

# The Beta-distribution

## Parametrisation

The Beta-distribution has the following density

$$\pi(y) = \frac{1}{B(a, b)} y^{a-1} (1-y)^{b-1}, \quad 0 < y < 1, \quad a > 0, \quad b > 0$$

where  $B(a, b)$  is the Beta-function

$$B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

and  $\Gamma(x)$  is the Gamma-function. The (re-)parameterisation used is

$$\mu = \frac{a}{a+b}, \quad 0 < \mu < 1$$

and

$$\phi = a+b, \quad \phi > 0,$$

as it makes

$$E(y) = \mu \quad \text{and} \quad \text{Var}(y) = \frac{\mu(1-\mu)}{1+\phi}.$$

The parameter  $\phi$  is known as the *precision parameter*, since for fixed  $\mu$ , the larger  $\phi$  the smaller the variance of  $y$ . The parameters  $\{a, b\}$  are given as  $\{\mu, \phi\}$  as follows,

$$a = \mu\phi \quad \text{and} \quad b = -\mu\phi + \phi.$$

## Link-function

The linear predictor  $\eta$  is linked to the mean  $\mu$  using a default logit-link

$$\mu = \frac{\exp(\eta)}{1 + \exp(\eta)}.$$

## Hyperparameter

The hyperparameter is the precision parameter  $\phi$ , which is represented as

$$\phi = \exp(\theta)$$

and the prior is defined on  $\theta$ .

## Specification

- family = **beta**
- Required arguments:  $y$ .

## Hyperparameter specification and default values

**hyper**

**theta**

**name** precision parameter

**short.name** phi

```

    initial 2.30258509299405
    fixed FALSE
    prior loggamma
    param 1 0.1
    to.theta
    from.theta

survival FALSE

discrete FALSE

link default logit probit cloglog

pdf beta

```

## Example

In the following example we estimate the parameters in a simulated example.

```

## the precision parameter in the beta distribution
phi = 5

## generate simulated data
n = 1000
z = rnorm(n, sd=0.2)
eta = 1 + z
mu = exp(eta)/(1+exp(eta))
a = mu * phi
b = -mu * phi + phi
y = rbeta(n, a, b)

## estimate the model
formula = y ~ 1 + z
r = inla(formula, data = data.frame(y, z), family = "beta")
summary(r)

```

## Notes

None.