1}{x^2}}{\sqrt{1 - \frac{1}{x^2}}} = \frac{1}{-x^2 \sqrt{1 - \frac{1}{x^2}}} = \frac{1}{-x^2 \sqrt{\frac{x^2-1}{x^2}}} = -\frac{1}{|x| \sqrt{x^2-1}}$$

$$\boxed{\text{답}}: y' = -\frac{1}{|x| \sqrt{x^2-1}}$$