

# Proper/Non-intrinsic Besag model for spatial effects

## Parametrization

The proper version of the Besag model for random vector  $\mathbf{x} = (x_1, \dots, x_n)$  is defined as

$$x_i | x_{-i}, \tau, d \sim \mathcal{N} \left( \frac{1}{d + n_i} \sum_{i \sim j} x_j, \frac{1}{\tau(d + n_i)} \right) \quad (1)$$

where  $n_i$  is the number of neighbours of node  $i$ ,  $i \sim j$  indicates that the two nodes  $i$  and  $j$  are neighbours,  $d > 0$  is an extra term added on the diagonal controlling the “properness” and  $\tau > 0$  is a “precision-like” (or scaling) parameter.

This parameterisation corresponds to this precision matrix  $Q = (Q_{ij})$ , where for  $j \neq i$

$$Q_{ii} = \tau(n_i + d) \quad \text{and} \quad Q_{ij} = -\tau.$$

## Hyperparameters

The precision parameter  $\tau$  is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on  $\theta_1$ . The diagonal parameter  $d$  is represented as

$$\theta_2 = \log d$$

and the prior is defined on  $\theta_2$ .

## Specification

The besag model is specified inside the `f()` function as

```
f(<whatever>, model="besagproper", graph.file=<graph file name>,  
  hyper=<hyper>)
```

The neighbourhood structure of  $\mathbf{x}$  is passed to the program through the `graph.file` argument. The structure of this file is described below.

## Hyperparameter spesification and default values

### hyper

#### theta1

```
name  log precision  
short.name  prec  
prior  loggamma  
param  1 5e-04  
initial  2  
fixed  FALSE  
to.theta  
from.theta
```

#### theta2

```
name  log diagonal
```

```

    short.name diag
    prior loggamma
    param 1 1
    initial 1
    fixed FALSE
    to.theta
    from.theta

constr FALSE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required TRUE

set.default.values TRUE

pdf besagproper

```

### Structure of the graph file

We describe the required format for the graph file using a small example. Let the file `gra.dat`, relative to a small graph of only 5 elements, be

```

5
1 1 2
2 2 1 3
3 3 2 4 5
4 1 3
5 1 3

```

Line 1 declares the total number of nodes in the graph (5), then, in lines 2-6 each node is described. For example, line 4 states that node 3 has 4 neighbours and these are nodes 2, 4 and 5.

The graph file can either have nodes indexed from 1 to  $n$ , or from 0 to  $n - 1$ . Note that in the latter case, node  $i$  seen from R corresponds to node  $i - 1$  in the 0-indexed graph.

### Example

```

## pick a graph
graph.file = system.file("demodata/germany.graph", package="INLA")
g = inla.read.graph(graph.file)

## we will use replicated samples in our testing
nrep = 5

## make life easy; use dense matrix algebra
d = 1.0
tau = 1.0
Q = matrix(0, g$n, g$n)

```

```

diag(Q) = tau * (d + g$nnbs)
for(i in 1:g$n) {
  if (g$nnbs[i] > 0) {
    Q[i, g$nnbs[[i]]] = -tau
    Q[g$nnbs[[i]], i] = -tau
  }
}
R = chol(Q) ## 'chol' returns the upper triangular

## simulate data with replications
y = c()
for(i in 1:nrep) {
  y = c(y, backsolve(R, rnorm(g$n)))
}

i = rep(1:g$n, nrep)
replicate = rep(1:nrep, each = g$n)
formula = y ~ f(i, model="besagproper", graph.file = graph.file,
  replicate=replicate,
  hyper = list(diag = list(param = c(1, 1)))) -1

## use 'exact' observations, so we fix the noise precisin to a high
## value
r = inla(formula,
  data = data.frame(y, i, replicate),
  family = "gaussian",
  control.data = list(
    hyper = list(
      prec = list(
        initial = 10,
        fixed=TRUE))))

```

## Notes

If  $d = 0$  and the parameter `rankdef=1` is set, then this model corresponds to the `besag` model. `constr=FALSE` is default for this model.