University of Houston, COSC 3340-6309, Midterm Review and Selected Solutions

Instructions: You are responsible for all the materials: (a) on the slides, (b) covered in class for lectures until and including October 6 and (c) Chapters 0, 1, 3, 4 and Sections 5.1, 5.2 of textbook with the exception of Algorithms for CFG/CFLs in section 5.1. Note that the Turing Machine schemas, Busy Beaver problem discussion is on the slides and the book Lewis and Papadimitriou but not the textbook. You are responsible for this as well.

Solution sketches for selected problems that were found to be difficult by the class are given below.

Reading Quiz 2 (e): The construction for closure under intersection is on the slides - it is the cross product construction. The claim to be shown is that if string w is in L(M) then $w \in L(M_1) \cap L(M_2)$. If w is in L(M), then by construction ((s_1, s_2), w) yields in 0 or more steps of M a final state, say (f1, f2) with w consumed. Here s_1 and s_2 are start states of M1 and M2 respectively and similarly for f1, f2. Since delta(M) applies delta(M1) to the 1st component of the states of M and delta(M2) to the 2nd components, this computation of M implies (s_1, w) yields final state f1 in 0 or more steps of M1 and (s_2, w) yields final state f2 in M2 similarly, so w is in L(M1) and w is in L(M2) so w is in the intersection of L(M1) and L(M2), QED.

Hw 2 Q 3: Definition of reflexive and transitive closure is in the book. For the given relation we observe that the required closure is the congruence relation x congruent to y modulo 3 or x - y is an integer multiple of 3,

Hw 2 Q 4: If L is a regular language, then Max(L) is regular since we can start with the DFA M = (K, Sigma, delta, s, F) for L and get a DFA M' as follows for Max(L): M' = (K, Sigma, delta, s, F') where F' = $F - \{f \in F | (f, w)$ yields via δ^* (f', ϵ) for some w of length at least one $\}$, i.e, every final state f of M that can reach a different final state f' of M (via a path of at least one edge) becomes non-final in M'. Now you still have to prove the correctness of this construction.

Hw 1 Q 4: Yes, reducibility is a reflexive relation since the identity transformation Id reduces every language L to itself and Id is obviously a computable function.

It is not a symmetric relation, since the Turing Decidable (TD) language $H = \{< M, w.k > \mid$ TM M halts on input w in k steps$\}$ can can be reduced to the language $L = \{< M, w > \mid$ M accepts w$\}$ (can you show this?), which is TA but not TD, but the other direction cannot work (i.e., L cannot be reduced to H) since then L would also be TD (this is an application of a result we proved in class). We also showed in class that L is not TD using diagonalization.

Hw 1 Q 3 The NTM just guesses a subset of the vertices say S of the required size and then checks that every pair (u, v) of vertices in S has an edge between u and v in the given graph G.

Reading Quiz 1 Q 3 We use diagonalization to show that all decimal numbers in the interval [0, 1] are uncountable. The since the real numbers is a superset of this set, it will also be uncountable. For decimal numbers in interval [0, 1], we do a proof by contradiction. Assume this set is countable infinite and list the numbers as n0, n1, etc., where n0 is the decimal number corresponding to 0, n1 corresponding to 1, etc., Now construct a new decimal number such that it differs from n0 in the 1st place after decimal point, it differs from n1 in the 2nd place after decimal point and so on. This is done by adding 1 modulo 10 at the correct position. This new decimal number is in the interval [0,1] but it does not correspond to any natural number. Because if it corresponded to say number k, then it would be nk. But then it would differ from itself in the k+1 th position (by construction), which is a contradiction.

The rest of this reading quiz was directly from the slides or the book. Good Luck!