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The Marketing of Cross-border E-commerce Enterprises in Foreign Trade Based on the Statistics of Mathematical Probability Theory

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Abstract

This article applies mathematical probability theory to statistical algorithms to conduct game analysis on the relevant marketing decisions of foreign trade cross-border e-commerce. We build a theoretical framework for the collaborative development of SMEs and cross-border e-commerce platforms. At the same time, build a trustworthy transaction margin mechanism for cross-border e-commerce transactions in a fuzzy market environment. The participation of the blockchain big data platform can promote the development of foreign trade cross-border e-commerce toward a more transparent and healthy partnership.

Keywords: Cross-border e-commerce; Probability theory statistics; Evolutionary game; Enterprise marketing
AMS 2010 codes: 91A80

1. Introduction

Before the New Deal, cross-border e-commerce bonded imports were implemented as a "negative list.". Commodities already in transit but not on the list have been stranded at the terminal and airport. These have brought huge losses to cross-border e-commerce companies [1]. If the business owner predicts that the relevant regulatory authorities will continue to improve, it will increase the stocking of goods not on the positive list. This leads to a monopoly of "people without me." Therefore, there will be a three-stage game from stocking to sales: In the first stage, there is a game of "stocking" and "supervision" between the company and the government. The second stage is that after the company has decided, there will be a network of strategies of multiple companies in the market. This article assumes that one company has adopted a strategy of stocking products other than the "positive list" to form a monopoly in the cross-border e-commerce market. Another company can only stock and sell products using traditional trading methods. This forms competition with its sales channels.

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The third stage is the pricing strategy of the sellers of the two channels in the market when facing rational consumers [2]. This strategy maximizes self-interest. The relevant suggestions put forward by this article through the three-stage game analysis between the government, enterprises, and consumers are more theoretical and practical.

2. The first stage: the evolutionary game between the enterprise and the government

Promoting the steady and healthy development of the cross-border e-commerce industry while meeting regulatory requirements and preventing and controlling risks is a crucial matter. This puts forward higher regulatory requirements for relevant regulatory agencies such as customs, taxation, and national inspection. The introduction of the "positive list" is a manifestation of increased supervision. From the relationship between relevant regulatory agencies and cross-border e-commerce companies, companies will always pursue excess profits and increase their supply types in the short term. In this way, the monopoly situation of "people without me" is realized. The government has stepped up supervision to prevent the risks of moving too fast. This reduces the profit margins of cross-border e-commerce companies [3]. When the government gradually enriches the positive list, cross-border e-commerce companies will start a new round of stocking decisions for products not on the positive list. The two sides promote each other in a dynamic game.

The subjects of the game are cross-border e-commerce companies and related regulatory agencies. The enterprise's decision strategy set is {stocking, not stocking}. The regulatory agency's decision-making strategy set is {included in the list, not included in the list}. Cross-border e-commerce companies normally stock up to obtain normal income I_{E1} and pay the cost C_{E1}. Relevant regulators who choose to be included in the list will obtain benefits I_{M1} and pay costs C_{M1}. When the company stocks goods out of the list and the regulator does not supervise it, the company obtains an excess income I_{E2}, pays a cost C_{E2}, and the regulator assumes the risk C_{M2}. In the case of not being included in the list by the supervisory authority, it will obtain income I_{M2}. At the same time, if the company adopts stocking, the regulatory agency will obtain additional penalty income I_{M3} and bear the cost C_{M3}. The combination of the game process is as follows:

(1) When a company chooses not to stock products not on the positive list, the regulator adopts the income function of the two parties when they are included in the list as: $R_{11}^{E}=I_{E1}-C_{M1}, R_{11}^{M}=I_{M1}-C_{M1}$.

(2) The company adopts the inventory strategy for the products outside the positive list, and the supervisory agency adopts the listing strategy. The income functions of the two parties at this time are: $R_{21}^{E}=I_{E1}+I_{E2}-C_{M1}-C_{E2}, R_{21}^{M}=I_{M1}-C_{M1}-C_{E2}$.

3) The enterprise adopts the non-stocking of the goods outside the positive list, and the supervisory agency adopts the non-listed list. The profit function of both parties is $R_{21}^{E}=I_{E1}-C_{E1}R_{21}^{M}=I_{M2}-C_{M1}$ respectively.

(4) The company adopts the strategy of not being included in the list when the company adopts the stocking of goods that are not on the positive list. The profit function of both parties is $R_{22}^{E}=I_{E1}-C_{E1}-C_{E2}-I_{M3}, R_{22}^{M}=I_{M2}+I_{M3}-C_{M1}-C_{M3}$ respectively.

In the game process, cross-border e-commerce companies adopt non-stocking and stocking strategies with a probability of 1-a, respectively [4]. The regulatory agency adopts the listing and non-listing strategies with the probability of b and 1-b. The income matrix of regulatory agencies and cross-border e-commerce companies is shown in Table 1.

Table 1 Revenue matrix of cross-border e-commerce companies and related regulatory agencies

		Regulatory Authority	
		Included in the list (b)	Not included in the list (1-b)
Cross-border e-commerce companies	Not stocking (a)	(IE1-CE1, IM1-CM1)	(IE1-CE1, IM2-CM1)
	Stocking (1-a)	(IE1+IE2-CE1-CE2, IM1-CM1-CM2)	(IE1-CE1-CE2-IM3, IM2+IM3-CM1-CM3)

The income of cross-border e-commerce companies adopting pure non-stocking and pure stocking strategies is $[E(R)]_1^E = R_{11}^E \times b + R_{12}^E \times (1-b), [E(R)]_2^E = R_{21}^E \times b + R_{22}^E \times (1-b)$ respectively. Among cross-border e-commerce companies, a proportion of a companies choose a non-stocking strategy, and a proportion of $1-a$ companies choose a stocking strategy. Then the average income is $E(R^E) = R_1^E \times a + R_5^E \times (1-b)$, and the income of the relevant regulatory agencies using the pure listing and pure non-listing strategies is $[E(R)]_1^G = R_{11}^G \times a + R_{12}^G \times (1-a), [E(R)]_2^G = R_{21}^G \times a + R_{22}^G \times (1-a)$ respectively [5]. If among the relevant regulatory agencies, an institution with a ratio of b adopts the listing strategy, and an institution with a ratio of $1-b$ adopts the non-listing strategy. Then the average return is $E(R^M) = R_2^M \times b + R_2^M \times (1-a)$.

Assuming each participant's bounded rationality and F(a)=0, the fixed point of the replicated dynamic equation can be solved. We get the dynamic replication equation of cross-border e-commerce companies $F(a) = da/dt = a(1-a)[b(R_{11}^E - R_{21}^E) + (1-b)(R_{12}^E - R_{22}^E)]$.

Assume that $F(a) = 0$ can solve the fixed point that replicates the dynamic equation. Available: $a_1^* = 0, a_2^* = 1, a_3^* = \dfrac{(R_{12}^E - R_{22}^E)}{(R_{12}^E + R_{21}^E - R_{11}^E - R_{22}^E)}$. In the same way, the fixed point of the dynamic replication equation of the relevant regulatory agency can be obtained: $b_1^* = 0, b_2^* = 1, b_3^* = \dfrac{(R_{12}^M - R_{22}^M)}{(R_{12}^M + R_{21}^M - R_{11}^M - R_{22}^M)}$. Simultaneous equations can get five equilibrium points of the dynamic system (Table 2).

Table 2 Distribution of equilibrium points of dynamic equations

Equilibrium point	Det(J)	Tr(J) symbol	Result
(0,0)	+	+	Unstable point
(1,0)	+	-	ESS
(0,1)	-	+	ESS
(1,1)	-	-	Unstable point
(a*,b*)	-	0	Saddle point

From Figure 1, we can see that the evolutionary stable equilibrium points in the dynamic evolutionary game between cross-border e-commerce companies and relevant regulatory agencies are $A(1,0)$ and $B(0,1)$ respectively [6]. There are four areas in the evolutionary dynamic game process, of which, in the area OBDE, it will converge to $B(0,1)$ (stocking, listing). The EDCA area will converge to $A(1,0)$ (not stocked, not included in the list). F is the saddle point. This means that cross-border e-commerce companies choose the strategy of not stocking with the probability of a, and the relevant regulatory agencies choose the strategy of not being included in the list with the probability of b.

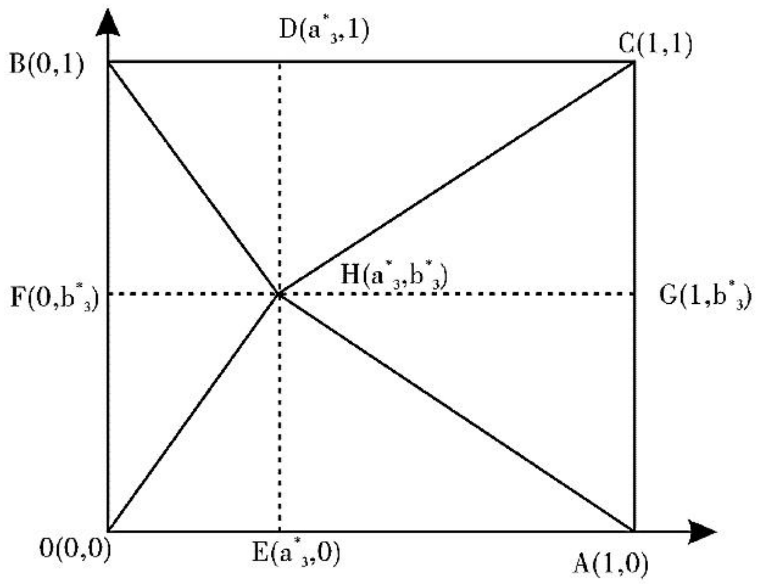

Figure 1 Phase diagram of the dynamic evolutionary process

From the perspective of the long-term and healthy development of cross-border e-commerce, the probability that relevant regulatory agencies will be included in the positive list is as follows:

$$b_3^* = \frac{(R_{12}^M - R_{22}^M)}{(R_{12}^M + R_{21}^M - R_{11}^M - R_{22}^M)} = 1 - \frac{C_{M2}}{I_{M3} - C_{M3} + C_{M2}} 1 - b_3^* \frac{C_{M2}}{I_{M3} - C_{M3} + C_{M2}} .$$

The above formula shows that the difference between the public opinion pressure of the relevant regulatory agency choosing not to be included in the positive list and the decline in reputation caused by the inclusion of the positive list is $C_{M3} - C_{M2}$ less than the penalty income I_{M3} when not included in the positive list $1 - b_3^* > 0$. The relevant regulatory agencies will choose not to be included in the list. $C_{M3} - C_{M2} = X, I_{M3} - X > 0$. When $\frac{d(1-b_3^*)}{dI_{M3}} > 0$, the relevant regulatory agency's choice not to be included in the list is proportional to I_{M3}. The greater the penalty income, the greater the probability of not being included in the positive list. When $\frac{d(1-b_3^*)}{dX} < 0$, the relevant regulatory agency's choice not to be included in the list is inversely proportional to $C_{M3} - C_{M2}$. The smaller the public opinion pressure of not being included in the positive list, the greater the reputational impact of being included in the positive list and the greater the probability of not being included in the positive list.

When $\frac{d(1-a_3^*)}{dI_{M2}} > 0$, the cross-border e-commerce company chooses to stock products that are not on the positive list in direct proportion to the excess income I_{M2}. The greater the probability that cross-border e-commerce companies choose to stock up. When $\frac{d(1-a_3^*)}{dC_{E1}} < 0$, cross-border e-commerce companies choose to stock products that are not on the positive list and C_{E1} is inversely proportional. The smaller the cost of stocking, the greater the probability that cross-border e-commerce companies will choose to stock.

Conclusion 1: The stocking probability of an enterprise is directly proportional to the excess income, and it is inversely proportional to the stocking cost. The probability that a regulatory agency chooses to monitor is directly proportional to the penalty income. It is inversely proportional to the pressure of public opinion not included in the list. It is directly proportional to the reputation impact of the list.

3. The second stage: the game of channel competition among enterprises

Under the condition that the government allows the sale of off-list goods, enterprises are automatically divided into two teams with an out-of-stock strategy due to stocking and not stock. Enterprise 1 can only purchase goods through traditional trading methods and sell them through traditional retail channels [7]. Enterprise 2 can stock up and sell on cross-border e-commerce channels and traditional trade channels. At this stage, this article makes the following assumptions:

(1) The market is a supply chain system composed of a cross-border e-commerce channel sales company and a traditional retail channel sales company. (2) The two companies pursue their profit maximization through price competition. (3) Two companies sell the same product, and the product itself is completely replaceable. (4) Each company has its own fixed loyal consumers. There are still a group of consumers in the market who are pursuing low prices.

In this stage, the Bertrand model is used to characterize the competition among enterprises. Let c_{ue} and c_{ut} denote the unit product inventory cost $(c_{ue} < c_{ut})$ of the enterprise through cross-border e-commerce channels and traditional trade channels, respectively. $f_{ci}(i=1,2)$ represents the fixed costs of enterprises 1 and 2, respectively. $p_{ti}(i=1,2)$ represents the sales prices of enterprises 1 and 2 in traditional trade channels, respectively. p_{e2} represents the sales price of enterprise 2 in the cross-border e-commerce channel. It is assumed here that the price influence of the commodity itself is greater than the mutual influence. Then the demand function faced by enterprise 1 can be expressed as follows:

$$D_1 = l_1 - m_1 p_{t1} + n_1 p_{t2} + n_2 p_{e2} \tag{1}$$

The demand function faced by enterprise 2 in traditional trade sales channels can be expressed as follows:

$$D_{t2} = l_{t2} - m_{t2} p_{t2} + n_1 p_{t1} + n_2 p_{e2} \tag{2}$$

The demand function faced by enterprise 2 in cross-border e-commerce sales channels can be expressed as follows:

$$D_{e2} = l_{e2} - m_{e2} p_{e2} + n_2 (p_{t1} + p_{t2}) \tag{3}$$

Among them, l_1, l_{t2}, l_{e2} represents the potential market size of companies 1 and 2 in their respective sales channels. n_1 represents the degree of substitution of two companies in traditional sales channels. n_2 represents the degree of substitution between traditional sales channels and cross-border e-commerce sales channels. Therefore, the profit function of enterprises 1 and 2 can be expressed as follows:

$$\begin{gathered} \pi_1 = (p_{t1} - c_{ut}) D_1 = (p_{t1} - c_{ut})(l_1 - m_1 p_{t1} + n_1 p_{t2} + n_2 p_{e2}) \\ \pi_{t2} = (p_{t2} - c_{ut}) D_{t2}, \pi_{e2} = (p_{e2} - c_{ue}) D_{e2} \end{gathered} \tag{4}$$

$$\begin{gathered} \pi_2 = \pi_{t2} + \pi_{e2} = (p_{t2} - c_{ut}) D_{t2} + (p_{e2} - c_{ue}) D_{e2} = (p_{t2} - c_{ut}) \\ (l_{t2} - m_{t2} p_{t2} + n_1 p_{t1} + n_2 p_{e2}) + (p_{e2} - c_{ue})[l_{e2} - m_{e2} p_{e2} + n_2 (p_{t1} + p_{t2}) \end{gathered} \tag{5}$$

Conclusion 2: The equilibrium price strategy of the Bertrand game between firm 1 and firm 2 is $(p_{t1}^*, p_{t2}^*, p_{e2}^*)^T = M_1^{-1} M_2$. Where $M_1 = \begin{pmatrix} 2m_1 & -n_1 & -n_2 \\ -n_1 & 2m_{t2} & -n_2 \\ 0 & -n_2 & m_{e2} \end{pmatrix}, M_2 = \begin{pmatrix} l_1 + c_{ut} \\ 1_{t2} - m_{t2} c_{ut} - n_2 c_{ue} \\ -n_2 c_{ut} + m_{e2} c_{ue} \end{pmatrix}$.

Prove that: π_1 and π_2 are equal to 0 when the partial derivative of p_{t1}, p_{t2}, p_{e2} is equal to 0. So we can get

$$\frac{\partial \pi_1}{\partial p_{t1}} = (l_1 - m_1 p_{t1} + n_1 p_{t2} + n_2 p_{e2}) - m_1(p_{t1} - c_{ut}) = 0$$

$$\frac{\partial \pi_2}{\partial p_{t2}} = (l_{t2} - m_{t2} p_{t2} + n_1 p_{t1} + n_2 p_{e2}) - m_{t2}(p_{t2} - c_{ut}) + n_2(p_{e2} - c_{ue}) = 0$$

$$\frac{\partial \pi_2}{\partial p_{e2}} = n_2(p_{t2} - c_{ut}) - m_{e2}(p_{e2} - c_{ue}) = 0$$

Combine the above formula to get:

$$\begin{pmatrix} 2m_1 & -n_1 & -n_2 \\ -n_1 & 2m_{t2} & -n_2 \\ 0 & -n_2 & m_{e2} \end{pmatrix} \begin{pmatrix} p_{t1} \\ p_{t2} \\ p_{e2} \end{pmatrix} = \begin{pmatrix} l_1 + c_{ut} \\ 1_{t2} - m_{t2}c_{ut} - n_2 c_{ue} \\ -n_2 c_{ut} + m_{e2} c_{ue} \end{pmatrix}$$

Assume $M_1 = \begin{pmatrix} 2m_1 & -n_1 & -n_2 \\ -n_1 & 2m_{t2} & -n_2 \\ 0 & -n_2 & m_{e2} \end{pmatrix}, M_2 = \begin{pmatrix} l_1 + c_{ut} \\ 1_{t2} - m_{t2}c_{ut} - n_2 c_{ue} \\ -n_2 c_{ut} + m_{e2} c_{ue} \end{pmatrix}, P = \begin{pmatrix} p_{t1} \\ p_{t2} \\ p_{e2} \end{pmatrix}$.

Since the sum of the modules of the non-diagonal elements in each row of M_1 is less than the module of the diagonal elements in this row, M_1 is the diagonal dominance matrix. In this way, we can easily obtain that M_1 is a non-singular positive definite matrix [8]. Therefore, the equilibrium price strategy $(p_{t1}^*, p_{t2}^*, p_{e2}^*)^T = M_1^{-1} M_2$ of the Bertrand game.

Corollary 1: When companies 1 and 2 play the Bertrand game, the proof of $\frac{\partial p_{t1}^*}{\partial m_1} < 0, \frac{\partial p_{t2}^*}{\partial m_1} < 0, \frac{\partial p_{e2}^*}{\partial m_1} < 0; \frac{\partial p_{t1}^*}{\partial m_{t2}} < 0, \frac{\partial p_{t2}^*}{\partial m_{t2}} < 0 : \frac{\partial p_{t1}^*}{\partial m_{e2}} < 0, \frac{\partial p_{t2}^*}{\partial m_{e2}} < 0, \frac{\partial p_{e2}^*}{\partial m_{e2}} > 0;$ is omitted.

In the supply chain of traditional trade channels, since commodities are substitutable, their demand elasticity is relatively large. Due to the low cost of cross-border e-commerce stocking channels, enterprise 2 chooses to cut prices in cross-border e-commerce stocking channels to gain market share [9]. At this time, enterprise 1, with only one stocking channel, chooses the price reduction strategy. Enterprise 2 chooses to reduce the price simultaneously. Therefore, it can be seen that after the introduction of the positive list, the prices of some products that are not on the positive list (such as the Kao steam eye mask) will have a certain degree of price reduction in supermarkets. This is to resist the advantages of cross-border e-commerce in stocking sales before and after introducing the positive list. Due to the monopoly of cross-border e-commerce stocking channels, price demand elasticity is small. When the demand elasticity of cross-border e-commerce stocking channels increases, Enterprise 2 can only use traditional trade stocking imports for products that are not on the positive list. There are still some consumers in the market who are keen to sell products through cross-border e-commerce stocking channels. The decline in the supply of this channel has caused Enterprise 2 to increase its profits by increasing prices. As the monopoly advantage of cross-border e-commerce stocking channels is gradually disappearing, in order to maintain market share, enterprises 2 have to adopt a strategy of lowering prices in traditional trade stocking sales.

Corollary 2: $\frac{\partial p_{t1}^*}{\partial n_i} > 0, \frac{\partial p_{t2}^*}{\partial n_i} > 0, \frac{\partial p_{e2}^*}{\partial n_i} > 0 (i = 1, 2)$ when companies 1 and 2 are playing Bertrand game.

Due to the substitutability of products and the greater flexibility of alternative channels, cross-border e-commerce stocking channels have to increase prices to maintain profit margins after the tax reform [10]. Corresponding enterprise 2 will keep the price advantage of cross-border e-commerce stocking channels in the minds of consumers, and will also increase prices correspondingly in traditional trade stocking channels. As a market follower, Enterprise 1 also raises prices to increase profits.

Conclusion 3: There is the following relationship $\frac{\partial \pi_1}{\partial m_1}<0, \frac{\partial \pi_1}{\partial m_{t2}}<0, \frac{\partial \pi_1}{\partial m_{e2}}>0$ between firm 1's profit function and price elasticity coefficient m_1, m_{t2}, m_{e2}.

Prove:

$$\frac{\partial \pi_1}{\partial m_1}=(p_{t1}-c_{ut})(-m_1\frac{\partial p_{t1}}{\partial m_1}-p_{t1}+n_1\frac{\partial p_{t2}}{\partial m_1}+n_2\frac{\partial p_{e2}}{\partial m_1})<0$$

$$\frac{\partial \pi_1}{\partial m_{t2}}=(p_{t1}-c_{ut})(-m_1\frac{\partial p_{t1}}{\partial m_{t2}}-p_{t1}+n_1\frac{\partial p_{t2}}{\partial m_{t2}}+n_2\frac{\partial p_{e2}}{\partial m_{t2}})<0$$

$$\frac{\partial \pi_1}{\partial m_{e2}}=(p_{t1}-c_{ut})(-m_1\frac{\partial p_{t1}}{\partial m_{e2}}-p_{t1}+n_1\frac{\partial p_{t2}}{\partial m_{e2}}+n_2\frac{\partial p_{e2}}{\partial m_{e2}})>0$$

Since the channel is not dominant, the demand of enterprise 1 is greatly affected by the price in the competition with enterprise 2, so at this time, the price and demand decrease at the same time, which leads to a decrease in profit. When the demand elasticity of cross-border e-commerce stocking channels increases, enterprise 1 seizes the opportunity to choose to reduce prices and increase market share at this time, which will lead to increased demand and profit recovery.

Corollary 3: There is the following relationship $\frac{\partial \pi_{t2}}{\partial m_1}<0, \frac{\partial \pi_{t2}}{\partial m_{t2}}<0, \frac{\partial \pi_{t2}}{\partial m_{e2}}>0, \frac{\partial \pi_{e2}}{\partial m_1}<0, \frac{\partial \pi_{e2}}{\partial m_{t2}}<0, \frac{\partial \pi_{e2}}{\partial m_{e2}}<0$ between firm 2's profit function and price elasticity coefficient m_1, m_{t2}, m_{e2}.

Prove:

$$\frac{\partial \pi_{t2}}{\partial m_1}=(p_{t2}-c_{ut})(-m_{t2}\frac{\partial p_{t2}}{\partial m_1}+n_1\frac{\partial p_{t2}}{\partial m_1}+n_2\frac{\partial p_{e2}}{\partial m_1})<0$$

$$\frac{\partial \pi_{t2}}{\partial m_{t2}}=(p_{t2}-c_{ut})(-m_{t2}\frac{\partial p_{t2}}{\partial m_{t2}}+n_1\frac{\partial p_{t1}}{\partial m_{t2}}+n_2\frac{\partial p_{e2}}{\partial m_{t2}})>0$$

$$\frac{\partial \pi_{t2}}{\partial m_{e2}}=(p_{t2}-c_{ut})(-m_{t2}\frac{\partial p_{t2}}{\partial m_{e2}}+n_1\frac{\partial p_{t1}}{\partial m_{e2}}+n_2\frac{\partial p_{e2}}{\partial m_{e2}})>0$$

$$\frac{\partial \pi_{e2}}{\partial m_1}=(p_{e2}-c_{ue})[-m_1\frac{\partial p_{e2}}{\partial m_1}+n_2(\frac{\partial p_{t1}}{\partial m_1}+n_2\frac{\partial p_{e2}}{\partial m_1})]<0$$

$$\frac{\partial \pi_{e2}}{\partial m_{t2}}=(p_{t2}-c_{ut})[-m_{e2}\frac{\partial p_{e2}}{\partial m_{t2}}+n_2(\frac{\partial p_{t1}}{\partial m_{t2}}+n_2\frac{\partial p_{t2}}{\partial m_{t2}})]<0$$

$$\frac{\partial \pi_{e2}}{\partial m_{e2}}=(p_{e2}-c_{ut})[-m_{e2}\frac{\partial p_{e2}}{\partial m_{e2}}+n_2(\frac{\partial p_{t1}}{\partial m_{e2}}+n_2\frac{\partial p_{e2}}{\partial m_{e2}})]<0$$

Since the demand of enterprise 1 in the competition with enterprise 2 is greatly affected by price, when enterprise 1 chooses to reduce the price, enterprise 2 will also choose to reduce the price. Because enterprise 1 has some loyal customers, in the price game, enterprise 2's demand and profit decrease when facing the market demand of enterprise 1. However, the dual stocking channels of enterprise 2 enable enterprise 2 to have an advantage in the

Bertrand game with enterprise 1. The demand market for traditional trade stocking channels faced by enterprise 2 is greater than that of enterprise 1. Profits of traditional trade stocking channels have risen. As sales progress, more companies adopt traditional trade to enter the market, which increases the elasticity of demand in the traditional trade market [11]. Enterprise 2 adopts a price reduction strategy to compete with Enterprise 1 completely Bertrand, achieving Bertrand equilibrium in price competition. This will lead to reduced profits. Corollary 1 shows that the existence of consumers who have a soft spot for cross-border e-commerce stocking channels makes enterprise 2 raise prices in cross-border e-commerce channels. However, due to the decrease in demand, profits will fall sharply.

4. The third stage: the game of channel and product competition among enterprises

In the second stage of Bertrand competition, this article only considers one commodity. Because this product is completely replaceable, the competition is only channel competition. In actual competition, companies often take the complementary cooperation between commodities and channels and complement each other to occupy the market. In this stage, this article considers that enterprise 1 uses complementary products to improve its competitiveness. Then the demand function faced by enterprise 1 for product A under the traditional trade stocking mode can be expressed as follows:

$$D_1^{'} = l_1^{'} - m_1^{'} p_{t1}^{'} + n_1^{'} p_{t2}^{'} + n_2^{'} p_{e2}^{'} - n_3^{'} p_{t3}^{'} \tag{6}$$

The demand function faced by enterprise 1 for product B under the traditional trade stocking mode can be expressed as follows:

$$D_2^{'} = l_2^{'} - m_{t3}^{'} p_{t3}^{'} + n_3^{'} (p_{t1}^{'} + p_{t2}^{'}) - n_4^{'} p_{e2}^{'} \tag{7}$$

The demand function faced by enterprise 2 for product A under the traditional trade stocking model can be expressed as follows:

$$D_{t2}^{'} = l_{t2}^{'} - m_{t2}^{'} p_{t2}^{'} + n_1^{'} p_{t1}^{'} + n_2^{'} p_{e2}^{'} - n_3^{'} p_{t3}^{'} \tag{8}$$

The demand function faced by enterprise 2 for product A in the cross-border e-commerce stocking mode can be expressed as follows:

$$D_{e2}^{'} = l_{e2}^{'} - m_{e2}^{'} p_{e2}^{'} + n_2^{'} (p_{t1}^{'} + p_{t2}^{'}) - n_4^{'} p_{t3}^{'} \tag{9}$$

Among them, $l_1^{'}, l_2^{'}, l_{t2}^{'}, l_{e2}^{'}$ represents the potential market size of companies 1 and 2 in their respective sales channels. $c_{ut}^{'}$ represents the stocking cost of product B for enterprise 1 in traditional channels, and other costs remain unchanged. $n_1^{'}$ represents the degree of substitution of product A by the two companies in the traditional sales channel. $n_2^{'}$ represents the degree of substitution of product A sales by traditional sales channels and cross-border e-commerce sales channels. The profit function of companies 1 and 2 can be expressed as follows:

$$\begin{aligned} \pi_1^{'} &= (p_{t1}^{'} - c_{ut}) D_1^{'} + (p_{t3}^{'} - c_{ut}^{'}) D_2^{'} = (p_{t1}^{'} - c_{ut})(l_1^{'} - \\ & m_1^{'} p_{t1}^{'} + n_1^{'} p_{t2}^{'} + n_2^{'} p_{e2}^{'} - n_3^{'} p_{t3}^{'}) + (p_{t3}^{'} - c_{ut}^{'})[l_2^{'} - m_{t3}^{'} p_{t3}^{'} - n_3^{'} (p_{t1}^{'} + p_{t2}^{'}) - n_4^{'} p_{e2}^{'}] \end{aligned} \tag{10}$$

$$\begin{aligned} \pi_2^{'} &= (p_{t2}^{'} - c_{ut}) D_{t2}^{'} + (p_{e2}^{'} - c_{ut}) D_{e2}^{'} = (p_{t2}^{'} - c_{ut})(l_{t2}^{'} - \\ & m_{t2}^{'} p_{t2}^{'} + n_1^{'} p_{t1}^{'} + n_2^{'} p_{e2}^{'} - n_3^{'} p_{t3}^{'}) + (p_{e2}^{'} - c_{ut}^{'})[l_{e2}^{'} - m_{e2}^{'} p_{e2}^{'} + n_2^{'} (p_{t1}^{'} + p_{t2}^{'}) - n_4^{'} p_{t3}^{'}] \end{aligned} \tag{11}$$

Conclusion 4: The equilibrium price strategy of the Bertrand game between firm 1 and firm 2 is $(p_{t1}^{*}, p_{t3}^{*}, p_{t2}^{*}, p_{e2}^{*})^{T} = M_1^{'-1} M_2^{'}$, where

$$M_1^{'} = \begin{pmatrix} 2m_1^{'} & 2n_3^{'} & -n_1^{'} & -n_2^{'} \\ 2n_3^{'} & 2m_{t3}^{'} & n_3^{'} & n_4^{'} \\ -n_1^{'} & n_3^{'} & 2m_{t3}^{'} & -2n_2^{'} \\ -n_2^{'} & n_4^{'} & -2n_2^{'} & 2m_{e2}^{'} \end{pmatrix}, M_2 = \begin{pmatrix} l_1^{'} + m_1^{'} c_{ut} + n_3^{'} c_{ut}^{'} \\ l_2^{'} + m_{t3}^{'} c_{ut}^{'} + n_3^{'} c_{ut} \\ l_{t2}^{'} + m_{t2}^{'} c_{ut}^{'} + n_2^{'} c_{ut} \\ l_{e2}^{'} + m_{e2}^{'} c_{ut}^{'} + n_2^{'} c_{ut} \end{pmatrix}$$ is.

Corollary4:

$$\frac{\partial p_{t1}^{'*}}{\partial m_1^{'}} > 0, \frac{\partial p_{t3}^{'*}}{\partial m_1^{'}} > 0, \frac{\partial p_{t2}^{'*}}{\partial m_1^{'}} < 0, \frac{\partial p_{e2}^{'*}}{\partial m_1^{'}} < 0; \frac{\partial p_{t1}^{'*}}{\partial m_{B}^{'}} > 0, \frac{\partial p_{t3}^{'*}}{\partial m_{B}^{'}} < 0, \frac{\partial p_{t2}^{'*}}{\partial m_{B}^{'}} > 0, \frac{\partial p_{e2}^{'*}}{\partial m_{B}^{'}} > 0;$$

$$\frac{\partial p_{t1}^{'*}}{\partial m_{t2}^{'}} > 0, \frac{\partial p_{t3}^{'*}}{\partial m_{t2}^{'}} > 0, \frac{\partial p_{t2}^{'*}}{\partial m_{t2}^{'}} < 0, \frac{\partial p_{e2}^{'*}}{\partial m_{t2}^{'}} < 0; \frac{\partial p_{t1}^{'*}}{\partial m_{e2}^{'}} < 0, \frac{\partial p_{t3}^{'*}}{\partial m_{e2}^{'}} > 0, \frac{\partial p_{t2}^{'*}}{\partial m_{e2}^{'}} < 0, \frac{\partial p_{e2}^{'*}}{\partial m_{e2}^{'}} > 0,$$

when companies 1 and 2 are playing Bertrand game.

Since the complementary commodity B has just entered the market and its demand elasticity is small, enterprise 1 will choose to increase the price in order to occupy the market to obtain excess profits. At the same time, company 1 will lower the price of commodity A as a bait to increase sales profits. Due to the competitive relationship of commodity A, enterprise 2 will respond to the price reduction in a timely manner. During the Bertrand game of price reduction between the two parties, the elasticity of demand for commodity A gradually increases. Since enterprise 1 uses complementary product B to occupy the market, it will use the bundling strategy to increase the prices of products A and B at the same time. When the demand elasticity of the cross-border e-commerce sales channel of commodity A increases, enterprise 1 will reduce the price to counter enterprise 2. At the same time, enterprise 1 still chooses to increase the price of commodity B as a supplement to the reduced profit of commodity A. Similarly, Enterprise 2 will also respond to price cuts in traditional channels. It will use cross-border e-commerce channels to increase prices to supplement profits.

5. Conclusion

This article establishes a three-stage game in the process of an import and export trade enterprise's transformation to cross-border e-commerce. We use the Bertrand game model to analyze the channel price game with market competitors in the second stage after the effect of the first stage is reached. The article makes certain theoretical suggestions for enterprises to rationally price in order to obtain high profits. Traditional trade stocking channels use the introduction of complementary products as a strategy to counter cross-border e-commerce stocking channels. Similarly, we use Bertrand game to analyze the pricing strategy of enterprises in the game process.

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