

# Proper Besag model for spatial effects

## Parametrization

The proper version of the Besag model for random vector  $\mathbf{x} = (x_1, \dots, x_n)$  is defined as

$$x_i | x_{-i}, \tau, \phi \sim \mathcal{N} \left( \frac{\phi}{1 + \phi n_i} \sum_{i \sim j} x_j, \frac{1}{(1 + \phi n_i) \tau} \right) \quad (1)$$

where  $n_i$  is the number of neighbours of node  $i$ ,  $i \sim j$  indicates that the two nodes  $i$  and  $j$  are neighbours,  $\phi > 0$  is as weight parameter and  $\tau > 0$  is a “precision-like” (or scaling) parameter.

## Hyperparameters

The precision parameter  $\tau$  is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on  $\theta_1$ .

The weight parameter  $\phi$  is represented as

$$\theta_2 = \log \phi$$

and the prior is defined on  $\theta_2$ .

## Specification

The besag model is specified inside the `f()` function as

```
f(<whatever>, model="besagproper", graph.file=<graph file name>,  
  hyper=<hyper>)
```

The neighbourhood structure of  $\mathbf{x}$  is passed to the program through the `graph.file` argument. The structure of this file is described below.

## Hyperparameter spesification and default values

### hyper

#### theta1

```
name  log precision  
short.name  prec  
prior  loggamma  
param  1 5e-04  
initial  2  
fixed  FALSE  
to.theta  function(x) log(x)  
from.theta  function(x) exp(x)
```

#### theta2

```
name  log weight  
short.name  weight  
prior  loggamma
```

```

    param 1 0.1
    initial 2.30258509299405
    fixed FALSE
    to.theta function(x) log(x)
    from.theta function(x) exp(x)

constr FALSE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required TRUE

set.default.values TRUE

pdf besagproper

```

### Structure of the graph file

We describe the required format for the graph file using a small example. Let the file `gra.dat`, relative to a small graph of only 5 elements, be

```

5
1 1 2
2 2 1 3
3 3 2 4 5
4 1 3
5 1 3

```

Line 1 declares the total number of nodes in the graph (5), then, in lines 2-6 each node is described. For example, line 4 states that node 3 has 4 neighbours and these are nodes 2, 4 and 5.

The graph file can either have nodes indexed from 1 to  $n$ , or from 0 to  $n - 1$ . Note that in the latter case, node  $i$  seen from R corresponds to node  $i - 1$  in the 0-indexed graph.

### Example

To be added

### Notes

Add notes later ?