

Besag model for spatial effects

Parametrization

The besag model for random vector $\mathbf{x} = (x_1, \dots, x_n)$ is defined as

$$x_i | x_j, i \neq j, \tau \sim \mathcal{N}\left(\frac{1}{n_i} \sum_{i \sim j} x_j, \frac{1}{n_i \tau}\right) \quad (1)$$

where n_i is the number of neighbours of node i , $i \sim j$ indicates that the two nodes i and j are neighbours.

Hyperparameters

The precision parameter τ is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on θ_1 .

Specification

The besag model is specified inside the `f()` function as

```
f(<whatever>, model="besag", graph.file=<graph file name>
    prior=c(<prior.model.theta1>),
    param=c(<param.prior.theta1>))
```

The neighbourhood structure of \mathbf{x} is passed to the program through the `graph.file` argument. The structure of this file is described below.

Structure of the graph file

We describe the required format for the graph file using a small example. Let the file `gra.dat`, relative to a small graph of only 5 elements, be

```
5
1 1 2
2 2 1 3
3 3 2 4 5
4 1 3
5 1 3
```

Line 1 declares the total number of nodes in the graph (5), then, in lines 2-6 each node is described. For example, line 4 states that node 3 has 4 neighbours and these are nodes 2, 4 and 5.

The graph file can either have nodes indexed from 1 to n , or from 0 to $n - 1$. Note that in the latter case, node i seen from R corresponds to node $i - 1$ in the 0-indexed graph.

Example

For examples of application of this model see the Bym, Munich, Zambia or Scotland examples in Volume I.

Notes

The besag model is a intrinsic random field.

The model is modified accordingly is the graph has more than one connected components.