Random walk model of order 1 (RW1)

Parametrization

The random walk model of order 1 (RW1) for the Gaussian vector $\mathbf{x} = (x_1, \dots, x_n)$ is constructed assuming independent increments:

$$\Delta x_i = x_i - x_{i+1} \sim \mathcal{N}(0, \tau^{-1})$$

The density for $\mathbf{x}$ is derived from its $n-1$ increments as

$$\begin{aligned} \pi(\mathbf{x}|\tau) &\propto \tau^{(n-1)/2} \exp\left\{-\frac{\tau}{2}\sum(\Delta x_i)^2\right\} && (1)\\ &= \tau^{(n-1)/2} \exp\left\{-\frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x}\right\} && (2) \end{aligned}$$

where $\mathbf{Q} = \tau\mathbf{R}$ and $\mathbf{R}$ is the structure matrix reflecting the neighbourhood structure of the model.

It is also possible to define a *cyclic* version of the RW1 model, in this case the graph is modified so that last node x_n is neighbour of x_{n-1} and x_1.

Hyperparameters

The precision parameter τ is represented as

$$\theta = \log \tau$$

and the prior is defined on θ.

Specification

The RW1 model is specified inside the `f()` function as

```
f(<whatever>, model="rw1", values=<values>, cyclic=<TRUE|FALSE>,
             hyper = <hyper>)
```

The (optional) argument `values` is a numeric or factor vector giving the values assumed by the covariate for which we want the effect to be estimated. See next example for an application.

Hyperparameter spesification and default values

hyper

theta

name log precision

short.name prec

prior loggamma

param 1 5e-05

initial 4

fixed FALSE

to.theta

from.theta

constr TRUE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values FALSE

pdf rw1

Example

```
n=100
z=seq(0,6,length.out=n)
y=sin(z)+rnorm(n,mean=0,sd=0.5)
data=data.frame(y=y,z=z)

formula=y~f(z,model="rw1",
            hyper = list(prec = list(prior="loggamma",param=c(1,0.01))))
result=inla(formula,data=data,family="gaussian")

#here we estimate the effect only for some of the values in z
formula1=y~f(z,model="rw1",
             hyper = list(prec = list(prior="loggamma",param=c(1,0.01))),
             values=z[seq(1,length(z),2)])
result1=inla(formula1,data=data,family="gaussian")
```

Notes

The RW1 is intrinsic with rank deficiency 1.

There exist also support to define irregular RW1 models.