Correlated random effects: `iid1d`, `iid2d`, `iid3d`

This model is available for dimensions $p = 1, 2, 3, 4$ and 5. We describe in detail the case for $p = 2$, and then the changes required for $p = 1$, $p = 3$, $p = 4$ and $p = 5$

Parametrization

The 2-dimensional Normal-Wishard model is used if one want to define two vectors of "random effects", u and v, say, for which (u_i, v_i) are iid bivariate Normals

$$\begin{pmatrix} u_i \\ v_i \end{pmatrix} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{W}^{-1}\right)$$

where the covariance matrix $\mathbf{W}^{-1}$ is

$$\mathbf{W}^{-1} = \begin{pmatrix} 1/\tau_a & \rho\sqrt{\tau_a \tau_b} \\ \rho\sqrt{\tau_a \tau_b} & 1/\tau_b \end{pmatrix} \tag{1}$$

and τ_a, τ_b and ρ are the hyperparameters. For these models the precision matrix $\mathbf{W}$ is Wishart distributed

$$\mathbf{W} \;\sim\; \text{Wishart}_p(r, \mathbf{R}^{-1}), \quad p = 2$$

with density

$$\pi(\mathbf{W}) = c^{-1}|\mathbf{W}|^{(r-(p+1))/2} \exp\left\{-\frac{1}{2}\text{Trace}(\mathbf{W}\mathbf{R})\right\}, \quad r > p+1$$

and

$$c = 2^{(rp)/2}|\mathbf{R}|^{-r/2}\pi^{(p(p-1))/4}\prod_{j=1}^{p}\Gamma((r+1-j)/2).$$

Then,

$$\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r-(p+1)).$$

Hyperparameters

The hyperparameters are

$$\theta = (\log \tau_a, \log \tau_b, \tilde{\rho})$$

where

$$\rho = 2\frac{\exp(\tilde{\rho})}{\exp(\tilde{\rho})+1} - 1$$

The prior-parameters are

$$(r, R_{11}, R_{22}, R_{12})$$

where

$$\mathbf{R} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

and $r_{12} = R_{21}$ due to symmetry.

The `inla` function reports posterior distribution for the hyperparameters τ_a, τ_b, ρ in equation (1).

The prior for θ is **fixed** to be `wishart2d`

Specification

The model `iid2d` is specified as

```
y ~ f(i, model="iid2d",n = <length>) + ...
```

and the `iid2d` model is represented internally as one vector of length n,

$$(u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_m)$$

where $n = 2m$, and n is the (required) argument in `f()`.

For this model the argument `constr=TRUE` is interpreted as

$$\sum u_i = 0, \qquad \text{and} \qquad \sum v_i = 0.$$

Hyperparameter spesification and default values

hyper

- **theta1**
  - **name** log precision1
  - **short.name** prec1
  - **initial** 4
  - **fixed** FALSE
  - **prior** wishart2d
  - **param** 4 1 1 0
  - **to.theta** `function(x) log(x)`
  - **from.theta** `function(x) exp(x)`
- **theta2**
  - **name** log precision2
  - **short.name** prec2
  - **initial** 4
  - **fixed** FALSE
  - **prior** none
  - **param**
  - **to.theta** `function(x) log(x)`
  - **from.theta** `function(x) exp(x)`
- **theta3**
  - **name** logit correlation
  - **short.name** cor
  - **initial** 4
  - **fixed** FALSE
  - **prior** none
  - **param**
  - **to.theta** `function(x) log((1+x)/(1-x))`
  - **from.theta** `function(x) 2*exp(x)/(1+exp(x))-1`

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 1

aug.constr 1 2

n.div.by 2

n.required TRUE

set.default.values TRUE

pdf iid123d

Example

In this example we implement the model

$$y|\eta \sim \text{Pois}(\exp(\eta))$$

where

$$\eta = a + b + 1$$

and a and b are correlated as described above.

```
n = 1000
N = 2*n
rho = 0.5
## set variances
Sigma = matrix(c(1/1, NA, NA, 1/2), 2, 2)
## and the correlation
Sigma[1,2] = Sigma[2,1] = rho*sqrt(Sigma[1,1]*Sigma[2,2])

## need it to simulate data
library(mvtnorm)

if (TRUE)
{
    ## first example

    y = yy = rmvnorm(n, sigma=Sigma)
    y = c(y[,1], y[,2])

    i = 1:N
    formula = y ~ f(i, model="iid2d", n=N)

    r = inla(formula, data = data.frame(i,y),
            control.data=list(initial=10,fixed=TRUE))
    print(summary(r))
```

```
    print(1/diag(cov(yy)))
    print(cor(yy)[1,2])
}

if (TRUE)
{
    ## second example

    y = yy = rmvnorm(n, sigma=Sigma)
    z = rnorm(n)
    zz = rnorm(n)
    y = y[,1] + z*y[,2] + zz
    i = 1:n
    j = n + 1:n
    formula = y ~ f(i, model="iid2d", n=N) + f(j,z,copy="i") + zz

    r = inla(formula, data = data.frame(i,j,y,z,zz),
            control.data=list(initial=10,fixed=TRUE),keep=T)
    print(summary(r))
    print(1/diag(cov(yy)))
    print(cor(yy)[1,2])
}
```

The case $p = 1$

For $p = 1$ the hyperparameter is the marginal precision

$$\theta = \log \tau_1$$

The prior is fixed to be `wishart1d` with parameters

$$param = \; r \; R_{11}$$

where

$$\mathbf{R} = \left[\; R_{11} \; \right]$$

Hyperparameter spesification and default values

hyper

theta

- **name** precision
- **short.name** prec
- **initial** 4
- **fixed** FALSE
- **prior** wishart1d
- **param** 2 1e-04
- **to.theta** `function(x) log(x)`
- **from.theta** `function(x) exp(x)`

constr FALSE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values TRUE

pdf iid123d

The case $p = 3$

For $p = 3$ the hyperparameters are

$$\theta = (\log \tau_1, \log \tau_2, \log \tau_3, \tilde{\rho}_{12}, \tilde{\rho}_{13}, \tilde{\rho}_{23})$$

The prior is fixed to be `wishart3d` with parameters

$$param = \; r \; R_{11} \; R_{22} \; R_{33} \; R_{12} \; R_{13} \; R_{23}$$

where

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{12} & R_{22} & R_{23} \\ R_{13} & R_{23} & R_{33} \end{bmatrix}$$

The reported hyperparameters are the marginal precisions τ_1, τ_2 and τ_3 and the correlations ρ_{12}, ρ_{13} and ρ_{23}.

In this case, the internal representation is given as

$$(u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_m, w_1, w_2, \ldots, w_m)$$

where $n = 3m$ is a required argument, and where (u_i, v_i, w_i) are trivariate iid Normal.

Hyperparameter spesification and default values

hyper

theta1

name log precision1

short.name prec1

initial 4

fixed FALSE

prior wishart3d

param 7 1 1 1 0 0 0

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

theta2

- **name** log precision2
- **short.name** prec2
- **initial** 4
- **fixed** FALSE
- **prior** none
- **param**
- **to.theta** `function(x) log(x)`
- **from.theta** `function(x) exp(x)`

theta3

- **name** log precision3
- **short.name** prec3
- **initial** 4
- **fixed** FALSE
- **prior** none
- **param**
- **to.theta** `function(x) log(x)`
- **from.theta** `function(x) exp(x)`

theta4

- **name** logit correlation12
- **short.name** cor12
- **initial** 0
- **fixed** FALSE
- **prior** none
- **param**
- **to.theta** `function(x) log((1+x)/(1-x))`
- **from.theta** `function(x) 2*exp(x)/(1+exp(x))-1`

theta5

- **name** logit correlation13
- **short.name** cor13
- **initial** 0
- **fixed** FALSE
- **prior** none
- **param**
- **to.theta** `function(x) log((1+x)/(1-x))`
- **from.theta** `function(x) 2*exp(x)/(1+exp(x))-1`

theta6

- **name** logit correlation23
- **short.name** cor23
- **initial** 0
- **fixed** FALSE
- **prior** none
- **param**

to.theta `function(x) log((1+x)/(1-x))`
from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 1

aug.constr 1 2 3

n.div.by 3

n.required TRUE

set.default.values TRUE

pdf iid123d

The case $p = 4$

For $p = 4$ the hyperparameters are

$$\theta = (\log \tau_1, \log \tau_2, \log \tau_3, \log \tau_4, \tilde{\rho}_{12}, \tilde{\rho}_{13}, \tilde{\rho}_{14}, \tilde{\rho}_{23}, \tilde{\rho}_{24}, \tilde{\rho}_{34})$$

The prior is fixed to be `wishart4d` with parameters

$$param = \; r \; R_{11} \; R_{22} \; R_{33} \; R_{44} \; R_{12} \; R_{13} \; R_{14} \; R_{23} \; R_{24} \; R_{34}$$

where

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{12} & R_{22} & R_{23} & R_{24} \\ R_{13} & R_{23} & R_{33} & R_{34} \\ R_{14} & R_{24} & R_{34} & R_{44} \end{bmatrix}$$

The reported hyperparameters are the marginal precisions τ_1, τ_2, τ_3 and τ_4, and the correlations ρ_{12}, ρ_{13}, ρ_{14}, ρ_{23}, ρ_{24} and ρ_{34}.

In this case, the internal representation is given as

$$(u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_m, w_1, w_2, \ldots, w_m, x_1, x_2, \ldots, x_m)$$

where $n = 4m$ is a required argument, and where (u_i, v_i, w_i, x_i) are fourvariate iid Normal.

Hyperparameter spesification and default values

hyper

theta1

name log precision1
short.name prec1
initial 4
fixed FALSE

prior wishart4d

param 11 1 1 1 1 0 0 0 0 0 0

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

theta2

name log precision2

short.name prec2

initial 4

fixed FALSE

prior none

param

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

theta3

name log precision3

short.name prec3

initial 4

fixed FALSE

prior none

param

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

theta4

name log precision4

short.name prec4

initial 4

fixed FALSE

prior none

param

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

theta5

name logit correlation12

short.name cor12

initial 0

fixed FALSE

prior none

param

to.theta `function(x) log((1+x)/(1-x))`

from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

theta6

name logit correlation13

short.name cor13

initial 0

fixed FALSE
prior none
param
to.theta `function(x) log((1+x)/(1-x))`
from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

theta7

name logit correlation14
short.name cor14
initial 0
fixed FALSE
prior none
param
to.theta `function(x) log((1+x)/(1-x))`
from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

theta8

name logit correlation23
short.name cor23
initial 0
fixed FALSE
prior none
param
to.theta `function(x) log((1+x)/(1-x))`
from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

theta9

name logit correlation24
short.name cor24
initial 0
fixed FALSE
prior none
param
to.theta `function(x) log((1+x)/(1-x))`
from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

theta10

name logit correlation34
short.name cor34
initial 0
fixed FALSE
prior none
param
to.theta `function(x) log((1+x)/(1-x))`
from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 1

aug.constr 1 2 3 4

n.div.by 4

n.required TRUE

set.default.values TRUE

pdf iid123d

The case $p = 5$

The case $p = 5$ follows by a direct extention of $p = 3$ and $p = 4$, and is therefore not included.

Hyperparameter spesification and default values

hyper

- **theta1**
  - **name** log precision1
  - **short.name** prec1
  - **initial** 4
  - **fixed** FALSE
  - **prior** wishart5d
  - **param** 16 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0
  - **to.theta** `function(x) log(x)`
  - **from.theta** `function(x) exp(x)`
- **theta2**
  - **name** log precision2
  - **short.name** prec2
  - **initial** 4
  - **fixed** FALSE
  - **prior** none
  - **param**
  - **to.theta** `function(x) log(x)`
  - **from.theta** `function(x) exp(x)`
- **theta3**
  - **name** log precision3
  - **short.name** prec3
  - **initial** 4
  - **fixed** FALSE
  - **prior** none
  - **param**
  - **to.theta** `function(x) log(x)`

from.theta `function(x) exp(x)`

theta4

- **name** log precision4
- **short.name** prec4
- **initial** 4
- **fixed** FALSE
- **prior** none
- **param**
- **to.theta** `function(x) log(x)`
- **from.theta** `function(x) exp(x)`

theta5

- **name** log precision5
- **short.name** prec5
- **initial** 4
- **fixed** FALSE
- **prior** none
- **param**
- **to.theta** `function(x) log(x)`
- **from.theta** `function(x) exp(x)`

theta6

- **name** logit correlation12
- **short.name** cor12
- **initial** 0
- **fixed** FALSE
- **prior** none
- **param**
- **to.theta** `function(x) log((1+x)/(1-x))`
- **from.theta** `function(x) 2*exp(x)/(1+exp(x))-1`

theta7

- **name** logit correlation13
- **short.name** cor13
- **initial** 0
- **fixed** FALSE
- **prior** none
- **param**
- **to.theta** `function(x) log((1+x)/(1-x))`
- **from.theta** `function(x) 2*exp(x)/(1+exp(x))-1`

theta8

- **name** logit correlation14
- **short.name** cor14
- **initial** 0
- **fixed** FALSE
- **prior** none
- **param**

to.theta `function(x) log((1+x)/(1-x))`

from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

theta9

name logit correlation15

short.name cor15

initial 0

fixed FALSE

prior none

param

to.theta `function(x) log((1+x)/(1-x))`

from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

theta10

name logit correlation23

short.name cor23

initial 0

fixed FALSE

prior none

param

to.theta `function(x) log((1+x)/(1-x))`

from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

theta11

name logit correlation24

short.name cor24

initial 0

fixed FALSE

prior none

param

to.theta `function(x) log((1+x)/(1-x))`

from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

theta12

name logit correlation25

short.name cor25

initial 0

fixed FALSE

prior none

param

to.theta `function(x) log((1+x)/(1-x))`

from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

theta13

name logit correlation34

short.name cor34

initial 0

fixed FALSE

prior none

param

to.theta `function(x) log((1+x)/(1-x))`

from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

theta14

name logit correlation35

short.name cor35

initial 0

fixed FALSE

prior none

param

to.theta `function(x) log((1+x)/(1-x))`

from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

theta15

name logit correlation45

short.name cor45

initial 0

fixed FALSE

prior none

param

to.theta `function(x) log((1+x)/(1-x))`

from.theta `function(x) 2*exp(x)/(1+exp(x))-1`

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 1

aug.constr 1 2 3 4 5

n.div.by 5

n.required TRUE

set.default.values TRUE

pdf iid123d

Notes

The model `iid1d` is similar to the model `iid` (and included for completeness only). The prior for `iid1d` is fixed to be Wishart-distributed, which reduces to a Gamma-distribution for the precision with parameters

$$a = r/2 \qquad \text{and} \qquad b = R_{11}/2$$

hence

```
y ~ f(i, model="iid1d", hyper = list(theta=list(param=c(3, 4))))
```

is equivalent to

```
y ~ f(i, model="iid", hyper = list(theta=list(param=c(1.5, 2), prior="loggamma")))
```