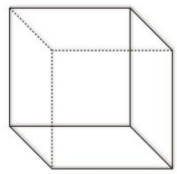
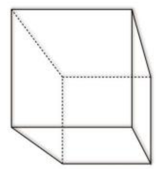
COSC4370 Midterm Example Solution

Problem 1

(8 points) One of the diagrams below shows a cube under orthographic projection, the other under perspective projection. Label which is which.



Orthographic



Perspective

Problem 2

Consider the projective transformation:

$$\begin{pmatrix} f_0 & 0 & 0 & 0 \\ 0 & f_0 & 0 & 0 \\ 0 & 0 & f_0 + f_1 & -f_0 f_1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

a. (6 points) Which points in R^3 get mapped to points at infinity?
The plane z=0 gets mapped into infinity. To see this, multiply the matrix by (x, y, z, 1) and examine the 4th coordinate. For the point to get mapped to infinity; the 4th coordinate should be 0 as z=0.

b. (6 points) Which points at infinity get mapped to points in R^3?
A general point at infinity (x, y, z, 0) gets mapped to (f_0x, f_0y, $(f_0+f_1)_2$, 2). By inspections, all points at infinity get mapped to R^3 except those points at infinity for which z=0. The points get mapped to other points at infinity.

Problem 3

a. (5 points) Give a 2x2 matrix that reflects (mirrors) any 2D point about the x-axis.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

b. (10 points) Is this 2x2 matrix a rotation matrix? Why or why not?

No, its determinant is -1.

Problem 4

(15 points) Assume homogeneous transform matrices, where
$T(t_x,t_y,t_z)$ gives general 3D translation
$S(s_x,s_y,s_z)$ gives uniform scaling, i.e. $s_x = s_y = s_z$

$R(\theta_x, \theta_y, \theta_z)$ gives general 3D rotation
Given the above definitions, which of the following 3D graphics transformations commute?

a. TS b. SR c. S_1S_2 d. R_1R_2 e. T_1T_2

Problem 5

(10 points) We are given the triangle with vertices $P_1=(1,2)$, $P_2=(4,2)$, $P_3=(1,6)$. We are also given (r,g,b) colors (in the range 0:255) at the three vertices $C_1=(200,200,0)$, $C_2=(0,50,200)$, $C_3=(200,50,100)$. What is the color at a point inside the triangle $Q = (2,3)$?

$$\lambda_1 = \frac{(y_2 - y_3)(x - x_3) + (x_3 - \chi_2)(y - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)} = \frac{(2-6)(2-1) + (1-4)(3-6)}{(2-6)(1-1) + (1-4)(2-6)} = \frac{5}{12}$$

$$\lambda_2 = \frac{(y_3 - y_1)(x - x_3) + (x_1 - \chi_3)(y - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)} = \frac{(6-2)(2-1) + (1-1)(3-6)}{(2-6)(1-1) + (1-4)(2-6)} = \frac{1}{3}$$

$$\lambda_3 = 1 - \lambda_1 - \lambda_2 = 1 - \frac{5}{12} - \frac{1}{3} = \frac{1}{4}$$

$$C = \lambda_1 C_1 + \lambda_2 C_2 + \lambda_3 C_3 = \frac{5}{12}(200, 200, 0) + \frac{1}{3}(0, 50, 200) + \frac{1}{4}(200, 50, 100)$$

Problem 6

(20 points) Let S be a 3D surface made up of points $p = (x,y,z)$ that satisfy the implicit equation

$$5x^2 + 3y^2 + 3xz - 4 = 0.$$

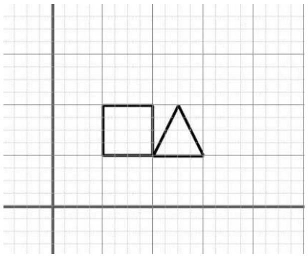
Find a vector that is normal to S at point (1,2,0). Show and explain your work.

Let $F(x, y, z) = x^2 + 3y^2 + 2xz - 4$, then the gradient $\nabla F = (2x+2z, 6y, 2x)$. The normal vector at $(1, 1, 1)$ is evaluated as $\nabla F = (1, 1, 1) = (2, 6, 2)$

Problem 7

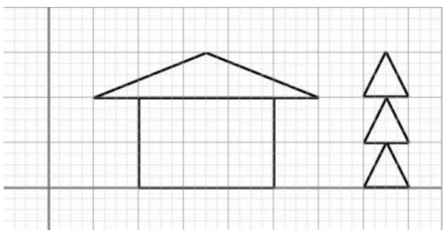
(20 points) Consider a simple graphics toolkit that works like OpenGL (that is, it has a matrix stack, and the transformation commands post-multiply themselves onto it):
The toolkit has the following commands

- translate(x,y) – post-multiplies a translation matrix onto the top of the matrix stack
- scale(x,y) – scales by x and y from the origin. BOTH X and Y MUST BE POSITIVE
- push() – pushes a copy of the top element on the matrix stack
- pop() – removes the top element from the matrix stack
- draw(triangle) – draws a triangle with unit base and unit height • draw(square) – draws a unit square



Sample:
translate(1,1) draw(square)
translate(1,0) draw(triangle)

Write down the sequence of commands to make the following drawing in a minimum number of steps. Assume the origin of triangle and square is bottom left.



translate(2, 0)
scale(3, 2)
draw(square)
scale($\frac{1}{3}$, $\frac{1}{2}$)
translate(5, 0)
draw(triangle)
translate(0, 1)
draw(triangle)
translate(0, 1)
draw(triangle)
translate(-6, 0)
scale(5, 1)
draw(triangle)