Tokyo rainfall data

The number of occurrences of rainfall over 1 mm in the Tokyo area for each calendar year during two years (1983-84) are registered. It is of interest to estimate the underlying probability p_t of rainfall for calendar day t which is, apriori, assumed to change gradually over time. The likelihood model is binomial

$$y_t|\eta_t \sim \text{Bin}(n_t, p_t)$$

with logit link function

$$p_t = \frac{\exp(\eta_t)}{1 + \exp(\eta_t)}.$$

The model for the latent variables can be written as

$$\eta_t = f(t)$$

where t is the observed time whose effect is modelled as a smooth function $f(\cdot)$. Following [Rue and Held, 2005], the random vector $\boldsymbol{f} = \{f_0, \ldots, f_{365}\}$ is assumed to have a *circular* random walk of order 2 (RW2) prior with unknown precision λ_f.

There is only one hyperparameter $\boldsymbol{\theta} = (\log \lambda_f)$ which we assign a LogGamma(a, b) prior distribution with $a = 1$ and $b = 0.0001$. The LogGamma distribution is defined such that if $X \sim$ LogGamma(a, b), the $Y = \exp(X) \sim$ Gamma(a, b) with $\text{E}(Y) = a/b$ and $\text{Var}(Y) = a/b^2$.

References

[Rue and Held, 2005] Rue, H. and Held, L. (2005). *Gaussian Markov Random Fields: Theory and Applications*, volume 104 of *Monographs on Statistics and Applied Probability*. Chapman & Hall, London.