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Game model of optimal quality experience strategy for Internet of Vehicles bandwidth service based on DAG blockchain

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***Abstract*—With the rapid development of the Internet of Vehicles (IoV), vehicle networks will generate large amounts of data. Vehicle-to-Vehicle (V2V) data sharing can improve driving experience and quality of service, and Vehicle-to-Roadside unit (V2R) communication can improve road transport efficiency. A certain amount of bandwidth is consumed in the process of data sharing between V2V and V2R, and when multiple vehicles request data from the RSU at the same time, there may be network delay or congestion. Therefore, the vehicle obtains data faster by requesting high bandwidth from the RSU. However, since the bandwidth resources of RSUs are limited, optimal bandwidth service strategies need to be investigated. In addition, in order to ensure the safety, efficiency and traceability of data sharing between V2V and V2R, a blockchain-supported IoV needs to be established. Therefore, we design a blockchain-based framework for V2V and V2R transaction framework that integrates DAG blockchain and modern cryptography. The framework ensures security and scalability during the execution of data exchange and completion of bandwidth transactions, improves the service quality of vehicles and realizes decentralized payment. In this work, a two-stage Stackelberg game is established under two pricing schemes of uniform pricing and discriminatory pricing to jointly maximize the benefits of vehicles and RSUs. In Stage I, the RSU sets the corresponding bandwidth service and pricing strategy. In Stage II, the vehicle decides the service demand according to the observed pricing strategy. The optimal bandwidth allocation and pricing strategy between the RSU and the vehicle is determined through the game of both parties, and the existence and uniqueness of the two-stage subgame Nash equilibrium is analyzed by applying backward induction. The correctness of the model is verified by experiments, and the comparative analysis shows that the proposed model has high throughput.**

***Index Terms*—Internet of Vehicles, DAG blockchain, bandwidth allocation, game theory, Stackelberg equilibrium.**

I. Introduction

A. Background and Motivations

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WITH the rapid development of 5G network and vehicle communication technology, low latency, high reliability and efficient communication rates [1] provide possibilities for advanced vehicle services and applications (e.g., autonomous driving and content delivery). They can improve the safety and comfort ability of transportation by providing ubiquitous access to information for drivers and passengers(e.g., providing road traffic conditions, accident prevention, payment services, safety applications, etc.) [2]. These offer unprecedented opportunities and capabilities for vehicle applications. In this context, Internet of Vehicles (IoV) serves as a complex system of dynamic mobile networks that enable information sharing between vehicles, surrounding sensors and the cloud. In the IoV scenario, data is ubiquitous and its data sharing process mainly occurs in Vehicle-to-Roadside unit (V2R) communication and Vehicle-to-Vehicle (V2V) communication, which enables each vehicle to obtain more comprehensive road traffic and other related information.

Nonetheless, there are still some issues waiting to be resolved in a large-scale IoV environment, namely the security of data sharing between V2V and V2R. Common IoV systems adopt a centralized architecture, but they are vulnerable to single points of failure, denial of service, privacy leaks, and malicious attacks [3]. Blockchain technology is considered as a promising decentralized technology to solve security problems with tamper-proof, traceability and decentralization properties. The classic PoW consensus mechanism is used in the blockchain, which has high computational cost and low confirmation speed [4], [5], and the ledgers are frequently synchronized, making it difficult be applied to large-scale vehicle networks [6]. Additionally, it generates and validates blocks in a sequential and synchronous manner, resulting in low throughput. In order to solve the problems faced by PoW, some alternative consistency algorithms have been proposed, such as Proof of Stake (PoS) and Delegated Proof of Stake (DPoS), among which PoS shortens the time for reaching consensus to a certain extent and also increases the cost of node evil through benefit bundling, but with low decentralization. DPoS as a variant of PoS has improved efficiency due to fewer nodes and decides who is out of the block based on voting and no waste of resources. But similar to POS there is a tendency of centralization and lack of incentive to improve the arithmetic power [7].

DAG blockchain as a new technology with asynchronous consistency mechanism and the ability to get rid of the PoW that miners must complete to execute each transaction,

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improves scalability and efficiency compared to traditional blockchain [8], and almost no transaction fees. Meanwhile, considering that V2V and V2R consume certain bandwidth resources in the process of data request and exchange, when multiple vehicles request data from RSU at the same time, network congestion and delay may occur. Accordingly, for faster data acquisition by each vehicle, the vehicles request high bandwidth from the RSU to meet the vehicle time-varying topology and strict latency requirements, but the bandwidth resources of the RSU are limited. Based on this, there are still some challenges to be solved for the bandwidth request problem. On the one hand, it lacks an effective way to ensure the security of the vehicle and RSU payment bandwidth transactions. On the other hand, from the bandwidth requestor's point of view, vehicles want to get higher bandwidth at the lowest price. From the bandwidth allocator's point of view, RSUs want to maximize their profits by providing bandwidth services. Therefore, an optimal strategy for vehicles and RSUs needs to be investigated, in which profit and cost should be considered to determine an optimal bandwidth allocation as a way to reach an equilibrium for each.

B. Solutions and Contributions

Based on the above problems, we have designed a V2V and V2R transaction framework based on DAG blockchain for data request transactions and bandwidth service transactions. The framework combines DAG blockchain and digital encryption technology to ensure transaction security while also improving transaction efficiency. And the problem of requesting bandwidth from a resource limited RSU is modeled as a two-stage Stackelberg game based on bandwidth auction. In Stage I, RSU sets the corresponding bandwidth service as well as the pricing strategy. In Stage II, the vehicle decides the service demand based on the observed pricing strategy. After knowing the RSU policy and the bandwidth requirements of all vehicles, the quality experience of vehicle is obtained by calculating the bandwidth that the RSU may allocate to the requesting vehicle and the actual bandwidth allocated. And the equilibrium of each stage of the subgame under both uniform and discriminatory pricing strategies is analyzed by using the backward induction method. For uniform pricing where the same price applies to all vehicles, the uniqueness of the Stackelberg equilibrium is validated by identifying the best response bandwidth strategies. For discriminatory pricing where the different prices are applied, the uniqueness of the Stackelberg equilibrium is proved by using the variational inequality theory.

Specifically, in this paper, we propose a novel transaction framework based on DAG blockchain, designed to support secure and efficient data and bandwidth transactions between V2V and V2R. We also design an auction-based two-stage Stackelberg game that maps providers and consumers performing bandwidth services in a way that is optimal in terms of profit and quality experience, and obtains the optimal service by finding the stable point of the Nash equilibrium. The main contributions of this paper are summarized as follows.

(1) We design a scheduling framework that includes V2V and V2R communication transactions and V2R bandwidth transactions that integrates DAG blockchain and modern cryptography to ensure security and efficiency of V2V to V2R communication, as well as security and scalability during execution of bandwidth scheduling services and completion of transactions.

(2) We propose a two-stage Stackelberg game based on bandwidth auctions. In Stage I, RSU sets the corresponding bandwidth service as well as the pricing strategy. In Stage II, the vehicle decides the service demand based on the observed pricing strategy. Through backward induction, the optimal pricing and bandwidth allocation strategy relationship between V2R is analyzed, and negotiation between vehicles and RSUs is carried out with the purpose of quality experience and profit maximization.

(3) We analyze the optimal bandwidth allocation of vehicles and the profit maximization problem of RSU in both uniform and discriminatory pricing schemes, and verify the existence and uniqueness of the Stackelberg game.

The rest of this paper is organized as follows: Related work is presented in Section II. We build a DAG blockchain-based vehicle network system model in Section III. In Section IV analyzes of the vehicle and RSU quality experience and profitability issues under the Stackelberg game model and its performance evaluation in Section V. Finally, we conclude this paper in Section VI.

II. Related Work

In this section, we will briefly discuss the related references on blockchain-based and game theory-based data transactions and bandwidth strategies (see Sections II-A and II-B).

A. Blockchain

There has been increasing interest in using blockchain to ensure the security of transactions. Kumaran *et al*. [9] proposed a blockchain innovation authorized IoV to ensure the reliability, accuracy and security of communication in the Internet of Vehicles. However, this study ignores the high efficiency of data sharing. On the contrary, our proposed DAG blockchain-based framework has fast transaction speed and high throughput compared to traditional blockchains. Yakubu *et al*. [10] integrate a given smart market scenario with blockchain technology by using the Proof of Authority (PoA) consensus mechanism. The simulation results show that the model effectively solves the waste of excess bandwidth in smart cities and provides a secure bandwidth transaction scheme. Unlike this study, we design a scheme that maximizes the benefits of participants while keeping transactions secure. Xiong *et al*. [11] presents a simple and self-adjusting blockchain protocol called ORIC, which allows many more blocks to be produced during the block-time so that the bandwidth can be exploited to its maximum extent. However, the purpose of the bandwidth strategy in this paper is to obtain the required data as quickly as possible. Xu *et al*. [12] propose an integrated blockchain and MEC framework based on a space-structured ledger to meet the transaction demands for IoT applications. And an alternate optimization algorithm is also proposed to further improve the performance

TABLE I
EXISTING BLOCKCHAIN-BASED AND GAME THEORY -BASED BANDWIDTH STRATEGIES: A COMPARATIVE SUMMARY

Scheme	Blockchain	DAG	Game theory	Dynamic Interaction	Bandwidth Scheduling Methods
[10]	✓	×	×	×	Integration in blockchain
[11]	✓	×	×	×	Self-adjusting protocol
[13]	✓	✓	✓	✓	non-cooperative game
[15]	×	×	✓	✓	Dynamic game
[17]	×	×	✓	×	Wtp-Game theory
[18]	×	×	✓	✓	non-cooperative game
[19]	×	×	✓	✓	Matching game
[Ours]	✓	✓	✓	✓	Stackelberg game+ non-cooperative game

of the scheme by jointly optimizing bandwidth allocation and computing resource allocation. Unlike this paper, a Stackelberg game algorithm is designed to find the optimal bandwidth allocation strategy that maximizes the profit of both sides of the game. Hassija *et al*. [13] proposes a new framework of Directed Acyclic IoV (DAGIoV), which solves the scalability problem in traditional blockchains. And an auction-based non-cooperative game algorithm is proposed, which considers the cost and profit of the vehicle bandwidth request, so that the price of the vehicle bandwidth is optimal. Different from [13], the auction-based Stackelberg game algorithm proposed in this paper maximizes the profit of the RSU under the condition of satisfying the optimal vehicle quality experience through the game between the vehicle and the RSU.

B. Game Theory

In recent years, a lot of research has been conducted on the optimization of mutual benefit through game theory. Wang *et al*. [14] designed a Stackelberg game-based incentive mechanism for resource allocation to maximize vehicle satisfaction and overall energy efficiency. However, the security of resource allocation is not considered in the designed scheme. Different from this work, the Stackelberg game in this paper is constructed on the basis of blockchain, which ensures the security of transactions. Kaur *et al*. [15] proposed a game-theoretic Dynamic Bandwidth Allocation (DBA) scheme, which achieved a fair way to allocate the available bandwidth. However, the bandwidth allocation of the Stackelberg game in this paper is auction-based, and the allocated bandwidth is determined based on the requested bandwidth and the requested price.

In order to maximize the benefits of both sides of the game, many game algorithms are proposed. Ng *et al*. [16] proposed a joint auction-alliance formation algorithm, aiming at maximizing the individual profits of UAVs, and the simulation results show that it is not always stable for all UAVs to join the alliance due to the profit maximization behavior of UAVs. The Stackelberg game algorithm in this paper aims at maximizing the overall interests of the vehicle and the RSU, and finally achieves the Nash stability of the algorithm by iterating the Stackelberg algorithm. Colonnese *et al*. [17] implements willingness to pay (wtp)-based optimal bandwidth allocation (WTP-GTOBA) through a greedy, application-layer transparent algorithm, which distributes the remaining bandwidth fairly after each user's minimum bandwidth requirement is met. For limited bandwidth resources, our paper proposes an auction-based Stackelberg game algorithm, which allocates bandwidth according to the bids of each vehicle. Within a certain range, the requested price is high and the bandwidth obtained is more. Zhou *et al*. [18] proposed a joint access selection and bandwidth allocation algorithm for UAV-assisted wireless communication networks, which models bandwidth allocation among UAVs as a non-cooperative game. The algorithm uses the Lagrangian method to solve the convex optimization problem after the selection has stopped. Unlike this work, our article uses the DAG blockchain technology to develop a safe and efficient trading environment, and proposes an auction-based Stackelberg game algorithm to allocate bandwidth. For the optimal allocation of bandwidth, the second-order derivation method is used to solve the convex optimization problem. Zhang *et al*. [19] proposed a dynamic many-to-many full matching algorithm based on three-layer bidding to effectively solve the problem of many-to-many bandwidth allocation between UAVs and users, which realizes the optimization of global network bandwidth resources. In this paper, we consider the many-to-one scenario through a two-stage Stackelberg game, that is, the scenario where multiple vehicles request bandwidth from one RSU, and our proposed algorithm also achieves the optimal resource utilization. A comparison of our work with other competing approaches is given in Table I.

TABLE II
SUMMARY OF NOTATIONS

Notation	Description
N	Total vehicles
B	Total bandwidth of RSU
w_i	Requested bandwidth by vehicle i
$\underline{w}$	Minimum bandwidth request
$\overline{w}$	Maximum bandwidth request
W_{-i}	Sum of bandwidth requests for vehicles other than vehicle i
$\sum w_j$	Bandwidth requests for all vehicles including vehicle i
β_i	Bids of vehicle i
ρ_i	Price offered by vehicle i to request bandwidth
Γ	Sum of prices offered for all vehicles
${\alpha_i}'$	Estimate the bandwidth allocated by the RSU to the vehicle
${w_i}'$	Actual bandwidth allocated to the vehicle by RSU
E_i	Quality of Experience of Vehicles i
ϕ_i	Valuation function of the vehicle i
Λ_i	Cost function of the vehicle i
c	Cost of RSU bandwidth service
$\prod(P)$	Function of RSU profit
${w_i}^*$	Best response function for vehicle i

TABLE III
CONTENT OF COMMUNICATION BETWEEN V2R AND V2V

service	**Information Collection**	**Data Service**	**Safe Traffic**	**Traffic efficiency**
V2R Communication	Real-time traffic congestion	High precision maps	Collision Warning	Best Driving Route Navigation
	Real-time traffic accidents	Traffic Control Information	Intersection warning	Traffic guidance
	Real-time signal matching	Weather condition	Signal warning	Speed guidance
	Real-time road icing conditions	Gas station service areas	Speed limit warning	Road Congestion Alert
service	**Collision Warning**	**Lane Change Alert**	**Ramp Exit Warning**	**Congested road warning**
V2V Communication	Vehicle Location	Incoming traffic from behind	Location of vehicles	Duration of congestion
	Workshop distance	Lane changes	Speed of vehicle	Recommended detour route

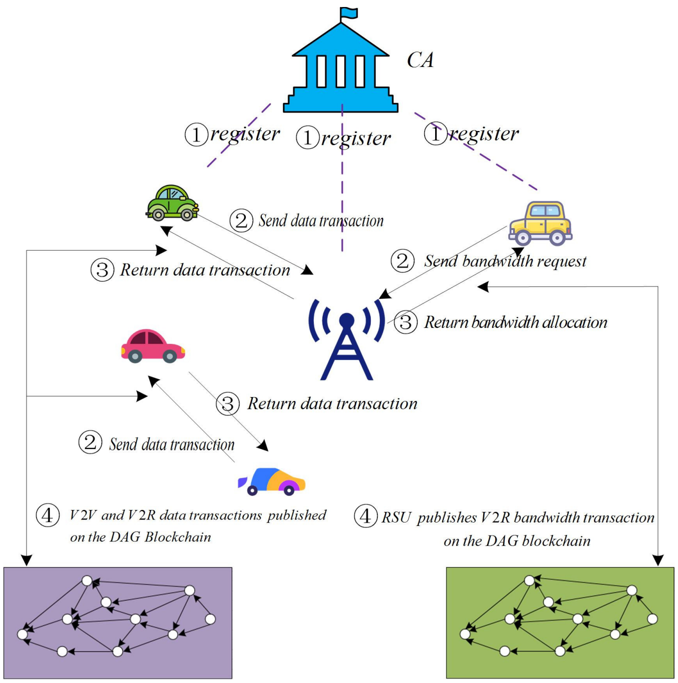

Fig. 1. DAG blockchain-based IoV transaction system model.

III. SYSTEM MODEL

In this section, we introduce the system model of the IoV based on the DAG blockchain, which consists of the network model, task model, blockchain model and Stackelberg game model, respectively. A summary of notations used in the remaining of this paper is also presented in Table II.

A. Network Model

This system mainly consists of vehicle, RSU, certificate authority CA and blockchain, as shown in Fig. 1.

Vehicle: The set of vehicles in the network is defined as $\mathcal{N} = \{1, 2, ..., N\}$. The vehicle requests data from the RSU and exchanges data with the vehicle for different purposes to obtain information such as traffic and road conditions, improve the driving quality and safety of the vehicle, and avoid traffic jams and traffic accidents. Due to the consumption of certain bandwidth resources in the process of V2V and V2R communication, when multiple vehicles request data from the RSU at the same time, they can request high bandwidth from the RSU to improve the speed at which the vehicle obtains data.

Roadside unit(RSU): Distributed on both sides of the road, it provides corresponding data and corresponding bandwidth for requesting vehicles, and its own bandwidth resources are limited. RSU has great computing power.

Certificate Authority(CA): The CA can be regarded as the trusted certificate authority center which means that it cannot be compromised. Both vehicles and RSUs in the system need to obtain their unique identity from the CA, and generate a pair of keys to obtain certificates for data sharing in V2V and V2R.

DAG blockchain: As a distributed ledger, it records data transactions and bandwidth transactions between the vehicle and RSU, and maintains the blockchain ledger through the vehicle and RSU. To prevent bias in the data stored between nodes, the DAG blockchain in the system uses the conflux protocol. Specifically, the block size in Conflux is set to 300KB, and the block confirmation time is 0.5s [20]. In the tree graph structure in Conflux, when a new block is generated, after pointing to the parent block, find the leaf blocks (blocks without child blocks and reference blocks) in the network, and then establish reference edges with them. As shown in Fig. 2, block C does not refer to block B, which means that B is not before block C, while block F refers to block B, which means that B is before F. So the block order is CBF. Thus, a fully sequential block structure can be obtained, and this fully sequential block structure can sort out the order of transactions in the network. When there are duplicate or conflicting transactions in the block, they can be excluded in order.

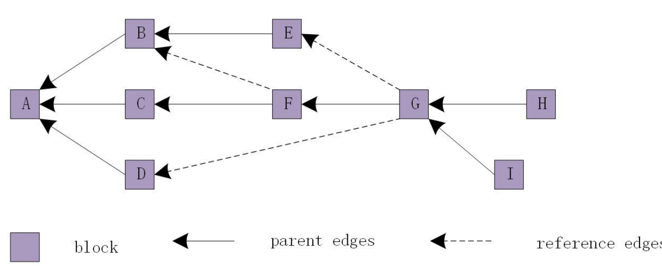

Fig. 2. Conflux tree diagram structure.

B. Task Model

In the vehicle network, data is mainly transmitted through V2V and V2R communication. The on-board units (OBUs) of the vehicle and the RSU communicate through a wireless channel, and the RSU communicates with the fixed safety

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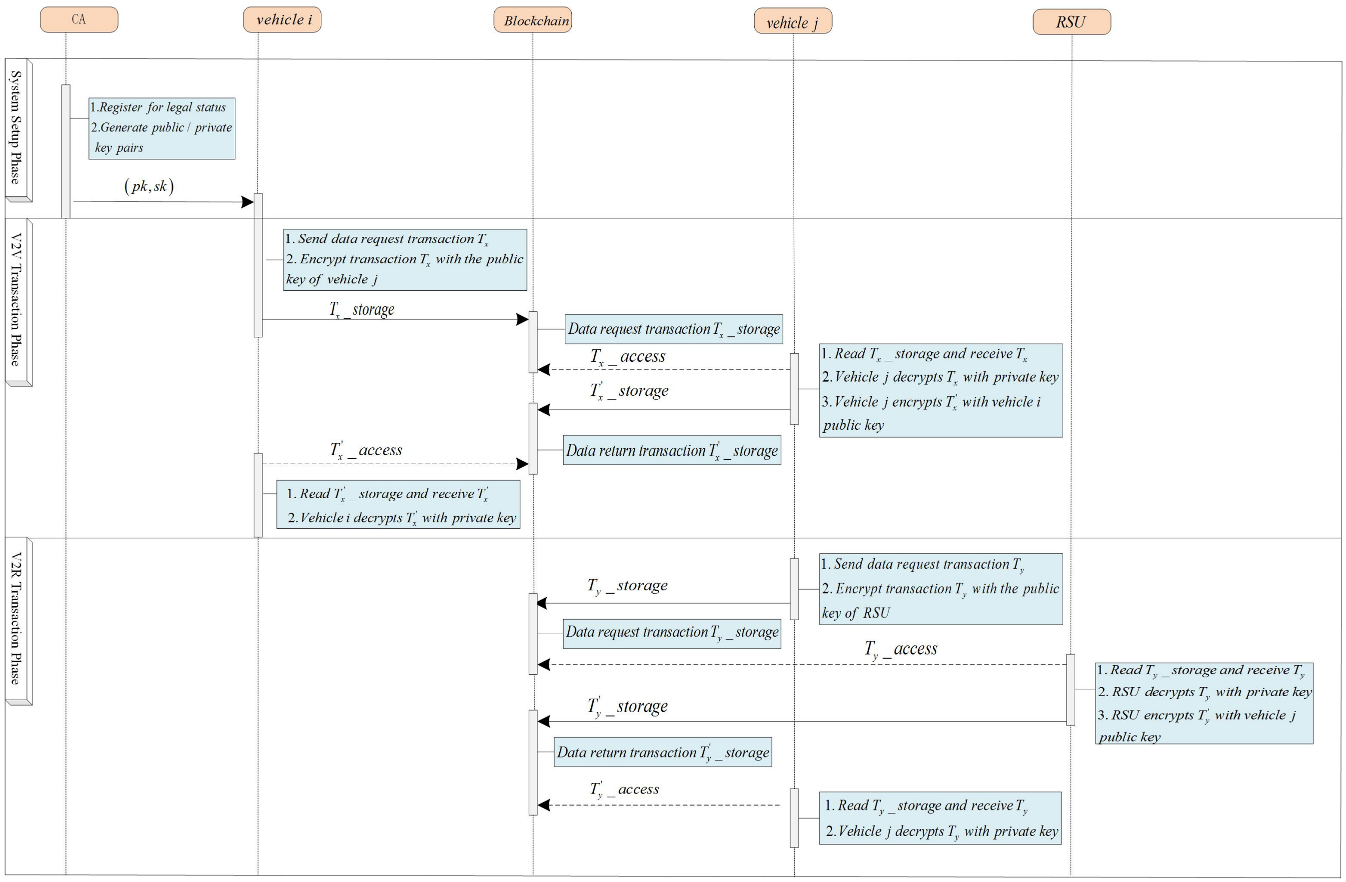

Fig. 3. Flow chart of V2V and V2R transaction system.

network to which it is connected. Through the communication between V2V and V2R, the vehicles in the IoV can obtain more data and information that are beneficial to the driving experience, as shown Table III. But since malicious actors may be present in V2V and V2R communications, i.e., malicious vehicles obtain data from other vehicles or RSUs without any cost and hindrance, dishonest vehicles provide false information, and malicious vehicles try to generate fake ID for data exchange, etc. In addition, the vehicle network has strict time-varying topology and strict latency requirements. Therefore, for data transactions between V2V and V2R in the IoV, it is necessary to ensure both security and high efficiency.

When multiple vehicles send requests to the RSU at the same time, problems such as network delays may occur. In order to enable each vehicle to obtain the required data in time, high bandwidth can be requested from the RSU to improve the speed of obtaining data. However, the bandwidth resource of RSU is limited, and each vehicle requesting bandwidth wants to get the maximum bandwidth at the lowest price, while RSU also wants to get the maximum profit. Therefore, for bandwidth transactions between V2Rs, it is important to ensure security and efficiency while satisfying maximum benefits.

C. DAG Blockchain Model

Initialization: In the vehicle network system, the blockchain is initialized by the CA. Specifically, the CA generates the system parameters and its keys (i.e., private key SK_{CA} and public key PK_{CA}), keeps the private key SK_{CA} secret, and publishes the public key and other parameters to the system.

Registration: After the initialization, the vehicle and the RSU then can register to the CA and obtain their public keys, private keys, digital certificates (DCs), and wallet addresses to become legitimate node in the blockchain. Meanwhile, since RSU has strong computing power, RSU acts as a super node, responsible for publishing and verifying transactions in a timely manner, storing and maintaining the whole blockchain to avoid the problem of uncontrollable transaction length in the DAG blockchain; the vehicle acts as a light node to release transactions.

First, describe the process of data transaction between V2V and V2R, as shown in Fig. 3.

Transactions between V2V: Vehicle i sends a data request transaction T_x to vehicle j, encrypts the transaction with vehicle j public key $PK_j\left(T_x\right)$. This request transaction is denoted by $T_x = \left\{PK_j\left(T_x\right), cer_i, sTime\right\}$ where cer_i represents the legal status of the of vehicle i and $sTime$ represents the timestamp for generating transaction T_x. At the same time, the transaction T_x is recorded on the DAG blockchain. Vehicle j gets the transaction related to itself by accessing the blockchain ledger and decrypts it with its own private key, while returning data transaction T_x' to vehicle i, encrypting the transaction with vehicle i public key $PK_i\left(T_x'\right)$. This return transaction is denoted by $T_x' = \left\{PK_i\left(T_x'\right), cer_j, sTime\right\}$

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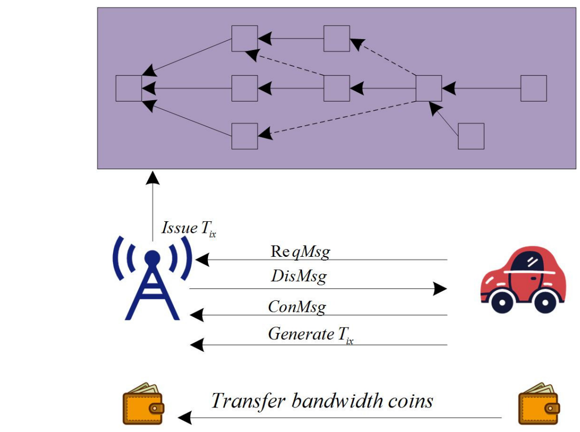

Fig. 4. V2R bandwidth transaction based on DAG blockchain..

where cer_j represents the legal status of the of vehicle j and $sTime$ represents the timestamp for generating transaction T_x'. At the same time, the transaction T_x' is recorded on the DAG blockchain.

Transactions between V2R: Vehicle j sends data request transaction T_y to RSU, encrypts the transaction with RSU public key $PK_R\left(T_y\right)$. This request transaction is denoted by $T_y = \{PK_R\left(T_y\right), cer_j, sTime\}$ where cer_j represents the legal status of the vehicle j and $sTime$ represents the timestamp for generating transaction T_y. At the same time, the transaction T_y is recorded on the DAG blockchain. RSU gets the transaction related to itself by accessing the blockchain ledger and decrypts it with its own private key, while returning data transaction T_y' to vehicle j, encrypting the return transaction with vehicle j public key $PK_j\left(T_y'\right)$. This return transaction is denoted by $T_y' = \left\{PK_j\left(T_y'\right), cer_R, sTime\right\}$ where cer_R represents the legal status of the of RSU and $sTime$ represents the timestamp for generating transaction T_y'. At the same time, the transaction T_y' is recorded on the DAG blockchain.

Second, for N vehicles in the system, i.e., $\mathcal{N} = \{1, 2, ..., N\}$, data are requested from the RSU simultaneously. Each vehicle $i \in \mathcal{N}$ determines its bandwidth service requirements, defined as w_i. Additionally, we Consider $w_i \in [\underline{w}, \overline{w}]$, where $\underline{w}$ is the minimum bandwidth requirement and $\overline{w}$ is the maximum bandwidth service requirement controlled by the RSU. Then, let $W \triangleq (w_1, w_2, ..., w_N)$ denote an overview of bandwidth service requirements for all vehicles. W_{-i} and $\sum w_j$ represent the bandwidth service demand of other vehicles except vehicle i and the bandwidth request of all vehicles including vehicle i, respectively.

Step 1, vehicles submit their bids, i.e., β_i as Eqn. (1).

$$\beta_i = (w_i, \rho_i) \tag{1}$$

Here, w_i represents the requested bandwidth of the ith vehicle and ρ_i represents the price offered by the vehicle when requesting bandwidth from RSU. Note that, the bandwidth requested by the vehicle from the RSU is smaller than the total bandwidth of the RSU itself, i.e., $(w_i < B)$.

Step 2, after collecting the bids of N vehicles, the bandwidth allocated by RSU to each vehicle is as shown in Eqn. (2).

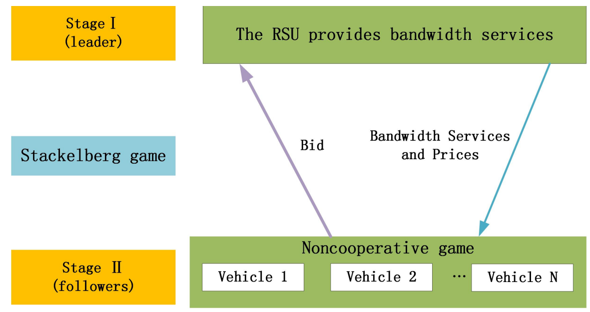

Fig. 5. V2R two-Stage Stackelberg Game.

$$w_i' = \min(w_i, \frac{\rho_i}{\Gamma}B) \forall \quad w_i < B \tag{2}$$

Step 3, the sum of the requested prices offered by all vehicles is as shown Eqn. (3).

$$\Gamma = \sum_{i}^{N} \rho_i \tag{3}$$

Third, the specific process of bandwidth transaction between V2Rs is shown in Fig. 4.

Bandwidth transactions between V2R:

(1) Bid request: Multiple vehicles send requests to the RSU simultaneously, where vehicle i sends its request information $\beta_i = (w_i, \rho_i)$, which includes the requested bandwidth and the requested price. Vehicle i signs the request message $sig_i\left(\beta_i\right)$, and sends both the request message and the signature to the RSU, denoted as $ReqMsg = \{sig_i\left(\beta_i\right), \beta_i\}$. RSU then returns the distribution information $w_i' = \min(w_i, \frac{\rho_i}{\Gamma}\beta)$ to the requesting vehicle, which includes the actual bandwidth distributed to the vehicle by RSU and the price information. RSU signs the returned distribution information $sig_R\left(w_i'\right)$, sends the signature and distribution information to the requesting vehicle i, denoted as $DisMsg = \{sig_R\left(w_i'\right), w_i'\}$.

(2) Verification message: When vehicle i receives the message from RSU, it will send a confirmation message E_i to RSU, which means "I have determined the requested bandwidth", denoted as $ConMsg = \{PK_R(E_i), Cer_i, sTime\}$, where $PK_R(E_i)$ means the confirmation message E_i is encrypted with RSU public key, Cer_i indicates the legal status of Vehicle i and $sTime$ represents the timestamp to confirm this message.

(3) Allocate bandwidth: Once the RSU receives a vehicle confirmation message, it first checks whether vehicle i is among the multiple requesting vehicles. If it is, RSU immediately provides bandwidth service to the vehicle, otherwise, it will refuse to provide service. After the allocation of bandwidth ends, vehicle i generates a new transaction T_{ix}, and vehicle i sends the transaction signature $sig_i\left(T_{ix}\right)$ to the RSU.

(4) Transaction: When the RSU receives a new transaction from a vehicle, it decrypts the transaction using the public key of the corresponding vehicle to view the transaction and signs it. Then, the RSU publishes a new transaction $sig_R\left(sig_i\left(T_{ix}\right)\right)$ on the DAG blockchain.

(5) Validation and payment: After the transaction T_{ix} is issued, it waits for the validation of the new added transaction

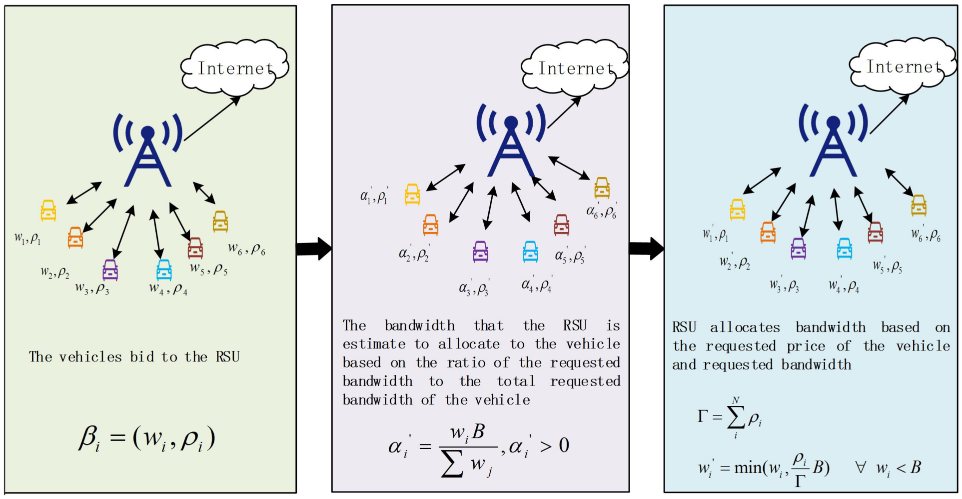

Fig. 6. Bandwidth auction between V2R.

later. When the transaction is successfully validated, the system transfers the cost of the bandwidth from N_i wallet to RSU wallet. The coins of the vehicle N_i will be deducted from the wallet $W_{i(k)}$ and permanently transferred to the $W_{R(k)}$ wallet.

D. Stackelberg Game Model

The bandwidth request between the vehicle and the RSU is modeled as a two-stage Stackelberg game, as shown in Fig. 5. In Stage I, RSU as the leader, displays the bandwidth and price it can provide. In Stage II, each vehicle acts as a follower to determine its own requested bandwidth and price based on the information provided by RSU. Vehicles are in competition with each other and all want to maximize their own profit, i.e., to find the equilibrium point of optimal bandwidth allocation so that the quality experience of each vehicle is maximum. As long as they deviate from the equilibrium point, their own quality experience will be reduced [21]. Meanwhile, RSU maximizes its own profit by providing bandwidth services to vehicles.

By recording the data transactions between V2V and V2R and the bandwidth transactions of V2R on the DAG blockchain, the security of the transactions is enhanced and the efficiency of the transactions is improved. However, for bandwidth transactions between V2R, we not only want to ensure the security and efficiency of the transactions, but also make the profits of both sides of the transactions optimal. Therefore, a two-stage Stackelberg game based on bandwidth auction is proposed, which satisfies to make the vehicle obtain the maximum bandwidth at the lowest price while maximizing the profit of RSU. The specific game is described in detail in Section IV.

IV. Equilibrium Analysis Of Optimal Bandwidth Allocation

In order to satisfy the requirement that the vehicle can obtain the maximum bandwidth from the RSU at the lowest price and maximize the profit of the RSU, we propose an auction-based two-stage Stackelberg game between the vehicle and the RSU. A scenario of a bandwidth auction between the RSU and the vehicle is shown in Fig. 6.

(1) The first box shows the vehicle submitting a bid to the RSU, which includes the requested bandwidth and the requested price. Among them, the bandwidth allocated to the vehicle by the RSU depends not only on the requesting vehicle but also on the bidding of other vehicles. Non-cooperative relationship between V2V.

(2) The second box shows a scenario for estimating how much bandwidth the RSU allocates to the vehicle. According to the bandwidth requests of all requesting vehicles, it is estimated that the RSU allocates the bandwidth according to the ratio of the bandwidth requested by the vehicle to the total bandwidth as $\alpha_i{}' = \frac{w_i B}{\sum w_j}, \alpha_i{}' > 0$.

(3) The third box shows a scenario of how much bandwidth the actual RSU allocates to the vehicle. According to the bandwidth requests of all requesting vehicles, RSU actually allocates the bandwidth according to the required bandwidth of the vehicle and the offered price as $w_i^{'} = \min(w_i, \frac{\rho_i}{\Gamma} B)$.

In the auction-based two-stage Stackelberg game, to better analyze the strategy problem between vehicles and RSUs, we use the backward induction method.

1) Stage II (Vehicle Request Bandwidth Policy): With known RSU prices and other vehicle policies, vehicle i determines its bandwidth requirements to maximize quality of experience as follows:

$$\begin{aligned} E_i(\rho_i, \rho_{-i}) &= \phi_i(\alpha_i{}'(\beta_i), \rho_i{}^*) - \Lambda_i(w_i{}'(\beta_i), \rho_i) \\ &= \rho_i{}^* \alpha_i{}' - \rho_i w_i{}' \end{aligned} \tag{4}$$

where $\alpha_i{}' = \frac{w_i B}{\sum w_j}$, $w_i{}' = \min(w_i, \frac{\rho_i}{\Gamma} B)$. $\phi_i(\alpha_i{}'(\beta_i), \rho_i{}^*)$ and $\Lambda_i(w_i{}'(\beta_i), \rho_i)$ represent the estimated cost price and the actual cost price, respectively.

2) Stage I (RSU Pricing Strategy): The profit of RSU is the fee obtained by providing bandwidth service to the vehicle minus the service cost, where the RSU service cost depends on the bandwidth demand w_i of the vehicle. Therefore, the price of the strategy space determined by RSU is $\left\{P = [\rho_i]_{i \in \mathcal{N}} : 0 \leq \rho_i \leq \overline{\rho}\right\}$, so that its profit maximization is expressed as:

$$\prod(P, W) = \sum\nolimits_{i \in \mathcal{N}} \rho_i w_i{}' - \sum\nolimits_{i \in \mathcal{N}} c w_i{}' \tag{5}$$

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Algorithm 1 Auction Based Stackelberg Game Theoretic Model.

Input: Bid price $\beta_i = (w_i, \rho_i)$ for all vehicles where w_i = requested bandwidth and ρ_i = bid price $\forall i \in \{1, 2, ..., N\}$
Output: The vehicle experience quality is optimal and the profit of RSU is maximum
for (i=1 : n) do
if $w_i' < w_i$ **then**
Follow the below mentioned steps:
1. compute estimate allocate bandwidth α' using: $\alpha_i' = \frac{w_i B}{\sum w_j}, \alpha_i' > 0$
2. compute actual allocate bandwidth w_i using: $w_i' = \min(w_i, \frac{\rho_i}{\Gamma} B)$
3. compute estimate the cost ϕ_i using: $\phi_i(\alpha_i'(\beta_i), \rho_i^*) = \rho_i^* \frac{w_i B}{\sum w_j}$
4. compute the actual cost Λ_i using: $\Lambda_i(w_i'(\beta_i), \rho_i) = \rho_i w_i'$
5. compute the vehicle quality experience E_i is optimal using: $E_i(\rho_i, \rho_{-i}) = \phi_i(\alpha_i'(\beta_i), \rho_i^*) - \Lambda_i(w_i'(\beta_i), \rho_i)$
6. compute the RSU profit $\prod(P, W)$ is maximum using: $\prod(P, W) = \sum_{i\in\mathcal{N}} \rho_i w_i' - \sum_{i\in\mathcal{N}} c w_i'$
end if
end for

In Algorithm 1, first the bid price of all vehicles in the RSU network is taken as the input. The output of the algorithm is the optimal quality experience of the vehicle and the maximum profit of the RSU. For this purpose, the actual bandwidth allocation $w_i{'}$, estimated bandwidth $\alpha_i{'}$, estimated cost ϕ and actual cost Λ of all vehicles are calculated.

Conclusion 1. *The proposed algorithm converges when* $\left\|\rho^{[n]} - \rho^{[n-1]}\right\| / \left\|\rho^{[n-1]}\right\| < \varepsilon$ *is satisfied.*

Proof. Starting from the initial bidding of any vehicle, in order to satisfy to make the best quality experience of the vehicle and the maximum profit of RSU, RSU allocates the bandwidth as in Eqn. (2) according to the bid of each vehicle as in Eqn. (1), and if the vehicle is not satisfied with the bandwidth allocated by RSU, the vehicle will adjust its bid and rebid to continue the iteration. After RSU gets a new bid for the vehicle, it will use the gradient-assisted search algorithm to update the price, i.e., $\rho(n+1) = \rho(n) + \delta\ \nabla\Pi(\rho(n))$, until $\left\|\rho^{[n]} - \rho^{[n-1]}\right\| / \left\|\rho^{[n-1]}\right\| < \varepsilon$ is satisfied and the algorithm converges, where $\rho(n)$ is the bandwidth price at the nth step of the vehicle iteration, ε is the iteration step size of the bandwidth price, and $\nabla\Pi(\rho(n))$ is the gradient. In this case, the quality experience of the vehicle and the profit of the RSU converge to the optimum.

Conclusion 2. *The time complexity of the proposed algorithm is $o(1)$ for one iteration, $o(n)$ for n iterations, and $o(\log n)$ for in-between iterations. The communication complexity of the algorithm is $o(|xN|)$.*

Proof. First, for the time complexity of the proposed algorithm, the vehicle evaluates its quality experience according to Eqn. (4) and the RSU evaluates its profit according to Eqn. (5) to determine whether to continue with the iteration. If the benefits of the vehicle and RSU are optimal in one bid, then the complexity is $o(1)$. The worst case is that it takes n iterations to find the vehicle and RSU Nash stable points with complexity $o(n)$, and in between these two cases, the complexity is $o(\log n)$. Second, for the communication complexity of the proposed algorithm, we denote the size of each bidding content as x, and there are N vehicles in total, then the communication complexity of these N vehicle is $o(|xN|)$.

Next, the bandwidth allocation problem of RSUs under both uniform and discriminatory pricing schemes is considered, and the Nash stability of the Stackelberg game is analyzed.

A. Uniform Price Scheme

We first consider a uniform pricing scheme, where RSU charges the same unit price for all vehicles, i.e., $\rho_i = \rho, \forall i$. And apply backward induction to analyze the two-stage Stackelberg game.

1) Stage II (Demand game for vehicles): Given a price ρ for RSUs, vehicles compete with each other by choosing their own bandwidth requirements to maximize their quality experience, forming a non-cooperative demand game (NDG) $G^u = \left\{\mathcal{N}, \{w_i\}_{i\in N}, \{E_i\}_{i\in N}\right\}$, where $\mathcal{N}$ is the set of vehicles, $\{w_i\}_{i\in\mathcal{N}}$ is the set of bandwidth policies, and E_i is the quality of experience function for the vehicle to obtain the optimal bandwidth service.

Theorem 1. *There exists a Nash equilibrium in (NDG)* $G^u = \left\{\mathcal{N}, \{w_i\}_{i\in N}, \{E_i\}_{i\in N}\right\}$.

Proof. First, the policy space $[\underline{w}, \overline{w}]$ of each vehicle is a non-empty convex compact subset of the Euclidean space. Further, we know E_i is apparently continuous in $[\underline{w}, \overline{w}]$. Then we derive the first order and second order derivatives of (4) with respect to w_i, which can be written as follows:

$$\frac{\partial E_i}{\partial w_i} = {\rho_i}^* \frac{\partial \alpha_i}{\partial w_i} - \rho_i \tag{6}$$

$$\frac{\partial^2 E_i}{\partial {w_i}^2} = {\rho_i}^* \frac{\partial^2 \alpha_i}{\partial {w_i}^2} \tag{7}$$

$$\begin{cases} \frac{\partial {\alpha_i}'}{\partial w_i} = \frac{B(\sum w_j - w_i)}{\sum {w_j}^2} = \frac{B\sum_{i\neq j} w_j}{\sum {w_j}^2} > 0 \\ \frac{\partial^2 \alpha_i}{\partial {w_i}^2} = -\frac{2B\sum_{i\neq j} w_j}{\sum {w_j}^3} < 0 \end{cases} \tag{8}$$

Therefore, we show that E_i is second-order derivable with respect to w_i, so the non-cooperative demand game NDG exists [22]. Meanwhile, according to the condition of the first order derivative in (6), the best response function of the vehicle is obtained, as shown in Eqn. (9).

Theorem 2. *Under the following conditions, the uniqueness of the NDG Nash equilibrium of non-cooperative games is guaranteed.*

$$\frac{2B(N-1)\rho_i}{{\rho_i}^* B} < \sum\nolimits_{i\in\mathcal{N}} \frac{\rho_i}{{\rho_i}^* B} \tag{10}$$

Proof. Refer to the appendix section for details.

$$w_i{}^* = F_i(w) = \begin{cases} \underline{w}, & \sqrt{\frac{\rho_i{}^* B \sum_{i\neq j} w_j}{\rho_i}} - \sum_{i\neq j} w_j < \underline{w} \\ \sqrt{\frac{\rho_i{}^* B \sum_{i\neq j} w_j}{\rho_i}} - \sum_{i\neq j} w_j, & \underline{w} \le \sqrt{\frac{\rho_i{}^* B \sum_{i\neq j} w_j}{\rho_i}} - \sum_{i\neq j} w_j \le \overline{w} \\ \overline{w}, & \sqrt{\frac{\rho_i{}^* B \sum_{i\neq j} w_j}{\rho_i}} - \sum_{i\neq j} w_j > \overline{w} \end{cases} \tag{9}$$

Theorem 3. *The unique Nash equilibrium of vehicle i at NDG is given by*

$$w_i{}^* = \frac{(N-1)}{\sum_{i\in\mathcal{N}} \frac{\rho_i}{\rho_i{}^* B}} - \left(\frac{(N-1)}{\sum_{i\in\mathcal{N}} \frac{\rho_i}{\rho_i{}^* B}}\right)^2 \frac{\rho_i}{\rho_i{}^* B}, \forall i \tag{11}$$

,which holds the condition in the given Eqn. (10).
Proof. Refer to the appendix section for details.

In summary, we use the best response of the vehicles to obtain the Nash equilibrium of the non-cooperative game of vehicles in stage II [22]. Next, we analyze the profit maximization problem of RSU under uniform pricing in stage I.

2) Stage I (RSU profit maximization): Based on the Nash equilibrium of the NDG in stage II, the RSU pricing strategy can be optimized in stage I to maximize the profit in Eqn. (5). Therefore, optimal pricing can be formulated as an optimization problem. Substituting (11) into (5), the profit maximization of RSU is expressed as Eqn. (12).

$$\begin{cases} \max_{P>0} imize & \prod(P) = (\rho - c)\frac{(N-1)}{\sum_{i\in\mathcal{N}} \frac{\rho}{\rho_i^* B}} \\ subject \;\; to & 0 \le \rho \le \overline{\rho} \end{cases} \tag{12}$$

Theorem 4. *Under uniform pricing $\rho_i = \rho\, \forall i$, RSU achieves profit maximization under unified optimal price.*
Proof. From (12), we have

$$\prod(P) = \frac{(\rho - c)}{\rho} \frac{(N-1)}{\sum_{i\in\mathcal{N}} \frac{1}{\rho_i^* B}} \tag{13}$$

Find the first and second derivative of profit $\prod(P)$ with respect to ρ.

$$\frac{d\prod(P)}{d\rho} = \frac{c}{\rho^2}\frac{(N-1)}{\sum_{i\in\mathcal{N}} \frac{1}{\rho_i^* B}} \tag{14}$$

$$\frac{d^2\prod(P)}{d\rho^2} = -\frac{2c}{\rho^3}\frac{(N-1)}{\sum_{i\in\mathcal{N}} \frac{1}{\rho_i^* B}} \tag{15}$$

Due to the negativity of (15), the strict concavity of the objective function is guaranteed. Therefore, RSU can get the maximum profit under the unified pricing.

Under uniform pricing conditions, we show that the Nash equilibrium in stage II is unique, and the maximum profit is also unique in stage I. Therefore, we can conclude that Stackelberg equilibrium is unique and the best response algorithm can achieve this unique Stackelberg equilibrium [22].

B. Discriminatory Pricing Schemes

In discriminatory pricing, RSU sets different unit prices for different vehicle needs. And backward induction is used to analyze the optimal bandwidth requirement of the vehicle and the profit maximization of the RSU.

1) Stage II (Demand game for vehicles): under the discriminatory pricing mechanism, the strategic space of RSU is $\left\{P = [\rho_i]_{i\in\mathcal{N}} : 0 \le \rho_i \le \overline{\rho}\right\}$. We prove the existence and uniqueness of NDG under unified pricing. Under discriminatory pricing, the existence and uniqueness of NDG can still be guaranteed. A slight variation on Theorem 3 leads to the following theorem.

Theorem 5. *Under discriminatory pricing, the unique Nash equilibrium of vehicle i can be obtained as follows:*

$$w_i{}^* = \frac{(N-1)}{\sum_{j\in\mathcal{N}} \frac{\rho_j}{\rho_j{}^* B}} - \left(\frac{(N-1)}{\sum_{j\in\mathcal{N}} \frac{\rho_j}{\rho_j{}^* B}}\right)^2 \frac{\rho_i}{\rho_i{}^* B}, \forall i \tag{16}$$

provided that the following condition

$$\frac{2(N-1)\rho_i}{\rho_i{}^* B} < \sum_{j\in\mathcal{N}} \frac{\rho_j}{\rho_j{}^* B} \tag{17}$$

holds.
Proof. The steps of proof are similar to those in the case of uniform pricing as shown in Section IV-A, and thus we omit them for brevity.

2) Stage I (RSU profit maximization): We use the analytical result of Theorem 5, i.e., the Nash equilibrium of the service demand in stage II, to analyze the profit maximization. Substituting (16) into (5), we have the following optimization

$$\begin{cases} \max_{P>0} imize & \prod(P) = \sum_{i\in\mathcal{N}} (\rho_i - c)\frac{(N-1)}{\sum_{j\in\mathcal{N}} \frac{\rho_j}{\rho_j{}^* B}} \\ subject \;\; to & 0 \le \rho_i \le \overline{\rho}, \forall i \end{cases} \tag{18}$$

Theorem 6. *$\prod(P)$ is concave on each ρ_i, when $\sum\limits_{i\neq j} (\tau_i + \tau_j)\left(1 - \frac{N\frac{\rho_j}{\tau_j}}{\sum\limits_{j\in\mathcal{N}} \frac{\rho_j}{\tau_j}}\right) \le 0, \forall i \in \mathcal{N}$, and monotonically decreasing at every ρ_i when $\sum\limits_{i\neq j} (\tau_i + \tau_j)\left(1 - \frac{N\frac{\rho_j}{\tau_j}}{\sum\limits_{j\in\mathcal{N}} \frac{\rho_j}{\tau_j}}\right) > 0, \forall i \in \mathcal{N}$, provided that the condition $\frac{\rho_i}{\tau_i} \ge \frac{\sum\limits_{j\in\mathcal{N}} \frac{\rho_j}{\tau_j}}{(N-1)^2}$ is satisfied, where $\tau_i = \rho_i{}^* B$.*
Proof. We firstly decompose the objective function in (18) into two components, i.e., $\sum_{i\in\mathcal{N}} \rho_i w_i{}^*$ and $\sum_{i\in\mathcal{N}} c w_i{}^*$. Then, analyze the properties of each part. We define

$$f(\rho) = -c\sum_{i\in\mathcal{N}} w_i{}^* = -c\frac{(N-1)}{\sum_{j\in\mathcal{N}} \frac{\rho_j}{\rho_j{}^* B}} \tag{19}$$

$$
\begin{aligned}
\frac{\partial h(\rho)}{\partial \rho_i} &= \sum\nolimits_{i\neq j}\left((\tau_i+\tau_j)\left(\left(-\frac{1}{\tau_i}\frac{N-1}{\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}+\frac{\rho_i}{\tau_i}\frac{\frac{N-1}{\tau_i}}{\left(\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}\right)^2}\right)\left(1-\frac{\rho_j}{\tau_j}\frac{N-1}{\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}\right)+\left(1-\frac{\rho_i}{\tau_i}\frac{N-1}{\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}\right)\left(\frac{\frac{N-1}{\tau_i}\frac{\rho_j}{\tau_j}}{\left(\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}\right)^2}\right)\right)\right)\\
&= \sum\nolimits_{i\neq j}\left((\tau_i+\tau_j)\left(\left(\frac{-\frac{(N-1)}{\tau_i}\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}+\frac{N-1}{\tau_i}\frac{\rho_i}{\tau_i}}{\left(\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}\right)^2}\right)\left(1-\frac{\rho_j}{\tau_j}\frac{N-1}{\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}\right)+\left(1-\frac{\rho_i}{\tau_i}\frac{N-1}{\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}\right)\left(\frac{\frac{N-1}{\tau_i}\frac{\rho_j}{\tau_j}}{\left(\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}\right)^2}\right)\right)\right) \qquad (26)\\
&= \sum\nolimits_{i\neq j}\left((\tau_i+\tau_j)\left(\left(\frac{-\frac{N-1}{\tau_i}\sum\limits_{t\neq i}\frac{\rho_t}{\tau_t}}{\left(\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}\right)^2}\right)\left(1-\frac{\rho_j}{\tau_j}\frac{N-1}{\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}\right)+\left(1-\frac{\rho_i}{\tau_i}\frac{N-1}{\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}\right)\left(\frac{\frac{N-1}{\tau_i}\frac{\rho_j}{\tau_j}}{\left(\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}\right)^2}\right)\right)\right)
\end{aligned}
$$

$$
\frac{\partial^2 h(\rho)}{\partial {\rho_i}^2} = \sum\nolimits_{i\neq j}\left((\tau_i+\tau_j)\left(\frac{2\frac{N-1}{{\tau_i}^2}\sum\limits_{t\neq i}\frac{\rho_t}{\tau_t}}{\left(\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}\right)^3}\left(1-\frac{\rho_j}{\tau_j}\frac{N-1}{\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}\right)-\left(1-\frac{\rho_i}{\tau_i}\frac{N-1}{\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}\right)\frac{2\frac{N-1}{{\tau_i}^2}\frac{\rho_j}{\tau_j}}{\left(\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}\right)^3}\right)\right) \qquad (27)
$$

Let $\tau_j = {\rho_j}^* B$, and we have $f(P) = -\frac{c(N-1)}{\sum_{j\in\mathcal{N}}\frac{\rho_j}{\tau_j}}$. Next, we find the first and second partial derivatives of (19) with respect to ρ_i:

$$\frac{\partial f(\rho)}{\partial \rho_i} = \frac{c(N-1)}{\tau_i\left(\sum_{j\in\mathcal{N}}\frac{\rho_j}{\tau_j}\right)^2} \qquad (20)$$

$$\frac{\partial^2 f(\rho)}{\partial {\rho_i}^2} = \frac{-2c(N-1)}{{\tau_i}^2\left(\sum_{j\in\mathcal{N}}\frac{\rho_j}{\tau_j}\right)^3} \qquad (21)$$

Further, we have:

$$\frac{\partial f(\rho)}{\partial \rho_i \rho_j} = \frac{-2c(N-1)}{\tau_i\tau_j\left(\sum_{j\in\mathcal{N}}\frac{\rho_j}{\tau_j}\right)^3} \qquad (22)$$

Thus, the Hessian matrix of $f(\rho)$ can be obtained, denoted as :

$$\nabla^2 f(\rho) = \frac{-2c(N-1)}{\left(\sum_{j\in\mathcal{N}}\frac{\rho_j}{\tau_j}\right)^3}\begin{bmatrix} \frac{1}{{\tau_1}^2} & \frac{1}{\tau_1\tau_2} & \cdots & \frac{1}{\tau_1\tau_N} \\ \frac{1}{\tau_2\tau_1} & \frac{1}{{\tau_2}^2} & \cdots & \frac{1}{\tau_2\tau_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\tau_N\tau_1} & \frac{1}{\tau_N\tau_1} & \cdots & \frac{1}{{\tau_N}^2} \end{bmatrix} \qquad (23)$$

For each $i \in \mathcal{N}$, we have $\frac{1}{{\tau_i}^2} > 0$. The diagonal elements of the Hessian matrix are all greater than zero, and the major dimension is equal to zero. Thus, the Hessian of $f(\rho)$ is semi-negative definite.

Then, we analyze the $\sum_{i\in\mathcal{N}}\rho_i {w_i}^*$.

$$h(\rho) = \sum\nolimits_{i\in\mathcal{N}}\rho_i {w_i}^* = \frac{\sum_{i\neq j}\tau_i w_i w_j}{\left(\sum w_j\right)^2} \qquad (24)$$

Substituting (16) into (24), the final expression for can be written as (25)

$$h(\rho) = \sum\nolimits_{j\neq t}\left(\tau_t\left(1-\frac{\rho_t}{\tau_t}\frac{N-1}{\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}\right)\left(1-\frac{\rho_j}{\tau_j}\frac{N-1}{\sum\limits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}\right)\right) \qquad (25)$$

The first-order and second-order partial derivatives for ρ_i in Eqn. (25) are shown in (26) and (27).

Since we have, $w_i = \frac{(N-1)}{\sum_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}\left(1-\frac{(N-1)}{\sum_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}\frac{\rho_i}{\tau_i}\right) > 0$ and $1-\frac{(N-1)}{\sum_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t}}\frac{\rho_i}{\tau_i} > 0$. For convenience, we define $\varsigma_{ij} = \sum\limits_{i\neq j}(\tau_i+\tau_j)\left(1-\frac{N\frac{\rho_j}{\tau_j}}{\sum\limits_{j\in\mathcal{N}}\frac{\rho_j}{\tau_j}}\right)$, when $\varsigma_{ij} \leq 0$, $\frac{\partial^2 h(\rho)}{\partial {\rho_i}^2} < 0$, every ρ_i on $h(\rho)$ is concave.

Furthermore, we show that $\prod(P)$ is a monotonically decreasing function of ρ_i for $\varsigma_{ij} > 0$. As shown in Eqn. (28), where $\rho_{\min} = \min\{\rho_1, \rho_2, \cdots, \rho_N\}$. In fact, $\rho_{\min} > c$. Thus, after a series of proofs, we can obtain when $\varsigma_{ij} > 0$, $\frac{\partial \prod(P)}{\partial \rho_i} < 0$.

Theorem 7. *Under discriminatory pricing, RSU maximizes profits by finding a unique optimal pricing vector.*

Proof. From Theorem 6, we know that every ρ_i on $\prod(p)$ is concave when $\varsigma_{ij} \leq 0$ and every ρ_i decreases when $\varsigma_{ij} > 0$. In other words, when ρ_i is concave on $\prod(\rho)$, ρ_i needs to be less than a certain threshold, and when ρ_i is greater than a certain threshold, ρ_i on $\prod(\rho)$ is decreasing. If the price is above the threshold, the vehicle is reluctant to purchase bandwidth services from the RSU. We know that when $\varsigma_{ij} \leq 0$, the optimal profit value of RSU, i.e., $\prod^*(\rho)$ is realized at the concave part, so ρ^* exists. Then, we use the variational inequality method to prove the existence of at most one optimal solution [22], and obtain the uniqueness of the optimal solution, i.e., the Stackelberg equilibrium. Let $S = \left\{\rho = [\rho_1, \cdots \rho_N] \mid \sum\limits_{i\neq i}(\tau_i+\tau_j)\left(1-\frac{N\frac{\rho_j}{\tau_j}}{\sum\limits_{j\in\mathcal{N}}\frac{\rho_j}{\tau_j}}\right) \leq 0, \forall i \in \mathcal{N}\right\}$ Constraints can be rewritten as follows:

$$\sum_{i\neq j}\left((\tau_i+\tau_j)\left(\sum\nolimits_{t\in\mathcal{N}}\frac{\rho_t}{\tau_t} - N\frac{\rho_j}{\tau_j}\right)\right) \leq 0 \qquad (29)$$

Then, we formulate the equivalence problem of (18) as:

$$
\begin{aligned}
\frac{\partial \prod (P)}{\partial \rho_i} &= \sum\nolimits_{i \neq j} \left((\tau_i + \tau_j) \left(\left(\frac{-\frac{N-1}{\tau_i} \sum\limits_{t \neq i} \frac{\rho_t}{\tau_t}}{\left(\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \right)^2} \right) \left(1 - \frac{\rho_j}{\tau_j} \frac{N-1}{\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t}} \right) + \left(1 - \frac{\rho_i}{\tau_i} \frac{N-1}{\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t}} \right) \left(\frac{\frac{N-1}{\tau_i} \frac{\rho_j}{\tau_j}}{\left(\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \right)^2} \right) \right) \right) + \frac{c \frac{(N-1)}{\tau_i}}{\left(\sum_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \right)^2} \\
&= \frac{\frac{N-1}{\tau_i}}{\left(\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \right)^2} \left(\sum\nolimits_{i \neq j} \left((\tau_i + \tau_j) \left(-\sum_{t \neq i} \frac{\rho_t}{\tau_t} \left(1 - \frac{\rho_j}{\tau_j} \frac{N-1}{\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t}} \right) + \frac{\rho_j}{\tau_j} \left(1 - \frac{\rho_i}{\tau_i} \frac{N-1}{\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t}} \right) \right) \right) + c \right) \\
&\leq \underbrace{- \frac{\frac{N-1}{\tau_i}}{\left(\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \right)^2} \sum\nolimits_{i \neq j} \left((\tau_i + \tau_j) \left(\sum_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \left(1 - \frac{\rho_j}{\tau_j} \frac{N-1}{\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t}} \right) \right) \right)}_{<0} + \frac{\frac{N-1}{\tau_i}}{\left(\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \right)^2} \left(c - \sum\nolimits_{i \neq j} \left((\tau_i + \tau_j) \frac{N-1}{\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t}} \frac{\rho_i}{\tau_i} \frac{\rho_j}{\tau_j} \right) \right) \\
&= - \frac{\frac{N-1}{\tau_i}}{\left(\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \right)^2} \sum\nolimits_{i \neq j} \left((\tau_i + \tau_j) \left(\sum_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \left(1 - \frac{\rho_j}{\tau_j} \frac{N-1}{\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t}} \right) \right) \right) + \frac{\frac{N-1}{\tau_i}}{\left(\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \right)^2} \left(c - \sum\nolimits_{i \neq j} \left(\underbrace{\frac{(\tau_i + \tau_j)}{\tau_j}}_{<1} \frac{N-1}{\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t}} \frac{\rho_i \rho_j}{\tau_i} \right) \right) \\
&\leq - \frac{\frac{N-1}{\tau_i}}{\left(\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \right)^2} \sum\nolimits_{i \neq j} \left((\tau_i + \tau_j) \left(\sum_{h \in \mathcal{N}} \frac{\rho_h}{\tau_h} \left(1 - \frac{\rho_j}{\tau_j} \frac{N-1}{\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t}} \right) \right) \right) + \frac{\frac{N-1}{\tau_i}}{\left(\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \right)^2} \left(c - \rho_{\min} \frac{N-1}{\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t}} \frac{N-1 \rho_i}{\tau_i} \right) \\
&= \underbrace{- \frac{\frac{N-1}{\tau_i}}{\left(\sum\limits_{t \in N} \frac{\rho_t}{\tau_t} \right)^2} \sum\nolimits_{i \neq j} \left((\tau_i + \tau_j) \left(\sum_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \left(1 - \frac{\rho_j}{\tau_j} \frac{N-1}{\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t}} \right) \right) \right)}_{<0} + \underbrace{\frac{\frac{N-1}{\tau_i}}{\left(\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t} \right)^2} \left(c - \rho_{\min} \frac{(N-1)^2}{\sum\limits_{t \in \mathcal{N}} \frac{\rho_t}{\tau_t}} \frac{\rho_i}{\tau_i} \right)}_{<0} < 0
\end{aligned}
\tag{28}
$$

$$
\begin{cases}
\min\limits_{\rho > 0} imize & -\prod (\rho) \\
subject \;\; to & \rho \in S
\end{cases}
\tag{30}
$$

Let $F(\rho) = \nabla(-\prod(\rho)) = -[\nabla_{p_i}\Pi]_{i \in \mathcal{N}}$, where the optimization problem in (30) is equivalent to finding a point set $\rho^* \in S$, i.e., $(\rho - \rho^*) F(\rho^*) \geq 0, \forall \rho \in S$. It is the variational inequality $\mathrm{VI}(S, F)$. Therefore, we can conclude that $\mathrm{VI}(S, F)$ has at most one solution by showing that F is strictly monotone and continuous on S [23]. The optimal unique solution of the equivalence problem, i.e., the Stackelberg equilibrium, is found.

Definition 1. *If F is strictly monotonic over S, VI(S, F) has at most one solution, where $S \in R^N$ is a concave closed set and the map $F : S \mapsto R^N$ is continuous [23].*

Proof. Let $\lambda \in (0, 1)$, $\rho_1, \rho_2 \in S$, can get $\lambda\rho_1 + (1-\lambda)\rho_2 \in S$. It can be known from (31) that S is a concave closed set. To prove that the map $F : S \mapsto R^N$ is strictly monotonic over S, we consider the positivity $(\rho_1 - \rho_2)(F(\rho_1) - F(\rho_2)) \geq 0, \forall \rho_1, \rho_2 \in S$ and $\rho_1 \neq \rho_2$. Also from the definition of the variational inequality it follows that

$$
\begin{aligned}
&(\rho_1 - \rho_2)(F(\rho_1) - F(\rho_2)) = \\
&\sum_{i \in \mathcal{N}} \left((\rho_1 - \rho_2) \left(-\nabla_{p_i}\Pi|_{\rho i = \rho_{1i}} + \nabla_{p_i}\Pi|_{\rho i = \rho_{2i}} \right) \right)
\end{aligned}
$$

From Theorem 6 we have $\frac{\partial^2 \prod(\rho)}{\partial \rho_i^2} = \frac{\partial^2 (f(\rho) + h(\rho))}{\partial \rho_i^2} < 0$. Therefore, $\nabla_{p_i}\Pi$ is decreasing on each ρ_i and $-\nabla_{p_i}\Pi$ is increasing on each ρ_i, which yields

$$
-\nabla_{p_i}\Pi|_{\rho_i = \rho_{1i}} + \nabla_{p_i}\Pi|_{\rho_i = \rho_{2i}} = \begin{cases} \geq 0, \rho_{1i} \geq \rho_{2i} \\ < 0, \rho_{1i} < \rho_{2i} \end{cases}
\tag{32}
$$

Based on the above analysis we have:

$$
\left((\rho_1 - \rho_2) \left(-\nabla_{p_i}\Pi|_{\rho_i = \rho_{1i}} + \nabla_{p_i}\Pi|_{\rho_i = \rho_{2i}} \right) \right) \geq 0, \forall i \in \mathcal{N}
\tag{33}
$$

We know that $\rho_1 \neq \rho_2$, and that there exists at least one j satisfying the restriction (33). Therefore, we prove that F is strictly monotone and continuous on S. So far, we have proved that $\mathrm{VI}(S, F)$ has at most one solution according to Definition 1 in [23]. Thus, the Stackelberg equilibrium is proved.

V. Numerical Analysis

A. Simulation Settings

Consider a distributed network of V2V and V2R. For the evaluation of V2R model, we first consider four vehicles in the RSU network, and assume that the bandwidth requested by the four vehicles is $w_i = \{6, 12, 18, 24\}$ MHz, and the corresponding request price offered by each vehicle is $\rho_i = \{35, 58, 71, 84\}$ per cent. For each iteration, by increasing the number of requested vehicles, the situation of bandwidth allocation and quality experience of these four vehicles are considered to make the vehicle quality experience optimal. In addition, we increase the number of requested vehicles by analyzing the impact of uniform and discriminatory pricing schemes on RSU profits, which makes the RSU the most profitable. Major parameters used in the simulation are given in Table IV, most of which are adopted from [24], [25].

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$$
\begin{aligned}
&\sum_{i\neq j}\left(\left(\tau_i+\tau_j\right)\left(\sum\nolimits_{t\in\mathcal{N}}\frac{\lambda\rho_{1t}+(1-\lambda)\,\rho_{2t}}{\tau_t}-N\frac{\lambda\rho_{1j}+(1-\lambda)\,\rho_{2j}}{\tau_j}\right)\right)\\
&=\sum_{i\neq j}\left(\left(\tau_i+\tau_j\right)\left(\lambda\sum\nolimits_{t\in\mathcal{N}}\frac{\rho_{1t}}{\tau_t}+(1-\lambda)\sum\nolimits_{t\in\mathcal{N}}\frac{\rho_{2t}}{\tau_t}-\lambda N\frac{\rho_{1j}}{\tau_j}-(1-\lambda)\,N\frac{\rho_{2j}}{\tau_j}\right)\right)\\
&=\lambda\sum_{i\neq j}\left(\left(\tau_i+\tau_j\right)\left(\sum\nolimits_{t\in\mathcal{N}}\frac{\rho_{1t}}{\tau_t}-N\frac{\rho_{2j}}{\tau_j}\right)\right)+(1-\lambda)\sum_{i\neq j}\left(\left(\tau_i+\tau_j\right)\left((1-\lambda)\sum\nolimits_{t\in\mathcal{N}}\frac{\rho_{1t}}{\tau_t}-N\frac{\rho_{2j}}{\tau_j}\right)\right)\leq 0
\end{aligned}
\tag{31}
$$

TABLE IV
PARAMETER SETTING IN SIMULATION

Parameter	Setting
Frequency of interaction between vehicle and RSU	[50,200]times/week
Coverage range of RSU	[300,500]m
Vehicle speed	[40,120]km/h
Vehicle to RSU bandwidth	20MHz
Number of vehicles	[4,40]
Transmission power	[10,23]dBm
Receiver power	14 dBm
Define parameters	$w_1=6,\ \rho_1=35, w_2=12,\ \rho_2=58$ $w_1=6,\ \rho_1=35, w_2=12,\ \rho_2=58$

B. Performance of the Proposed Game Model

Fig. 7 shows that as the number of requested vehicles in the network increases, the bandwidth allocation of the RSU to the vehicles decreases. This is because the bandwidth of the RSU is fixed and is divided among the vehicles that are in the network of RSU.

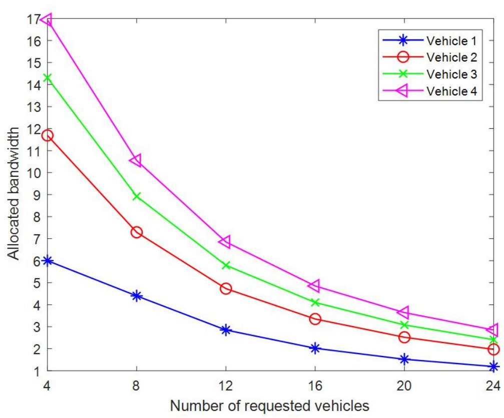

Fig. 7. Variation in allocated bandwidth based on the number of requested vehicles.

Fig. 8 shows that as the number of requested vehicles changes the impact on bandwidth allocation and bid prices. As bid prices increase, the quality of the vehicle experience decreases as bids for other vehicles increase. This is because the quality experience of a vehicle depends on the decisions of all vehicles in the network. And in a certain situation, the vehicle will get the maximum quality experience based on the required bandwidth and bid price. After that, the quality experience of the vehicle will not increase as the bid price of the vehicle increases.

Fig. 9 shows that as the number of requested vehicles increases, the profit of RSU tends to increase. This is because

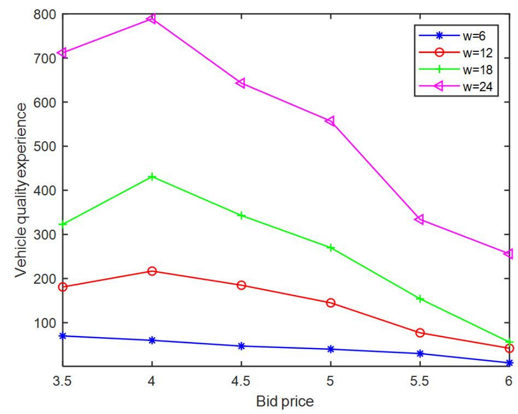

Fig. 8. Variation in Quality experience of vehicle with increase in bid price

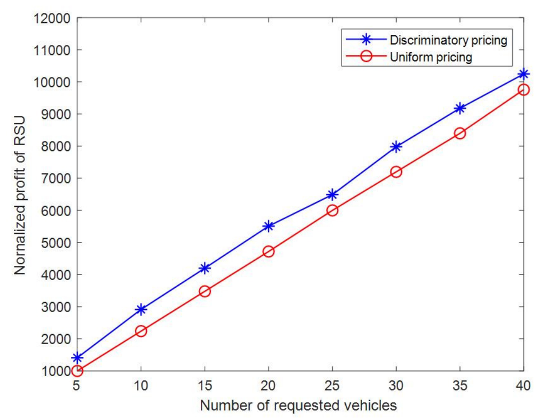

Fig. 9. Normalized the profit of RSU versus the number of requested vehicles.

the more vehicles are requested in the network and the more competition among the vehicles, the more profit the RSU can make. Further, we find that RSU is slightly more profitable under discriminatory pricing than under uniform pricing. This is because discriminatory pricing can set different service demand unit prices for vehicles. In this case, RSU can significantly encourage vehicles to obtain higher total service demand under discriminatory pricing and gain greater profit gains.

C. Performance Analysis and Comparison of the Proposed DAG blockchain

The traditional blockchains such as Bitcoin [26], Ethereum [27] and DAG blockchains such as IOTA [28], Byteball [29], NANO [30], Conflux [31], Hashgraph [32], SPECTRE [33] and PHANTOM [34] are compared in terms of throughput,

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TABLE V
COMPARISON OF DAG BLOCKCHAIN AND TRADITIONAL BLOCKCHAIN

Blockchain technology	Consensus mechanism	TPS	Whether practical application	Whether the node requires arithmetic power	Whether there are incentives	Whether to support smart contracts
IOTA	Tangle	100	✓	✓	✓	×
Byteball	Main Chain	100	✓	×	×	✓
NANO	DPoS	1000	✓	×	×	×
Conflux	GHAST	6400	✓	✓	✓	✓
Hashgraph	BFT	10000	×	×	✓	✓
SPECTRE	Block+DAG	3000	×	✓	✓	×
PHANTOM	Block+DAG	1500	×	✓	✓	✓
Bitcoin	Pow	7	✓	✓	✓	×
Ethereum	Pow	15	✓	✓	✓	✓

etc. The results are shown in Table V. It is observed that DAG blockchain has a significant advantage in throughput, which is tens of times higher than the traditional blockchain.

Specifically, no mining and no transaction fees are required in IOTA [28], and the throughput of the entire network is high. It does not support smart contracts itself. Byteball [29] introduces the master chain and witness to avoid the double spend problem. Smart contracts are supported. However, the time of transaction confirmation is uncertain, which limits its scalability and speed to some extent. NANO [30] guarantees a reasonably low power consumption operation of the block through the DPOS mechanism. However, NANO is not fully tested, the consensus algorithm may be at risk of serious flaws, and does not support smart contracts. Conflux [31] uses the GHAST consensus protocol to make full use of network bandwidth for higher throughput, and dynamically determines the weight of each block based on the tree graph structure to ensure the confirmation speed and activity of the consensus system. Hashgraph [32] introduces the Gossip protocol to avoid causing massive messaging storms at consensus and supports smart contracts. However, it has not yet been operated in large-scale public chain environments. SPECTRE [33] can be mined in parallel to obtain greater throughput and faster transaction confirmation. However, it cannot integrate smart contracts because it can only do the relative ordering of conflicting transactions and cannot do an absolute ordering. Finally, PHANTOM [34] uses a greedy algorithm to implement a full sequence of settings verified by honest nodes. It enhances the computational power of PHANTOM and also makes smart contracts compatible with it. However, it has not been specifically applied due to technical difficulties.

Based on the above analysis, Conflux its achieved decentralization while ensuring TPS and supporting smart contracts. Therefore, the DAG chain in our scheme is chosen to be the conflux protocol, which has a throughput of 6400 TPS per second, much higher than the 7 TPS per second of Bitcoin and 15 TPS per second of Ethereum, and can meet the demand of transactions in the Internet of Vehicles. Meanwhile, the average of 4 blocks generated per second, each block size is about 300KB, the amount of data transferred per second is 1.2MB, which is about 4GB per hour, then the new transaction history data can be up to 30TB per year, and storing 30TB of data in industrial grade applications is not a particularly difficult task [31]. Also, since a DAG is used in conflux, the computational overhead is negligible compared to POW.

VI. CONCLUSION

In this paper, we propose a DAG blockchain-based game model of optimal bandwidth allocation strategy for IoV. The model combines the communication between V2V and V2R to improve the diversity of data flow in the vehicle network. By recording the communication and bandwidth transactions between them in the DAG blockchain, the security and efficiency of the communication between V2V and V2R in the network is guaranteed. Further, the bandwidth service between V2Rs is modeled as a two-stage Stackelberg game model, and the RSU profit maximization and vehicle quality experience maximization problems are studied by backward induction. Meanwhile, the bandwidth pricing schemes of RSUs are analyzed, including the uniform pricing scheme and the discriminatory pricing scheme. For uniform pricing applicable to all vehicles, the uniqueness of the Stackelberg equilibrium is verified by determining the optimal request bandwidth of the vehicles. For vehicles that apply to different prices, the uniqueness of Stackelberg equilibrium is proved using variational inequality theory. The correctness of the model is verified by simulated experimental data, and the comparative analysis shows that the model has a high throughput.

APPENDIX

Proof of Theorem 2.

Proof. Based on the first-order derivative condition in (6), the optimal response function of vehicle with respect to the bandwidth is obtained as shown in (9). The uniqueness of the Nash equilibrium is proved and the optimal response function is given [22], i.e., as shown in (9).

First, for the positivity of the response function, under the conditions of (10), we have (given by Lemma 1 in [21])

$$\sum\nolimits_{i\neq j} w_j < \frac{{\rho_i}^* B}{4\rho_i} < \frac{{\rho_i}^* B}{\rho_i} \tag{34}$$

Thus $\sum_{i\neq j} w_j < \sqrt{\frac{{\rho_i}^* B \sum_{i\neq j} w_j}{\rho_i}}$, so we can get $\sqrt{\frac{{\rho_i}^* B \sum_{i\neq j} w_j}{\rho_i}} - \sum_{i\neq j} w_j > 0$. Positivity is proven.

Second, we prove continuity with respect to (9). Let $w' > w$, we can further prove that $F_i(w') > F_i(w)$, as shown in (35).

Specifically, we know that $\sqrt{\sum_{i\neq j} {w'}_j} - \sqrt{\sum_{i\neq j} w_j} > 0$, and in addition

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$$
\begin{aligned}
F_i(w') - F_i(w) &= \sqrt{\frac{{\rho_i}^* B \sum_{i \neq j} {w'}_j}{\rho_i}} - \sum_{i \neq j} {w'}_j - \left(\sqrt{\frac{{\rho_i}^* B \sum_{i \neq j} w_j}{\rho_i}} - \sum_{i \neq j} w_j \right) \\
&= \left(\sqrt{\frac{{\rho_i}^* B}{\rho_i}} - \sqrt{\sum_{i \neq j} {w'}_j} - \sqrt{\sum_{i \neq j} w_j} \right) \left(\sqrt{\sum_{i \neq j} {w'}_j} - \sqrt{\sum_{i \neq j} w_j} \right)
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
\lambda F_i(w) - F_i(\lambda w) &= \lambda \sqrt{\frac{{\rho_i}^* B \sum_{i \neq j} w_j}{\rho_i}} - \lambda \sum_{i \neq j} w_j - \left(\sqrt{\frac{{\rho_i}^* B \sum_{i \neq j} \lambda w_j}{\rho_i}} - \sum_{i \neq j} \lambda w_j \right) \\
&= \left(\lambda - \sqrt{\lambda} \right) \sqrt{\frac{{\rho_i}^* B \sum_{i \neq j} w_j}{\rho_i}} > 0, \forall \lambda > 1
\end{aligned} \tag{36}
$$

$$
\sqrt{\frac{{\rho_i}^* B}{\rho_i}} - \sqrt{\sum_{i \neq j} {w'}_j} - \sqrt{\sum_{i \neq j} w_j} \in \left(\sqrt{\frac{{\rho_i}^* B}{\rho_i}} - 2\sqrt{\sum_{i \neq j} {w'}_j}, \sqrt{\frac{{\rho_i}^* B}{\rho_i}} - 2\sqrt{\sum_{i \neq j} w_j} \right)
$$

We can prove by condition (44) that $\sqrt{\frac{{\rho_i}^* B}{\rho_i}} - 2\sqrt{\sum_{i \neq j} w_j} > 0, \forall w_j$, then $\sqrt{\frac{{\rho_i}^* B}{\rho_i}} - \sqrt{\sum_{i \neq j} {w'}_j} - \sqrt{\sum_{i \neq j} w_j} > 0$. Therefore $F_i(w') > F_i(w)$ holds, which means that the best response function (9) for vehicle i is always positive.

Finally, we need to prove $\lambda F_i(w) > F_i(\lambda w)$, for $\lambda > 1$ scalability in Eqn. (36).

So far, we have shown that the best response function in (9) satisfies the three properties of the standard function described in [21]. Thus, the Nash equilibrium of NDG $G^u = \left\{ \mathcal{N}, \{w_i\}_{i \in N}, \{E_i\}_{i \in N} \right\}$ is unique.

Proof of Theorem 3.

Proof. According to (6), for each vehicle i , we have

$$
\frac{\sum_{i \neq j} w_j}{\sum {w_j}^2} = \frac{\rho_i}{{\rho_i}^* B} \tag{37}
$$

Then calculate the total expression for all vehicles

$$
\frac{(N-1) \sum w_j}{\sum {w_j}^2} = \sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^* B} \tag{38}
$$

This indicates: $\frac{(N-1)}{\sum w_j} = \sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^* B}$. Thus, we have

$$
\sum w_j = \frac{(N-1)}{\sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^* B}} \tag{39}
$$

Recalling (9), according to the condition of the first derivative of (6), we have

$$
\sum w_j = \sqrt{\frac{{\rho_i}^* B \sum_{i \neq j} w_j}{\rho_i}} \tag{40}
$$

We substitute (40) into (39) for a simple calculation to get

$$
\frac{(N-1)}{\sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^* B}} = \sqrt{\frac{{\rho_i}^* B}{\rho_i} \left(\frac{(N-1)}{\sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^* B}} - w_i \right)} \tag{41}
$$

After squaring both sides, we have:

$$
\left(\frac{(N-1)}{\sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^* B}} \right)^2 = \frac{{\rho_i}^* B}{\rho_i} \left(\frac{(N-1)}{\sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^* B}} - w_i \right) \tag{42}
$$

With a simple transformation, we obtain the Nash equilibrium of the vehicle as shown in (11).

Proof of Lemma 1.

Give

$$
\frac{2B(N-1)\rho_i}{{\rho_i}^* B} < \sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^* B} \tag{43}
$$

Meet the condition

$$
\sum_{i \neq j} w_j < \frac{{\rho_i}^* B}{4\rho_i} \tag{44}
$$

Proof. According to (11) and (39), we can obtain

$$
\sum_{i \neq j} w_j = \left(\frac{B(N-1)}{\sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^*}} \right)^2 \frac{\rho_i}{{\rho_i}^* B} \tag{45}
$$

Substituting (44) into (45), we have

$$
\left(\frac{(N-1)}{\sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^* B}} \right)^2 \frac{\rho_i}{{\rho_i}^* B} < \frac{{\rho_i}^* B}{4\rho_i} \rightarrow \frac{(N-1)^2}{\left(\sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^* B} \right)^2} < \frac{{\rho_i}^* B}{4\rho_i} \frac{{\rho_i}^* B}{\rho_i} \rightarrow \frac{4(N-1)^2}{\left(\frac{{\rho_i}^* B}{\rho_i} \right)^2} < \left(\sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^* B} \right)^2 \rightarrow \frac{2(N-1)}{\frac{{\rho_i}^* B}{\rho_i}} < \sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^* B} \rightarrow \frac{2(N-1)\rho_i}{{\rho_i}^* B} < \sum_{i \in \mathcal{N}} \frac{\rho_i}{{\rho_i}^* B}
$$

This means that the condition in (43) is guaranteed.

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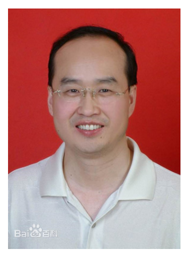

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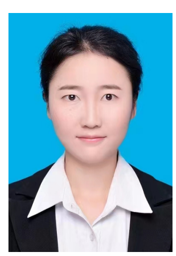

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