On Strategic Interactions in Blockchain Markets: A Three-stage Stackelberg Game Approach

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***Abstract*—Blockchain technology is a promising approach for solving the security and personal privacy problems in Internet applications. The successful commercial deployment of Blockchain markets relies on a comprehensive understanding of the economic and strategic interactions among different entities involved. In this paper, we focus on a blockchain market consisting of a blockchain platform (BP), multiple miners, and blockchain users (BUs), and formulate their interactions as a three-stage Stackelberg game. In Stage I, the BP strategizes the rewards granted to the miners, so as to attract the miners to contribute more computing power used for improving the security and privacy of the blockchain. In Stage II, each miner strategizes its computing power individually for winning the mining competition, which is modeled as a non-cooperative game. In Stage III, the BUs strategize the transaction fee to acquire a corresponding service experience. With the objective of utility maximization, we develop a theoretical framework to analyze the hierarchical interactive behaviors among the entities in a backward inductive way. By solving the Stackelberg equilibrium, we determine the optimal strategies of entities in closed-form. Numerical results are provided to demonstrate the performance of the strategic interactions in the blockchain market.**

***Index Terms*—Blockchain market, strategic interactions, Stackelberg game, equilibrium, PoW competition.**

I. Introduction

The world-changing blockchain technology is a promising approach for solving the security and personal privacy problems in Internet applications, via establishing distributed tamper-proof ledgers and platforms [1]. As the most famous blockchain application, Bitcoin was invented by Nakamoto in 2008, which adopts a distributed consensus mechanism proof-of-work (PoW) to achieve trusted interactions in distributed systems without the involvement of any third party [2]. With the increasing popularity of Bitcoin, as well as due to the capability of providing low-cost, efficient, and secure data storage in distributed systems, blockchain technology has been attracting tremendous attention from both industry and academia.

Over the last decade, great efforts have been made to study blockchain technology from diverse perspectives, e.g., security and privacy protection, consensus protocol design, blockchain applications, etc. Although these existing works have remarkably promoted the development of blockchain, the economic and strategic interactions among different blockchain entities still remain largely uninvestigated. Actually, such interactive behaviors are pervasive in practical blockchain markets. For example, miners join the block discovery competition for financial incentives, while users pay fees for packing their transactions into the blockchain. Therefore, to facilitate the successful commercial deployment of Blockchain markets, a comprehensive understanding of the entities' interactions is of great significance.

By now, some preliminary works have been conducted to explore the blockchain entities' interactions. Specifically, Feng *et al.* [3] proposed a cyber risk management approach for the blockchain market by modeling the interactions between the blockchain provider and users. Considering the competition among miners, Jiao *et al.* [4] designed an auction-based edge computing allocation mechanism to support the mobile blockchain. Taghizadeh *et al.* [5] exploited the mean-field game theory to analyze the mining computational power equilibrium of miners in a PoW-based blockchain network. Focusing on the interactions between miners and users, Liu *et al.* [6] explored the economics of blockchain storage and identified some incentive issues related to the storage cost. More recently, Ding *et al.* [7] designed an incentive mechanism that drives IoT devices to purchase more computational power to join the mining process, so as to build a secure blockchain network while guaranteeing the profits of the blockchain platform.

In this paper, we focus on a PoW-based blockchain market composed of a blockchain platform (BP), miners and blockchain users (BUs). To guarantee the security and privacy of the blockchain, the BP attracts miners to contribute more computing power by granting the PoW winner rewards. For winning the PoW competition, each miner strategizes its computing power individually. The winner not only receives the BP's rewards but also charges transaction fees from the BUs for packing their transactions into the blockchain. The BUs strategize the transaction fee to acquire a corresponding service experience. We formulate these strategic interactions

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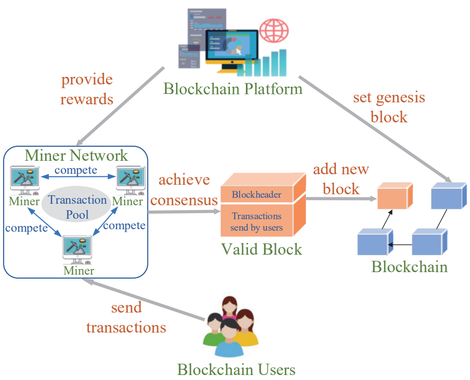


Fig. 1. PoW-based Blockchain System.

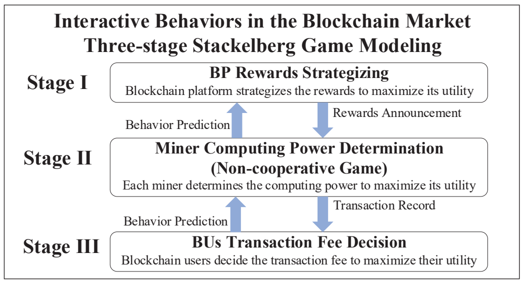


Fig. 2. Three-stage Stackelberg game modeling for interactive behaviors within the blockchain market.

among entities as a three-stage Stackelberg game. By analyzing the Stackelberg equilibrium in a backward inductive way, we determine the optimal strategies which enable the entities to achieve their maximal utilities.

The contributions of this paper are summarized as follows.

- We propose a three-stage Stackelberg game modeling to formulate the strategic interactions among the BP, miners, and BUs in a blockchain market.
- We develop an analytical framework to solve the Stackelberg equilibrium, such that the optimal strategies of all entities to be determined in closed form. In particular, the PoW competition is modeled as a non-cooperative game, and the existence and uniqueness of its Nash equilibrium are proved theoretically.
- We provide numerical results to demonstrate the strategic interactive behaviors among the blockchain entities. The results could give guidelines for the economic operations in practical blockchain markets.

The rest of this paper is organized as follows. Section II describes the system model and formulates the Stackelberg game. We analyze the Stackelberg equilibrium in Section III. Numerical results are provided in Section IV, followed by the conclusion in Section V.

II. System Model and Stackelberg Game Formulation

In this section, we first introduce the PoW-based blockchain system model involved in this study, and then formulate the interactive behaviors among entities in the system as a three-stage Stackelberg game.

A. PoW-based Blockchain System

As illustrated in Fig. 1, we consider a PoW-based blockchain service market that consists of three types of entities, i.e., a blockchain platform (BP), multiple miners, and blockchain users (BUs). In the PoW-based blockchain service market, the BP maintains the PoW consensus and offers blockchain services to the BUs. The larger computing power the whole blockchain market owns, the harder it is for attackers to tamper with the blockchain and then the more secure the BP will be. Therefore, to realize sustainable development for the blockchain market, the BP encourages the miners to adopt larger computing power, by providing rewards (i.e., incentive payments) to the miner who wins the PoW competition. Let R, $\mathcal{M} = \{m_1, m_2, \cdots, m_N\}$ and P_{m_i} denote the value of rewards, the set of miners and the computing power of the i-th miner, respectively, where N represents the number of miners in the blockchain market. The PoW consensus scheme deploys a transaction pool to deposit transactions sent by the BUs. The miner who wins the PoW competition is able to satisfy the requirements of BUs for recording transactions to the blockchain, and it has the right to charge a fee for every transaction. We assume the number of transactions included in a block is constant and the fee paid by BUs for recording every transaction is the same, which are denoted as B and f_u, respectively. The BUs expect a low delay, denoted as d_u, for adding the block which contains their transactions into the blockchain, and d_u is assumed to be inversely proportional to the fee f_u paid by BUs.

B. Stackelberg Game Formulation

As illustrated in Fig. 2, we model the interactive behaviors among the BP, miners, and BUs in the blockchain market as a three-stage Stackelberg game [8], [9].

1) Stage I (BP Rewards Strategizing): For security enhancement, the BP stimulates the miners to contribute their computing power by granting rewards to the miner who wins the PoW competition. Thus, the benefit that BP receives increases as the miners' computing power increases. However, the incremental speed of BP's benefit should gradually diminish when the computing power gets larger, due to the generic economic law of "diminishing marginal return" [10]. Therefore, we formulate the BP's utility function as

$$U_p = \frac{\lambda}{1 + \frac{1}{\sum_{m_i \in \mathcal{M}} P_{m_i}}} - R, \tag{1}$$

where λ is a positive system parameter that reflects the degree of importance of miners' computing power viewed by the BP. By strategizing the value of rewards R appropriately, the BP aims to achieve its maximal utility.

2) Stage II (Miner Computing Power Determination): Miners join the PoW competition by contributing their computing power. Thus, the strategy of miner m_i is its computer power

P_{m_i}. We use $\mathbf{P}_m = \{P_{m_1}, P_{m_2}, \cdots, P_{m_N}\}$ to denote the strategy profile including all miners' strategies, and $\mathbf{P}_{-m_i}$ to denote the strategy profile excluding P_{m_i}, i.e., $\mathbf{P}_m = (P_{m_i}, \mathbf{P}_{-m_i})$. We consider that the probability of winning the PoW competition is proportional to the computing power a miner adopts, and the unit cost of computing power is c. Then, taking both the return (i.e., BP's rewards and BUs' transaction fee) and cost (i.e., power consumption) into consideration, we formulate the utility function U_{m_i} of miner m_i as

$$U_{m_i} = \frac{P_{m_i}}{\sum_{j\in\mathcal{M}} P_{m_j}}(R + Bf_u) - cP_{m_i}. \tag{2}$$

Note that in the PoW competition, miners compete with each other individually to maximize their own utilities by determining computing power strategically. Thus, the PoW competition among miners can be modeled as a non-cooperative game. We use $G = (\mathcal{M}, \{\mathbf{P}_m\}, \{U_{m_i}\})$ to denote the PoW competition, where $\mathcal{M}$, $\{\mathbf{P}_m\}$, $\{U_{m_i}\}$ represent the sets of players (i.e., miners), strategy profiles and utility functions, respectively. A fundamental problem arising from the non-cooperative game is that whether there is a stable strategy profile such that no miner can gain more utility by unilaterally changing its current strategy. The stable strategy profile corresponds to the concept of Nash equilibrium (NE) in Game Theory [11], which can be defined as follow.

Definition 1 (Nash Equilibrium): A strategy profile $\mathbf{P}_m^{ne} = (P_{m_1}^{ne}, P_{m_2}^{ne}, \cdots, P_{m_N}^{ne})$ is a Nash Equilibrium of the PoW competition, if for any miner m_i and any strategy $P_{m_i} \geq 0$ we have

$$U_{m_i}(P_{m_i}^{ne}, \mathbf{P}_{-m_i}^{ne}) \geq U_{m_i}(P_{m_i}, \mathbf{P}_{-m_i}^{ne}). \tag{3}$$

3) Stage III (BUs Transaction Fee Decision): The BUs pay fees to the miner for packing their transactions into the blockchain. The corresponding delay d_u is inversely proportional to the transaction fee f_u, i.e., $d_u = \frac{\theta}{f_u}$, where $\theta \in (0, 1]$ is a parameter that controls the BUs' sensitivity to the delay for packing the transactions into the blockchain. That is, BUs with a larger θ are more sensitive to the delay and need to pay more fees to receive a satisfactory effect. By following [12], we formulate the BUs' utility function as

$$U_u = \alpha \ln(\beta - d_u) - f_u = \alpha \ln(\beta - \frac{\theta}{f_u}) - f_u, \tag{4}$$

where α and β are positive system parameters, and the first term in logarithmic form is widely adopted to evaluate the satisfaction degree of BUs. Equation (4) indicates the BUs' satisfaction degree is negatively correlated with the service delay and will deteriorate more seriously as the delay becomes higher, which is in accordance with the intuition. By deciding an appropriate transaction fee f_u, the BUs aim to achieve the maximal utility.

III. Stackelberg Equilibrium Analysis

In this section, we analyze the Stackelberg equilibrium (SE) for the three-stage Stackelberg game in a backward inductive way. Specifically, we first analyze the BUs' optimal strategy in Stage III. Then, we go backward and analyze the optimal strategy profile of miners in Stage II based on the result in Stage III. Finally, we reach Stage I and analyze the BP's optimal strategy. The joint results in the three stages constitute the SE.

A. Stage III: BUs' Optimal Strategy

The goal of BUs is to maximize their utility by deciding the optimal transaction fee strategically. Therefore, the utility maximization for BUs can be formulated as the following mathematical optimization problem:

$$\max_{f_u} \quad U_u(f_u) = \alpha \ln(\beta - \frac{\theta}{f_u}) - f_u \tag{5a}$$

$$\text{s.t.} \quad \frac{\theta}{\beta - 1} < f_u < \alpha \ln \beta, \tag{5b}$$

where the constraint (5b) follows from the non-negativity of the feasible domain of a logarithmic function as well as the non-negativity of the BUs' utility (otherwise, the BUs will not purchase the service from the blockchain market).

To solve the optimization problem (5), we utilize the method of Lagrange multipliers with KKT conditions. By introducing two Lagrange multipliers κ and γ as well as two slack variables η and ω, we construct the Lagrangian function for the above optimization problem as

$$\begin{aligned} L(f_u, \kappa, \gamma, \eta, \omega) = \alpha \ln(\beta - \frac{\theta}{f_u}) - f_u + \kappa(f_u - \frac{\theta}{\beta - 1} - \eta^2) \\ + \gamma(f_u - \alpha \ln \beta + \omega^2). \end{aligned} \tag{6}$$

By setting the first-order derivative of L with respect to each variable equal to zero, we obtain a system of equations. Then, by solving the system of equations and mapping the solutions into the feasible domain, the optimal strategy of BUs, i.e., the optimal transaction fee f_u^* paid by BUs, can be decided as

$$f_u^* = \frac{\theta + \sqrt{\theta}\sqrt{\theta + 4\alpha\beta}}{2\beta}. \tag{7}$$

The detailed algebraic operations for solving the system of equations are omitted here due to the space limitation.

B. Stage II: Miners' Optimal Strategy Profile

The PoW competition among miners is modeled as a non-cooperative game $G = (\mathcal{M}, \{\mathbf{P}_m\}, \{U_{m_i}\})$. To achieve its maximal utility, each miner will adopt its best response strategy in the PoW competition, which is defined as follows.

Definition 2 (Best Response Strategy): Given $\mathbf{P}_{-m_i}$, a strategy is the best response strategy of miner m_i, denoted as $b_i(\mathbf{P}_{-m_i})$, if it satisfies $U_{m_i}(b_i(\mathbf{P}_{-m_i}), \mathbf{P}_{-m_i}) \geq U_{m_i}(P_{m_i}, \mathbf{P}_{-m_i})$ for all $P_{m_i} \geq 0$.

By reviewing the definition of NE, we know that the strategy a miner adopts in an NE is its best response strategy. Therefore, the strategy profile of all miners in the PoW competition will converge to an NE, if it exists. Let $\mathbf{b}(\mathbf{P}_m) = (b_1(\mathbf{P}_{-m_1}), b_2(\mathbf{P}_{-m_2}), \cdots, b_N(\mathbf{P}_{-m_N}))$, which is termed as the best response correspondence of the strategy profile $\mathbf{P}_m$. Then, an NE of the game is actually a fixed point of the best response correspondence, i.e., $\mathbf{P}_m^{ne} = \mathbf{b}(\mathbf{P}_m^{ne})$. Before proving the existence and uniqueness of NE in the PoW competition, we provide the following propositions.

Proposition 1: An NE exists in a non-cooperative game, if: 1) the set of strategy profiles of players is a non-empty, compact, and convex subset of the N-dimensional Euclidean space $\mathbb{R}^N$ and 2) the utility function of a player is concave on the strategy of this player, for every player in the game [13, Theorem 1].

Proposition 2: Function $\mathbf{b}(\mathbf{P}_m)$ is *standard* if for all $\mathbf{P}_m > 0$, the following properties are satisfied [14]:

- *Positivity*: $\mathbf{b}(\mathbf{P}_m) > 0$.
- *Monotonicity*: if $\mathbf{P}_m \geq \mathbf{P}_m^{'}$, then $\mathbf{b}(\mathbf{P}_m) \geq \mathbf{b}(\mathbf{P}_m^{'})$.
- *Scalability*: for all $\phi > 1$, $\phi\mathbf{b}(\mathbf{P}_m) > \mathbf{b}(\phi\mathbf{P}_m)$.

Proposition 3: If the function $\mathbf{b}(\mathbf{P}_m)$ is standard, then its fixed point is unique [15, Theorem 1].

Then, we have the following theorem.

Theorem 1: The PoW competition game has a unique NE.

Proof: We first prove the existence of NE and then prove the uniqueness of NE.

1) Existence: Note that the strategy of miner m_i is $P_{m_i} \geq 0$. Thus, the strategy space of the PoW competition game $\{\mathbf{P}_m\}$ is a non-empty, compact, and convex subset of the N-dimensional Euclidean space $\mathbb{R}^N$. Given the optimal strategy of BUs f_u^* and any strategy of BP R, we take the first- and second-order derivatives of U_{m_i} with respect to P_{m_i}, which yields

$$\frac{\partial U_{m_i}}{\partial P_{m_i}} = \frac{(R + Bf_u^*)\sum_{j\neq i} P_{m_j}}{(\sum_{j\in\mathcal{M}} P_{m_j})^2} - c, \tag{8}$$

$$\frac{\partial^2 U_{m_i}}{\partial P_{m_i}^2} = -\frac{2(R + Bf_u^*)\sum_{j\neq i} P_{m_j}}{(\sum_{j\in\mathcal{M}} P_{m_j})^3} < 0. \tag{9}$$

(8) shows U_{m_i} is continuous and differentiable on P_{m_i}, and (9) indicates the second-order derivative of U_{m_i} with respect to P_{m_i} is negative. Thus, U_{m_i} is a concave function of P_{m_i}. According to Proposition 1, the PoW competition game exists at least an NE.

2) Uniqueness: Since U_{m_i} is a concave function of P_{m_i}, we can obtain the best response strategy $b_i(\mathbf{P}_{-m_i})$ by solving $\frac{\partial U_{m_i}}{\partial P_{m_i}} = 0$, that is

$$\frac{(R + Bf_u^*)\sum_{j\neq i} P_{m_j}}{(\sum_{j\in\mathcal{M}} P_{m_j})^2} - c = 0, \tag{10}$$

$$\Rightarrow P_{m_i} = \sqrt{\frac{(R + Bf_u^*)\sum_{j\neq i} P_{m_j}}{c}} - \sum_{j\neq i} P_{m_j}. \tag{11}$$

If $P_{m_i} > 0$, it is the best response strategy of miner m_i due to the concavity of U_{m_i}. When $P_{m_i} < 0$, it indicates that the miner will not join the PoW competition by setting $P_{m_i} = 0$. Hence, $b_i(\mathbf{P}_{-m_i})$ can be determined as (12), as shown at the bottom of this page. We can see that for a miner m_i who joins the PoW competition, its best response function $b_i(\mathbf{P}_{-m_i})$ is always positive and monotonic. In addition, for $\forall\phi > 1$ we have

$$\begin{aligned}&\phi b_i(\mathbf{P}_{-m_i}) - b_i(\phi\mathbf{P}_{-m_i}) \\ =&(\phi - \sqrt{\phi})\sqrt{\frac{(R + Bf_u^*)\sum_{j\neq i} P_{m_j}}{c}} > 0,\end{aligned} \tag{13}$$

which indicates $\phi\mathbf{b}(\mathbf{P}_m) > \mathbf{b}(\phi\mathbf{P}_m)$ holds. Therefore, $\mathbf{b}(\mathbf{P}_m)$ is standard and its fixed point is unique. Accordingly, the PoW competition game has a unique NE. ■

Then, we have the following theorem regarding the optimal strategy of a miner in the PoW competition.

Theorem 2: The optimal strategy of miner m_i in the PoW competition is given by

$$P_{m_i}^{ne} = \frac{(R + Bf_u^*)(N-1)}{c \cdot N^2}. \tag{14}$$

Proof: Note that the optimal strategy profile of miners in the PoW competition converges to the unique NE, which satisfies equation (10). Performing some algebraic transformations on (10) yields

$$P_{m_i}^{ne} = \sum_{m_j\in\mathcal{M}} P_{m_j}^{ne} - \frac{c\left(\sum_{m_j\in\mathcal{M}} P_{m_j}^{ne}\right)^2}{R + Bf_u^*}. \tag{15}$$

By calculating the sum on both sides of (15) over the set of miners $\mathcal{M}$, we have

$$\sum_{m_i\in\mathcal{M}} P_{m_i}^{ne} = N \cdot \sum_{m_j\in\mathcal{M}} P_{m_j}^{ne} - \frac{cN\left(\sum_{m_j\in\mathcal{M}} P_{m_j}^{ne}\right)^2}{R + Bf_u^*}, \tag{16}$$

and thus

$$\sum_{m_j\in\mathcal{M}} P_{m_j}^{ne} = \frac{(R + Bf_u^*)(N-1)}{cN}. \tag{17}$$

Substituting (17) into (15) and performing some algebraic transformations, equation (14) can be obtained.

This completes the proof. ■

C. Stage I: BP's Optimal Strategy

Based on the above analysis, for any given BP's rewards R, the optimal strategies of BUs and miners are determinable. That is, the BP can anticipate the behaviors of BUs and miners, and then optimize its own strategy accordingly. Substituting

$$b_i(\mathbf{P}_{-m_i}) = \begin{cases} 0, & R + Bf_u^* \leq c\sum_{j\neq i} P_{m_j} \\ \sqrt{\frac{(R+Bf_u^*)\sum_{j\neq i} P_{m_j}}{c}} - \sum_{j\neq i} P_{m_j}, & \text{otherwise} \end{cases} \tag{12}$$

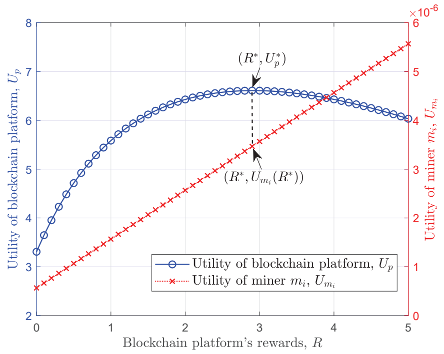


Fig. 3. BP's and miners' utilities versus BP's rewards.

(17) into (1), the utility maximization for BP can be expressed as the following optimization issue:

$$\max_{R \geq 0} U_p(R) = \frac{\lambda}{1 + \frac{cN}{(R+Bf_u^*)(N-1)}} - R. \tag{18}$$

Taking the first-order and second-order derivatives of $U_p(R)$ with respect to R, we have

$$\frac{\partial U_p}{\partial R} = \frac{\lambda c N}{N-1}\left(R + Bf_u^* + \frac{cN}{N-1}\right)^{-2} - 1, \tag{19}$$

$$\frac{\partial^2 U_p}{\partial R^2} = -\frac{2\lambda c N}{N-1}\left(R + Bf_u^* + \frac{cN}{N-1}\right)^{-3}. \tag{20}$$

It indicates that U_p is continuous and differentiable on R and $\frac{\partial^2 U_p}{\partial R^2} < 0$. Therefore, U_p is concave on R, and then the BP's optimal strategy R^* can be determined by letting $\frac{\partial U_p}{\partial R} = 0$, which yields

$$R^2 + 2\left(Bf_u^* + \frac{cN}{N-1}\right)^2 \cdot R - \frac{\lambda c N}{N-1} + \left(Bf_u^* + \frac{cN}{N-1}\right)^2 = 0. \tag{21}$$

When $\frac{\lambda c N}{N-1} - \left(Bf_u^* + \frac{cN}{N-1}\right)^2 < 0$, (21) has no solution within the BP's feasible strategy domain. We only consider the feasible case where the system parameters satisfy

$$\lambda \geq \frac{N-1}{cN}\left(Bf_u^* + \frac{cN}{N-1}\right)^2. \tag{22}$$

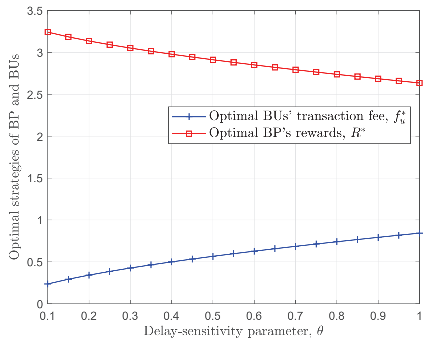
Then, projecting the solution of (21) into the BP's feasible strategy domain, the optimal BP's strategy R^* is given by

$$R^* = \frac{\sqrt{\lambda c N(N-1)} - cN}{N-1} - Bf_u^*. \tag{23}$$

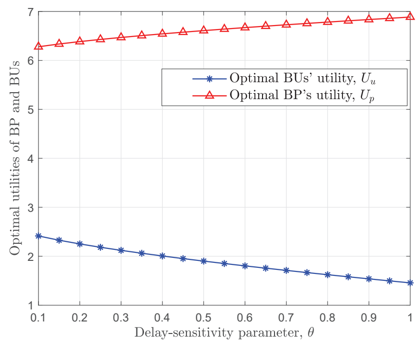
IV. Numerical Results

In this section, we provide numerical results to demonstrate the interaction behaviors among the entities in the blockchain market under the variation of system parameters. Without loss of generality, we set $\lambda = 15$, $\alpha = 2$, $\beta = 4$ and $B = 1$ in the simulations.

We plot Fig. 3 to show how the utilities of the BP and miners varies with the BP's rewards R, where we set the number of miners in the blockchain market $N = 1000$ and the BUs' delay-sensitivity parameter $\theta = 0.5$. It can be observed from Fig. 3 that the miner's utility increases linearly as the BP's rewards R increases. This is because the growth of rewards can raise the expected profit of every miner in the market. On the other hand, the BP's utility first increases and then decreases as R gets larger. There exists an optimal strategy R^* for the BP to maximize its utility. Accordingly, the miner in fact can only achieve its maximal utility under the given rewards R^*.



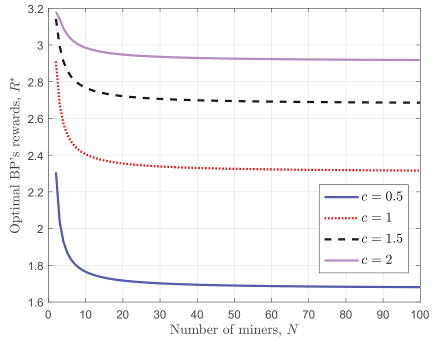
(a) Optimal strategies vs. θ.



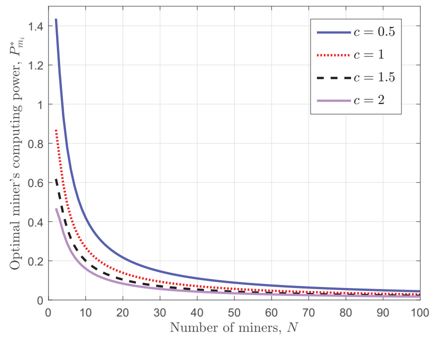
(b) Optimal utilities vs. θ.

Fig. 4. Effects of BUs' delay-sensitivity parameter θ on the optimal strategies and utilities.

We summarize in Fig. 4 the effects of BUs' delay sensitivity parameter θ on the optimal strategies and utilities. We can observe from Fig. 4(a) that as the BUs become more sensitive to the delay for packing their transactions into the blockchain, the BUs' optimal strategy f_u^* monotonically increases, while the optimal BP's rewards R^* monotonically decreases. This is because as θ gets larger, the BUs need to pay more transaction fees to receive a lower delay and so as to achieve the maximal utility. As a result that the miners have received more profits from the BUs, the BP can reduce its rewards so as to save its costs. Fig. 4(b) shows that a larger θ contributes to a higher utility for the BP but reduces the utility for the BUs. Such behaviors correspond to the trends in Fig. 4(a), as an entity's utility is negatively correlated to its costs. Fig 4 indicates that in the blockchain market, the BP prefers to serve the BUs who require a lower delay.



(a) R^* vs. N.



(b) $P^*_{m_i}$ vs. N.

Fig. 5. Effects of the number of miners on the optimal BP's and miner's strategies.

We summarize in Fig. 5 the behaviors of the BP's optimal rewards and the miner's optimal computing power under the variation of the number of miners in the blockchain market, where we set the unit cost of computing power $c \in \{0.5, 1, 1.5, 2\}$ and the delay-sensitivity parameter $\theta = 0.5$. We can observe from Fig. 5(a) that as the number of miners scales up, the rewards offered by the BP, i.e., the BP's optimal strategy, first goes down and then tends to be steady gradually. It can be intuitively explained that as N increases the computing power in the market becomes larger and then saturated, so the BP doesn't need to make efforts for stimulating miners and thus can reduce its rewards. Fig. 5(b) shows that the optimal computing power adopted by a miner in the PoW competition monotonically decreases as the number of miners in the market gets larger. This is because as N increases the competition for winning the PoW game becomes more intense and the BP's rewards will be reduced, and then each miner needs to save its costs for achieving its maximal utility. Another observation of Fig. 5 is that a higher unit cost of computing power will lead to a larger R^* but a smaller $P^*_{m_i}$. This is due to the fact that when c is large, the miner will reduce the computing power to save its costs, while the BP will offer more rewards to compensate the miners' costs such that the miners can provide sufficient computing power for the blockchain market.

V. Conclusion

In this paper, we have investigated the interactions among the entities in a blockchain market from a game-theoretic perspective. By identifying the utilities of the blockchain platform, miners and blockchain users, we have constructed a three-stage Stackelberg game-based modeling. For the Stackelberg game, we have proposed an analytical framework to solve the equilibrium in a backward inductive way, such that the optimal strategies of different entities were obtained. The results of this study reveal the essential behaviors of blockchain entities and provide insights into economic operations in practical blockchain markets.

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