

## Correlated random effects: iid1d, iid2d, iid3d

This model is available for dimensions  $p = 1, 2$  and  $3$ . We describe in detail the case for  $p = 2$ , and then the changes required for  $p = 1$  and  $p = 3$ .

### Parametrization

The 2-dimensional Normal-Wishard model is used if one want to define two vectors of “random effects”,  $u$  and  $v$ , say, for which  $(u_i, v_i)$  are iid bivariate Normals

$$\begin{pmatrix} u_i \\ v_i \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{W}^{-1})$$

where the covariance matrix  $\mathbf{W}^{-1}$  is

$$\mathbf{W}^{-1} = \begin{pmatrix} 1/\tau_a & \rho\sqrt{\tau_a\tau_b} \\ \rho\sqrt{\tau_a\tau_b} & 1/\tau_b \end{pmatrix} \quad (1)$$

and  $\tau_a$ ,  $\tau_b$  and  $\rho$  are the hyperparameters. For these models the precision matrix  $\mathbf{W}$  is Wishart distributed

$$\mathbf{W} \sim \text{Wishart}_p(r, \mathbf{R}^{-1}), \quad p = 2$$

with density

$$\pi(\mathbf{W}) = c^{-1} |\mathbf{W}|^{(r-(p+1))/2} \exp \left\{ -\frac{1}{2} \text{Trace}(\mathbf{W}\mathbf{R}) \right\}, \quad r > p + 1$$

and

$$c = 2^{(rp)/2} |\mathbf{R}|^{-r/2} \pi^{(p(p-1))/4} \prod_{j=1}^p \Gamma((r+1-j)/2).$$

Then,

$$\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r - (p + 1)).$$

### Hyperparameters

The hyperparameters are

$$\theta = (\log \tau_a, \log \tau_b, \tilde{\rho})$$

where

$$\rho = 2 \frac{\exp(\tilde{\rho})}{\exp(\tilde{\rho}) + 1} - 1$$

The prior-parameters are

$$(r, R_{11}, R_{22}, R_{12})$$

where

$$\mathbf{R} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

and  $r_{12} = R_{21}$  due to symmetry.

The `inla` function reports posterior distribution for the hyperparameters  $\tau_a, \tau_b, \rho$  in equation (1).

The prior for  $\theta$  is **fixed** to be `wishart2d`

## Specification

The model `iid2d` is specified as

```
y ~ f(i, model="iid2d", n = <length>) + ...
```

and the `iid2d` model is represented internally as one vector of length  $n$ ,

$$(u_1, u_2, \dots, u_m, v_1, v_2, \dots, v_m)$$

where  $n = 2m$ , and  $n$  is the (required) argument in `f()`.

For this model the argument `constr=TRUE` is interpreted as

$$\sum u_i = 0, \quad \text{and} \quad \sum v_i = 0.$$

## Hyperparameter spesification and default values

**hyper**

**theta1**

```
name precision1
short.name prec1
initial 4
fixed FALSE
prior wishart2d
param c(4, 1, 1, 0)
```

**theta2**

```
name precision2
short.name prec2
initial 4
fixed FALSE
prior none
param numeric(0)
```

**theta3**

```
name correlation
short.name cor
initial 4
fixed FALSE
prior none
param numeric(0)
```

**constr** FALSE

**nrow.ncol** FALSE

**augmented** TRUE

**aug.factor** 1

```

aug.constr  c(1, 2)
n.div.by  2
n.required  TRUE
set.default.values  TRUE

```

## Example

In this example we implement the model

$$y|\eta \sim \text{Pois}(\exp(\eta))$$

where

$$\eta = a + b + 1$$

and  $a$  and  $b$  are correlated as described above.

```

n = 1000
N = 2*n
rho = 0.5
## set variances
Sigma = matrix(c(1/1, NA, NA, 1/2), 2, 2)
## and the correlation
Sigma[1,2] = Sigma[2,1] = rho*sqrt(Sigma[1,1]*Sigma[2,2])

## need it to simulate data
library(mvtnorm)

if (TRUE)
{
  ## first example

  y = yy = rmvnorm(n, sigma=Sigma)
  y = c(y[,1], y[,2])

  i = 1:N
  formula = y ~ f(i, model="iid2d", n=N)

  r = inla(formula, data = data.frame(i,y),
           control.data=list(initial=10,fixed=TRUE))
  print(summary(r))

  print(1/diag(cov(yy)))
  print(cor(yy)[1,2])
}

if (TRUE)
{
  ## second example

```

```

y = yy = rmvnorm(n, sigma=Sigma)
z = rnorm(n)
zz = rnorm(n)
y = y[,1] + z*y[,2] + zz
i = 1:n
j = n + 1:n
formula = y ~ f(i, model="iid2d", n=N) + f(j,z,copy="i") + zz

r = inla(formula, data = data.frame(i,j,y,z,zz),
         control.data=list(initial=10,fixed=TRUE),keep=T)
print(summary(r))
print(1/diag(cov(yy)))
print(cor(yy)[1,2])
}

```

## The case $p = 1$

For  $p = 1$  the hyperparameter is the marginal precision

$$\theta = \log \tau_1$$

The prior is fixed to be `wishart1d` with parameters

$$param = r R_{11}$$

where

$$\mathbf{R} = \begin{bmatrix} R_{11} \end{bmatrix}$$

## Hyperparameter spesification and default values

**hyper**

**theta**

```

name precision
short.name prec
initial 4
fixed FALSE
prior wishart1d
param c(2, 2e-04)

```

**constr** FALSE

**nrow.ncol** FALSE

**augmented** FALSE

**aug.factor** 1

**aug.constr** NULL

**n.div.by** NULL

**n.required** TRUE

**set.default.values** TRUE

## The case $p = 3$

For  $p = 3$  the hyperparameters are

$$\theta = (\log \tau_1, \log \tau_2, \log \tau_3, \tilde{\rho}_{12}, \tilde{\rho}_{13}, \tilde{\rho}_{23})$$

The prior is fixed to be **wishart3d** with parameters

$$param = r \ R_{11} \ R_{22} \ R_{33} \ R_{12} \ R_{13} \ R_{23}$$

where

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{12} & R_{22} & R_{23} \\ R_{13} & R_{23} & R_{33} \end{bmatrix}$$

The reported hyperparameters are the marginal precisions  $\tau_1$ ,  $\tau_2$  and  $\tau_3$  and the correlations  $\rho_{12}$ ,  $\rho_{13}$  and  $\rho_{23}$ .

In this case, the internal representation is given as

$$(u_1, u_2, \dots, u_m, v_1, v_2, \dots, v_m, w_1, w_2, \dots, w_m)$$

where  $n = 3m$  is a required argument, and where  $(u_i, v_i, w_i)$  are trivariate iid Normal.

## Hyperparameter spesification and default values

**hyper**

**theta1**

**name** precision1  
**short.name** prec1  
**initial** 4  
**fixed** FALSE  
**prior** wishart3d  
**param** c(7, 1, 1, 1, 0, 0, 0)

**theta2**

**name** precision2  
**short.name** prec2  
**initial** 4  
**fixed** FALSE  
**prior** none  
**param** numeric(0)

**theta3**

**name** precision3  
**short.name** prec3  
**initial** 4  
**fixed** FALSE  
**prior** none  
**param** numeric(0)

```

theta4
  name correlation12
  short.name cor12
  initial 0
  fixed FALSE
  prior none
  param numeric(0)
theta5
  name correlation13
  short.name cor13
  initial 0
  fixed FALSE
  prior none
  param numeric(0)
theta6
  name correlation23
  short.name cor23
  initial 0
  fixed FALSE
  prior none
  param numeric(0)
constr FALSE
nrow.ncol FALSE
augmented TRUE
aug.factor 1
aug.constr c(1, 2, 3)
n.div.by 3
n.required TRUE
set.default.values TRUE

```

## Notes

The model `iid1d` is similar to the model `iid` (and included for completeness only). The prior for `iid1d` is fixed to be Wishart-distributed, which reduces to a Gamma-distribution for the precision with parameters

$$a = r/2 \quad \text{and} \quad b = R_{11}/2$$

hence

```
y ~ f(i, model="iid1d", hyper = list(theta=list(param=c(3, 4))))
```

is equivalent to

```
y ~ f(i, model="iid", hyper = list(theta=list(param=c(1.5, 2), prior="loggamma")))
```