Vote Delegation and Malicious Parties

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***Abstract*—We initiate the study of vote delegation in a costly voting setting and compare its performance to conventional voting. Our central insight is that if the number of malicious voters is comparatively low, delegation dominates conventional voting. If the number of malicious voters is moderate, then conventional voting tends to be better than vote delegation. If the number of malicious voters is high, both voting methods fail to deliver a positive outcome.**

***Index Terms*—Governance; Delegation; Game Theory;**

I. Introduction

One of the reasons why voters do not vote is that voting entails a cost, e.g., time to acquire information and to vote. Rational voting is a topic of the following papers [5], [7]. Vote delegation in the network and its dangers is studied in [2], [4]. Vote delegation can be seen as a part of blockchain governance, [3].

In this communication, we study intertemporal and temporal vote delegation when a minority of malicious voters attempt to disrupt the functioning of an ecosystem (e.g., a blockchain community, or a committee). First, we briefly discuss intertemporal vote delegation. In this setting, any pair of voters (v_i, v_j) can write a contract about a vote exchange. A voter v_i gives a vote to a voter v_j in round r in exchange for a vote in round w. So much freedom regarding vote delegation can introduce an attack, even with a single malicious voter. A malicious voter can choose a voting round t in the future. If t is larger than the half of all voters, the malicious voter can engineer the vote exchange contracts with the majority of the other voters such that at voting round t, she has the majority of votes, and therefore, compromises the whole system. This simple yet efficient attack called "t-period attack" shows that a free intertemporal vote exchange can be very dangerous, and that some restrictions on the vote delegation and contracting should be imposed.

Next, in the main part, we consider vote delegation in the same voting round.

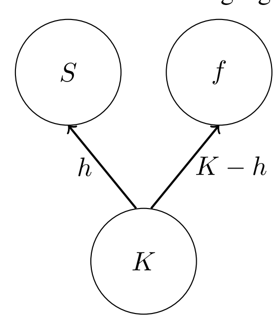
II. Preliminaries

We consider a collective decision problem in which voters decide between two alternatives, A and B. One of them is correct, and the other one is wrong. Honest voters want the correct alternative to be implemented, while malicious voters prefer the opposite. We assume that honest voters obtain utility 1 if the correct alternative is implemented and utility 0 if the wrong alternative is implemented. Informational assumption: voters do not know the type of other voters, i.e., whether other voters are honest or malicious. The collective decision problem has the following elements:

- N: total number of honest voters is distributed as a Poisson random variable with parameter n. Hence, n is the expected number of honest voters.
- f: a finite number of malicious voters.
- c: cost of voting for every honest voter. We assume that c includes the cost of finding the correct alternative.
- Malicious voters always vote (or equivalently, they are not deterred from voting by costs).

A. Delegation

In this section, we consider a game in which the strategy set of each voter consists of voting and delegation. Since the identities of the malicious voters are not known, we assume that delegation is done uniformly at random. Let γ denote the delegation probability. From the *decomposition property* of the Poisson random variable, see [6], we have that K, the number of honest voters delegating, is distributed as a Poisson random variable with parameter $n\gamma$. On the other hand, S, the number of honest voters voting, is distributed as a Poisson random variable with parameter $n(1-\gamma)$. Hence, K votes are delegated to a group of $S+f$ voters consisting of the remaining S honest voters and f malicious voters. Let h denote the number of votes that the honest voters obtain. Then, $h \sim Binomial(K, \frac{S+1}{S+1+f})$. The malicious voters receive the remaining $K-h$ votes. The following figure illustrates this.



The function g denotes the gain for honest voters if they receive x votes, and malicious voters receive y votes. $g(x, y) := 1$, if $x > y$, $\frac{1}{2}$, if $x = y$ and 0 if $x < y$.

We next determine mixed strategy equilibrium values for γ ($\gamma \in (0, 1)$) (and thus S and K). A value of γ represents an

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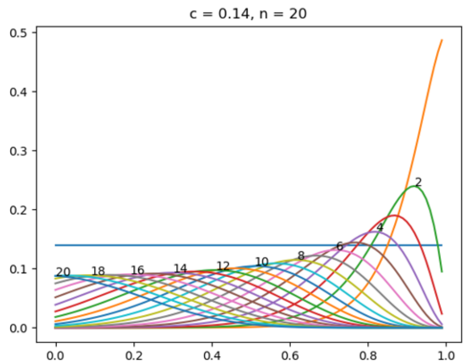

Fig. 1. Expected utility gain from voting for different values of f.

equilibrium if honest voters are indifferent between delegation and voting. Let us consider deviations from a potential equilibrium constellation by an honest voter. From the *environmental equivalence*, [6], the honest voter group size is $S+1$ if he votes. In case of delegating, a voter either delegates to S honest voters or to f malicious voters. Therefore, we have the following equilibrium condition:

$$c = \mathbb{E}[U(\text{voting})] - \mathbb{E}[U(\text{delegating})] = \sum_{K=0}^{\infty}\sum_{S=0}^{\infty}\sum_{h=0}^{K}\sum_{d=0}^{h} \frac{(n\gamma)^K}{e^{n\gamma}K!}\frac{(n(1-\gamma))^S}{e^{n(1-\gamma)}S!}\binom{K}{h}\left(\frac{S+1}{S+1+f}\right)^h \left(\frac{f}{S+1+f}\right)^{K-h}\binom{h}{d}\left(\frac{1}{S+1}\right)^d \left(1-\frac{1}{S+1}\right)^{h-d} \left[g(S+1+h, f+K-h) - \left(\frac{S}{S+f}g(S+1+h, f+K-h) + \frac{f}{S+f}g(S+h-d, f+K-h+d+1)\right)\right].$$

The probability p that a majority of votes is cast by honest voters is equal to:

$$p(n,f,\gamma) = \sum_{K=0}^{\infty}\sum_{S=0}^{\infty}\sum_{h=0}^{K} \frac{(n\gamma)^K}{e^{n\gamma}K!}\frac{(n(1-\gamma))^S}{e^{n(1-\gamma)}S!}\binom{K}{h} \left(\frac{S+1}{S+1+f}\right)^h \left(\frac{f}{S+1+f}\right)^{K-h} g(S+1+h, f+K-h).$$

We obtain our first result.

Proposition 1: There is a threshold $f(c) \in \Theta(\frac{1}{c})$, so that there are mixed strategy equilibria in the delegation game for $f < f(c)$ and no mixed strategy equilibria for $f > f(c)$. In particular, for high enough values of f, everyone delegates, and thus malicious voters win.

The result is illustrated in Figure 1. The x-axis represents values of γ, and the y-axis displays expected utility gains from voting, with the straight blue line representing cost c.

B. Conventional Voting

In this section, we consider conventional voting in which honest voters only decide between voting and abstention. Let α be the probability of voting. We obtain the following indifference condition between voting and abstaining:

$$2c = \frac{(n\alpha)^f}{e^{n\alpha}f!} + \frac{(n\alpha)^{f-1}}{e^{n\alpha}(f-1)!}.$$

The probability of honest voters winning is given by: $q(n,f,\alpha) = \sum_{k=0}^{\infty} \frac{(n\alpha)^k}{e^{n\alpha}k!}g(k,f)$.

We know from [1] that if f is larger than $r\frac{1}{c^2}$ for some positive constant r, then the conventional voting game has no mixed equilibria. The only equilibrium solution is that no honest voter votes and malicious voters win. For low values of f, we obtain a similar structural result as for vote delegation.

Proposition 2: There are mixed equilibria solutions to the game with abstentions for sufficiently low f.

C. Comparison of Voting Rules

The following table displays the equilibrium probabilities of honest voters wining for delegation and conventional voting.

f	p_1	p_2	q_1	q_2
1	0.97	-	0.80	-
2	0.92	0.30	0.76	0.01
3	0.85	0.31	0.73	0.05
4	0.78	0.39	0.68	0.10
5	0.67	0.47	0.63	0.13
6	-	-	0.60	0.19
7	-	-	0.53	0.24
8	-	-	0.45	0.34
9-20	-	-	-	-

TABLE I
$n = 20, c = 0.14$

Numerical experiments support the pattern in Table I. They suggest that two different equilibrium probabilities of honest voters winning in the delegation game are higher than the probabilities in the conventional voting game for sufficiently low values of f. For moderate values of f, however, the conventional voting game yields higher probabilities that the correct alternative is chosen. Finally, for high values of f, malicious voters win with certainty in both delegation and conventional voting games.

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