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연습문제 4.3.

3. $\lim_{x \to \infty} x(x-\sqrt{x^2-4})$

sol) $x(x-\sqrt{x^2-4}) = \frac{x(x-\sqrt{x^2-4})(x+\sqrt{x^2-4})}{x+\sqrt{x^2-4}} = \frac{4x}{x+\sqrt{x^2-4}} = \frac{4}{1+\sqrt{1-\frac{4}{x^2}}}$

$\lim_{x \to \infty} \frac{4}{1+\sqrt{1-\frac{4}{x^2}}} = \frac{4}{2} = 2.$

$\therefore \lim_{x \to \infty} x(x-\sqrt{x^2-4}) = 2.$

5) $\lim_{x \to 2} \frac{x^2+x-6}{x^2-4}$

sol) $\frac{x^2+x-6}{x^2-4} = \frac{(x-2)(x+3)}{(x+2)(x-2)} = \frac{x+3}{x+2}.$

$\lim_{x \to 2} \frac{x+3}{x+2} = \frac{5}{4}$

$\therefore \lim_{x \to 2} \frac{x^2+x-6}{x^2-4} = \frac{5}{4}.$

(17) $\lim\limits_{h \to 0} \frac{1}{h}\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right)$

$$\frac{1}{h}\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right) = \frac{1}{h} \times \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}$$

$$= \frac{1}{h} \times \frac{(\sqrt{x} - \sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}{(\sqrt{x}\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}$$

$$= \frac{1}{h} \times \frac{x - x - h}{(\sqrt{x}\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}$$

$$= \frac{1}{\cancel{h}} \cdot \frac{-\cancel{h}}{(\sqrt{x}\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}$$

$$= -\frac{1}{(\sqrt{x}\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}$$

$$-\lim_{h \to 0} \frac{1}{(\sqrt{x}\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}$$

$$= -\frac{1}{(\sqrt{x}\sqrt{x+0})(\sqrt{x} + \sqrt{x+0})}$$

$$= -\frac{1}{x \times (\sqrt{x} + \sqrt{x})}$$

$$= -\frac{1}{2\sqrt{x}\,x}$$

$$= -\frac{\sqrt{x}}{2x^2}$$

$\therefore \lim\limits_{h \to 0} \frac{1}{h}\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right)$

$$= -\frac{\sqrt{x}}{2x^2}$$