

# Besag model for spatial effects

## Parametrization

The besag model for random vector  $\mathbf{x} = (x_1, \dots, x_n)$  is defined as

$$x_i | x_j, i \neq j, \tau \sim \mathcal{N}\left(\frac{1}{n_i} \sum_{i \sim j} x_j, \frac{1}{n_i \tau}\right) \quad (1)$$

where  $n_i$  is the number of neighbours of node  $i$ ,  $i \sim j$  indicates that the two nodes  $i$  and  $j$  are neighbours.

## Hyperparameters

The precision parameter  $\tau$  is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on  $\theta_1$ .

## Specification

The besag model is specified inside the `f()` function as

```
f(<whatever>, model="besag", graph.file=<graph file name>, hyper=<hyper>)
```

The neighbourhood structure of  $\mathbf{x}$  is passed to the program through the `graph.file` argument. The structure of this file is described below.

## Hyperparameter specification and defaults

**hyper**

**theta**

```
name precision
short.name prec
initial 4
fixed FALSE
prior loggamma
param c(1, 1e-04)
```

**constr** TRUE

**nrow.ncol** FALSE

**augmented** FALSE

**aug.factor** 1

**aug.constr** NULL

**n.div.by** NULL

**n.required** TRUE

**set.default.values** TRUE

## Structure of the graph file

We describe the required format for the graph file using a small example. Let the file `gra.dat`, relative to a small graph of only 5 elements, be

```
5
1 1 2
2 2 1 3
3 3 2 4 5
4 1 3
5 1 3
```

Line 1 declares the total number of nodes in the graph (5), then, in lines 2-6 each node is described. For example, line 4 states that node 3 has 4 neighbours and these are nodes 2, 4 and 5.

The graph file can either have nodes indexed from 1 to  $n$ , or from 0 to  $n - 1$ . Note that in the latter case, node  $i$  seen from R corresponds to node  $i - 1$  in the 0-indexed graph.

## Example

For examples of application of this model see the Bym, Munich, Zambia or Scotland examples in Volume I.

## Notes

The besag model intrinsic with rankdef 1.

The model is modified accordingly is the graph has more than one connected components.