## The Matérn-model

#### Parametrisation

This model is the Gaussian field with Matérn correlation function on a regular nrow x ncol -lattice

$$\operatorname{Corr}(d) \propto (\kappa d)^{\nu} K_{\nu}(\kappa d), \qquad \alpha = \nu + d/2,$$

where  $K_{\nu}$  is the modified Bessel function. The range is defined as

$$r = \sqrt{8}/\kappa$$

which about the distance where the covariance function becomes "small".

The boundary conditions are so that the values are taken to be 0 outside the lattice. No further boundary options are available at this time.

## Hyperparameters

The hyperparameters are the precision parameter  $\tau$  and the range r,

$$\theta = (\tau, r)$$

The latent field has marginal variance  $1/\tau$  and range (as defined above) r. Note that  $\nu$  is fixed parameter and the model is available only for  $\nu = 1, 2, 3$  ( $\nu = 0$  is not yet ready). The hyperparameters are represented internally as

$$(\log \tau, \log r)$$

the prior are assigned to these quantities.

### Specification

The matern2d model is specified inside the f() function as:

# Hyperparameter spesification and defaults

### hyper

```
theta1
```

```
name precision
short.name prec
initial 4
fixed FALSE
prior loggamma
param c(1, 1e-04)
```

## theta2

name range short.name range initial 2 fixed FALSE prior loggamma

```
param c(1, 0.01)
 constr TRUE
nrow.ncol TRUE
augmented FALSE
aug.factor 1
aug.constr NULL
n.div.by NULL
n.required FALSE
set.default.values TRUE
Example
nrow=20
ncol=30
n = nrow*ncol
s.noise = 1
zi.mat = matrix(NA,nrow=nrow,ncol=ncol)
i=1:nrow
for(j in 1:ncol)
    zi.mat[i,j] = 3*exp(-(i-j)^2/4)
## iid noise
noise.mat=matrix(rnorm(nrow*ncol, sd=s.noise),nrow,ncol)
## make simulated data with no spatial component
y.mat = zi.mat + noise.mat
## convert matrices to the internal representation in INLA
y = inla.matrix2vector(y.mat)
node = 1:n
formula= y ~ 1+ f(node, model="matern2d", nu=1, nrow=nrow, ncol=ncol,
        hyper = list(range = list(param =c(1, 1),
                                  prior = "loggamma",
                                  initial=1),
                     prec = list(param=c(1, 1))))
data=data.frame(y=y,node=node)
## fit the model
result=inla(formula, family="gaussian", data=data, verbose=TRUE,
        control.predictor = list(compute = TRUE),
        control.data = list(hyper = list(theta = list(initial = log(1/s.noise^2),
                                                 fixed = FALSE))),
        keep=T)
## plot the posterior mean for 'predictor' and compare with the truth
```

```
dev.new()
inla.display.matrix(zi.mat)
dev.new()
inla.display.matrix(inla.vector2matrix(result$summary.linear.predictor$mean,nrow,ncol))
```

## Notes

All indexes in the R-INLA library are one-dimensional so an appropriate mapping is required to get it into the ordering defined internally in inla; see ?inla.matrix2vector, ?inla.vector2matrix, ?inla.node2lattice and ?inla.lattice2node.

This model has much similarity with rw2d; please read the documentation for rw2d.