Besag model for spatial effects

Parametrization

The besag model for random vector $\mathbf{x} = (x_1, \ldots, x_n)$ is defined as

$$x_i | x_j, i \neq j, \tau \sim \mathcal{N}(\frac{1}{n_i} \sum_{i \sim j} x_j, \frac{1}{n_i \tau}) \tag{1}$$

where n_i is the number of neighbours of node i, $i \sim j$ indicates that the two nodes i and j are neighbours.

Hyperparameters

The precision parameter τ is represented as

$$\theta = \log \tau$$

and the prior is defined on θ.

Specification

The besag model is specified inside the `f()` function as

```
f(<whatever>,model="besag",graph.file=<graph file name>
          prior=c(<prior.model.theta>),
          param=c(<param.prior.theta1>))
```

The neighbourhood structure of $\mathbf{x}$ is passed to the program through the `graph.file` argument. The structure of this file is described below.

Structure of the graph file

We describe the required format for the graph file using a small example. Let the file `gra.dat`, relative to a small graph of only 5 elements, be

```
5
1  1  2
2  2  1  3
3  3  2  4  5
4  1  3
5  1  3
```

Line 1 declares the total number of nodes in the graph (5), then, in lines 2-6 each node is described. For example, line 4 states that node 3 has 4 neighbours and these are nodes 2, 4 and 5.

The graph file can either have nodes indexed from 1 to n, or from 0 to $n-1$.

Example

For examples of application of this model see the `Bym`, `Munich`, `Zambia` or `Scotland` examples in Volume I.

Notes

The besag model is a intrinsic random field.