

## The Wishart model for correlated effects

This model is available for dimensions  $p = 2, 3$ , we describe in detail the case for  $p = 2$  and the case for  $p = 3$ .

### Parametrization

The 2-dimensional Wishart model is used if one want to define the model for the linear predictor  $\eta$  as:

$$\eta = a + b$$

where  $a$  and  $b$  are correlated

$$\begin{pmatrix} a \\ b \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{W}^{-1})$$

with covariance matrix  $\mathbf{W}^{-1}$

$$\mathbf{W}^{-1} = \begin{pmatrix} 1/\tau_a & \rho\sqrt{\tau_a\tau_b} \\ \rho\sqrt{\tau_a\tau_b} & 1/\tau_b \end{pmatrix} \quad (1)$$

and  $\tau_a$ ,  $\tau_b$  and  $\rho$  are the hyperparameters. In this case the following model is implemented for the precision matrix  $\mathbf{W}$

$$\mathbf{W} \sim \text{Wishart}_p(r, \mathbf{R}^{-1}), \quad p = 2$$

where the Wishart distribution has density

$$\pi(\mathbf{W}) = c^{-1} |\mathbf{W}|^{(r-(p+1))/2} \exp \left\{ -\frac{1}{2} \text{Trace}(\mathbf{W}\mathbf{R}) \right\}, \quad r > p + 1$$

and

$$c = 2^{(rp)/2} |\mathbf{R}|^{-r/2} \pi^{(p(p-1))/4} \prod_{j=1}^p \Gamma((r+1-j)/2).$$

Then,

$$\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r - (p + 1)).$$

### Hyperparameters

The hyperparameters are

$$\theta = (\log \tau_a, \log \tau_b, \tilde{\rho})$$

where

$$\rho = 2 \frac{\exp(\tilde{\rho})}{\exp(\tilde{\rho}) + 1} - 1$$

The prior-parameters are

$$(r, R_{11}, R_{22}, R_{12})$$

where

$$\mathbf{R} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

and  $r_{12} = R_{21}$  due to symmetry.

The `inla` function reports posterior distribution for the hyperparameters  $\tau_a, \tau_b, \rho$  in equation (1).

The prior for  $\theta$  is **fixed** to be **wishart**

## Specification

The model 2d wishart for

$$\eta = a + b$$

is specified as

```
y~f(a,model="2diidwishartp1",param=<param.vector(4 elements)>)+f(b,model="2diidwishartp2")
```

The parameters for the Wishart distribution are specified *only* for 2diidwishartp1

## Example

In this example we implement the model

$$y|\eta \sim \text{Pois}(\exp(\eta))$$

where

$$\eta = a + b + c$$

and  $b$  and  $c$  are correlated as described above.

```
n=100
#set hyperparameters
r=4
R11=1
R22=2
R12=0.1
R=matrix(c(R11,R12,R12,R22),2,2)
S=solve(R)
#these are needed to simulate from a wishart prior
# and sample from a multivariate normal
library(MCMCpack)
library(mvtnorm)
W=rwish(r, S)
cc=rmvnorm(n, mean=c(0,0), sigma=solve(W))
a=cc[,1]
b=cc[,2]
#simulate data
x1=1:n
x2=1:n
eta=0.1+a+b
y=rpois(n,exp(eta))
data=data.frame(y=y,x1=1:n,x2=1:n)
#fit the model
formula=y~f(x1,model="2diidwishartp1",param=c(4,1,2,0.1))+f(x2,model="2diidwishartp2")
result=inla(formula,family="poisson",data=data)
```

## Notes

If more than one pair of 2diidwishartp1/2 is defined, the following rule is used to determine the match between  $p(\text{art})1$  and  $p(\text{art})2$ .

The first occurrence of 2diidwishartp1 belongs with the first occurrence of 2diidwishartp2.

The second occurrence of 2diidwishartp1 belongs with the second occurrence of 2diidwishartp2 and so on.

## Three dimensional case

The previous formulation is also available for 3D. In this case the hyperparameters are

$$\theta = (\log \tau_1, \log \tau_2, \log \tau_3, \tilde{\rho}_{12}, \tilde{\rho}_{13}, \tilde{\rho}_{23})$$

In this case the name the prior is **fixed** to be Wishart3d. The parameters in the prior are

$$parameters = r \ R_{11} \ R_{22} \ R_{33} \ R_{12} \ R_{13} \ R_{23}$$

where

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{12} & R_{22} & R_{23} \\ R_{13} & R_{23} & R_{33} \end{bmatrix}$$

The reported hyperparameters are the marginal precisions  $\tau_1$ ,  $\tau_2$  and  $\tau_3$  and the correlations  $\rho_{12}$ ,  $\rho_{13}$  and  $\rho_{23}$ . The model names are as given in the following example.

```
formula2 <- Y ~ f(diid.part0,model="3diidwishartp1",
  param=c(7,1,2,3,0.1,0.2,0.3))
  f(diid.part1,model="3diidwishartp2") +
  f(diid.part2,model="3diidwishartp3") +
```

If more than one pair of 3diidwishartp1/2/3 is defined, the following rule is used to determine the match between  $p(\text{art})1$ ,  $p(\text{art})2$  and  $p(\text{art})3$ .

The first occurrence of 3diidwishartp1 belongs with the first occurrence of 3diidwishartp2 and 3diidwishartp3. The second occurrence of 3diidwishartp1 belongs with the second occurrence of 3diidwishartp2 and the second occurrence of 3diidwishartp3, and so on.