

A Blockchain-based Distributed Non-cooperative Transactive Energy Market

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Abstract—Existing electricity networks and markets are experiencing and exploring the transition from operating in a conventional centralized or pseudo-decentralized systems to a fully distributed one. This research contributes to that transition by proposing a blockchain-based distributed non-cooperative transactive energy market that can be easily integrated with existing electricity market framework. We first introduce an abstracted market where generators, knowing competitor's strategy, will rationally place bids in order to seek their maximized profits and loads are strategically updating offered prices basing on power demand. Under certain assumptions, it is proved there exists a Nash equilibrium state that optimizes the social welfare of the whole market. The details of blockchain-based market architecture including primary components and trading process are subsequently presented. Last but not least, a numerical experiment on a simplified microgrid market is conducted to illustrate the effectiveness of our proposed non-cooperative market framework.

Index Terms—Transactive energy market, blockchain, distributed generation, microgrid market, Nash equilibrium, multi-agent system.

I. INTRODUCTION

A. Blockchain in Electricity Market

The increasing popularity of renewable energy[1], distributed generation[2] and microgrid formation[3] has introduced both opportunities and challenges to our existing electricity markets frameworks. Xu *et al.* [4] points out the fact that current market regulation, energy market design and business model are dominated by centralized or pseudo-decentralized schemes. To provide more flexibilities, better integration and improved efficiency, it is critical to adapt to the transition of fully decentralized systems which potentially could be enabled by blockchain particularly for network integrated microgrid.

Blockchain, by design, is an open and distributed ledger that can permanently record transactions between different parties as blocks on the network[5], [6]. Upon generation, new blocks are only accepted and populated after being verified by majority nodes on the network. Due to its nature of fault-tolerance and distributed architecture, blockchain has gained increasing attention in electric power systems for handling increased transactions due to microgrids trading amongst themselves. Verma *et al.* [7] provide a brief introduction of the Irish peer-to-peer (P2P) transactive energy trading project named EnerPort. This report also remarks some key challenges

faced by existing prosumer market models and explains the advantages of adopting blockchain to enable a highly distributed architecture.

In terms of detailed blockchain-based market models and trading schemes, different design and implementations have been proposed in several literature. Conceptual machine-to-machine (M2M) electricity trading markets can be found in [8], [9] where the trading mechanisms are relatively simple. Energy producer, in those markets, publishes blockchain contracts and consumers choose the most economical ones and place direct transactions through target contracts. Mengelkamp *et al.* presents a blockchain-based market where a uniform clearing price is adopted by all market participants[10]. Within the system, each electricity supplier and consumer places bids or prices to a blockchain contract which is acting as the market price clearing mechanism. However, it is widely acknowledged that a uniform clearing prices is inefficient as power suppliers are less motivated to improve efficiency and quality of services. With increasing penetration of electric vehicles (EVs), research in [11] proposes a blockchain-based P2P EV trading model for a local power network. Participants in the system include plug-in hybrid electric vehicles (PHEVs), acting as electricity buyers and sellers, and auctioneers that are responsible for allocating power supply and demand. Similarly in [12] the utilization of blockchain enables EV's autonomy of energy trading in a decentralized transactional energy market.

B. Contribution

Existing trading schemes in blockchain-based energy market are constrained by ease of implementation and intervention of third parties, whereas in practical scenarios electricity markets are transitioning to competition-oriented[13] where participants are actively competing with others and constantly pursuing maximized profits. Existing wholesale electricity market structures primarily deal with large-scale generation and load entities. Hence in this work we explore a blockchain-based distributed competitive energy market by adopting game theoretic constructs that in future will be able to handle increased localized peer-to-peer transactions alongside existing and mature electricity market grid trading framework.

The contribution of this study is twofold:

- 1) A distributed non-cooperative market pricing scheme is presented and proved. In this market, the microgrid

participants compete with others by strategically placing bids or prices in order to maximize their own benefits. Iteratively, the market will converge to a Nash equilibrium where system's social welfare is optimized and each participant's benefit is simultaneously met.

- 2) A blockchain-based distributed transactive energy market is designed. Participants in the market are abstracted as virtual agents and actively performing transactions through a smart contract. An illustrative numerical experiment is conducted to validate our trading scheme for a limited number of transactions.

II. SYSTEM MODEL

A. Nash Equilibrium

Assume that a microgrid consists of N distributed generators and M electricity consumers and all entities are able to perform multi-player energy transactions. An electricity market is formed within the microgrid and it operates in the way that agreements containing price and quantity information are automatically reached between power suppliers and consumers.

Each consumer is associated with a total electricity consumption d_j ($j \in M$). In order to fulfil its power demand, a Price-of-Purchase p is issued. Knowing the prices from each consumer, a generator i is willing to provide $q_{ij} = b_i p_j$ electricity to consumer j where b_i is a bid and it represents generator i 's willingness against price p_j . After receiving bids from all potential suppliers, a power customer j updates its new price as $p_j = d_j / \sum b_i$. The price and quantity between generators and consumers will settle once all power demand is fulfilled and no generator is willing to change its bid. Furthermore, given generator i 's total power supply is $q_i = \sum_j q_{ij}$, there is a continuously differentiable and strictly convex cost function $C_i(q_i)$ associated with it.

Assumption In this research we assume that each distributed generator in the microgrid market is price-taking and actively pursuing optimized profits. All generators' bids are transparent and accessible to others. Upon receiving prices p_j ($j \in M$) from consumers and knowing other competitors' bids \mathbf{b}_{-i} , a generator i strategically issues a new bid b_i that will maximize its profit function $R_i(b_i, \mathbf{b}_{-i})$ where

$$\begin{aligned} R_i(b_i, \mathbf{b}_{-i}) &= \sum_j p_j q_{ij} - C_i(q_i) \\ &= b_i \sum_j \frac{d_j^2}{(b_i + B_{-i})^2} - C_i\left(\frac{b_i \sum_j d_j}{b_i + B_{-i}}\right) \end{aligned} \quad (1)$$

Here $\mathbf{b}_{-i} = [b_1, b_2, \dots, b_{i-1}, b_{i+1}, \dots, b_n]$ and denote $B_{-i} = \sum_{l \neq i}^n b_l$ where $l \in N$.

From the above assumption, we can get the derivative of each generator's profit function

$$\frac{dR_i(b_i, \mathbf{b}_{-i})}{db_i} = \frac{1}{(b_i + B_{-i})^2} \left[\frac{B_{-i} - b_i}{B_{-i} + b_i} \tilde{D} - B_{-i} DC'_i\left(\frac{b_i D}{b_i + B_{-i}}\right) \right] \quad (2)$$

where $D = \sum_j d_j$ and $\tilde{D} = \sum_j d_j^2$.

Note that $\frac{B_{-i} - b_i}{B_{-i} + b_i} < 1$, we can get that $\frac{dR_i(b_i, \mathbf{b}_{-i})}{db_i} < 0$ if $B_{-i} DC'_i(q_i) \geq \tilde{D} > \frac{B_{-i} - b_i}{B_{-i} + b_i} \tilde{D}$, and $b_i^* = 0$ maximizes generator i 's profit function. If $B_{-i} DC'_i(q_i) < \tilde{D}$, we can see from the above equation that $\frac{dR_i(b_i, \mathbf{b}_{-i})}{db_i} \Big|_{b_i=0} > 0$ and $\frac{dR_i(b_i, \mathbf{b}_{-i})}{db_i} \Big|_{b_i=B_{-i}} < 0$, then there exists a point b_i^* in range $(0, B_{-i})$ satisfying $\frac{B_{-i} - b_i^*}{B_{-i} + b_i^*} \tilde{D} - B_{-i} DC'_i\left(\frac{b_i^* D}{b_i^* + B_{-i}}\right) = 0$. Combining the two cases, b_i^* satisfies the following

$$\left[\frac{\tilde{D}}{B_{-i} + b_i^*} - \frac{B_{-i} D}{B_{-i} - b_i^*} C'_i\left(\frac{b_i^* D}{b_i^* + B_{-i}}\right) \right] (b_i - b_i^*) \leq 0 \quad (3)$$

Recall that $p_j = d_j / \sum b_i$, the above can be transformed into

$$\left[\frac{\sum_j p_j^{*2}}{\sum_j p_j^*} - \frac{B_{-i}}{B_{-i} - b_i} C'_i(q_i^*) \right] (b_i \sum_j p_j^* - q_i^*) \leq 0 \quad (4)$$

Note that $b_i^* < B_{-i}$, then $p_j^* = d_j / \sum_i b_i^* = d_j / (b_i^* + B_{-i}) < d_j / (2b_i^*)$, and $q_{ij}^* = b_i^* p_j^* < d_j / 2$. Therefore for each load d_j there will be at least 3 generators providing positive power supplies ($q_{ij} > 0$) to it. Moreover, it can be seen that $q_i^* = \sum_j q_{ij}^* < \sum_j d_j / 2 = D / 2$, which implies in order to fulfil all loads' demand D the number of generators (N) existing in the microgrid is greater than 2.

Theorem 1. Given a microgrid market with $N \geq 3$ generators following optimization Equ. 4 and $M \geq 1$ consumers responding prices based on $p_j = d_j / \sum b_i$, there exists a Nash equilibrium state. Moreover, the equilibrium vector $(\mathbf{b}^*, \mathbf{p}^*)$ solves the following convex social welfare optimization problem

$$\min_{q_i \geq 0} \sum_{i \in N} F_i(q_i) \quad (5a)$$

$$s.t. \quad \sum_i q_{ij} = d_j \quad \forall j \in M \quad (5b)$$

where $F_i(q_i)$ is

$$\begin{aligned} F_i(q_i) &= \left(1 + \frac{q_i}{D - 2q_i}\right) C_i(q_i) \\ &\quad - \int_0^{q_i} \frac{D}{(D - 2x_i)^2} C_i(x_i) dx_i \end{aligned} \quad (6)$$

and $F_i(q_i)$ is the sum of generation cost and some efficiency loss that are necessary for reaching agreements.

Proof. Considering the above convex optimization problem, we can construct its dual problem as shown below:

$$\max \quad g(\mathbf{u}) \quad (7a)$$

$$s.t. \quad \mathbf{u} \in R^m \quad (7b)$$

where the dual function $g(\mathbf{u})$ is in the following form

$$\begin{aligned} \min L(\mathbf{q}, \mathbf{u}) &= \sum_i F_i(q_i) - \sum_j u_j \left(\sum_i q_{ij} - d_j \right) \\ &= \sum_i \left(F_i(q_i) - \sum_j q_{ij} u_j + \frac{\sum_j u_j d_j}{n} \right) \end{aligned} \quad (8)$$

Note that the dual function $L(\mathbf{q}, \mathbf{u})$ is separable and it can be decomposed into N independent one-dimensional problems [14], [15] with each as the following

$$\min f_i(q_i) := F_i(q_i) - \sum_j q_{ij} u_j + \frac{\sum_j u_j d_j}{n} \quad (9a)$$

$$s.t. \quad 0 \leq q_i < D/2 \quad (9b)$$

Since $F_i(q_i)$ is strictly convex in $[0, D/2)$ and $q_i = b_i \sum_j p_j$, so is each $f_i(q_i)$. Hence each sub-problem has its unique optimal solution b_i^* and its optimal condition is as following

$$\left[\sum_j p_j^* u_j^* - F_i'(q_i^*) \left(\sum_j p_j^* \right) \right] (b_i - b_i^*) \leq 0 \quad (10)$$

After some mathematical manipulations, the above can be transformed into

$$\left[\frac{\sum_j p_j^* u_j^*}{\sum_j p_j^*} - F_i'(q_i^*) \right] (b_i \sum_j p_j^* - q_i^*) \leq 0 \quad (11)$$

Recall Equation 6, it can be easily verified that $F_i'(q_i) = (1 + \frac{q_i}{D-2q_i})C_i'(q_i) = \frac{B-i}{B-i-b_i}C_i'(q_i)$, and Equation 11 and 4 are identical when $\mathbf{u}^* = \mathbf{p}^*$, which states that the Nash equilibrium vector $(\mathbf{b}^*, \mathbf{p}^*)$ minimizes the target social welfare problem.

We have stated it as a social welfare problem objective, though it benefits the competitive behaviour of the microgrid players, since all these participants do operate in the bigger scheme of the overall electricity market structure where the distribution utility is participating. \square

B. Distributed Power Supply Agreement Scheme

Given an electricity market where a large number of generators and consumers are simultaneously performing power transactions, massive computing and networking resources are required for the system to operate sufficiently. Nevertheless concerns regarding complexity of operation and maintenance, security and scalability also exist in a centralized architecture, which in the last 3 decades have been managed in wholesale electricity market structures. In this work to handle the scalability challenge, a distributed power supply agreement scheme is proposed.

Note from Equ. 8 that the dual function of social welfare optimization can be decomposed into independent sub-problems where each sub-problem's optimal solution is a component of the market's Nash equilibrium vector and this makes a distributed computation scheme an ideal solution for solving our global welfare optimization. Therefore in this formulation we propose a distributed scheme where generators calculate and submit their own optimized bids and consumers adjust their offering prices based on the mismatch between proposed supply and demand. During market trading process, information is constantly exchanged between different entities and the global welfare optimization can be achieved iteratively. The proposed distributed agreement reaching scheme is presented

Algorithm 1 Distributed Market Trading Scheme

Initialization: $k = 1, u_j^k = 0$ for $j = 1, 2, 3, \dots, M$, choose a suitable step-size $s > 0$ and tolerance $\delta > 0$,

while $|t^k| > \delta$ **do**

for Generator $i = 1$ to N **do**

 Retrieving \mathbf{u}^k and \mathbf{b}_{-i} ,

 Computing b_i^{k+1} as the optimal solution of Equ. 9,

$$b_i^{k+1} = \frac{(C_i')^{-1} \left(\frac{\sum_j u_j^k}{\sum_j u_j^k} \left(\frac{B-i-b_i}{B-i} \right) \right)}{\sum_j u_j^k} \quad (12)$$

 Broadcast b_i^{k+1} ,

end for

for Load $j = 1$ to M **do**

 Receiving \mathbf{b}^{k+1} from generators,

 Updating its price u_j as following

$$t_j^{k+1} = u_j^k \left(\sum_i b_i^{k+1} \right) - d_j \quad (13a)$$

$$u_j^{k+1} = u_j^k + s \cdot t_j^{k+1} \quad (13b)$$

 Broadcast u_j^{k+1} ,

end for

$k = k + 1$

end while

in Algorithm 1. Specifically the dual gradient algorithm is utilized to converge our solution to the global optimization.

It can be seen from the algorithm that each entity within the market is solving a relatively simpler optimization problem, thus requiring only limited amount of computing resources. For the information exchange, low bandwidth network is sufficient as entities only need to broadcast their latest bids or prices. By transforming a global welfare optimization problem that demands intensive central computing and networking performance into a distributed solving scheme that supports devices with finite resources, our consensus reaching mechanism is suitable for future electricity distribution system that features high penetration of distributed generators, smart meters and other smart Internet-of-Things (IoT) appliances.

III. MARKET ARCHITECTURE

As a distributed computing system, blockchain features resistance of record modification and decentralized consensus between majority nodes in the network, which makes it an ideal platform for our distributed energy market. Hence in this section, we present a blockchain-based trading system which in general is a virtual environment where different trading entities in the actual power distribution network and systems are abstracted as virtual agents on the blockchain network and continuously performing market operations. The market architecture and components are illustrated in Figure 1.

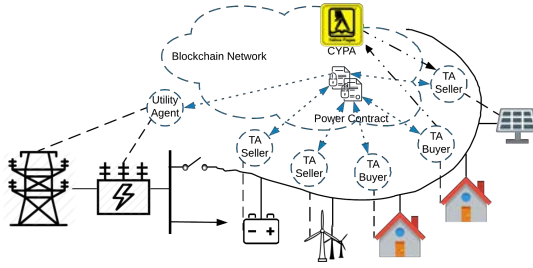


Fig. 1: Blockchain-based Transactive Energy Market

A. Market Components

1) *Trading Agent (TA)*: Trading Agents are the representatives of market participants that are primarily conducting energy transactions. Specifically a distributed generator is represented as an intelligent TA seller in the market that is constantly placing bids in order to maximize its profit. Correspondingly a TA buyer, created as the virtual agent of a consumer, offers purchasing prices to fulfil its power demand. A TA is consisted of a power controller, a smart measurement unit and a blockchain market trading module.

2) *Power Contract (PC)*: An energy contract is an agreement for supplying and purchasing specified quantity of power between different TA buyers and TA sellers. Issued by a buyer agent, its blockchain address is publicly accessible and sellers who are willing to supply can place bids within the contract meanwhile the issuer is also updating its price of purchase. From trading entities' points of view, a power contract is an information mediator and obligation enforcer between buyers and sellers. Once agreement reached, the fund from buyer will be stored within the contract and it will only be released if the contract has verified that a generator has fulfilled its obligation.

3) *Utility Agent (UA)*: Utility Agents on blockchain represent electricity utilities in an actual distribution system. By collecting real-time measurements across the system, the utility is constantly monitoring and maintaining power network's stability so that all other parties can function properly. Hence an UA in our market is acting on behalf of an utility company and charging fees for all established contracts in order to compensate its operation cost. Therefore our proposed peer-to-peer microgrid market structure can be easily integrated with existing wholesale electricity market under which the local distribution utility (UA) operates upon.

4) *Contract Yellow Page Agent (CYPA)*: A CYPA acts as a market information centre that keeps record of existing power contracts in the market. Externally it provides a list of open contract addresses so that potential TA sellers can retrieve more details from the network providing the blockchain address of that contract. Moreover, a TA buyer will voluntarily add its contract address to CYPA's list as it could potentially attract more TA sellers and decrease its consumption cost.

B. Trading Process

As shown in Figure 2, the trading process consists of three phases: initialization, iterative agreement reaching and

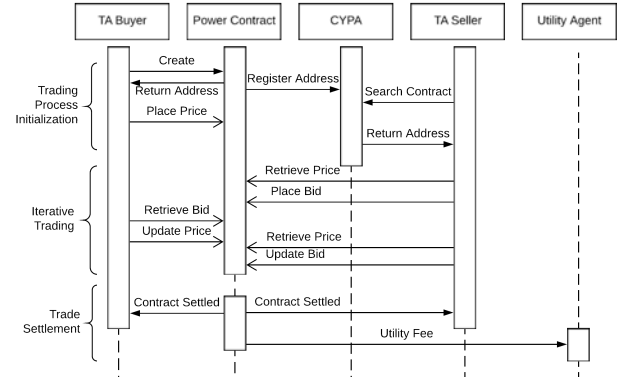


Fig. 2: Market Trading Flows

settlement. To initialize the process, a TA buyer starts issuing a power contract with desired quantity of power and registers the returned contract address with a CYPA and this makes it publicly accessible to all potential sellers. Asynchronously, the buyer also places an initial price within that contract. During the iterative trading phase, TA sellers retrieve current contract price and competitors' bids, strategically calculate new bids from Equ. 12, and then update their latest bids in their contracts. Simultaneously the price of purchase is updated by the TA buyer using Equ. 13. This iterative approach will drive current market status towards the Nash equilibrium eventually. Once converged, events are automatically triggered and emitted from contract to all involved parties to inform the settled price, bids and utility fee. After being notified, a generator needs to supply specified quantity of electricity so that the contract will release the fund to its account.

Note that the market trading process in this section is generally similar to Algorithm 1 except that, instead of broadcasting, all trading entities will send new price or bids to the blockchain contract directly. In this way, our trading process benefits from blockchain's decentralized consensus as any updates in the contract will be automatically populated, verified and synchronized among the majority nodes on the network.

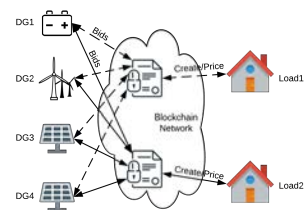
IV. NUMERICAL RESULT

To demonstrate the effectiveness of our study, a numerical experiment is conducted and presented in this section. Firstly we abstract a simplified microgrid market consisting of 4 distributed generators and 2 household consumers (Figure 3) from its physical system and correspondingly create virtual agents for all these new market participants. Each agent is assigned with a unique account address within a private Ethereum network. By doing so, agents are able to perform blockchain transactions, create smart power contracts, place bids and update prices.

To simplify our model further, each generator's cost function is in the form $C_i(q_i) = \alpha_i q_i^2 + \beta_i q_i$, and each generator's optimal bid can be easily calculated by substituting the cost function $C_i(q_i)$ into Equ. 12. Each consumer follows Equ. 13 to update its price and all other parameters are provided in Table I. The result of our experimental market trading is

TABLE I: Transactive Energy Market Participant Parameters

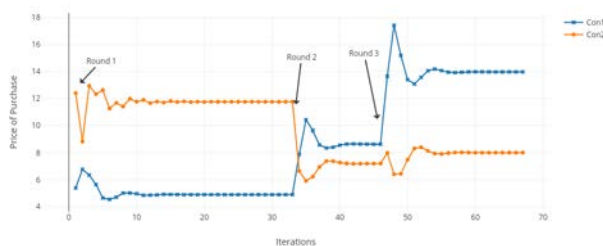
Participant	α	β	s	δ	d_1	d_2	d_3
DG1	0.2	0.3	—	—	—	—	—
DG2	0.3	0.5	—	—	—	—	—
DG3	0.7	0.2	—	—	—	—	—
DG4	0.4	0.5	—	—	—	—	—
Load1	—	—	0.5	0.001	10.0	15.0	26.5
Load2	—	—	0.5	0.001	24.0	12.5	15.2



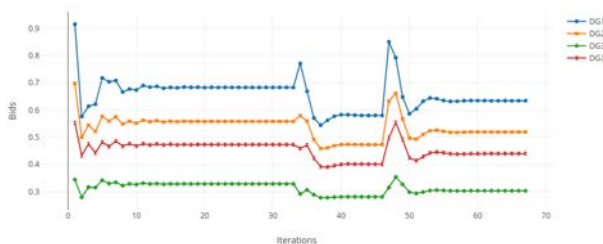
(a) Microgrid Market Participants

[illegible]

(b) A snippet of blockchain transactions



(c) Price Of Purchase from Consumers



(d) Supply Bids from Generators

Fig. 3: Numerical result

shown in Figure 3. During this numerical test, three rounds of trading are conducted and it can be seen that our distributed trading scheme can quickly converge to the Nash equilibrium. Substituting generators' bids and consumers' prices within $p_j = d_j / \sum b_i$ it can be verified that each household's demand is fulfilled. A snippet of the transaction log is also provided. Please note that the CYPA and UA proposed in our architecture, though important for realistic implementation, are not factored in the simulation.

V. CONCLUSION

In this paper, a blockchain-based non-cooperative distributed transactive energy market is formulated and analysed. We firstly prove that under the assumed market game there exists a Nash equilibrium where the market entities' power demand, maximized profit as well as the entire market's optimal social welfare are concurrently achieved. A distributed consensus reaching scheme that utilizes the dual gradient algorithm is correspondingly proposed to achieve the requirement of distributed computing. Furthermore, the paper illustrates a blockchain-based energy market where market participants are abstracted as intelligent virtual agents and perform transactions on blockchain network following the proposed distributed trading scheme.

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