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Analyzing Bitcoin transaction fees using a queueing game model

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Abstract

In the Bitcoin system, large numbers of miners invest massive computing resources in the blockchain mining process in pursuit of Bitcoin rewards, which are comprised of a fixed amount of system-generated new block reward and a variable amount of user-submitted transaction fees. Here, transaction fees serve as the important tuner for the Bitcoin system to define the priorities in users' transaction confirmation. In this paper, we aim to study the priority rule for queueing transactions based on their associated fees, and in turn users' strategies in formulating their fees in the transaction confirmation game. We first establish a full-information game-theoretical model to study users' equilibrium fee decisions; and then discuss three types of Nash equilibria, under which no, all and some users submit transaction fees. Moreover, we conduct empirical studies and design computational experiments to validate our theoretical analysis. The experimental results show that (1) users' fee decisions will be significantly affected by their waiting time; (2) the reduced time costs, instead of transaction values, are the basis for users to evaluate their revenues; (3) longer waiting time and higher unit time cost drive users to submit transaction fees in pursuit of desired priorities; (4) with the required transaction fee increasing, the proportion of fee-submitting users decreases slowly at first followed by a sharp decline, and over-high required fees will make the transaction confirmation game end up with no users submitting fees.

Keywords Bitcoin · Blockchain · Transaction fees · Transaction confirmation queueing game

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1 Introduction

As a typical decentralized and trustless network designed by Satoshi Nakomoto [1], the blockchain-powered Bitcoin system is capable of dealing with the secured transfer of values, thus has attracted intensive attentions [2–5]. Within the Bitcoin system, miners (both individual-level solo miners and group-level mining pools) compete to solve computational puzzles via searching a random number that satisfies specific difficulty requirements using a brute force approach, and this process is widely known as proof-of-work mining [6]. New blocks are created via mining and appended to the main-chain of previously agreed upon blocks, creating a complete record of all data updating that has ever taken place [7].

The transaction confirmation process in the Bitcoin system is described as follows (as shown in Fig. 1). Essentially, the Bitcoin system can be viewed as a queueing system of transactions pending for confirmation [8–11]. Users typically submit transactions with a certain amount of associated fees to get their desired priority and stimulate miners to confirm their transactions preferentially. All the transactions pending for confirmation are stored in the memory pool. Since the block size is limited, the number of transactions that miners can record into a block is restricted [12]. As such, revenue-maximizing miners naturally first select and pack those transactions with higher priorities as their mining basis. During the mining process, the miner who first solves the computational puzzle will be rewarded. Meanwhile, those transactions packaged by the winning miner will be recorded into the new block after being successfully confirmed by all or a majority of miners [13, 14].

In the transaction confirmation process, transaction fees impose great impacts on the participants' individual decisions and even the system-level policy making or mechanism design.

From the perspective of miners, transaction fees serve as the important basis of their transaction confirmation decisions. Their revenues include a fixed number of system-generated Bitcoins issued in the Coinbase transactions, and also the user-submitted transaction fees. Although the incentive of the mining process relies largely on the new block reward so far [15, 16], the amount of block reward is preset

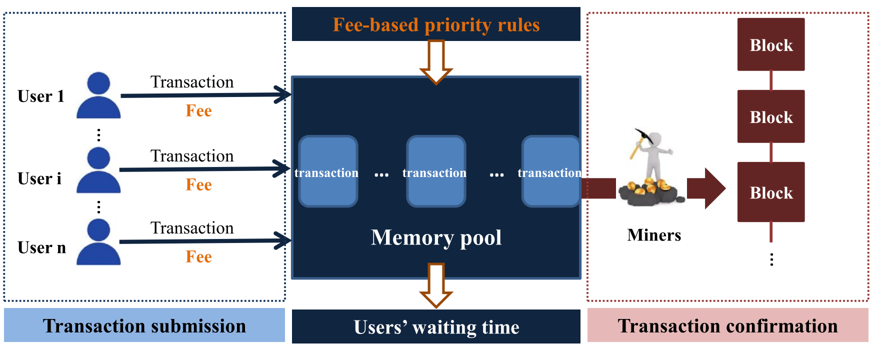

Fig. 1 The transaction confirmation in the Bitcoin system

to be halved approximately every 4 years until 21 millions Bitcoins are created finally. On the contrary, transaction fees are expected to gradually increase. In the future, in case that the system eventually stops creating new Bitcoins, transaction fees will serve as the major way to stimulate miners to keep mining and provide confirmation service [12, 17].

From the perspective of users, transaction fees are the main tuner for their transaction confirmation priorities. With the average transaction size getting larger, the confirmation rate gradually decreases if the block size is not enlarged. As a result, there will be large numbers of transactions congested in the memory pool and users will wait for longer time until their transactions are confirmed. Under this situation, users are confronted with great challenges to determine the proper fees with the purpose of achieving the successful transaction confirmation within the desired waiting time.

Besides, transaction fees are the important input of the Bitcoin's system-level policy making, aiming to guarantee the security, vitality and sustainability of the blockchain-powered system [18]. Obviously, the transaction fee policy making should be done based on the analysis of users' fee decisions. As such, there is a critical need for us to study transaction fees' impacts and in turn users' equilibrium transaction fee decisions in the Bitcoin ecosystem.

The transaction fees' impacts are typically realized by designing transaction priority rules based upon them, and different priorities will lead to different waiting time of users. In general, users' willingness to pay transaction fees mainly depends on the trade-off between the required transaction fees and the expected benefits from reduced waiting time. In view of these facts, we aim to study users' waiting time under the specific priority rule for queueing transactions as well as users' fee decisions in the transaction confirmation game. Usually, one Bitcoin block can record multiple transactions. For these transactions recorded into the same block, we can assume they are confirmed almost simultaneously. Accordingly, we formulate the Bitcoin transaction confirmation queueing problem under the non-preemptive priority rule based on transaction fees in the multi-server systems, namely the multi-server multi-priority non-preemptive priority (MSMP) rule. Based on this consideration, we establish the game-theoretical model to analyze users' transaction fee decisions. Besides, we conduct both theoretical analysis and experimental validation of the formulated transaction confirmation model.

The main contributions of our research are summarized as follows. First, we explicitly characterize the fee-based priority rule and in turn analyze the equilibrium fee decisions utilizing a queueing game model. Then, we conduct the empirical study and design computational experiments to validate the theoretical models and analysis. The theoretical and experimental results derived from this work can not only help us understand users' equilibrium decisions under the fee-based priority rule, but also provide good support for the Bitcoin system to determine the proper fee given the optimization objective. Besides, this research can also be extended to study users' fee decisions in other blockchain-powered crypto-currency systems, e.g., we only need to adjust the priority rule according to their transaction confirmation characteristics and compute users' waiting time accordingly, since the waiting time is the main influential factor of users' fee decisions in our formulated model.

The remainder of this paper is organized as follows. Section 2 briefly reviews the related literature. Section 3 investigates the transaction confirmation queueing game and studies its equilibrium. Section 4 conducts both empirical analysis and computational experiments. Section 5 discusses the managerial implications. Section 6 concludes this paper and discusses the future work.

2 Related literatures

Bitcoin economics has been widely studied in literature [7, 19–21], while the transaction fee economics still receives very limited research attentions. In literature, [22] considered that transaction fees have only little importance, and their research on Bitcoin mining games thus does not take transaction fees into consideration. However, other researchers argued that it is of great values to study transaction fees in the Bitcoin system considering both the systematic ecosystem and the individual participants.

From the macro-scopic perspective, transaction fees impose an important impact on the Bitcoin system. However, [23] argued that Bitcoin's current fee market does not extract revenue well when blocks are not congested. Houy [12] studied the economics of Bitcoin's transaction fees in a simple partial equilibrium setting, and showed that the fixed transaction fee is equivalent to setting a maximum block size instead. Easley et al. [8] investigated the role that transaction fees play in the Bitcoin blockchain's evolution from a mining-based structure to a market-based ecology. Iyidogan [24] developed a blockchain economic model in the presence of endogenously determined mining difficulty and proposed a fee structure, and the results showed that both the increasing number of miners and the developed technology reduced the optimal transaction fee.

From the micro-scopic perspective, transaction fees not only serve as the incentives to miners' decisions on confirming transactions [8] but also can be used to adjust users' priorities as well as their waiting time for transaction confirmation [10]. Consequently, transaction fees impose influences on each participant's decisions and revenue maximization [9]. From the angle of miners, transaction fees currently serving as an alternative reward scheme will inevitably develop to be the most important incentive as the new block reward gradually decreases [12]. From the angle of users, transaction fees indeed usually account for only a small percentage of the Bitcoins transferred by their transactions; However, it is possible that transaction fees might reach or even exceed the trading Bitcoins, especially in micro-payment scenarios. Generally speaking, the larger the required transaction fee is, the longer a transaction could reside in the memory pool [25]; adversely, the higher the submitted fee is, the faster a transaction will be confirmed [18]. However, exorbitant transaction fees will render Bitcoin uneconomical for micro payments [16, 26].

With the purpose to understand how users determine their transaction fees, [9] analyzed the implied congestion queueing game, calculated each user's trade-off between the transaction fee and the delay cost, and concluded that each user's equilibrium transaction fee equals the externality that his transaction imposes. Thus, the equilibrium transaction fees should coincide with the payments that result from selling priority of service in a Vickrey–Clarke–Groves (VCG) auction. According to [12], if the transaction fee is totally determined by a decentralized market and the maximum block size is not constrained, the transaction fee will eventually go to zero and miners will not have the sufficient incentives to keep mining, and hence to keep the Bitcoin system viable. Lavi et al. [23] proposed two alternative auction mechanisms: the monopolistic price mechanism, and the random sampling optimal price mechanism. They proved that the monopolistic price mechanism is nearly incentive compatible and can extract revenues better for miners, under which users submit fees equal to their true valuations. Abdullah [27] used the linear perceptron machine learning classification algorithm to estimate the fees in reference of memory pool state, which is with the aim to help users save fees when building multisig transactions.

Overall, the research efforts devoted to studying the Bitcoin transaction fees are still limited. Also, the existing literatures mainly focus on the transaction fee determination or the mechanism design with respect to transaction fees, but seldom study users' fee decisions in the transaction confirmation queueing game except for [8, 9]. However, they both did not discuss the fee-based priority rule in line with the Bitcoin practice for queueing transactions, which serves as the fundamental evidence for the game-theoretical transaction fee decisions. Also, [9] focused on the theoretical analysis of the transaction fee determination from the mechanism-design perspective, without considering miners' transaction confirmation strategies; while [8] mainly considered the transaction fee evolution under relatively strong assumptions, including homogenous users, single priority, and identical transaction fees, which are still far from enough to explain the real Bitcoin system.

Our research is distinct from previous researches mainly in the following aspects. Unlike other studies of homogenous users offering identical fees, we relax the assumptions to characterize more practical research environment, namely, taking into consideration limited block space recording multiple transactions with different sizes. Another distinct feature of our study is that we investigate Bitcoin users' fee decisions using a game-theoretical queueing model. In addition, we make use of the available real-world data to conduct both the empirical study and computational experiments to validate our theoretical models and analysis.

3 The transaction confirmation queueing game

In what follows, we study the fee-based priority rule as well as users' fee decisions in the transaction confirmation queueing game. Notations in this paper are listed in Table 1.

In the Bitcoin system, all information regarding each transaction will be broadcasted in public as soon as it is submitted, which includes size, fee, input amount, output amount, address, and submission time, etc. After each block is mined, we are free to access the information of confirmed transactions in the block, which means the equilibrium results of the transaction fee auction are totally public. Besides, Bitcoin transactions are basically transfer transactions; thus it is natural to use the transfer amount to represent the transaction's value to the user. Since the transfer amount

Table 1 List of notations

Notations	Definitions
R^i	The user i's payoff
v^i	The value of user i's transaction
α^i	The user i's unit time cost
f^i	The user i's transaction fee
$k(i)$	The user i's priority
γ_k	The submission rate of the priority k
Γ	Total transaction submission rate
Λ	The new block generation rate
ρ	The transaction confirmation intensity
s^i	The user i's transaction size
χ	The optimal number of transactions recorded into the upcoming new block
d_k	The waiting time of users with the kth priority under MSMP rule

is public information, we can then consider that the transaction's value is also public information [5]. As such, we consider a single-round transaction confirmation queueing game in the full-information scenario in this paper.

3.1 Users' payoffs

In the memory pool, users play the transaction confirmation queueing game through submitting proper transaction fees to maximize their expected payoffs. For user $i \in N = \{1, \ldots, n\}$, only if he/she wins the game and gets the transaction confirmed and recorded into the blockchain, the revenue v^i can be earned. Normally, we have $f^i \leq v^i$ to consider that all users are conservative [28]. However, the revenue of the winning user will be then subtracted by the transaction fee f^i and the time cost $\alpha^i d^i$. Accordingly, we formulate the payoff function as

$$R^i = v^i - f^i - \alpha^i d^i. \tag{1}$$

here α^i and d^i represent the user's unit time cost and waiting time, respectively.

3.2 Users' waiting time

Different from the traditional multi-server queueing problem, the number of the service systems is not fixed in the Bitcoin system. Because, it is determined by miners' transaction confirmation strategies. We assume that each user submits only one transaction, so the user's priority is also the transaction's priority.

In general, restricted by the block size, miners selectively confirm either larger-size transactions with higher fees or smaller-size transactions with lower fees in pursuit of maximal revenues. Accordingly, the optimal number χ of transactions recorded into the upcoming new block is computed by

$$\chi = \underset{x \in [1,n]}{argmax} \left(\sum_{k(i)=1}^{x} f^i \mid \sum_{k(i)=1}^{x} s^i \leq X \right), \tag{2}$$

where X is the upper limit of the block size, and $k(i)$ is the priority of the user i's transaction.

Each miner has different personal computing power, which will lead to different cost per unit time. However, their major purpose to provide computing powers is to maximize their revenues from mining and transaction confirmation. Generally, for each miner who successfully mines out the new block, he/she will confirm transactions as many as possible to fill up the mined new block, because more transactions will bring in more transaction fees under the given priority rule. As such, for the miner's transaction confirmation decision, his/her cost per unit time is not an influence factor. Based on this consideration, we will figure out the number χ of the transactions recorded into the upcoming new block through Eq. (2).

For Bitcoin miners, higher computing power will generally result in higher computing speed and higher probability of first mining out the new block. However, the computing power will not affect the speed of generating new block. Because the system will adjust the mining difficulty according to the invested computing power, to make sure that the generation rate of new block keeps stable at the predetermined level. As such, we consider the exogenously determined new block generation rate Λ in this paper. According to the above analysis, we formulate the transaction confirmation rate as $\chi\Lambda$.

The MSMP rule is not completely determined by the transaction fee, but also affected by the transaction size. The user submits the fee considering the transaction size, and then miners assign him/her with the corresponding priority according to the unit fee $g^i = f^i/s^i$, where s^i is the transaction size. Under this rule, users with the same priority may offer different fees.

Suppose there are K priorities of submitted transactions pending for confirmation. Users with the kth priority submit transactions according to the independent Poisson process with the arrival rate of γ_k, and thus the total submission rate is $\Gamma = \sum_k \gamma_k$. The transaction with the kth priority has non-preemptive priority over the priority $k + \Delta, \Delta > 0$. Within a priority class, transactions are confirmed following the first-come-first-served (FCFS) rule. In addition, only if the transaction confirmation rate exceeds the submission rate, i.e. $\chi\Lambda > \Gamma$, the memory pool can converge to be steady in the long run, which serves as the key prerequisite for our following analysis.

Let p_n be the probability that the number of unconfirmed transactions waiting in the memory pool is n, and we have

$$p_n = p_0 \frac{\Gamma^n}{\prod_{i=1}^{n} \Lambda_i}. \tag{3}$$

Here, p_0 is the probability of the case that the upcoming new block is empty, and it follows that

$$p_0 = \left[\sum_{n=0}^{\chi-1} \frac{1}{n!} (\chi\rho)^n + \frac{(\chi\rho)^\chi}{\chi!(1-\rho)} \right]^{-1}, \tag{4}$$

where ρ is the average computing intensity and we have $\rho = \Gamma / \chi\Lambda$. The probability that transactions under confirmation are enough to fill up the upcoming new block and the coming new transactions need to keep waiting for the next new block is

$$p_w = \sum_{n=\chi}^{\infty} p_n. \tag{5}$$

Following the calculation process in the classical queueing theory, we derive the average waiting time of all users as

$$d = \frac{p_w}{\chi\Lambda(1-\rho)}. \tag{6}$$

For the user with the highest priority, his/her waiting time is equal to the classic multi-server non-preemptive multi-priority case with the submission rate of γ_1. Therefore, the expected waiting time of the user in the highest priority is

$$d_1 = \frac{p_w}{\chi\Lambda(1-\rho_1)}. \tag{7}$$

As for the expected waiting time of the user with the kth priority, we can derive its Laplace–Stieltjes transform as

$$\tilde{d}_k(s) = (1-p_w) + p_w \frac{\chi\Lambda(1 - \sum_{y=1}^{k} \rho_y)(1 - \tilde{B}(s))}{s - \gamma_k + \gamma_k \tilde{B}(s)}, \tag{8}$$

where B is the length of the period from the moment when transactions with priorities higher than k are under confirmation but the block has no more space to support the confirmation of kth-priority transactions until the moment when miners start to confirm the first transaction with the kth priority [29]. The Laplace–Stieltjes transform of B is given as

$$\begin{aligned} \tilde{B}(s) &\equiv E\left[e^{-sB}\right] \\ &= \frac{s + \gamma_h + \chi\Lambda - \sqrt{(s+\gamma_h+\chi\Lambda)^2 - 4\gamma_h\chi\Lambda}}{2\gamma_h}, \end{aligned} \tag{9}$$

where $\gamma_h = \sum_{y<k} \gamma_y$ represents the total submission rate of transactions with the priority higher than k. Through differentiating $\tilde{d}_k(s)$, the expected waiting time d_k can be derived.

3.3 Users' fee decisions

Users' fee decisions mainly depend on the trade-off between transaction fees and saved time costs resulting from the reduced waiting time. If the waiting time is not long or the user is patient enough (i.e., the unit time cost is very low), he/she may prefer not to submit the transaction fee; while if the waiting time is significantly long or the user is impatient (i.e. the unit time cost is very high), he/she may be willing to submit the transaction fee to get the desired priority so as to shorten the waiting time. As follows, we will analyze the users' equilibrium fee strategies in the transaction confirmation queueing game under the above-mentioned MSMP rule (MSMP game).

The user's payoff function in the MSMP game is given as

$$R^i = v^i - f^i - \alpha^i d_{k(i)}. \tag{10}$$

As shown above, the calculation complexity of the waiting time d_k grows significantly with the increasing k. For simplicity, we take the case of two priorities as the example to conduct the following studies, under which transactions with the fees have the high priority while those with no fee have the low priority. Then, the expected waiting time of the low-priority transactions is derived as

$$d_2 = \frac{p_w(1-\rho_1\rho)}{\chi\Lambda(1-\rho)(1-\rho_1)^2}. \tag{11}$$

Theorem 1 *The full-information MSMP game with two priorities has three types of Nash equilibria described as follows*:

- *If* $\forall i \in N,\ f > \frac{\alpha^i p_w(\Gamma-1)}{\chi\Lambda(1-\rho)(\Lambda-1)}$, *there will exist the Nash equilibrium of no user submitting the transaction fee.*
- *If* $\forall i \in N,\ 0 < f < \frac{\alpha^i p_w(2\Lambda-\Gamma-1)}{\chi\Lambda(1-\rho)(\Lambda-1)^2}$, *there will exist the Nash equilibrium of all users submitting transaction fees.*
- *If* $\exists i \in N,\ \frac{\alpha^i p_w(2\Lambda-\Gamma-1)}{\chi\Lambda(1-\rho)(\Lambda-1)^2} \le f \le \frac{\alpha^i p_w(\Gamma-1)}{\chi\Lambda(1-\rho)(\Lambda-1)}$, *there will exist the Nash equilibrium of some users submitting transaction fees.*

Proof The MSMP game is a finite game where each user has finite fee strategies, because no user can submit a fee less than zero or high than the transaction amount; as such we can conclude that it has the Nash equilibrium [30]. Furthermore, we discuss the equilibria under which no, all and some users submit transaction fees.

First, the equilibrium of no user submitting the fee exists when the average waiting time is very short or all users' unit time costs are very low; otherwise, they would like to deviate from the equilibrium strategy to offer a fee in the interest of a higher payoff. Under this type of equilibrium, any user turning to submit the fee to get the first priority will suffer a certain loss. That is, the following condition is satisfied for every user:

$$v^i - f - \alpha^i d_1 < v^i - \alpha^i d, \tag{12}$$

where

$$d_1 = \frac{p_w}{\chi \Lambda (1 - \frac{1}{\Lambda})}. \tag{13}$$

Calculations yield that

$$f > \frac{\alpha^i p_w (\Gamma - 1)}{\chi \Lambda (1 - \rho)(\Lambda - 1)}, \quad \forall i \in N. \tag{14}$$

Second, if the waiting time is sufficiently long, all users prefer to offer transaction fees in exchange for the reduced waiting time. The one who does not submit the fee will have the low priority, and his/her waiting time will be prolonged. This type of equilibrium exists if the following condition is possessed by all users:

$$v^i - f - \alpha^i d > v^i - \alpha^i d_2, \tag{15}$$

where

$$d_2 = \frac{p_w (1 - \frac{\rho}{\Lambda})}{\chi \Lambda (1 - \rho)(1 - \frac{1}{\Lambda})^2}. \tag{16}$$

Calculations yield that

$$0 < f < \frac{\alpha^i p_w (2\Lambda - \Gamma - 1)}{\chi \Lambda (1 - \rho)(\Lambda - 1)^2}, \quad \forall i \in N. \tag{17}$$

Third, under the Nash equilibrium of some users submitting transaction fees, we have the following conditions. For the users with the high priority, there is

$$v^{1(i)} - f - \alpha^{1(i)} d_1 \geq v^{1(i)} - \alpha^{1(i)} d_2; \tag{18}$$

while, for the users with the low priority, there is

$$v^{2(i)} - \alpha^{2(i)} d_2 \geq v^{2(i)} - f - \alpha^{2(i)} d_1. \tag{19}$$

Otherwise, users have incentives to change their equilibrium fee decisions. Then, we derive that the equilibrium fee required for the high priority should satisfy the following condition:

$$\alpha^{2(i)} (d_2 - d_1) \leq f \leq \alpha^{1(i)} (d_2 - d_1). \tag{20}$$

This condition indicates that over-high transaction fee leads to fee-submitting users under equilibrium state forego fees; while over-low transaction fee encourages more than the equilibrium number of users to submit fees. According to discussions of the aforementioned two types of equilibria, the MSMP game has the equilibrium of

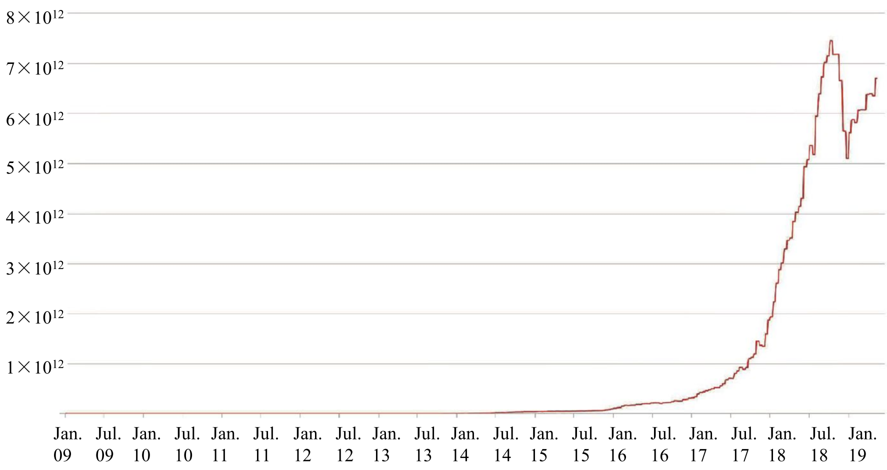

Fig. 2 Difficulty

some users submitting the transaction fee when there exists at least one transaction satisfying the following condition:

$$\frac{\alpha^i p_w(2\Lambda - \Gamma - 1)}{\chi\Lambda(1-\rho)(\Lambda-1)^2} \le f \le \frac{\alpha^i p_w(\Gamma - 1)}{\chi\Lambda(1-\rho)(\Lambda-1)}. \tag{21}$$

□

4 Experiments

In this section, we conduct experiments to make further study on users' waiting time under the MSMP rule and also their equilibrium fee decisions, with the purpose to validate the above theoretical analysis.

4.1 Empirical analysis

First, we collect the real-world data from the Bitcoin system during January 2009 and April 2019 as the experimental database. Until April 2019, the total invested computing power peaks at the hash rate 50 Eh/s, and the height of block in the Bitcoin system is 573,996; more than 17 millions Bitcoins have been mined and enter circulation, and the Bitcoin price is 5228.18 USD. Besides, the user size has increased to 7.5 millions, and the active address number has exceeded 35 millions. The overall trend of some important factors during this period are shown in

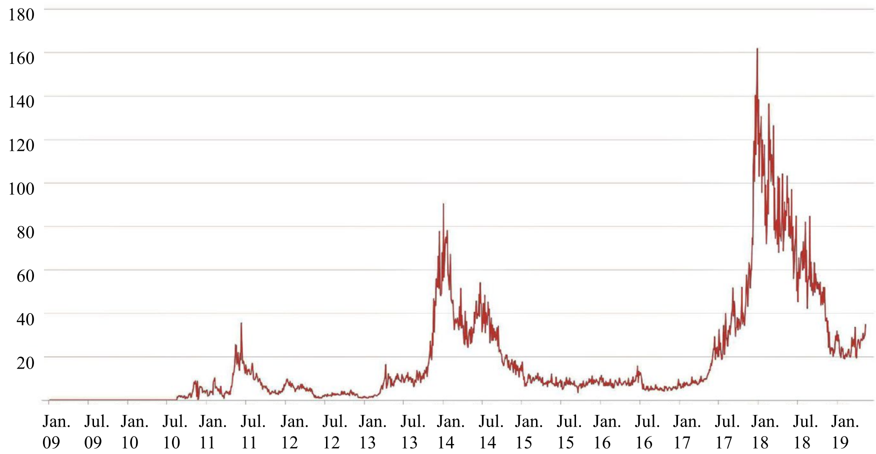

Fig. 3 Cost per transaction (unit: USD)

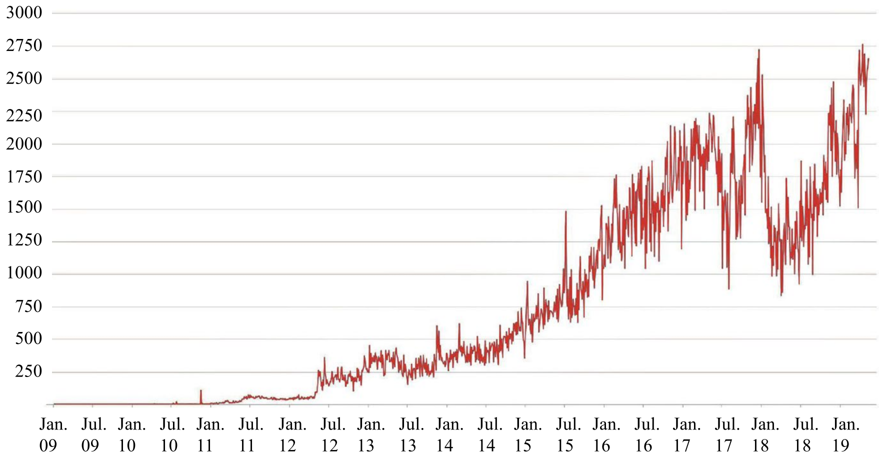

Fig. 4 Average number of transactions per block

Figs. 2, 3, 4, 5, 6 and 7,[1] including mining difficulty, transaction fees, miners' revenues, cost per transaction, median confirmation time, etc.

In the Bitcoin system, more and more miners are joining in mining, which make the computing power improve significantly; but meanwhile the mining difficulty increases sharply as shown in Fig. 2. The new block generation rate nearly keeps fixed under the compound effects of these two factors. As such, it is rational to consider an exogenously fixed new block generation rate in the memory pool. Besides,

[1] Source: https://www.blockchain.com.

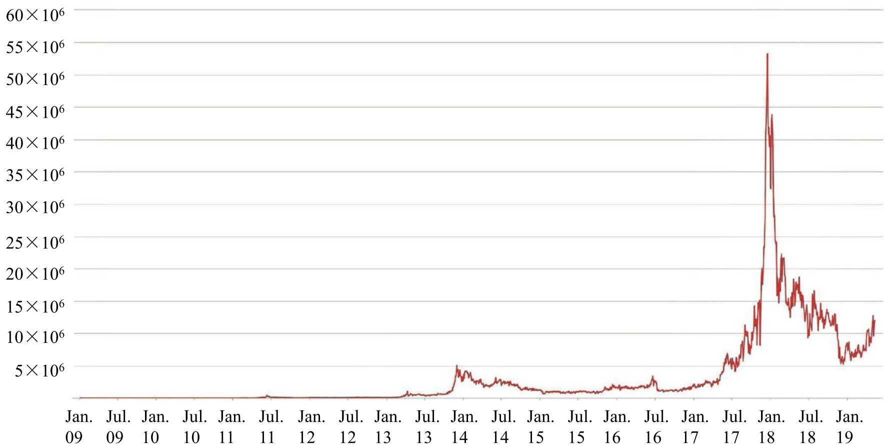

Fig. 5 Miners' revenues (unit: USD)

Fig. 6 Transaction fees (unit: USD)

we look into the stability of the Bitcoin memory pool. Comparing the number of daily confirmed transactions to the number of transactions pending for confirmation in the memory pool, we can find that the submission rate is lower than the confirmation rate in the long run, which is in conformity with the memory pool steady condition in Sect. 3.

Miners' revenues mainly rely on the Coinbase reward currently, however it is predetermined to be halved about every four years. Although transaction fees take up very limited shares of miners' revenues, i.e. about 5%, they show an upward trend in general and once hit up to over 30% in December 2017. Both the cost per transaction and average transactions per block have the fluctuating upward trend as shown in

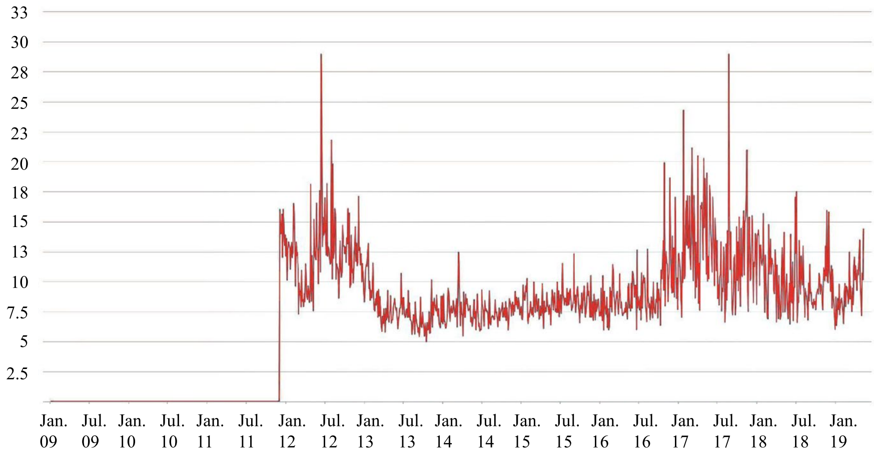

Fig. 7 Median confirmation time of transactions with fees

Figs. 3 and 4. In addition, from Figs. 5 and 6, we can also find that miners' revenues and transaction fees show a generally similar trend. Hence, it is natural to predict that transaction fees will develop to be miners' dominant economic incentives in the future.

In this paper, we use the median confirmation time to represent users' waiting time, which is described by Fig. 7. Before July 2010, no user needs to submit fees in the transaction confirmation queueing game; and until December 2011, the waiting time still kept to be zero for those who submitted transaction fees. This indicates that when the waiting time is very short (even equal to zero), the transaction confirmation queueing game has the equilibrium of no user submitting transaction fees. Since January 2012, the equilibrium of the transaction confirmation queueing game has evolved to have some users submitting transaction fees. According to [8], the equilibrium of all user submitting transaction fees are suggested by the late 2014. Although the waiting time fluctuates over time, transaction fees show a upward trend, which means the growing transaction fees are required to keep the relatively short waiting time as the Bitcoin system develops. The empirical analysis confirms that the practical equilibrium evolution of the transaction confirmation queueing game is in the conformity with our theoretical analysis in Sect. 3.

Then, we conduct the econometric analysis to explain the rationality of our transaction confirmation queueing game model formulation. According to discussions in Sect. 3, we establish the following simultaneous equations of the waiting time and transaction fees.

$$\begin{aligned} d_t &= \pi_{10} + \pi_{11} f_t + \pi_{12} s_t + \tau_{13} ms_t + \pi_{14} \Gamma_t + \pi_{15} S_t + \pi_{16} H_t + \pi_{17} \Lambda_t + u_{1t} \\ f_t &= \pi_{20} + \pi_{21} d_t + \pi_{22} s_t + \tau_{23} ms_t + \tau_{24} \Gamma_t + u_{2t} \end{aligned} \tag{22}$$

In the equations, the waiting time d is endogenous, which results in the non-conformance estimation when applying the ordinary least square (OLS) regression.

Table 2 2SLS regression of transaction fees on the waiting time

Variable	Coef.	SE	t	Sig.
d_t	21.083	7.583	2.780	0.006
s_t	– 102.26	125.536	– 0.815	0.416
ms_t	3.553	0.452	7.867	0.000
Γ_t	0.001	0.000	0.584	0.560
Constant	– 58.425	56.994	– 1.025	0.306

Adjusted-R^2 = 0.596, F = 134.288, Sig. = 0.000

Also, the first equation is under-identified, while the second equation is over-identified. In view of these considerations, we utilize the two-stage least squares (2SLS) regression to gets rid of the possible correlation of the endogenous variable and the error through introducing the instrument variable $\hat{d}_t$. Accordingly, we run a 2SLS regression to get the following equations.

$$\begin{aligned} \hat{d}_t &= \hat{\tau}_0 + \hat{\tau}_1 s_t + +\hat{\tau}_2 ms_t + \hat{\tau}_3 \Gamma_t + \hat{\tau}_4 S_t + \hat{\tau}_5 H_t + \hat{\tau}_6 \Lambda_t \\ f_t &= \tau_{20} + \tau_{21}\hat{d}_t + \tau_{22} s_t + \tau_{23} ms_t + \tau_{24} \Gamma_t + u_{2t}, \end{aligned} \tag{23}$$

here the predicted $\hat{d}_t$ is totally determined by the predetermined variables Γ_t, S_t and H_t, which is then used in the second-stage regression.

We utilize the real-world data to conduct estimations of these parameters in Eq. (23), where s_t is the average transaction size computed by the average block size divided by the number of transactions per block, and ms_t is the memory pool size, Γ_t is the number of unconfirmed transactions, Λ_t is the number of daily confirmed transactions, and d_t is the median confirmation time.

The estimated results are shown in Table 2. The results indicate that our formulated model has good explanatory power on the research problem. Besides, they confirm the significantly positive effect of the waiting time as well as the memory pool size on transaction fees. However, although the average transaction size shows the negative effect on transaction fees, its coefficient cannot pass the significance test. It indicates that the average transaction size does not influence transaction fees in the transaction confirmation queueing game currently. In the same manner, the number of unconfirmed transactions also does not have significant effect on transaction fees. In conclusion, the econometric analysis can supportively validate our theoretical discussions concerning users' fees and waiting time in Sect. 3.

4.2 Computational experiments

In what follows, we design computational experiments to evaluate the game-theoretical transaction confirmation queueing model as well as the corresponding equilibrium analysis. For simplicity, we take the transaction confirmation queueing game with two priorities for example, where the high priority is got through submitting the transaction fee while the low priority needs no fee. Using the real-world data, we can figure out that the daily submission rate is 257,987 and the daily confirmation

Table 3 Users' waiting time in the MSMP game (unit: min)

	Mean	Range	SE
d_1	0.0321	[0.0055, 1.8266]	0.0952
d_2	12.4386	[1.8288, 1207.2719]	49.2100

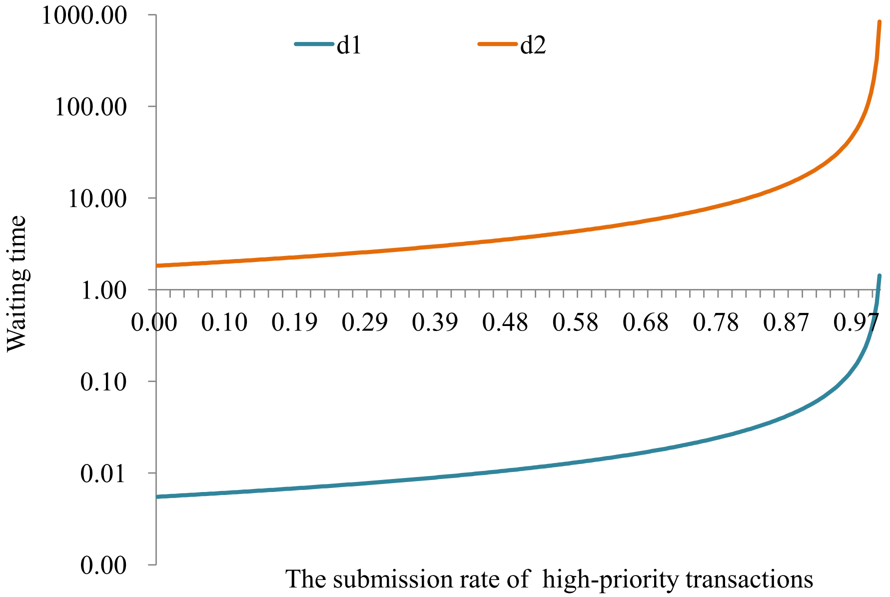

Fig. 8 High-priority users' waiting time in the MSMP games

rate is 258,204. Then, we randomly distribute the transaction submission rates of these two priorities. In addition, based on the results of econometric analysis, it is reasonable for us to consider $s^i = 1$ to calculate the optimal number of transactions recorded into a block.

In our computational experiments, users' waiting time are shown in Table 3 and Fig. 8,[2] where d_1 is the waiting time of high-priority users and d_2 is the waiting time of low-priority users. The average waiting time of all users is 1.8286. From experimental results, we can find that:

- The average waiting time of high-priority users is 0.0321, which is distinctly shorter than the average waiting time of 12.4386 for low-priority users. Moreover, high-priority users can expect for the shorter waiting time under any submission rate distribution, even the shortest d_2 exceeds the longest d_1. These

[2] Y-axis is logarithmic scale.

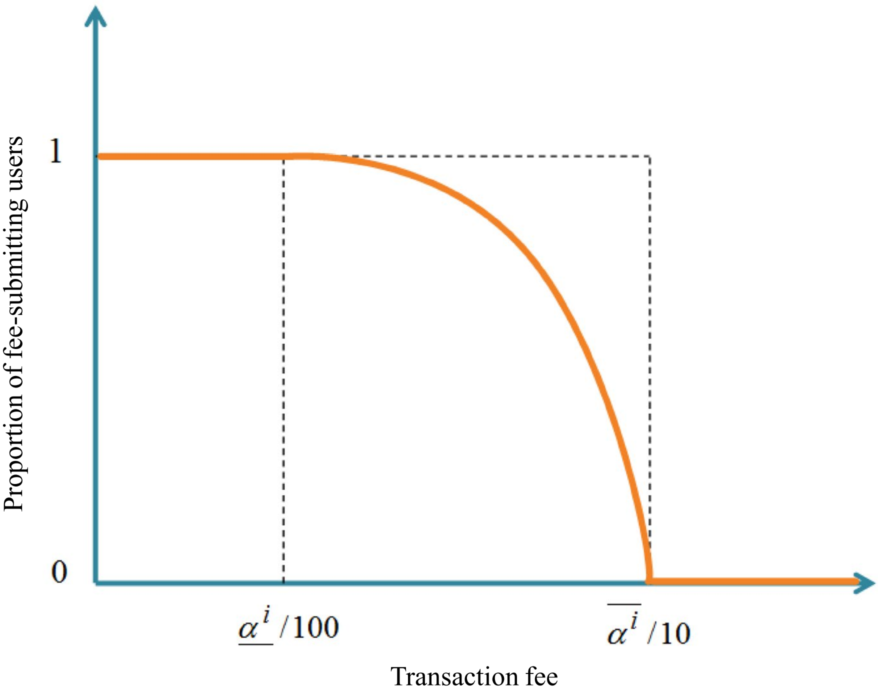

Fig. 9 The equilibrium of the MSMP game

indicate that the high priority takes overwhelming advantages in the transaction confirmation game in terms of reducing the waiting time.

- The submission rate distribution produces great influences on the waiting time of both priorities, and results in much higher variances of low-priority users' waiting time. This indicates that high-priority users can expect for more stably short waiting time than low-priority users.
- With the increasing submission rate of high-priority transactions, the waiting time of both priorities get longer. It indicates that the MSMP rule can not exert ideal effects on reducing users' waiting time when a lot of transactions flow into the high priority.

Next, we investigate the equilibrium of the MSMP game, which is shown in Fig. 9, where $\overline{\alpha}^i$ and $\underline{\alpha}^i$ represent the highest and lowest unit time cost of all users, respectively. More formally, the equilibrium of the MSMP game can be described as:

- If $\forall i \in N, f > \alpha^i/10$, the MSMP game has the equilibrium of no user submitting transaction fees;
- If $\forall i \in N, f < \alpha^i/100$, the MSMP game has the equilibrium of all users submitting transaction fees;

- If $\exists i \in N$, $\alpha^i/100 \leq f \leq \alpha^i/10$, the MSMP game has the equilibrium of some users submitting transaction fees. Besides, with the required transaction fee increasing, the proportion of fee-submitting users decreases slowly at first followed by a sharp decline.

5 Discussion and implications

Users' fee decisions depend on their trade-off between the required fees and the expected revenues, and they depend on time costs instead of transaction values to evaluate their revenues. The low time cost may result from either low unit time cost or the reduced waiting time, that is, users' fee decisions depend on the comprehensive effect of their unit time costs and the waiting time. As indicated by the econometric analysis, users' fee decisions are significantly affected by the waiting time. Generally, the high priority takes overwhelming advantages to the low priority in terms of more stably shorter waiting time as shown in the experimental results. Since the high priority is usually accompanied with high transaction fee, longer waiting time drives users to submit transaction fees in pursuit of desired priorities. Besides, the more impatient users with higher unit time costs have incentives to provide higher transaction fees to get higher priority with the purpose to reduce the time costs. These results provide good explanation of users' fee decisions in the transaction confirmation queueing game, and also suggest that the Bitcoin system's fee policy should take both users' unit time costs and their waiting time into consideration.

Our research has shown the conditions of three types of Nash equilibria in the full-information game, i.e. the equilibrium of no, all and some users submitting transaction fees, using both the theoretical analysis and computational experiments. Besides, the experimental results show that with the required transaction fee increasing, the proportion of fee-submitting users decreases slowly at first followed by a sharp decline. In addition, over-high required fees will make the transaction confirmation game end up with no users submitting fees, which will make miners lose the necessary economic incentive to provide computing power to maintain the security and sustainability of the Bitcoin system. Based on the above analysis, we can also find the optimal fee of the Bitcoin system aiming to maximize total transaction fees paid for miners, and the computing process can be described as follows:

Given a transaction fee f, we can get the expected waiting time d_1 and d_2, under which the equilibrium transaction fee of each user is achieved as f^i. Then, the total transaction fee collected by the miners is $\sum_i f^i$.

With the purpose of profit maximization, the Nash equilibrium with no user paying the transaction fee is not desirable for miners. Consequently, the optimal transaction fee f^* should satisfy the condition that

$$0 < f^* \leq \alpha^i(d - d_1). \tag{24}$$

Consider that $s^i = 1$, and then all fee-paying users are submitting the identical transaction fee in the MSMP game. So, we have

$$\sum_i f^i = \theta(f)Nf, \tag{25}$$

where $\theta(f)$ is the proportion of fee-paying users under the transaction fee f. Therefore, we obtain the optimal transaction fee as

$$f^* = \underset{0<f\leq\alpha^i(d-d_1)}{argmax}\ [\theta(f)Nf]. \tag{26}$$

The optimal transaction fee may be a relatively low fee making all users pay or a relatively high fee making some users pay. Under the former case, we have the optimal transaction fee as

$$f^*(1) = \min[\alpha^i(d_2 - d)] - \varepsilon, \tag{27}$$

where ε is an infinitely small positive number. Meanwhile, the corresponding maximal profit is $P^*(1) = Nf^*(1)$. Under the latter case, the optimal transaction fee $f^*(2)$ should be set to achieve the maximal profit $\theta(f^*(2))Nf^*(2)$, thus we have

$$f^*(2) \in [\alpha^i(d_2 - d), \alpha^i(d - d_1)]. \tag{28}$$

As such, we can get the optimal transaction fee

$$f^* = argmax\left[Nf^*(1), \theta(f^*(2))Nf^*(2)\right]. \tag{29}$$

In this paper, we conduct the equilibrium analysis of the transaction confirmation game to figure out the conditions of different portfolio of users' equilibrium strategies. Moreover, given all users' unit time costs, we can extend our model to derive the optimal transaction fee under different objectives on the basis of our equilibrium analysis, e.g. maximizing miners' revenues, all users' total payoffs, or the system-level welfare and so on. Also, we can extend to study the priority rule according to different transaction confirmation characteristics in other blockchain systems and compute users' waiting time accordingly.

To summarize, our research can not only help understand users' fee decisions in the transaction confirmation game; but also provide useful references for the Bitcoin system to optimize the transaction fee policies. Therefore, our research can offer useful managerial insights to the transaction fee management in the Bitcoin transaction confirmation queueing game.

6 Conclusions and future work

This paper highlights the importance of transaction fees in both the individual decisions and the system-wide sustainability in the Bitcoin system. We build a game-theoretical queueing model to investigate users' fee decisions on transaction confirmation. Also, we conduct the empirical study and computational experiments to validate our theoretical analysis. Through discussing users' fee decisions on maximizing their payoffs, we show the conditions of three types of

Nash equilibria in the full-information game, i.e. the equilibrium of no, all and some users submitting transaction fees. Our research can help understand users' fee decisions, which depend on the unit time cost and priority-based waiting time rather than the value of submitted transactions. Besides, we find that longer waiting time encourages users to submit transaction fees, but with the required transaction fee increasing, the proportion of fee-submitting users decreases slowly at first followed by a sharp decline until no user submits fee. This can provide good support for Bitcoin system's fee policy selection.

In the future work, we plan to address the limitations of the current research. In fact, the monetary price of the Bitcoin is vital for the participants' decisions due to its high exchange rate with US dollars and its high volatility over time. Therefore, we will incorporate the dynamic Bitcoin price into the game-theoretical model to make further study of users' transaction fee decisions. Second, on the basis of current research, we will further design novel priority rules to achieve some useful goals, e.g., reducing the waiting time difference between different priorities, shortening the longest waiting time, etc.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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