# Zero-inflated models: Beta-Binomial

#### Parameterisation

There is support for a further zero-inflated model of type 2 (see zero-inflated.pdf), the zero-inflated beta-binomial. It is only defined for type 2.

## Type 2

The likelihood is defined as

$$Prob(y \mid ...) = p \times 1_{[y=0]} + (1-p) \times Beta-binomial(y)$$

where:

$$p = 1 - \left(\frac{\exp(x)}{1 + \exp(x)}\right)^{\alpha}$$

### Link-function

As for the Binomial (see Zero-inflated.pdf).

## Hyperparameters

The Beta-binomial distribution has two arguments ( $\beta_1 \& \beta_2$ ) which we assume are a (specific) function of an underlying hyperparameter ( $\delta$ ) & x. There is a further hyperparameter,  $\alpha$ , governing zero-inflation where:

The parameter controlling the degree of overdispersion,  $\delta$ , is represented as

$$\theta_1 = \log(\delta)$$

and the prior is defined on  $\theta_1$ .

The zero-inflation parameter  $\alpha$ , is represented as

$$\theta_2 = \log(\alpha)$$

and the prior and initial value is is given for  $\theta_2$ .

# **Specification**

- family = zeroinflatedbetabinomial2
- Required arguments: As for the zero-inflated-nbinomial likelihood.

## Hyperparameter spesification and default values

### hyper

#### theta1

name log alpha short.name a initial 0.693147180559945 fixed FALSE prior gaussian

```
param 0.693147180559945 1
          to.theta function(x) log(x)
          from.theta function(x) exp(x)
     theta2
          name beta
          short.name b
          initial 0
          fixed FALSE
          prior gaussian
          param 01
          to.theta function(x) log(x)
          from.theta function(x) exp(x)
survival FALSE
discrete FALSE
link default logit probit cloglog
pdf zeroinflated
Example
In the following we estimate the parameters in a simulated example.
Example-zero-inflated-beta-binomial2.R
nx = 1000
                          # number of x's to consider
n.trial = 20
                          # size of each binomial trial
x = rnorm(nx)
                          # generating x
delta = 10
                              #hyperparameter 1
p = \exp(1+x)/(1+\exp(1+x))
                              #hyperparameter 2
alpha = 2
                                #ZI parameter
q = p^alpha
                                #prob presence
beta_1=delta*p
                                   #beta-bin parameter 1
beta_2=delta*(1-p)
                                   #beta-bin parameter 2
rb = rbeta(nx, beta_1, beta_2, ncp = 0)
y = rep(0,nx)
                                     #generating data
abs.pres = rbinom(nx,1,q)
y[abs.pres==1] = rbinom( sum(abs.pres>0), n.trial, rb[abs.pres==1])
formula = y \sim x + 1
r = inla(formula, data = data.frame(x,y), family = "zeroinflatedbetabinomial2",
        control.data = list(hyper=list(a = list(prior = "flat", param=numeric(0)),
                                        b = list(prior = "flat", param=numeric(0)))),
        Ntrials = rep(n.trial, nx),
```

verbose=TRUE)