Chapter 9: Dynamics of Employment Adjustment

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Focus attention on labor demand

- The usual textbook model of labor demand depicts a firm as choosing the number of workers and their hours given a wage rate
- The determination of wages, employment, and hours is much more complex than this
- The key is to recognize that the adjustment of many factors of production, including labor, is not costless

Understanding the nature of adjustment costs matters

- Many competing models of the business cycle depend crucially on the operation of labor markets
- Attempts to forecast macroeconomic conditions often resort to consideration of observed movements in hours and employment to infer the state of economic activity
- Policy interventions in the labor market are numerous and widespread
  - eg: restrictions on wages, restrictions on hours, costs of firing workers

Understanding the nature of adjustment costs matters

Outline of this chapter

- Baseline model
- Quadratic Adjustment Costs
- Richer Models of Adjustment
  - Piecewise Linear Adjustment Costs
  - Nonconvex Adjustment Costs
- The Gap Approach
- Estimation

General Model of Dynamic Labor Demand

- We consider variants of the following dynamic programming problem:

$$\begin{aligned} V(A, e_{-1}) = \max_{h,e} & \, R(A, e, h) - \omega(e, h, A) - C(e, e_{-1}) \\ & + \beta E_{A'|A} V(A', e) \qquad \text{for all } (A, e_{-1}) \end{aligned} \tag{1}$$

- meanings of variables
  - A: a shock to the profitability of the plant and/or firm
  - A(a,ε): common component across plants a; plant specific ε
  - e: number of workers
  - h: work hours

Baseline model

- $R(A, e, h)$: the revenues of a firm/plant
- $\omega(e, h, A)$: total cost of hiring workers
  - increase in both of its arguments and is convex with respect to hours
  - state dependent
- $C(e, e_{-1})$: the cost of adjusting the number of workers
  - Search and recruiting
  - Training
  - Explicit firing costs
  - Variations in complementary activities (capital accumulation, reorganization of production activities, etc.)

Quadratic Adjustment Costs

- suppose that the cost of adjustment is given by:

$$C\left(e, e_{-1}\right) = \frac{\eta}{2}\left(e - (1 - q)e_{-1}\right)^2 \tag{2}$$

- convex in e and continuously differentiable
- q: exogenous quit rate

Quadratic Adjustment Costs

- First order condition for h:

$$R_h(A, e, h) = \omega_h(e, h, A) \tag{3}$$

- First order condition for e using (2):

$$R_e(A, e, h) - \omega_e(e, h, A) - \eta\left(e - (1-q)e_{-1}\right) + \beta EV_e\left(A', e\right) = 0 \tag{4}$$

- $EV_e\left(A', e'\right)$ can be evaluated using (1):

$$\begin{aligned} R_e(A, e, h) - \omega_e(e, h, A) - \eta\left(e - (1-q)e_{-1}\right) \\ +\beta E\left[\eta\left(e' - (1-q)e\right)(1-q)\right] = 0 \end{aligned} \tag{5}$$

Quadratic Adjustment Costs: no cost with employment adjustment

- The employment policy function $e = \phi(A, e_{-1})$
- The hours policy function $h = H(A, e_{-1})$
- As a benchmark, suppose no adjustment costs: $\eta \equiv 0$
  - suppose the compensation function:

$$\omega(e, h, A) = e\tilde{\omega}(h)$$

  - suppose revenues depend on the product eh

$$R(A, e, h) = A\tilde{R}(eh)$$

  - first-order conditions can be manipulated to imply that

$$1 = h\frac{\tilde{\omega}'(h)}{\tilde{\omega}(h)}$$

  - variations in the labor input arise from variations in the number of workers rather than hours

Quadratic Adjustment Costs: no cost with hour adjustment

- suppose $\eta \neq 0$, suppose that compensation is simply:

$$\omega(e, h, A) = eh$$

- suppose no costs to hours variation, from eq.(3)
-

$$A\tilde{R}'(eh) = 1$$

- Using the above equation, we can see eq.(5) is satisfied by a constant level of e.
- Hence the variation in the labor input would be only in terms of hours. No employment variations.

A Simulated Example

The revenue function is:

$$R(A, e, h) = A(eh)^{\alpha}$$

The wage rate function is:

$$w(e, h) = w * e * [w_0 + h + w_1(h - 40) + w_2(h - 40)^2]$$

The employment adjustment cost is:

$$C(e, e_{-1}) = \frac{\eta}{2} \frac{(e - e_{-1})^2}{e_{-1}}$$

Hence, there are both hour and employment adjustment cost

A Simulated Example

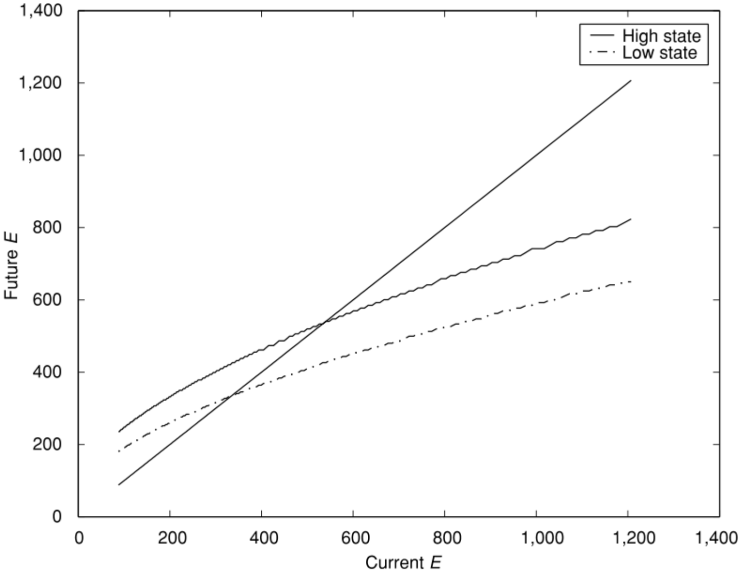


Fig.1:Employment policy functions: Quadratic costs

Sargent: Linear Quadratic Specification

$$R(A, e) = (R_0 + A)e - \frac{R_1}{2}e^2$$

wage rate follows and exogenous process

$$w_t = v_0 + \sum_{i=1}^{n} v_i w_{t-i} + \kappa_t$$

The first-order condition with respect to employment is

$$\beta E_t e_{t+1} - e_t(\frac{R_1}{\eta} + (1 + \beta)) + e_{t-1} = \frac{1}{\eta}(w_t - R_0 - A_t)$$

Sargent finds evidence of employment adjustment costs when estimating the above VAR.

Richer Models of Adjustment

- Quadratic adjustment cost is tractable
- but cannot capture inactivity and bursts at the plant level
- we need something richer
- at the plant level, employment adjustment is more erratic than the pattern implied by the quadratic model

Richer Models of Adjustment

- Piecewise Linear Adjustment Costs:

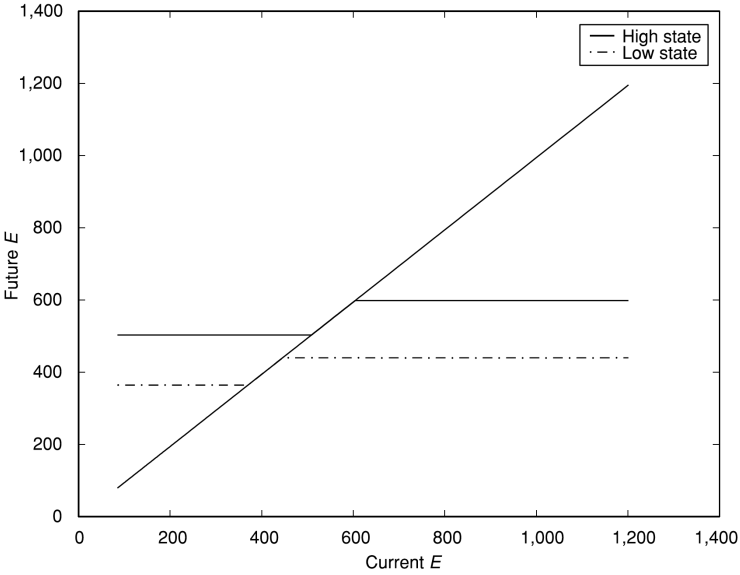
$$C(e, e_{-1}) = \begin{cases} \gamma^{+}\Delta e & \text{if } \Delta e > 0 \\ \gamma^{-}\Delta e & \text{if } \Delta e < 0 \end{cases}$$

- The optimal policy is characterized by two boundaries:

$$e^{-}(A); e^{+}(A)$$

- If $e_{-1} \in [e^{-}(A), e^{+}(A)]$, then there is no adjustment
- If there is adjustment, the optimal adjustment is to $e^{-}(A)$ if $e_{-1} < e^{-}(A)$ and is to $e^{+}(A)$ if $e_{-1} > e^{+}(A)$

Richer Models of Adjustment



Nonconvex Adjustment Costs

- consider the model:

$$V(A, e_{-1}) = \max\left[V^a(A, e_{-1}), V^n(A, e_{-1})\right] \quad \text{for all } (A, e_{-1})$$

- $V^a(A, e_{-1})$ represents the value of adjusting employment:

$$V^a(A, e_{-1}) = \max_{h,e} R(A, e, h) - \omega(e, h) - F + \beta E_{A'|A} V(A', e)$$

- $V^n(A, e_{-1})$ represents the value of not adjusting employment:

$$V^n(A, e_{-1}) = \max_{h} R(A, e_{-1}, h) - \omega(e_{-1}, h) + \beta E_{A'|A} V(A', e_{-1})$$

Nonconvex Adjustment Costs

- Rich trade-offs between hours and employment variations
- Suppose that there is a positive shock to profitability: A rises
  - If this variation is large and permanent, then adjusting employment. Hours will vary only slightly
  - If the shock to profitability is not large or permanent enough to trigger adjustment, then employment will remain fixed. The main variation will be in worker hours

Asymmetries

Hiring costs may be different than firing costs, Pfann and Palm (1993) use the following adjustment cost function:

$$C(e, e_{-1}) = -1 + e^{\gamma \Delta e} - \gamma \Delta e + \frac{1}{\eta}(\Delta e)^2$$

If $\gamma < 0$, then firing costs exceed hiring costs.

The Gap Approach

- An alternative approach to studying dynamic labor adjustment(Caballero & Engel ,1993; Caballero et al.,1997)
- Labor adjustment will respond to a gap between the actual and desired employment level at a plant
- Gap approach simplifies the dynamic optimization problem as the target level of employment summarizes the current state

Partial Adjustment Model

- labor adjustment

$$e_t - e_{t-1} = \lambda\left(e^* - e_{t-1}\right)$$

- Cooper and Willis (2001) consider a dynamic programming problem given by:

$$L\left(e^*, e_{-1}\right) = \min_e \frac{\left(e - e^*\right)^2}{2} + \frac{\kappa}{2}\left(e - e_{-1}\right)^2 + \beta E_{e^{*\prime}|e^*} L\left(e^{*\prime}, e\right)$$

- e^*: target workers, follows an AR(1) process with serial correlation of ρ; λ parameterizes how quickly the gap is closed

- The first-order condition:

$$\left(e - e^{*}\right) + \kappa \left(e - e_{-1}\right) - \beta \kappa E \left(e' - e\right) = 0$$

- Conjecture a policy function:

$$e = \lambda_1 e^{*} + \lambda_2 e_{-1}$$

- Taking expectations of the future value of e^{*} yields:

$$\left(e - e^{*}\right) + \kappa \left(e - e_{-1}\right) - \beta \kappa \left(\lambda_1 \rho e^{*} + \left(\lambda_2 - 1\right) e\right) = 0$$

- Solve for e

$$\lambda_1 = \frac{1 + \beta \kappa \lambda_1 \rho}{1 + \kappa - \beta \kappa \left(\lambda_2 - 1\right)}; \quad \lambda_2 = \frac{\kappa}{\left(1 + \kappa - \beta \kappa \left(\lambda_2 - 1\right)\right)}$$

Estimation of a Rich Model of Adjustment Costs

- Evaluate models that may have both convex and nonconvex adjustment costs
- Letting A represent the profitability of a production unit, considering:

$$V(A, e_{-1}) = \max_{h,e} R(A, e, h) - \omega(e, h, A) - C(A, e_{-1}, e) \\ + \beta E_{A'|A} V(A', e)$$

- Let $R(A, e, h) = A(eh)^{\alpha}$
  - α: shares of capital and labor in the production function as well as the elasticity of demand

- $\omega(e, h, A)$: compensation function
- The costs of adjustment function:

$$C(A, e_{-1}, e) = \begin{cases} F^H + \frac{v}{2}\left(\frac{e - e_{-1}}{e_{-1}}\right)^2 e_{-1} & \text{if } e > e_{-1} \\ F^F + \frac{v}{2}\left(\frac{e - e_{-1}}{e_{-1}}\right)^2 e_{-1} & \text{if } e < e_{-1} \end{cases}$$

  - F^H and F^F represent the respective fixed costs of hiring and firing workers
  - v parameterizes the level of the adjust- ment cost function

- This adjustment cost function yields the following dynamic optimization problem:

$$V\left(A, e_{-1}\right) = \max\left\{V^{H}\left(A, e_{-1}\right), V^{F}\left(A, e_{-1}\right), V^{N}\left(A, e_{-1}\right)\right\}$$

  where N refers to the choice of no adjustment of employment
- These options are defined by:

$$\begin{aligned}
V^{H}\left(A, e_{-1}\right) = &\max_{h,e} R(A, e, h) - \omega(e, h, A) - F^{H} \\
&- \frac{v}{2}\left(\frac{e - e_{-1}}{e_{-1}}\right)^{2} e_{-1} + \beta E_{A'|A} V\left(A', e\right) \qquad \text{if } e > e_{-1} \\
V^{F}\left(A, e_{-1}\right) = &\max_{h,e} R(A, e, h) - \omega(e, h, A) - F^{F} \\
&- \frac{v}{2}\left(\frac{e - e_{-1}}{e_{-1}}\right)^{2} e_{-1} + \beta E_{A'|A} V\left(A', e\right) \qquad \text{if } e < e_{-1} \\
V^{N}\left(A, e_{-1}\right) = &\max_{h} R\left(A, e_{-1}, h\right) - \omega\left(e_{-1}, h, A\right) + \beta E_{A'|A} V\left(A', e_{-1}\right)
\end{aligned}$$

An example

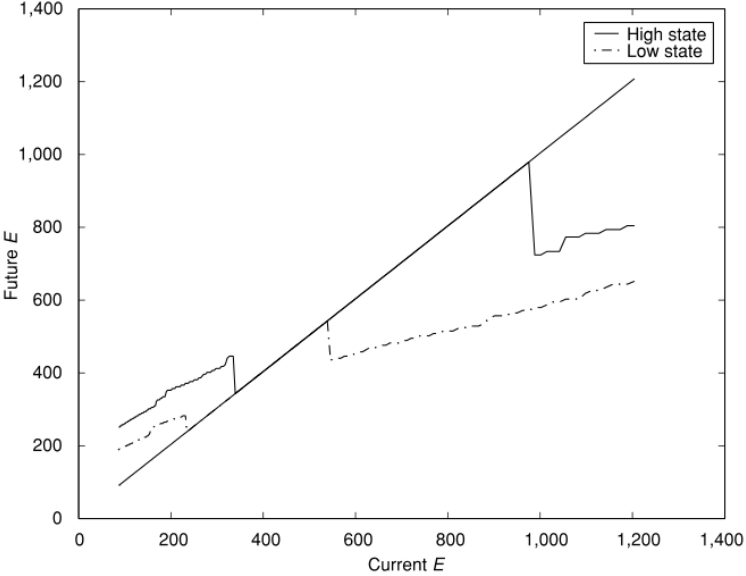


Fig.2:Employment policy functions: Mixed adjustment costs

Summary

- In the presence of adjustment costs, the conventional model of static labor demand is replaced by a possibly complex dynamic optimization problem
- Governments often impose restrictions on employment and hours. The dynamic optimization framework facilitates the analysis of those interventions
- These policies (such as restrictions on hours and/or the introduction of firing costs) help to infer key structural parameters characterizing labor adjustment costs