Negative Binomial

Parametrisation

The negative Binomial distribution is

$$\text{Prob}(y) = \frac{\Gamma(y+n)}{\Gamma(n)\Gamma(y+1)} p^n (1-p)^y$$

for responses $y = 0, 1, 2, \ldots$, where

n**:** number of successful trials, or dispersion parameter. Must be strictly positive, need not be integer.

p**:** probability of success in each trial.

Link-function

The mean and variance of y are given as

$$\mu = n\frac{1-p}{p} \qquad \text{and} \qquad \sigma^2 = \mu(1+\frac{\mu}{n})$$

and the mean is linked to the linear predictor by

$$\mu = E\exp(\eta)$$

where the hyperparameter n (or the *size*) plays the role of an overdispersion parameter. E represents knows constant and $\log(E)$ is the offset of η.

Hyperparameters

The overdispersion parameter n is represented as

$$\theta = \log(n)$$

and the prior is defined on θ.

Specification

- family = `nbinomial`
- Required arguments: y and E (default $E = 1$).

Hyperparameter spesification and default values

hyper

theta

name size
short.name size
initial 2.30258509299405
fixed FALSE
prior loggamma
param c(1, 100)

survival FALSE

discrete TRUE

Example

In the following example we estimate the parameters in a simulated example with negative binomial responses and assign the hyperparameter θ a Gaussian prior with mean 0 and precision 0.01

```
n=100
a = 1
b = 1
E = rep(1,n)
z = rnorm(n)
eta = a + b*z
mu = E*exp(eta)
size = 15
prob = size/(size + mu)
y = rnbinom(n, size=size, prob = prob)

data = list(y=y,z=z)
formula = y ~ 1+z
result = inla(formula, family = "nbinomial", data = data, E=E,
              control.data = list(hyper = list(
                                         theta = list(
                                                 prior="gaussian",
                                                 param = c(0,0.01)))))
summary(result)
```

Notes

As $n \to \infty$, the negative Binomial converges to the Poisson distribution. For numerical reasons, if n is too large:

$$\frac{\mu}{n} < 10^{-4},$$

then the Poisson limit is used.