Ray Tracing III
(Light Transport & Global Illumination)

Last Lectures

- Basic ray tracing
  - Ray generation
  - Ray object intersection
- Acceleration
  - Ray AABB intersection
  - Spatial partitions vs object partitions
  - BVH traversal

Today

- Basic radiometry
- Light transport
  - The reflection equation
  - The rendering equation
- Global illumination

Measurement system and units for illumination

Accurately measure the spatial properties of light

- New terms: Radiant flux, intensity, irradiance, radiance

Perform lighting calculations **in a physically correct manner**

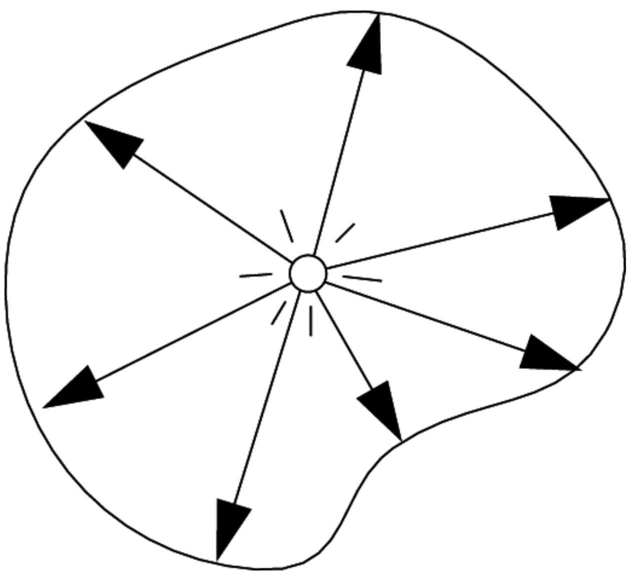
Radiant Energy and Flux (Power)

Definition: Radiant energy is the energy of electromagnetic radiation. It is measured in units of joules, and denoted by the symbol:

$$Q \ [\mathrm{J} = \mathrm{Joule}]$$

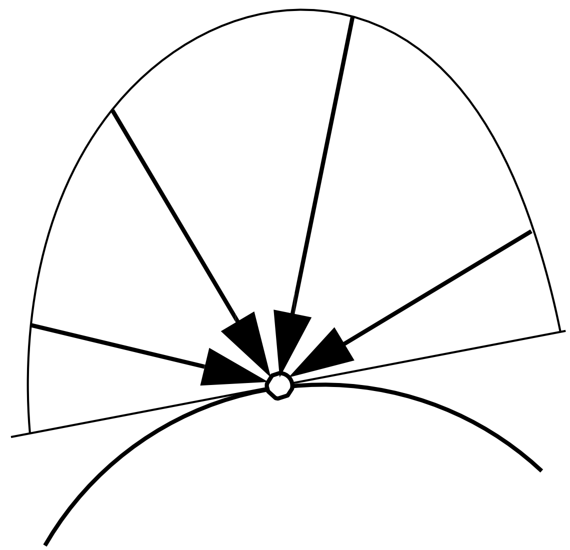
Definition: Radiant flux (power) is the energy emitted, reflected, transmitted or received, <u>per unit time</u>.

$$\Phi \equiv \frac{\mathrm{d}Q}{\mathrm{d}t} \ [\mathrm{W} = \mathrm{Watt}] \ [\mathrm{lm} = \mathrm{lumen}]^{*}$$



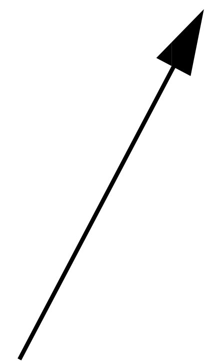
Light Emitted
From A Source

“Radiant Intensity”



Light Falling
On A Surface

“Irradiance”

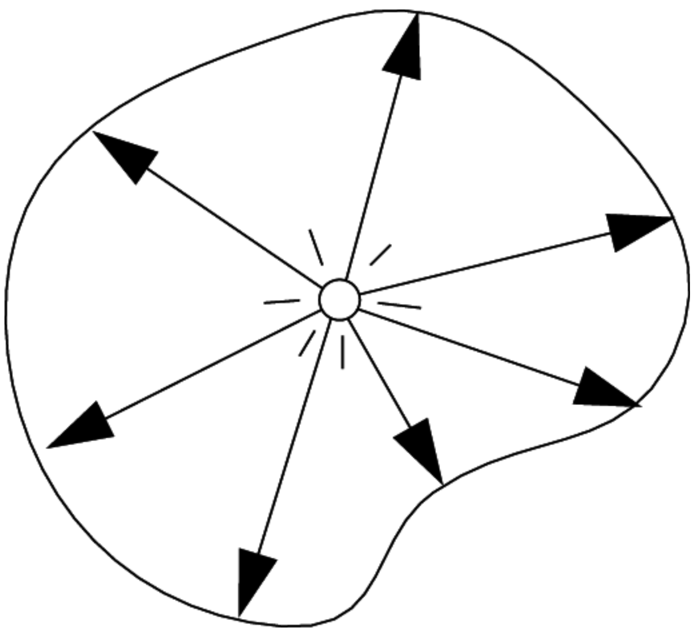


Light Traveling
Along A Ray

“Radiance”

Radiant Intensity

Definition: The radiant (luminous) intensity is the power **per unit solid angle (?)** emitted by a point light source.



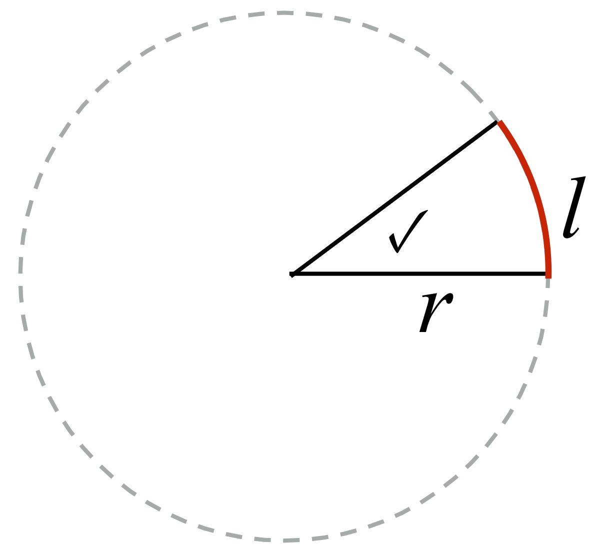
$$I(\omega) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$$

$$\left[\frac{\mathrm{W}}{\mathrm{sr}}\right] \left[\frac{\mathrm{lm}}{\mathrm{sr}} = \mathrm{cd} = \mathrm{candela}\right]$$

The candela is one of the seven SI base units.

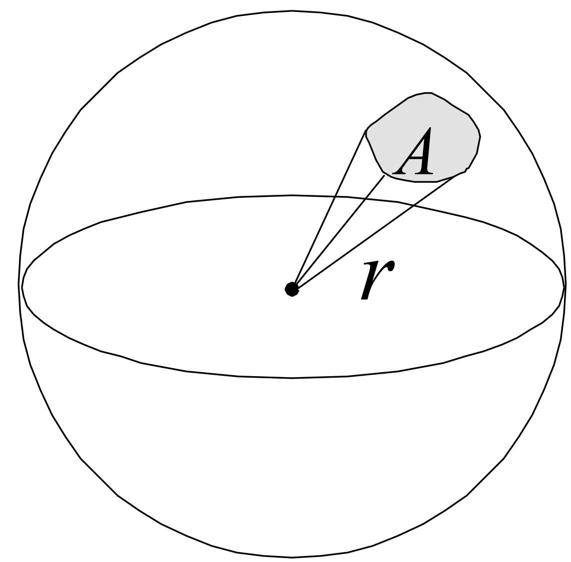
Angle: ratio of subtended arc length on circle to radius

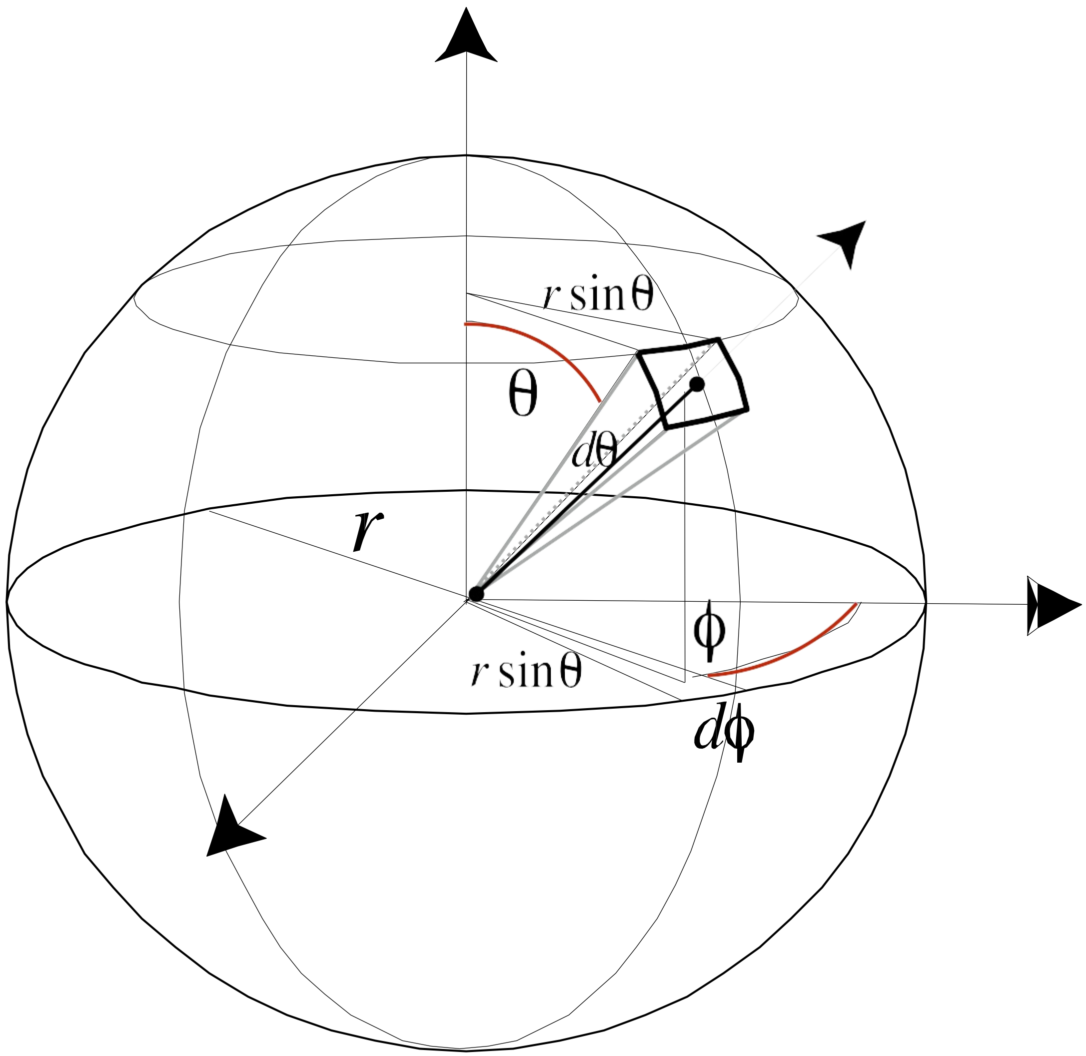
- $\theta = \frac{l}{r}$
- Circle has 2π radians



Solid angle: ratio of subtended area on sphere to radius squared

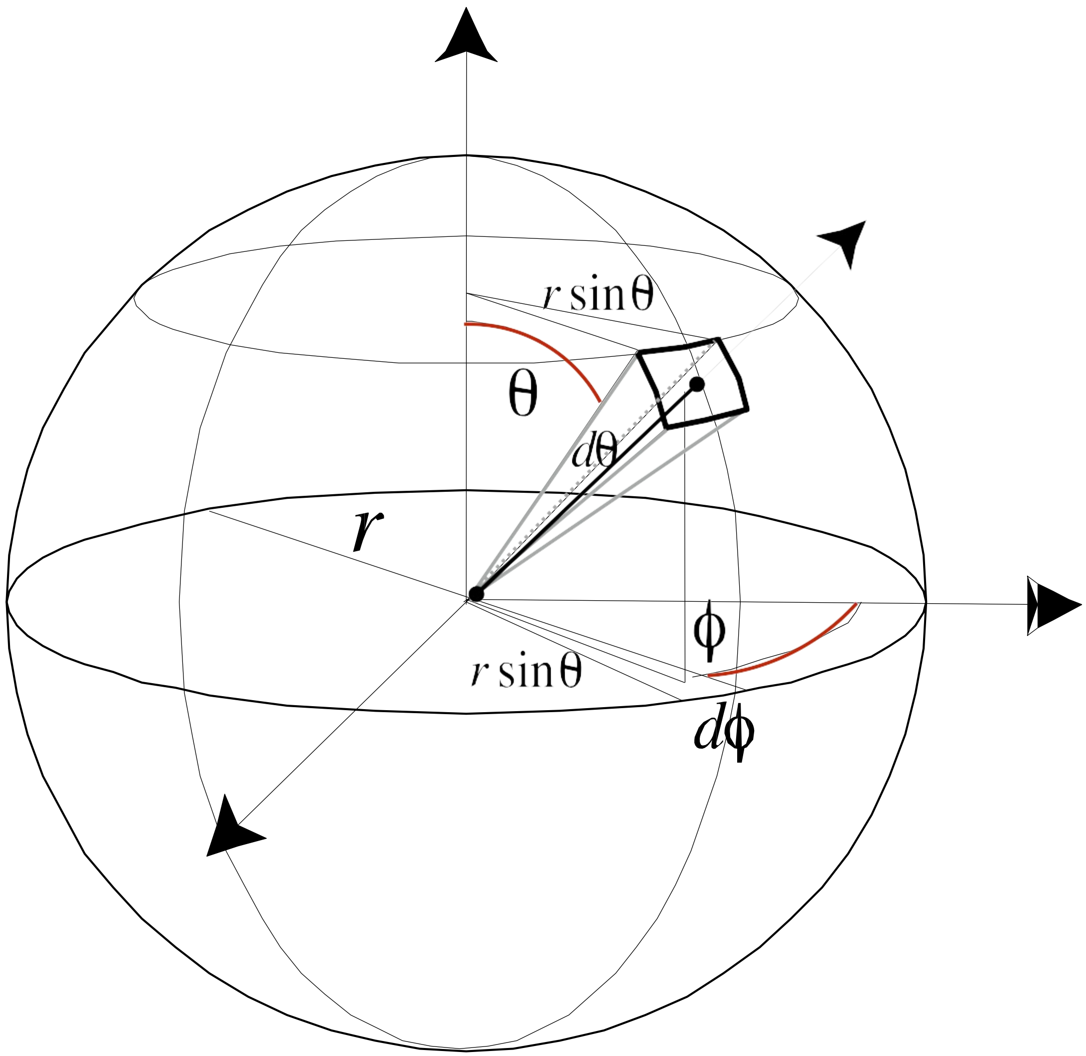
- $\Omega = \frac{A}{r^2}$
- Sphere has 4π steradians





$$\mathrm{d}A = (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi)$$
$$= r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

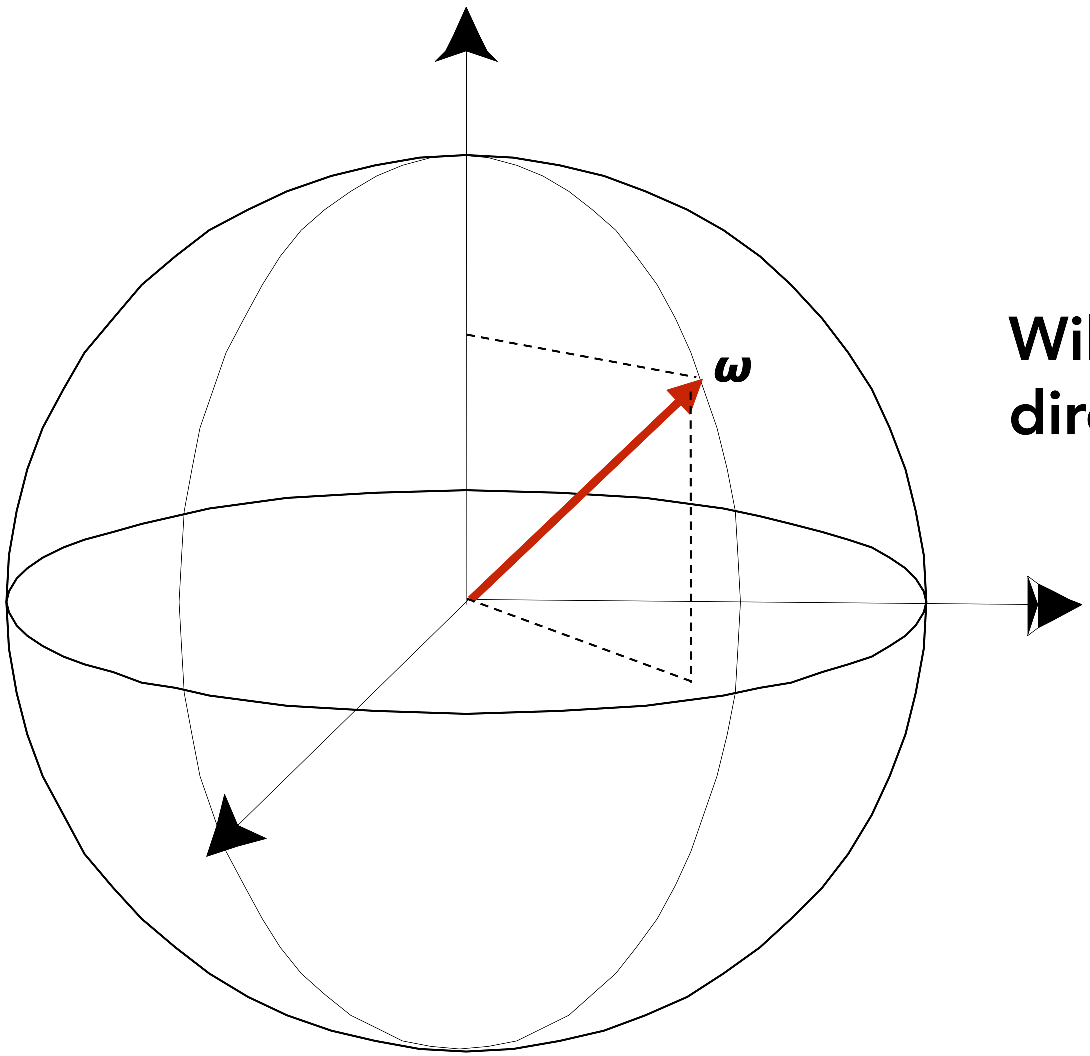


Sphere: S^2

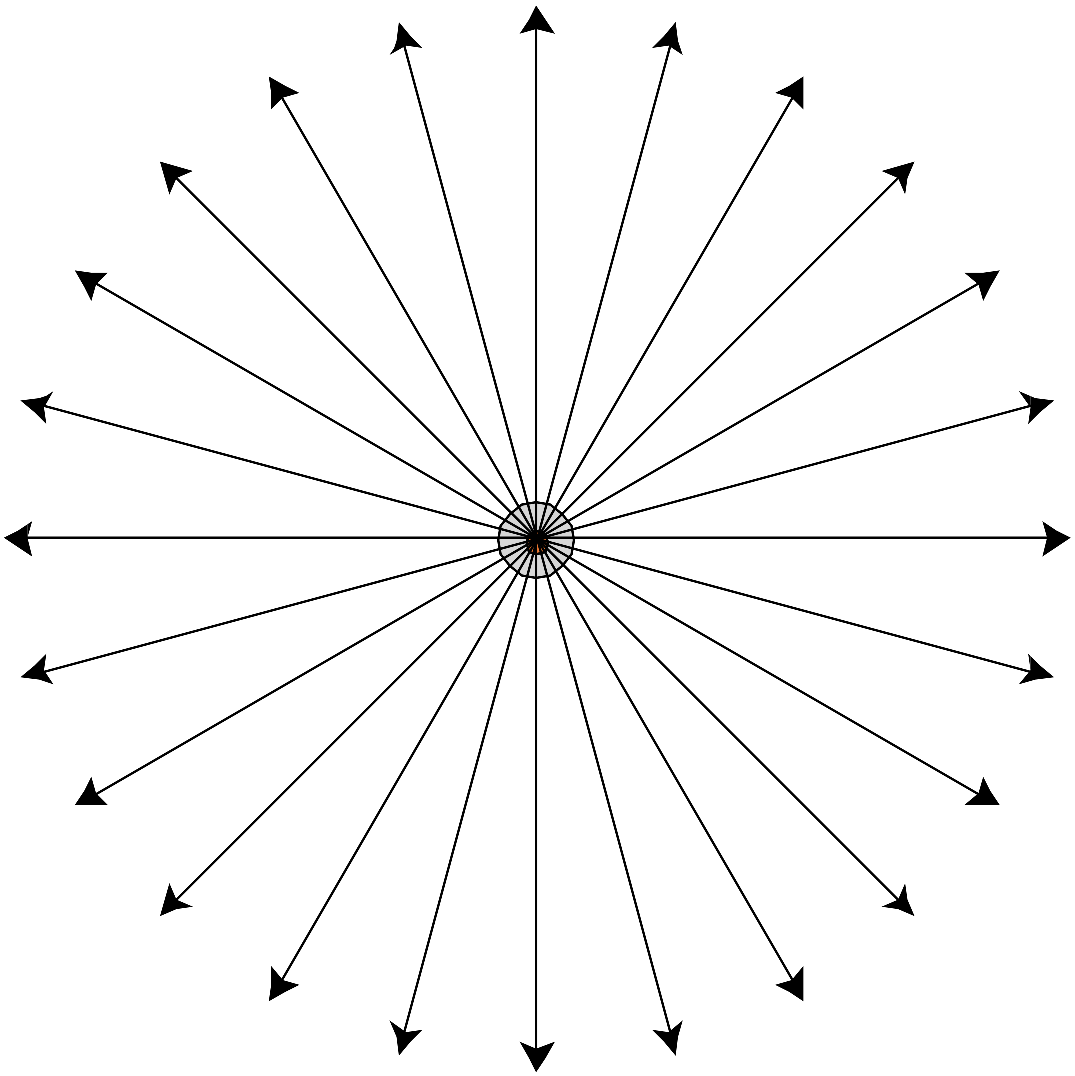
$$\Omega = \int_{S^2} \mathrm{d}\omega$$

$$= \int_0^{2\pi} \int_0^{\pi} \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

$$= 4\pi$$



Will use $\boldsymbol{\omega}$ to denote a direction vector (unit length)



$$\Phi = \int_{S^2} I \, \mathrm{d}\omega$$
$$= 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

Output: 815 lumens

(11W LED replacement
for 60W incandescent)

Radiant intensity?

Assume isotropic:

Intensity = 815 lumens / 4pi sr
= 65 candelas

Reviewing Concepts

Radiant energy Q [J = Joule] (barely used in CG)

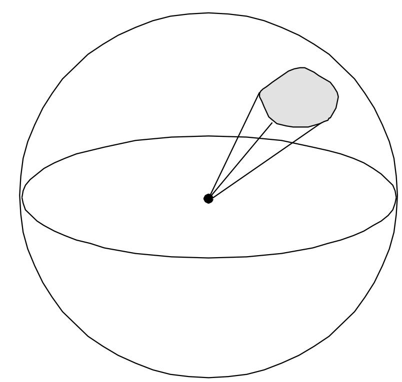
- the energy of electromagnetic radiation

Radiant flux (power) $\Phi \equiv \frac{\mathrm{d}Q}{\mathrm{d}t}$ [W = Watt] [lm = lumen]

- Energy per unit time

Radiant intensity $I(\omega) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$

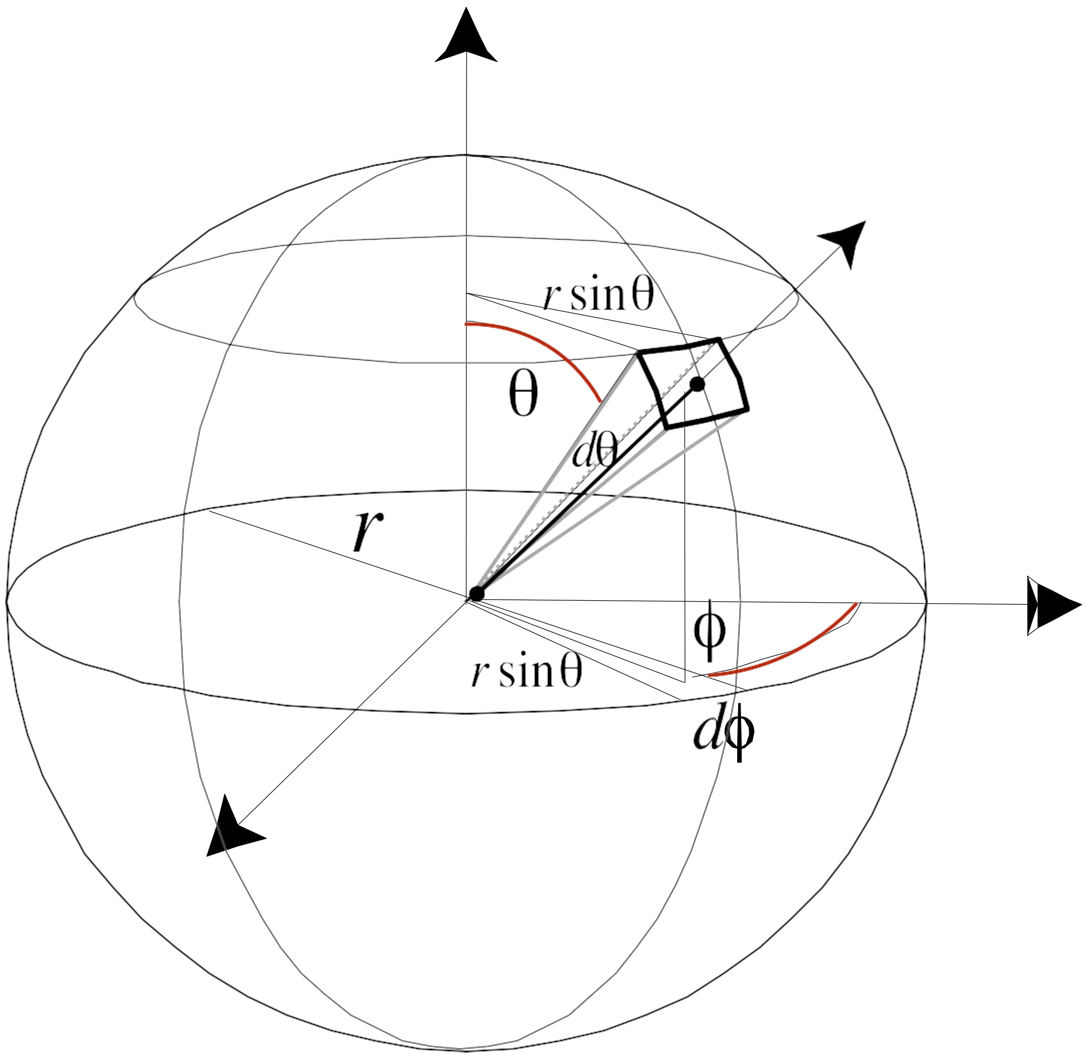
- power per unit solid angle



Solid Angle $\Omega = \frac{A}{r^2}$

- ratio of subtended area on sphere to radius squared

Differential Solid Angles



$$\mathrm{d}A = (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi)$$
$$= r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

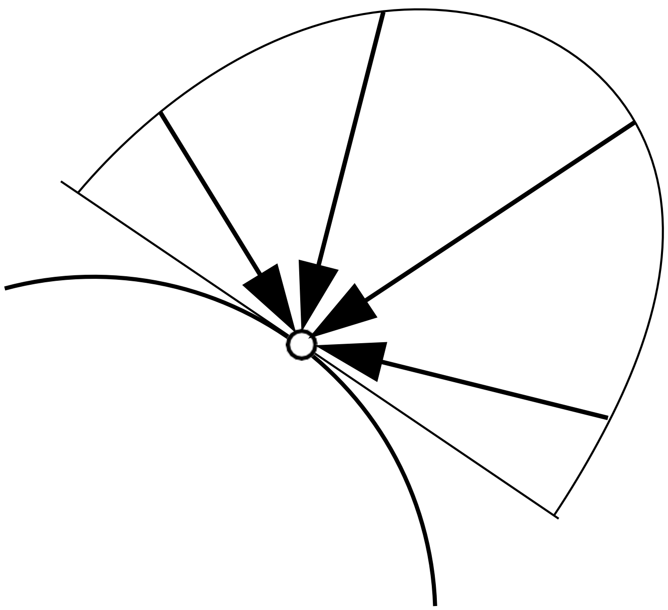
$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

Irradiance

Irradiance

Definition: The irradiance is the power per unit area incident on a surface point.

$$E(\mathbf{x}) \equiv \frac{\mathrm{d}\Phi(\mathbf{x})}{\mathrm{d}A}$$

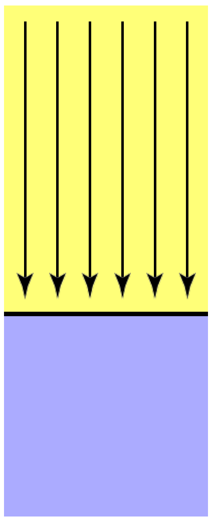


$$\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right] \; \left[\frac{\mathrm{lm}}{\mathrm{m}^2} = \mathrm{lux}\right]$$

Lambert's Cosine Law

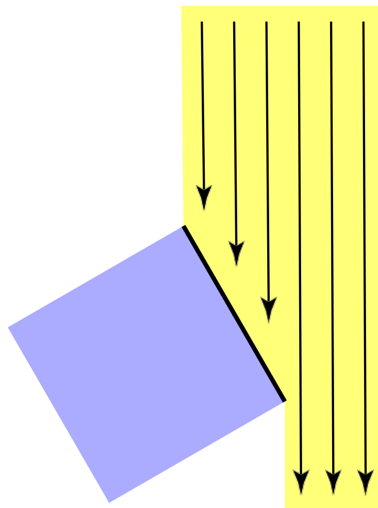
Irradiance at surface is proportional to cosine of angle between light direction and surface normal.

(Note: always use a unit area, the cosine applies on Φ)



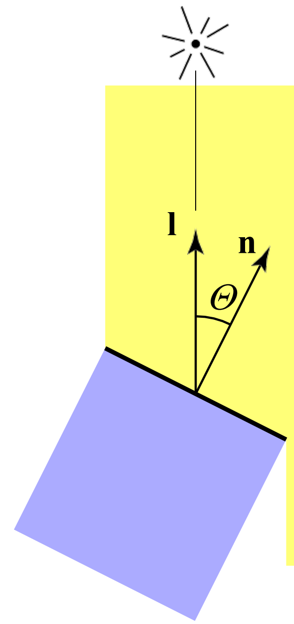
Top face of cube receives a certain amount of power

$$E = \frac{\Phi}{A}$$



Top face of 60° rotated cube receives half power

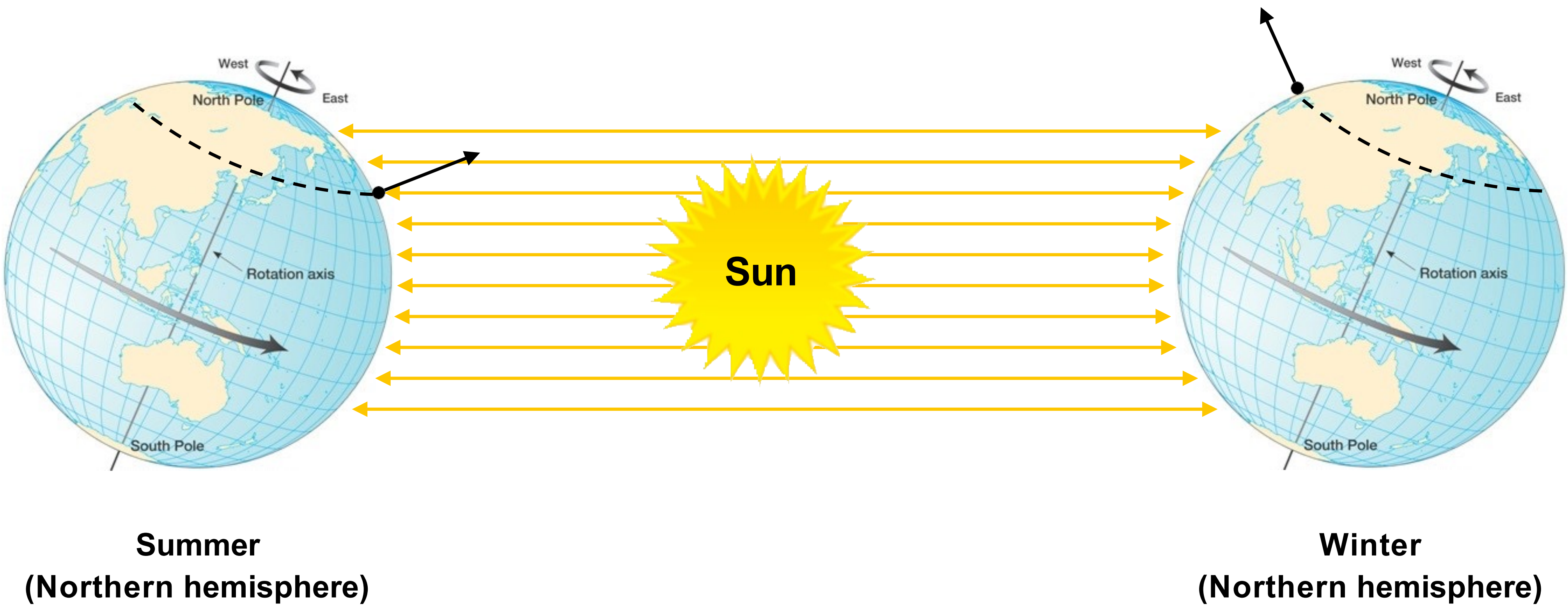
$$E = \frac{1}{2}\frac{\Phi}{A}$$



In general, power per unit area is proportional to $\cos \boldsymbol{\theta} = l \,.\, n$

$$E = \frac{\Phi}{A}\cos\theta$$

Why Do We Have Seasons?



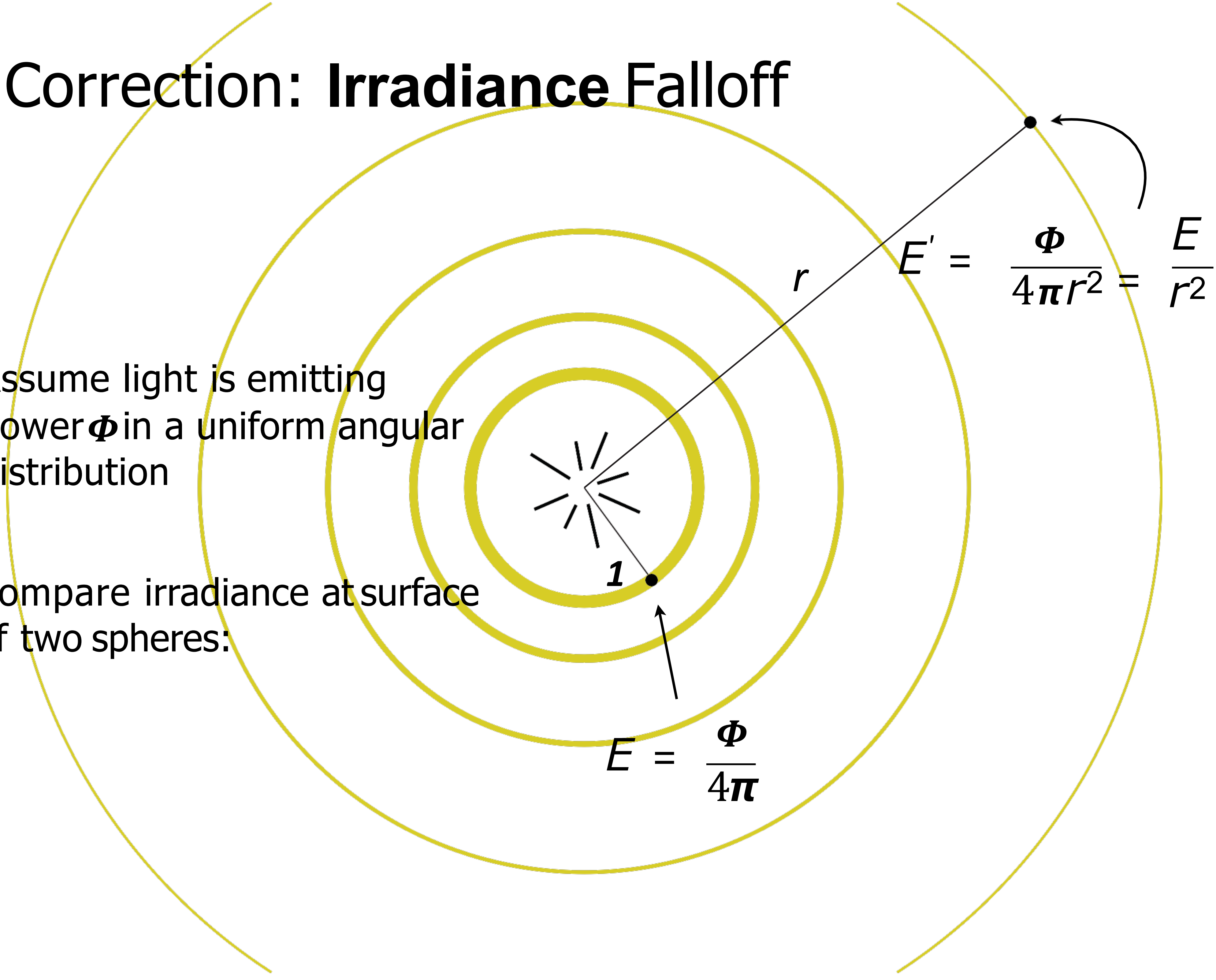
[Image credit: Pearson Prentice Hall]

Earth's axis of rotation: ~23.5° off axis

Correction: **Irradiance** Falloff

Assume light is emitting power $\boldsymbol{\Phi}$ in a uniform angular distribution

Compare irradiance at surface of two spheres:



$$E' = \frac{\boldsymbol{\Phi}}{4\boldsymbol{\pi} r^2} = \frac{E}{r^2}$$

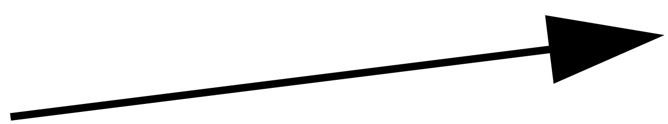
$$E = \frac{\boldsymbol{\Phi}}{4\boldsymbol{\pi}}$$

Radiance

Radiance

Radiance is the fundamental field quantity that describes the distribution of light in an environment

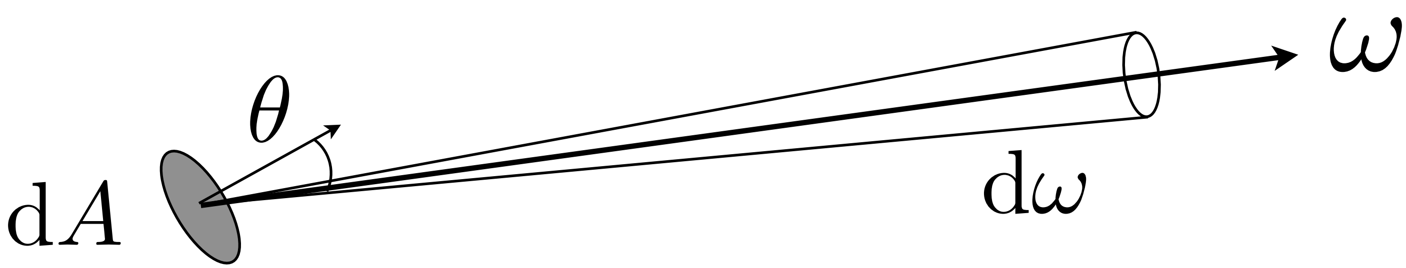
- Radiance is the quantity associated with a ray
- Rendering is all about computing radiance



Light Traveling Along A Ray

Radiance

Definition: The **radiance (luminance)** is the power emitted, reflected, transmitted or received by a surface, per unit solid angle, per projected unit area.



$$L(\mathrm{p}, \omega) \equiv \frac{\mathrm{d}^2 \Phi(\mathrm{p}, \omega)}{\mathrm{d}\omega \, \mathrm{d}A \cos\theta}$$

$\cos\theta$ accounts for projected surface area

$$\left[\frac{\mathrm{W}}{\mathrm{sr\,m}^2}\right] \quad \left[\frac{\mathrm{cd}}{\mathrm{m}^2} = \frac{\mathrm{lm}}{\mathrm{sr\,m}^2} = \mathrm{nit}\right]$$

Radiance

Definition: power per unit solid angle per projected unit area.

$$L(\mathrm{p}, \omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p}, \omega)}{\mathrm{d}\omega \, \mathrm{d}A \cos\theta}$$

Recall

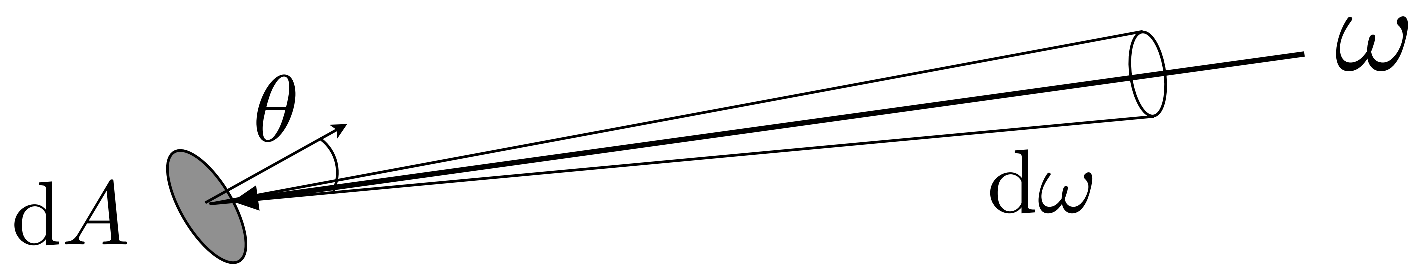
- Irradiance: power per projected unit area
- Intensity: power per solid angle

So

- Radiance: Irradiance per solid angle
- Radiance: Intensity per projected unit area

Incident Radiance

Incident radiance is the **irradiance** per unit solid angle arriving at the surface.

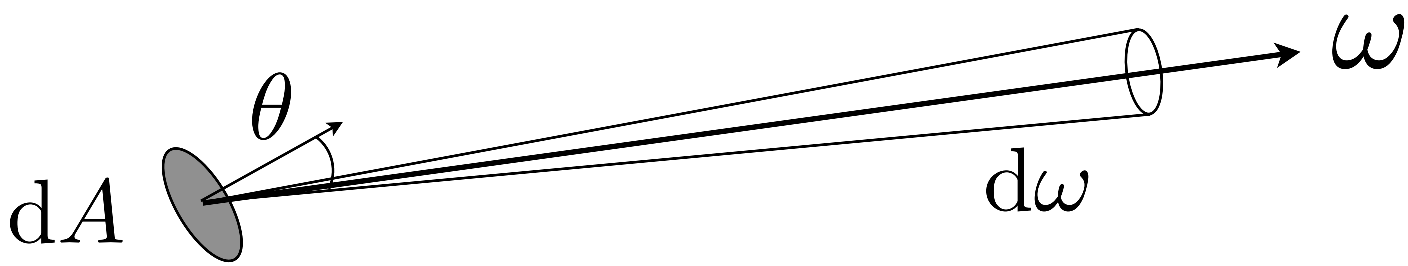


$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}E(\mathrm{p})}{\mathrm{d}\omega \cos\theta}$$

i.e. it is the light arriving at the surface along a given ray (point on surface and incident direction).

Exiting Radiance

Exiting surface radiance is the **intensity** per unit projected area leaving the surface.



$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}I(\mathrm{p}, \omega)}{\mathrm{d}A \cos\theta}$$

e.g. for an area light it is the light emitted along a given ray (point on surface and exit direction).

Irradiance vs. Radiance

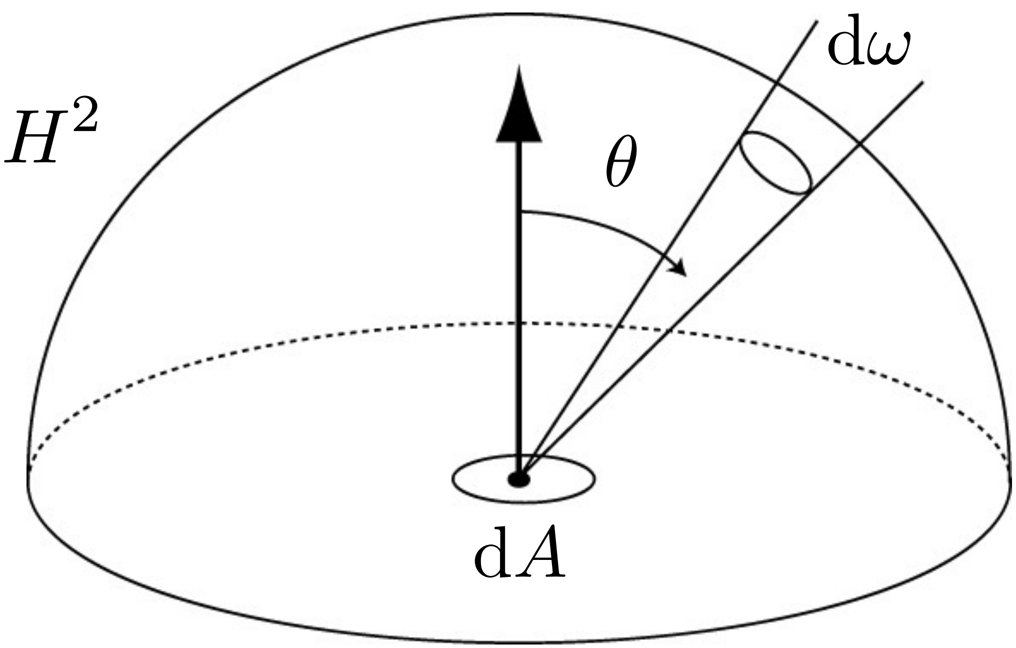
Irradiance E(p): total power received by area dA

Radiance L_i: power received by area dA from "direction" dω

$$dE(\mathrm{p},\omega) = L_i(\mathrm{p},\omega)\cos\theta\,\mathrm{d}\omega$$

$$E(\mathrm{p}) = \int_{H^2} L_i(\mathrm{p},\omega)\cos\theta\,\mathrm{d}\omega$$

Unit Hemisphere: H^2

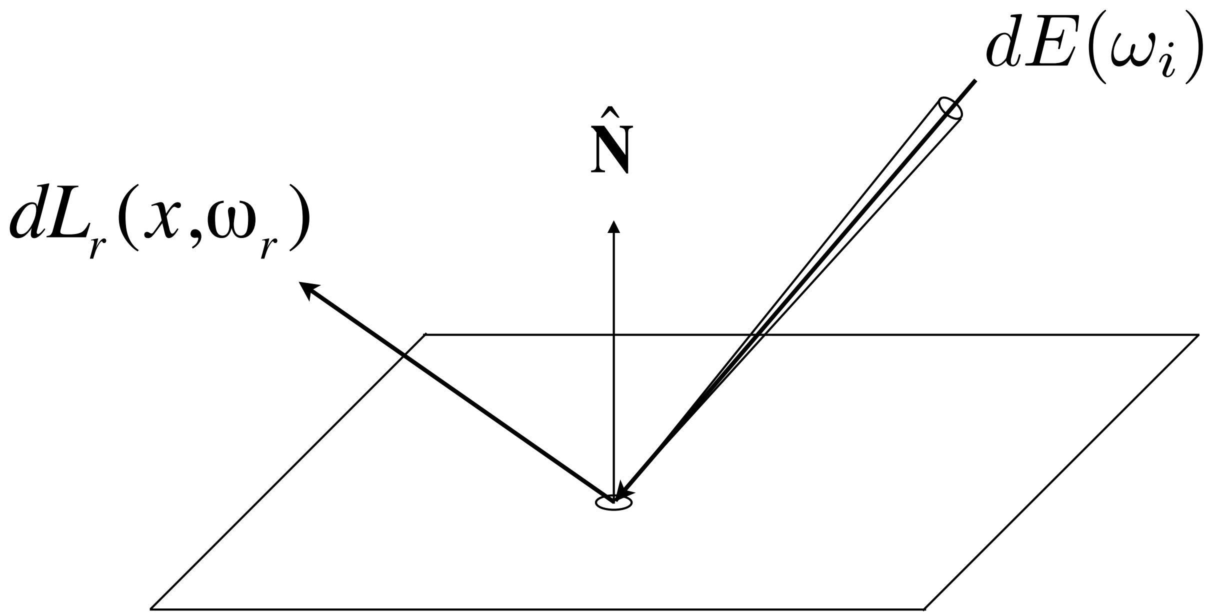


Bidirectional Reflectance Distribution Function (BRDF)

Reflection at a Point

Radiance from direction $\boldsymbol{\omega}_i$ turns into the power E that dA receives

Then power E will become the radiance to any other direction $\boldsymbol{\omega}_o$

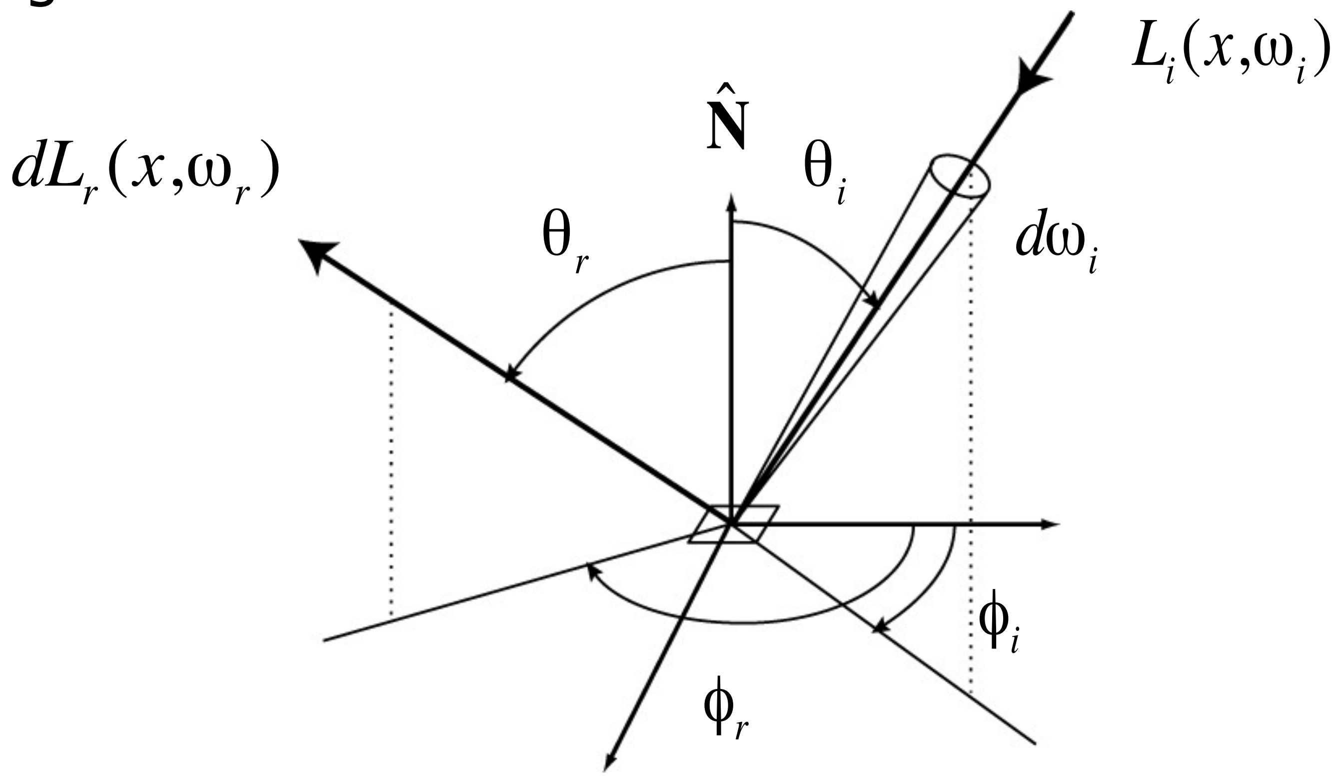


Differential irradiance incoming: $dE(\omega_i) = L(\omega_i)\, cos\,\theta_i\, d\omega_i$

Differential radiance exiting (due to $dE(\omega_i)$): $dL_r(\omega_r)$

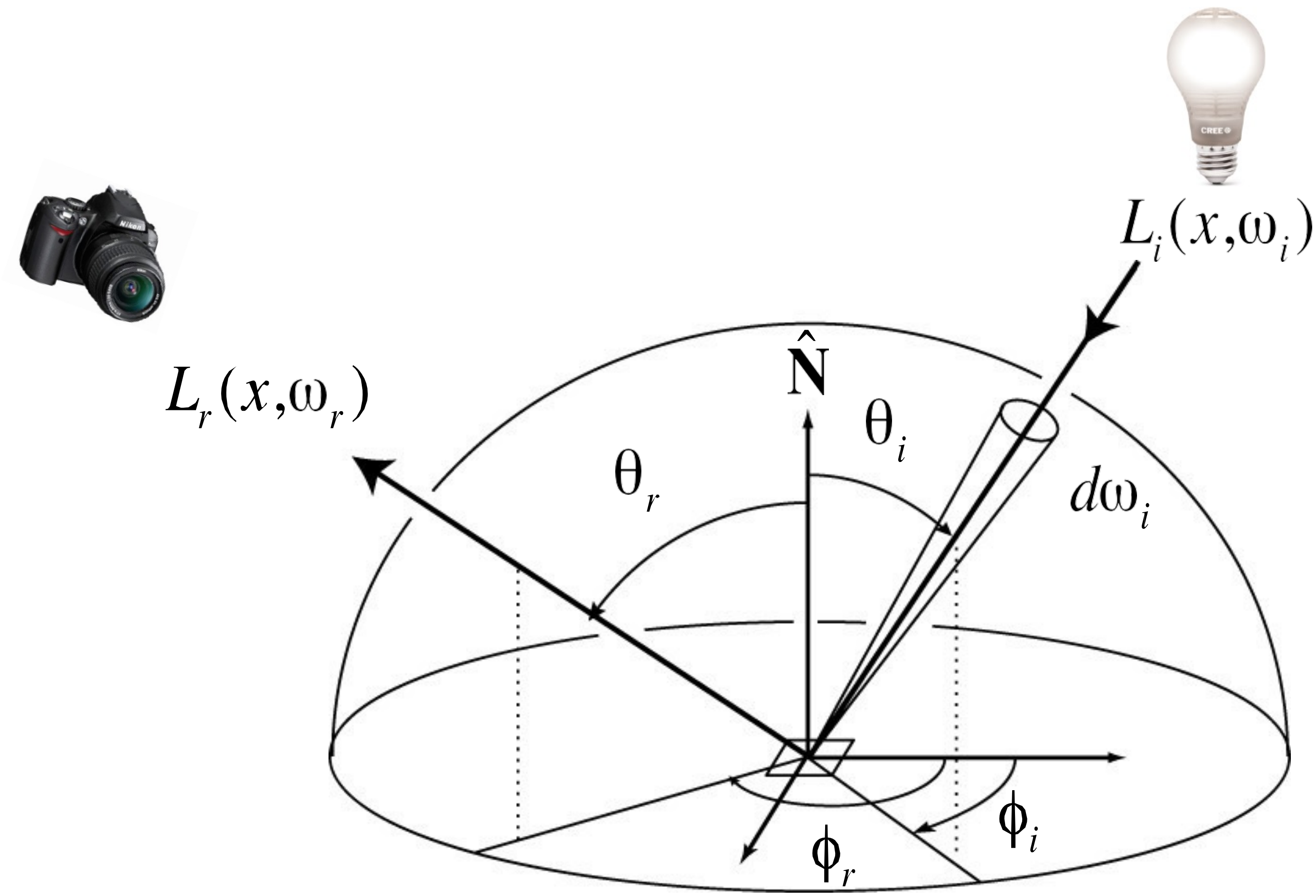
BRDF

The Bidirectional Reflectance Distribution Function (BRDF) represents how much light is reflected into each outgoing direction $\boldsymbol{\omega}_r$ from each incoming direction $\boldsymbol{\omega}_i$



$$f_r(\omega_i \to \omega_r) = \frac{\mathrm{d}L_r(\omega_r)}{\mathrm{d}E_i(\omega_i)} = \frac{\mathrm{d}L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i\,\mathrm{d}\omega_i} \quad \left[\frac{1}{\mathrm{sr}}\right]$$

The Reflection Equation

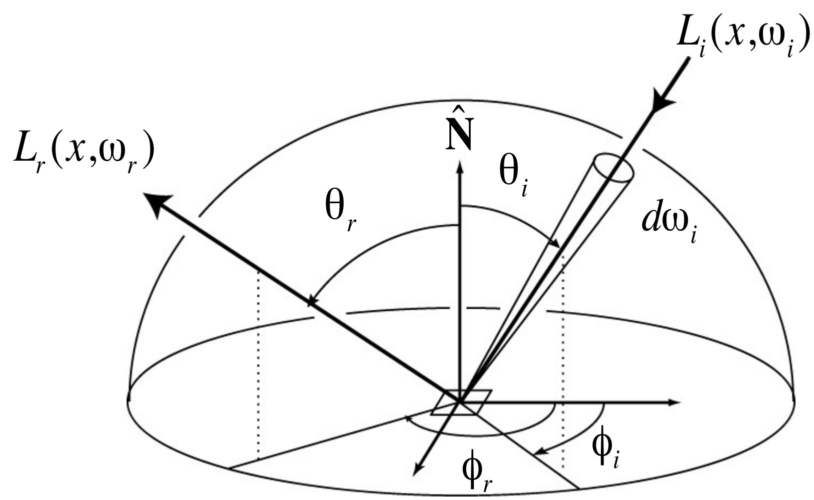


$$L_r(\mathrm{p}, \omega_r) = \int_{H^2} f_r(\mathrm{p}, \omega_i \rightarrow \omega_r)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i \, \mathrm{d}\omega_i$$

Challenge: Recursive Equation

Reflected radiance depends on incoming radiance

$$\boxed{L_r(\mathrm{p}, \omega_r)} = \int_{H^2} f_r(\mathrm{p}, \omega_i \rightarrow \omega_r) \boxed{L_i(\mathrm{p}, \omega_i)} \cos\theta_i \, \mathrm{d}\omega_i$$



But incoming radiance depends on reflected radiance (at another point in the scene)

The Rendering Equation

Re-write the reflection equation:

$$L_r(\mathrm{p}, \omega_r) = \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_r)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i \,\mathrm{d}\omega_i$$

by adding an Emission term to make it general!

The Rendering Equation

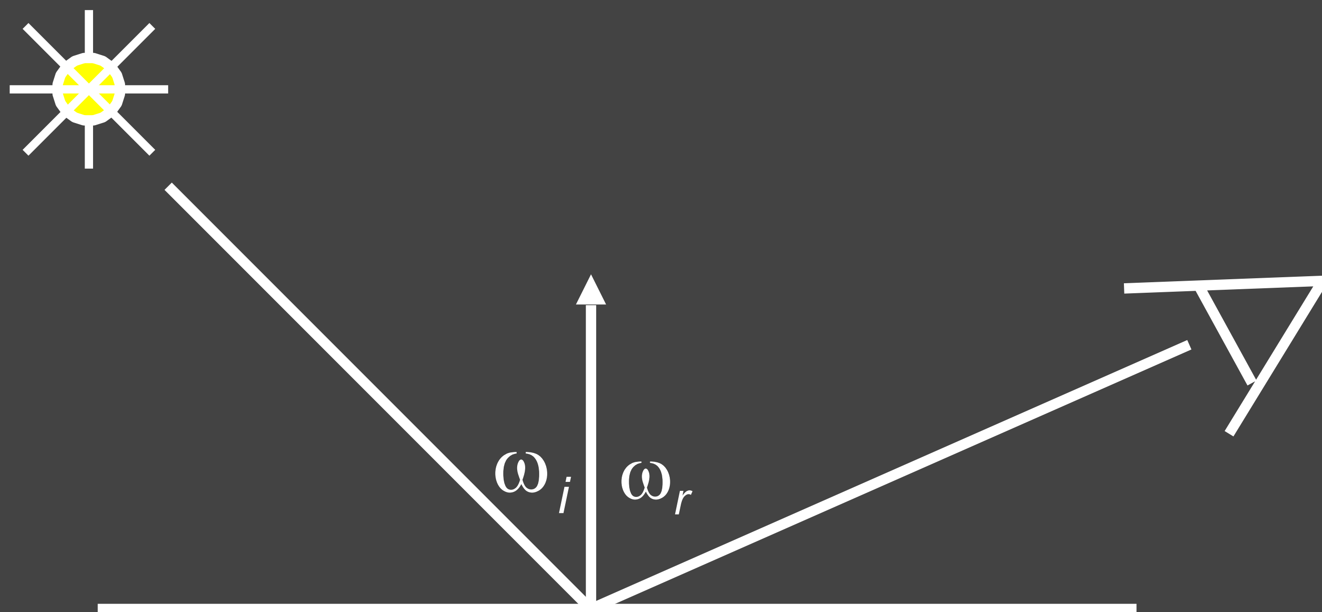
$$\boxed{L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\Omega^+} L_i(p, \omega_i) f_r(p, \omega_i, \omega_o)(n \cdot \omega_i)\,\mathrm{d}\omega_i}$$

How to solve?

Note: now, we assume that all directions are pointing outwards!

Understanding the rendering equation

Reflection Equation



$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ (\omega_i,n)$$

Reflected Light (Output Image)

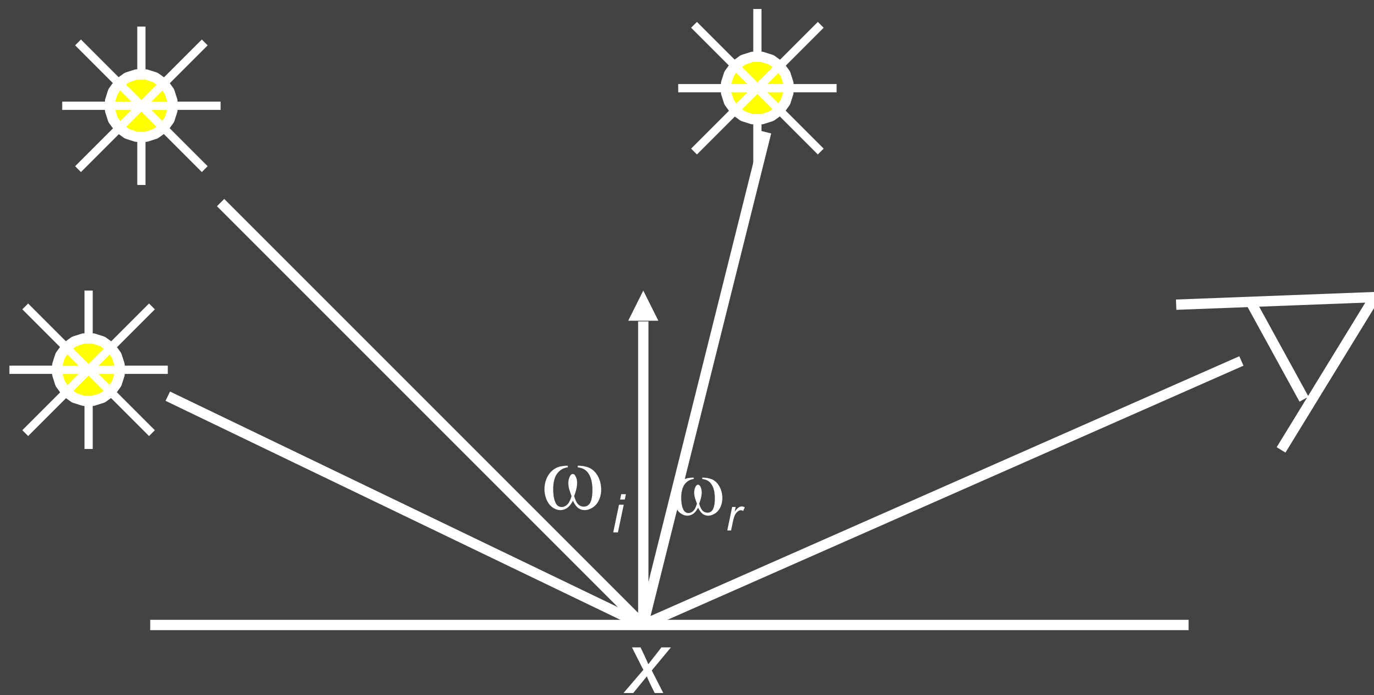
Emission

Incident Light (from light source)

BRDF

Cosine of Incident angle

Reflection Equation



Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ (\omega_i,n)$$

Reflected Light (Output Image)

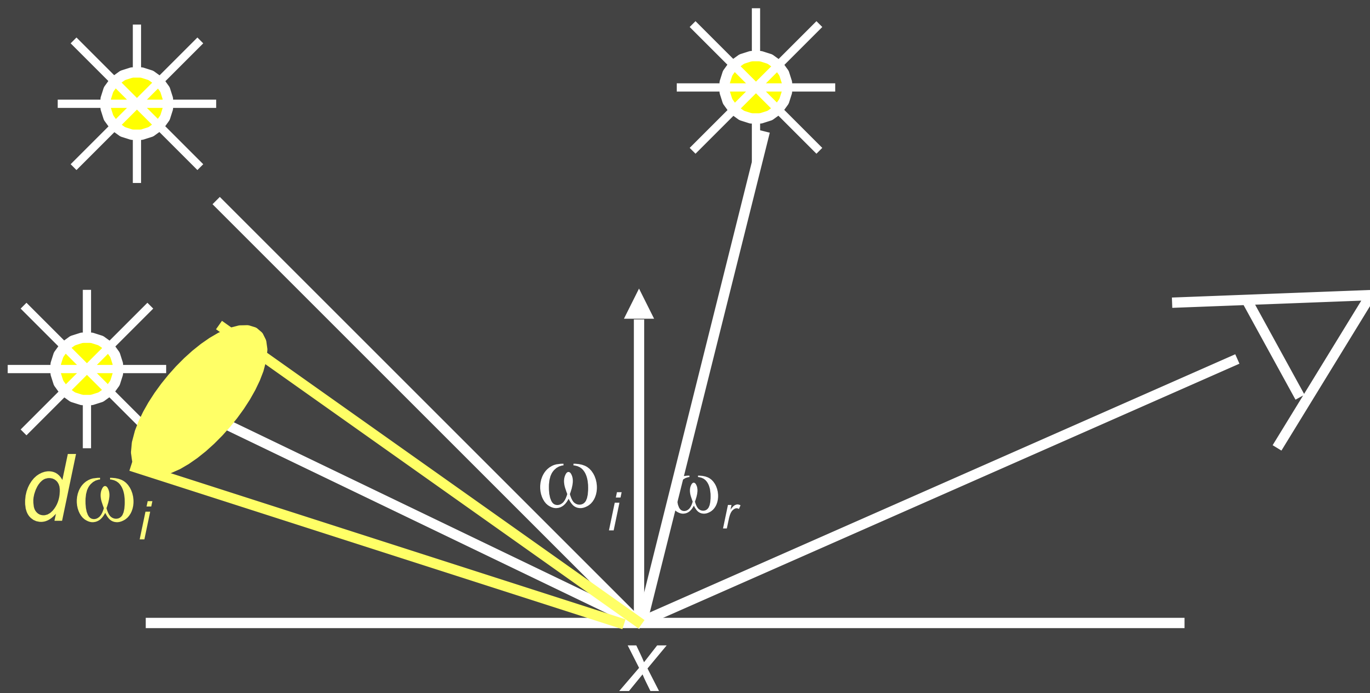
Emission

Incident Light (from light source)

BRDF

Cosine of Incident angle

Reflection Equation



Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ \cos\theta_i\, \mathrm{d}\omega_i$$

Reflected Light (Output Image)

Emission

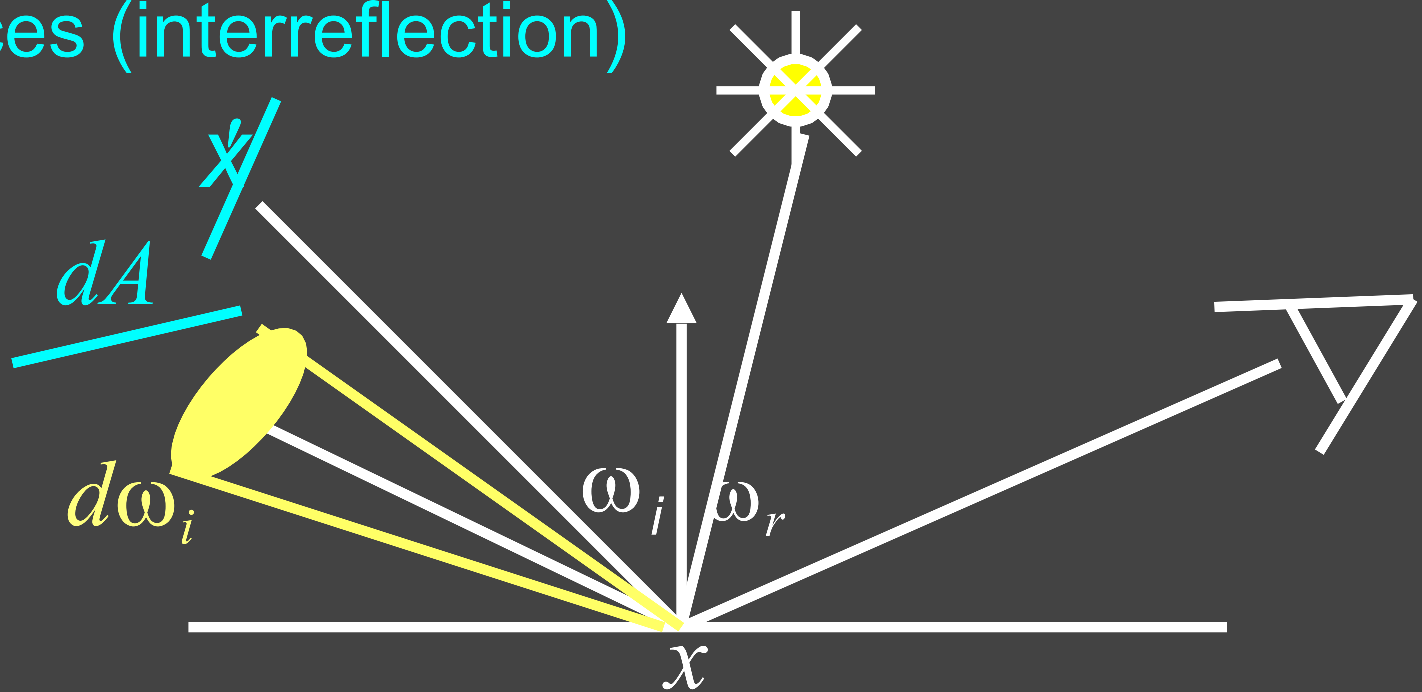
Incident Light (from light source)

BRDF

Cosine of Incident angle

Rendering Equation

Surfaces (interreflection)



$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i) f(x,\omega_i,\omega_r)\cos\theta_i \, d\omega_i$$

Reflected Light (Output Image)	Emission	reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Rendering Equation (Kajiya 86)

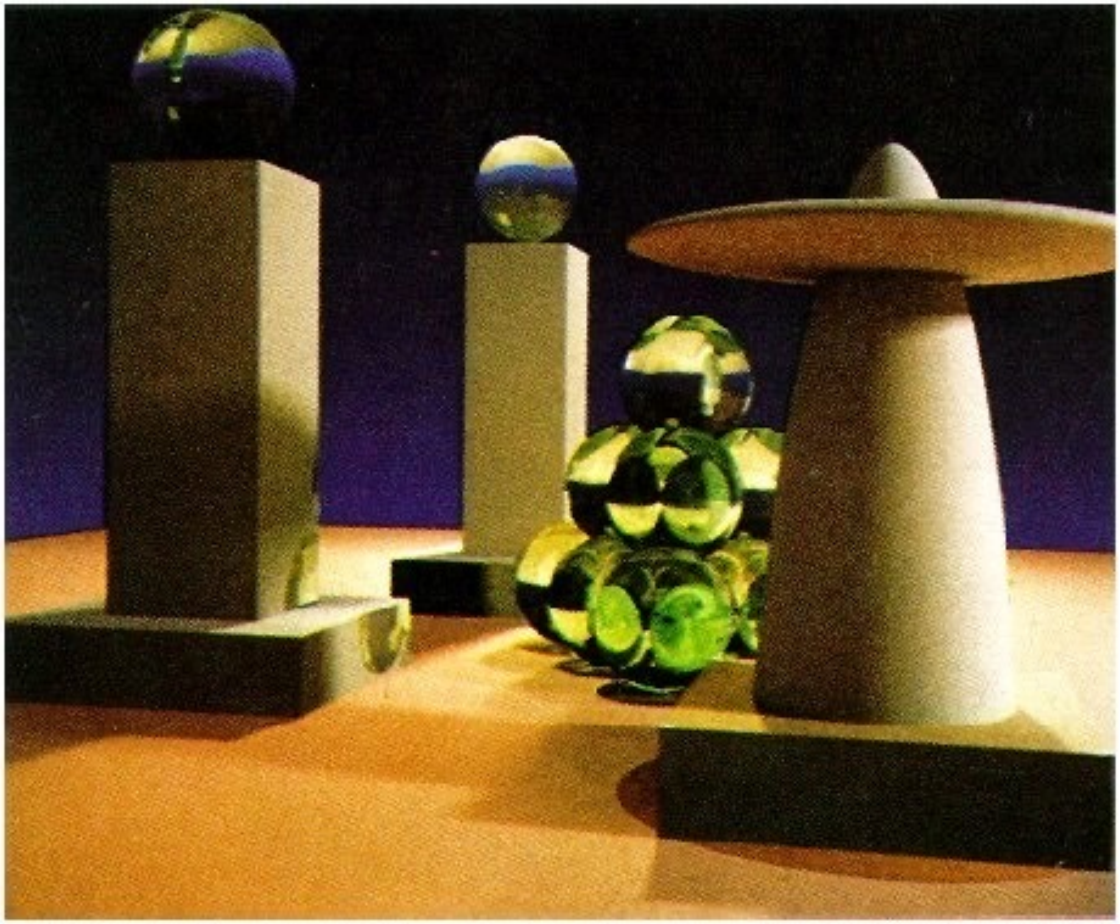


Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

Rendering Equation as Integral Equation

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i)\ \boxed{f(x,\omega_i,\omega_r)\cos\theta_i\, d\omega_i}$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

$$l(u) = e(u) + \int l(v)\ \boxed{K(u,v)dv}$$

Kernel of equation

Linear Operator Equation

$$l(u) = e(u) + \int l(v) \boxed{K(u,v) dv}$$

Kernel of equation

Light Transport Operator

$$L = E + KL$$

Can be discretized to a simple matrix equation [or system of simultaneous linear equations] (L, E are vectors, K is the light transport matrix)

Ray Tracing and extensions

- General class numerical Monte Carlo methods
- Approximate set of all paths of light in scene

$$L = E + KL$$

$$IL - KL = E$$

$$(I - K)L = E$$

$$L = (I - K)^{-1}E$$

Binomial Theorem

$$L = (I + K + K^2 + K^3 + \ldots)E$$

$$L = E + KE + K^2E + K^3E + \ldots$$

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

Direct Illumination
on surfaces

Indirect Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect illum.)

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

Direct Illumination
on surfaces

Indirect Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect illum.)

Shading in
Rasterization

p
Direct illumination


p
One-bounce global illumination (dir+indir)


p
Two-bounce global illumination


p
Four-bounce global illumination

p
Eight-bounce global illumination


p
Sixteen-bounce global illumination

Thank you!

(And thank Prof. Lingqi Yan (UCSB), Prof. Ravi Ramamoorthi (UCSD) and Prof. Ren Ng (UC Berkeley) for many of the slides!)