Model for seasonal variation

Parametrization

A model for seasonal variation with periodicity m for the random vector $\S = (x_1, \ldots, x_n)$, $n > m$ is obtained assuming that the sums $x_i + x_{i+1} + \ldots + x_{i+m-1}$ are independent Gaussian with preciosion τ.

The density for $\mathbf{x}$ is derived from the $n - m + 1$ increments as

$$\pi(\mathbf{x}|\tau) \propto \tau^{\frac{(n-m+1)}{2}} \exp\left\{-\frac{\tau}{2}\sum(x_i + x_{i+1} + \ldots + x_{i+m-1})^2\right\} \quad (1)$$

$$= \tau^{\frac{(n-m+1)}{2}} \exp\left\{-\frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x}\right\} \quad (2)$$

where $\mathbf{Q} = \tau\mathbf{R}$ and $\mathbf{R}$ is the structure matrix reflecting the neighbourhood structure of the model.

Hyperparameters

The precision parameter τ is represented as

$$\theta = \log\tau$$

and the prior is defined on θ.

Specification

The seasonal model is specified inside the `f()` function as

```
f(<whatever>,model="seasonal",season.length=<season.length>,
              prior=c(<prior.model.theta>),
              param=c(<param.prior.theta1>))
```

Example

```
n=203
n.seas=12

trend=seq(1:n)
seasonal=rep(1:n.seas, ceiling(n/n.seas))[1:n]

a=1
b=0.5
y = rnorm(n,a+b*trend,1)+rnorm(n,0.2*seasonal,1)

data=data.frame(y=y,trend=trend,seasonal=trend)

formula = y~f(trend,model="rw2")+f(seasonal,model="seasonal",
              season.length=n.seas,param=c(1,0.1))
result=inla(formula,family="gaussian",data=data)
```

Notes

The seasonal is a intrinsic random field with rank deficiency of $m - 1$.