

## Generic 2 model

### Parametrization

The generic2 model implements the following precision matrix

$$\mathbf{Q} = \begin{bmatrix} \tau_u \mathbf{I} & -\tau_u \mathbf{I} \\ -\tau_u \mathbf{I} & \tau_u \mathbf{I} + \tau_v \mathbf{C} \end{bmatrix} \quad (1)$$

where  $\mathbf{C}$  is (a given) symmetric matrix. This model arrives from the hierarchical model,

$$\mathbf{v} \sim \mathcal{N}(\mathbf{0}, \tau_v \mathbf{C})$$

and

$$\mathbf{u} \mid \mathbf{v} \sim \mathcal{N}(\mathbf{v}, \tau_u \mathbf{I})$$

and the precision matrix in Eq. (1) implements the joint precision matrix of

$$\begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix}$$

using the following hyperparameters

$$\tau_v \quad \text{and} \quad h^2 = \frac{1/\tau_v}{1/\tau_v + 1/\tau_u}.$$

### Hyperparameters

The two parameters in the `generic2` model are represented as

$$\begin{aligned} \theta_1 &= \log(\tau_v) \\ \theta_2 &= \log(\tau_u) \end{aligned}$$

and priors are assigned to  $(\theta_1, \theta_2)$ .

**YES THIS IS CORRECT!** This is because the prior specification would then be similar to another specification, but allows one to compute the posterior marginal for  $h^2$  more easily.

### Specification

The `generic2model` is specified inside the `f()` function as

```
f(<whatever>, model="generic2", Cmatrix = <Cmat>, hyper = <hyper>)
```

where `<Cmat>` can be given in two different ways:

- a dense matrix or a sparse-matrix defined by `Matrix::sparseMatrix()`.
- the name of a file giving the structure matrix. The file should have the following format

$$i \quad j \quad \mathbf{C}_{ij}$$

where  $i$  and  $j$  are the row and column index and  $\mathbf{C}_{ij}$  is the corresponding element of the precision matrix. Only the non-zero elements of the precision matrix need to be stored in the file.

## Hyperparameter spesification and default values

hyper

theta1

```
name log precision cmatrix
short.name prec
initial 4
fixed FALSE
prior loggamma
param 1 5e-05
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

theta2

```
name log precision random
short.name prec.random
initial 4
fixed FALSE
prior loggamma
param 0 0.001
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

constr FALSE

nrow.ncol FALSE

augmented FALSE

aug.factor 2

aug.constr 2

n.div.by

n.required TRUE

set.default.values TRUE

## Example

```
require(mvtnorm)
n = 200
Cm = matrix(runif(n^2,min=-1,max=1),n,n)
Cm = Cm %*% t(Cm)
Sigma = solve(Cm)

sd = 0.001
z = rnorm(n,sd=sd)
eta = rmvnorm(n=1,sigma = Sigma)
s = 0.1
y = c(eta) + rnorm(n,sd=s) + z
idx = 1:n
```

```

##
## Alternative 1
##
file = "Cmatrix.dat"
cat("",file=file, append = FALSE)
for(i in 1:n)
{
  j = i
  cat(i,j,Cm[i,j], "\n", sep = " ", file=file, append=TRUE)
  if (i < n)
    for(j in (i+1):n)
      cat(i, j, Cm[i,j], "\n", sep = " ", file=file, append=TRUE)
}
formula = y ~ f(idx, model = "generic2", Cmatrix = file,
              initial=c(0,0), fixed=c(F,F))

## Alternative 2
## formula = y ~ f(idx, model = "generic2", Cmatrix = Cm,
##              initial=c(0,0), fixed=c(F,F))

## Alternative 3
## Cm.sparse = as(Cm, "dgTMatrix")
## formula = y ~ f(idx, model = "generic2", Cmatrix = Cm.sparse,
##              initial=c(0,0), fixed=c(F,F))

#####

result = inla(formula, data=data.frame(y,idx),
              control.data = list(initial = log(1/sd^2), fixed=TRUE),
              verbose = TRUE)

## tau.u should be about  $1/s^2 = 100$ . increase 'n' above to get
## it...
tau.u = result$summary.hyperpar["Precision-cmatrix for idx", "mean"]
h2 = result$summary.hyperpar["h2 for idx", "mean"]
tau.v = h2/(1-h2)*tau.u
print(paste("tau.v", tau.v, "should be (for large n)",  $1/s^2$ ))

```

## Notes

The option `constr=TRUE` will impose a sum-to-zero constraint on  $v$  only.