## Stochastic volatility models

The data consist in 945 observed logarithms of the daily difference of the dollar-pound exchange rate from October 1st, to June 28th, 1985. We analyse this data set using a univariate stochastic volatility model ([Taylor, 1986]). The likelihood of the data, conditional on the latent variables is:

$$y_t | \eta_t \sim \mathcal{N}(0, \exp(\eta_t)), \quad t = 0, \dots, n_d - 1$$
 (1)

and the model for the latent variables:

$$\eta_t = \mu + f_t \quad t = 0, \quad , n_\eta - 1 \tag{2}$$

where  $\mu$  is an unknown common mean with vague Gaussian prior and  $\mathbf{f} = (f_0, \dots, f_{n_{\eta}-1})$  is modelled as an auto regressive process of order 1 (AR1) with persistence parameter  $\phi \in (-1,1)$  to ensure stationarity, and precision parameter  $\lambda_f$ . The model has two hyperparameters,  $(\log \lambda_f, \phi)$ . We re-parametrise the persistence parameter  $\phi$  as

$$\kappa = \operatorname{logit}\left(\frac{\phi + 1}{2}\right)$$

and assign the following prior distributions

$$\log \lambda_f \sim \text{LogGamma}(1, 0.0005)$$
  
$$\kappa \sim \mathcal{N}(0, 1/0.0001)$$

## Student-t distribution

An alternative model for the response variable  $y_t$  is a Student-t. This allows heavier tail, a feature which is often observed in financial time series. The observation model in equation (1) then becomes

$$y_t = \exp(\eta_t/2) \ \mathcal{T}_t(\nu) \quad t = 1, \dots, T \tag{3}$$

where  $\mathcal{T}_t(\nu)$  is a random variable having a Student-t distribution having  $\nu$  degree of freedom and standardised so that its variance is 1 for any value of  $\nu > 2$ .

## Student-t NIG distribution

Yet another model is the normal inverse Gaussian (NIG) distribution, for which

$$y_t = \exp(\eta_t/2) NIG, \quad t = 1, \dots, T \tag{4}$$

where NIG is a standardised NIG distribution with two parameters, which (essentially) are skewness and shape-parameters.

## References

[Taylor, 1986] Taylor, S. J. (1986). Modelling Stochastic volatility. John Wiley.