Economic Analysis of Loot Box Market in Blockchain Games

Yu Jiang
The Chinese University of Hong Kong, Shenzhen
Shenzhen Institute of Artificial Intelligence and Robotics for Society
Shenzhen, China
yujiang1@link.cuhk.edu.cn

Sizheng Fan
The Chinese University of Hong Kong, Shenzhen
Shenzhen Institute of Artificial Intelligence and Robotics for Society
Shenzhen, China
sizhengfan@link.cuhk.edu.cn

Wei Cai*
The Chinese University of Hong Kong, Shenzhen
Shenzhen Institute of Artificial Intelligence and Robotics for Society
Shenzhen, China
caiwei@cuhk.edu.cn

ABSTRACT

As an emerging blockchain technique, the blockchain-based loot box has received significant attention recently due to its promising characters in transparency and decentralization. Because all virtual assets are bound to the players' own address, which allows the players to control and manage everything, players can trade virtual items via the 2nd exChange Market (2CM) directly. To understand the players' optimal strategies and the game provider's optimal pricing, we conduct an economic analysis of both the game provider's and players' behaviors. In addition, gas fee, a unique factor in blockchain, is taken into consideration. Specifically, we model the interactions between the game provider and players as a two-stage Stackelberg game. In Stage I, we model the game provider's optimal pricing problem to maximize his utility using prospect theory (PT) due to the intrinsic demand uncertainty. In Stage II, the players choose the market which can maximize their utility to derive their preferred items. Moreover, our analysis and numerical results show that a game provider who considers the PT modeling should adopt a conservative pricing mechanism to increase his utility. Besides, the primary market is more susceptible to gas fees than the 2CM.

CCS CONCEPTS

• **Networks** → **Network economics**; • **Human-centered computing** → *Collaborative and social computing*.

KEYWORDS

Blockchain game, loot box, Stackelberg game, prospect theory

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*Wei Cai is the corresponding author (caiwei@cuhk.edu.cn).

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1 INTRODUCTION

Along with the rise and prosperity of online and mobile games over the past decades, loot box, as a probabilistic good involving a probability of obtaining one or more multiple distinct in-game virtual items [22], has been a solid and robust source of revenue for the game industry. For example, in the case of Activision Blizzard, a US-based gaming company, approximately 51% of its revenue, US $6.5 billion, came from in-game net bookings in 2019, which covered loot boxes and other in-app purchases [2]. However, the loot box has caused controversy and criticism [19]. There are two limitations to the traditional loot box: 1) *Probability opacity*: Loot box items can be common or rare, and the probabilities of obtaining different items can be unknown to the player before opening a loot box. Chen *et al.* [4] showed that the seller could obtain additional revenue by purposely misrepresenting the allocation probabilities in the absence of regulation. 2) *Meaningless Ownership*: All the virtual properties in the loot boxes, including credits, items, and avatars, are stored in the game providers' server rather than owned by the players [16, 18]. Also, players sell unwanted or redundant virtual items in the loot box through centralized third-party platforms like Steam market[1], which sometimes can not be converted into fiat. Hence, the players call for more freedom and want true ownership, which allows them to buy, sell, trade, and use their items with no restrictions from third parties. According to a survey by Worldwide Asset Exchange (WAX), a blockchain platform focused on virtual items [1], 75% of game players would be more likely to invest in digital assets if they could use them in multiple games, which is made possible by true ownership. Moreover, 69% of game developers believe when items are freely tradeable, it increases the value of those items, and 84% would create cross-compatible in-game items if the technology allowed them to do so.

Fortunately, Ethereum, known as the Blockchain 2.0 platform, provides a promising solution to the problems mentioned above. In addition to the classic application of distributed public ledger, the smart contracts on Ethereum are open-source programs that can be automatically executed without any centralized control [6, 28]. Following are the major benefits that blockchain has brought to the loot box. 1) *Probability transparency*: Due to the transparent characteristic of blockchain data, players or third-party organizations can audit the smart contract-based probability, which will enhance the trustworthiness of the loot box. 2) *Asset Ownership*: The players own the loot box and corresponding virtual assets in the blockchain games because virtual assets are bound to their addresses. Hence, the ownership enables the virtual assets to be independent of game

[1]https://steamcommunity.com/market/

providers, allowing players to retain their digital properties even after the game stops running. Besides, game players can trade the virtual assets in the loot box with others simply by listing their respective prices via the on-chain 2nd exChange Market (2CM). So, when a buyer is willing to buy the item with the corresponding price, he will transfer the corresponding price and receive the item. In this case, the loot box for blockchain games has attracted much interest from industry and academia in recent years. Besides, many blockchain games which are significant to Metaverse[5, 17], such as *Gods Unchained*[2], *Total War: Three Kingdoms*[3], and *NBA Top Shot*[4], set a loot box mechanism.

However, the blockchain-based loot box also poses new problems for buyers and sellers. Unlike off-chain loot boxes, purchasing a loot box in blockchain games requires a significant commission, known as the gas fee. Due to the rapid development of blockchain, games and on-chain applications are booming, which caused the evolution of gas prices during the second half of 2020. For example, in September 2019, the average price was 0.0225ETH ($4.8 at the time), and one year later, it was 0.193ETH ($74.9 at the time), that is, an 8500% increase [8]. Moreover, gas fees fluctuate continuously, which will be much higher than usual during network congestion. Besides, a freer blockchain-based 2CM will bring uncertainty to loot box sales in the primary market, which the game providers should consider. We aim to solve three critical questions in such a market: 1) How should the game provider set every loot box's price to maximize his revenue under the uncertainty of the 2CM? 2) Given a fixed price of each loot box, how should the players choose their strategies to maximize their utility? 3) How does a gas fee affect players' behavior?

To answer the above questions, we model the interactions between the game provider and players as a two-stage Stackelberg game. In Stage I, the game provider decides the selling price of each loot box in the primary market. In Stage II, we model the players' preferences based on the Hotelling line. Each player needs to decide among the primary market, 2CM, and non-participation to maximize utility. On account of the price and demand uncertainty from the 2CM, it is significantly challenging for the game provider to solve problems in Stage I. For the above problem, Expected Utility Theory (EUT) [24] is always applied to solve uncertain decision problems. However, EUT can not reflect the complex psychology in humans' decision-making mechanisms, which leads to significant inconsistencies with observations from reality. Luckily, Prospect Theory (PT) was proposed by researchers in behavioral economics, which can more accurately describe decision-making under uncertainty. Moreover, quite a few human behaviors that seem to be illogical under EUT have been explained legitimately [13]. We summarize our key results and contributions as follows.

- *Blockchain-based Virtual Assets Trading Market Modeling*: To the best of our knowledge, this is the first work to consider the blockchain-based virtual assets trading market, including the primary market and 2CM at the same time. Besides, we take the transparency of blockchain and the cost of the gas fee into account and use the Stackelberg game to model the interactions between the game provider.
- *PT-based Game Provider's Behavior Analysis*: Since the game provider decides the price of every loot box involving uncertain demand in the future due to the appearance of 2CM, we adopt the PT mechanism, which is a more accurate description of the decision-making uncertainty than EUT. We consider the particularity of blockchain to show the effects of the 2CM on players and compare the optimal decision of the game provider under the PT model and traditional EUT model.

[2]https://godsunchaiend.com
[3]https://ht.twtk.finance
[4]https://nbatopshot.com

The rest of the paper is organized as follows. In section 2, we present related works and analyze their limitations. We put forward the blockchain-based virtual assets trading market model and formulate the two-stage Stackelberg game in Section 3. We analyze the Nash equilibrium (NE) of Stage I and present the players' best response in Section 4. In Section 5, we solve the loot box pricing optimization problem. We show the numerical results to analyze players' strategy and the impact of the PT model on the game provider in Section 6, and conclude in Section 7.

2 RELATED WORK

2.1 Blockchain-based New Business Model

Many studies focused on building new business models on the blockchain. Cai *et al.* [3] revealed the direction of blockchain development by surveying the state-of-the-art decentralized applications (dApps). The work in [26] considered a blockchain-based ranking system to incentivize using renewable energy for electric vehicles. Similarly, the study in [27] used cryptocurrency as a trustful and privacy-preserving incentive method to reward the contribution of users in the crowdsourcing platform. The authors in [20] implemented the cryptocurrency for the store-carry-forwarding security in vehicular networks. A study in [21] applied cryptocurrency to create a secure service providing and data sharing between the Internet of things devices. The paper [25] focused on using cryptocurrency to guide and influence the big data industry. Fan *et al.* [7] designed and implemented a smart contract to facilitate an automatic, autonomous, and auditable auction that was transparent and decentralized. Specifically, anyone participating in the auction can check the code and results of the auction in the smart contract.

2.2 Loot Box

A loot box is a kind of opaque selling, which is the practice of selling items where some features of the item are hidden from the customer until after purchase. A series of works on opaque selling has emerged recently. Most of these studies have focused on opaque selling as a tool to manage imbalanced customer demand or induce opportunities for price discrimination[10, 11]. However, the loot boxes have zero marginal cost compared with the traditional industries. Chen *et al.* [4] provided the first formal economics analysis of the loot boxes. Specifically, they considered how to optimally price the two kinds of loot boxes, including traditional loot boxes and unique loot boxes, from the perspective of a revenue-maximizing video game company. However, they did not consider the impact of the 2CM on pricing and designing loot boxes from the perspective of the game provider and players.

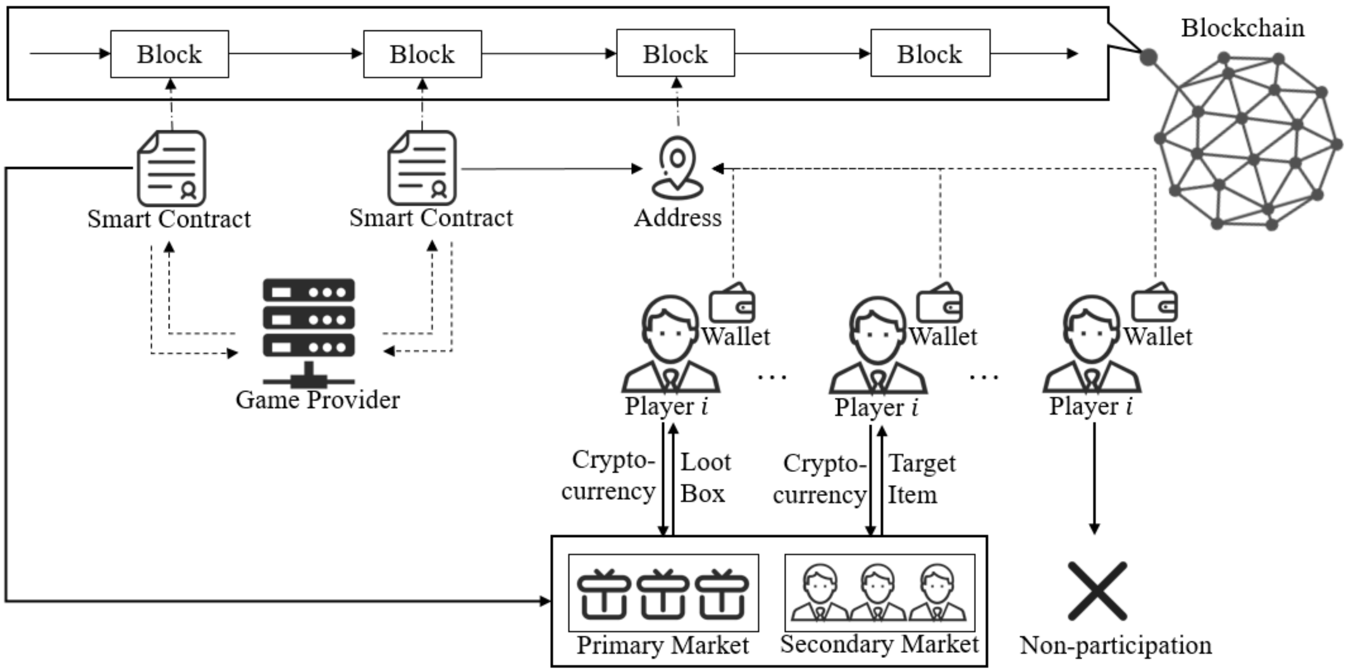


Figure 1: System Model

3 SYSTEM MODEL

In this section, we model the game provider's and the players' strategies in Section 3.1 and Section 3.2, respectively. And we formulate the two-stage Stackelberg game in Section 3.3.

3.1 Game Provider

Figure 1 illustrates the architecture for the virtual assets trading market for blockchain games. The game provider deploys smart contracts, which can be audited by the players, regarding the loot box's price and probability. Before playing the blockchain games, the player should register an address on the blockchain.

We consider a revenue-maximizing game provider sells a catalog of $\mathcal{M} = \{1, 2, \cdots, M\}$ specific virtual items with different classes or rarities. To simplify the presentation and better illustrate the insights, we assume two different items ($M = 2$), item A and item B, in a set of loot boxes for the rest of the paper. The probability that the loot box is actually item A is $\phi_A = \phi \in [0.5, 1]$, while the probability that the loot box is actually item B is $\phi_B = 1-\phi$. Besides, we scale the market size to 1[5], which is divided into three parts: the primary market, 2CM, and non-participant market[6]. The game provider's revenue comes from selling loot boxes, and the virtual items have zero marginal cost compared with the other industrial products. Hence, the game provider's utility can be calculated as the product of the single loot box price and the number of loot boxes.

$$U_p = P \cdot D_p \tag{1}$$

where P refers to the price of single loot box and $D_p \in [0, 1]$[7] refers to the demand of the loot boxes in the primary market.

3.2 Players

We consider a continuum of players in the virtual assets trading market and let $\mathcal{I} = \{1, 2, \cdots, I\}$ denote the set of players and i be the index of the player set.

We use x to denote the player's type, which parameterizes its preference for virtual items. We assume that x is continues variable and distributed uniformly over $[0, 1]$ (i.e., $x_i \in [0, 1]$). We consider using the Hotelling model to represent players' preference [9]. Over the Hotelling line $[0, 1]$, item A and item B are located at the endpoints 0 and 1, respectively. The maximum values of virtual item A and virtual item B are V_A and V_B, respectively, such that $V_B > V_A$[8]. Besides, there exists a fit-cost-coefficient t for modeling each player's item preference. So, the valuation of item m for the player i can be represented as follows.

$$u_j(x_i) = \begin{cases} V_A - tx_i & \text{if } j = A, i \in N. \\ V_B - t(1 - x_i) & \text{if } j = B, i \in N. \end{cases} \tag{2}$$

where V_A, V_B, t satisfy three basic assumptions [14]:

- $V_A \geq t$ and $V_B \geq t$ so that the buyer's valuation $u_j(x_i)$ of item j is positive for any $x_i \in [0, 1]$

[5] To simplify the presentation and derive the insights, we scale the market size to 1, which has been widely used in literature [14].
[6] The non-participant market refers to players who choose to give up obtaining virtual items because of the price.
[7] The total demand is the same as the market size, which is scaled to 1. Hence, the demand of the primary market D_p satisfies $D_p \in [0, 1]$.
[8] This assumption is based on the fact that, for example, a Rare item is more difficult to acquire and more expensive to buy in the 2CM than a Common one.

- $V_A > V_B - t$ so that the buyer located at $x_i = 0$ would prefer item A over item B.
- $V_B > V_A - t$ so that the buyer located at $x_i = 1$ would prefer item B over item A.

As shown in Figure 1, players can obtain the target items from both the primary market and 2CM. Besides, players can also choose non-participation if they think their preferred items' prices are higher than their expectations in the above two markets. So, we use α_i to denote the strategy of the player i. The strategy of player $i \in \mathcal{I}$ is to decide to choose the primary market with $\alpha_i = 1$, the 2CM with $\alpha_i = 2$ and the non-participation with $\alpha_i = 0$, respectively.

A type-x_i players' payoff is

- When player i chooses to participate the primary market to buy one loot box (i.e., $\alpha_i = 1$), the expected valuation of the loot box for player i located at $x_i \in [0, 1]$ can be represented as follows.

$$\omega(x_i) = \phi_A \cdot u_A(x_i) + \phi_B \cdot u_B(x_i) \tag{3}$$

Hence, the utility of the player i who chooses the primary market is given by $U_i(1)$, where

$$U_i(1) = \omega(x_i) - P - G \tag{4}$$

where P refer to the price of each loot box which is determined by the game provider. In order to buy loot boxes, players must pay a fee to call the smart contract, known as "gas fee" denoted by G[9].

- When the player i decides to take part in the 2CM to buy the prefered virtual item from the other players directly (i.e., $\alpha_i = 2$), we use π_A and π_B to denote the price of item A and item B in the 2CM, respectively. Similarly, to buy the virtual item from the 2CM in blockchain games, players also need to pay gas fee G' to call the smart contract, which assumed to be equal to the gas fee to buy the loot box, such that $G' = G$. As a rational seller, the price of the virtual item is larger than the price of a loot box (i.e., $\pi_A \geq P$ and $\pi_B \geq P$) and smaller than the maximal value of the corresponding virtual item (i.e., $\pi_A \in [V_A - t, V_A]$ [10] and $\pi_B \in [V_B - t, V_B]$ [11]). Hence, the player i's utility can be represented as follows.

$$U_i(2) = \begin{cases} u_A(x_i) - \pi_A - G, & \text{if player } i \text{ buy item } A, \\ u_B(x_i) - \pi_B - G, & \text{if player } i \text{ buy item } B. \end{cases} \tag{5}$$

- If the player i chooses non-participation (i.e., $\alpha_i = 0$) and the corresponding utility is

$$U_i(0) = 0. \tag{6}$$

3.3 Two-Stage Stackelberg Problem Formulation

In this subsection, we formulate the overall problem as a two-stage *Stackelberg* game.

- In Stage I, the game provider decides the selling price of each loot box in the primary market to maximize his utility.
- In Stage II, each player chooses among the primary market, 2CM and non-participation.

We will use backward induction to solve this two-stage optimization problem. In Section 4, We first derive the solution to the player's strategy problem in Stage II based on the selling price in the primary market and 2CM. Then we deal with the game provider's utility maximization problem in Stage I based on the solution to the player's strategy problem in Section 5.

4 STAGE II: MARKET SELECTION GAME

In this section, we first formulate the players' market selection game in Section 4.1 and analyze the strategy of the players in Section 4.2.

4.1 Player Market Selection Game Formulation

The *players*[12] of the market selection game is the set $\mathcal{I}$.

The strategies and payoff functions of players have been defined in Section 3.2.

Hence, the market selection game is defined as follows.

Definition 1.(Market Selection Game)

- *Players*: the set $\mathcal{I}$.
- Stategies: $\alpha_i \in \{0, 1, 2\}, \forall i \in \mathcal{I}$.
- Payoff: $U_i(\alpha_i), \forall i \in \mathcal{I}, \alpha_i \in 0, 1, 2$.

Definition 2.(Nash Equilibrium) A NE of market selection game is a profile $\boldsymbol{\alpha^*} = \{\alpha_i, i \in \mathcal{I}\}$ such that for each player $i \in \mathcal{I}$,

$$U_i(\alpha_i^*) \geq U_i(\alpha_i)$$

Player i makes a decision in the market selection game to maximize their payoff. The objective function can be represented as follows.

$$\begin{aligned} \max_{\alpha_i} \quad & U_i(\alpha_i) \\ s.t. \; & \alpha_i \in \{0, 1, 2\} \end{aligned} \tag{7}$$

4.2 Analysis of Player's Strategy

Now we study the Nash equilibrium of the above market selection game in stage II. First, we consider the players' preference under Hotelling model in 3.2.

Theorem 1. *The player i located at $x_i \in [0, x^*]$ in the Hotelling line will prefer item A rather than item B, and the player located at $x_i \in (x^*, 1]$ will prefer item B rather than item A, where $x^* = \frac{V_A - V_B + t}{2t}$.*

Proof. We assume that there are only two virtual items and use the Hotelling model to represent the players' preferences. We use x^* to denote the player whose valuation of the two products is the same. Hence, x^* can be calculated as follows.

$$u_A(x^*) = u_B(x^*) \tag{8}$$

where $u_A(x^*)$ and $u_B(x^*)$ means the valuation of item A and B valued by player located at x_i, respectively. And, we can obtain that $x^* \equiv \frac{V_A - V_B + t}{2t}$.

According to the Eq. (2), $u_A(x_i)$ is decreasing with x_i, while $u_B(x_i)$ is increasing with x_i. Hence, the player i located at $x_i \in [0, x^*]$ buys item A, while the player i located at $x_i \in (x^*, 1]$ buys item B. □

[9]Gas fee varies with time and is uncertain. To simplify the presentation and better illustrate the insights, we use G to represent the average gas in a period.

[10]$V_A - t$ represents the valuation valued by the player located at $x_i = 1$ who prefer item A least. There is no reason for the seller to set a price below $V_A - t$ to sell item A.

[11]$V_B - t$ represents the valuation valued by the player located at $x_i = 0$ to prefer item B least. There is no reason for the seller who sets a price below $V_B - t$ to sell item B.

[12]The game theory participants are the same as the participants in the game, which is the player. To differentiate, we use *player* to denote the participants in game theory.

Table 1: The Player i's Optimal Strategy $\alpha_i = 2$ with Corresponding Utility for Choosing the 2CM to Buy Item A

Target Item	Condition	Location of the Player i x_i	Optimal Utility U_i
Item A	$0 \le \pi_A - P \le (1-\phi)(V_A - V_B + t)$ $0 < G < \frac{(1-\phi)(V_A+V_B-t)+(2\phi-1)\pi_A - P}{2(1-\phi)}$	$x_i \in [0, \frac{(1-\phi)(V_A-V_B+t)-(\pi_A-P)}{2t(1-\phi)}]$	$U_i = V_A - tx_i - \pi_A - G$
	$0 \le \pi_A - P \le (1-\phi)(V_A - V_B + t)$ $\frac{(1-\phi)(V_A+V_B-t)+(2\phi-1)\pi_A - P}{2(1-\phi)} \le G \le V_A - \pi_A$	$x_i \in [0, \frac{V_A-\pi_A-G}{t}]$	

We now consider the players' optimal strategies in the market selection game.

4.2.1 Player's Decision of Participating in the 2CM ($\alpha_i = 2$). In this scenario, the players can directly buy the in-game virtual items from other players through the 2CM, in which players can pay a higher fee to remove the loot box's uncertainty. Hence, we consider which player's best decision is to choose the 2CM. Specifically, we consider two preferences as discussed in Theorem 1: (a) which player prefers item A and considers buying it from the 2CM; and (b) which player prefers item B and considers buying it from the 2CM.

4.2.1.1 Case I- Consideration about Buying Item A. In this part, we consider that the player i prefers item A and will buy it from the 2CM.

Theorem 2. *The player i located at $x_i \in [0, \min\{x_A, x'_A\}]$ will purchase item A from other players in the 2CM, where*

$$x_A = \frac{(1-\phi)(V_A - V_B + t) - (\pi_A - P)}{2t(1-\phi)}, x'_A = \frac{V_A - \pi_A - G}{t}.$$

The details are summarized in the Table 1.

Proof. According to Theorem 1, the player i located at $x_i \in [0, x^*]$ will prefer item A, and he can obtain item A for sure if he choose the 2CM. When $U_i(2) \ge U_i(1)$ and $U_i(2) \ge 0$, the player i' s best strategy is to choose the 2CM to buy item A.

When $U_i(2) \ge U_i(1)$, we have

$$\begin{aligned} u_A(x_i) - \pi_A - G &\ge \phi u_A(x_i) + (1-\phi)u_B(x_i) - P - G \\ x_i &\le \frac{(1-\phi)(V_A - V_B + t) - (\pi_A - P)}{2t(1-\phi)} \end{aligned} \quad (9)$$

We define $x_A \equiv \frac{(1-\phi)(V_A-V_B+t)-(\pi_A-P)}{2t(1-\phi)}$, and consider three different conditions:

- If $x_A < 0$, we have $\pi_A - P > (1-\phi)(v_A - v_B + t)$. Because $x_i \in [0,1]$, none of the players choose the 2CM under this condition.
- If $x_A \in [0, x^*]$, we have $0 \le \pi_A - P \le (1-\phi)(V_A - V_B + t)$. Under this condition, the player i is located at $x_i \in [0, x_A]$ will consider to participate in the 2CM rather than the primary market.
- If $x_A > x^*$, we have $\pi_A < P$, which conflicts with the condition that $\pi_A \ge P$. This condition does not exist.

Then, when $x_A \in [0, x^*]$, we consider another condition that $U_i(2) \ge 0$. Combined with Eq.(5), we have

$$\begin{aligned} U_i(2) = u_A(x_i) - \pi_A - G \ge 0 \\ x_i \le \frac{V_A - \pi_A - G}{t} \end{aligned} \quad (10)$$

We define $x'_A \equiv \frac{V_A-\pi_A-G}{t}$, and consider the following conditions:

- If $x'_A < 0$, we have $G > V_A - \pi_A$. Under this condition, the players who consider to buy item A will have negative utility. A high gas fee will cause players to give up.
- If $x'_A \ge 0$, we have $0 < G \le V_A - \pi_A$. Besides, $U_i(2)$ decreases with x_i. Hence, the utility of the player i located at $x_i \in [0, x'_A]$ choosing the 2CM is greater than zero.

Then, we consider $x'_A \ge 0$, so that the player i located at $x_i \in [0, \min\{x_A, x'_A\}]$ will purchase item A from other players in the 2CM. Next, we discuss the relationship between x_A and x'_A. We can obtain that

$$\begin{aligned} & x_A - x'_A \\ &= \frac{-(1-\phi)(V_A + V_B - t - 2G) - (2\phi - 1)\pi_A + P}{2t(1-\phi)}. \end{aligned} \quad (11)$$

To define the relationship, we need to discuss the gas fee:

- If $x_A < x'_A$, we have $0 < G < \frac{(1-\phi)(V_A+V_B-t)+(2\phi-1)\pi_A-P}{2(1-\phi)}$.
- Otherwise, we have $G \ge \frac{(1-\phi)(V_A+V_B-t)+(2\phi-1)\pi_A-P}{2(1-\phi)}$.

In conclusion, when

$$0 \le \pi_A - P \le (1-\phi)(V_A - V_B + t), \text{and}$$
$$0 < G < \frac{(1-\phi)(V_A + V_B - t) + (2\phi - 1)\pi_A - P}{2(1-\phi)},$$

the player i located at $x_i \in [0, x_A]$ will purchase the item A in the 2CM; when

$$0 \le \pi_A - P \le (1-\phi)(V_A - V_B + t), \text{and}$$
$$\frac{(1-\phi)(V_A + V_B - t) + (2\phi - 1)\pi_A - P}{2(1-\phi)} \le G \le V_A - \pi_A,$$

the player i located at $x_i \in [0, x'_A]$ will purchase the item A in the 2CM.

□

4.2.1.2 Case II- Consideration about Buying Item B. In this part, we analyze the case that the player prefers item B and will buy it in the 2CM.

Theorem 3. *The player i located at $x_i \in [x'_B, 1]$ will purchase item B from other players in the 2CM, where*

$$x'_B = \frac{\pi_B - V_B + t + G}{t}.$$

Table 2: The Player i's Optimal Strategy $\alpha_i = 2$ with Corresponding Utility for Choosing the 2CM to Buy Item B

Target Item	Condition	Location of the Player i x_i	Optimal Utility U_i
Item B	$\phi(V_A + V_B - t) - (2\phi - 1)\pi_B \leq P \leq \pi_B$ $0 < G < V_B - \pi_B$	$x_i \in [\frac{\pi_B - V_B + t + G}{t}, 1]$	$U_i = V_B - t(1 - x_i) - \pi_B - G$

The details are summarized in the Table 2.

Proof. According to the Theorem 1, the player i located at $x_i \in (x^*, 1]$ prefers item B, and he will buy item B for sure if he choose the 2CM. When $U_i(2) \geq U_i(1)$ and $U_i(2) \geq 0$, the player i's best strategy is to choose the 2CM to buy item B.

When $U_i(2) \geq U_i(1)$, $x_i \in (x^*, 1]$, we have

$$u_B(x_i) - \pi_B - G \geq \phi u_A(x_i) + (1 - \phi)u_B(x_i) - P - G$$
$$x_i \geq \frac{\phi(V_A - V_B + t) + (\pi_B - P)}{2t\phi} \tag{12}$$

We define $x_B \equiv \frac{\phi(V_A - V_B + t) + (\pi_B - P)}{2t\phi}$ and consider two different conditions:

- If $x_B < x^*$, we have $\pi_B < P$, which conflicts with the condition that $\pi_B \geq P$. This condition doesn't exist.
- If $x_B \in [x^*, 1]$, we have $0 \leq \pi_B - P \leq -\phi(V_A - V_B - t)$. Under this condition, the player i is located at $x_i \in [x_B, 1]$ will consider to participate in the 2CM other than the primary market.
- If $x_B > 1$, we have $\pi_B - P > -\phi(V_A - V_B - t)$. Because $x_i \in [0, 1]$, none of the players choose the 2CM under this condition.

Then, when $x_B \in [x^*, 1]$, we consider that $U_i(2) \geq 0, x_i \in (x^*, 1]$. When $u_B(x_i) - \pi_B - G \geq 0$, we have

$$x_i \geq \frac{\pi_B - V_B + t + G}{t} \tag{13}$$

We define $x'_B \equiv \frac{\pi_B - V_B + t + G}{t}$, and consider the following conditions:

- If $x'_B > 1$, we have $G > V_B - \pi_B$ so that all the players who consider to buy item B in the 2CM will have the negative utility due to the high gas fee.
- If $x'_B \leq 1$, we have $0 < G \leq V_B - \pi_B$. The utility of the player i located at $x_i \in [x'_B, 1]$ choosing the item B in the 2CM is greater than zero.

Then, when $x'_B \leq 1$, the player i is located at $x_i \in [\max\{x_B, x'_B\}, 1]$, he will purchase item B from other players in the 2CM. Next, we discuss the relationship between x_B and x'_B:

$$x_B - x'_B = \frac{\phi(V_A + V_B - t - 2G) - (2\phi - 1)\pi_B - P}{2t\phi}. \tag{14}$$

To define the relationship, we need to discuss the selling price P in the primary market, the selling price π_B of item B in the 2CM and gas fee.

Firstly, let's talk a little bit more about the range of P:

From Eq. (3), the expected valuation of the loot box to the player i who chooses the primary market is

$$\omega(x_i) = -(2\phi - 1)tx_i + V_A - (1 - \phi)(V_A - V_B + t), x_i \in (x_A, x_B) \tag{15}$$

We can see that $\omega(x_i)$ is a linear function and decreases in x_i. $\omega(x_i)$ is bounded below by $\omega(x_B)$. Hence, there is no reason for the game provider to set a price below $\omega(x_B) = \phi(V_A + V_B - t) - (2\phi - 1)\pi_B$. The price of the loot box must satisfy:

$$P \geq \phi(V_A + V_B - t) - (2\phi - 1)\pi_B \tag{16}$$

Next, we combine the discussion of the gas fee:

Due to $0 < G \leq V_B - \pi_B$ and $P \geq \phi(V_A + V_B - t) - (2\phi - 1)\pi_B$, we always have $x_B - x'_B < 0$ such that $x_B < x'_B$.

In summary, when

$$\phi(V_A + V_B - t) - (2\phi - 1)\pi_B \leq P \leq \pi_B, \text{and}$$
$$0 < G \leq V_B - \pi_B,$$

the player i located at $x_i \in [x'_B, 1]$ will purchase the item B in the 2CM.

□

4.2.2 *Player's Decision of Entering the Primary Market ($\alpha_i = 1$).*

According to the Theorem 2 and Theorem 3, when $x_A < 0$ and $x_B > 1$, no player will participate the 2CM. To analysis the players' strategies under two markets at the same time, we only consider the condition that $x_A \in [0, x^*]$ and $x_B \in [x^*, 1]$ in the following theorem.

Theorem 4. *The player i located at $x_i \in (x_A, x_L)$ will purchase one loot box in the primary market, where*

$$x_L = \frac{-(1 - \phi)(V_A - V_B + t) + V_A - P - G}{(2\phi - 1)t}.$$

The details are summarized in the Table 3.

Proof. When $U_i(1) > U_i(2)$ and $U_i(1) \geq 0$, the player i's best strategy is to buy one loot box in the primary market. We first consider the condition $U_i(1) > U_i(2)$. Based on Theorem 2 and Theorem 3, we have $0 < x_i < x_A$ and $x_B < x_i < 1$ when $U_i(2) > U_i(1)$. According to Eq. (4), we have

$$\begin{aligned} U_i(1) &= \omega(x_i) - P - G \\ &= -(2\phi - 1)tx_i + V_A - (1 - \phi)(V_A - V_B + t) - P - G \end{aligned} \tag{17}$$

where $0 \leq \pi_A - P \leq (1 - \phi)(V_A - V_B + t)$ and $0 \leq \pi_B - P \leq -\phi(V_A - V_B - t)$. Hence, $U_i(1)$ is a linear function and decreases with x_i. Hence, we have $x_A < x_i < x_B$ when $U_i(2) < U_i(1)$.

Then, we consider that $U_i(1) \geq 0$. When

$$\phi u_A(x_i) + (1 - \phi)u_B(x_i) - P - G \geq 0,$$

we have

$$x_i \leq \frac{-(1 - \phi)(V_A - V_B + t) + V_A - P - G}{(2\phi - 1)t} \tag{18}$$

We define $x_L \equiv \frac{-(1-\phi)(V_A - V_B + t) + V_A - P - G}{(2\phi - 1)t} < 1$, and consider the following conditions:

Table 3: The Player i's Optimal Strategy $\alpha_i = 1$ with Corresponding Utility for Choosing the Primary Market

Condition	Location of the Player i x_i	Optimal Utility U_i
$\pi_A - (1-\phi)(V_A - V_B + t) \le P \le \pi_A$ $0 < G < \frac{(1-\phi)(V_A+V_B-t)+(2\phi-1)\pi_A-P}{2(1-\phi)}$	$x_i \in (\frac{(1-\phi)(V_A-V_B+t)-(\pi_A-P)}{2t(1-\phi)}, \frac{-(1-\phi)(V_A-V_B+t)+V_A-P-G}{(2\phi-1)t}]$	$U_i = \phi(V_A - tx_i) + (1-\phi)[V_B - t(1-x_i)] - P - G$

Table 4: The Player i's Optimal Strategy $\alpha_i = 0$ with Corresponding Utility for Non-participation

Condition	Location of the Player i x_i	Optimal Utility U_i
$0 < G < \frac{(1-\phi)(V_A+V_B-t)+(2\phi-1)\pi_A-P}{2(1-\phi)}$	$x_i \in (x_L, x_B)$	$U_i = 0$
$\frac{(1-\phi)(V_A+V_B-t)+(2\phi-1)\pi_A-P}{2(1-\phi)} \le G \le V_A - \pi_A$	$x_i \in (x'_A, x_B)$	
$G > V_A - \pi_A$	$x_i \in (0, x_B)$	
$0 < G \le V_B - \pi_B$	$x_i \in (x_B, x'_B)$	
$G > V_B - \pi_B$	$x_i \in (x_B, 1)$	

- If $x_L \le x_A$, we have $G \ge \frac{(1-\phi)(V_A+V_B-t)+(2\phi-1)\pi_A-P}{2(1-\phi)}$ so that all the players who consider to buy the loot box in the primary market will have the negative utility due to the high gas fee.
- If $x_L > x_A$, we have $0 < G < \frac{(1-\phi)(V_A+V_B-t)+(2\phi-1)\pi_A-P}{2(1-\phi)}$. The player i located at $x_i \in (x_A, x_L]$ choosing the primary market.

Then, when $x_L > x_A$, the player i located at $x_i \in (x_A, \min\{x_B, x_L\}]$ will purchase the loot box from the primary market when $x_L > x_A$.

Next, we discuss the relationship between x_B and x_L:

$$x_B - x_L = \frac{-\phi(V_A + V_B - t - 2G) + (2\phi - 1)\pi_B + P}{2t\phi(2\phi - 1)} \tag{19}$$

where

$$\phi(V_A + V_B - t) - (2\phi - 1)\pi_B \le P \le \pi_B, \text{and}$$
$$0 < G < \frac{(1-\phi)(V_A + V_B - t) + (2\phi - 1)\pi_A - P}{2(1-\phi)}.$$

Hence, we can always get $x_B - x_L > 0$ such that $x_B > x_L$.

In conclusion, when

$$\pi_A - (1-\phi)(V_A - V_B + t) \le P \le \pi_A, \text{and}$$
$$0 < G < \frac{(1-\phi)(V_A + V_B - t) + (2\phi - 1)\pi_A - P}{2(1-\phi)}.$$

the player i located at $x_i \in (x_A, x_L]$ will choose the primary market to buy a loot box.

□

4.2.3 Player's Decision of Non-participation ($\alpha_i = 0$).

If the loot box prices in the primary market and the target item in the 2CM exceed the player i's expectation, the player i may give up purchasing.

Theorem 5. *When the player i is located at $x_i \in (\max\{x_L, x'_A, 0\}, x_B) \cup (x_B, \min\{x'_B, 1\})$, he will choose non-participation. The details are summarized in Table 4.*

Proof. When $U_i(1) < 0$ and $U_i(2) < 0$, a rational player i choose non-participation. Besides, in this part, we discuss the condition based on $0 < x_A < x^*$ and $x^* < x_B < 1$.

According to Theorem 2 and Theorem 3, we can obtain that the player located at $[0, \min\{x_A, x'_A\}]$ and $[x'_B, 1]$ choose the 2CM. Based on Theorem 4, we can get that the players located at $(x_A, x_L]$ choose the primary market. Hence, the rest of players choose the non-participation.

Firstly, we need to compare the values of x_A and x'_A. Through proving the Theorem 2, we can notice that

- If $x'_A < 0$ such that $G > V_A - \pi_A$, all the players who consider to buy item A will have negative utility. Under this condition, the player i located at $x_i \in (0, x_A)$ will choose non-participation.
- If $0 \le x'_A < x^*$, the player i located at $x_i \in (\min\{x_A, x'_A\}, x_A)$ will abandon purchase. When $\frac{(1-\phi)(V_A+V_B-t)+(2\phi-1)\pi_A-P}{2(1-\phi)} \le G \le V_A - \pi_A$, the player i located at $x_i \in (x'_A, x_A)$ will abandon purchase.

Next, we talk about the value of x_B and x'_B. Through proving the Theorem 3, we can notice that

- If $x'_B > 1$ such that $G > V_B - \pi_B$, no player will choose to buy item B on the 2CM. Under this condition, the player i located at $x_i \in (x_B, 1)$ will choose non-participation.
- If $x^* < x'_B \le 1$, the player located at $x_i \in (x_B, \max\{x_B, x'_B\})$. Specifically, when $0 < G \le V_B - \pi_B$, the player i located at $x_i \in (x_B, x'_B)$ will choose non-participation.

Then, we should talk about the primary market. Through proving Theorem 4, we can find that

- If $x_L \le x_A$ such that $G \ge \frac{(1-\phi)(V_A+V_B-t)+(2\phi-1)\pi_A-P}{2(1-\phi)}$, no players will enter into the primary market. Specifically, the player i located at $x_i \in (x_A, x_B)$ will give up the primary market.
- If $x_L > x_A$ such that $0 < G < \frac{(1-\phi)(V_A+V_B-t)+(2\phi-1)\pi_A-P}{2(1-\phi)}$, the player i located at $x_i \in (x_L, x_B)$ will abandon the purchase.

In summary, gas fee has a huge impact on players' decisions about the participation. When

$$0 < G < \frac{(1-\phi)(V_A + V_B - t) + (2\phi - 1)\pi_A - P}{2(1-\phi)},$$

the player i located at $x_i \in (x_L, x_B)$ will choose non-participation. When

$$\frac{(1-\phi)(V_A+V_B-t)+(2\phi-1)\pi_A-P}{2(1-\phi)} \leq G \leq V_A-\pi_A,$$

the player i located at $x_i \in (x'_A, x_B)$ will choose non-participation. When

$$G > V_A - \pi_A,$$

the player i located at $x_i \in (0, x_B)$ will choose non-participation. When

$$0 < G \leq V_B - \pi_B,$$

the player i located at $x_i \in (x_B, x'_B)$ will choose non-participation. When

$$G > V_B - \pi_B,$$

the player i located at $x_i \in (x_B, 1)$ will choose non-participation.

□

5 STAGE I: LOOT BOX PRICING GAME

In this section, we aim to study the game provider's best pricing mechanism in stage I. The game provider's payoff comes from the selling of loot boxes which depends on the player's strategies in stage II. Specifically, the players who participate in the primary market provide the game provider revenue. However, the number of players choosing the primary market is related to the price of virtual items in the 2CM. Hence, The game provider designs the loot box pricing mechanism based on his anticipation of the price of virtual items in the 2CM.

5.1 Loot Box Pricing Game Formulation

The *player* of loot box pricing game is the game provider.

The strategies and the payoff function are defined in Section 3.1. Hence, the loot box pricing game is defined as follows.

Definition 3.(Loot Box Pricing Game)

- *Player*: The game provider.
- Strategies: The price of every loot box $P \in [0, \pi_A]$.
- Payoff: $U_p(P)$, $P \in [0, \pi_A]$.

Definition 4.(Nash Equilibrium) A NE of loot box pricing game is a profile p^* such that for the game provider,

$$U_p(P^*) \geq U_p(P)$$

Hence, the game provider needs to deal with the following optimization problem:

$$\begin{aligned} \max \quad & U_p = P \cdot D_p \\ s.t.\ & P \in [0, \pi_A],\ D_p \in [0,1] \end{aligned} \tag{20}$$

where D_p is estimated by the game provider because the game provider can not know the exact prices of items in the 2CM before he sets the price of the loot box.

5.2 Analysis of the Game Provider's Strategy

In this part, we will solve the NE of the above pricing problem in stage I. Specifically, we derive the game provider's expected utility under both EUT and PT.

5.2.1 *Utility Under Expected Utility Theory (EUT).*

When the game provider sets the price, he does not know the price of the secondary market. So he needs to estimate the price of goods in the 2CM.

In our setup, the price of item j ($j \in \{A, B\}$) is continuous. In other words, $\pi_A \in [V_A - t, V_A]$ as the price of item A and $\pi_B \in [V_B - t, V_B]$ as the price of item B are continuous. We adopt the result from [23] to approximate infinite continuous price of item j with finite discrete price of item j. We take item A as an example. Specifically, we divide the set of all possible continuous prices $[V_A - t, V_A]$ of item A into $K (K \geq 2)$ discrete prices $V_A + t(\frac{k}{K-1} - 1)$, with the corresponding probabilities p_k^A, $k = 0, 1, 2, \ldots, K-1$. Then the price of item A on the 2CM is the summation of weighted valuation of all discrete outcomes:

$$\pi_{EUT}^A = \sum_{k=0}^{K-1} p_k^A [V_A + t(\frac{k}{K-1} - 1)] \tag{21}$$

According to the Theorem 4, when the player i located at $x_i \in (x_A, x_L]$ will buy the loot box in the primary market, the demand of the loot box in the primary market is:

$$\begin{aligned} D_{EUT} &= x_L - x_A \\ &= \frac{(1-\phi)(V_A+V_B-t-2G)+(2\phi-1)\pi_{EUT}^A - P}{2t(2\phi-1)(1-\phi)} \end{aligned} \tag{22}$$

where $\pi_{EUT}^A - (1-\phi)(V_A - V_B + t) \leq P \leq \pi_{EUT}^A$.

Next we derive the utility of the game provider under EUT. The game provider's expected utility is

$$\begin{aligned} U_{EUT} &= P \cdot D_{EUT} \\ &= \frac{[(1-\phi)(V_A+V_B-t-2G)+(2\phi-1)\pi_{EUT}^A]P - P^2}{2t(2\phi-1)(1-\phi)} \end{aligned} \tag{23}$$

The second order partial derivative of U_{EUT} with respect to P is

$$\frac{\partial^2 U_{EUT}}{\partial P^2} = \frac{-2}{2t(2\phi-1)(1-\phi)} < 0 \tag{24}$$

which implies that U_{EUT} is a concave function in P. Hence, the optimal solution P^* satisfies the first order condition or lies at the boundary point. Then, we calculate

$$\begin{aligned} \frac{\partial U_{EUT}}{\partial P} &= \frac{[(1-\phi)(V_A+V_B-t-2G)+(2\phi-1)\pi_{EUT}^A] - 2P}{2t(2\phi-1)(1-\phi)} \\ &= 0. \end{aligned} \tag{25}$$

so that

$$P = \frac{(1-\phi)(V_A+V_B-t-2G)+(2\phi-1)\pi_{EUT}^A}{2} \tag{26}$$

We define $P_{EUT} \equiv \frac{(1-\phi)(V_A+V_B-t-2G)+(2\phi-1)\pi_{EUT}^A}{2}$.

To solve the optimal pricing problem, we should consider the gad fee G as follows. Let $G'_{EUT} \equiv \frac{(1-\phi)(V_A+V_B-t)+(2\phi-3)\pi_{EUT}^A}{2(1-\phi)}$ and $G''_{EUT} \equiv \frac{(1-\phi)(3V_A-V_B+t)+(2\phi-3)\pi_{EUT}^A}{2(1-\phi)}$.

- If $P_{EUT} \geq \pi_{EUT}^A$, we have $G \leq G'_{EUT}$. The game provider should set the optimal price $P^* = \pi_{EUT}^A$ to earn the maximum utility.

- If $\pi^A_{EUT} - (1-\phi)(V_A - V_B + t) < P_{EUT} < \pi^A_{EUT}$, we have $G'_{EUT} < G < G''_{EUT}$. The game provider should set the optimal price $P^* = \frac{(1-\phi)(V_A+V_B-t-2G)+(2\phi-1)\pi^A_{EUT}}{2}$ to realize the maximum utility.
- If $P_{EUT} \leq \pi^A_{EUT} - (1-\phi)(V_A - V_B + t)$, we have $G \geq G''_{EUT}$.The game provider should set the optimal price $P^* = \pi^A_{EUT} - (1-\phi)(V_A - V_B + t)$ to win the optimal utility.

5.2.2 *Utility Under Prospect Theory (PT).*

The uncertainty of the prices of item A and item B in the 2CM plays a significant role to address the game provider's utility maximization problem. This motivates us to use PT as the modeling tool. According to PT, the game provider's cognitive has three following characteristic:

- *Loss Aversion*: The game provider is more sensitive to losses than to gains. In other words, he prefers avoiding losses to achieving gains.
- *Decision Asymmetry:* The game provider is risk-averse in gains and risk-seeking in losses.
- *Weighting Distortion:* The game provider psychologically overestimates low probability events and underestimates high probability events.

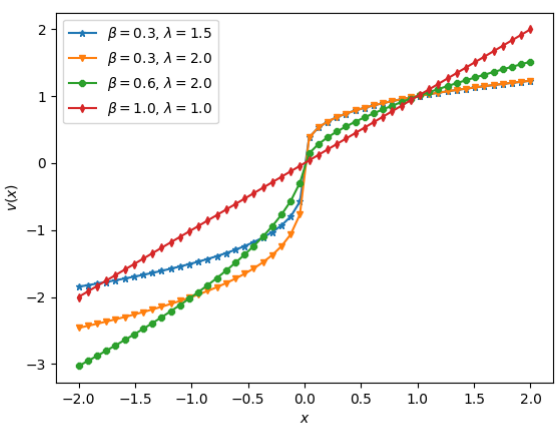


Figure 2: S-shape Asymmetric Value Function

Under PT, the S-shape asymmetric value function [12] which maps an outcome x to the user's subjective valuation $v(x)$, is given by $v(x)$, where:

$$v(x) = \begin{cases} x^{\beta}, & x \geq 0, \\ -\lambda(-x)^{\beta}, & x < 0. \end{cases} \tag{27}$$

where $0 < \beta \leq 1, \lambda \geq 1$. Figure 2 shows the function under different parameters. As the risk aversion parameter, β is the coefficient of diminishing marginal utility, and it suggests that as more gains (losses) are added to the current wealth, the subjective utility of an additional gain (loss) becomes more insignificant. The parameter λ describes the game provider's psychology of loss aversion, which indicates that the impact of loss is larger than that of the same absolute value [15]. A larger λ means that the game provider is more loss averse.

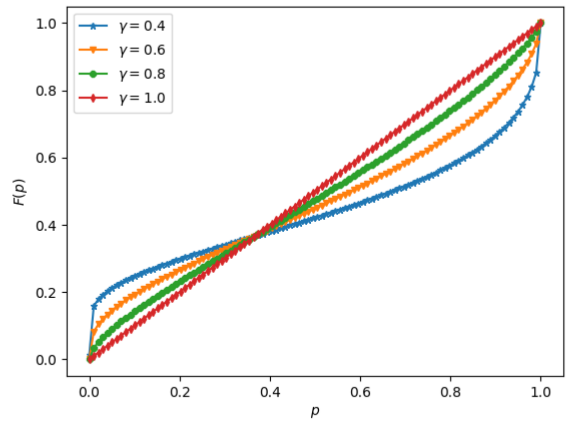


Figure 3: Probability Weighting Function

Besides, a probability weighting function captures the game provider's distorted perception of probability, which is:

$$F(p) = \exp\left(-(-\ln p)^{\gamma}\right), 0 < \gamma \leq 1. \tag{28}$$

where γ reveals how the game provider distorts the objective probability. Figure 3 shows the function under different γ. A smaller γ leads to a larger distortion.

From the above, we set

$$\pi^A_{PT} = \sum_{k=0}^{K-1} F(p^A_k) \cdot v[V_A + t(\frac{k}{K-1} - 1)] \tag{29}$$

Next, we derive the utility of the game provider under PT. The game provider's prospected utility is

$$\begin{aligned} U_{PT} &= P \cdot D_{PT} \\ &= \frac{[(1-\phi)(V_A + V_B - t - 2G) + (2\phi - 1)\pi^A_{PT}]P - P^2}{2t(2\phi-1)(1-\phi)} \end{aligned} \tag{30}$$

where $\pi^A_{PT} - (1-\phi)(V_A - V_B + t) \leq P \leq \pi^A_{PT}$. The second order partial derivative of U_{PT} with respect to P is

$$\frac{\partial^2 U_{PT}}{\partial P^2} = \frac{-2}{2t(2\phi-1)(1-\phi)} < 0 \tag{31}$$

which implies that U_{PT} is a concave function in P. Hence, the optimal solution P^* satisfies the first order condition or lies at the boundary point. Then, we calculate

$$\begin{aligned} \frac{\partial U_{PT}}{\partial P} &= \frac{[(1-\phi)(V_A + V_B - t - 2G) + (2\phi - 1)\pi^A_{PT}] - 2P}{2t(2\phi-1)(1-\phi)} \\ &= 0. \end{aligned} \tag{32}$$

so that

$$P = \frac{(1-\phi)(V_A + V_B - t - 2G) + (2\phi - 1)\pi^A_{PT}}{2} \tag{33}$$

We define $P_{PT} \equiv \frac{(1-\phi)(V_A+V_B-t-2G)+(2\phi-1)\pi^A_{PT}}{2}$.

(a) The Impact of π_A and ϕ on the Players' Strategies

(b) The Impact of π_B and ϕ on the Players' Strategies

(c) The Impact of P and ϕ on the Players' Strategies

Figure 4: The Impact of π_A, π_B, P and ϕ on the Players' Strategies

The solution of optimal pricing problem under PT is similar to that under EUT. Let $G'_{PT} \equiv \frac{(1-\phi)(V_A+V_B-t)+(2\phi-3)\pi^A_{PT}}{2(1-\phi)}$ and $G''_{PT} \equiv \frac{(1-\phi)(3V_A-V_B+t)+(2\phi-3)\pi^A_{PT}}{2(1-\phi)}$.

- If $P_{PT} \geq \pi^A_{PT}$, we have $G \leq G'_{PT}$. The game provider should set the optimal price $P^* = \pi^A_{PT}$ to earn the maximum utility.
- If $\pi^A_{PT} - (1-\phi)(V_A - V_B + t) < P_{PT} < \pi^A_{PT}$, we have $G'_{PT} < G < G''_{PT}$. The game provider should set the optimal price $P^* = \frac{(1-\phi)(V_A+V_B-t-2G)+(2\phi-1)\pi^A_{PT}}{2}$ to realize the maximum utility.
- If $P_{PT} \leq \pi^A_{PT} - (1-\phi)(V_A - V_B + t)$, we have $G \geq G''_{PT}$.The game provider should set the optimal price $P^* = \pi^A_{PT} - (1-\phi)(V_A - V_B + t)$ to win the optimal utility.

6 SIMULATION RESULTS

In this section, we provide numerical results to illustrate players' behavior and analyze the impact of the PT model on game provider's pricing strategy. For a better presentation, we set $V_A = 2.2$, $V_B = 2.5$ and $t = 0.9$, and scale the loot box trading market to size 1.

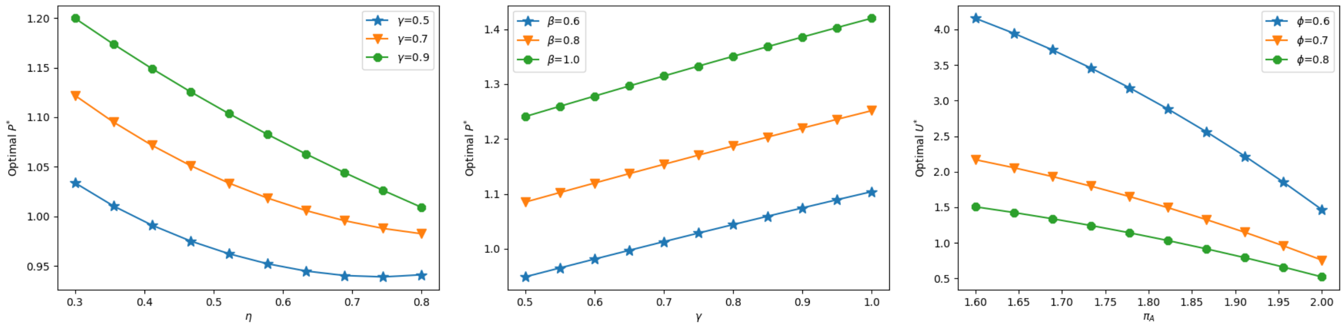
6.1 How Should Players Decide on the Market Selection?

In this part, we illustrate the impact of item A's price π_A and item B's price π_B in the 2CM, and the loot box's price P in the primary market on the players' optimal decision, shown in Figure 4. Besides, we set $G = 0.01$.

Impact of the item A's price π_A in the 2CM and loot box probability parameter ϕ on the players' strategies. We set $\pi_B = 2.2$ and $P = 1.88$. We considers three selling prices π_A (i.e., $\pi_A = 1.9$, $\pi_A = 2.0$ and $\pi_A = 2.1$) and the probabilities to win item A in the primary market (i.e., $\phi = 0.6$, $\phi = 0.7$ and $\phi = 0.8$), respectively. From Figure 4(a), we can observe that with the fixed probability ϕ, the number of people choosing to buy loot boxes grows along with the increase in π_A. The reason is that when the price in 2CM is large enough, players tend to obtain target items via buying loot boxes. We can also see that with the fixed price π_A and increasing probability ϕ, the number of players who choose to buy item A from 2CM decreases, while the number of non-participation players increases. The reason is that with the increasing ϕ, players can obtain item A easily, but item B hardly from the loot box. In this case, some players who prefer item B will choose non-participation. It's worth mentioning that the player's choice to buy an item B in the 2CM will not be affected by the price of item A.

Impact of the item B's price π_B in the 2CM and loot box probability parameter ϕ on the players' strategies. We set $\pi_A = 1.9$ and $P = 1.88$. We considers three selling prices π_B (i.e., $\pi_B = 2.0$, $\pi_B = 2.2$ and $\pi_B = 2.4$) and the probabilities to win item A in the primary market (i.e., $\phi = 0.6$, $\phi = 0.7$ and $\phi = 0.8$), respectively. From Figure 4(b), we can observe that the number of players who choose to buy item B from 2CM decreases with the fixed probability ϕ and increasing price π_B, while the number of players who decide non-participation increases. The reason is that with the increasing π_B, some players who prefer item B will choose non-participation. Besides, with the fixed price π_B, the number of players who choose non-participation increases along with the increasing probability ϕ. In this case, players can hardly win item B in the loot box, which leads to an increase in non-participation. Besides, the players who choose item A are not affected by the price of item B in the 2CM.

Impact of the loot box's price P in the primary market and loot box probability parameter ϕ on the players' strategies. We set $\pi_A = 1.9$ and $\pi_B = 2.2$. We considers three P (i.e., $P = 1.84$, $P = 1.87$ and $P = 1.90$) and the probabilities to win item A in the primary market (i.e., $\phi = 0.6$, $\phi = 0.7$ and $\phi = 0.8$), respectively. From Figure 4(c), we can see that more players will give up the primary market with the increasing price P and fixed probability ϕ, while the number of players who choose to buy item A from 2CM increased. Recall that the trading market is mature, which means that item A and item B have already been listed on the secondary market, while loot boxes are still being sold on the primary market. Hence, the primary market will be abandoned by players as the price of loot boxes in the primary market increases to a certain level. Besides, the player who prefer item A will buy it in the 2CM when



(a) The Impact of η and γ on the Game Provider's Strategy (b) The Impact of γ and β on the Game Provider's Strategy (c) The Impact of π_A and ϕ on the Game Provider's Utility

Figure 5: The Impact of η, γ, β, π_A and ϕ on the Game Provider's Pricing Strategy and Utility

the price of the loot box increases. We can also see that the number of players who buy item A from 2CM decreases with the fixed price P and increasing probability ϕ, while the number of players who choose non-participation increases. The reason is that when the probability of getting item A increases, players who prefer item A will choose the loot box rather than the 2CM, while some of the players who prefer item B will choose non-participation.

6.2 How Should the Game Provider Set the Optimal Price?

In this part, we will show the impact of the 2CM (π_A), PT model parameter (γ and β), and loot box probability parameter (ϕ) on the game provider's pricing strategy and utility, shown in Figure 5. For a better presentation, we assume $K = 2$ for the rest discussion. Specifically, the item A has two possible values: π_l^A and π_h^A. The probability of a low price π_l^A is η, while the probability of a high price π_h^A is $1 - \eta$.

Impact of the probability η of π_l^A and weighting distortion parameter γ on the game provider's pricing strategy. We set $\beta = 0.6$ and $\phi = 0.6$. Figure 5(a) considers three different weighting distortion parameters: $\gamma = 0.5$, $\gamma = 0.7$ and $\gamma = 0.9$. η is large, so the probability of a low price π_l^A is high. Relatively, the probability of a high price π_h^A is low. Hence, the price of item A is relatively low, which causes a lower optimal price. Besides, the game provider psychologically overestimates low probability events and underestimates high probability events. The smaller γ is, the greater distortion is. When the γ is higher, the optimal price is higher. According to the Figure 3, the difference in optimal price between $\gamma = 0.5$ and $\gamma = 0.7$ is larger than that between $\gamma = 0.7$ and $\gamma = 0.9$.

Impact of the weighting distortion parameter γ and risk aversion parameter β on the game provider's pricing strategy. We set $\eta = 0.6$ and $\phi = 0.6$. Figure 5(b) considers three different risk aversion parameters: $\beta = 0.6$, $\beta = 0.8$, and $\beta = 1.0$. We can observe that optimal price P^* rises with increasing γ. According to the Figure 3, when $\eta = 0.6$, the game provider will underestimate the probability. Besides, when γ is smaller, the distortion is greater, which leads to a smaller optimal price. Then, we can observe the impact of β on the optimal price: the β is smaller, while the optimal price is smaller. EUT is a special case of PT where $\beta = 1$ and $\gamma = 1$. So, the game provider's optimal price under PT is lower than that under EUT. Under PT, the game provider will decide on a more conservative pricing mechanism when considering his loss aversion and risk aversion. Appropriate consideration of the game provider's psychological factors will improve his utility.

Impact of the selling price π_A of item A in the 2CM and loot box probability parameter ϕ on the game provider's utility. We set $\gamma = 1.0$ and $\beta = 1.0$. Figure 5(c) considers three different loot box probability parameters: $\phi = 0.6$, $\phi = 0.7$, and $\phi = 0.8$. We can observe that when the π_A increase, the optimal utility decreases. Specifically, when the game provider estimates that the price of item A will be higher, he will always set a higher selling price of the loot box, which will cause fewer players to participate in the primary market to buy the loot box. In this case, the game provider will have lower utility. Besides, a higher ϕ will decrease the utility. Recall that ϕ is the probability to win item A after revealing the loot box, which is less valuable than item B. When the players are more likely to get low-value item A, some of them may give up the purchase. Hence, if the game provider sets a high probability to get item A, he will reduce the utility.

6.3 How Do Gas Fees Affect Players' Behavior?

In this part, we will show the extent to which gas fee (G) affects the players' market selections.

Impact of gas fees (G) on players' market selections. We set $P = 1.88$, $\pi_A = 1.93$ and $\pi_B = 2.2$. We considers four different gas fees G (i.e., $G = 0.01$, $G = 0.10$, $G = 0.27$ and $G = 1.0$) and the probabilities to win item A in the primary market (i.e., $\phi = 0.6$, $\phi = 0.7$ and $\phi = 0.8$), respectively. From Figure 6, we can observe that both the number of players choosing the primary market and those choosing the secondary market decline when the gas fee increases. Specifically, when $G = 0.1$ and $\phi = 0.6$, we can see that no players will choose the primary market. Next, players give up buying item A in the 2CM when $G = 0.27$. When G reaches a certain value, all players will drop out of the market, neither the primary

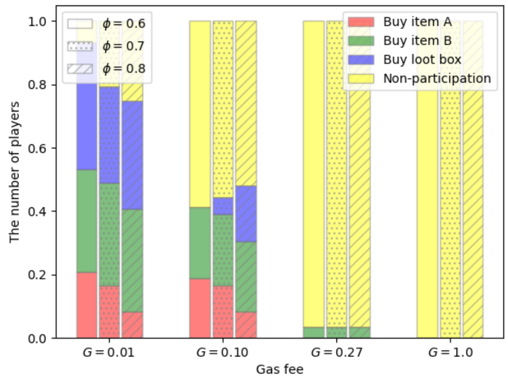


Figure 6: The Impact of Gas Fees on Players' Behavior

market nor the 2CM. When $G = 1.0$, no one will buy the item or loot box. Hence, gas gee has the most significant impact on the primary market. When the gas fee is too high, players will abandon the primary market first. The reason is that the players will bear the uncertainty when purchasing the loot boxes. Then, the players who choose to buy item A in the 2CM will give up the purchase when the gas fee increases. Due to the high value of item B, players who choose item B in the 2CM can afford a relatively high gas fee.

7 CONCLUSION

In this paper, we presented the first blockchain-based loot box market model considering the primary market and 2CM. We show that the players who strongly prefer one item will prioritize the 2CM. Besides, when the prices in the primary market and 2CM and the probability of acquiring high-value items increase, some players will abandon purchases. In addition, compared with EUT, the game provider in PT, considering his aversion to loss and risk, should adopt a conservative pricing mechanism to boost his utility. Properly considering the game provider's behavioral characteristics can increase the utility. It is worth mentioning that the gas fee is a significant factor in the loot box trading market for blockchain games. When gas fee increases, the primary market will be the first to be affected due to more significant uncertainty. Then, when the gas fee increases, players who buy item A in the 2CM will choose to give up. Because item B is relatively valuable, players who prefer item B can afford a higher gas fee. We conclude that the gas fee has a more substantial effect on the primary market than the 2CM, and players who prefer high-value items can tolerate high gas fees.

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