Mean-Field-Type Games for Blockchain-Based Distributed Power Networks

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Abstract. In this paper we examine mean-field-type games in blockchain-based distributed power networks with several different entities: investors, consumers, prosumers, producers and miners. Under a simple model of jump-diffusion and regime switching processes, we identify risk-aware mean-field-type optimal strategies for the decision-makers.

Keywords: Blockchain · Bond · Cryptocurrency · Mean-field game Oligopoly · Power network · Stock

1 Introduction

This paper introduces mean-field-type games for blockchain-based smart energy systems. The cryptocurrency system consists in a peer to peer electronic payment platform in which the transactions are made without the need of a centralized entity in charge of authorizing them. Therefore, the aforementioned transactions are validated/verified by means of a coded scheme called *blockchain* [1]. In addition, the blockchain is maintained by its participants, which are called *miners*.

Blockchain or distributed ledger technology is an emerging technology for peer-to-peer transaction platforms that uses decentralized storage to record all transaction data [2]. One of the first blockchain applications was developed in the e-commerce sector to serve as the basis for the cryptocurrency "Bitcoin" [3]. Since then, several other altcoins and cryptocurrencies including Ethereum, Litecoin, Dash, Ripple, Solarcoin, Bitshare etc have been widely adopted and are all based on blockchain. More and more new applications have recently been emerging that add to the technology's core functionality - decentralized storage of transaction data - by integrating mechanisms that allow for the actual transactions to be implemented on a decentralized basis. The lack of a centralized entity, that could have control over the security of transactions, requires

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the development of a sophisticated verification procedure to validate transactions. Such task is known as *Proof-of-Work*, which brings new technological and algorithmic challenges as presented in [4]. For instance, [5] discusses the sustainability of bitcoin and blockchain in terms of the needed energy in order to perform the verification procedure. In [6], algorithms to validate transactions are studied by considering propagation delays. On the other hand, alternative directions are explored in order to enhance the blockchain, e.g., [7] discusses how the blockchain-based identity and access management systems can be improved by using an *Internet of Things* security approach.

In this paper the possibility of implementing distributed power networks on the blockchain and its pros and contras are presented. The core model (Fig. 1) uses a Bayesian mean-field-type game theory on the blockchain. The base interaction model considers producers, consumers and a new important element of distributed power networks called prosumers. A prosumer (producer-consumer) is a user that not only consumes electricity, but can also produce and store electricity [8,9]. We identify and formulate the key interactions between consumers, prosumers and producers on the blockchain. Based on forecasted demand generated from the blockchain, each producer determines its production quantity, its mismatch cost, and engages an auction mechanism to the prosumer market on the blockchain. The resulting supply is completed by the prosumers auction market. This determines a market price, and the consumers react to the offers and the price and generate a certain demand. The consistency relationship between demand and supply provides a fixed-point system, whose solution is a mean-field-type equilibrium [10].

The rest of paper is organized as follows. The next subsection presents the emergence of decentralized platform. Section 3 focuses on the game model. Section 4 presents risk-awareness and price stability analysis. Section 5 focuses on consumption-insurance and investment tradeoffs.

2 Towards a Decentralized Platform

The distributed ledger technology is a peer-to-peer transaction platform that integrates mechanisms that allow decentralized transactions or decentralized and distributed exchange system. These mechanisms, called "smart contracts", operate on the basis of individually defined rules (e.g. specifications as to quantity, quality, price, location) that enable an autonomous matching of distributed producers and their prospective customers. Recently the energy sector is also moving towards a semi-decentralized platform with the integration of prosumers' market and aggregators to the power grid. Distributed power is a power generated at or near the point of use. This includes technologies that supply both electric power and mechanical power. In electrical applications, distributed power systems stand in contrast to central power stations that supply electricity from a centralized location, often far from users. The rise of distributed power is being driven by broader decentralization movement of smarter cities. With blockchain transaction, every participant in a network can transact directly with every other network participant without involving a third-party intermediary (aggregator, operator). In other words, aggregators and the third parties are replaced by the blockchain. All transaction data is stored on a distributed blockchain, with all relevant information being stored identically on the computers of all participants, all transactions are made on the basis of smart contracts, i.e., based on predefined individual rules concerning quality, price, quantity, location, feasibility etc.

2.1 A Blockchain for Underserved Areas

One of the first questions that rises in blockchain is the service to Society. An authentication service offering to make environment-friendly (solar/wind/hydro) energy certificates available via a blockchain. The new service works by connecting solar panels and wind farms to an Internet of Things (IoT)-enabled device that measures the quality (of the infrastructure), quantity and the location of the power produced and fed into the grid. Certificates supporting PV growth and wind power can be bought and sold anonymously via a blockchain platform. Then, solar and wind energy produced by prosumers in undeserved areas can be transmitted to end-users. SolarCoin [11] was developed following that idea, with blockchain technology to generate an additional reward for solar electricity producers. Solar installation owners registering to the SolarCoin network receive one SolarCoin for each MWh of solar electricity that they produce. This digital asset will allow solar electricity producers to receive an additional reward for their contribution to the energy transition, which will develop itself through network effect. SolarCoin is freely distributed to any owner of a solar installation owner. Participating in the SolarCoin program can be done online, directly on the SolarCoin website. As of October 2017, more than 2,134,893 MWh of solar energy have been incentivized through SolarCoin across 44 countries. The ElectriCChain aims to provide the bulk of Blockchain recording for the solar installation owners in order to micro-finance the solar installation, incentivize it (through the SolarCoin tool), and monitor the install production. The idea of Wattcoin is to build this scheme for other renewable energies such as wind, thermo, hydro power plants to incentivize global electricity generation from several renewable energy sources. The incentive scheme influences the prosumers decision because they will be rewarded in WattCoins as an additional incentive to initiate the energy transition and possibly to compensate a fraction of the peak-hours energy demand.

2.2 Security, Energy Theft and Regulation Issues

If fully adopted, blockchain-based distributed power networks (b-DIPONET) is not without challenge. One of the challenges is security. This includes not only network security but also robustness, double spending and false/fake accounts. Stokens are regulated securities tokens built on the blockchain using smart contracts. They provide a way for accredited investors to interact with regulated companies through a digital ecosystem. Currently, the cryptocurrency industry has enormous potential - but it needs to be accompanied properly.

The blockchain technology can be used to reduce energy theft and unpaid bills by means of the automation of the prosumers who are connected to the power grid and their produced energy data is monitored in the network.

3 Mean-Field-Type Game Analysis

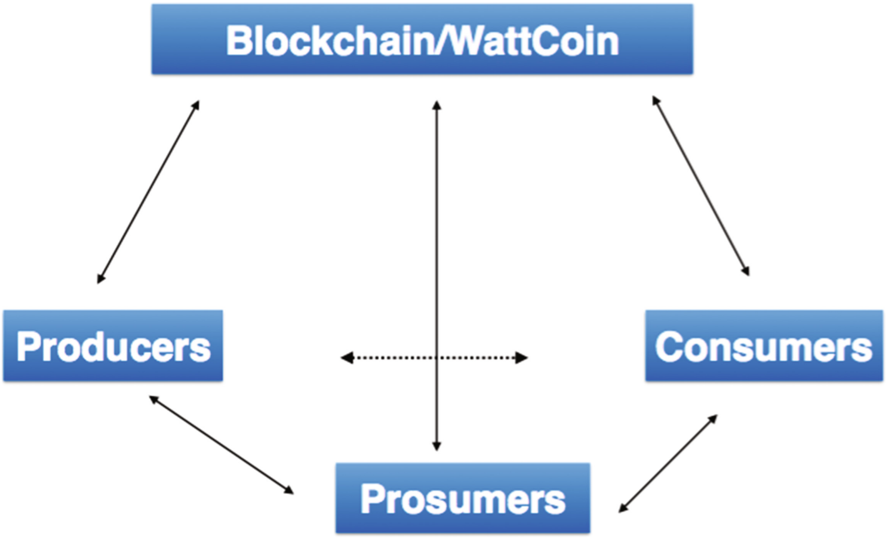

Fig. 1. Interaction blocks for blockchain-based distributed power networks.

This section presents the base mean-field-type game model. We identify and formulate the key interactions between consumers, prosumers and producers (see Fig. 1). Based on the forecasted demand from the blockchain-based history matching, each prosumer determines its production quantity, its mismatch cost, and use the blockchain to respond directly to consumers. All the energy producers together are engaged in a competitive energy market share. The resulting supply is completed by the prosumers energy market. This determines a market price, and the consumers react to the price and generate a demand. The consistency relationship between demand and supply of the three components provides a fixed-point system, whose solution is a mean-field equilibrium.

3.1 The Game Setup

Consumer i can decide to install a solar panel on her roof or a wind power station. Depending on sunlight or wind speed consumer i may produce surplus energy. She is no longer just an energy consumer but a prosumer. *A prosumer can decide to participate or not to the blockchain.* If the prosumer decides to participate to the blockchain to sell her surplus energy, the energy produced by this prosumer is measured by a dedicated meter which is connected and linked to the blockchain. The measurement and the validation is done ex-post from the quality-of-experience of the consumers of prosumer i. The characteristics and the bidding price of the energy produced by the prosumer are registered in the blockchain. This allows to give a certain score or Wattcoin to that prosumer for incentivization and participation level. This data is public if in the public blockchain's distributed register. All the transactions are verified and validated by the users of the blockchain ex-post. If the energy transaction does not happen in the blockchain platform, the proof-of-validation is simply an ex-post quality-experience measurement and therefore it does not need to use the heavy proof-of-work used by some crypto-currencies. The adoption of energy transactions to be blockchain requires a significantly reduction of the energy consumption of the proof-of-work itself. If the proof-of-work is energy consuming (and costly) then the energy transactions is kept to the traditional channel and only proof-of-validation is used as a recommendation system to monitor and to incentivize the prosumers. The blockchain technology makes it public and more transparent. If j and k are neighbors of the location of where i produced the energy, j and k can buy electricity off him and the consumption needs recorded in the blockchain ex-post. The transactions need to be technically secure and automated. Once prosumer i reaches a total of 1 MWh of energy sold to its neighbors, consumer i gets an equivalent of a certain unit of blockchain cryptocurrency such as Wattcoin, WindCoin, Solarcoin etc. It is an extra reward to the revenue of the prosumer. This scheme incentivizes prosumers to participate and promotes environment-friendly energy. Instead of a digitally mined product (transaction), the WattCoin proof-of-validity happens in the physical world, and those who have wind/thermo/photovoltaic arrays can earn Wattcoin just for generating electricity and serving it successfully. It is essentially a global rewarding/loyalty program, and is designed to help incentivize more renewable electricity production, while also serving as a lower-carbon cryptocurrency than Bitcoin and similar alternative currencies.

Each entity can

- Purchase and supply energy and have automated and verifiable proof of the amounts of green energy purchased/supplied via the information stored on the blockchain.
- Ensure that local generation (and feasibility) is supported, as it becomes possible to track the exact geographical origin of each energy MWh produced. For example, it becomes possible to pay additional premiums for green energy if it is generated locally, to promote further local energy generation capacity. Since the incentive reward is received only ex-post by the prosumer after checking the quality-of-experience, the proof-of-validity will improve the feasibility status of the energy supply and demand.

- Spatial energy price (price field) is publicly available to the consumers and prosumers who would like to purchase. This includes production cost and migration/distribution fee for moving energy from its point of production to its point of use.
- Each producer can supply energy on the platform and make smart contract for the delivery.
- Miners can decide to mine environment-friendly energy blocks. Honest miners are entities or people who validate the proof-of-work or proof-of-stakes (or other scheme). This can be individual, a pool or a coalition. There should be an incentive for them to mine. Selfish miners are those who may aim to pool their effort to maximize their own-interest. This can be individual, a pool or a coalition. Deviators or Malicious miners are entities or people who buy tokens for market and vote to impose their version of blockchain (different assigns at different block).

The game is described by the following four key elements:

- Platform: A Blockchain
- Players: Investors, consumers, prosumers, producers, miners.
- Decisions: Each player can decide and act via the blockchain.
- Outcomes: The outcome is given by gain minus loss for each participant.

Note that in this model, there is no energy trading option on the blockchain. However, the model can be modified to include trading at some part of the private blockchain. The electricity price dynamics regulation and stability will be discussed below.

3.2 Analysis

How can blockchain improve the penetration rate of renewable energy?
Thanks to the blockchain-based incentive, a non-negligible portion of prosumers will participate to the program. This will increase the produced renewable energy volumes. A basic rewarding scheme is that simple and easy to implement is a Tullock-like scheme, where probabilities to win a winner-take-all contest are considered, defining some *constest success functions* [12–14]. It consists of taking a spatial rewarding scheme to be added to the prosumers if a certain number of criteria are satisfied. In terms of incentives, a prosumer producing energy from location x will be rewarded ex-post $R(x)$ with probability $\frac{h_j(x,a_j)}{\sum_{i=1}^n h_i(x,a_i)}$ if $\sum_{i=1}^n h_i(x,a_i) > R(x) > 0$, where h_i is non-decreasing in its second component. Clearly, with this incentive scheme, a non-negligible portion of producers can reinvest more funds in the renewable energy production.

Implementation Cost
We identify basic costs for the blockchain-based energy system need to be implemented properly with largest coverage. As the next generation wireless communication and internet-of-everything is moving toward advanced devices with

high-speed, well-connected and more security and reliability than the previous version, blockchain technology should take advantage of it to decentralized operation. The wireless communication devices can be used as hotspots to connect to the blockchain as mobile calls are using wireless access points and hotspots as relays. Thus, a large coverage of the technology as related to the wireless coverage and connectivity of the location. Thus, the cost is reflected to the consumers and to the producers from their internet subscription fees. In addition to that cost, miners operations consume energy and powers. Supercomputers (CPUs, GPUs) and operating machines cost should be added to.

Demand-Supply Mismatch Cost

Let $\mathscr{T} := [t_0, t_1]$ be the time horizon with $t_0 < t_1$. In presence of blockchain, prosumers aim to anticipate their production strategies by solving the following problem:

$$\begin{cases} \inf_s \mathbb{E}L(s, e, \mathscr{T}) \\ L(s, e, \mathscr{T}) = l_{t_1}(e(t_1)) + \int_{t_0}^{t_1} l(t, D(t) - S(t))\, dt \\ \frac{d}{dt} e_{jk}(t) = x_{jk}(t) \mathbb{1}_{\{k \in A_j(t)\}} - s_{jk}(t), \\ n \geq 1, \\ j \in \{1, \ldots, n\}, \\ k \in \{1, \ldots, K_j\}, \\ K_j \geq 1, \\ x_{jk}(t) \geq 0, \\ s_{jk}(t) \in [0, \bar{s}_{jk}],\ \forall j, k, t \\ \bar{s}_{jk} \geq 0, \\ e_{jk}(t_0) \text{ given}, \end{cases} \tag{1}$$

where

- the instant loss is $l(t, D(t) - S(t))$, l_{t_1} is the terminal loss function.
- the energy supply at time t is

  $$S(t) = \sum_{j=1}^{n} \sum_{k=1}^{K_j} s_{jk}(t),$$

  $s_{jk}(t)$ is the production rate of power plant/generator k of prosumer j at time t, $\bar{s}_{jk}$ is an upper bound for s_{jk} which will be used as a control action.
- The stock of energy $e_{jk}(t)$ of prosumer j at power plant k at time t is given by the following classical motion dynamics:

  $$\frac{d}{dt} e_{jk}(t) = \text{incoming flow}_{jk}(t) - \text{outgoing flow}_{jk}(t), \tag{2}$$

  The incoming flow happens only when the power station is active. In that case, the arrival rate is $x_{jk}(t)\mathbb{1}_{\{k \in A_j(t)\}}$ where $x_{jk}(t) \geq 0$, and the set of active power plant of j is defined by $A_j(t)$, the set of all active power plants is $A(t) = \cup_j A_j(t)$. $D(t)$ is the demand on the blockchain at time t.

In general, the demand needs to be anticipated/estimated/predicted so that the produced quantity is enough to serve the consumers. If the supply S is less than

D some of the consumers will not be served, hence it is costly for the operator. If the supply S is greater that D then the operator needs to store the exceed amount of energy. It will be lost if the storage is enough. Thus, it is costly in both cases, and the cost is represented by $l(\cdot, D-S)$. The demand-supply mismatch cost is determined by solving (1).

3.3 Oligopoly with Incomplete Information

There are $n \geq 2$ potential interacting energy producers over the horizon $\mathscr{T}$. At time $t \in \mathscr{T}$, producer i's output is $u_i(t) \geq 0$. The dynamics of the log-price, $p(t) :=$ logarithm of the price of energy at time t, is given by $p(t_0) = p_0$ and

$$dp(t) = \eta[a - D(t) - p(t)]dt + \left(\sigma dB(t) + \int_{\theta\in\Theta} \mu(\theta)\tilde{N}(dt, d\theta)\right) + \sigma_o dB_o(t), \quad (3)$$

where

$$D(t) := \sum_{i=1}^{n} u_i(t),$$

is the supply at time $t \in \mathscr{T}$, and B_o is standard Brownian motion representing a global uncertainty observed by all participant to the market. The processes B and N describe local uncertainties or noises. B is a standard Brownian motion, N is a jump process with Lévy measure $\nu(\mathrm{d}\theta)$ defined over Θ. It is assumed that ν is a Radon measure over Θ (the jump space) which is subset of $\mathbb{R}^m$. The process

$$\tilde{N}(\mathrm{d}t, \mathrm{d}\theta) = N(\mathrm{d}t, \mathrm{d}\theta) - \nu(\mathrm{d}\theta)\mathrm{d}t$$

is the compensated martingale. We assume that all these processes are mutually independent. Denote by $\mathscr{F}_t^{B,N,B_o}$ the natural filtration generated by the union of events $\{B, N, B_o\}$ up to time t, and by $(\mathscr{F}_t^{B_o}, t \in \mathscr{T})$ the natural filtration generated by the observed common noise, where $\mathscr{F}_t^{B_o} = \sigma(B_0(s), s \leq t)$ is the smallest σ-field generated by the process B_0 up to time t (see e.g. [15]).

The number η is positive. For larger values of the real number η the market price adjusts quicker along the inverse demand, all in the logarithmic scale. The terms a, σ, σ_o are fixed constant parameters. The jump rate size $\mu(\cdot)$ is in $L^2_\nu(\Theta, \mathbb{R})$ i.e.

$$\int_\Theta \mu^2(\theta)\nu(\mathrm{d}\theta) < +\infty.$$

The initial distribution of $p(0)$ is square integrable: $\mathbb{E}[p_0^2] < \infty$.

Producers know only their own types $(c_i, r_i, \bar{r}_i)$ but not the types of the others $(c_j, r_j, \bar{r}_j)_{j\neq i}$. We define a game with incomplete information denoted by G_ξ. The game G_ξ has n producers. A strategy for producer j is a map $\tilde{u}_j : I_j \to U_j$ prescribing an action for each possible type of producer j. We denote the set of actions of producer j by $\tilde{\mathscr{U}}_j$. Let ξ_j denote the distribution on the type vector $(c_j, r_j, \bar{r}_j)$ from the perspective of the jth producer. Given ξ_j, producer j can compute the conditional distribution $\xi_{-j}(c_{-j}, r_{-j}, \bar{r}_{-j}|c_j, r_j, \bar{r}_j)$, where

$$c_{-j} = (c_1, \ldots, c_{j-1}, c_{j+1}, \ldots, c_n) \in \mathbb{R}^{n-1}.$$

Producer j can then evaluate her expected payoff based on the expected types of other producers. We call a Nash equilibrium of G_ξ Bayesian equilibrium as.

At time $t \in \mathcal{T}$, producer i receives $\hat{p}(t)u_i - C_i(u_i)$ where $C_i : \mathbb{R} \to \mathbb{R}$, given by

$$C_i(u_i) = c_i u_i + \frac{1}{2} r_i u_i^2 + \frac{1}{2} \bar{r}_i \hat{u}_i^2,$$

is the instant cost function of i. The term $\hat{u}_i = \mathbb{E}[u_i \mid \mathcal{F}_t^{B_o}]$ is the conditional expectation of producer i's output given the global uncertainty B_o observed in the market. The last term $\frac{1}{2}\bar{r}_i \hat{u}_i^2$, in the expression of the instant cost C_i, aims to capture the risk-sensitivity of producer i. The conditional expectation of the price given the global uncertainty B_o up to time t is $\hat{p}(t) = \mathbb{E}[p(t) \mid \mathcal{F}_t^{B_o}]$. At the terminal time t_1 the revenue is $-\frac{q}{2} e^{-\lambda_i t_1} \left(p(t_1) - \hat{p}(t_1)\right)^2$. The long-term revenue of producer i is

$$R_{i,\mathcal{T}}(p_0, u) = -\frac{q}{2} e^{-\lambda_i t_1} \left(p(t_1) - \hat{p}(t_1)\right)^2 + \int_{t_0}^{t_1} e^{-\lambda_i t} \left[\hat{p} u_i - C_i(u_i)\right] \mathrm{d}t,$$

where λ_i is a discount factor of producer i. Finally, each producer optimizes her long-term expected revenue. The case of deterministic complete information was investigated in [16, 17]. Extension of the complete information to the stochastic case with mean-field term was done recently in [18]. Below, we investigate the equilibrium solution under incomplete information.

3.3.1 Bayesian Mean-Field-Type Equilibria

A Bayesian-Nash Mean-Field-Type Equilibrium is defined as a strategy profile and beliefs specified for each producer about the types of the other producers that minimizes the expected performance functional for each producer given their beliefs about the other producers' types and given the strategies played by the other producers. We compute the generic expression of the Bayesian mean-field-type equilibria.

Any strategy $u_i^* \in \tilde{\mathcal{U}_i}$ satisfying the maximum in

$$\begin{cases} \max_{u_i \in \tilde{\mathcal{U}_i}} \mathbb{E}\left[R_{i,\mathcal{T}}(p_0, u) \mid c_i, r_i, \bar{r}_i, \xi\right], \\ dp(t) = \eta \left[a - D(t) - p(t)\right] dt + \left(\sigma dB(t) + \int_\Theta \mu(\theta) \tilde{N}(dt, d\theta)\right) \\ \qquad\qquad + \sigma_o dB_o(t), \\ p(t_0) = p_0, \end{cases} \tag{4}$$

is called a Bayesian best-response strategy of producer i to the other producers strategy $u_{-i} \in \prod_{j \neq i} \tilde{\mathcal{U}_j}$.

Generically, Problem (4) has the following interior solution:

The Bayesian equilibrium strategy in state-and-conditional mean-field feedback form and is given by

$$\tilde{u}_i^*(t) = -\frac{\eta \hat{\alpha}_i(t)}{r_i} (p(t) - \hat{p}(t)) + \frac{\hat{p}(t)(1 - \eta \hat{\beta}_i(t)) - (c_i + \eta \hat{\gamma}_i)(t)}{r_i + \bar{r}_i},$$

where the conditional equilibrium price $\hat{p}$ is

$$\begin{cases} d\hat{p}(t) = \eta\left\{a + \frac{c_i+\eta\hat{\gamma}_i(t)}{r_i+\bar{r}_i} + \int \sum_{j\neq i} \frac{c_j+\eta\hat{\gamma}_j(t)}{r_j+\bar{r}_j} d\xi_{-i}(.|c_i, r_i, \bar{r}_i)\right. \\ \left. -\hat{p}(t)\left(1 + \frac{1-\eta\hat{\beta}_i(t)}{r_i+\bar{r}_i} + \int \sum_{j\neq i} \frac{1-\eta\hat{\beta}_j(t)}{r_j+\bar{r}_j} d\xi_{-i}(.|c_i, r_i, \bar{r}_i)\right)\right\} dt + \sigma_o \mathrm{d}B_o(t), \\ \hat{p}(t_0) = \hat{p}_0, \end{cases}$$

and the random parameters $\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\delta}$ solve the stochastic Bayesian Riccati system:

$$\begin{cases} d\hat{\alpha}_i(t) = \left\{(\lambda_i + 2\eta)\hat{\alpha}_i(t) - \frac{\eta^2}{r_i}\hat{\alpha}_i^2(t) - 2\eta^2\hat{\alpha}_i(t) \int \sum_{j\neq i} \frac{\hat{\alpha}_j(t)}{r_j} d\xi_{-i}(.|c_i, r_i, \bar{r}_i)\right\} dt \\ \qquad + \hat{\alpha}_{i,o}(t)dB_o(t), \\ \hat{\alpha}_i(t_1) = -q, \\ \\ d\hat{\beta}_i(t) = \left\{(\lambda_i + 2\eta)\hat{\beta}_i(t) - \frac{(1-\eta\hat{\beta}_i(t))^2}{r_i+\bar{r}_i} + 2\eta\hat{\beta}_i(t) \int \sum_{j\neq i} \frac{1-\eta\hat{\beta}_j(t)}{r_j+\bar{r}_j} d\xi_{-i}(.|c_i, r_i, \bar{r}_i)\right\} dt \\ \qquad +\hat{\beta}_{i,o}(t)dB_o(t), \\ \hat{\beta}_i(t_1) = 0, \\ \\ d\hat{\gamma}_i(t) = \left\{(\lambda_i + \eta)\hat{\gamma}_i(t) - \eta a\hat{\beta}_i(t) - \hat{\beta}_{i,o}(t)\sigma_o + \frac{(1-\eta\hat{\beta}_i(t))(c_i+s\hat{\gamma}_i(t))}{r_i+\bar{r}_i}\right. \\ \left. + \eta\hat{\gamma}_i(t) \int \sum_{j\neq i} \frac{1-\eta\hat{\beta}_j(t)}{r_j+\bar{r}_j} d\xi_{-i}(.|c_i, r_i, \bar{r}_i) - \eta\hat{\beta}_i(t) \int \sum_{j\neq i} \frac{c_j+\eta\hat{\gamma}_j(t)}{r_j+\bar{r}_j} d\xi_{-i}(.|c_i, r_i, \bar{r}_i)\right\} dt \\ \qquad - \hat{\beta}_i(t)\sigma_o dB_o(t), \\ \hat{\gamma}_i(0) = 0, \\ \\ d\hat{\delta}_i(t) = -\left\{-\lambda_i\hat{\delta}_i(t) + \frac{1}{2}\sigma_o^2\hat{\beta}_i(t) + \frac{1}{2}\hat{\alpha}_i(t)\left(\sigma^2 + \int_\Theta \mu^2(\theta)\nu(d\theta)\right) + \eta a\hat{\gamma}_i(t)\right. \\ \qquad \left. +\hat{\gamma}_{i,o}(t)\sigma_o + \frac{1}{2}\frac{(c_i+\eta\hat{\gamma}_i(t))^2}{r_i+\bar{r}_i} + \eta\hat{\gamma}_i(t) \int \sum_{j\neq i} \frac{c_j+s\hat{\gamma}_j(t)}{r_j+\bar{r}_j} d\xi_{-i}(.|c_i, r_i, \bar{r}_i)\right\} dt \\ \qquad -\sigma_o\hat{\gamma}_i(t)\mathrm{d}B_o(t), \\ \hat{\delta}_i(t_1) = 0, \end{cases}$$

and the equilibrium revenue of producer i is

$$\mathbb{E}\left[\frac{1}{2}\hat{\alpha}_i(t_0)(p(t_0) - \hat{p}_0)^2 + \frac{1}{2}\hat{\beta}_i(t_0)\hat{p}_0^2 + \hat{\gamma}_i(t_0)\hat{p}_0 + \hat{\delta}_i(t_0)\right].$$

The proof of the Bayesian Riccati system follows from a Direct Method by conditioning on the type $(c_i, r_i, \bar{r}_i, \xi)$.

Noting that the Riccati system of the Bayesian mean-field-type game is different from the Riccati system of mean-field-type game, it follows that the Bayesian equilibrium costs are different. They become equal when $\xi_{-j} = \delta_{(c_{-j}, r_{-j}, \bar{r}_{-j})}$. This also shows that there is a value of information in this game. Note that the equilibrium supply is

$$\sum_i \tilde{u}_i^*(t) = -\eta(p(t) - \hat{p}(t)) \sum_i \frac{\hat{\alpha}_i(t)}{r_i} + \sum_i \frac{\hat{p}(t)(1 - \eta\hat{\beta}_i(t)) - (c_i + s\hat{\gamma}_i(t))}{r_i + \bar{r}_i}.$$

3.3.2 Ex-Post Resilience

Definition 1. We define a strategy profile $\tilde{u}$ as ex-post resilient if for every type profile $(c_j, r_j, \bar{r}_j)_j$, and for each producer i,

$$\begin{aligned}\text{argmax}_{\tilde{u}_i \in \tilde{\mathscr{U}}_i} \mathbb{E} \int R_{i,\mathscr{T}}(p_0, c_i, , r_i, \bar{r}_i, \tilde{u}_i, \tilde{u}_{-i}) \xi_{-i}(dc_{-i} dr_{-i} d\bar{r}_{-i} \mid c_i, r_i, \bar{r}_i) \\ = \text{argmax}_{\tilde{u}_i \in \tilde{\mathscr{U}}_i} \mathbb{E} R_{i,\mathscr{T}}(p_0, \tilde{u}_i, \tilde{u}_{-i}).\end{aligned}$$

We show that generically the Bayesian equilibrium is not ex-post resilient. An $n-$tuple of strategies is said to be ex-post resilient if each producer's strategy is a best response to the other producers' strategies, under all possible realizations of the others' types. An ex-post resilient strategy must be an equilibrium of every game with the realized type profile $(c, r, \bar{r})$. Thus, any ex-post resilient strategy is a robust strategy of the game in which all the parameters $(c, r, \bar{r})$ are taken. Here, each producer makes her ex-ante decision based on ex-ante information, that is, distribution and expectation, which is not necessarily identical to her ex-post information, that is, the realized actions and types of other producers. Thus, ex-post, or after the producer observes the actually produced quantities of energy of all the other producers, she may prefer to alter her ex-ante optimal production decision.

4 Price Stability and Risk-Awareness

This section examines the price stability of a stylized blockchain-based market under regulation designs. As a first step we design a target price dynamics that allows a high volume of transactions while fulfilling the regulation requirement. However, the target price is not the market price. In a second step, we propose and examine a simple price market dynamics under jump-diffusion process. The market price model builds on the market demand, supply and token quantity. We use three different token supply strategies to evaluate the proposed market price motion. The first strategy designs a supply of tokens to the market more frequently balancing the mismatch between market supply and market demand. The second strategy is a mean-field control strategy. The third strategy is a mean-field-type control strategy that incorporates the risk of deviating from the regulation bounds.

4.1 Unstable and High Variance Market

As an illustration of high variance price, we take the fluctuations of bitcoin price between December 2017 and February 2018. The data is from coindesk (https://www.coindesk.com/price/). The price went from 10 K USD to 20 K USD and back to 7 K USD within 3 months. The variance was extremely high within that period, which implied very high risks in the market (Fig. 1). This extremely high variance and unstable market is far beyond the risk-sensitivity index distributions of users and investors. Therefore the market needs to be re-designed to fit investors and users risk-sensitivity distributions.

Fig. 2. Coindesk database: the price of bitcoin went from 10K USD to 20 K USD and back to below 7 K USD within 2–3 months in 2017–2018.

4.2 Fully Stable and Zero Variance

We have seen that the above example is too risky and is beyond the risk-sensitivity index of the many users. Thus, it is important to have a more stable market price in the blockchain.

A fully stable situation is the case of constant price. For that case the variance is zero and there is no risk on that market. However, this case may not be interesting for producers, and investors: if they know that the price will not vary they will not buy. Thus, the volume of transactions will be significantly reduced which is not convenient for the blockchain technology which aims to be a place of innovations and investments.

Electricity market price cannot be constant because demand is variable on a daily basis or from one season to another within the same year. Peak hours price may be different from off-peak hours price as it is already the case in most countries.

Below we propose a price dynamics that is somehow in between the two scenarios: it is of relatively low variance and it allows several transaction opportunities.

4.3 What Is a More Stable Price Dynamics?

An example of a more stable cryptocurrency within similar time frame as the bitcoin is the tether USD (USDT) which oscillates between 0.99 and 1.01 but with an important volume of transactions (see Fig. 2). The maximum magnitude variation of the price remains very small while the number oscillations in between is large, allowing several investment, buying/selling opportunities (Fig. 3).

Is token supply possible in the blockchain?
Tokens in blockchain-based cryptocurrencies are generated by blockchain algorithms. Token supply is a decision process that can be incorporated in the algorithm. Thus, token supply can be used to influence the market price. In our model below we will use it as a control action variable.

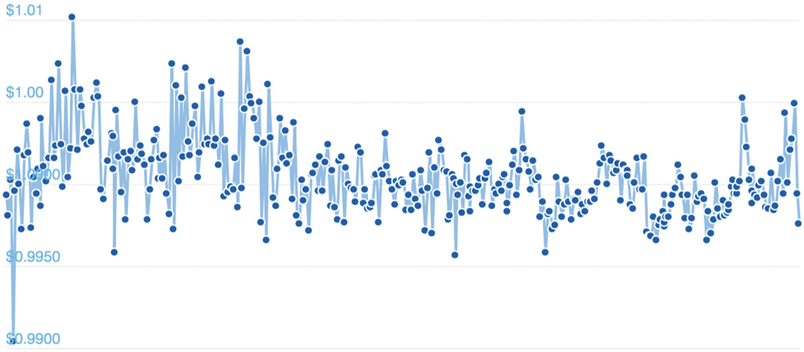

Fig. 3. Coindesk database: the price of tether USD went from 0.99 USD to 1.01 USD

4.4 A More Stable and Regulated Market Price

Let $\mathscr{T} := [t_0, t_1]$ be the time horizon with $t_0 < t_1$. There are n potential interacting regulated blockchain-based technologies over the horizon $\mathscr{T}$. The regulation authority of each blockchain-based technology has to choose the regulation bounds: the price of cryptocurrency i should be between $[\underline{p}_i, \bar{p}_i]$, $\quad \underline{p}_i < \bar{p}_i$. We construct a target price $p_{tp,i}$ from an historical data-driven price dynamics of i. The target price should stay within the interval $[\underline{p}_i, \bar{p}_i]$ target range. The market price $p_{mp,i}$ depends on the quantity of token supplied, demanded and is given by a simple price adjustment dynamics obtained from Roos 1925 (see [16,17]). The idea of the Roos's model is very simple: Suppose that the cryptocurrency authority supplies a very small number of token in total, it will result in high prices and if the authorities expect these high price conditions not to continue in the following period, they will raise the number of tokens and, as a result, the market price will decrease a bit. If low prices are expected to continue, the authorities will decrease the number of token, resulting again in higher prices. Thus, oscillating between periods of low number of tokens with high prices and high number of tokens with low prices, the set price-quantity traces out an oscillatory phenomenon (which will allow large volume of transactions).

4.4.1 Designing a Regulated Price Dynamics

For any given $\underline{p}_i < \bar{p}_i$ one can choose the coefficients $c, \hat{c}$ such that the target price $p_{tp,i}(t) \in [\underline{p}_i, \bar{p}_i]$ for all time t. An example of such an oscillatory function is as follows:

$$p_{tp,i}(t) = c_{i0} + \sum_{k=1}^{2} c_{ik} \cos(2\pi kt) + \hat{c}_{ik} \sin(2\pi kt),$$

with $c_{ik}, \hat{c}_{ik}$ to be designed to fulfill the regulation requirement. Let $c_{i0} := \frac{\underline{p}_i + \bar{p}_i}{2}$, $c_1 := \frac{\bar{p}_i - \underline{p}_i}{100}$, $\hat{c}_{i1} := \frac{\bar{p}_i - \underline{p}_i}{150}$, $c_{12} := \frac{\bar{p}_i - \underline{p}_i}{200}$, $\hat{c}_{12} := \frac{\bar{p}_i - \underline{p}_i}{250}$. We want the target function to stay between 0.98 USD and 1.02 USD we set $\underline{p}_i = 0.98, \bar{p}_i = 1.02$. Figure 4 plots such a target function.

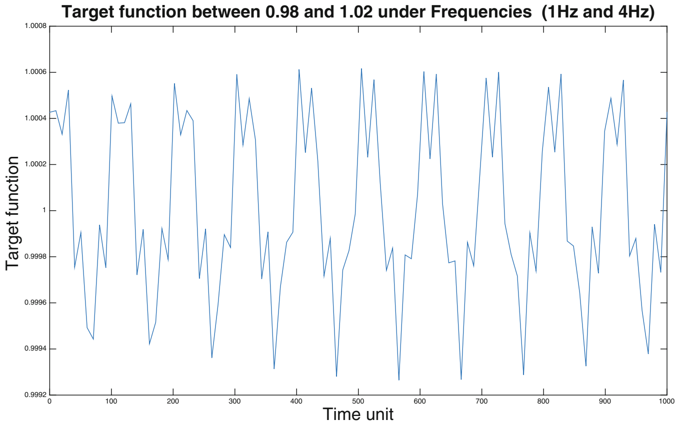

Fig. 4. Target price function $p_{tp,i}(t)$ between 0.98 and 1.02 under Frequencies (1 Hz and 4 Hz)

Note that this target price is not the market price. In order to incorporate a more realistic market behavior we introduce a dependence on demand and supply of tokens.

4.4.2 Proposed Price Model for Regulated Monopoly

We propose a market price dynamics that takes into consideration the market demand and the market supply. The blockchain-based market log-price (i.e. the logarithm of the price) dynamics is given by $p_i(t_0) = p_0$ and

$$\begin{aligned} dp_i(t) &= \eta_i[D_i(t) - p_i(t) - (S_i(t) + u_i(t))]dt \\ &\quad + \left(\sigma_i dB_i(t) + \int_{\theta\in\Theta} \mu_i(\theta)\tilde{N}_i(dt, d\theta)\right) + \sigma_o dB_o(t), \end{aligned} \tag{5}$$

where $u_i(t)$ is the total token injected to the market at time t, B_o is standard Brownian motion representing a global uncertainty observed by all participant to the market. As above, the processes B and N are local uncertainty or noise. B is a standard Brownian motion, N is a jump process with Lévy measure $\nu(\mathrm{d}\theta)$ defined over Θ. It is assumed that ν is a Radon measure over Θ (the jump space). The process

$$\tilde{N}(dt, d\theta) = N(dt, d\theta) - \nu(d\theta)dt,$$

is the compensated martingale. We assume that all these processes are mutually independent. Denote by $(\mathscr{F}_t^{B_o}, t \in \mathscr{T})$ the filtration generated by the observed common noise B_0 (see Sect. 3.3). The number η_i is positive. For larger values of

η_i the market price adjusts quicker along the inverse demand. a, σ, σ_o are fixed constant parameters. The jump rate size $\mu(.)$ is in $L^2_\nu(\Theta, \mathbb{R})$ i.e.

$$\int_\Theta \mu^2(\theta)\nu(\mathrm{d}\theta) < +\infty.$$

The initial distribution p_0 is square integrable: $\mathbb{E}[p_0^2] < \infty$.

4.4.3 A Control Design that Tracks the Past Price

We formulate a basic control design that tracks the past price and the trend. A typical example is to choose the control action $u_{ol,i}(t) = -p_{tp,i}(t) + D_i(t) - S_i(t)$. This is an open-loop control strategy if D_i and S_i are explicit functions of time. Then the price dynamics becomes

$$\begin{aligned} dp_i(t) = \eta_i[p_{tp,i}(t) - p_i(t)]dt \\ + \sigma_i dB_i(t) + \int_{\theta\in\Theta} \mu_i(\theta)\tilde{N}_i(dt, d\theta) + \sigma_o dB_o(t). \end{aligned} \tag{6}$$

Figure 5 illustrates an example of real price evolution from prosumer electricity markets in which we have incorporated a simulation of a regulated price dynamics as a continuation of real market. We observe that the open-loop control action $u_{ol,i}(t)$ decreases the magnitude of the fluctuations under similar circumstances.

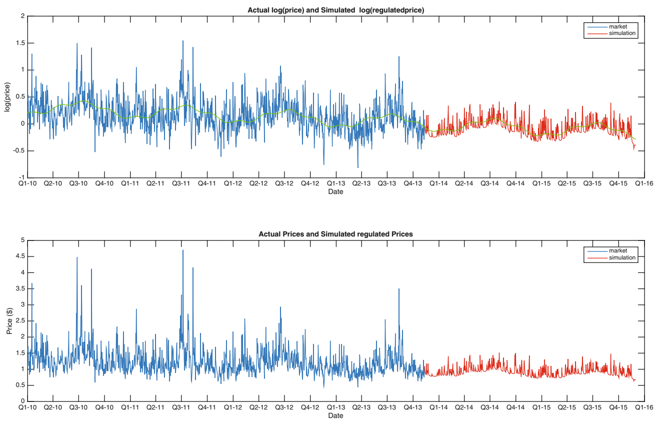

Fig. 5. Real market price and simulation of the regulated price dynamics as a continuation price under open-loop strategy.

4.4.4 An LQR Control Design

We formulate a basic LQR problem to a control strategy. Choose the control action that minimize $\mathbb{E}\{(p_i(t_1) - p_{tp,i}(t_1))^2 + \int_{t_0}^{t_1}(p_i(t) - p_{tp,i}(t))^2 dt\}$. Then the price dynamics becomes

$$\begin{aligned} dp_i(t) = \eta_i[D_i(t) - p_i(t) - (S_i(t) + u_i(t))]dt \\ + \sigma_i dB_i(t) + \int_{\theta\in\Theta} \mu_i(\theta)\tilde{N}_i(dt, d\theta) + \sigma_o dB_o(t). \end{aligned} \tag{7}$$

4.4.5 A Mean-Field Game Strategy

The mean-field game strategy is obtained by freezing the mean-field term $\mathbb{E}p_i(t) := m(t)$ resulting from other cryptocurrencies and choosing the control action that minimizes

$$\begin{aligned}\mathbb{E}q(t_1)(p_i(t_1) - f(t_1))^2 + \bar{q}(t_1)[m(t_1) - f(t_1)]^2 \\ + \mathbb{E}\int_{t_0}^{t_1} q(t)(p_i(t) - f(t))^2 + \bar{q}(t)[m(t) - f(t)]^2 dt.\end{aligned} \tag{8}$$

The mean-field term $\mathbb{E}p_i(t) := m(t)$ is a frozen quantity and does not depend on the individual control action $u_{mfg,i}$. Then, the price dynamics becomes

$$\begin{aligned}dp_i(t) = \eta[D_i(t) - p_i(t) - (S_i(t) + u_{mfg,i}(t))]dt \\ + \sigma_i dB_i(t) + \int_{\theta\in\Theta} \mu_i(\theta)\tilde{N}_i(dt, d\theta) + \sigma_o dB_o(t).\end{aligned} \tag{9}$$

4.4.6 A Mean-Field-Type Game Strategy

A mean-field-type game strategy consists of a choice of a control action $u_{mftg,i}$ that minimizes

$$\begin{aligned}L_{mftg} = \mathbb{E}q_i(t_1)(p_i(t_1) - p_{tp,i}(t_1))^2 + \bar{q}_i(t_1)[\mathbb{E}(p_i(t_1) - p_{tp,i}(t_1))]^2 \\ + \mathbb{E}\int_{t_0}^{t_1} q_i(t)(p_i(t) - p_{tp,i}(t))^2 + \bar{q}_i(t)[\mathbb{E}p_i(t) - p_{tp,i}(t)]^2\ dt.\end{aligned} \tag{10}$$

Note that here the mean-field-type term $\mathbb{E}p_i(t)$ is not a frozen quantity. It depends significantly on the control action $u_{mftg,i}$. The performance index can be rewritten in terms of variance as

$$\begin{aligned}L_{mftg} = \mathbb{E}q_i(t_1)var(p_i(t_1) - p_{tp,i}(t_1)) + [q_i(t_1) + \bar{q}_i(t_1)][\mathbb{E}p_i(t_1) - p_{tp,i}(t_1)]^2 \\ + \int_{t_0}^{t_1} q_i(t)\mathrm{Var}(p_i(t) - p_{tp,i}(t))dt \\ + \mathbb{E}\int_{t_0}^{t_1} [q_i(t) + \bar{q}_i(t)][\mathbb{E}p_i(t) - p_{tp,i}(t)]^2\ dt.\end{aligned} \tag{11}$$

Then the price dynamics becomes

$$\begin{aligned}dp_i(t) = \eta_i[D_i(t) - p_i(t) - (S_i(t) + u_{mftg,i})(t)]dt \\ + \left(\sigma_i dB_i(t) + \int_{\theta\in\Theta} \mu_i(\theta)\tilde{N}_i(dt, d\theta)\right) + \sigma_o dB_o(t),\end{aligned} \tag{12}$$

The cost to be paid to the regulation authority if the price does not stay within $[\underline{p}_i, \bar{p}_i]$ is $\bar{c}_i(1 - \mathbb{1}_{[\underline{p}_i, \bar{p}_i]}(p_i(t)))$, $\bar{c}_i > 0$. Since the market price is stochastic due to demand, exchange and random events, there is still a probability to be out of the regulation range $[\underline{p}_i, \bar{p}_i]$. The outage probabilities under the three strategies $u_{ol,i}, u_{mfg,i}, u_{mftg,i}$ can be computed and used as a decision-support with respect to the regulation bounds. However, these continuous time strategies may not be convenient.

Very often, the supply of tokens decision is made in fixed times τ_i and not continuously. We look for a simpler strategy that is piecewise constant and takes a finite number of values within the horizon $\mathscr{T}$. Since the price may fluctuates very quickly due the jump terms, we propose an adjustment based on the recent moving average called the trend: $y(t) = \int_{t-\tau_i}^{t} x(t')\phi(t, t')\lambda(dt')$, implemented at different discrete time block units.

Different regulated blockchain technologies may choose different ranges $[\underline{p}_i, \bar{p}_i]$, so that investors and users can diversify their portfolios depending on their risk-sensitivity index distribution across the assets. This means that there will be an interaction between the cryptocurrencies and the altcoins. For example, the demand $D = \sum_{i=1}^{n} D_i$ will be shared between them. Users may exchange between coins and switch into another altcoins. The payoff of the blockchain-based technology i is $R_i = \hat{p}_i D_i - \bar{c}_i(1 - \mathbb{1}_{[\underline{p}_i, \bar{p}_i]}(\hat{p}_i(t)))$, where $\hat{p}_i(t) = \mathbb{E}[p_i(t) \mid \mathscr{F}_t^{B_o}]$ is the conditional expectation of the market price with respect to $\mathscr{F}_t^{B_o}$.

4.5 Handling Positive Constraints

The price of the energy asset under cryptocurrency k is $x_k = e^{p_k} \geq 0$. The wealth of decision-maker i is $x = \sum_{k=0}^{d} \kappa_k x_k$.

Set $u_k^I = \kappa_k x_k$ to get the state dynamics. The sum of all the u_k is x. The variation is

$$\begin{aligned} dx = {} & [\kappa_0(r_0 + \hat{\mu}_0)x + \textstyle\sum_{k=1}^{d}[\hat{\mu}_k - (r_0 + \hat{\mu}_0)\kappa_0]u_k^I]dt \\ & + \textstyle\sum_{k=1}^{d} u_k^I\{Drift_k + Diffusion_k + Jump_k\}, \end{aligned} \tag{13}$$

where

$$\begin{aligned} & Drift_k = \eta_k[D_k - p_k - (S_k + u_{mftg,k})]dt + \tfrac{1}{2}(\sigma_i^2 + \sigma_o^2)dt \\ & \qquad + \textstyle\int_{\Theta}[e^{\gamma_k} - 1 - \gamma_k]\nu(d\theta)dt, \\ & Diffusion_k = (\sigma_k dB_k + \sigma_o dB_o), \\ & Jump_k = \textstyle\int_{\Theta}[e^{\gamma_k} - 1]\tilde{N}_k(dt, d\theta). \end{aligned} \tag{14}$$

5 Consumption-Investment-Insurance

A generic agent wants to decide between consumption-Investment-Insurance [19–21] when the blockchain market is constituted of a bond with price p_0 and several stocks with prices $p_k, k > 0$ and is under different switching regime defined over a complete probability space $(\Omega, \mathscr{F}, \mathbb{P})$ in which a standard Brownian motion B, a jump process N, an observable Brownian motion B_o and an observable continuous-time finite-state Markov chain $\tilde{s}(t)$ representing a regime switching, with $\tilde{S}$ being the set of regimes, and $\tilde{q}_{\tilde{s}\tilde{s}'}$ a generator (intensity matrix) of $\tilde{s}(t)$. The log-price processes are the ones given above. The total wealth of the generic agent follows the dynamics

$$\begin{aligned} dx = {} & \kappa_0(r_0(\tilde{s}) + \hat{\mu}_0(\tilde{s}))xdt \\ & + \textstyle\sum_{k=1}^{d}[\hat{\mu}_k - (r_0(\tilde{s}) + \hat{\mu}_0(\tilde{s}))\kappa_0 + \mathrm{Drift}_k(\tilde{s})]u_k^I dt - u^c dt \\ & - \bar{\lambda}(\tilde{s})(1 + \bar{\theta}(\tilde{s}))E[u^{ins}]dt + \textstyle\sum_{k=1}^{d} u_k^I \mathrm{Diffusion}_k(\tilde{s}) \\ & + \textstyle\sum_{k=1}^{d} u_k^I \mathrm{Jump}_k(\tilde{s}) - (L - u^{ins})dN, \end{aligned} \tag{15}$$

where $L = l(\tilde{s})x$.

In the dynamics (15) we have considered per-claim insurance of u^{ins}. That is, if the agent suffers a loss L at time t, the indemnity pays $u^{ins}(L)$. Such indemnity arrangements are common in private insurance at the individual level, among others. Motivated by new blockchain-based insurance products, we allow not only the cryptocurrency market but also the insurable loss to depend on the regime of the cryptocurrency economy and mean-field terms.

The payoff functional of the generic agent is

$$R = -qe^{-\lambda t_1}\{\hat{x}(t_1) - [x(t_1) - \hat{x}(t_1)]^2\} + \int_{t_0}^{t_1} e^{-\lambda t} \log u^c(t)\; dt,$$

where the process $\hat{x}$ denotes $\hat{x}(t) = \mathbb{E}[x(t) \mid \mathscr{F}_t^{\tilde{s}_0, B_o}]$. The generic agent seeks for a strategy $u = (u^c, u^I, u^{ins})$ that optimizes the expected value of R given $x(t_0), \tilde{s}(t_0)$ and the filtration generated by the common noise B_o.

For $q = 0$ an explicit solution can be found. To prove it, we choose a guess functional of the form

$$f = \alpha_1(t, \tilde{s}(t)) \log x(t) + \alpha_2(t, \tilde{s}(t)).$$

Applying Itô's formula for jump-diffusion-regime switching yields

$$\begin{array}{l}
f(t,x,\tilde{s}) = f(t_0, x_0, \tilde{s}_0) + \int_{t_0}^t \{\dot{\alpha}_1 \log x + \dot{\alpha}_2 + \frac{\alpha_1}{x}\kappa_0(r_0(\tilde{s}) + \hat{\mu}_0(\tilde{s}))x \\
+\frac{\alpha_1}{x}\sum_{k=1}^d [\hat{\mu}_k - (r_0(\tilde{s}) + \hat{\mu}_0(\tilde{s}))\kappa_0 + \mathrm{Drift}_k(\tilde{s})]u_k^I \\
-\frac{\alpha_1}{x}u^c - \frac{\alpha_1}{x}\bar{\lambda}(\tilde{s})(1+\bar{\theta}(\tilde{s}))E[u^{ins}] - \frac{\alpha_1}{x^2}\frac{1}{2}\sum_{k=1}^d\{(u_k^I\sigma_k)^2 + (u_k^I\sigma_o)^2\} \\
+\sum_{k=1}^d \int_\Theta \alpha_1 \log\{x + u_k^I(e^{\gamma_k} - 1)\} - \alpha_1 \log x - \frac{\alpha_1}{x}u_k^I(e^{\gamma_k} - 1)\nu(d\theta) \\
+\bar{\lambda}[\alpha_1 \log(x - (L - u^{ins})) - \alpha_1 \log x + \frac{\alpha_1}{x}(L - u^{ins})] \\
\sum_{\tilde{s}'}[\alpha_1(t,\tilde{s}') - \alpha_1(t,\tilde{s})]\log x + \sum_{\tilde{s}'}\alpha_2(t,\tilde{s}') - \alpha_2(t,\tilde{s})\ \}\,dt + \int_{t_0}^t d\tilde{\varepsilon},
\end{array} \tag{16}$$

where $\tilde{\varepsilon}$ is a martingale. The term $\bar{\theta}(\tilde{s})$ represents $\frac{\bar{\theta}(\tilde{s})}{1+\bar{m}(t)}$ where $\bar{m}(t)$ the average amount invested by other agents for insurance.

$$\begin{array}{l}
R - f(t_0, x_0, \tilde{s}_0) = -f(t_1, x(t_1), \tilde{s}(t_1)) - qe^{-\lambda t_1}[x(t_1) - \hat{x}(t_1)]^2 \\
+\int_{t_0}^{t_1} \{\dot{\alpha}_1 \log x + \dot{\alpha}_2 + \frac{\alpha_1}{x}\kappa_0(r_0(\tilde{s}) + \hat{\mu}_0(\tilde{s}))x + e^{-\lambda t}\log u^c - \frac{\alpha_1}{x}u^c \\
+\frac{\alpha_1}{x}\sum_{k=1}^d [\hat{\mu}_k - (r_0(\tilde{s}) + \hat{\mu}_0(\tilde{s}))\kappa_0 + \mathrm{Drift}_k(\tilde{s})]u_k^I \\
-\frac{\alpha_1}{x^2}\frac{1}{2}\sum_{k=1}^d\{(u_k^I\sigma_k)^2 + (u_k^I\sigma_o)^2\} \\
+\sum_{k=1}^d \int_\Theta \alpha_1 \log\{x + u_k^I(e^{\gamma_k} - 1)\} - \alpha_1 \log x - \frac{\alpha_1}{x}u_k^I(e^{\gamma_k} - 1)\nu(d\theta) \\
-\frac{\alpha_1}{x}\bar{\lambda}(\tilde{s})(1+\bar{\theta}(\tilde{s}))E[u^{ins}] \\
+\bar{\lambda}[\alpha_1 \log(x - (L - u^{ins})) - \alpha_1 \log x + \frac{\alpha_1}{x}(L - u^{ins})] \\
+\sum_{\tilde{s}'}[\alpha_1(t,\tilde{s}') - \alpha_1(t,\tilde{s})]\log x + \sum_{\tilde{s}'}\alpha_2(t,\tilde{s}') - \alpha_2(t,\tilde{s})\ \}\,dt + \int_{t_0}^{t_1} d\tilde{\varepsilon}.
\end{array} \tag{17}$$

The optimal u^c is obtained by direct optimization of $e^{-\lambda t}\log u^c - \frac{\alpha_1}{x}u^c$. This is a strictly concave function and its maximum is achieved at $u^c = \frac{e^{-\lambda t}}{\alpha_1}x$, provided that $\alpha_1(t,\cdot) > 0$ and $x(\cdot) > 0$.

This latter result can be interpreted as follows. The optimal consumption strategy process is proportional to the wealth process, i.e., the ratio $\frac{u^c(t)}{x^*(t)} > 0$.

This means that the blockchain-based cryptocurrency investors will consume proportionally more when they become wealthier in the market. Similarly, the insurance strategy u^{ins} can be obtained by optimizing

$$-\frac{1}{x}(1+\bar{\theta}(\tilde{s}))\mathbb{E}[u^{ins}(\tilde{s})] + \log(x-(L(\tilde{s})-u^{ins}(\tilde{s}))) + \frac{1}{x}(L(\tilde{s})-u^{ins}(\tilde{s}))],$$

which yields that

$$\frac{1}{x-L+u^{ins}} = \frac{1}{x}(2+\bar{\theta}).$$

Thus, noting that we have set $L(\tilde{s}) = l(\tilde{s})x$, we obtain

$$u^{ins}(\tilde{s}) = \left[l(\tilde{s}) - \frac{1+\bar{\theta}(\tilde{s})}{2+\bar{\theta}(\tilde{s})}\right]^{+} x = \max\left\{0, l(\tilde{s}) - \frac{1+\bar{\theta}(\tilde{s})}{2+\bar{\theta}(\tilde{s})}\right\} x.$$

We observe that, for each fixed regime $\tilde{s}$, the optimal insurance is proportional to the blockchain investor's wealth x. We note that it is optimal to buy insurance only if $l(\tilde{s}) > \frac{1+\bar{\theta}(\tilde{s})}{2+\bar{\theta}(\tilde{s})}$. When this condition is satisfied, the insurance strategy is $u^{ins}(\tilde{s}) := \left[l(\tilde{s}) - \frac{1+\bar{\theta}(\tilde{s})}{2+\bar{\theta}(\tilde{s})}\right] x$ which is a decreasing and convex function of $\bar{\theta}$. This monotonicity property means that, as the premium loading $\bar{\theta}$ increases, it is optimal to reduce the purchase of insurance.

The optimal investment strategy u_k^I can be found explicitly by mean-field-type optimization. Incorporating all together, a system of backward ordinary differential equations can be found for the coefficient functions $\{\alpha(t,\tilde{s})\}_{\tilde{s}\in\tilde{S}}$. Lastly, a fixed-point problem is solved by computing the total wealth invested in insurance to match with $\bar{m}$.

6 Concluding Remarks

In this paper we have examined mean-field-type games in blockchain-based distributed power networks with several different entities: investors, consumers, prosumers, producers and miners. We have identified a simple class of mean-field-type strategies under a rather simple model of jump-diffusion and regime switching processes. In our future work, we plan to extend these works to higher moments and predictive strategies.

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