

Security Analysis of Paxos Mechanism Design Based on Game Theory

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Abstract—At present, a large number of token-less blockchain system use Paxos algorithm to reach a consensus. Based on game theory, this paper finds that when Paxos algorithm runs, each node in system may produce a kind of selfish behavior. The selfish behavior will lead to a game, which is called the computing resource dilemma game in this paper. By analyzing the payoff matrix and establishing the utility function model, it is found that Pareto optimal Nash equilibrium in the game needs to meet certain security conditions. Otherwise, each node will choose an uncooperative strategy, which will eventually lead to the system stop working. By solving the perfect Nash equilibrium solution of the subgame in the game, the security conditions for achieving the purpose(each node adheres to the cooperative strategy) of Paxos mechanism design are obtained. And the effectiveness of the security conditions is proved by simulation experiments.

Keywords—game theory; Paxos; Nash equilibrium; security; mechanism design

I. INTRODUCTION

Consensus mechanism is the key technology of blockchain, consensus is the set of block generation, validation and reorder rules. If there are rules, there will be games. Mechanism design is an engineering problem of game theory. Players want to achieve a social goal[1], and each of the players will have their own behaviors. In this case, mechanism design is needed to urge everyone to restrict their behaviors in order to achieve social goals. Here mechanism design just is an algorithm, which makes every player willing to provide their own real information and take the design behavior to achieve social goals. As a classical consensus

algorithm, Paxos also has mechanism design problems. Next, based on the game theory, the mechanism design of Paxos[2] is analyzed to verify whether it is reasonable and secure[3].

Game theory is to study the strategies and the equilibrium of strategies when the behaviors of decision-makers interact directly. In essence, game theory studies strategy, it is not only a method to study human behavior, but also an effective tool to study collaborative problems[4]. And game theory originated from the field of economics and behavior science, it is not the main research category in the field of distributed system(blockchain is a kind of distributed system), but because the behavior of distributed system nodes can be the result of decision-making of system nodes for a certain motivation, which is similar to human behavior, so game theory is also suitable for studying the behavior of distributed system nodes.

II. BASIC CONCEPTION OF GAME THEORY

A. The classification of game

Game can be divided into different types according to the number of games, game process, game structure, the number of players, the way of profit and other dimensions. This section only focuses on the types of game involved in this paper.

Definition 1. Subgame[5] is a part of the original game, which can be used as an independent game analysis. It is composed of a subsequent game stage starting from a stage other than the first stage of the dynamic game. It has the exact initial information set and all the information needed for the game.

Definition 2. Dynamic game of complete information[6] means that the information in the game is complete, and all players have a complete understanding of the characteristics, the strategic space and the utility function under the strategy combination of other players. The actions are in order, so the latter can observe the actions of the former, understand all the information of the former, and generally last for a long time.

Definition 3. Infinite repeated game[7] in mathematics can be regarded as a long-term game relationship between players in reality. Here, long-term is not narrowly understood as the length of time or the number of stages, but refers to the game scenario where players cannot predict when the game will end.

B. Solutions to game

The solution of the game problem, which is the most likely result of the game, is called equilibrium. The solution of game problem is defined as the result predicted by all players, i.e. the consistency prediction of players. It should be noted that this kind of consistent prediction is not only all players can predict the certain result, but also all players can predict that all players predicted the a certain result. So this kind of consistent prediction is common knowledge. At present, people have a relatively consistent understanding—take the research findings of John Nash as the solution of general game problems. For the dynamic game of complete information with subgame, Reinhard Selten's research shows that the subgame perfect Nash equilibrium[9] is the most reasonable solution among all the Nash equilibrium solutions of the dynamic game of complete information.

Nash equilibrium[8] refers to such a strategy combination, which is composed of the optimal strategies of all players. Under the premise that other players do not change the current strategy, no one can obtain higher benefits by unilaterally changing their own strategy.

Definition 4. Nash equilibrium[8] is formally defined as : For game $G=\{N,\{S_i\},\{U_i\}\}$, if a certain strategy combination is composed of any strategies of each player (s_1^*, \dots, s_n^*) , the strategy s_i^* of any player i is the best strategy for the strategy combination $s_{-i}^* = (s_1^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_n^*)$ of

other players. If $U_i(s_i^*, s_{-i}^*) \geq U_i(s_i, s_{-i}^*)$ holds for any $s_i \in S_i$, it is called (s_1^*, \dots, s_n^*) is a Nash equilibrium solution of G .

Perfect Nash equilibrium is to eliminate the unbelievable threat strategies contained in Nash equilibrium. It requires that the decision-making of the players is optimal at any time point, and the players should adapt to the circumstances and consider future gains and losses, rather than stick to the old strategy.

Definition 5. Subgame perfect Nash equilibrium[9] is defined as: For the strategy combination $s^*=(s_1^*, \dots, s_i^*, \dots, s_n^*)$ of the extended game, if it is the Nash equilibrium of the original game and it also constitutes the Nash equilibrium in each subgame, then it is a subgame perfect Nash equilibrium.

If players' preferences are complete, reflexive, transitive, continuous and strongly monotonic, then there is a continuous utility function that can represent the preferences.

Definition 6. Utility function[10] is a function used to express the quantitative relationship between the utility obtained and the combination of goods consumed. It is used to measure the degree of satisfaction that players get from consuming a given combination of goods.

Definition 7. Pareto optimality[11] refers to an ideal state of resource allocation, which assumes that an inherent group of people and allocable resources, from one allocation state to another, make at least one person better without making anyone worse. Pareto optimal state is that there is no more room for Pareto improvement.

III. ANALYSIS OF NODE'S BEHAVIOR IN PAXOS

Paxos is a protocol for data consistency in unreliable networks[12]. Nodes in Paxos can be divided into three roles (proposer, acceptor and learner), and their execution process can be divided into three phases. The following is the analysis of each role's behavior in each phase[13] to show mechanism design of Paxos. From the analysis, we can see that proposers actively participate in the proposal is the key to keep Paxos run.

A. Role-phase analysis of Paxos

(1) Specific description of proposers behavior

Phase 1.a Send Prepare(n) requests to all acceptors (participation in proposals).

Phase 2.a If a Reject(\max_N) message is received, resend Prepare(new(n)), where different proposers have a different n , i. e. the numbers are partially ordered, guaranteeing that no two proposals are ever issued with the same number.

Phase 2.a If Pok(\max_N, n, v) messages from a majority of the acceptors have been received, then the proposer is free to choose a v (typically the client's request) and send Accept(n, v) if each V_a is null, or set $v = V_a$ (V_a corresponds to the largest N_a in Pok).

Phase 2.b If Nack(\max_N) is received, go back to Phase 1.a to send Prepare(new (n)).

Phase 3 If Aok(\max_N, N_a, V_a) messages from a majority of the acceptors have been received, broadcast the news of becoming a leader and send the accepted value(chosen proposal) to the learners.

(2) Specific description of acceptors behavior

Acceptors maintain a table of state records. Each row of the table represents a tuple (\max_N, N_a, V_a)

Phase 1.b Receive Prepare(n); if $n > \max_N$, then set $\max_N = n$ to reply to Pok(\max_N, N_a, V_a) or reply Reject(\max_N).

Phase 2.b Receive Accept(n, v); if $n < \max_N$, then reply Nack(\max_N), indicating that at least one other proposer broadcasts a higher numbered proposal after the proposal is proposed, or set $N_a = n$ and $V_a = v$ and reply Aok(\max_N, N_a, V_a).

(3) Specific description of learners behavior

Phase 3 Tasks for learners is to monitor broadcast in this phase, so as to eventually confirm and update the accepted value.

B. Paxos application scenario

(1) The protocol is implemented in a unreliable communication environment. During the asynchronous communication process[14], transmitted data may be lost, delayed and repeated, but not tampered with.

(2) Nodes do not suffer from Byzantine failures[15]. But each node can run at any speed, allowing stop and restart errors.

(3) Nodes are allowed to assume multiple role.

C. Selfishness analysis of nodes running Paxos

Each node not only ensures its normal communication, but also consumes its own resources to provide other nodes of the network with services such as proposal review and messages broadcast. However, not all nodes in the network have good cooperation as designers imagine. Considering their own interests, some nodes will produce selfish behavior. They want to get updated data for free, but are not willing to provide additional services for other nodes. Nodes themselves are limited by various resources[16] such as power, bandwidth, computing power, memory space, etc. If a node does not provide broadcast services to other nodes all the time, it will save a lot of computing resources and reduce cost for itself. From the above Paxos application scenario, Paxos allows nodes to play the roles of proposer, acceptor and learner, and Paxos allows nodes to stop working or a role function to stop working. Therefore, nodes may have the following selfish behaviors:

Not take the proposer function, avoid being selected as a leader, because the leader has to take the function of processing resolutions and data broadcasting.

IV. COMPUTING RESOURCE DILEMMA GAME

A distributed system running Paxos may have a network topology as shown in Fig. 1. P_1 , P_2 and P_3 are all with the roles of proposer, acceptor and learner (PAL nodes), they need to learn new values to maintain their own updates, but they are not willing to take up more resources to submit proposals, participate in the election and broadcast messages to learners. As long as one of the nodes is elected as Leader, the other two nodes can save a certain amount of resource cost and still get value update.

Definition 8 In Paxos, the nodes with the roles of proposer do not perform the function of proposer in order to reduce the cost, but hope that others can submit proposals and be selected as leader all the time. This kind of game is called computing resource dilemma game in this paper.

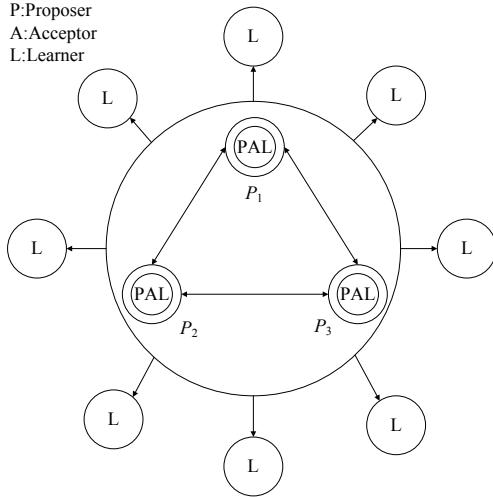


Fig. 1 Topology of computing resource dilemma

The game of computing resource dilemma game (hereinafter referred to as CRD game) in token-less blockchain is just like the famous prisoner's dilemma game in game theory[17], which is caused by the selfish behavior of nodes (Only enjoying the resources in the system, but not contributing their own resources to the system). It will destroy the fair sharing of nodes in the network. In extreme cases, no player is willing to propose and participate in the election, and no one can get any profit in the future, which will affect the security of the system[18].

V. FIND A SOLUTION OF CRD GAME

From **Definition 2** and **Definition 3**, we can see that CRD game belongs to dynamic game of complete information and infinite repeated game. In this section, we will analyze the classification attribute of the game, and try to solve CRD game by using payment matrix and establishing utility function model.

A. The payoff matrix for CRD game

There is a long-term benefit value and extra resource cost as leader in the computing resource dilemma game. Therefore, in the current round of the game, each player P_i should consider that it can't cause other players with negative attitude for proposals in the later round. One player P_i makes a cooperative attitude, which will make other players ($P_1, \dots, P_{i-1}, P_{i+1}, \dots, P_n$) take a cooperative attitude in the future round, so as to realize common long-term benefits. Here we assume that there are

proposals for each round of active players. When there is more than one node participating in the election, the probability of each node being elected is the same, so we can get a payoff matrix of CDR game problem.

First, consider the case of three PAL nodes. There are eight(2^3) possible strategy combinations in this game, which can be represented by a pseudo three-dimensional payoff matrix[19], as shown in Table 1.

TABLE I PAYOFF MATRIX OF CALCULATION RESOURCE DILEMMA GAME

		P_1				
		active	negative			
P_2	active	$v-c/3$	v	active	P_3	
	active	$v-c/3$	$v-c/2$			
	active	$v-c/3$	$v-c/2$			
	negative	$v-c/2$	v	negative		
	negative	$v-c/2$	v			
	negative	$v-c/2$	$v-c$			
P_3	negative	$v-c$	0	active		
	negative	v	0			
	negative	v	0			

In Table 1, v represents the benefit value after the election, c represents the additional resource consumption of broadcast messages after the leader is elected, and the probability[20] of being elected is 1/3, 1/2 or 1, which is represented by p .

In the real world, it can be understood that c is regarded as the cost of power consumption of a company, and v represents the indirect income obtained by the company through Paxos distributed system. Then v and c can be calculated and compared.

Obviously, if P_1 , P_2 and P_3 don't run for election, they will get the most. They can wait for other players to win the election and get value for free. Therefore, if each game is considered separately, all players will choose not to cooperate, i.e. not to participate in the election. The never cooperate strategy is a Nash equilibrium. But now that all

players are in a long-term(infinite repeated) game relationship. If players find that the strategy chosen in the previous stage is not (active, active, active), the resource consumption of the players in the round will increase (change to $c/2$ or c). If the resource consumption is too high for all players, there will be no leader and no value generated in the end, which is what everyone does not want to see. In order to avoid the situation of non-cooperation caused by vicious circle, it is necessary to analyze the relationship between c and v , i.e. the security conditions required for everyone to maintain active attitude towards cooperation[21].

B. Utility function modeling to solve CRD game

Now we abstract a Paxos distributed system into a game model, because token-less blockchain is a distributed system. According to the three elements of the game, the players of the game correspond to the nodes in the distributed system, the strategy set of the players in the game such as actively or negatively participate in leader election correspond to the relevant action set, and the utility function of the game corresponds to the benefit value and resource cost[22]. Table 2 shows the corresponding relationship between Paxos distributed system and game model.

TABLE II CORRESPONDING RELATIONSHIP BETWEEN PAXOS DISTRIBUTED SYSTEM AND GAME MODEL

The three elements	Paxos distributed system components
players	nodes
strategy set	active or negative
utility function	value, cost

CDR game can be defined as an array of three elements[23], including three elements of the game model, $G=\{N, \{S_i\}, \{U_i\}\}$, where:

$N:=\{1, \dots, N\}$ represents the set of all players. In the Paxos distributed system it represents the set of all PAL type nodes;

$S:=\{S_1, S_2, \dots, S_i, \dots, S_{|N|}\}$ represents the strategy space set of all nodes. In CRD game model, S_i is a boolean variable, with a value of 1 or 0, representing participating in the

election(active) and not participating in the election(negative) respectively;

$U:=\{U_1, U_2, \dots, U_i, \dots, U_{|N|}\}$ represents the set of utility functions of all nodes. Utility is benefit and cost, i.e. updata(value) and resource consumption. The difference between benefit function and cost function of node i is its utility function, can be expressed as follow:

$$U_i = v_i - c_i \quad (1)$$

According to Table 1, the benefit function of node i in round r can be expressed as follow:

$$v_{i(r)} = \begin{cases} v, & \exists S_{i(r)} = 1 \\ 0, & \forall S_{i(r)} = 0 \end{cases} \quad (2)$$

Meanwhile, the cost function of node i in round r can be expressed as:

$$c_{i(r)} = \begin{cases} \frac{c * S_{i(r)}}{\sum_{j=1}^N S_{j(r)}}, & \exists S_{i(r)} = 1 \\ 0, & \forall S_{i(r)} = 0 \end{cases} \quad (3)$$

Then the utility function of node i in the r round of CDR game is expressed as follow:

$$U_{i(r)}(S_{i(r)}) = \begin{cases} v - \frac{c * S_{i(r)}}{\sum_{j=1}^N S_{j(r)}}, & \exists S_{i(r)} = 1 \\ 0, & \forall S_{i(r)} = 0 \end{cases} \quad (4)$$

According to **Definition 4**, when all nodes in the system running Paxos adopt the strategy of negative ($S_{1(r)}=S_{2(r)}=\dots=S_{i(r)}=0$), the system will maintain this "never cooperate" Nash equilibrium state. At this time, the system stops working and $U_{i(r)}(S_{i(r)}) = 0$, which is obviously not the result that the designer wants. Therefore, to solve the above problems, we need to set security conditions to determine the relationship between v and c , to ensure the system can achieve the Pareto optimal Nash equilibrium.

From the utility function derived above, it can be seen that if the security condition is effective, for node i , $U_{i(r)}(S_{i(r)}) = 0$ is certainly not the maximum utility when everyone adopt the strategy of negative($\forall S_{i(r)} = 0$). As long as there are nodes participating in the election, the value of benefit function for each node i is a fixed v . Only when all nodes actively participate in the election to maximize the denominator $\sum_{j=1}^N S_{j(r)}$ of $c_{i(r)}$, and minimize the cost function. In this way, the utility of each node can be maximized.

VI. SECURITY CONDITIONS OF PERFECT NASH EQUILIBRIUM SOLUTION IN CRD GAME

According to the analysis of payment matrix and utility function, we know that the Nash equilibrium with optimal strategy in CRD game needs certain security conditions. Because CRD game meets **Definition 3**, this section will deduce this security condition by solving the perfect Nash equilibrium solution of the subgame of CRD game, and verify the validity of the security condition through simulation experiments.

A. Find the security conditions of CRD game

According to reference[23], if the discount rate[24] is δ , then the payoff π_i of players in $G(\infty, \delta)$ can be expressed as formula (5), where π_i^t is the payoff of player i in stage $t(t=1,2,\dots,\infty)$ of $G(\infty, \delta)$.

$$\pi_i(G(\infty, \delta)) = \pi_i^1 + \delta\pi_i^2 + \dots + \delta^{t-1}\pi_i^t + \dots, \delta \in (0,1) \quad (5)$$

First, analyze the three participating nodes P_1 , P_2 and P_3 in Fig. 1. According to section 5, the participating nodes adopt the trigger strategy[25] (s_1, s_2, s_3) . If one node adopts non-cooperation strategy, the other nodes immediately adopt non-cooperation strategy and will always adopt non-cooperation strategy thereafter. So strategy (s_1, s_2, s_3) are (active, active, active) in the first stage; in the $t(t > 1)$ stage, if the result of the previous stage is (active, active, active), the election will continue; otherwise, strategy of negative is the only choice in the future.

For the infinite repeated game $G(\infty, \delta)$ of CRD game, there are two kinds of subgame(Γ_1 and Γ_2) under the strategy combination (s_1, s_2, s_3) .

Definition 9. Γ_1 is a kind of subgame that the results of all previous stages are (active, active, active).

Definition 10. Γ_2 is a kind of subgame that the results of the previous stage are not all (active, active, active).

In order to prove that the strategy combination (s_1, s_2, s_3) constitutes the subgame perfect Nash equilibrium of $G(\infty, \delta)$ in infinite repeated games, it is necessary to prove that (s_1, s_2, s_3) constitutes the Nash equilibrium for both kinds of games. Obviously, (s_1, s_2, s_3) constitutes the Nash equilibrium in Γ_2 . In the meantime, if we can prove (s_1, s_2, s_3)

is the Nash equilibrium for the first game, we can also prove (s_1, s_2, s_3) also is the Nash equilibrium for Γ_2 .

In order to prove (s_1, s_2, s_3) is the Nash equilibrium for Γ_1 , we only need to prove that: Suppose that when player k chooses the above triggering strategy, the optimal strategy[26] of players i and j is also the above triggering strategy. Let P_1 choose negative in the $t(t \geq 1)$ stage of the game, then its payoff in this stage is v . Next, its opportunistic behavior[27] will make P_2 and P_3 deviate from the choice (active, active), and they will choose the punishment behavior[28] of negative forever. So the payoff of P_1 will be 0 in every stage from now on. Therefore, the discounted payoff(relative to stage t) from the above opportunistic behavior is: $v+0+0+\dots=v$.

If P_1 does not deviate from the trigger strategy in the game and always chooses active, the payoff in the t stage will be $v-c/3$, P_2 and P_3 will never deviate from the trigger strategy and always choose active. So P_1 can get exactly the same payoff $v-qc$ at every stage in the future. In this case, the payoff of P_1 (relative to stage t) is derived as follows:

$$(v - c/3) + (v - c/3)\delta + (v - c/3)\delta^2 + \dots = \frac{v - c/3}{1 - \delta} \quad (6)$$

Therefore, if formula (7) is met, i.e. the discount payoff of continuous cooperation is greater than or equal to the discount payoff of opportunism, given P_2 or P_3 choose active, P_1 will choose active.

$$\frac{v - c/3}{1 - \delta} \geq v \quad (7)$$

By solving the above inequality, we get $\delta \geq c/3v$. Therefore, when security condition $\delta \geq c/3v$ is met, CRD game can continue in the cooperative state, and the trigger strategy combination above constitutes the Nash equilibrium of Γ_1 . Further reasoning is available, in the normal case of n players(the probability of election p is $1/n$) when security condition $\delta \geq c/nv$ is met, CRD game can keep in the cooperative state, triggering strategy combination (s_1, s_2, \dots, s_n) constitutes the Nash equilibrium of the game. According to the principles of economics, the more the discount rate δ approaches to 1, the higher the attention of players to future payoff; otherwise, the more the discount rate δ approaches to 0, the more the attention of players to current payoff. When the actual situation is sure to meet formula (8), then $0 < \delta < 1$

can be deduced. According to economic principles, the discount rate δ tends to be closer to 1, and the players pay more attention to future payoff. Otherwise, the players pay more attention to current payoff[29]. And the larger the δ , the higher the security of the system, the next section of the simulation experiment will be verified.

$$v \geq c/n, v > 0 \text{ and } c > 0 \quad (8)$$

And the security condition show that the smaller n is, the larger δ is, i.e. the higher the players attach importance to the future, that is to say, the less players are, the higher the enthusiasm of players for cooperation is. Conversely, the more players, the more likely they are to hitchhike, which is also in line with people's common sense.

B. Experimental results and analysis

The experiment is to simulate the network topology shown in Figure 1 through multiple virtual machines on a single high-performance computer. The three cluster servers in the center all assume the three roles of proposer, acceptor and learner (PAL node) in Paxos, while the peripheral servers only assume the role of learner. Paxos protocol with security condition judgment and trigger strategy is implemented by C language.

The system runs 19 times in total, and the PAL3 node will be down at the 10th time. The trigger strategy is enabled. The number of messages transfer and the computing resource consumption of each PAL node are compared when security condition 1 and security condition 2 are met separately.

$$\text{Security condition 1: } c/2v \geq \delta \geq c/3v \quad (9)$$

$$\text{Security condition 2: } 1 \geq \delta \geq c/2v \quad (10)$$

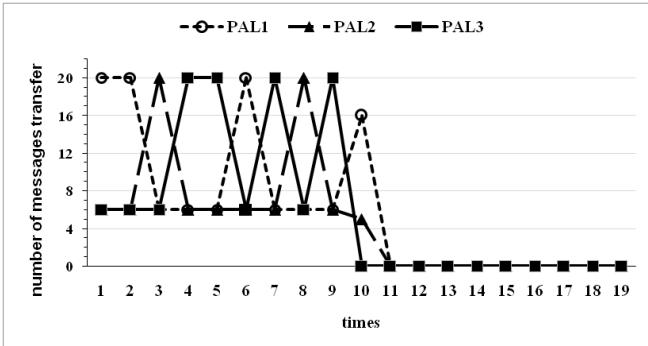


Fig. 2 The number of message transfer when only security condition 1 is met

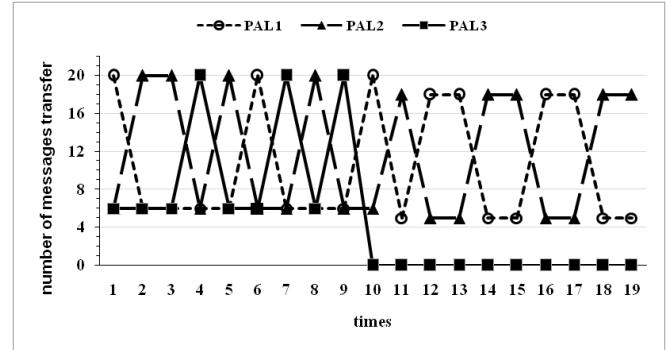


Fig. 3 The number of message transfer when security condition 2 is met

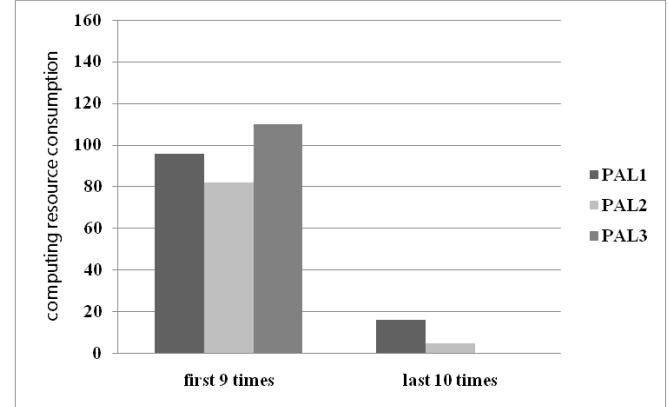


Fig. 4 The computing resource consumption when only security condition 1 is met

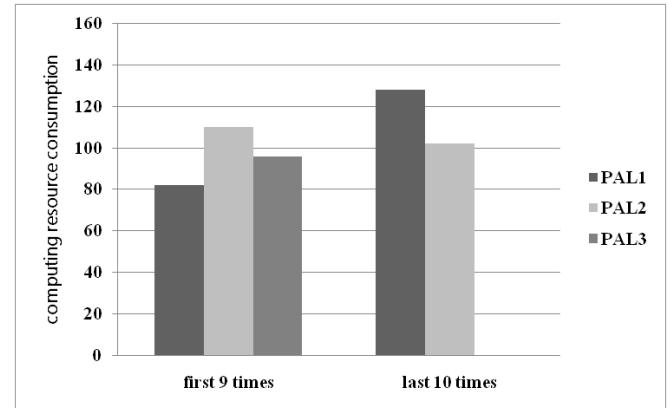


Fig. 5 The computing resource consumption when security condition 2 is met

Analysis and comparison of the experimental results in Fig. 2, Fig. 3, Fig. 4 and Fig. 5 can draw the following conclusions:

- (1) When only security condition 1 is met, the system cannot continue to work normally after the PAL3 goes down.
- (2) When the security condition 2 is met, in the event of a node downtime, each PAL node continues to work normally, while computing resource consumption is

basically balanced, there are no idle nodes and nodes with too much load, thus realizing the balanced distribution of network resources[30].

VII. CONCLUSIONS

This paper studies and analyzes the nodes of token-less blockchain running Paxos algorithm through game theory. The research results are helpful for the designers of the alliance chain using Paxos as the consensus mechanism to establish the system access conditions, and also help to weigh the actual benefits and consumption costs of the units who want to join the alliance chain. The next work will focus on the following two aspects: one is to study the security conditions and limitations of PAL nodes when they adopt other strategies (such as two-phase strategy); the other is to explore other selfish behaviors that may exist in the system running Paxos and the game caused by the selfish behavior.

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