Exponential and Weibull Models

We consider the data of times to infection of kidney dialysis patients. In a study Nahman et al. (1992), (given in the book by Klein and Moeschberger (2003)) designed to assess the time to first exitsite infection (in months) in patients with renal insufficiency, 43 patients utilized a surgically placed catheter (Group 1), and 76 patients utilized a percutaneous placement of their catheter (Group 2), a total of 119 patients.

The variables represented in the data set are time to infection in months/10 denoted by t, infection indicator or event (0=no, 1=yes) denoted by δ and catheter placement (1=surgically, 2=percutaneously) denoted by trt. We analyse the data set using exponential model and Weibull model.

The **exponential model** for this example can be specified as:

$$t_i \sim E(\lambda_i)$$

Where each survival time follows an exponential distribution with parameter λ_i and i is from 1 to 119. For this example we have only one covariate, catheter placement (trt) and therefore $\beta = (\beta_0, \beta_1)'$, where β_0 denotes the intercept term and β_1 denotes the coefficient for the placement covariate (trt). Here, the latent field is

$$\lambda_i = \exp(\eta_i)$$

with

$$\eta_i = \beta_0 + trt_i\beta_1$$

where both β_0 and β_1 are assign the following priors distributions

$$\beta_0 \sim N(0, 0.001)$$

$$\beta_1 \sim N(0, 0.001)$$

There is no hyperparameter used in this model.

The **Weibull model** for this example can be specified as:

$$t_i \sim \text{Weibull}(\alpha, \lambda_i)$$

Here also, the latent field is

$$\lambda_i = \exp(\eta_i)$$

with

$$\eta_i = \beta_0 + trt_i\beta_1$$

where β_0 and β_1 are assign the following priors distributions

$$\beta_0 \sim N(0, 0.001)$$

$$\beta_1 \sim N(0, 0.001)$$

The model has one hyperparameter, α , we assign the following prior distribution

$$\alpha \sim \text{Gamma}(1, 0.001)$$

References

Klein, J. P. and Moeschberger, M. L. (2003). *Survival Analysis: Techniques for Censored and Truncated Data*. Springer, New York, 2nd edition.

Nahman, N. S., Middendorf, D. F., Bay, W. H., McElligott, R., Powell, S., and Anderson, J. (1992). Modification of the percutaneous approach to peritoneal dialysis catheter placement under peritoneoscopic visualization: Clinical results in 78 patients. *Journal of The American Society of Nephrology*, 3:103–107.