Strategic Prosumers: How to Set the Prices in a Tiered Market?

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***Abstract*—We consider users who may have renewable energy harvesting devices or distributed generators. Such users can behave as consumers or producers (hence, we denote them as prosumers) at different time instances. We consider a tiered market where the grid selects a price function, which reveals price in the real time based on the total demand to the grid. In the real time, a prosumer can buy from another prosumer in an exchange market knowing the price from the grid. The exchange price is set by a platform and can be different for different sellers. A prosumer is a selfish entity, which selects the amount of energy it wants to buy either from the grid or from other prosumers or the amount of excess energy it wants to sell to other prosumers by maximizing its own payoff. However, the strategy and the payoff of a prosumer inherently depend on the strategy of other prosumers as a prosumer can only buy if the other prosumers are willing to sell. We formulate the problem as a coupled constrained game and seek to obtain the generalized Nash equilibrium. We show that the game is a concave potential game and show that there exists a unique generalized Nash equilibrium. We propose a distributed algorithm that converges to the exchange price, which clears the market and achieves the generalized Nash equilibrium. We, finally, show how the grid should select the price function in a day-ahead scenario by computing the estimated demand from the history. Our numerical result shows that the tiered market can reduce the peak load and increase the prosumers' total payoffs.**

***Index Terms*—Blockchain, distributed generators, Nash equilibrium, peer-to-peer energy trading, potential game.**

I. Introduction

THE problems of the peak demand and the uncertainty of the demand and supply are challenging the traditional power grid. The integration of the renewable energy into the system has increased the uncertainty because of the randomness of renewable energy generation. The ever-increasing use of the electric vehicles (EVs) has also increased the demand of the users. A demand response mechanism has been proposed, where time-of-use price is primarily used to shift the peak load during the off-peak times. However, those approaches assume that the users are *price takers* and do not consider the strategy of the other users. In the real time, a consumer may consume a higher load compared to the estimated value, which can lead to instability in the grid. The total load can also be small, which should influence the users to consume more to eliminate the instability in the grid. Thus, a dynamic pricing mechanism is required, which also incentivizes the users to reduce the demand in the real time.

The continuing proliferation of the distributed energy resources (e.g., photovoltaic arrays, solar rooftops, and energy storage units) has transformed the notion of traditional users of energy. The consumers can now also produce and reduce the burden of the grid. If those producers share energy with the consumers at a certain time in a geographical region, it would enhance the renewable energy usage and also decrease the conventional energy demand from the grid. Such a local exchange is also useful as the transmission loss will be lower as compared to the scenario where the grid has to serve the users. Thus, such a kind of exchange between the prosumers (producers of distributed energy) and consumers has a lot of potential. However, the prosumers have to be incentivized to participate in the exchange market. The design of the exchange market inherently depends on the prices of the grid. If the prices for the conventional energy are low, the exchange market may not be useful. Higher price of the conventional energy may facilitate the exchange; it may increase the user's cost in case it has to buy from the grid. The higher electric prices will reduce the overall user's surplus.

Thus, a proper pricing mechanism is required, which will simultaneously increase the social welfare, facilitate the exchange of energies among the prosumers, and decrease the demand of the conventional energy from the grid. Note that the exchange price has to be computationally simple because the exchange has to be taken place in real time. The determination of an optimal exchange price at a certain time is challenging. The exchange between the prosumers will not depend on the price at a certain time but also on the exchange prices at other times. For example, if the exchange price is high at peak time, users will seldom opt for the exchange market during the peak time, which does not serve the purpose of the exchange market. On the other hand, if the exchange price is low, it may not incentivize the prosumers to sell their exchange energy. The prices for the grid have to be selected judiciously along with the prices for the

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exchange market. The grid itself is unaware of the utilities of the users and the excess energy the users possess *a priori*. We seek to propose pricing mechanisms, which will simultaneously maximize the social welfare, facilitate the exchange of energies among the prosumers in an exchange market, and the grid's cost of procuring conventional energy.

We consider a tiered market structure, where, first, the grid decides the price function, which is a price based on the real-time demand (see Section III). Note that in the dynamic price, the exact price is only revealed when the real-time demand is realized. We assume that the price function is linear and is a function of the total demand. Thus, the price (or the payment) made by a consumer not only depends on its own demand, but also on the demand of other consumers.

We consider a local exchange market, where the prosumers can sell energies to other prosumers in an exchange market (see Section III). Each prosumer may also have a renewable energy harvesting device and a storage device. Given the exchange price, the price function of the grid, and the renewable energy prediction, each prosumer decides how much to buy from the grid, how much to store, how much to sell (if any) to other prosumers, and how much to buy from other prosumers at each instance. *The prosumers are selfish entities, which only want to maximize their own payoffs.* Each prosumer achieves a utility for consuming a certain amount of load. Each prosumer's payoff depends on the utility it achieves, the profit it makes (if it sells energy in the exchange market), or the cost it has to pay.

We consider that there exists a platform, which sets the exchange price. Examples of platform may be the load serving entity, the aggregator, and a private retailer.[1] The platform selects different selling prices for different prosumers. Note that different prosumers may have different amount of excess energy to sell; thus, the platform selects different prices to incentivize them to sell their energies. The platform selects not only prices for the current time slot, but also prices for future time slots, since the decision of the prosumers also depends on the future time slots. The platform wants to maximize the overall utility of the users. However, selecting a price that will maximize the sum of the utilities of all the users is inherently challenging, since the platform is unaware of the utilities of the users.

Since the prosumers are selfish entities, we use a game-theoretic model to characterize the interaction among the prosumers (see Section IV-A). Each prosumer selects its best response by anticipating the behavior of others. However, the strategy space is also dependent on the strategy of the other prosumers. For example, the maximum energy bought by a prosumer (A, say) from a prosumer (B, say) is limited by the amount the prosumer B wants to sell at a certain time. Different prosumers may not sell all the excess energy; rather, they can store the energy and sell it only during the peak period. Thus, finding an equilibrium strategy is inherently challenging. We resort to the concept of the generalized Nash equilibrium as the equilibrium concept, since the strategy space of a player inherently depends on the strategies of others.

[1] Recently, there have been private retailers, which are providing exchange service among the distributed generators [1].

We show that the game is a potential game (see Theorem 1). In fact, the potential game is strictly concave when the utility functions of the users are strictly concave and, thus, admits a unique generalized Nash equilibrium. However, the above equilibrium is not an optimal solution of the scenario, where the sum of the prosumers' utilities is maximized. In other words, the equilibrium is not an efficient one; however, if the renewable energy generation is high, the efficiency is also high.

Subsequently, we propose a distributed algorithm to find the equilibrium (see Section IV-C) and the optimal price the platform should set for the exchange of energy. In the algorithm, the platform first sets a low price for each prosumer. Each prosumer then decides the total load, the amount of energy to be bought from other prosumers, and the amount of energy to be sold. While taking its decision, a prosumer takes the strategy of the previous iteration of other prosumers as an estimate of the current strategy of the prosumers. Thus, a prosumer does not need to know the utilities of other prosumers. The platform then increases the exchange price by an ϵ amount for those prosumers for which the supply is less than the demand. The process continues till the supply from all the prosumers matches the demand of the consumers at each time slot.

We show that such a simple distributed algorithm converges to the equilibrium strategy of the prosumers using the concept of the *fictitious play* (see Theorem 2). The exchange price also converges to the optimal price, which maximizes the overall exchange of energy among the prosumers. The convergence is also exponential.

Finally, we investigate the price function the grid has to select for the price of the real-time demand. Too low price for the conventional energy may enhance the user's satisfaction (surplus); however, it will not facilitate the exchange among the prosumers and may not be enough for the grid to procure the conventional energy. On the other hand, too high price may decrease the user's surplus. The grid should select prices, which have to be profitable while maximizing the user's surplus. We consider such a scenario in Section V, where the grid maximizes the expected total surpluses of the prosumers while maintaining a fixed profit (β) to the generators. We, empirically, show that our mechanism reduces the conventional energy consumption, decreases the price, and increases the renewable energy consumption in Section VI. Our main contributions can be summarized as follows.

1) We formulate a tiered market architecture, where, in the first stage, the grid selects a linear price function depending on the total consumers' demand. The price is realized only when the total consumption of the prosumers is revealed. In the second stage, the platform selects a price for each seller in the exchange market. The proposed formulation considers the tiered market operating on a time horizon consisting of multiple time slots. Each seller and buyer decide the amount of energy to sell and buy, respectively, in the exchange market.
2) The prosumers are strategic; they only want to maximize their own utilities. We show that there exists a unique generalized Nash equilibrium (GNE) in the prosumers' strategy profile if the utility functions are concave.

3) We show that there exists a readily implementable price mechanism for the platform, which achieves the unique GNE in an iterative process.
4) We determine the optimal price function the grid should select in order to maximize the prosumer's profit while maintaining a fixed profit of the grid.

II. Related Literature

Demand response pricing has already been studied [2]–[8]. The analysis of all these papers is based on the assumption that consumers are price takers. They find the optimal price based on the estimated demand profile of the consumers. However, in real time, the consumers may consume more as compared to the estimate. The consumers will still pay a marginal price according to the demand, which has been estimated *a priori*. We consider a real-time pricing mechanism, where the price is set based on the real-time total demand. Thus, the users (prosumers) have to strategize their demand based on the anticipation of the strategy of the other users. The above papers also did not consider the scenario, where the users can sell energies among themselves in a separate market setting.

Real-time pricing has been considered recently [9]–[13]. Eksin *et al.* [14] proposed a game-theoretic model, where the consumers face a price, which varies depending on the total load to the grid. However, the above papers did not consider the exchange market, which is considered in our work. The game is now a coupled constrained game, where the constraints of each user (prosumer) also depend on the other prosumers' decisions, since a consumer can only know the amount a prosumer wants to share. Thus, an optimal exchange price is required, which will enhance the exchange of energy among the prosumers. Finally, Eksin *et al.* [14] did not consider the temporal correlation of the demand of the users. The utility of the users and the demand of the users often have temporal correlation. For example, in the EV charging, the users may need a certain amount of charged battery before a certain deadline. The consumption of conventional energy, and thus the exchange of energy, will also depend on the prices across a certain horizon. Finally, each prosumer may also have a storage device and, thus, may defer the selling of energy at a later time if the selling price is higher.

Energy exchange among the users in a microgrid setting has been considered [15]–[18]. The papers considered that the buyers and sellers will exchange energy among each other by bidding asking price and selling price, respectively. The authors of [19]–[22] considered a microgrid scenario, where the prosumers within the microgrid exchange energy. However, none of the above-cited papers consider that the prosumers may have a storage unit. Thus, a prosumer may strategically delay selling energy if it gets a favorable price in the future by storing energy. Furthermore, the above-cited papers assumed that the demand consists of nondeferrable loads. However, in practice, users may defer the consumption of energy if it is better to exchange energy when the exchange price is high. Thus, unlike the above-cited papers, we consider *temporal correlation* among the strategies of the prosumers. The above-cited papers did not consider the price selection of the grid. We consider a tiered architecture, where, at the first stage, the grid selects a price function for the conventional energy. The price is realized only when the amount of conventional energy is realized. We optimize the price the grid should select in order to maximize the utilities of the prosumers with a fixed profit margin.

III. System Model and Prosumer's Problem Formulation

In this section, we will describe the pricing behavior (see Section III-A), prosumer's utility functions (see Section III-B), and the constraints every prosumer has to satisfy (see Section III-C). Finally, we formulate the optimization problem for the prosumer decision (see Section III-D).

A. Pricing Structure

We assume that the time is slotted. Each prosumer can act either as a producer or a consumer in a slot. Prosumers also decide how much to sell (producer) or buy (consumer) in the exchange market. The prosumer wants to decide its demand over a certain time horizon T (e.g., over a day or over a 8–9-h period). Let the number of prosumers be N. Note that a prosumer does not need to have energy generation capability; rather, it can be a traditional consumer, which consumes energy.

Let $l_{i,t}$ be the demand of prosumer i to the grid at during time $[t, t+1)$. The price selected by the independent system operator (ISO) for the consumption of conventional energy is denoted as

$$p_t\left(\sum_i l_{i,t}\right) = \gamma_t \sum_i l_{i,t} \quad (1)$$

where $l_{i,t}$ is the demand of prosumer i to the grid at time t. Note that $\sum_i l_{i,t}$ is the total load of the grid at time t. γ_t is the parameter that is selected by the grid and depends on the time of the day. Note that setting a price that varies linearly with the total demand is quite common in the literature [14], [23]. *In Section V, we discuss how the grid should select* γ_t.

Note that prosumer i can buy energy from the grid at time t. Thus, the load $l_{i,t}$ can be positive for those prosumers. The prosumer may buy energy from the grid if it does not produce enough energy, and the energy bought in the local exchange market is not sufficient to meet its demand. Also, note that we do not rule out the scenario, where the prosumer may sell energy in the local exchange market; however, it may buy energy from the grid. Our analysis shows that in our designed market, the above-mentioned scenario does not arise.

The prosumer is different from the microgrid. The microgrid consists of a cluster of loads and distributed generators. However, the prosumer is a term reserved for an individual user. A cluster of prosumers can constitute a microgrid if it can act independently or in synchronization with the macrogrid.

Note that the price of the grid not only depends on the load of the user i, but also on the loads of the other prosumers. The price is realized only when the load of all the prosumers is known. Thus, it leads to a game among the users, where the payoff inherently depends on the strategies chosen by the other users. A user is not aware of the exact demand of the other users; thus, it leads to an *incomplete information game*. Note that unlike the price that is independent of the total load, in the price strategy

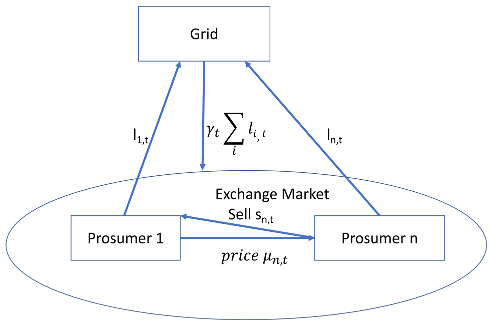

Fig. 1. Tiered market structure: price for the conventional energy $\gamma_t \sum_i l_{i,t}$, where $l_{i,t}$ is the demand of conventional energy. $\mu_{j,t}$ is the selling price for prosumer j in the exchange market. Prosumer j sells $s_{j,t}$ amount during time slot t.

stated in (1), there will always be an upper limit of the load. This is because a user will try to resist consuming too much energy. *The price is the same to all buyers at any given time t.* Fig. 1 depicts the tiered market structure we envision.

Exchange price: At time t, we assume that there is a platform, which sets the prices for exchange among the prosumers. Each prosumer decides how much energy to be sold not only at time t, but also for entire time horizon T. The consumer also selects the amount of energy it wants to consume. If prosumer i buys energy from prosumer j, it pays an amount $\mu_{j,t}$ per unit of energy during time $[t, t+1)$. Thus, the selling prices may be different for different prosumers. However, the buyers pay the same per unit price to each seller. The platform wants to maximize the overall utilities of the users by selecting an optimal price. Fig. 1 depicts that prosumer 1 gets energy from prosumer n at price $\mu_{n,t}$ during time slot t.

B. Prosumer's Payoff

We, now, compute the expression of each prosumer's payoff. Let prosumer i buy a $q_{i,j,t}$ amount of energy from prosumer j during $[t, t+1)$. Let prosumer i sell an $s_{i,t}$ amount of energy during the time duration. Let the prosumer i's demand (or total consumption) during time duration $[t, t+1)$ be $d_{i,t}$. The user's utility $U_{i,t}$ only depends on the consumption $d_{i,t}$. Note that the utility function is time dependent, as the values of the parameters of the function may change over time.

Let the prosumer i's demand (or total consumption) during time duration $[t, t+1)$ be $d_{i,t}$. Thus, the user i's profit or total payoff is

$$\sum_{t=1}^{T}\left[\mu_{i,t}s_{i,t} - \sum_{j\neq i}\mu_{j,t}q_{i,j,t} - l_{i,t}\gamma_t\left(\sum_i l_{i,t}\right) + U_{i,t}(d_{i,t})\right]. \tag{2}$$

The user's[2] payoff inherently depends on the decision of the other users. Note that the amount of energy a consumer buys from the other producers in the exchange market also inherently depends on the amount of energy sold by the other producers.

Assumption 1: We assume that $U_{i,t}(\cdot)$ is concave and continuous.

[2]In the following, we use the notation user and prosumer interchangeably.

User's preference (or comfort) increases with the consumption. However, the rate of increase of preference decreases with the demand $d_{i,t}$. The utility function is randomly drawn from a distribution. *The ISO or the other prosumers are unaware of the exact utility of a prosumer.* However, there may be a correlation among the prosumers.

The utility may also have a temporal correlation. For example, a prosumer can only attain a utility if the total demand is satisfied within a deadline. The prosumer will have zero utility if the demand is unsatisfied within the deadline. Such a kind of utility arises for the deferrable loads such as EV charging and dishwasher.

C. Problem Formulation: Constraints

In this subsection, we describe the system of constraints that each prosumer has to satisfy while taking its own decision.

Some users may have a deferrable load. The demand only needs to be fulfilled within a certain time. For example, the EV needs to be ready before 8 A.M. (e.g., if the user is going to work). However, the individual load may vary over time. We denote the set of deferrable appliances as $\mathcal{A}_i$. Suppose that the load assigned to appliance j of user i is $x_{i,j,t}$ for the time duration $[t, t+1)$. Hence, we have

$$\sum_{t=1}^{T_j} x_{i,j,t} \geq X_j \forall i, \forall j \in \mathcal{A}_i \tag{3}$$

where X_j is the amount of load required for appliance j.

Let the set of nondeferrable load of prosumer i be $\mathcal{B}_i$. Let $x_{i,j,t}$ be the load for appliance $j \in \mathcal{B}_i$. The user's total consumption during time $[t, t+1)$ is thus

$$d_{i,t} = \sum_{j\in\mathcal{A}_i\cup\mathcal{B}_i} x_{i,j,t}. \tag{4}$$

The demand of prosumer i has to exceed the demand for nondeferrable appliances at each instance. Thus, we have

$$d_{i,t,\max} \geq d_{i,t} \geq d_{i,t,\min} \tag{5}$$

where $d_{i,t,\max}$ and $d_{i,t,\min}$ are known beforehand.

Let the amount of energy sold by prosumer j be $s_{j,t}$ during time slot t. Let the energy bought by prosumer i from prosumer j is denoted as $q_{i,j,t}$ during time $[t, t+1)$. Thus, we have

$$q_{i,t} = \sum_j q_{i,j,t}. \tag{6}$$

The energy bought from user j must be smaller than the total energy sold by user j. Also note that the sum of energies bought by all the prosumers has to be equal to the energy sold by prosumer j. Thus, we have

$$\sum_i q_{i,j,t} = s_{j,t} \forall j. \tag{7}$$

Each prosumer has a renewable energy harvesting device, which harvests $\bar{E}_{i,t}$ amount of energy[3] during time $[t, t+1)$.

[3]A prosumer may not have any renewable energy harvesting device. In that case, the renewable energy will be 0.

Prosumer i may also have a battery with capacity $B_{i,\max}$. If the prosumer does not have any battery, then $B_{i,\max}$ is 0. The state of the battery is $B^{i,t}$. The amount of energy discharged from the battery is $e_{i,t}$, and that charged to the battery is $b_{i,t}$. Thus, we have

$$B^{t+1} = B^t + \bar{E}_{i,t} - e_{i,t} + b_{i,t}. \tag{8}$$

Note that the renewable energy generation is a random process. Hence, a prosumer will only have the estimate of $\bar{E}_{i,t}$, rather than the exact realized value.

The state of the battery cannot be less than 0. The state of the battery is also required to be a specific value at the end of the horizon. Most often, the state of the battery is kept to be the same as the start of the day. Thus, we have

$$B^{T+1} = B^1, \quad 0 \leq B^t \leq B_{\max}. \tag{9}$$

Note that the energy bought from the grid as well as from the other prosumers also has a transmission loss. Let $r_{i,j} \leq 1$ be the transmission efficiency between the users i and j, and r_i be the transmission efficiency between user i and the grid. Thus, the total consumption of prosumer i during time $[t, t+1)$ is given by

$$d_{i,t} = \sum_j q_{i,j,t} r_{i,j} - s_{i,t} + l_{i,t} r_i - b_{i,t}\eta_d + e_{i,t}\eta_c \tag{10}$$

where $\eta_d \leq 1$ and $\eta_c \leq 1$ are the discharging and charging efficiency from the battery, respectively. Note that only a portion of energy bought by the prosumer can be used because of the transmission loss.

Now, we assume the following.

Assumption 2: $r_{i,j} > r_i$ for all i and j.

We consider a geographically colocated prosumers. The extension of our work for prosumers situated in a vast geographical area is left for the future. Generally, the transmission efficiency from a neighbor prosumer should be high compared to obtaining energy from the grid. This is because the grid often times obtain energy from a far greater distance than the distance between local neighbors.

D. Formulated Problem

We are now ready to specify the optimization problem that each prosumer solves for every time horizon $[t, t+T)$ as

$$\begin{aligned} P_i : \text{maximize} \quad & \sum_{t=1}^{T} \Bigg[\mu_{i,t} s_{i,t} - \sum_{j \neq i} \mu_{j,t} q_{i,j,t} \\ & \qquad - l_{i,t}\gamma_t \left(\sum_i l_{i,t} \right) + U_{i,t}(d_{i,t}) \Bigg] \\ \text{subject to} \quad & (3)\text{–}(10) \\ \text{var} \quad & d_{i,t}, l_{i,t}, q_{i,t}, s_{i,t}, e_{i,t}, \\ & b_{i,t}, q_{i,j,t}, s_{i,j,t} \geq 0. \end{aligned} \tag{11}$$

The problem is convex when the prosumer knows others' strategies. However, a prosumer is unaware of the strategies set by other prosumers.

Note that we must have $q_{i,t}s_{i,t} = 0$, since a prosumer cannot buy and sell simultaneously. Later, we show that in an optimal solution, we indeed have $q_{i,t}s_{i,t} = 0$.

Here, $U_{i,t}(\cdot)$ is assumed to be separable across the demand for each time period t. Our analysis can also be applied to the scenario, where $U_{i,t}(\cdot)$ is not separable across time. Also note that each prosumer optimizes over the time slots $t = 1, \ldots, T$. It is extended to the scenario for any time horizon starting from $t = \tau$ rather from $t = 1$.

Also note that the exchange market is local. We did not consider the ac reactive power constraint and the network effect due to the Kirchoff's laws.

E. Platform's Objective

The platform wants to maximize the total payoff of the prosumers. Thus, the objective of the platform is the following:

$$\begin{aligned} \text{maximize} \quad & \sum_{t=1}^{T} \sum_i \mu_{i,t} s_{i,t} - \sum_i \sum_{j \neq i} \mu_{j,t} q_{i,j,t} \\ & - \sum_i l_{i,t}\gamma_t \left(\sum_i l_{i,t} \right) + U_{i,t}(d_{i,t}) \\ \text{subject to} \quad & (3)\text{–}(10) \\ \text{var :} \quad & \mu_{j,t} \geq 0 \quad \forall j. \end{aligned} \tag{12}$$

Note from the constraint in (7) that $\sum_j q_{j,i,t} = s_{i,t}$ and $q_{i,i,t} = 0$. Hence, we have

$$\begin{aligned} \sum_i \mu_{i,t} s_{i,t} - \sum_i \sum_{j \neq i} \mu_{j,t} q_{i,j,t} &= \sum_i \mu_{i,t} - \sum_j \sum_{i \neq j} \mu_{i,t} q_{j,i,t} \\ &= 0. \end{aligned} \tag{13}$$

Thus, the optimization problem the platform solves is the following:

$$\begin{aligned} \mathcal{P}_{\text{plat}} : \text{maximize} \quad & \sum_{t=1}^{T} \sum_i U_{i,t}(d_{i,t}) - \sum_i l_{i,t}\gamma_t \left(\sum_i l_{i,t} \right) \\ \text{subject to} \quad & (3)\text{–}(10) \\ \text{var :} \quad & \mu_{j,t} \geq 0 \quad \forall j. \end{aligned} \tag{14}$$

Note that though $\mu_{j,t}$ does not appear in the objective value, $\mu_{j,t}$ controls the amount of energy to be exchanged among the prosumers as prosumer i maximizes P_i. Thus, $|mu_{j,t}$ inherently depends on the decisions of the prosumers. However, the platform does not know exact utility parameter and the user specific parameters such as the renewable energy generation and the battery capacity of the prosumers. In Section IV-C, we show how the prosumers should select the prices, which will optimize the above problem.

IV. Solution Methodology

Each prosumer is a selfish entity. It is only entitled to maximize its own payoff. The prosumer's payoff inherently depends on the other prosumers' strategies. We, thus, formulate the problem of prosumer's decision as a game-theoretic model (see Section IV-A). We show that the problem is coupled constrained game, since the constraints of the prosumers are coupled. We seek to obtain the generalized Nash equilibrium. In Section IV-B, we show that the game admits a concave potential function, and thus, there exists a unique generalized Nash equilibrium. Leveraging on the concave potential game, we propose a distributed algorithm (see Section IV-C), where each prosumer updates its strategy based on the decision taken by the other prosumers in the previous period. However, the equilibrium is *not efficient* as it does not solve the problem, where the objective is to maximize the sum of the payoffs of the prosumers. We show that as the renewable energy generation increases, the efficiency increases.

A. Generalized Nash Equilibrium

We, first, start with a notation which we use throughout.

Definition 1: Let a_i be the strategy of player i, and a_{-i} be the strategy vector of all players except player i.

The Nash equilibrium is generally to describe the equilibrium strategy profile for strategic users. In the following, we define the Nash equilibrium.

Definition 2 (see [24]*):* The strategy profile (a_i, a_{-i}) is a Nash equilibrium strategy profile if the following holds: $E[u_i(a_i, a_{-i})] \geq E[u_i(a'_i, a_{-i}]$ for all i and $a'_i \in S_i$, where S_i is the set of strategies of the player i.

Specifically, in a Nash equilibrium strategy profile, any player cannot have higher payoff by deviating unilaterally from the prescribed strategy profile. We next introduce a class of games known as a coupled constrained game.

Definition 3: If the strategy space of a player depends on the other players, the game is known as a coupled constrained game. The generalized Nash equilibrium is the corresponding equilibrium concept in the coupled constrained game.

Note that because of the constraints in (7), the strategy space of a prosumer also depends on the strategy of other prosumers. Hence, the our model belongs to the coupled constrained game. A generalized Nash equilibrium is used as an equilibrium concept for the coupled constrained game. We now define the generalized Nash equilibrium.

We denote the set of strategy of prosumer i as $\mathbf{a}_{i,t}$, where $\mathbf{a}_{i,t} = \{d_{i,t}, \mathbf{Q}_{i,t}, q_{i,t}, s_{i,t}, e_{i,t}, b_{i,t}, l_{i,t}\}$. $\mathbf{Q}_{i,t}$ consists of the component $q_{i,j,t}$ ($s_{i,j,t}$, respectively) for all j.

Definition 4: Let $\mathcal{A}_i$ be the set of feasible solution of P_i. A strategy profile $\mathbf{a}_t^* = (\mathbf{a}_{1,t}, \ldots, \mathbf{a}_{N,t})$ is a generalized Nash equilibrium if $\mathbf{a}_t^* \in \mathcal{A}_i$ for all i, and $\mathbf{a}_t^*$ is a Nash equilibrium strategy profile.

In many games, a Nash equilibrium may not exist. Even if the Nash equilibrium exists, the question arises whether it is unique. If it is unique, the question is whether it is practically implementable. In the following section, we answer the above questions in an affirmative sense by applying the theory of potential game [25], [26].

B. Potential Game

We, now, show that the game defined in the previous section, where user i maximizes P_i, is a potential game. First, we introduce the definition of the potential game.

Definition 5: Suppose user i's payoff function is $u_i(a_i, a_{-i})$, where a_i is the strategy of user i, and a_{-i} denotes strategies of other users apart from user i. Then, a game is potential if and only if there exists a function Φ such that $u_i(a'_i, a_{-i}) - u_i(a_i, a_{-i}) = \Phi(a'_i, a_{-i}) - \Phi(a_i, a_{-i}), \forall i$ [25].

If $\Phi(\cdot)$ is concave (strictly), then the game is (strictly) concave potential game [25].

In the following, we show that the game we introduced is a potential game.

Theorem 1: The game is potential. If $U_i(\cdot)$ is strictly concave $\forall i$, then it is a strictly concave potential game, and thus, it has a unique pure generalized Nash equilibrium.

Proof: Let us assume that $u_i(a_i, a_{-i}) = \sum_{t=1}^{T} \mu_{i,t} s_{i,t} + U_{i,t}(d_{i,t}) - \sum_{j \neq i} \mu_{j,t} q_{i,j,t} - l_{i,t}\gamma_t(\sum_i l_{i,t})$, where a_i is itself a vector consisting of $d_{i,t}, l_{i,t}, q_{i,t}, s_{i,t}, e_t, b_t$. Now, consider the following function:

$$\Phi(a_i, a_{-i}) = \sum_{t=1}^{T} \Bigg[\sum_i \mu_i s_{i,t} + \sum_i U_{i,t}(d_{i,t}) - \sum_i \sum_{j \neq i} \mu_j q_{i,j,t} - \sum_i \gamma_t l_{i,t}^2 - \gamma_t \sum_i \sum_{j > i} l_{i,t} l_{j,t} \Bigg]. \tag{15}$$

$\Phi(\cdot)$ is a potential function since

$$u_i(a', a_{-i}) - u_i(a_i, a_{-i}) = \Phi(a', a_{-i}) - \Phi(a_i, a_{-i}). \tag{16}$$

$\Phi(\cdot)$ is concave since $U_{i,t}(\cdot)$ is concave.

Since the potential function is strictly concave, there exists a unique solution. Hence, the generalized Nash equilibrium is unique. ∎

Thus, the equilibrium of the game is given by the optimal solution of the potential function subject to the constraints.

We, now, state the optimization problem, the solution of which will give the generalized Nash equilibrium:

$$\begin{aligned} P_{\text{potential}} : \text{maximize} \quad & \sum_{t=1}^{T} \Bigg[\sum_i \mu_i s_{i,t} + \sum_i U_{i,t}(d_{i,t}) \\ & - \sum_i \sum_{j \neq i} \mu_j q_{i,j,t} - \sum_i \gamma_t l_{i,t}^2 \\ & - \gamma_t \sum_i \sum_{j > i} l_{i,t} l_{j,t} \Bigg] \\ \text{subject to} \quad & (3)\text{–}(11) \\ \text{var :} \quad & s_{i,t}, q_{i,j,t}, d_{i,t}, e_{i,t}, b_{i,t} \geq 0. \end{aligned} \tag{17}$$

Note from (7) that

$$\sum_i \mu_i s_{i,t} = \sum_{j\neq i}\sum_i \mu_i q_{j,i,t}. \tag{18}$$

Hence, $P_{\text{potential}}$ can be written as

$$P_{\text{potential}} : \text{maximize} \quad \sum_{t=1}^{T}\left[\sum_i U_{i,t}\left(d_{i,t}\right) - \sum_i \gamma_t l_{i,t}^2 - \gamma_t \sum_i \sum_{j>i} l_{i,t} l_{j,t}\right]$$
$$\text{subject to} \quad (3)\text{–}(11)$$
$$\text{var}: \quad s_{i,t}, q_{i,j,t}, d_{i,t}, e_{i,t}, b_{i,t} \geq 0. \tag{19}$$

Intuitively, the payment made by a buyer is received by a seller. Thus, the total payoffs of the prosumers were devoid of the total exchange payment made in the exchange market. Since the potential game is concave, the solution of the problem can be obtained using the convex optimization tool.

C. Distributed Solution

Though $P_{\text{potential}}$ is a convex optimization problem, one needs to know each prosumer's utility and constraints. Thus, a centralized solution is difficult to obtain in practice. In this section, we show that how a distributed algorithm converges to the generalized Nash equilibrium.

Each user updates its strategy for a certain time horizon T, which we denote as epoch. At epoch k, the prosumer decides for the time slots $kT+1,\ldots,(k+1)T$. The platform initially selects exchange prices $\mu_{j,t}$ for each prosumer j. The prosumer then updates its strategy profile for each time slot in the epoch. The platform then again updates the price until the process converge. The algorithm **ALGO-DIST** is detailed in the following

ALGO-DIST:

1) Initialization: For each prosumer i, the price is set at $\mu_{i,t}^1$ at a some minimum possible value.
2) At iteration $k \geq 1$, each prosumer $i = 1,\ldots,N$ updates its strategy by obtaining $\mathbf{a}_{i,t}^k = (l_{i,t}^k, d_{i,t}^k, q_{i,t}^k, s_{i,t}^k, e_{i,t}^k, b_{i,t}^k)$, while solving the following problem:

$$\text{maximize} \sum_{t=1}^{T} \mu_{i,t}^k s_{i,t} + U_{i,t}(d_{i,t}) - \sum_{j\neq i} \mu_{j,t}^k q_{i,j,t} - l_{i,t}\gamma_t l_{i,t} - \gamma_t \sum_{j\neq i} l_{i,t}^{k-1} U_{i,t}(d_{i,t}) - 1/\alpha_k \sum_{t=1}^{T} \left\|\mathbf{a}_{i,t} - \mathbf{a}_{i,t}^{k-1}\right\|^2$$

subject to (3)–(6), (8)–(11).

3) If $s_{i,t} < \sum_{j\neq i} q_{j,i,t}$, set $\mu_{i,t}^{k+1} = \mu_{i,t}^k + \epsilon$; if $s_{i,t} \geq \sum_{j\neq i} q_{j,i,t}$, set $\mu_{i,t}^{k+1} = \mu_{i,t}^k$, and go to step 2.
4) If $s_{i,t} \geq \sum_{j\neq i} q_{j,i,t}$ for all i, then exit.
5) The user i pays $l_{i,t}^k \gamma_t \sum_j l_{j,t}^k$ at time t to the grid and obtains the value of other users' load.

Here, α_k is the learning parameter, and it is set at $1/(k+1)$. The parameter puts a weight α_k on the optimal decision of the current stage. The above also ensures that the optimal decision at the current iteration is not very far away from the decision of the previous iteration.

Note that a prosumer does not have a constraint in (7), since the user does not know the amount to be sold by other prosumers. Rather, the exchange amount is set in a dynamic fashion, where each prosumer updates the amount of energy it wants to buy or sell depending on the price. The price converges to the value, where the market is clear. This is because the price is reduced till the point the supply is enough to satisfy the demand for each prosumer in the exchange market. In other words, the constraint in (7) is always satisfied in the converged solution.

The following shows that the above algorithm converges to an equilibrium.

Theorem 2: The distributed algorithm **ALGO-DIST** terminates after a finite number of iterations. The solution converges to the optimal solution of the potential game for small enough $\epsilon > 0$.

The constraint in (7) is satisfied in the converged solution.

The proof of the above theorem readily follows from the Jacobi algorithm [27]. Hence, we omit it here.

It is apparent that as ϵ becomes smaller, the algorithm converges to the solution of the potential game. Specifically, the algorithm converges to the generalized Nash equilibrium.

Note that in the algorithm, each prosumer only needs to know the load of the other users during the time slots $(k-1)T+1,\ldots,kT$ for deciding its own decision. The prosumer can obtain the information from the blockchain, where each prosumer updates its strategy. It does not need to know the utility functions of the other users, their energy surplus, or even the battery capacities. Thus, the algorithm is easy to implement, and yet, it converges to the generalized Nash equilibrium.

The user uses the data over the last epoch to obtain its own decision in the current epoch. Surprisingly, a simple algorithm converges to an equilibrium strategy. The above type of strategy, where a prosumer learns about the strategy of the other prosumers, is known as *fictitious play*. Since the potential function is strictly concave, the convergence is exponential [28].

The platform updates the price for each prosumer depending on the supply and demand for each prosumer. If the supply exceeds demand for a prosumer, it will decrease the corresponding price. On the other hand, if the demand is higher than the supply, it will increase the price.

Note that each prosumer knows the value of γ_t while updating its decision. γ_t is set by the grid or the ISO on behalf of the grid and changes in a longer time scale. Since the estimate of the renewable energy generation changes in real time, the prosumer can update its strategy at a minute scale. The convergence is exponential; thus, the algorithm can be adopted dynamically to obtain the prices. The initial price can be set at the lowest possible price for the conventional energy, which will ensure that the prosumers always try to buy from the exchange market.

The exchange prices are dynamic and are different for each prosumer. In the numerical section, we show that during the peak time, the conventional energy price is likely to be more.

The prosumers will be more likely to sell energies during the peak time. The above has threefold advantages. First, the prosumers will earn more. Second, the peak conventional energy demand will also reduce. Finally, the price for the conventional energy will also go down.

We consider a posted price mechanism, which is inherently truthful. The prices correspond to the Lagrangian of a convex optimization problem. A prosumer only selects the amount of energy to be sold or to be bought. Thus, a prosumer does not have any market power unlike the scenario where the distributed generators are bidding.

D. Efficiency of the Equilibrium

We, now, investigate the efficiency of the generalized Nash equilibrium obtained in the last section. Our result shows that the game is not efficient. *Specifically, the equilibrium strategy profile of the game is not optimal for maximizing the sum of the profits of the users.* In the following, we compute the efficiency ratio.

Let

$$P: \text{maximize} \quad \sum_{t=1}^{T}\sum_{i}\left[\mu_{i,t}s_{i,t} + U_{i,t}\left(d_{i,t}\right) - \sum_{j\neq i}\mu_{j,t}q_{i,j,t} - l_{i,t}\gamma_t\left(\sum_i l_{i,t}\right)\right]$$
$$\text{subject to} \quad (3)\text{–}(11)$$
$$\text{var}: \quad s_{i,t}, q_{i,j,t}, d_{i,t}, e_{i,t}, b_{i,t} \geq 0. \tag{20}$$

Since $\sum_i l_{i,t}\gamma_t(\sum_i l_{i,t}) = \gamma_t(\sum_i l_{i,t})^2$, and using (13), P can be equivalently represented as

$$P: \text{maximize} \quad \sum_{t=1}^{T}\left[\sum_i U_{i,t}\left(d_{i,t}\right) - \gamma_t\left(\sum_i l_{i,t}\right)^2\right]$$
$$\text{subject to} \quad (3)\text{–}(11)$$
$$\text{var}: \quad s_{i,t}, q_{i,j,t}, d_{i,t}, e_{i,t}, b_{i,t} \geq 0. \tag{21}$$

Note that the objective function of P denotes the sum of the payoffs of the prosumers. Thus, the optimal solution of P maximizes the sum of the prosumer's payoffs. Thus, P is the same as the platform's objective.

Let $\mathbf{a}_{i,t} = (l_{i,t}, s_{i,t}, q_{i,t}, b_{i,t}, e_{i,t}, d_{i,t})$ be the decision vector for each user i at time t. The problem P is a convex optimization problem as the objective function is concave, and the constraints are linear.

We, next, introduce some notations.

Definition 6: Let $\mathbf{a}^*_{i,t}$ be the optimal solution of the problem P. Let $\mathbf{a}^{eq}_{i,t}$ be the equilibrium strategy profile. Specifically, it is the optimal solution of $P_{\text{potential}}$ [cf., (19)].

Now, we are ready to characterize the efficiency ratio of the equilibrium strategy profile.

Definition 7: Let the objective value attained by the strategy profile $\mathbf{a}^{eq}_{i,t}$ in the problem P be P^{eq}, and let P^* be the optimal value of P [cf., (21)]. Then, the efficiency is

$$\eta = \frac{P^{eq}}{P^*}. \tag{22}$$

Note that since each prosumer selfishly optimizes its own payoff, the generalized Nash equilibrium does not necessarily maximize the sum of the payoffs of the prosumers. The efficiency is always less than or equal to 1 since $\mathbf{a}^{eq}_{i,t}$ may not be equal to the optimal solution of P. Note that in the equilibrium strategy profile, the user consumes larger energy as compared to the optimal solution from the grid because of the additional term.

Proposition 1: The efficiency η increases as the amount of renewable energy increases.

The above proposition shows that the efficiency increases when the renewable energy supply increases. Thus, as the renewable energy penetration increases, the equilibrium becomes closer to the optimal solution of the sum of the prosumers' utility maximization problem.

If the load to the grid increases, the efficiency decreases. However, the efficiency is never 0. It is always upper bounded above 0. However, as the number of user increases, the efficiency decreases.

V. Grid's Objective

In this section, we now consider how the grid should select γ_t. We consider that the grid incurs a cost $C_t(\cdot)$ for the conventional energy during time slot t. In general, the generators or conventional energy providers bid the selling price as a piecewise linear function. We assume that $C_t(\cdot)$ is a convex function.

The grid would like to increase the user's satisfaction. Thus, the grid's main objective is to maximize the total consumer's (or, prosumer's surplus). Thus, from (13), the grid wants to maximize

$$\sum_t\sum_i U_{i,t}\left(d_{I,t}\right) - \sum_t \gamma_t\left(\sum_i l_{i,t}\right)^2. \tag{23}$$

The grid also has to make sure that it pays the bidding price to the generators if the grid obtains conventional energy from the generator. The grid has to make sure that the system is at least *weakly budget balanced*, i.e., the total revenue generated from the users has to be greater than or equal to the payment made to the grid. In practice, the grid has to provide incentives to the conventional grid operators for the conventional energy. Thus, we consider that a profit of β amount has to be provided to the conventional energy providers for per unit of energy sold by them. Hence, the grid needs to solve the following problem:

$$\mathcal{P}_g \ \text{maximize}_{\ \gamma_t} \quad \sum_t\sum_i U_{i,t}\left(d_{I,t}\right) - \sum_t \gamma_t\left(\sum_i l_{i,t}\right)^2$$
$$\text{subject to} \quad C_t\left(\sum_i l_{i,t}\right) + \beta\sum_i l_{i,t} \leq \gamma_t\left(\sum_i l_{i,t}\right)^2 \ \forall t. \tag{24}$$

The constraint in (24) ensures that the revenue generated from the users must exceed the cost and the profit margin $\beta\sum_i l_{i,t}$.

Note that $l_{i,t}, d_{i,t}$ inherently depend on γ_t as it specifies the price one has to pay. Since $U_{i,t}(\cdot)$ is an increasing function, the objective will be maximized when $\gamma_t = 0$. However, setting $\gamma_t = 0$ will not satisfy the constraint unless $l_{i,t}$ is 0. In the following, we state the price strategy γ_t the grid should select to optimize the problem in $\mathcal{P}_g$.

Observation 1: Suppose $\gamma_t^* = \frac{C_t(\sum_i l_{i,t})+\beta\sum_i l_{i,t}}{(\sum_i l_{i,t})^2}$; it is an optimal solution of the problem in $\mathcal{P}_g$.

Proof: Note that the user's utility increases as the demand $d_{i,t}$ increases. $d_{i,t}$ increases as γ_t decreases. Hence, the objective is a decreasing function of γ_t. Thus, the price that optimizes the objective is the minimum price that satisfied the constraint. Hence, the result follows. $\square$

Note that the grid selects γ_t before the exchange of energy among the prosumers occur. Hence, the grid does not know the exact amount of exchange, which will take place before selecting γ_t. Thus, the grid is also unaware of the total load $l_{i,t}$ *a priori*.

Thus, we consider the following problem where the grid optimizes the expected value of the objective value:

$$\mathcal{P}_g : \text{maximize}_{\gamma_t} \ \sum_t \sum_i U_i(d_{I,t}) - \sum_t \gamma_t E\left[\left(\sum_i l_{i,t}\right)^2\right]$$
$$\text{subject to} \quad E\left[C_t\left(\sum_i l_{i,t}\right)\right] + \beta E\left[\sum_i l_{i,t}\right] \le \gamma_t E\left[\left(\sum_i l_{i,t}\right)^2\right]. \tag{25}$$

The expectation is taken over the distribution of $\sum_i l_{i,t}$, which is a random entity because of the uncertainty of the demand, and the renewable energy. However, the grid may be able to predict the demand from the historical data.

Since $C_t(\cdot)$ is convex, thus, note from Jensen's inequality that

$$E\left[C_t\left(\sum_i l_{i,t}\right)\right] \ge C_t\left(E\left[\sum_i l_{i,t}\right]\right). \tag{26}$$

Hence, if we solve the following optimization problem:

$$\mathcal{P}_{\exp} : \text{maximize}_{\gamma_t} \ \sum_t \sum_i U_i(d_{I,t}) - \sum_t \gamma_t E\left[\left(\sum_i l_{i,t}\right)^2\right]$$
$$\text{subject to} \quad C_t\left(E\left[\sum_i l_{i,t}\right]\right) + \beta E\left[\sum_i l_{i,t}\right] \le \gamma_t E\left[\left(\sum_i l_{i,t}\right)^2\right] \tag{27}$$

a solution of $\mathcal{P}_g$ is a feasible solution of $\mathcal{P}_{\exp}$. Thus, the optimal value of $\mathcal{P}_{\exp}$ provides an upper bound of $\mathcal{P}_g$. Computing $C_t(E[\sum_i l_{i,t}])$ is much easier compared to $E[C_t(\sum_i l_{i,t})]$, since we do not have to consider expectation over the parameter values. We compute $C_t(E[\sum_i l_{i,t}])$ by knowing the expected total load. Hence, we focus on solving $\mathcal{P}_{\exp}$.

Similar to Observation 1, the optimal γ_t the grid has to select is the following.

Observation 2: Suppose $\gamma_t^* = \frac{C_t(E[\sum_i l_{i,t}])+\beta E[\sum_i l_{i,t}]}{E[\sum_i l_{i,t}^2]}$; it is an optimal solution of the problem in $\mathcal{P}_{\exp}$ [cf., (27)].

Remark 1: Note that if $C_t(x) = \alpha_t x^2$ for some $\alpha_t > 0$, from Observation 2, $\gamma_t = \alpha_t + \frac{\beta E[\sum_i l_{i,t}]}{E[\sum_i l_{i,t}^2]}$. If variance is small, γ_t is small when the expected value of the total load is large. Thus, γ_t should be small when the total load is large (peak time). However, the price for the conventional energy can still be high. Since the price p_t for the conventional energy is $\gamma_t E[\sum_i l_{i,t}]$, the price for the peak period may still be high even though γ_t is small. In fact, if variance is small, the price for the peak period will always be high.

VI. Numerical Analysis

In this section, we show the equilibrium load profile, the reduction of the conventional energy consumption, and the efficiency of the generalized Nash equilibrium in an example setting.

A. Parameter Setup

The conventional energy generation incurs a cost to the grid. We assume that the cost function $C(\cdot)$ is a piecewise quadratic function

$$C(L) = \begin{cases} a_1 L^2 + c_1, & \text{if } L \le L_1 \\ a_2 L^2 + c_2, & \text{else.} \end{cases}$$

For the simulation, L_1 is assumed to be 2 MW. a_1 and c_1 are taken as 15\$/MWh^2 and 1\$, respectively. a_2 and c_2 are taken as 20\$/MWh^2 and 1\$, respectively. The utility function of the prosumers is defined in the following manner:

$$U_{i,t}(d) = \alpha_t d - \zeta d^2 \tag{28}$$

where α_t is a time-dependent random parameter, which has a Gaussian distribution of mean 0.3\$/kW during the off-peak period (for $t \in [6, 9]$ and $t \in [13, 15]$) and has a Gaussian distribution of mean 0.6\$/kW during the peak period (for $t \in [9, 12]$ and $t \in [16, 18]$). α_t has a variance of 30 cents for all t. ζ is assumed to be 0.1 \$/kWh^2.

We assume that the renewable energy is one-sided normally distributed with a mean of 2 kW and a variance of 1 kW^2 [29], [30]. We assume that the battery capacity is assumed to be 10 kWh. We use the algorithm **ALGO-DIST** with $\epsilon = 10^{-6}$ to obtain the solution for our approach.

The transmission loss is assumed to be symmetric and equal for each prosumer. Specifically, $r_{i,j} = 0.9$ for all i and j. The transmission loss between the grid and prosumers is also assumed to be the same with $r_i = 0.8$ for all i. The exchange market operates every 15 min. The time horizon T is set at 24 h.

Setting γ_t: We run the simulation for 14 consecutive days or 336 h. Since T is 24 h, thus γ_t is changed after each 24 h. We, now, describe how γ_t is set at the beginning of each day. γ_t is set according to Observation 2, where the expected value of load $l_{i,t}$ is taken to be the average of all the total load during time t at the previous days. For example, the expected value of $l_{i,t}$ for day 8 is considered to be the mean value of $l_{i,t}$ for

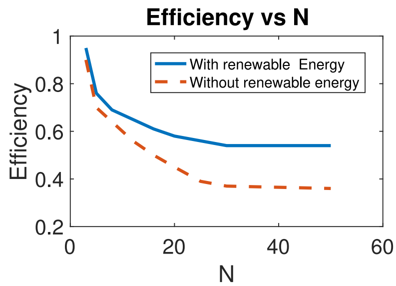

Fig. 2. Variation of efficiency of user's equilibrium strategy profile.

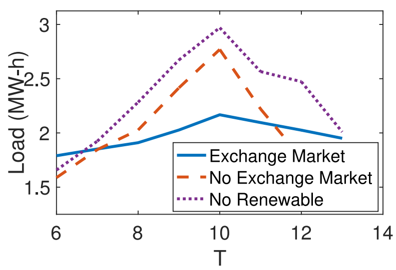

Fig. 3. Variation of the demand of conventional energy with T with exchange market and no exchange market for $N = 50$. The time $[9, 12]$ is the peak period; rest is the off-peak period.

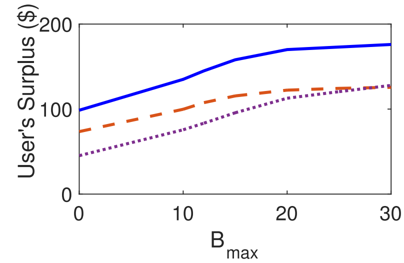

Fig. 4. Variation of social welfare when there is an exchange market and when there is no exchange market with the battery capacity.

seven days. β is assumed to be 5\$/MWh. Initial estimate of $l_{i,t}$ is assumed to be 20 kWh for the peak period (for $t \in [9, 12]$ and $t \in [16, 18]$) and $l_{i,t}$ is assumed to be 10 kWh for the off-peak period.

B. Results

1) Efficiency With the Number of Prosumers: Our first result (see Fig. 2) shows the efficiency of the equilibrium strategy profile with the number of prosumers. As the number of prosumers increases, the efficiency decreases; however, the decrement slows down when the number of prosumers exceeds a certain threshold.

Fig. 2 also shows that if the renewable energy generation is zero, the efficiency is much less. Note that Proposition 1 states that if the renewable energy generation is small, the efficiency decreases. Fig. 2 shows that the empirical findings are in accordance with the theoretical results.

2) Impact of Exchange Market on the Peak Load: Fig. 3 shows that the exchange market reduces the peak demand by at least 40%. When there is no exchange market, we assume that the prosumers solve the problem, with $\mu_{i,t}$ being set at a high value for all i and t in the optimization problem $\mathcal{P}_{\text{exp}}$ [cf., (27)]. Thus, the prosumers can only sell excess energies to the grid or use it at later time by storing in their batteries. However, even the prosumers have the storage device, the exchange market can greatly enhance the reduction of the peak load. This is because the prosumers can sell energies to the other prosumers in the peak time, which reduces the peak load. The exchange prices are also high during the peak time, as the demand to the prosumers increases during the peak time.

Fig. 3 also shows that if the renewable energy generation is zero, the demand of the conventional energy is higher. Intuitively, if the renewable energy is zero, all the prosumers are consumers; hence, the demand for conventional energy is higher.

3) Increase of User's Surplus: The total user's surplus is represented in the objective of the problem $\mathcal{P}_{\text{exp}}$ [cf., (27)]. Fig. 4 shows the variation user's surplus importance of the exchange market as compared to the one where there is no exchange market. Fig. 4 shows that the user's surplus is at least 25% higher when there is no exchange market. We assume that when there is no exchange market, the prosumers optimize, with $\mu_{i,t}$ being set at a high value in the optimization problem $\mathcal{P}_{\text{exp}}$ [cf., (27)], which precludes any exchange among the prosumers. The reduction of the user's surplus is because of two factors. First, the transmission efficiency is poorer between the grid and the prosumers. Second, the price paid by the prosumers is much higher when there is no exchange market. As the storage capacity increases, the difference is more significant. However, when the storage capacity increases beyond a certain threshold, the difference decreases. This is because for higher storage, the prosumers become self-sufficient.

Fig. 4 shows that if the renewable energy generation is zero, the user's surplus is smaller than both scenarios where there is exchange market and no exchange market if the capacity of storage unit is small. Since each user has to buy all the energy from the grid if the renewable energy generation is zero, hence, the user's surplus decreases. However, if the capacity increases, when the renewable energy generation is zero, since the users may buy energy at the off-peak period and utilize it or exchange among each other when $B_{\max}$ increases. When the $B_{\max}$ is very large, the user's surplus becomes higher compared to the scenario where there is no exchange market. This is because when the storage is high, a user may store excess energy during the off-peak period and sell it to the other users in the exchange market. Thus, a user is now able to attain a higher profit. Hence, the user's surplus becomes higher than the scenario where there is no exchange market. The above shows that even if the renewable energy generation is small, the exchange market may increase the user's surplus if the capacities of the storage units at the users' premises are high.

4) Prices of the Grid: We now state the variation of prices over t during a day. Note that the meant price $p_t = \gamma_t E[\sum_i l_{i,t}]$, where $E[\sum_i l_{i,t}]$ is the estimated total load. Fig. 5 shows the variation of price per kilowatthour. Note that though the peak period prices are high in the exchange market, the ratio of the peak period and the off-peak period is only around 1.4. However, the ratio of the peak period and off-period prices is almost 2 (similar to the current practice in Pacific Gas & Electricity) for the market where there is no exchange. Fig. 5 shows that the

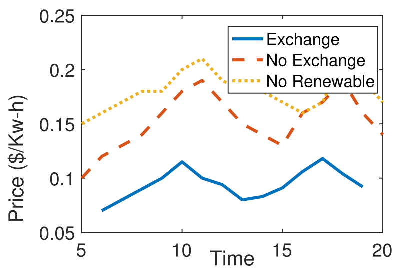

Fig. 5. Variation of prices in ($/kWh) with time for the exchange market and without any exchange market. The peak periods are $[9, 12]$ and $[16, 18]$.

prices are reduced in our proposed tiered market compared to the scenario where there is no exchange.

Fig. 5 shows that if there is no renewable energy, the prices are higher, since the users now have to obtain energy from the grid.

VII. Conclusion and Future Work

We consider a system of prosumers that can share their excess energies among themselves in addition to buying conventional energy from the market. We consider a linear dynamic price function for the conventional energy, where the price for the conventional energy depends on the total demand for the conventional energy. The optimal strategy of a prosumer depends on the strategies of other prosumers. We, thus, model the strategy selection problem of a prosumer as a game-theoretic problem. The strategy space of a prosumer inherently depends on the strategy of other prosumers, since the amount of energy a prosumer can buy inherently depends on the amount of energy other prosumers want to sell. We, thus, resort to the generalized Nash equilibrium as an equilibrium concept. We show that the game admits a concave potential function. We propose a distributed algorithm, where each prosumer only selects its own optima strategy. The platform selects the exchange price for each prosumer. A prosumer does not need to know the other prosumer's utilities. It only needs to know the strategy taken by a prosumer in the previous iteration to find its own in the current iteration. The platform updates the price depending on the supply and the demand. We show that the distributed algorithm converges to the unique generalized Nash equilibrium. We also integrate the grid's decision with the exchange market and compute how the grid should select price, which will also maintain a fixed profit to the conventional generators. We show that our approach reduces the total consumption of the conventional energy and the peak load. The prosumers' total social welfare also increase.

Note that when there are multiple locations, we need to consider the Kirchoff's laws and the reactance constraints. The characterization of a GNE when the exchange market operates over several locations is left for the future. We consider a real-time price, which varies linearly with the total demand. The characterization of the generalized Nash equilibrium for other nonlinear function and its impact on the efficiency ratio also constitutes an interesting research direction. The impact of the uncertainty of the renewable energy on the equilibrium strategy profile is also left for the future.

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