

Model for seasonal variation

Parametrization

A model for seasonal variation with periodicity m for the random vector $\mathbf{x} = (x_1, \dots, x_n)$, $n > m$ is obtained assuming that the sums $x_i + x_{i+1} + \dots + x_{i+m-1}$ are independent Gaussian with precision τ .

The density for \mathbf{x} is derived from the $n - m + 1$ increments as

$$\pi(\mathbf{x}|\tau) \propto \tau^{\frac{(n-m+1)}{2}} \exp \left\{ -\frac{\tau}{2} \sum (x_i + x_{i+1} + \dots + x_{i+m-1})^2 \right\} \quad (1)$$

$$= \tau^{\frac{(n-m+1)}{2}} \exp \left\{ -\frac{1}{2} \mathbf{x}^T \mathbf{Q} \mathbf{x} \right\} \quad (2)$$

where $\mathbf{Q} = \tau \mathbf{R}$ and \mathbf{R} is the structure matrix reflecting the neighbourhood structure of the model.

Hyperparameters

The precision parameter τ is represented as

$$\theta = \log \tau$$

and the prior is defined on θ .

Specification

The seasonal model is specified inside the `f()` function as

```
f(<whatever>, model="seasonal", season.length=<season.length>,  
    prior=c(<prior.model.theta>),  
    param=c(<param.prior.theta1>))
```

Example

```
n=204  
n.seas=12  
  
trend=seq(1:n)  
seasonal=rep(1:n.seas,n/n.seas)  
  
a=1  
b=0.5  
y = rnorm(n,a+b*trend,1)+rnorm(n,0.2*seasonal,1)  
  
data=data.frame(y=y,trend=trend,seasonal=seasonal)  
  
formula = y~f(trend,model="rw2")+f(seasonal,model="seasonal",season.length=n.seas,param=c(1,0.2))  
result=inla(formula,family="gaussian",data=data)
```

Notes

The seasonal is a intrinsic random field with rank deficiency of $m - 1$.