

# The Matérn-model

## Parametrisation

This model is the Gaussian field with Matérn correlation function on a regular `nrow` x `ncol` -lattice

$$\text{Corr}(d) \propto (\kappa d)^\nu K_\nu(\kappa d), \quad \alpha = \nu + d/2,$$

where  $K_\nu$  is the modified Bessel function. The range is *defined* as

$$r = \sqrt{8}/\kappa$$

which about the distance where the covariance function becomes “small”.

The boundary conditions are so that the values are taken to be 0 outside the lattice. No further boundary options are available at this time.

## Hyperparameters

The hyperparameters are the precision parameter  $\tau$  and the range  $r$ ,

$$\theta = (\tau, r)$$

The latent field has marginal variance  $1/\tau$  and range (as defined above)  $r$ . Note that  $\nu$  is fixed parameter and the model is available only for  $\nu = 1, 2, 3$  ( $\nu = 0$  is not yet ready). The hyperparameters are represented internally as

$$(\log \tau, \log r)$$

the prior are assigned to these quantities.

## Specification

The `matern2d` model is specified inside the `f()` function as:

```
f(<whatever>, model="matern2d",nrow=<n.of rows>,ncol=<n.of columns>,  
  nu = <value for nu, one of 1,2 or 3>,  
  prior=c(<prior.model.log.tau>, <prior.model.log.range>),  
  param=c(<prior.param.log.tau.1>, <prior.param.log.tau.2>,  
          <prior.param.log.range.1>, <prior.param.log.range.2>)
```

## Example

```
nrow=20  
ncol=30  
n = nrow*ncol  
  
## two covariates  
zi.mat = matrix(NA,nrow=nrow,ncol=ncol)  
i=1:nrow  
for(j in 1:ncol)  
  zi.mat[i,j] = rnorm(nrow, mean = i, sd=1)  
  
zj.mat = matrix(NA,nrow=nrow,ncol=ncol)  
j=1:ncol  
for(i in 1:nrow)  
  zj.mat[i,j] = rnorm(ncol, mean = j, sd=1)
```

```
## iid noise
noise.mat=matrix(rnorm(nrow*ncol, sd=1),nrow,ncol)

## make simulated data with no spatial component
y.mat = zi.mat + zj.mat + noise.mat

## convert matrices to the internal representation in INLA
y = inla.matrix2vector(y.mat)
zi = inla.matrix2vector(zi.mat)
zj = inla.matrix2vector(zj.mat)
node = 1:n
formula= y ~ 1+zi+zj + f(node, model="matern2d", nu=1, nrow=nrow, ncol=ncol,
                        param=c(NA,NA,1,1))
data=data.frame(y=y,node=node,zi=zi,zj=zj)

## fit the model
result=inla(formula, family="gaussian", data=data, verbose=TRUE,
            control.predictor = list(compute = TRUE))

#plot the posterior mean for 'predictor' and compare with the truth
par(mfrow=c(2,1))
image(s.mat)
image(inla.vector2matrix(result$summary.linear.predictor$mean,nrow,ncol))
```

## Notes

All indexes in the R-INLA library are one-dimensional so an appropriate mapping is required to get it into the ordering defined internally in `inla`; see `?inla.matrix2vector`, `?inla.vector2matrix`, `?inla.node2lattice` and `?inla.lattice2node`.

This model has much similarity with `rw2d`; please read the documentation for `rw2d`.

## References