Salamander mating data: Model B

We consider the salamander mating experiments reported and analysed by **?**, Section 14.5. Data consist of three separate experiments. The first one, conducted in the summer of 1986, used two groups of 20 salamanders. Each group comprised five male Rough Butt, five male Whiteside, five female Rough Butt and five female Whiteside salamanders. Within each group, each animal was paired to three animals of the opposite sex from the same and from the other population. Therefore, 60 male-female pairs were formed within one group leading to 120 observations in one experiment. Two further experiments with equal design were conducted in the fall of 1987. The animals used for the first and the second experiment were identical. A new set of salamanders was utilised for the third experiment.
The main scientific question addressed in the study was whether the mating of both geographically isolated species of salamanders was as successful as the one between the animals from the same population. Moreover, there was some interest if a seasonal effect could be identified. Therefore, two factors `wsf` (Whiteside female "yes": 1, "no": 0) and `wsm` (Whiteside male "yes": 1, "no": 0) together with their interaction `ww` and a seasonal effect `fall` (experiment conducted in fall "yes": 1, "no": 0) were coded.

Modelling

Let Y_{ijk} denote the binary outcome (0 = failure, 1 = success) of the mating for female $i = 1, \ldots, 20$ and male $j = 1, \ldots, 20$ in experiment $k = 1, \ldots, 3$. Thus

$$Y_{ijk}|\pi_{ijk} \sim \text{Bin}(1, \pi_{ijk}) \quad \text{with} \quad \text{logit}(\pi_{ijk}) = \boldsymbol{x}_{ijk}^\top \boldsymbol{\beta} + b_{ik}^F + b_{jk}^M$$

where x_{ijk} is a vector comprising an intercept, $\texttt{wsf}_{ik}$, $\texttt{wsm}_{jk}$, $\texttt{ww}_{ijk}$ and $\texttt{fall}_{ijk}$ variables, $\boldsymbol{\beta}$ is the corresponding vector of the fixed effects parameters, and b_{ik}^F and b_{jk}^M are female and male random effects respectively.

Prior distributions

We assume that $b_{i3}^F \sim \mathcal{N}(0, \kappa^F)$, $b_{i3}^M \sim \mathcal{N}(0, \kappa^M)$ and

$$\begin{pmatrix} b_{i1}^F \\ b_{i2}^F \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{W}_F^{-1}) \qquad \begin{pmatrix} b_{i1}^M \\ b_{i2}^M \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{W}_M^{-1})$$

where the covariance matrices $\mathbf{W}_F^{-1}$ and $\mathbf{W}_M^{-1}$ are parameterised in terms of precisions and correlations leading to a total of eight hyperparameters in the model.

Hyper-prior distributions

We assume that both κ^F and κ^M follow a gamma prior distribution with shape equal to 1 and rate equal to 0.622. The precision matrices $\mathbf{W}_M$ and $\mathbf{W}_F$ are Wishart distributed:

$$\mathbf{W}_M \sim \text{Wishart}_p(r, \mathbf{R}^{-1}) \qquad \mathbf{W}_F \sim \text{Wishart}_p(r, \mathbf{R}^{-1})$$

with $p = 2$,

$$\mathbf{R} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

and parameters $r = 3$, $R_{11} = 1.244$, $R_{22} = 1.244$ and $R_{12} = R_{21} = 0$.

References

McCullagh, P. and Nelder, J. A. (1983). *Generalized Linear Models*, 2 edn, Chapman & Hall/CRC Press, London.