Prosumer Community: A Risk Aversion Energy Sharing Model

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***Abstract*—Household photovoltaic (HPV) prosumers and the community photovoltaic (CPV) system have been growing rapidly with the development of the sustainable technology. The uncertainties of these distributed renewable energy resources bring a significant challenge to the design of power-market mechanism and the energy dispatch of the power system for promoting energy efficiency. It is essential to develop a novel efficient energy management strategy for addressing this challenge from the perspective of community prosumers. Accordingly, a risk aversion energy sharing model based on a devised local energy market is presented. A stochastic game is established to minimize prosumers' energy costs and the weighted conditional value-at-risk of energy sharing loss of uncertain CPV through optimal energy sharing profiles. The household loads and HPV outputs are considered as stochastic parameters in the game model. Moreover, a sample weighted average approximation (SWAA) method is proposed for a better estimation of the stochastic game while the SWAA equilibrium is obtained by a relaxation method-based algorithm with theoretical proof. In addition, the blockchain technology is introduced as a distributed and secure way to facilitate the energy sharing model. The case studies show the efficiency of the proposed energy sharing model and the algorithm.**

***Index Terms*—Prosumer, community photovoltaic, risk aversion, energy sharing, stochastic game, SWAA equilibrium, blockchain.**

NOMENCLATURE

Abbreviations:

CPV	Community photovoltaic.
EV	Electric vehicle.
HESS	Household energy storage system.
HPV	Household photovoltaic.
PV	Photovoltaic.
SAA	Sample average approximation.
SWAA	Sample weighted average approximation.

Common parameters and variables:

$\alpha^{\omega_0,t}$	Coefficient for real-time power prices.
$\alpha^{DA,t}$	Coefficient for day-ahead power prices.
$\beta^{\omega_0,t}$	Coefficient for real-time power prices.
$\beta^{DA,t}$	Coefficient for day-ahead power prices.
Γ	The stochastic game.
$\mathcal{K}$	Set of sample scenarios.
$\mathcal{N}$	Set of prosumers.
$\mathcal{T}$	Set of all hours.
ω	Subscript to denote stochastic variables.
τ_k	Weight of sample scenario k.
θ^t	Confidence level for CVaR.
$G^{\omega,t}$	Energy generation of CPV.
K	Number of sample scenarios.
k	Index of sample scenario.
$L^{\omega,t}$	Real-time net energy demand of the community.
$L^{DA,t}$	Day-ahead net energy demand of the community.
N	Number of prosumers.
n	Index of prosumer.
$p^{\omega,t}$	Real-time prices in local community market.
$p^{DA,t}$	Day-ahead prices in local community market.
p_{cpv}	Prices for prosumers of CPV energy.
S^t	Power demand for Community service.
T	Number of hours.
t	Index of hour.

Prosumer parameters and variables:

η_n^{EV}	Power efficiency of HPEV.
η_n^{HESS}	Power efficiency of HESS.
$\mathcal{X}$	Feasible set.
$\pi_n(\cdot)$	Cost function in game.
ψ_n	Energy cost.
ρ_n^t	Weight for prosumer n evaluating CVaR.
a_n^t	Variable for CVaR.
$b_n^{\omega,t}$	Real-time net energy demand.
$b_n^{DA,t}$	Day-ahead net energy demand.
$C_n^{\max}$	Maximum capacity of HESS.
$C_n^{\min}$	Minimum capacity of HESS.
$c_n^{\omega,t}$	Charging power for HESS.

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$Chr_n^{\max}$	Maximum charging power of HESS.
$d_n^{\omega,t}$	Discharging power for HESS.
$Dis_n^{\max}$	Minimum discharging power of HESS.
$e_n^{\omega,t}$	Charging power for HPEV.
E_n^{ω}	Power charging demand of EV.
Ech_n	Rated charging power of HPEV.
$g_n^{\omega,t}$	Household PV generation.
$l_n^{\omega,t}$	Household demand.
r_n^t	Expected energy share from CPV.
X_n	Decision vector.
x_n^t	Decision vector.

I. Introduction

UP TO now, renewable energy resources (RES) have been developing rapidly worldwide to handle energy crisis and environmental issues. Nevertheless, the uncertainty and the intermittence of large-scale RES integrating into the main grid bring challenges for the energy scheduling and the operation of distribution network [1]. Therefore, household consumers are encouraged to install small-scale household photovoltaic (HPV) systems on rooftop for local utilization [2], and these consumers are named as prosumers in [3] due to their abilities of both consuming and generating energy. Furthermore, middle-scale RES installations such as the community photovoltaic (CPV) with lower average capital costs are also getting an increasing attention for providing more access to renewable energy for prosumers in a community with open spaces [4], [5]. Even so, the temporal mismatch between the renewable energy generation of HPV and CPV and the energy demands leads to a low efficiency of local renewable energy utilization, not to mention the uncertainties of renewable energy generation and household demands. Therefore, efficient energy management for local-area prosumers and CPV with uncertainties is indeed essential for achieving a sustainable green community with higher energy efficiency, lower energy cost, and less dependence on the main grid.

A promising way to deal with the energy management problem of community prosumers is to broaden individual prosumers to a community-wide system by organizing a peer-to-peer group model for local energy sharing among prosumers [3], [6]. The model allows prosumers to directly buy/sell local resources from/to other prosumers under certain market rules. The related studies can be classified into two groups: *the market bidding models based on auction mechanism* [7]–[9] and *the dynamical pricing models with price-driven mechanism* [10]–[12]. For the former group, each participant has a fixed role which provides energy as a seller or consumes energy as a buyer according to bidding profile. However, prosumers should be capable of making their own decisions of selling or buying energy based on their energy generations, load schedules and power prices to achieve a high energy efficiency and low energy costs [10], [11]. In fact, this is natural and rational since each prosumer just requires to make its own decision for its own interest individually, which can also ensure its privacy security [11]. Therefore, *the dynamical pricing models* with the essence of price-driven mechanisms are more flexible and more suitable to organize the peer-to-peer energy sharing prosumer group, which is also verified by the related existing works [10], [13]–[21]. For a better literature review, we will introduce these works in detail in the next section. In these works, the dynamic prices models generally lead to local energy sharing games where prosumers decide their energy sharing profiles to minimize their own costs in response to different prices [12], [15], [21]. However, the uncertainties of household loads and PV generations have not been fully considered, and efficient distributed algorithms to seek the game equilibrium with a convergence proof are insufficient. Therefore, in this paper, we introduce two-stage dynamic prices, i.e., day-ahead prices and real-time prices, varying with the community net demands in day-ahead stage and real-time stage respectively. Based on the two-stage dynamic prices, we formulate a stochastic energy sharing game where each prosumer decides its two-stage energy sharing profiles to minimize the energy cost while the uncertainties are considered as real-time stochastic parameters. Then we propose a sample weighted average approximation (SWAA) equilibrium for estimating the stochastic Nash equilibrium of the proposed stochastic game, which can provide economical guidelines for practical energy sharing. Moreover, an efficient distributed algorithm with a convergence proof is presented to seek the SWAA equilibrium.

To further improve the efficiency of the energy management, the energy management of CPV is also integrated into the energy sharing model in this paper. The energy management of CPV can be regarded as the energy sharing of common resources among prosumers. [22] dispatches the common resource to prosumers according to their investments. Each prosumer has a fixed share of common resources, which cannot achieve a full and flexible utilization of the resources. Inspired by the risk management theory of a stochastic optimization [23], [24], for the uncertain CPV, we present a risk aversion energy sharing method, where each prosumer requires to decide its expected energy share from CPV by evaluating the conditional value-at-risk (CVaR) for the over-expecting loss for actual power generation of the uncertain CPV. In this way, the energy sharing of CPV is negotiated and accepted by all prosumers, and the common resources can also be fully utilized. The energy sharing of CPV is integrated into the proposed stochastic game model to achieve an efficient energy sharing within the prosumer community.

The presented energy sharing model of prosumers and CPV is shown to be effective since the local energy efficiency can be promoted and the model can be implemented in a distributed way. Therefore, a secure and distributed technology is also required for both reducing the cost of the computation center and ensuring the information security. Hence, we introduce an emerging information technology as one of the most promising ways to meet the requirement: blockchain with smart contracts [25]. Blockchain with smart contracts is a kind of technology on which transactions can succeed without the need of having a trusted third party. It is distributed to all nodes of the network to form a secure decentralized market platform. With extensive interesting financial applications for dealing with trust issues [26], blockchains have also gained growing attention in the smart energy market and the energy trading [9], [27]. [9] derives the components for a blockchain-based prosumer microgrid, and introduces a practical microgrid for verification.

[27] investigates the opportunity of blockchain-based microgrid market to the growth of prosumer. Inspired by [9], [27], we introduce the blockchain technology to implement the presented model. Specifically, each prosumer assigned with a unique index acts as a node on the blockchain, and then facilitates the proposed peer-to-peer energy sharing model through mutual communication. The equilibrium results containing energy sharing profiles and prices will be executed and cleared on the blockchain. This distributed implementation way can spread the calculation load over multiple prosumers and can provide information security guarantee for the energy system.

Finally, the contributions of the paper are summarized as follows:

- A novel energy sharing model of prosumers and CPV is proposed to facilitate a sustainable prosumer community.
- A CVaR based energy sharing model of uncertain CPV is presented to fully and reliably utilize the uncertain resources.
- The energy sharing model is formulated as a stochastic game based on the presented two-stage dynamic prices, where the household loads and PV generations are considered as real-time stochastic parameters. The modeling method is novel compared with the existing literature. A SWAA equilibrium is proposed to estimate the stochastic game with higher accuracy. A distributed practical algorithm is presented to seek the SWAA equilibrium.

The rest of the paper is organized as follows. Section II introduces the related literature on price-driven mechanisms. Section III models the community energy system. Section IV introduces the stochastic game and the equilibrium analysis for community energy sharing. Section V presents the case studies. Section VI concludes the paper and points out the limitation of the work and the future research focus.

II. Literature of Price-Driven Mechanisms

In this section, the existing literature on the price-driven mechanisms for energy management as well as the comparisons with our work will be introduced.

Generally speaking, the related literature can be classified into two kinds according to their modeling structures: *leader-follower structure* and *peer-to-peer structure*. For the *leader-follower structure*, there is a leader acting as the price maker and the users are price takers adjusting their energy demands. Typical solutions contains bi-level optimization and Stackelberg game. [13] formulates retailer-users of leader-follower structure as a bi-level problem where the retailer holds the upper level to decide prices and users being capable of executing load shifting hold the lower level to decide energy demands. [14] uses the bi-level scheme to investigate the energy sharing and pricing problem to facilitate a renewable and commercial solution for an apartment building. However, bi-level optimization problems are known to be strongly NP-hard to solve with classical numerical methods [28]. From the perspective of game theory, [10] sets a shared facility controller as the leader and prosumers as the game followers for a renewable city. Similarly, [15], [16] investigate the energy sharing within a PV prosumer microgrid setting the operator of microgrid as the price maker with detailed modeling, and the works analyse the game equilibrium strictly. In [15], [16], the real-time uncertainties such as energy generation and demands are discussed. However, the demand response models in [14]–[16] are based on an ideal natural logarithm for utilities of energy consumptions and may not be practically applicable. In general, in the *leader-follower structure*, the leader intends to maximize its profit function, and prosumers minimize their energy bills or discomfort associated with engaging in demand response actions. Therefore, the overall energy efficiency mainly depends on the profit function of the leader that controls the system energy schedule with trustworthy concerns and information security.

In the *peer-to-peer structure*, a certain global mechanism is usually required to drive the energy scheduling of individual peer-to-peer prosumers to achieve global expected efficiency. There are two widely used practices: *designing prices functions* and *reducing total costs*. In the first practice, [17] uses a linear price function and formulates a generalized equilibrium problem for the demand-side energy management of multiple users, and the randomness of renewable sources is considered through presenting an expected cumulative expense, however, the work requires exact distribution of uncertain parameters. [18] designs a price function based on the energy supply and demand ratio inside the community, and autonomous dynamic demand response is integrated to adjust the energy scheduling of prosumers as the prices changes. [19] discusses the energy sharing among neighborhood users with the appliance-level demand response to reduce individual energy costs. [20] presents a linear price function increasing with the overall net demands, and proposes a distributed algorithm for peer-to-peer demand response game. However, [18]–[20] investigate the ideal demand response programmes and haven't considered the uncertain factors. The other practice is to let users *minimize the overall energy costs*. [21] proposes a two-stage game for prosumers managing charging and discharging of their HESS and electric vehicles (EV) to reduce the community quadratic energy cost in day-ahead scheduling stage and real-time adjusting stage. Note that the price-driven *peer-to-peer structure* always leads to aggregative games [29]. Nevertheless, we believe designing a price function based on the demand and supply relationship is more applicable since it is natural for users or prosumers to receive prices signals rather than to persuade them to reduce the overall energy costs. Therefore, we adopt the former practice as our scheme in this paper. Moreover, the uncertainties of PV generation and loads have not been fully taken into account in day-ahead scheduling in these works, which may lead to a poor performance in real applications. In our paper, we treat the uncertain factors as stochastic parameters and formulate a stochastic energy sharing game by integrating the risk aversion energy sharing of CPV into it. The equilibrium analysis and proof for the convergence of the algorithm are also provided.

III. System Modeling

A schematic diagram of the proposed community energy sharing model is shown in Fig. 1. The community contains multiple prosumers, CPV and community electrical services including streetlights, and is connected to the main grid. To achieve a

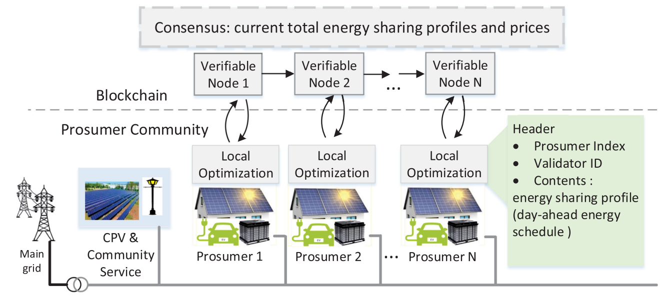


Fig. 1. Community energy sharing diagram: multiple prosumers, CPV, and community electric services, as well the implementation of the energy sharing model on the blockchain technology.

sustainable energy community, the proposed way is to promote price-driven energy sharing within the community with the existence of multiple uncertainties. The local two-stage dynamic prices include day-ahead prices and real-time prices varying respectively with day-ahead and real-time net demands of the community. The dynamic prices provide economic incentives for prosumers to commit themselves to adjust the net demand through scheduling their energy sharing profiles. The blockchain technology is used to implement the energy sharing model as Fig. 1 shows. Each prosumer in the community is assigned with an index and utilizes its unique validator identification (ID) to perform a verifiable node on the blockchain. Under certain distributed communication protocols, prosumers carry out the proposed energy sharing model in a secure and distributed way to obtain an equilibrium as the final energy sharing profiles and prices in the local market. The equilibrium serves as the smart contracts to be executed and cleared on the blockchain. Note that the main work of this paper is to investigate the community energy sharing model, the blockchain technology is referred as a promising way to implementing the model, and details on the implementation will be given in the end of section IV.

Let $\mathcal{N} = \{1, 2, \ldots, n, \ldots, N\}$ denote the set of prosumers in the community and $\mathcal{T} = \{1, 2, \ldots, t, \ldots, T\}$ denote the set of hours of the scheduled day. The prosumers can share their energy demands/supplies from/to the community, and can also negotiate with each other for their expected energy shares which refer to their day-ahead energy quantities shared from the CPV. Thus, the energy sharing profiles of prosumer $n \in \mathcal{N}$ include its day-ahead net demand $b_n^{DA,t}, \forall t \in \mathcal{T}$, real-time net demand $b_n^{\omega,t}, \forall t \in \mathcal{T}$, and the expected energy share $r_n^t, \forall t \in \mathcal{T}$, from CPV. Considering the real-time uncertainties, we adopt the two-stage stochastic optimization model for energy scheduling of prosumers [30], [31]. Thus in this section, we model the community energy system from three aspects. We firstly introduce the local two-stage dynamic prices. Then we formulate the over-expecting risk of energy sharing of uncertain CPV. At last we formulate the energy cost function of each prosumer based on two-stage stochastic optimization. Considering that the prosumers are located close to each other, the loss of energy distribution of the power network is negligible.

A. Price Functions

The day-ahead price $p^{DA,t}$ (\$/kWh) is supposed to be an increasing function with respect to the day-ahead net demand $L^{DA,t}$ of the community, and a linear form is presented as follows:

$$\begin{aligned} p^{DA,t}(L^{DA,t}) &= \alpha^{DA} L^{DA,t} + \beta^{DA}, \\ L^{DA,t} &= \sum_{n \in \mathcal{N}} b_n^{DA,t}, \end{aligned} \tag{1}$$

where α^{DA} and β^{DA} are given parameters based on the original maximum and minimum net demand of the community and the market prices bounds. The net power demand $L^{DA,t}$ is the summation of the day-ahead net demands $b_n^{DA,t}, \forall n \in \mathcal{N}$, of all prosumers. As an increasing function with respect to the community net power demand, the proposed price function is able to motivate prosumers to reduce their energy demands in peak hours and increase their energy demands in off-peak hours, and thus can smooth the net demand of the community, reduce the influence on main grid, and release the energy distribution complexity for power systems.

Similarly, the real-time price $p^{\omega,t}(L^{\omega,t})$ is a linear increasing function of the real-time net power demand $L^{\omega,t}$ (\$/kWh) equaling to the sum of real-time net demands $b_n^{\omega,t}, \forall n \in \mathcal{N}$, of prosumers as follows:

$$p^{\omega,t}(L^{\omega,t}) = \alpha^{\omega_0} L^{\omega,t} + \beta^{\omega_0},\ L^{\omega,t} = \sum_{n \in \mathcal{N}} b_n^{\omega,t}, \tag{2}$$

where $\alpha^{\omega_0,t}$ and $\beta^{\omega_0,t}$ are given parameters, and ω is a superscript to indicate the uncertain variables. In this paper, we use $\omega_1, \omega_2, \ldots$ to denote different scenarios. The real-time price is used to clear the real-time demands due to the mismatch between the day-ahead energy supplies and real-time uncertainties.

Different from the price mechanism currently in effect in the traditional power grid, the presented price functions can dynamically reflect the economical law of demand and supply. Note that once the net demands are settled, the prices are determined.

B. Risk-Averse Energy Sharing of CPV

CPV as the community common resources provides flexible access of renewable energy to energy consumption of the prosumer community including public electrical services and prosumers. However, the energy generation $G^{\omega,t}$ of CPV is uncertain, and the uncertainty leads to low energy efficiency and the difficulty for a fair distribution of common resources. Inspired by the risk management theory of stochastic programming [23], we propose a risk-averse energy sharing method to deal with the challenge. A different scenario of ω will have a different CPV generation $G_{\omega,t}$, and prosumers must reach consensus on their expected shares of the community resources $r_n^t, \forall n \in \mathcal{N}$, by minimizing the risk of total energy sharing loss due to the existence of these stochastic parameters. Thus we model the over-expecting risk by introducing CVaR [24] as follows:

$$\begin{aligned} &\mathrm{CVaR}_\theta \left(\sum_{n \in \mathcal{N}} r_n^t, S^{\omega,t}, G^{\omega,t} \right) = \\ &\arg\min_{a^t \geq 0} \left\{ a^t + \frac{\mathbb{E}[\sum_{n \in \mathcal{N}} r_n^t + S^{\omega,t} - G^{\omega,t} - a^t]^+}{1 - \theta} \right\}, \end{aligned} \tag{3}$$

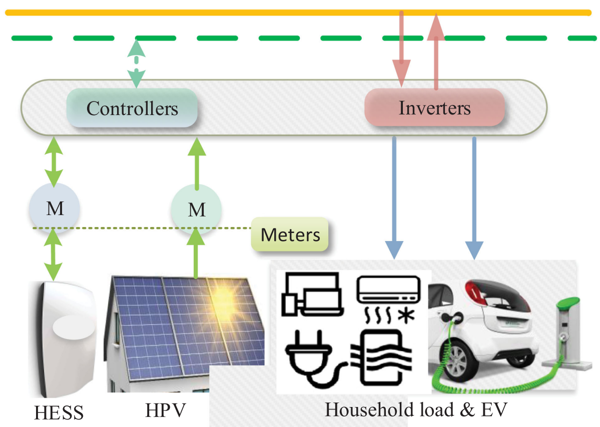


Fig. 2. Energy management for a prosumer.

where a^t is the auxiliary variables, $S^{\omega,t}$ is the community service energy demand, $G^{\omega,t}$ is the energy generation of CPV, θ is the confidence level, and $[\sum_{n\in\mathcal{N}} r_n^t + S^{\omega,t} - G^{\omega,t} - a^t]^+$ is the loss function for the total expected energy shares considering the uncertain energy outputs of CPV system. Moreover, $[\cdot]^+$ denotes the projector $\max\{0,\cdot\}$, and $\mathbb{E}[\cdot]$ denotes the mathematical expectation of the elements. Let $a = [a^1, a^2, \ldots, a^T]$, and the corresponding constraint set is defined as $\mathcal{A} = \{a^t \geq 0,\ t \in \mathcal{T}\}$.

C. Cost Function for Prosumers

The individual energy system of each prosumer is shown in Fig. 2. Smart meters, controllers and inverters are hardware supports for the energy management of a prosumer. A prosumer n owns HPV, EV, HESS, and household electrical appliances, and schedules its day-ahead and real-time energy demands of $b_n^{DA,t}, b_n^{\omega,t}$, expected CPV share r_n^t, charging rate $e_n^{\omega,t}$ of EV for the charging demand Ecn_n, charging and discharging power $(c_n^{\omega,t}, d_n^{\omega,t})$ of HESS based on HPV generation $g_n^{\omega,t}$ and household load $l_n^{\omega,t}$ so as to minimize its energy cost.

Under the local price mechanism, prosumer n always expects to minimize the following energy cost function by two-stage stochastic optimization:

$$\begin{aligned}&\sum_{t\in\mathcal{T}} \psi_n(b_n^{DA,t}, r_n^t, b_n^{\omega,t}, e_n^{\omega,t}, c_n^{\omega,t}, d_n^{\omega,t})\\ &= \sum_{t\in\mathcal{T}} \left\{p^{DA,t} b_n^{DA,t} + p_{cpv} r_n^t + \mathbb{E}\left[p^{\omega,t} b_n^{\omega,t}\right]\right\},\end{aligned} \tag{4}$$

where p_{cpv} denotes the power price of CPV which is set to be very low for fully utilization of CPV and compensating the construction and maintenance expenses. Since the price functions $p^{DA,t}$ and $p^{\omega,t}$ are linear increasing functions, the cost function $\psi_n(\cdot)$ is also convex with respect its decision variables. In a scenario of ω, the prosumer must keep the balance within itself, and the balance can be formulated as follows:

$$b_n^{DA,t} + r_n^t + b_n^{\omega,t} - e_n^{\omega,t} - c_n^{\omega,t} + d_n^{\omega,t} \geq l_n^{\omega,t} - g_n^{\omega,t}, \forall t, \tag{5}$$

$$r_n^t, e_n^{\omega,t}, c_n^{\omega,t}, d_n^{\omega,t}, b_n^{\omega,t} \geq 0, \forall t, \tag{6}$$

where $r_n^t, e_n^{\omega,t}, c_n^{\omega,t}, d_n^{\omega,t}$ are non-negative due to their physical implication. $b_n^{\omega,t}$ is set to be non-negative to avoid arbitrage. $b_n^{DA,t}$ can be non-negative or negative since prosumers can buy/sell energy from/to the community market and they can flexibly schedule their energy including HPV, HESS and EV. The constraints of charging limits of EV for prosumer n are given as follows:

$$0 \leq e_n^{\omega,t} \leq \varepsilon_n^t Ech_n, \forall t, \tag{7a}$$

$$\sum_{t\in\mathcal{T}} \eta_n^{EV} e_n^{\omega,t} = E_n^{\omega}, \tag{7b}$$

where η_n^{EV} is the charging efficiency of EV, Ech_n denotes the rated charging power and ε_n^t is a binary parameter indicating the plug-in state of EV n at hour t. When $\varepsilon_n^t = 1$, EV n is connected to the household charging point at hour t and thus is charged. E_n^{ω} is the energy usage of EV in current day at scenario of ω, and the energy consumption is assumed to be fully compensated in plug-in hours. This assumption will guarantee EV to be full charged when leaving the house and thus will avoid the abrupt energy demand of EV during the charging day. The constraints of HESS are shown below:

$$0 \leq c_n^{\omega,t} \leq Chr_n^{\max}, \forall t, \tag{8a}$$

$$0 \leq d_n^{\omega,t} \leq Dis_n^{\max}, \forall t, \tag{8b}$$

$$C_n^{\min} \leq C_n^{\omega,0} + \sum_{i=1}^{t} \left(\eta_n^{HESS} c_n^{\omega,i} - \frac{d_n^{\omega,i}}{\eta_n^{HESS}}\right) \leq C_n^{\max}, \forall t, \tag{8c}$$

$$C_n^{\omega,0} + \sum_{t\in T} \left(\eta_n^{HESS} c_n^{\omega,t} - \frac{d_n^{\omega,t}}{\eta_n^{HESS}}\right) \geq C_n^{\omega,0}, \tag{8d}$$

where η_n^{HESS} and $\frac{1}{\eta_n^{HESS}}$ are the charging and discharging efficiency of HESS, $Chr_n^{\max}$ and $Dis_n^{\max}$ are the charging and discharging rate limits of HESS. (8c) ensures that the energy level of HESS at each hour must be limited between the minimum capacity $C_n^{\min}$ and the maximum capacity $C_n^{\max}$ to avoid an overuse of the battery. (8d) ensures that the left energy level at the end of the day must not be lower than the initial energy level.

IV. Non-Cooperative Game and Equilibrium Analysis

In this section, we will show the proposed local market mechanism with dynamic prices will lead to a non-cooperative game among prosumers for energy sharing within the community. The main problem is how to deal with the energy sharing of CPV. Let each prosumer decide its expectation r_n^t for CPV by making a tradeoff between the expected share of CPV at low prices and the over-expecting risk of uncertain CPV. Then we can formulate the objective function of prosumer n by adding the risk model (3) with a weight parameter ρ_n to prosumers' energy cost function (4) as follows:

$$\begin{aligned}&\pi_n(X_n, X_{-n}; \omega) = \sum_{t\in\mathcal{T}}\\ &\left\{\psi_n(x_n^t) + \rho_n \mathrm{CVaR}_\theta\left(\sum_{m\in\mathcal{N}/\{n\}} r_m^t + r_n^t, S^{\omega,t}, G^{\omega,t}\right)\right\},\end{aligned} \tag{9}$$

where $x_n^t = [b_n^{DA,t}, r_n^t, b_n^{\omega,t}, e_n^{\omega,t}, c_n^{\omega,t}, d_n^{\omega,t}, a_n^t]$, and $X_n = [x_n^1, x_n^2, \ldots, x_n^T]$ is the decision variable vector of prosumer n for a whole period $[1, 2, \ldots, T]$ and $X_{-n} = [X_1, \ldots, X_{n-1}, X_{n+1}, \ldots, X_N]$ is all the strategies except prosumer n. Let

$X = [X_1, \ldots, X_n, ...X_N]$. (9) is designed to be the game model for prosumer n in response to the decisions of other prosumers. The decision of the game model is limited in the constraint set $\mathcal{X}_n := \{(5), (6), (7), (8), \forall t \in \mathcal{T}\} \cup \mathcal{A}$.

A. Formulation of the Non-Cooperative Game

In a non-cooperative game, each participant usually minimizes its cost according to other participants' decisions. In this paper, we formulate a stochastic non-cooperative game denoted by $\Gamma = \{\mathcal{N}, \pi_n, X_n \in \mathcal{X}_n, \forall n \in \mathcal{N}, \omega\}$ consisting of the following components:

1) All prosumers in the residential prosumer community are the players of the non-cooperative game;
2) Prosumer $n \in \mathcal{N}$ in the game always minimizes its game objective π_n to make its decision X_n within the set of strategies $\mathcal{X}_n$, which is formulated as follows:

$$\min_{X_n \in \mathcal{X}_n} \pi_n(X_n, X_{-n}; \omega).$$

Define the variational inequality [31], [32] as the game of seeking $x^* \in \mathcal{X}$ such that

$$(X - X^*)^\top F(X^*; \omega) \geq 0, \omega \in \Omega \text{ a.s.}, \tag{10}$$

where a.s. means *almost surely* under given probability measure Ω, $F(X; \omega) = (-\nabla \pi_n(X_n, X_{-n}; \omega)), n \in \mathcal{N}$, and $\mathcal{X} := \prod_{n \in \mathcal{N}} \mathcal{X}_n$.

B. SWAA Nash Equilibrium

Generally, it is difficult to ensure that there exits a solution X^* satisfying (10) with the existence of the uncertain variable ω. However, the Monte Carlo simulation based discretization can be applied to generate an independent and identically discrete sample $\omega_k, k = 1, 2, \ldots, K$, to approximates the expectation with sample average [33] rather than exploring the expectation of prosumers' stochastic game models.

Sample average approximation (SAA) as the average estimation of stochastic model is usually acceptable without any posterior knowledge. SAA can be improved once we have some posterior knowledge of the stochastic variable ω. Let $\bar{\omega}$ denote the scenario to be estimated, and $\omega_k, k = 1, 2, \ldots, K$, the historical scenarios. Supposing we have the observation for $\bar{\omega}$, we can define the posterior probability distribution according to the Bayesian inference [34]:

$$\mathrm{P}(\omega_k|\bar{\omega}) = \frac{\mathrm{P}(\bar{\omega}|\omega_k)\mathrm{P}(\omega_k)}{\mathrm{P}(\bar{\omega})}, k = 1, 2, \ldots, K, \tag{11}$$

where $\mathrm{P}(\omega_k|\bar{\omega})$ is the probability of ω_k with the observation of $\bar{\omega}$. $\mathrm{P}(\bar{\omega}|\omega_k)$ is the likehood for the prior scenario ω_k, $\mathrm{P}(\omega_\mathrm{k})$ the prior probability of scenario ω_k, and $\mathrm{P}(\bar{\omega})$ the evidence of observation $\bar{\omega}$. Based on the posterior estimation $\bar{\omega}$, we can estimate $\pi_n(X_n, X_{-n}; \bar{\omega})$ with the posterior probabilities $\mathrm{P}(\omega_\mathrm{k}|\bar{\omega}), \mathrm{k} = 1, 2, \ldots, \mathrm{K}$, as follows:

$$\pi_n^{\mathrm{SWAA}}(X_n, X_{-n}) = \sum_{k \in \mathcal{K}} \mathrm{P}(\omega_k|\bar{\omega})\pi_n(X_n, X_{-n}; \omega_k). \tag{12}$$

Note that (12) is an improvement of SAA by introducing $\mathrm{P}(\omega_k|\bar{\omega})$ as the weight of different scenario ω_k. This method is called SWAA in this paper. In order to guarantee the convergence and consistency of SWAA theoretically, we present the following theorem to illustrate the performance of SWAA with the proof provided in appendix A. For convenience, let $\pi^{\mathrm{SWAA}}(X)$ denote $\pi_n^{\mathrm{SWAA}}(X_n, X_{-n})$, and $\pi(X, \bar{\omega})$ denote $\pi_n(X_n, X_{-n}; \bar{\omega})$.

Theorem 1: The SWAA for stochastic optimization has the following properties:

i) $\pi^{\mathrm{SWAA}}(X) \to \pi(X, \bar{\omega})$ w.p. 1, as $K \to \infty$ at an exponential rate of convergence;
ii) let $\pi(X, \omega)$ be a Carathéodory function (i.e., continuous in X and measurable in ω), then the functions $\pi(X, \omega)$ and $\pi^{\mathrm{SWAA}}(X)$ are lower semicontinuous, and for any $\bar{X} \in \mathcal{X}$, $\pi^{\mathrm{SWAA}}(\bar{X}) \xrightarrow{e} \pi(\bar{X}, \bar{\omega})$ w.p. 1.

However, such improvement is still impractical as it requires knowledge of posterior weights distribution which cannot be obtained directly before the scenario $\bar{\omega}$ happens. Generally, the renewable energy generation and energy demand have a strong correlation with the weather conditions. Therefore, we can approximate the posterior probability in (11) by the likehood $\tau_k, k = 1, 2, .., K$, of predicted weather conditions of $\bar{\omega}$ to those of historical scenarios $\omega_k, k = 1, 2, .., K$. With this consideration, we propose a SWAA equilibrium as follows:

$$\begin{aligned} &\pi_n^K(X_n^{*,K}, X_{-n}^{*,K}) = \\ &\min_{X_n \in \mathcal{X}_n} \sum_{k \in \mathcal{K}} \tau_k \pi_n(X_n, X_{-n}^{*,K}; \omega_k), \forall n \in \mathcal{N}, \end{aligned} \tag{13}$$

where $\tau_k, k = 1, 2, \ldots, K$, are the weight parameters or occurrence probabilities of the scenarios $\omega_k, k = 1, 2, \ldots, K$. Besides, we have $\sum_{k \in \mathcal{K}} \tau_k = 1, 0 \leq \tau_k \leq 1, \forall k \in \mathcal{K}$. In this paper, we select the weight $\{\tau_1, \tau_2, \ldots, \tau_K\}$ through least square deviation according to the similarity of weather condition, and the detail will be shown in the simulation section. Therefore, the corresponding game models for all prosumers ($\forall n \in \mathcal{N}$) can be formulated as:

$$\begin{aligned} &\pi_n^K(X_n, X_{-n}) = \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} \tau_k \times \\ &\left\{ \psi_n(x_n^t) + \rho_n \mathrm{CVaR}_\theta \left(\sum_{m \in \mathcal{N}/\{n\}} r_m^t + r_n^t, S^{\omega_k, t}, G^{\omega_k, t} \right) \right\}. \end{aligned} \tag{14}$$

As mentioned before, the cost function $\psi_n, \forall n$, is convex with respect to its decision variables X_n. Similarly, the CVaR measurement $\mathrm{CVaR}_\theta(\sum_{n \in \mathcal{N}} r_n^t, S^{\omega_k, t}, G^{\omega_k, t})$ is also convex in $\mathcal{X}_n, \forall n$. Hence, the model (14) and its feasible solution sets are convex with respect to its decision variables, which guarantees the existence of SWAA equilibrium of the stochastic game.

C. Relaxation Method to Seek the Equilibrium

We introduce the relaxation method [32], [35] to seek the SWAA equilibrium through the regularized Nikaido-Isoda function $\Phi_\gamma(X, Y) : ((\prod_{n \in \mathcal{N}} \mathcal{X}_n) \times (\prod_{n \in \mathcal{N}} \mathcal{X}_n)) \to \mathbb{R}$ shown as follows:

$$\begin{aligned} &\Phi_\gamma(X, Y) = \\ &\sum_{n \in \mathcal{N}} \left[\pi_n^K(X_n, X_{-n}) - \pi_n^K(Y_n, X_{-n}) \right] - \frac{\gamma}{2} \|X - Y\|^2, \end{aligned} \tag{15}$$

where $Y = [Y_1, Y_2, \ldots, Y_N] \in \prod_{n \in \mathcal{N}} \mathcal{X}_n$ and γ is a given parameter. The regularized Nikaido-Isoda function is a measure to

Algorithm 1: Relaxation Method to Seek SWAA Equilibrium.

1: **for** each $n \in \mathcal{N}$ **do**
2: Prosumer n initializes its strategy $X_n(1) \in \mathcal{X}_n$;
3: **end for**
4: Initialize iteration time $i = 1$;
5: The maximum and minimum market energy prices in a day are given by pr^{max}, pr^{min};
6: Initialize α^{DA} and β^{DA} by the linear interpolation between pr^{max} and pr^{min} with respect to the maximum and minimum net demands of the initial strategy;
7: Initialize parameters: $\theta, \rho_1, \rho_2, \ldots, \rho_N, \gamma, \lambda, \epsilon$;
8: **repeat**
9: **for** each prosumer $n \in \mathcal{N}$ **do**
10: Prosumer n makes its best response $Y_n(i)$ by:
$$Y_n(i) = \arg\min_{Y_n \in \mathcal{X}_n} \left[\pi_n^K(Y_n, X_{-n}(i)) + \frac{\gamma}{2}(Y_n - X_n(i))^2\right]$$
11: **end for**
12: % Relaxation method to update strategies;
13: **for** each prosumer $n \in \mathcal{N}$ **do**
14: Prosumer n updates its strategy as follows:
$$X_n(i+1) = (1 - \lambda(i))X_n(i) + \lambda(i)Y_n(i);$$
15: **end for**
16: Update: $i = i + 1, \lambda(i) = \frac{1}{i+1}$;
17: **until** $\|Y(i) - X(i)\| \leq \epsilon$

indicate how much a player gains if the player changes its strategy to a new vector y_n while all other players continue to hold their strategies X_{-n}. Let Y^γ denote the maximizer of function (15), i.e. $Y^\gamma(X) = \arg\max_{Y \in \prod_{n\in\mathcal{N}} \mathcal{X}_n} \Phi_\gamma(X, Y)$, which also means:

$$Y^\gamma(X) = \arg\min_{Y \in \prod\limits_{n\in\mathcal{N}} \mathcal{X}_n} \sum_{n\in\mathcal{N}} \left[\pi_n^K(Y_n, X_{-n})\right] + \frac{\gamma}{2}\|X - Y\|^2. \tag{16}$$

Let $V_\gamma(X) := \Phi_\gamma(X, Y^\gamma(X))$. A point $X^* \in \prod_{n\in N} X_n$ is defined as the normalized equilibrium point that is also the SWAA equilibrium point of the proposed stochastic game Γ if $V_\gamma(X^*) = 0$ holds [35], [36]. By seeking the best responses $Y^\gamma(X)$ continually, the equilibrium X^* will be obtained [35], [36]. The relaxation method to seek the SWAA equilibrium is shown in Algorithm 1, where $\lambda(i), i = 1, 2, ...,$ are the relaxation parameters. The following theorem with the proof provided in appendix B can guarantee the convergence of the algorithm:

Theorem 2: There exists a unique normalized equilibrium point to which Algorithm 1 converges if:

1) $\prod_{n\in\mathcal{N}} \mathcal{X}_n$ is a convex subset of $\mathbb{R}$.
2) the Nikaido-Isoda function $\Phi(X, Y) : ((\prod_{n\in\mathcal{N}} \mathcal{X}_n) \times (\prod_{n\in\mathcal{N}} \mathcal{X}_n)) \to \mathbb{R}$ is convex and $\Phi(X, X) = 0$.
3) the relaxation parameters $\lambda(i)$ satisfy: $\lambda(i) > 0, \lambda(i) \to 0$ as $i \to \infty$, and $\sum_{i=1}^{\infty} \lambda(i) = \infty$.

According to the theorem, we simply set $\lambda(i) = \frac{1}{i+1}$ as the iteration proceeds. In each iteration, each player only needs to make its best response and update its strategy according to other players' strategies since (16) can be decomposed into subproblems for individual prosumers as shown in Step 10 in Algorithm 1. Thus, without a calculation center to gather all the information, the presented algorithm can be implemented in a distributed way where the privacy security of prosumers can be guaranteed. Moreover, the subproblems for prosumers are convex optimization problems with linear constraints which can be solved by the existing convex optimization algorithms. These advantages make Algorithm 1 practical for implementation.

Note that the energy sharing game model is peer-to-peer and totally distributed, and thus can be facilitated through the distributed secure blockchain technology. Specifically, each prosumer is assigned with an index and utilizes its unique ID to perform a node on the blockchain. During the iteration process of the proposed algorithm, a node will fulfill the following tasks in each iteration on the blockchain: 1) calculate its own new energy sharing profile based on the total energy sharing profiles recorded in the last iteration; 2) broadcast its new profile without other private information marked with its ID to other nodes on the blockchain; 3) receive information from other nodes with their ID marks and update the records of all energy sharing profiles in the iteration; 4) check the stopping criteria for equilibrium. Once the stopping criteria is satisfied, the iteration process will stop, and the nodes will record the equilibrium containing the final energy sharing profiles and prices in the local market. The equilibrium serves as the smart contracts to be executed and cleared on the blockchain.

V. Simulation Cases

In this section, we carry out case studies to show the advances of the proposed energy sharing model and to verify the efficiency of the algorithm for seeking the stochastic game equilibrium.

A. Data Preparation

Before the simulation, we prepare essential data such as renewable energy generation, the community service demand, the prosumers' information about EV, HESS, HPV and household demands. At first, a period is set to be from 7:00 am in one day to 7:00 am of the next day, since this period can represent an energy cycle especially considering EV charging/discharging process. We obtain the meteorological data of Melbourne city from December 1, 2016 to February 31, 2017 (the summer season) including temperature, humidity, outdoor air pressure and wind speed through the Australia Bureau of Meteorology [37]. The solar power outputs of PV systems are calculated through the following equation [38]:

$$P_s = \eta SI(1 - 0.005(H + 25))[kWm^{-2}], \tag{17}$$

where η is the conversion efficiency of PV array $(\%)$ setting 15.7%, S is the PV array area (m^2), I is the solar irradiance (kWm^{-2}), and H is the outdoor temperature (°C). In general, $1kW$ capacity of PV system needs about $10\,\mathrm{m}^2$ of PV array. Thus, we can calculate PV outputs with capacity and meteorological information through (17). The historical energy demands of household prosumers are obtained from their residential smart meters in Australia. The configurations for both EV and HESS [21], [39] (such as Tesla, Nissan, Toyota) are

TABLE I
HESS AND EV CONFIGURATIONS

Type	HESS	EV
Capacity (kWh)	13.5, 10, 6.4	90, 24, 53.6
Min Capacity (kWh)	0.68, 0.5, 0.32	9, 2.4, 5
Max Capacity (kWh)	12.8, 9.5, 6.08	90, 24, 53.6
Max Charging (kW)	5, 3, 2.5	9, 3.2 , 5.95
Max Discharging (kW)	5, 3, 2.5	*

shown in Table. I. According to [40]–[42], an EV usually consumes $0.13-0.20$ kWh/km and the daily trip length of most EVs ranges from 10 km to 20 km. Each EV's daily energy consumption is set by randomly choosing its energy consumption profile per kilometer and daily trip length. Normally, EVs return to their homes around 17:00 to 20:00. The arrival time (also plug-in time) of each EV is randomly selected from 17:00 to 20:00, and EV can be charged from its arrival time to its departure time 7:00.

Next we decide the weight $\{\tau_1, \tau_2, \ldots, \tau_K\}$ with least square deviation. Let $[w_1^{tem}, w_2^{tem}, \ldots, w_K^{tem}]$ be normalized hourly temperature vector of the K sample scenarios, and similarly we have $[w_1^{hum}, w_2^{hum}, \ldots, w_K^{hum}]$ being normalized hourly humidity vector, $[w_1^{pre}, w_2^{pre}, \ldots, w_K^{pre}]$ normalized hourly air pressure vector and $[w_1^{win}, w_2^{win}, \ldots, w_K^{win}]$ normalized hourly wind speed vector. Let $W_k = [w_k^{tem}, w_k^{hum}, w_k^{pre}, w_k^{win}], k = 1, 2, \ldots, K$, denote the corresponding weather condition of the sample scenarios. The predicted weather condition of the scheduled day provided by online meteorological services is denoted by $\bar{W}$ after normalization. Then $\tau_k, k = 1, 2, \ldots, K$, can be obtained through solving the following least square deviation problem:

$$\begin{aligned} &\min_{\tau_1,\tau_2,\ldots\tau_K} \sum_{k\in K} \|\tau_k W_k - \bar{W}\|^2 \\ &\text{s.t.} \sum_{k\in K} \tau_k = 1,\ \tau_1, \tau_2, \ldots\tau_K \geq 0. \end{aligned} \tag{18}$$

Other parameters are set as follows. The confidence level for CVaR is selected as 0.95. The regularization parameter γ is selected as 0.1. $\rho = 0.10$ is the initial setting. The market prices are $pr^{max} = 0.30$ and $pr^{min} = 0.05$. All the simulations are implemented in Matlab 2015b environment running on Intel-i5 personal computer with 8 GB RAM.

B. Numerical Results

1) Basic Result: To show the basic result, we assume a medium-scale community with $N = 50$ prosumers and CPV with the capacity of 50 kW. Prosumers are not equipped with HPVs. We select $K = 30$ scenarios whose detailed data such as meteorological information and household demands are chosen from the data set mentioned in subsection IV-A.

After executing Algorithm 1 with the settings, we obtain the numerical results shown in Fig. 3. The solid line in Fig. 3 represents the day-ahead energy demand of all prosumers without the support of CPV and HESS, and the dotted line represents the energy demand with CPV. In the period of 7:00 am–17:00 pm with renewable energy generation, the net demands of the community indeed can be reduced significantly, but the net demands also shows large fluctuations over the day.

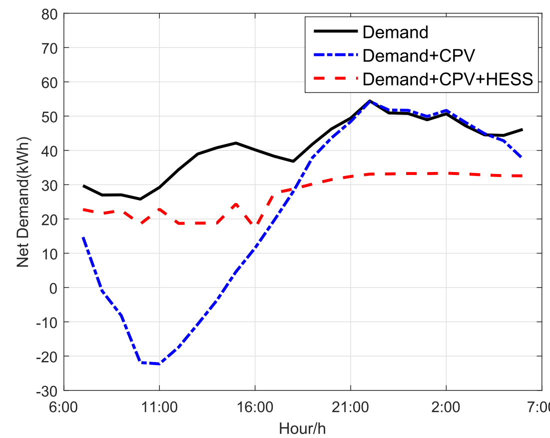


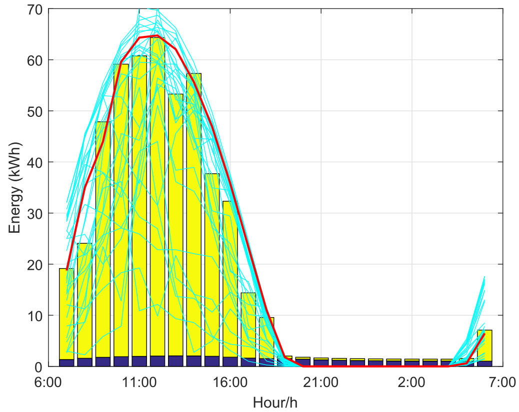
Fig. 3. Net energy demands of the prosumer community: net demand, net demand with CPV, and net demand with both CPV and HESS.

Moreover, the demand peak still exists during 21:00 pm-2:00 am, which shows that CPV is not capable of reducing the peak in periods of low or no renewable energy output. The dash line shows a smooth net energy demand with both CPV and HESS, which verifies the advantages of the proposed energy sharing model compared to the former two cases. The satisfactory result is mainly due to the spatial and temporal energy transferring abilities of HESS of prosumers. Basically, HESS stores locally surplus renewable energy for later utilization. It shows that the proposed model promotes an autonomous energy sharing among CPV and prosumers with their interest incentive.

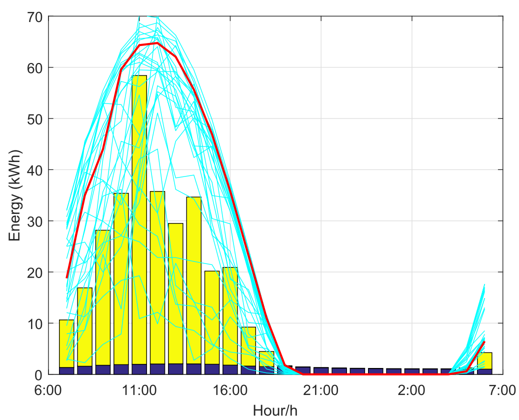
2) SWAA and SAA: We compare the results of SWAA equilibrium and SAA equilibrium in Fig. 4. Theoretically, with the estimated weight distribution, prosumers will definitely have a more accurate estimation for CPV energy generation compared to average estimation. As Fig. 4 shows, the energy expectation for CPV in SWAA denoted by the bars is closer to the actual output of CPV than that of SAA, which is in line with the analysis. In SWAA equilibrium, prosumers can make better utilization of the local random renewable energy compared to SAA equilibrium. Thus, SWAA equilibrium brings a higher energy efficiency for the community. SWAA shows a better performance than SAA approach by making the most of CPV, which indicates the advantage of the proposed SWAA over SAA.

3) Influence of CVaR weight: We carry out numerous simulations to analyse the influence of the CVaR weight ρ. In system modeling, we assume each prosumer has it own weight ρ_n on CVaR to make a tradeoff between the the energy cost and over-expecting risk of CPV. For convenience, we use the same weight ρ for all prosumers. Let $\rho = 0.05, 0.10, 0.15, 0.20$. By keeping all other parameters fixed, we can obtain the energy distributions of CPV for the four cases as shown in Fig. 5.

The solid lines are the actual energy generation profiles of CPV, and other four lines are the total energy expectation plus the community service demand profiles under different weight scales. Theoretically, if the weight ρ is small, it will lead to an overestimation of power generation of CPV, as a result, prosumers will expect too much energy from CPV which may not be able to provide in real time. This will lead to a large real-time energy shortage for the community, which may bring about high



(a) SWAA



(b) SAA

Fig. 4. Energy distribution of CPV of SWAA and SAA. The yellow and blue bars denote the total expected share of all prosumers for CPV energy and actual community service demand, and the red line and cyan lines denote the actual outputs and sample outputs of CPV.

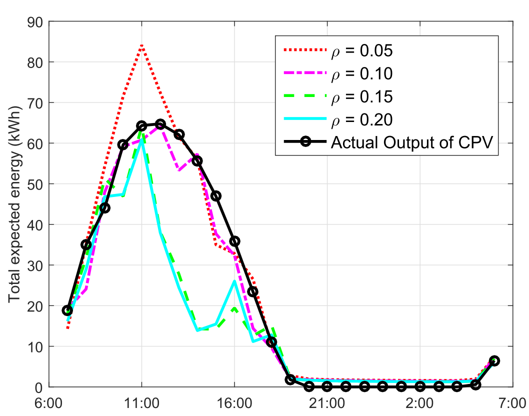


Fig. 5. Energy distribution of CPV in four cases of $\rho = 0.05, 0.100.15, 0.20$.

real-time energy expense. If the weight ρ becomes larger, prosumers will definitely behave more conservatively and tend to reduce the expected share of CPV as a tradeoff between the energy cost and over-expecting risk. Obviously, this is confirmed by the comparison shown in Fig. 5. Specifically, when $\rho = 0.05$, the expected energy from CPV is likely to exceed the actual energy generation of CPV, and when $\rho = 0.15, 0.20$ the renewable energy of CPV can not be fully utilized. $\rho = 0.10$ shows the best energy efficiency in the four cases. Different settings for the weight ρ have different energy efficiency and different conservatism on the usage of the uncertain resources. Therefore, by adjusting ρ, the proposed risk-averse energy sharing method for CPV can also regulate the conservatism on uncertain energy generation of CPV and the energy efficiency in the community. A suitable setting for ρ will both reduce the real-time demands and make full utilization of the renewable energy of CPV.

TABLE II
CALCULATION PERFORMANCE

Number	Iter	Tot.Time/s	Accuracy	Dis.Time/s	Ave.Time/s
10	14	652.1	0.0018	65.21	4.658
10	19	867.5	0.0005	86.75	4.566
50	14	2778	0.0119	55.56	3.969
50	19	3683	0.0068	73.66	3.890
100	14	7659	0.0327	76.59	5.471
100	19	9353	0.0076	93.53	4.923

C. Algorithm Evaluation

We evaluate the calculation performance of the proposed algorithm through simulation cases with different numbers of prosumers. We show the calculation detail for communities with 10, 50, 100 prosumers in Table II, where 'Number' means the number of prosumers in the community, 'Tot.Time' means the total calculation time in given iteration time denoted by 'Iter', 'Accuracy' denotes the norm of the difference of price vectors in the last two iterations. We also calculate the average running time denoted as 'Dis.Time' of single prosumer to be the quotient of 'Tot.Time' and 'Number'. Moreover, we estimate the average time denoted as 'Ave.Time' of solving the subproblem in Step 10 of Algorithm 1 through dividing 'Tot.Time' with 'Number' and iteration times.

Several conclusions can be obtained from Table II. Firstly, the algorithm is definitely applicable for the community with different scales since the algorithm requires few iteration times to reach a high accuracy and the number of prosumers has a slight influence. In practice, we just need a small number of iterations to reach a relatively high and acceptable accuracy. Secondly, the algorithm can be satisfactory if it is implemented in a distributed manner. 'Dis.Time', the average running time for single prosumer, is proportional to iteration times, and 'Dis.Time' barely changes as the number of prosumers increases from 10 to 100 in given Iter times. Thus the running time of distributed implementation with an ideal communication environment will be acceptable. Thirdly, the subproblems are easily solvable which is reflected by their average calculation time as 'Ave.Time'. In general, we can see that the proposed distributed algorithm is practically feasible and has a good computational performance.

VI. CONCLUSION

This paper presents a risk aversion energy sharing model based on a devised local energy market for a prosumer community with CPV installations. To be more specific, a stochastic energy sharing game is modeled where each prosumer tends to minimize its own objective where there is a tradeoff between the energy cost and the risk of energy sharing of uncertain CPV and the household loads and HPV outputs are considered as stochastic parameters. The risk is modeled by CVaR. A sample weighted average approximation (SWAA) equilibrium is proposed for better estimation of the stochastic game, and is achieved by a relaxation method based algorithm. The theoretical analysis is provided. To facilitate the energy sharing model, the blockchain technology as a distributed and secure way is introduced. The case studies show the efficiency of the proposed community energy sharing model and the algorithm. In future work, we intend to introduce other novel approaches to handle stochastic parameters such as autoregressive models. We will also consider the practical engineering realization of the blockchain-based energy sharing for a prosumer community.

Appendix A
Proof of Theorem 1

A. Proof for the Convergence

Suppose $\bar{\omega}$ is a random variable with the induced measure of μ, and ν the induced measure for the random variable ω. Let $\pi(X)$ denote the expectation of $\pi(X, \bar{\omega})$. Similar with the definition in [43], let the likelihood $L(\cdot) := \frac{d\mu}{d\nu}(\cdot)$. We define the following estimation:

$$\bar{\pi}(X) := \frac{1}{K}\sum_{k=1}^{K}\pi(X,\omega_k)L(\omega_k)$$

Thus we have $E_\omega\left[\bar{\pi}(X)\right] = \frac{1}{K}\sum_{k=1}^{K}E_\omega\left[\pi(X,\omega_k)L(\omega_k)\right] = \frac{1}{K}\sum_{k=1}^{K}E_{\bar{\omega}}\left[\pi(X,\bar{\omega})\right] = \pi(X)$. [23] proves that the variance of the estimation $\bar{\pi}(X)$ has smaller variance than SAA. We can apply the results to the proposed SWAA with some adjustments. The specific scenario $\bar{\omega}$ is deterministic, thus μ can be 0. Accordingly, the likelihood function can be used to approximate the posterior probability distribution. When $K \to \infty$, we have $\pi^{\mathrm{SWAA}}(X) := \sum_{k=1}^{K}P(\omega_k)\pi(X,\omega_k) \to \pi(X,\bar{\omega})$ since the posterior probability distribution will have a more accurate approximation for $L(\cdot)$ according to the Strong Law of Large Numbers. The exponential convergence rate with respect to given samples of number K can be obtained from the Theorem 5.16 in [23] of SAA convergence rate by considering $f(X,\omega_k) := P(\omega_k)\pi(X,\omega_k)$ as random samples.

B. Proof for the Consistency

The consistency means the optimal value and the solution set of the approximate problem converge to the corresponding deterministic problem. Let $f(X,\omega_k) := P(\omega_k)\pi(X,\omega_k)$ and $g(X,\omega_k) := \pi(X,\omega_k)$. The properties of lower semicontinuity of $g(X,\omega)$, $f(X,\omega)$ and $\pi(X,\bar{\omega})$ can be obtained as in [43]. The lower semicontinuity of $\pi^{\mathrm{SWAA}}(X)$ has been investigated following the Fatou's Lemma, since for every $\bar{X}$ we have $\lim_{X\to\bar{X}}\inf\pi^{\mathrm{SWAA}}(X) = \lim_{X\to\bar{X}}\inf\sum_{k=1}^{K}f(X,\omega_k) \geq \sum_{k=1}^{K}\lim_{X\to\bar{X}}\inf f(X,\omega_k) \geq \sum_{k=1}^{K}f(\bar{X},\omega_k) = g(\bar{X},\omega_k)$. Then the epiconvergence $\pi^{\mathrm{SWAA}}(\bar{X}) \xrightarrow{e} \pi(\bar{X},\bar{\omega})$ is directly obtained from the Theorem 2.3 of [44].

Appendix B
Proof of Theorem 2

The existence of a normalized equilibrium point X^* follows from the Kakutani's Theorem [45]. The convergence of the algorithm can be proved using the optimal value function $V_\gamma(X) := \Phi_\gamma(X, Y^\gamma(X))$. The function $V_\gamma(X)$ equals zero at the normalized equilibrium point X^*: $V_\gamma(X^*) = 0$ and $V_\gamma(X^*) > 0, X \in \mathcal{X}, X \neq X^*$. Suppose the statement does not holds, according to Lemma 4.1 of [46], there exists a subsequence i_m and $\delta > 0$ such that

$$V_\gamma(X(i_{m+1})) > V_\gamma(X(i_m)),$$

and

$$\|X(i_m) - Y^\gamma(X(i_m))\| \geq \delta > 0.$$

While the following inequality has been investigated [46]:

$$V_\gamma(X(i_{m+1})) \leq V_\gamma(X(i_m)) - \frac{1}{2}(1-\lambda(i_m))\lambda(i_m)\beta(\delta)$$

for sufficiently large numbers m. Thus the contradiction proves Theorem 2.

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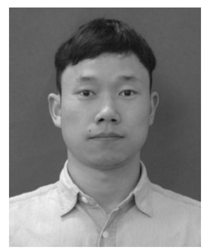

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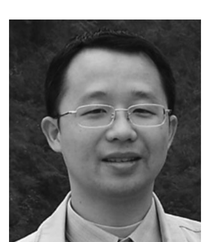

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