

Measurability of the epidemic reproduction number in data-driven contact networks

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The basic reproduction number is one of the conceptual cornerstones of mathematical epidemiology. Its classical definition as the number of secondary cases generated by a typical infected individual in a fully susceptible population finds a clear analytical expression in homogeneous and stratified mixing models. Along with the generation time (the interval between primary and secondary cases), the reproduction number allows for the characterization of the dynamics of an epidemic. A clear-cut theoretical picture, however, is hardly found in real data. Here, we infer from highly detailed sociodemographic data two multiplex contact networks representative of a subset of the Italian and Dutch populations. We then simulate an infection transmission process on these networks accounting for the natural history of influenza and calibrated on empirical epidemiological data. We explicitly measure the reproduction number and generation time, recording all individual-level transmission events. We find that the classical concept of the basic reproduction number is untenable in realistic populations, and it does not provide any conceptual understanding of the epidemic evolution. This departure from the classical theoretical picture is not due to behavioral changes and other exogenous epidemiological determinants. Rather, it can be simply explained by the (clustered) contact structure of the population. Finally, we provide evidence that methodologies aimed at estimating the instantaneous reproduction number can operationally be used to characterize the correct epidemic dynamics from incidence data.

computational modeling | infectious diseases | multiplex networks | reproduction number | generation time

Mathematical and computational models of infectious diseases are increasingly recognized as relevant quantitative support to epidemic preparedness and response (1–3). Independent of the type of modeling approach, our understanding of epidemic models is generally tied to two fundamental concepts. One is the basic reproduction number R_0 , which is the average number of secondary cases generated by a typical infectious individual over the entire course of the infectious period in a fully susceptible population (4). The other is the generation time T_g , the time interval between the infection time of the infector and her/his infectees (5, 6). These quantities are the cornerstones of our understanding of basic epidemic models, as they encompass the condition for the occurrence of an epidemic outbreak ($R_0 > 1$) and the epidemic doubling time. Both the reproduction number and generation time are determined by the biological characteristics of the pathogen (e.g., probability of transmission given a contact), the pathogen–host system (e.g., timeline of pathogen replication inside the host), and the contact patterns of the population in which the infection spreads (4, 7). The concept of the reproduction number has been extended to stratified models (8) and to heterogeneous contact networks (9) to account for more complex interaction patterns. Furthermore, the definition of R_0 has been generalized by the effective reproduction number $R(t)$ (i.e., the average number of secondary cases generated by

an infectious individual at time t), thus relaxing the hypothesis of a fully susceptible population (10).

R_0 and T_g are mathematically well defined in the early stages of the epidemic in homogeneous models and are widely used in predictive approaches (10, 11). However, several studies have cautioned on the importance of the local contact structures (e.g., households, extended families, communities) in estimating the value of R_0 (12–17). Theoretical work has explored the definition of the reproduction number in models entailing community and household structure (18–24), although an operational way to compute R_0 is still lacking. For more complex models closely resembling the actual structure of the human populations (e.g., accounting for households, schools, workplaces) (25–34), the estimation of R_0 mainly relied on the direct count of secondary cases generated by the index case of the outbreak (R_0^{index}) and/or the analysis of the growth rate of the simulated epidemics. However, both methods have limitations and show marked differences in the estimated values (31, 32). Indeed, R_0^{index} is computed from the analysis of the index case of an outbreak, which is not necessarily representative of a “typical” infectious individual. In addition to this, growing evidence shows that the classic exponential early growth of the epidemic is an oversimplification often not backed by real-world data (35, 36). For this reason, statistical methods were developed for the analysis of $R(t)$, assuming that the variations of this quantity are mostly due to the impact of the performed intervention strategies and behavioral changes in the population (10, 16, 17, 37). Unfortunately, for these methods, disentangling the role of

Significance

The analysis of real epidemiological data has raised issues of the adequacy of the classic homogeneous modeling framework and quantities, such as the basic reproduction number in real-world situations. Based on high-quality sociodemographic data, here we generate a multiplex network describing the contact pattern of the Italian and Dutch populations. By using a microsimulation approach, we show that, for epidemics spreading on realistic contact networks, it is not possible to define a steady exponential growth phase and a basic reproduction number. We show the operational use of the instantaneous reproduction rate as a good descriptor of the transmission dynamics.

Author contributions: Q.-H.L., M.A., A.A., S.M., Y.M., and A.V. designed research, performed research, analyzed data, and wrote the paper.

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A closer look at the transmission process in the different layers of the multiplex network helps in understanding the origin

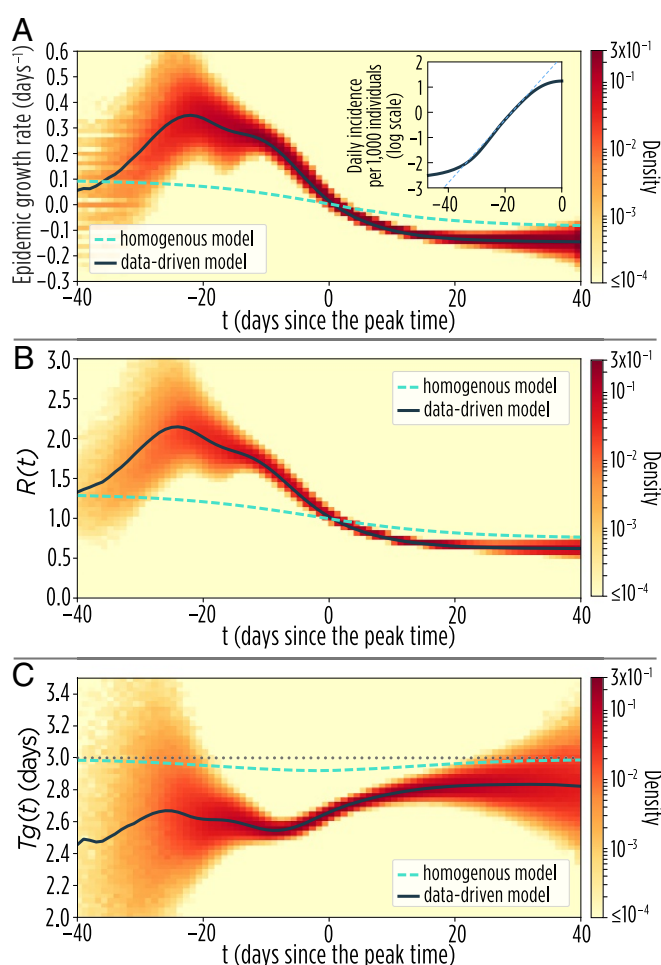


Fig. 2. Fundamental epidemiological indicators. (A) Mean daily exponential epidemic growth rate, r , over time of the data-driven and homogeneous models. The colored area shows the density distribution of $r(t)$ values obtained in the single realizations of the data-driven model. Results are based on 50,000 realizations of each model. Results are aligned at the epidemic peak, which corresponds to time $t = 0$. *Inset* shows the logarithm of the mean daily incidence of new influenza infections over time, which does not follow a linear trend. (B) Mean $R(t)$ of data-driven and homogeneous models. The colored area shows the density distribution of $R(t)$ values obtained in the single realizations of the data-driven model. (C) The three lines represent the mean $Tg(t)$ of data-driven and homogeneous models. The colored area shows the density distribution of $Tg(t)$ values obtained in the single realizations of the data-driven model. The horizontal dotted gray line represent the constant value of the duration of the infectious period.

of the deviations of $R(t)$ and $Tg(t)$ from the classical theory (Fig. 3). Specifically, we found that the average degree of infectious nodes as well as $R(t)$ tend to peak in the workplace layer (*SI Appendix, section 2.2*), and at least to some extent, the same happens in the school layer. However, $R(t)$ generally decreases in the household and community layers (Fig. 3A). Indeed, $R(t)$ in the household layer tends to be more uniform across the different nodes and thus, follows a general decreasing trend simply led by the depletion of susceptible contacts. We also found that, in the household layer, Tg is remarkably shorter than in all other layers (Fig. 3B)—with an average fluctuating around 2.6 d, close to the value reported by analyzing real data for household transmission (26). To provide a simple illustration of the saturation effect in households, let us consider a household of three, with one index case and two susceptible members. If, at time t , the index case infects exactly one of the two susceptibles,

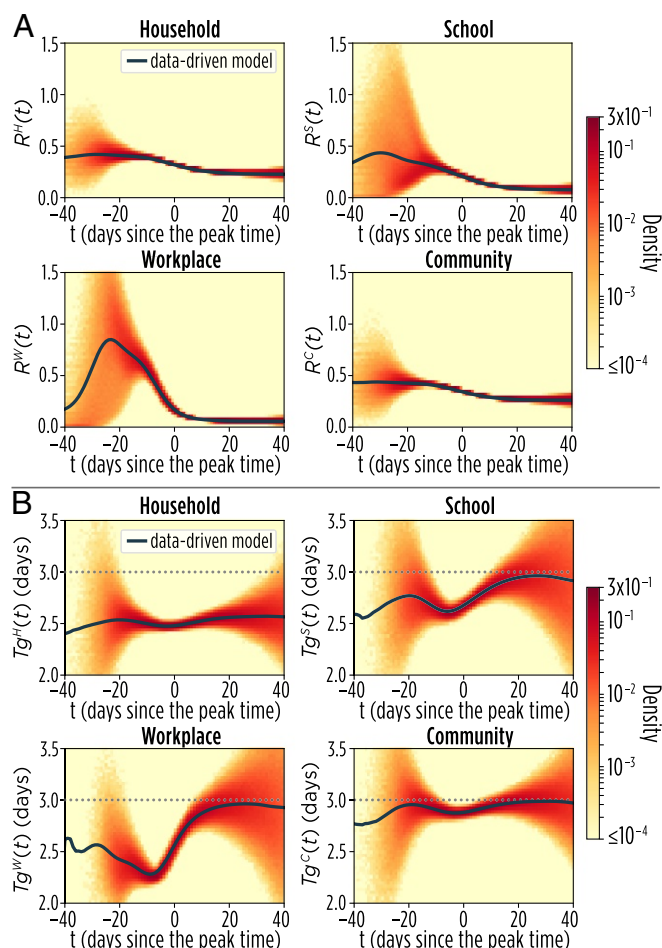


Fig. 3. Layer-specific patterns. (A) Mean $R(t)$ for the data-driven model in the four layers. The colored area shows the density distribution of $R(t)$ values obtained in the single realizations. (B) Mean $Tg(t)$ for the data-driven model in the four layers. The colored area shows the density distribution of $Tg(t)$ values obtained in the single realizations.

then at time $t + 1$, the index case has to compete with the other infectious individual to transmit the infection; the resulting generation time is shorter, simply because she/he cannot infect any other household member, although she/he may still be infectious. A similar argument was used in ref. 52 to explain why observed generation times are shorter than the infectious period. This evidence calls for considering within household competition effects when providing empirical estimates of the generation time (or serial interval) from household studies. Saturation effects are also responsible for shortening Tg in the other transmission settings, with the exception of the general community (Fig. 3B). All of the observed patterns of $R(t)$ and $Tg(t)$ are robust with respect to changes in the sociodemographic structure of the population (i.e., we simulated the infection spreading in both the Italian and Dutch synthetic populations), influenza transmission intensity (measured in terms of R^{index}), and the distribution of the removal time (we tested exponential and gamma distributions). These analyses are reported in *SI Appendix*, sections 2.4–2.6.

The 2009 H1N1 Influenza Pandemic in Italy. To test the robustness of the results in a more realistic epidemic transmission model, we used the data-driven modeling framework to model the 2009 influenza pandemic in Italy. One of the characteristic signatures of the 2009 H1N1 pandemic was the presence of a differential susceptibility by age (34, 53, 54); this is included in the model

by using values estimated for Italy as reported in the literature (55). We also consider prepandemic immunity by age in the population according to serological data (55). Vaccination is not considered, as vaccination started only during the tail of the pandemic and had a very limited uptake in Italy (vaccination coverage $< 1\%$) (56). As in the previous section, the model has four unknown parameters: the four layer-specific transmission rates. They are calibrated through a Bayesian Markov chain Monte Carlo (MCMC) approach on seroprevalence data by age collected in Italy before and after the 2009 H1N1 influenza pandemic (55). Model details are provided in *SI Appendix*, section 1.1.

The calibrated model is able to well capture the seropositive rates by age at the end of the pandemic (Fig. 4A). The estimated growth rate from the influenza-like illness (ILI) cases reported in Italy over the course of the 2009 H1N1 influenza pandemic clearly shows an increasing trend during the early phase of the epidemic followed by a sharp drop about 3 weeks before the epidemic peak (Fig. 4B). The trend observed in the data is consistent with that obtained in model simulations (Fig. 4C). Fitting a

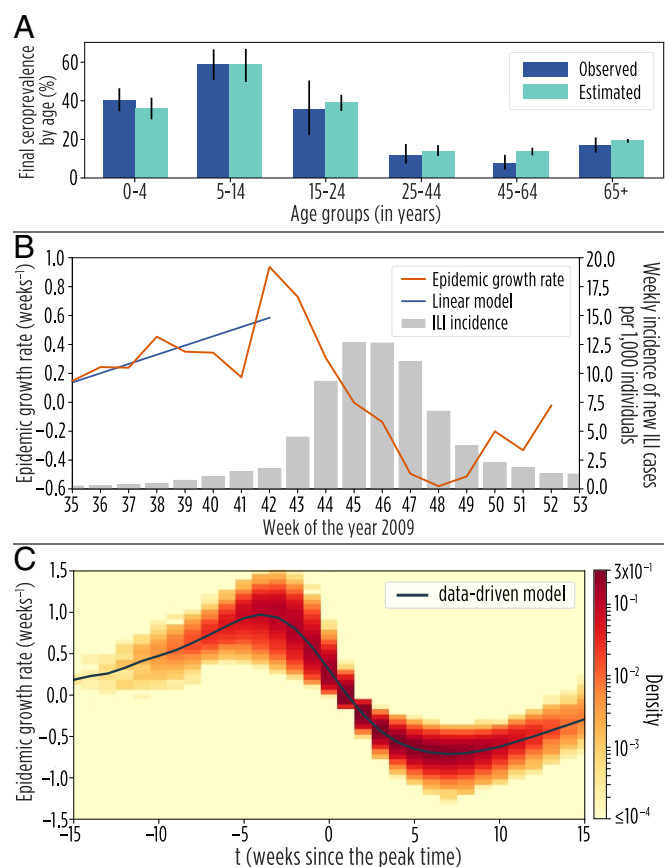


Fig. 4. The 2009 H1N1 influenza pandemic in Italy. (A) Seroprevalence rates by age as observed in a serosurvey conducted at the end of the 2009 H1N1 influenza pandemic in Italy (55) and as estimated by the calibrated model. (B) Epidemic growth rate over time $r(t)$ as estimated from the weekly incidence of new ILI cases in Italy over the course of the 2009 H1N1 influenza pandemic and the best-fitting linear model from week 35 to week 41 in 2009 (scale on the left axis). Weekly incidence of new ILI cases in Italy over the course of the 2009 H1N1 pandemic (scale on the right axis). Data are available at the ISS Influnet website (old.iss.it/flue/). Note that, over the period from week 35 to week 51 in 2009, schools were regularly open in Italy. (C) Temporal pattern of the mean weekly exponential epidemic growth rate (r) resulting from the analysis of the data-driven model calibrated on the 2009 H1N1 influenza seroprevalence data. The colored area shows the density distribution of $r(t)$ values obtained in the single realizations of the data-driven model.

$$L = \prod_{t=1}^T P \left(C(t), R(t) \sum_{s=1}^t \phi(s) C(t-s) \right),$$

where here, $P(k, \lambda)$ is the probability mass function of a Poisson distribution (i.e., the probability of observing k events if these events occur with a known rate λ). The posterior distribution of $R(t)$ is then explored using MCMC sampling.

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