

# Besag model for spatial effects

## Parametrization

The besag model for random vector  $\mathbf{x} = (x_1, \dots, x_n)$  is defined as

$$x_i | x_j, i \neq j, \tau \sim \mathcal{N}(\frac{1}{n_i} \sum_{i \sim j} x_j, \frac{1}{n_i \tau}) \quad (1)$$

where  $n_i$  is the number of neighbours of node  $i$ ,  $i \sim j$  indicates that the two nodes  $i$  and  $j$  are neighbours.

## Hyperparameters

The precision parameter  $\tau$  is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on  $\theta_1$ .

## Specification

The besag model is specified inside the `f()` function as

```
f(<whatever>, model="besag", graph.file=<graph file name>,  
  hyper=<hyper>, adjust.for.con.comp = TRUE)
```

The neighbourhood structure of  $\mathbf{x}$  is passed to the program through the `graph.file` argument. The structure of this file is described below.

The option `adjust.for.con.comp` adjust the model if the graph has more than one connected compoment, and this adjustment can be disabled setting this option to `FALSE`. This means that `constr=TRUE` is interpreted as a sum-to-zero constraint on *each* connected component and the `rankdef` parameter is set accordingly.

## Hyperparameter spesification and default values

### hyper

#### theta

```
name  log precision  
short.name  prec  
prior  loggamma  
param  1 5e-05  
initial  4  
fixed  FALSE  
to.theta  
from.theta
```

```
constr  TRUE
```

```
nrow.ncol  FALSE
```

```
augmented  FALSE
```

```
aug.factor  1
```

```
aug.constr
n.div.by
n.required TRUE
set.default.values TRUE
pdf besag
```

### Structure of the graph file

We describe the required format for the graph file using a small example. Let the file `gra.dat`, relative to a small graph of only 5 elements, be

```
5
1 1 2
2 2 1 3
3 3 2 4 5
4 1 3
5 1 3
```

Line 1 declares the total number of nodes in the graph (5), then, in lines 2-6 each node is described. For example, line 4 states that node 3 has 4 neighbours and these are nodes 2, 4 and 5.

The graph file can either have nodes indexed from 1 to  $n$ , or from 0 to  $n - 1$ . Note that in the latter case, node  $i$  seen from R corresponds to node  $i - 1$  in the 0-indexed graph.

### Example

For examples of application of this model see the `Bym`, `Munich`, `Zambia` or `Scotland` examples in Volume I.

### Notes

The besag model intrinsic with rankdef 1.