The Matérn-model

Parametrisation

This model is the Gaussian field with Matérn correlation function on a regular `nrow x ncol` -lattice

$$\mathrm{Corr}(d) \propto (\kappa\, d)^{\nu}\ K_{\nu}(\kappa d), \qquad \alpha = \nu + \mathrm{d}/2,$$

where K_ν is the modified Bessel function. The range is *defined* as

$$r = \sqrt{8}/\kappa$$

which about the distance where the covariance function becomes "small".

The boundary conditions are so that the values are taken to be 0 outside the lattice. No further boundary options are available at this time.

Hyperparameters

The hyperparameters are the precision parameter τ and the range r,

$$\theta = (\tau, r)$$

The latent field has marginal variance $1/\tau$ and range (as defined above) r. Note that ν is fixed parameter and the model is available only for $\nu = 1, 2, 3$ ($\nu = 0$ is not yet ready). The hyperparameters are represented internally as

$$(\log \tau, \log r)$$

the prior are assigned to these quantities.

Specification

The `matern2d` model is specified inside the `f()` function as:

```
f(<whatever>, model="matern2d",nrow=<n.of rows>,ncol=<n.of columns>,
              nu = <value for nu, one of 1,2 or 3>,
              prior=c(<prior.model.log.tau>, <prior.model.log.range>),
              param=c(<prior.param.log.tau.1>, <prior.param.log.tau.2>,
                      <prior.param.log.range.1>, <prior.param.log.range.2>)
```

Example

```
nrow=20
ncol=30
n = nrow*ncol

## two covariates
zi.mat = matrix(NA,nrow=nrow,ncol=ncol)
i=1:nrow
for(j in 1:ncol)
    zi.mat[i,j] = rnorm(nrow, mean = i, sd=1)

zj.mat = matrix(NA,nrow=nrow,ncol=ncol)
j=1:ncol
for(i in 1:nrow)
    zj.mat[i,j] = rnorm(ncol, mean = j, sd=1)
```

```
## iid noise
noise.mat=matrix(rnorm(nrow*ncol, sd=1),nrow,ncol)

## make simulated data with no spatial component
y.mat = zi.mat + zj.mat + noise.mat

## convert matrices to the internal representation in INLA
y = inla.matrix2vector(y.mat)
zi = inla.matrix2vector(zi.mat)
zj = inla.matrix2vector(zj.mat)
node = 1:n
formula= y ~ 1+zi+zj + f(node, model="matern2d", nu=1, nrow=nrow, ncol=ncol,
                          param=c(NA,NA,1,1))
data=data.frame(y=y,node=node,zi=zi,zj=zj)

## fit the model
result=inla(formula, family="gaussian", data=data, verbose=TRUE,
            control.predictor = list(compute = TRUE))

#plot the posterior mean for 'predictor' and compare with the truth
par(mfrow=c(2,1))
image(zi.mat + zj.mat)
image(inla.vector2matrix(result$summary.linear.predictor$mean,nrow,ncol))
```

Notes

All indexes in the R-INLA library are one-dimensional so an appropriate mapping is required to get it into the ordering defined internally in `inla`; see `?inla.matrix2vector`, `?inla.vector2matrix`, `?inla.node2lattice` and `?inla.lattice2node`.

This model has much similarity with `rw2d`; please read the documentation for `rw2d`.