Transaction Queuing Game in Bitcoin BlockChain

Juanjuan Li[1] *(Member, IEEE)*, Yong Yuan[2] *(Senior Member, IEEE)*,
Shuai Wang[3], Fei-Yue Wang[4]*(Fellow, IEEE)*

Abstract—Abstract- **Bitcoin is a novel protocol with the potential of enabling a decentralized and trustless cryptographic currency, and its underlying technology named blockchain operates on a worldwide basis via a complex set of rules originally proposed by Nakomoto in 2008. In Bitcoin blockchain, miners provide computational services (i.e. mining) to get profits from the fixed rewards of newly found block and also transaction fees from recording the users' transactions to the blocks. With the decreasing of the fixed new block reward, transaction fees will play the role as the main profit source of miners, thus provide important supports for the sustainability and vitality of the Bitcoin system. Therefore, it is of great necessity to research transaction fees. In this paper, we investigate transaction fees in a queuing game with non-preemptive priority, in which both the miners' mining rewards and the users' time cost are highlighted. Then, we conduct theoretical analysis of the game, getting five types of Nash equilibria of the game. We also find that the over-long waiting time will bring negative marginal profits on transaction fees to some users with low time cost, therefore, they will not be willing to offer transaction fees.**

Keywords- **Bitcoin blockchain; Transaction Fee; Miner; User; Transaction Queuing Game**

I. INTRODUCTION

Bitcoin blockchain was designed by Nakamoto [1] as a peer-to-peer decentralized trustless network [2], [3], in which any update of the data could be recorded into a block after being agreed by all participants. A new block is appended to a chain of previously agreed upon blocks, creating a complete record of all the data updating that have ever taken place [4]. In the Bitcoin blockchain, miners compete to solve the computational problem and provide the requisite security [5]. The computational problem involves a brute

[1]Juanjuan Li is with The State Key Laboratory for Management and Control of Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China; Qingdao Academy of Intelligent Industries, Qingdao, China. `juanjuan.li@ia.ac.cn`
[2]Yong Yuan is the corresponding author and is with The State Key Laboratory for Management and Control of Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China; Qingdao Academy of Intelligent Industries, Qingdao, China. `yong.yuan@ia.ac.cn`
[3]Shuai Wang is with University of Chinese Academy of Sciences, Beijing, China; The State Key Laboratory for Management and Control of Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China. `wangshuai2015@ia.ac.cn`
[4]Fei-Yue Wang is with The State Key Laboratory for Management and Control of Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China; Research Center of Military Computational Experiments and Parallel Systems, National University of Defense Technology, Changsha, China. `feiyue.wang@ia.ac.cn`
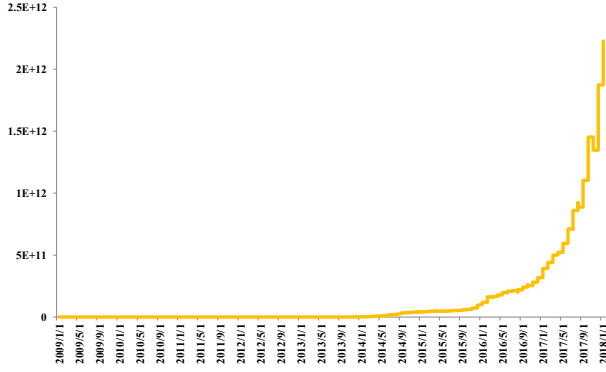
force approach to find a specific "hash function" or the string of numbers, which is called mining [6].

During the mining process, miners should first pick out the unconfirmed transactions pending in the memory pool (mempool), which significantly influences their mining difficulties and also revenues. The winning miner or pool can determine the transaction volume within the range of zero to the maximum block size of 1 MB currently [7]. Generally, the larger is the transaction volume, the higher will be the transaction fees. Miners can choose group mining through joining a pool or individual mining just by themselves. The one who first finds the specific hash value that solves the computational problem will be rewarded with a fixed number of newly issued Bitcoins known as a block reward, and also the transaction fees from recording the packed transactions to the new block. The fixed reward was originally set to 50 bitcoins while the protocol determines that it will be halved each time 210,000 blocks are permanently added to the blockchain, which happens approximately every 4 years. When the protocol eventually stops creating new Bitcoins, the fixed rewards for mining will be entirely replaced by transaction fees [8].

In the past, the mining incentive is almost only due to the new block reward creation [9], but the situation will not last forever [10]. It is foreseeable that with the decreasing of the fixed new block reward and the increasing mining difficulty (Figure 1), the continuous increase of miners' revenues (Figure 2) should be greatly attributed to the growing transaction fees (Figure3)[1]. As such, transaction fees are the only way Bitcoin users can encourage miners keep mining in the long run [7]. The long-term level of fees is uncertain, yet they are highly relevant to the security and sustainability of the system [11]. Therefore, it is necessary for the Bitcoin blockchain to view high of transaction fees, since they play the crucial role to encourage miners and mining pools to work on computational problems thus keep the vitality of the system.

In this paper, we strive to study the role of transaction fees in the transaction queuing game played by miners and users. First, miners' mining strategies are researched to probe the transactions flow-out rate in a stable mempool. Usually, miners pick up transactions to the new block according to transaction fees, which means the transaction queuing is with non-preemptive priority determined by transaction fees. Then, we study the flow-in rate of mempool, which is a

[1]From the data statistics in https://charts.bitcoin.com/

Fig. 1.   Mining Difficulty in Bitcoin Blockchain

Fig. 2.   Miners' Revenue in Bitcoin Blockchain

Fig. 3.   Transaction Fees in Bitcoin Blockchain

and showed that the fixed transaction fee is equivalently to setting a maximum block size instead. If the transaction fee is totally determined by a decentralized market and the maximum block size is not constrained, the transaction fee will eventually go to zero and miners will not have the necessary incentives to keep mining, hence to keep Bitcoin viable. Chalkias & Dionysiou (2016)[17] viewed transaction fee as alternative reward schemes for miners in blockchain systems. The larger the processing fee, the longer a transaction could reside in the blockchain before being considered dormant. To understand how users select their transaction fees, Huberman et al. (2017)[18] analyzed the implied congestion queuing game, calculated each user's trade-off between transaction fees and delay cost, and derived that each user's equilibrium transaction fee equals the externality his transaction imposes. Thus, equilibrium transaction fees coincide with the payments that result from selling priority of service in a VCG (Vickrey-Clarke-Groves) auction. Easley et al. (2017)[5] investigated the role that transaction fees play in the Bitcoin blockchains evolution from a mining-based structure to a market-based ecology. Lavi et al. (2017)[16] proposed two alternative auction mechanisms: The monopolistic price mechanism, and the random sampling optimal price mechanism. They proved that the monopolistic price mechanism extracts revenue better from users, and that it is nearly incentive compatible.

Our work is the extension of the study in [5] to consider the research setting where miners are with different hashing powers and the users are with different transaction fees, which makes the transaction queuing game much more complicated and the closer to the real Bitcoin system.

III. Model

In Bitcoin blockchain, the main players are users and miners. Here, we consider the common case that M miners operate in a pool, and they solve computational problems to get the new block's reward S and also the transaction fees collection F from recording transactions in the blocks, and
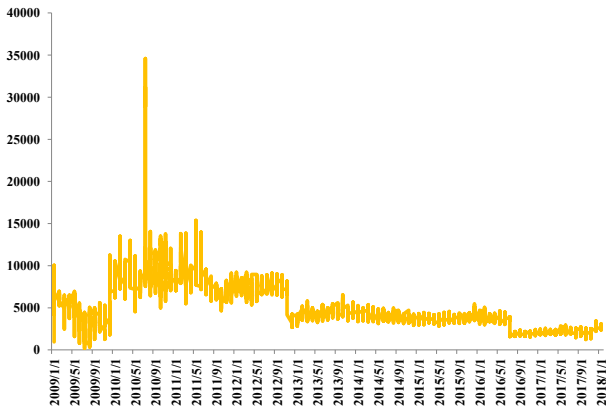
set of different flow-in rate of users with different priorities. Base on the research of the flow-out and flow-in rates of mempool, we compute users' waiting time, and formulate a game-theoretical model to consider their expected profits. Also, the Nash equilibria of the game are analyzed.

The paper is organized as follows. Section II briefly reviews the related literature. In Section III, a game-theoretical model will be proposed to research the transaction queuing of users offering transaction fees. Equilibrium analysis will also be conducted. Section IV concludes this paper.

II. Literature Review

Bitcoin blockchain economics have attracted wide research interests [4], [13], [14], but transaction fees economics still received very limited attentions. Kroll et al. (2013)[15] considered that transaction fees have little importance, although their mining game research did not include it. It has been speculated that higher fees will lead to faster confirmation [11], but over-high transaction fees will render Bitcoin uneconomical for micro payments [10], [12]. Lavi et al. (2017)[16] argued that Bitcoin's current fee market does not extract revenue well when blocks are not congested.

Houy (2014)[7] studied the economics of Bitcoin's transaction fees in a very simple partial equilibrium setting,

we have

$$F = \sum_{i=1}^{N} f^i$$

where f^i is the transaction fee from user i, and there are N users who has transactions unconfirmed in the mempool. With the purpose of optimizing the expected profits, miners pick up transactions with high fees, and users offer transaction fees to move forward in the queue and thus shorten the waiting time.

A. Miners and Mining Profit

Assume the hash power of a miner j is h_j, and the fixed mining cost is c_j, and the variant unit cost of time spent on mining is e_j. In the pool, miners work independently to find the particular hash, and get rewards according to their contributions, therefore, the probability that the miner j is the first one to find the hash value is h_j/H, and

$$H = \sum_{j=1}^{M} h_j.$$

Therefore, the miner j has the expected revenue as:

$$P_j = \frac{h_j}{H}(S+F)$$

We assume that the success of each miner to find the particular hash value is subject to a Poisson distribution with the arrival rate λ_j, so the expected time until the first success is $1/\sum_j \lambda_j$. Then, the expected profit for the miner j is

$$\tilde{P}_j = \frac{h_j}{H}(S+F) - \frac{1}{\sum_{j=1}^{M} \lambda_j} e_j - c_j$$

The mining game of these miners are free entry-exit, therefore, the Nash equilibria should have zero profit, otherwise there will be miners quit from or join in the game. As such, at the equilibria with stable M^* miners in the pool, the condition that $\tilde{P}_j(M^*) = 0, \forall j \in M^*$ must be satisfied.

In the mempool, the arrival rate of a miner's successful mining λ_j is greatly determined by the miners' hashing power, therefore, it is not exogenous. However, from the perspective of the whole pool, the difficulty of mining problem set by protocol infers an approximately exogenous arrival rate of a new block as Λ. Thus, we have

$$\begin{aligned} \Lambda &= \sum_{j=1}^{M^*} \lambda_j \\ &= \frac{e_j}{\frac{h_j}{H^*}(S+\sum_{i=1}^{N} f_i) - c_j} \end{aligned}$$

We can say that Λ stays unchanged because more competitive miners are accompanied with higher mining difficulty for each miner, while less active miners will also reduce the individual difficulty of mining.

B. Non-preemptive priority of transaction fee

Assume that a block consists of only one transaction. The processing order of all users' transactions is completely dependent on transaction fee f. Users who do not pay transaction fees for miners to solve the computational problems have the lowest priority. Sort all N users' transaction fees in a descending order, we have $f_1 >, ..., > f_k, ..., > f_K$. Thus, there are K priorities for arriving transactions. A user with the kth priority has non-preemptive priority over all users with the $k+1$th priority. Within a priority type, users are served on a first-come-first-served (FCFS) basis.

Considering the flow-out of transactions in the mempool at rate Λ. Users with kth priority submit transactions according to a Poisson process with rate γ_k, thus the arrival rate of mempool is $\Gamma = \sum_k \gamma_k$. First, we analyze the stability of the mempool. If the arrival rate of transactions Γ is greater than the removing rate of transactions Λ, the size of the mempool will grow infinitely. If $\Gamma < \Lambda$, the mempool will be stable in the long run. The mean mempool size is

$$\bar{L} = \frac{\Gamma}{\Lambda - \Gamma}$$

Since users of each priority are independent Poisson flows, the probability that a user arriving at any time belongs to kth priority is γ_k/Γ. The average processing time of each transaction can be computed as

$$\bar{s} = \sum_{k=1}^{K} \frac{\gamma_k}{\Gamma} E(s_k) = \frac{1}{\Lambda}\frac{1}{\Gamma}\sum_{k=1}^{K} \gamma_k = \frac{1}{\Lambda}$$

Furthermore, we have $\rho = \Gamma/\Lambda < 1$ representing the transaction intensity in the mempool, and $p_0 = 1 - \rho$ representing the probability that the miners are all idle. According to Little's Law, the average time, including the waiting time and processing time, for a user's transactions being recorded in the block can be given as

$$\bar{w} = \bar{L}/\Gamma = \frac{1}{\Lambda - \Gamma}$$

and the waiting time is $w = \bar{w} - \bar{s}$.

In the stable mempool, we can see that the priority only captures the change in the waiting time distributions but not the average waiting time of all users. The waiting time of high-priority users are shortened, while that of low-priority users are prolonged. With the purpose of probing the influence of the non-preemptive priority on miners' transaction processing, we should compute the average waiting time w_k and numbers of users L_k at each priority. Since the waiting time of users with highest priority will not be influenced by other classes, thus w_1 is the same under situations with different $\gamma_k, k = 2, ..., K$. Therefore, we consider the special case that $\gamma_2 = \gamma_3 =, ..., \gamma_K = 0$ to compute w_1.

When a highest-priority user submits the transaction processing request in the mempool, his waiting time is equal to the sum of time for miners to solve the computational

problems for in-service users. Therefore, we have his waiting time as:

$$w_1 = \frac{\Gamma}{\Lambda^2(1-\rho_1)}$$

where $\rho_1 = \gamma_1/\Lambda$. According to the Little's Law, the waiting users with the first priority is $L_1 = w_1\gamma_1$.

Similarly, we can compute the average waiting time for the users with kth priority as

$$w_k = \frac{\Gamma}{\Lambda^2(1-\sum\limits_{z=1}^{k}\rho_z)(1-\sum\limits_{i=z}^{k-1}\rho_z)}$$

We have $\rho_k = \gamma_k/\Lambda$ and $L_k = w_k\gamma_k$. It is easy to find that $w_1 < w_2,...,< w_K$.

From above, we can see that the waiting time of a user depends heavily on the numbers of users with higher priorities than himself/herself.

C. Users' expected revenues

For the user i, only if his transaction is written to the blockchain, the revenue v^i can be generated. α^i represents the time cost on the delayed computational service provided by miners. As such, the expected profit is formulated as:

$$R^i = v^i - f^i - \alpha^i w^i$$

If the user does not pay transaction fee, he still will get the revenue if the transaction is recorded to the blockchain, but will be reduced by a long waiting time. If the user pays a high enough transaction fee to get the first place, and his transaction will be processed immediately after its arrival, then his revenue will not be reduced by the time delay, but be subtracted by the transaction fee. Therefore, the user should make the trad-off between the cost and revenue from paying the transaction fee.

D. Transaction queuing game

In this section, we investigate transaction fees in the transaction queuing game played by users. In principle, the game of N users has Nash equilibria for the reason that each user has finite choices of transaction fees. Nash equilibria can be realized under the cases that no users, all users and a part of users pay transaction fees.

First, we consider the case that no users pay transaction fees. That means there is no priority, which can happen when the mempool size $\bar{L}$ is small enough and the average waiting time w is very short. In this case, for a user i who pays transaction fee f_i to get the first place in the waiting queue, his expected profit should be

$$R^i = v^i - f^i$$

So the Nash equilibria with no user paying transaction fee exist if for any f^i, there is

$$v^i - f^i < v^i - \alpha^i w$$

That is, $\forall f^i > 0$, the following inequality holds in a stable mempool,

$$\frac{f^i}{\alpha^i} > \frac{\rho}{\Lambda(1-\rho)}$$

Then, we analyze the case that all users pay transaction fees. In this case, the volume of unconfirmed transactions in the mempool should be large enough and the average waiting time should be long enough, thus every user wants to pay the transaction fee to move forward in the queue. In this case, a user i who does not pay the transaction fee will be with the lowest priority, his/her expected profit should be

$$R^i = v^i - \alpha^i w_K$$

So the Nash equilibria with all users paying transaction fees exist if for any f^i, there is

$$v^i - \alpha^i w_K < v^i - f^i - \alpha^i w_{K-1}$$

That is, $\forall f^i > 0$, the following inequality holds in a stable mempool,

$$\frac{f^i}{\alpha^i} < \frac{\rho(\rho_{K-1}+\rho_K)}{\Lambda x}$$

where,

$$x = (1-\sum_{z=1}^{K-2}\rho_z)(1-\sum_{z=1}^{K-1}\rho_z)(1-\sum_{z=1}^{K}\rho_z)$$

There are two types of equilibria for this case: 1) all users pay the same transaction fee, 2) users pay different transaction fees.

Finally, we study the equilibria in case that only a part of users pay transaction fees but others don't. Those who do not pay transaction fees will be at the lowest priority K, and those who pay transaction fees will be at a certain priority $k \leq K - 1$. According to the above analysis, we can get that the Nash equilibria in this case should be achieved under the condition that

$$\frac{\rho(\rho_{K-1}+\rho_K)}{\Lambda x} \leq \frac{f^i}{\alpha^i} \leq \frac{\rho}{\Lambda(1-\rho)}$$

Under the Nash equilibria, no user can improve the expected profit through increasing or reducing transaction fees. If the equilibrium transaction fee of the user i is $f^{i*} = f_k$, and the waiting time should be w_k, then the equilibrium profit for him/her is

$$R^{i*} = v^i - f_k - \alpha^i w_k$$

If he/she changes the transaction fee to access to priority k', $k' = (0,..,k-1,k+1,...,K)$, the expected profit will be

$$R_{k'}^i = v^i - f_{k'} - \alpha^i w_{k'}$$

Since $R^{i*} \geq R_{k'}^i$, calculations show that

$$f_k - f_{k'} \leq \alpha^i(w_{k'} - w_k)$$

Thus, the Nash equilibria should satisfy the condition that for each user, the increase (decrease) of the transaction fee

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must be no more than the decrease (increase) of time cost under the optimal transaction fee.

Consider the competitive user i' in the mempool, who is with the k'th priority under the Nash equilibria. Similarly, we have $R^{i'}* \geq R_k^{i'}$, where

$$R^{i'}* = v^{i'} - f_{k'} - \alpha^{i'} w_{k'}$$

$$R_k^{i'} = v^{i'} - f_k - \alpha^{i'} w_k$$

From the above two inequalities, we can get that

$$(\alpha^{i'} - \alpha^i)w_k \geq (\alpha^{i'} - \alpha^i)w_{k'}$$

If $k' > k$, there is $w_{k'} > w_k$, therefore, we have $\alpha^{i'} \leq \alpha^i$; and if $k' < k$, there is $w_{k'} < w_k$, therefore, we have $\alpha^{i'} \geq \alpha^i$. The equilibria conditions indicate the trade-off between costs and benefits from paying the transaction fee.

From the above analysis, there are five types of Nash equilibria of the transaction queuing game shown in Figure 4:

- If $f^i > \alpha^i \rho / \Lambda(1-\rho)$, $\forall i$, there is a the unique equilibrium in which no user pays the transaction fee;
- If $f^i < \alpha^i \rho(\rho_{K-1} + \rho_K) / \Lambda x$, $\forall i$, there exist two types of equilibria, in which all users pay the same transaction fee or different transaction fees;
- If $\alpha^i \rho(\rho_{K-1} + \rho_K) / \Lambda x \leq f^i \leq \alpha^i \rho / \Lambda(1-\rho)$, $\forall i$, there are two types of equilibria, in which are a part of users pay the same transaction fee or different transaction fees.
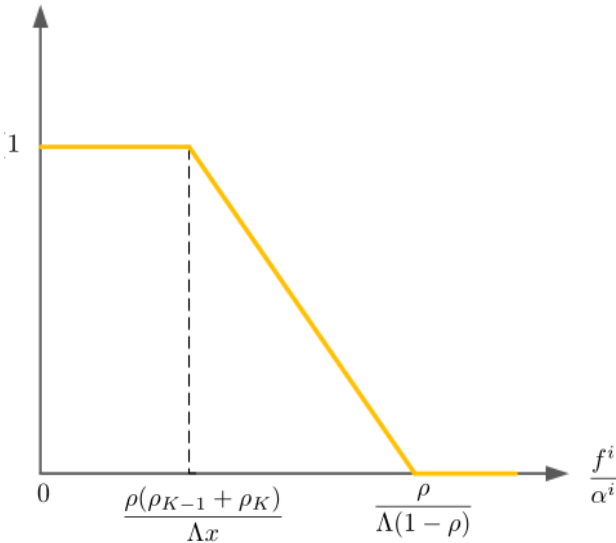

Fig. 4. Equilibria of transaction queuing game

Generally, the equilibrium transaction fees of all users are determined by his/her waiting time and time cost.

The user's waiting time is influenced not only by his/her own transaction fee, but also the flow-in rate of competitive

users with higher transaction fees than him/her and the flow-out rate of the mempool. The transaction fee determines the user's queue position thus his/her waiting time, and a higher transaction fee will lead to a shorter waiting time. Also, the flow-in rate of higher-priority users and the flow-out rate of the mempool determine the queue length in front of him/her and thus affect his/her waiting time. For the user, a higher flow-in rate of higher-priority users's transactions will lead to a longer waiting time, and a higher flow-out rate will lead to a shorter waiting time.

The time cost is endogenous and individual for the user. For the impatient user with higher time cost, he may be willing to offer a higher transaction fee to shorten the waiting time which has also been validated in the research of Moser et al. (2015)[11] through a longitudinal study. However, there is no evidence that the transaction fee is influenced by the transaction value.
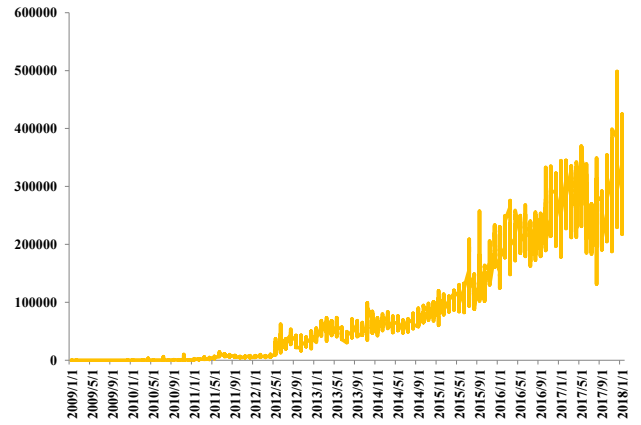

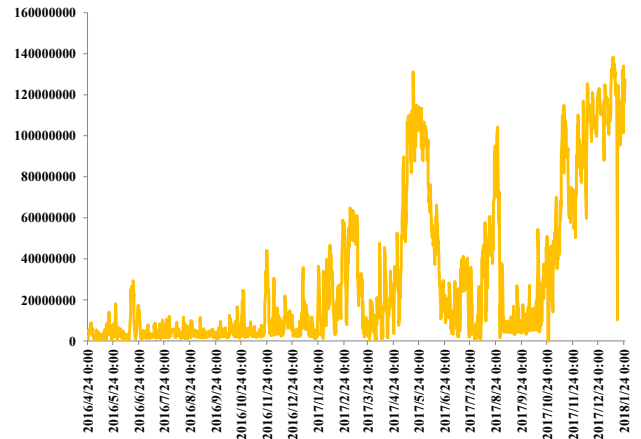Fig. 5. Daily Transactions in Bitcoin Blockchain

Fig. 6. Mempool Size of Bitcoin Blockchain

The user's marginal profit on the transaction fee is

$$R^{i'} = -1 - \alpha^i \frac{dw^i}{df^i}$$

It is known that $dw^i/df^i < 0$. The marginal profit is determined by the fact that how long the waiting time is shorten due to a certain amount of transaction fee.

As time goes by, the transactions flow-in rate increases greatly as shown in Figure 5, but the flow-out rate does not increase commensurately. As a result, the size of the mempool with unconfirmed transactions keeps growing and even clogs up in year 2017 (Figure 6). With the service strength ρ grows, the waiting time for users is prolonged. Since the average waiting time will not decrease, the continuously growing ρ may result in a situation that the marginal profits for some users with the low time cost α will turn to be negative when the waiting time is over-long or even divergent. Accordingly, in the equilibrium state, the users with the enough high time cost will increase transaction fees while those with the enough low time cost probably refuse to offer transaction fees. Therefore, the continuous congestion of the mempool may not produce over-high total transaction fees. Also, if the scalability of Bitcoin block size is realized, miners' revenue from transaction fees may decrease, since the average waiting time for all users will decrease.

IV. CONCLUSION

Transaction fees play a critical role for the stability and vitality of the Bitcoin system. In this paper, we highlight the critical role of transaction fees, and formulate the computational service of miners for users as a queuing problem with non-preemptive priority completely determined by transaction fees. Then, we investigate the problem through a game-theoretical model, in which the miners' mining rewards and users' time cost are emphasized. Also, we conducted equilibrium analysis of the model, and find that there are five types of Nash equilibria of the transaction queuing game. We also find that the over-long waiting time caused by continued congestions of the mempool may make some users with low enough time cost give up paying transaction fees.

When a specific priority has a lot of users, the average waiting time for them will not decrease significantly, instead, the average waiting time for those who have lower and no priority will be greatly prolonged. The practical Bitcoin transactions in mempool are precisely in line with this situation that most users pay transaction fees. Therefore, in our ongoing work, we will probe the problem to take more factors, including waiting time and transaction size into consideration to explore the limited priority of transaction fees.

Also, in the future, we will adopt ACP (Artificial systems + Computational experiments + Parallel execution) approach [19], [20], [21], [22] to study the transaction fee problem in Bitcoin blockchain. We will first construct an artificial Bitcoin blockchain network, design computational experiments in the artificial system, and then conduct parallel execution bridging the real-world system and the artificial system to make an in-depth research of the role and the influence of transaction fees [23], [24].

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