

Proper Besag model for spatial effects

Parametrization

The proper version of the Besag model for random vector $\mathbf{x} = (x_1, \dots, x_n)$ is defined as

$$x_i | x_{-i}, \tau, \phi \sim \mathcal{N} \left(\frac{\phi}{1 + \phi n_i} \sum_{i \sim j} x_j, \frac{1}{(1 + \phi n_i) \tau} \right) \quad (1)$$

where n_i is the number of neighbours of node i , $i \sim j$ indicates that the two nodes i and j are neighbours, $\phi > 0$ is as weight parameter and $\tau > 0$ is a “precision-like” (or scaling) parameter.

Hyperparameters

The precision parameter τ is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on θ_1 .

The weight parameter ϕ is represented as

$$\theta_2 = \log \phi$$

and the prior is defined on θ_2 .

Specification

The besag model is specified inside the `f()` function as

```
f(<whatever>, model="besagproper", graph.file=<graph file name>,  
  hyper=<hyper>)
```

The neighbourhood structure of \mathbf{x} is passed to the program through the `graph.file` argument. The structure of this file is described below.

Hyperparameter spesification and default values

hyper

theta1

```
name  log precision  
short.name  prec  
prior  loggamma  
param  1 5e-04  
initial  2  
fixed  FALSE  
to.theta  function(x) log(x)  
from.theta  function(x) exp(x)
```

theta2

```
name  log diagonal  
short.name  diag  
prior  loggamma
```

```

    param 1 1
    initial 1
    fixed FALSE
    to.theta function(x) log(x)
    from.theta function(x) exp(x)

constr FALSE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required TRUE

set.default.values TRUE

pdf besagproper

```

Structure of the graph file

We describe the required format for the graph file using a small example. Let the file `gra.dat`, relative to a small graph of only 5 elements, be

```

5
1 1 2
2 2 1 3
3 3 2 4 5
4 1 3
5 1 3

```

Line 1 declares the total number of nodes in the graph (5), then, in lines 2-6 each node is described. For example, line 4 states that node 3 has 4 neighbours and these are nodes 2, 4 and 5.

The graph file can either have nodes indexed from 1 to n , or from 0 to $n - 1$. Note that in the latter case, node i seen from R corresponds to node $i - 1$ in the 0-indexed graph.

Example

To be added

Notes

Add notes later ?