# Asymmetric Laplace

## Parametrisation

The asymmetric Laplace distribution is

$$f(y) = \delta\tau(1-\tau)\exp\{-\delta\rho_\tau(y-\mu)\}$$

for continuously responses $y$ where $\rho_\tau(u) = \{\tau - I(u<0)\}u$ is the so-called check function in Koenker and Bassett (1978), and

**$\mu$:** is the the location parameter ($-\infty < \mu < \infty$)

**$\tau$:** is the fixed skewness parameter ($0 < \tau < 1$)

**$\delta$:** is the inverse scale parameter ($\delta > 0$).

## Scale mixtures of normal representation

The asymmetric Laplace random variable $y$ can be represented as follows:

$$y = \mu + \xi w + \sigma\sqrt{w/\delta}z,$$

where $\xi = \frac{1-2\tau}{\tau(1-\tau)}$ and $\sigma^2 = \frac{2}{\tau(1-\tau)}$ are two scalars depending on $\tau$. The random variables $w > 0$ and $z$ are independent and have exponential distribution with mean $\delta^{-1}$ and standard normal distribution, respectively. As a result, $y$ has the following hierarchical structure:

$$y \mid w \sim N\left(\mu + \xi w, \sigma^2\delta^{-1}w\right) \quad \text{and} \quad w \sim \text{Exp}(\delta).$$

## Approximating check function

The log likelihood of asymmetric Laplace distribution has zero second-order derivative everywhere. To implement INLA, we approximate the check function as follows:

$$\tilde{\rho}_{\tau,\gamma}(u) = \begin{cases} \gamma^{-1}\log(\cosh(\tau\gamma|u|)) & \text{if } u \geq 0 \\ \gamma^{-1}\log(\cosh((1-\tau)\gamma|u|)) & \text{if } u < 0, \end{cases}$$

where the parameter $\gamma > 0$ is fixed and precision of the approximation increases as $\gamma \to \infty$.

## Link-function

The location parameter is linked to the linear predictor by

$$\mu = \eta$$

## Hyperparameters

The prior is defined on inverse scale $\delta$.

## Specification

- family = `laplace`

### Hyperparameter spesification and default values

## Example

## Notes

None.