Correlated random effects: `iid1d`, `iid2d`, `iid3d`

This model is available for dimensions $p = 1$, 2 and 3. We describe in detail the case for $p = 2$, and then the changes required for $p = 1$ and $p = 3$.

Parametrization

The 2-dimensional Normal-Wishard model is used if one want to define two vectors of "random effects", u and v, say, for which (u_i, v_i) are iid bivariate Normals

$$\begin{pmatrix} u_i \\ v_i \end{pmatrix} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{W}^{-1}\right)$$

where the covariance matrix $\mathbf{W}^{-1}$ is

$$\mathbf{W}^{-1} = \begin{pmatrix} 1/\tau_a & \rho\sqrt{\tau_a \tau_b} \\ \rho\sqrt{\tau_a \tau_b} & 1/\tau_b \end{pmatrix} \tag{1}$$

and τ_a, τ_b and ρ are the hyperparameters. For these models the precision matrix $\mathbf{W}$ is Wishart distributed

$$\mathbf{W} \;\sim\; \text{Wishart}_p(r, \mathbf{R}^{-1}), \quad p = 2$$

with density

$$\pi(\mathbf{W}) = c^{-1} |\mathbf{W}|^{(r-(p+1))/2} \exp\left\{-\frac{1}{2}\text{Trace}(\mathbf{W}\mathbf{R})\right\}, \quad r > p+1$$

and

$$c = 2^{(rp)/2} |\mathbf{R}|^{-r/2} \pi^{(p(p-1))/4} \prod_{j=1}^{p} \Gamma((r+1-j)/2).$$

Then,

$$\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r-(p+1)).$$

Hyperparameters

The hyperparameters are

$$\theta = (\log \tau_a, \log \tau_b, \tilde{\rho})$$

where

$$\rho = 2\frac{\exp(\tilde{\rho})}{\exp(\tilde{\rho}) + 1} - 1$$

The prior-parameters are

$$(r, R_{11}, R_{22}, R_{12})$$

where

$$\mathbf{R} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

and $r_{12} = R_{21}$ due to symmetry.

The `inla` function reports posterior distribution for the hyperparameters τ_a, τ_b, ρ in equation (1).

The prior for θ is **fixed** to be `wishart2d`

Specification

The model `iid2d` is specified as

```
    y ~ f(i, model="iid2d",n = <length>, param=<param.vector(4 elements)>) + ...
```

and the `iid2d` model is represented internally as one vector of length n,

$$(u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_m)$$

where $n = 2m$, and n is the (required) argument in `f()`.

For this model the argument `constr=TRUE` is interpreted as

$$\sum u_i = 0, \qquad \text{and} \qquad \sum v_i = 0.$$

Example

In this example we implement the model

$$y|\eta \sim \text{Pois}(\exp(\eta))$$

where

$$\eta = a + b + 1$$

and a and b are correlated as described above.

```
n = 1000
N = 2*n
rho = 0.5
## set variances
Sigma = matrix(c(1/1, NA, NA, 1/2), 2, 2)
## and the correlation
Sigma[1,2] = Sigma[2,1] = rho*sqrt(Sigma[1,1]*Sigma[2,2])

## need it to simulate data
library(mvtnorm)

if (TRUE)
{
    ## first example

    y = yy = rmvnorm(n, sigma=Sigma)
    y = c(y[,1], y[,2])

    i = 1:N
    formula = y ~ f(i, model="iid2d", n=N)

    r = inla(formula, data = data.frame(i,y),
            control.data=list(initial=10,fixed=TRUE))
    print(summary(r))
```

```
    print(1/diag(cov(yy)))
    print(cor(yy)[1,2])
}

if (TRUE)
{
    ## second example

    y = yy = rmvnorm(n, sigma=Sigma)
    z = rnorm(n)
    zz = rnorm(n)
    y = y[,1] + z*y[,2] + zz
    i = 1:n
    j = n + 1:n
    formula = y ~ f(i, model="iid2d", n=N) + f(j,z,copy="i") + zz

    r = inla(formula, data = data.frame(i,j,y,z,zz),
            control.data=list(initial=10,fixed=TRUE))
    print(summary(r))
    print(1/diag(cov(yy)))
    print(cor(yy)[1,2])
}
```

The case $p = 1$

For $p = 1$ the hyperparameter is the marginal precision

$$\theta = \log \tau_1$$

The prior is fixed to be `wishart1d` with parameters

$$param = \; r \; R_{11}$$

where

$$\mathbf{R} = \begin{bmatrix} R_{11} \end{bmatrix}$$

The case $p = 3$

For $p = 3$ the hyperparameters are

$$\theta = (\log \tau_1, \log \tau_2, \log \tau_3, \tilde{\rho}_{12}, \tilde{\rho}_{13}, \tilde{\rho}_{23})$$

The prior is fixed to be `wishart3d` with parameters

$$param = \; r \; R_{11} \; R_{22} \; R_{33} \; R_{12} \; R_{13} \; R_{23}$$

where

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{12} & R_{22} & R_{23} \\ R_{13} & R_{23} & R_{33} \end{bmatrix}$$

The reported hyperparameters are the marginal precisions τ_1, τ_2 and τ_3 and the correlations ρ_{12}, ρ_{13} and ρ_{23}.

In this case, the internal representation is given as

$$(u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_m, w_1, w_2, \ldots, w_m)$$

where $n = 3m$ is a required argument, and where (u_i, v_i, w_i) are trivariate iid Normal.

Notes

The model `iid1d` is similar to the model `iid` (and included for completeness only). The prior for `iid1d` is fixed to be Wishart-distributed, which reduces to a Gamma-distribution for the precision with parameters

$$a = r/2 \qquad \text{and} \qquad b = R_{11}/2$$

hence

```
y ~ f(i, model="iid1d", param=c(3, 4))
```

is equivalent to

```
y ~ f(i, model="iid", param=c(1.5, 2), prior="loggamma")
```