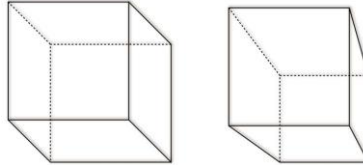# COSC4370 Midterm Example

Problem 1

(8 points) One of the diagrams below shows a cube under orthographic projection, the other under perspective projection. Label which is which.

Problem 2

Consider the projective transformation:

$$\begin{pmatrix} f_0 & 0 & 0 & 0 \\ 0 & f_0 & 0 & 0 \\ 0 & 0 & f_0 + f_1 & -f_0 f_1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

a. (6 points) Which points in R^3 get mapped to points at infinity?

b. (6 points) Which points at infinity get mapped to points in R^3?

Problem 3

a. (5 points) Give a 2x2 matrix that reflects (mirrors) any 2D point about the x-axis.

b. (10 points) Is this 2x2 matrix a rotation matrix? Why or why not?

Problem 4

(15 points) Assume homogeneous transform matrices, where
$T(t_x, t_y, t_z)$ gives general 3D translation
$S(s_x, s_y, s_z)$ gives uniform scaling, i.e. $s_x = s_y = s_z$
$R(\theta_x, \theta_y, \theta_z)$ gives general 3D rotation
Given the above definitions, which of the following 3D graphics transformations commute?
a. TS   b. SR   c. $S_1 S_2$ d. $R_1 R_2$ e. $T_1 T_2$

Problem 5

(10 points) We are given the triangle with vertices $P_1=(1,2)$, $P_2=(4,2)$, $P_3=(1,6)$. We are also given (r,g,b) colors (in the range 0:255) at the three vertices $C_1=(200,200,0)$, $C_2=(0,50,200)$, $C_3=(200,50,100)$. What is the color at a point inside the triangle $Q = (2,3)$?

Problem 6

(20 points) Let S be a 3D surface made up of points $p = (x,y,z)$ that satisfy the implicit equation

$$5x^2 + 3y^2 + 3xz - 4 = 0.$$

Find a vector that is normal to S at point $(1,2,0)$. Show and explain your work.

Problem 7

(20 points) Consider a simple graphics toolkit that works like OpenGL (that is, it has a matrix stack, and the transformation commands post-multiply themselves onto it):
The toolkit has the following commands

- translate(x,y) – post-multiplies a translation matrix onto the top of the matrix stack

- scale(x,y) – scales by x and y from the origin. BOTH X and Y MUST BE POSITIVE

- push() – pushes a copy of the top element on the matrix stack

- pop() – removes the top element from the matrix stack

- draw(triangle) – draws a triangle with unit base and unit height • draw(square) – draws a unit square
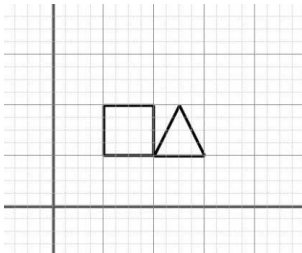
Sample:
translate(1,1) draw(square)
translate(1,0) draw(triangle)

Write down the sequence of commands to make the following drawing in a minimum number of steps. Assume the origin of triangle and square is bottom left.