Sistemas Complexos
TP17 – Predator – Preys (I)

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Abstract. In the study of simple dynamics of a single populations' growth, we typically only take into account factors such as natural growth rate and the carrying capacity of the local environment, but in reality, for a study to yield realistic data, these problems require the study of multiple non-isolated populations that interact with each other, thereby affecting their overall growth rates. In this paper we will study a very specific case of such interactions, in which there are exactly three species, one of which – the predators – eat the other – the prey. This is a variation of the well known Predator-Prey model based on Lotka-Volterras' equations [4]. This model attempts to mimic nature, where an apex-predator feeds on several types of prey. A very familiar example would be the Iberian Lynx, which eat mice, rabbits, and ducks[3]. The goal of this paper is to study how this variant of the predator-Prey problem behaves over time, studying the time evolution of the three species and their dynamics, while keeping in mind that things are never this simple in Nature.

Keywords: Complex Systems · Modelling and Simulation · Continuous Models · Predator-Prey Model · Lotka-Volterra Equations · Interaction and Influence · Behaviour Dynamics

1 Introduction

There are many instances where we are faced with the need to analyse data as a means to understand a specific behaviour. To aid in the understanding of said data, Modelling was developed. Models are a simplified representation of a system. They define a mathematical model of a system (components, interconnections, rules of change) so as to allow the study of its behavior over a period of time[9, 10]. Modelling and Simulation have become central to both science and engineering as they are used to analyse physical systems, helping to gain a better understanding of how these systems function by predicting their possible behaviors to different stimuli[9]. Modelling, along with Simulation, are effectively the only techniques available that allow an accurate analysis of systems under varying experimental conditions[11]. There are many applications for a model, and we hope that the one studied in this paper could provide a useful example to simulation approach and, therefore, a better understanding of any of the realities being modeled.

As stated earlier, this paper will research a variant of a well known Model, the Predator-Prey model[4] based on Lotka-Volterras' equations [6, 7, 15]. This means that in this paper we will focus in on Continuous modelling, where data has a potentially infinite number, and divisibility of attributes[8]. Continuous modelling often uses differential equations as well, which will be the basis of our mathematical description of the preys' and predator's behaviour.

The remainder of the report is structured as follows: Section 2, State of Art, introduces the common methodologies employed at the time. Section 3, Methodology, describes our own methodology and scientific approach, and details the decisions made to implement the problem at hand, while Section 4, Results and Discussion, describes the main results of the experiment. Finally, Section 5 gathers the main conclusions.

2 State of Art

In this section, we will try to make a resume of the most up to date methodologies used in the kind of model simulation this paper was based on.

2.1 Continuous Modelling

Continuous System Modelling introduces us to an important subclass of modelling techniques. They deal with the analysis of systems described through a set of ordinary or partial differential equations or through a set of difference equations. It is notable as one of the first uses ever put to computers, dating back to the Eniac in 1946. Continuous simulation must be clearly differentiated from discrete event simulation, which produces a system which changes its behaviour only in response to specific events, and typically models changes to a system resulting from a finite number of events distributed over time. The volume "Continuous System Modeling" by Cellier, François E. and Greifeneder, Jurgen [9] introduces concepts of Modelling physical systems through a set of differential and/or difference equations. The purpose is twofold: it enhances the scientific understanding of our physical world by codifying (organizing) knowledge about this world, and it supports engineering design by allowing us to assess the consequences of a particular design alternative before it is actually built.

2.2 Phase Space Diagrams

In dynamical system theory, a phase space is a space in which all possible states of a system are represented, with each possible state corresponding to one unique point in the phase space. [14]. A one-dimensional system is called a phase line, while a two-dimensional system is called a phase plane. For every possible state of the system or allowed combination of values of the system's parameters, a point is included in the multidimensional space. The system's evolving state over time traces a path (a phase space trajectory for the system) through the high-dimensional space. The motion of an ensemble of systems in this space is studied by classical statistical mechanics as it enables us to calculate the state of the system at any given time in the future or the past, through integration of Hamilton's or Lagrange's equations of motion.

3 Methodology

The experimental setup developed in order to examine the behaviour of the system previously described consisted of a python application, more specifically a custom iterator using the Simcx library[12] based on Eulers' model. This custom iterator was based on code given through PL3 to PL6. The simulations were then run extensively for each group of parameters so as to allow a higher diversity of behaviour for our experimental analysis.

3.1 Problem Analysis and Preparation

Our experiment began with an analysis of the standard Predator - Prey system [5] and the three discrete equations given to us:

$$\dot{p} = -p\,\eta_p + p\,w_a a + p\,w_b b$$
$$\dot{a} = \alpha_1\,a - \alpha_2\,p\,a$$
$$\dot{b} = \beta_1\,b - \beta_2\,p\,b$$

where p is the predator, a and b are the preys. All parameters are positive.

Fig. 1. Predator - Preys model used in this experiment [13]

These equations symbolize our model, and were the basis of all our research and assumptions. To keep our model simple, we made some assumptions that would be unrealistic in most predator-prey situations. Specifically, we assume that, on the behaviour of the species:

1. the predator species is totally dependent on the two prey species as its only food supply;
2. the prey species have an unlimited food supply and habitat, and therefore no maximum population sizes;
3. there is no threat to the prey other than the single modelled species of predator;

We also assume that, should one of the species become extinct, the others would behave in the following matter:

1. Should the predator become extinct, the prey population would increase exponentially, to an infinite value;

2. Should the prey become extinct, the predator population would decrease exponentially, until reaching 0;

After these assumptions, we felt that a real life example would serve as an adequate preview of what to expect from the behaviour of a predator prey system, so we researched on the Canadian Lynx and Snow Hare population dynamics, which has been well documented for over two centuries. We also researched the relation between the Iberian Lynx and the wild Hare [2, 1].

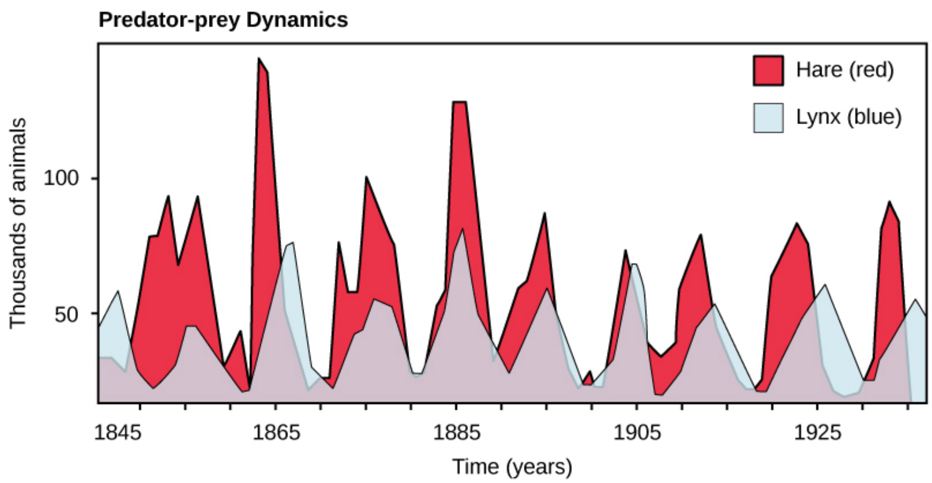

Fig. 2. Real life Predator - Prey system of the Canadian Lynx and Snow Hare [2]

This external research effectively gives us an expected behaviour, and an honest point of comparison: The populations are proportionally sized. Should the prey population decline, so would the predators' shortly after, and vice versa.

Finally, after this initial research, we proceeded to select the parameters that make up our model in the next subsection.

3.2 Parameters

These parameters were changed throughout the experiment at the start of each run so as to study the behaviour of the model towards different parameters. These parameters do not change every iteration. Since every test had different parameters, we instead explain the meaning of these parameters. Below is a summary of the parameters used in this experiment:

Initial population:

- p0 - Initial Predator Population
- a0 - Initial Prey A Population
- b0 - Initial Prey B Population

On the Predator equation:

- Wa - Weight of Prey A on Predator birthrate
- Wb - Weight of Prey B on Predator birthrate
- Np - Weight of Predator deaths by natural causes

On the Prey equations:

- a1 - Weight of Prey A birthrate
- a2 - Weight of Prey A deaths by natural causes
- b1 - Weight of Prey B birthrate
- b2 - Weight of Prey B deaths by natural causes

These three equations further use the current population of each species (a, b, p), which change every iteration due to updates to their respective populations. The initial population is only used on the first iteration.

3.3 Experimentation method

Our experimental method was based on exhaustive testing of all of the parameters previously explained. We started by confirming our expectations of the effects of the parameters on the system. Then afterwards we moved to finding adequate values that did not yield an exaggerated growth rate of single individual species over our iteration limit. Finally, after becoming familiar with the reactions of the system to different parameters, we were able to pinpoint where values being tested achieved a realistic equilibrium of population growth and decline, where each species' population was proportional and co-dependent with the other species. We used our Python *Simcx* application to simulate all the tests depicted on the next section of this paper, and then using the library *matplotlib* to see the state of a simulation at a certain point in time.

4 Results and Discussion

While we experimented numerous times to point out which parameters resulted in a stable, real world-like result that showed growth and decay, we decided to only show the final experiments since the previous tests resulted in unrealistic and exponential graphs, where the populations were barely interacting in any meaningful way.

4.1 Stability Experiment 1

On this specific experiment the initial added population of both Preys was equal to the population of Predators. We also used the following Parameters:

Parameter List:

- p0 = 3
- a0 = 2
- b0 = 1
- a1 = 1.5
- a2 = 0.5
- b1 = 1.8
- b2 = 0.6
- np = 4/7
- wa = 2/7
- wb = 1/7

We also calculated the respective fixed points, which resulted in

- p = a = b = 0
- a + 0.5*b = 2 and p = 3

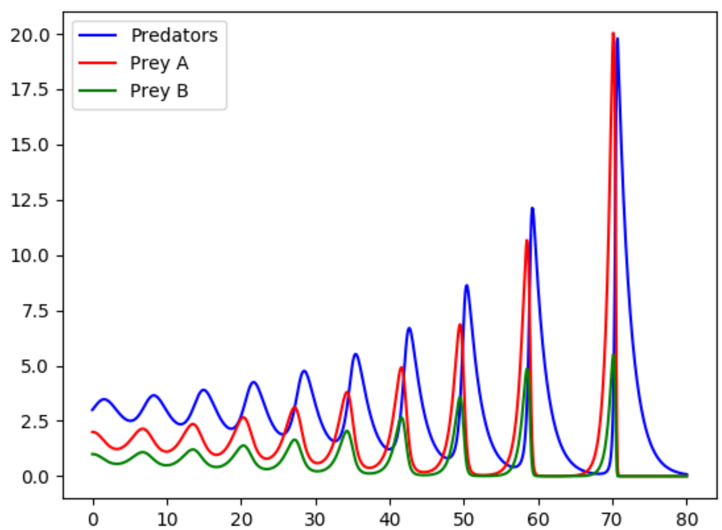

Fig. 3. Experiment Graph 1 - Red and Green depict Preys' A and B population respectively, while Blue depicts the Predator Population

4.2 Stability Experiment 2

On this specific experiment the initial added population of both Preys was double the population of Predators. We also used the following Parameters:

Parameter List:

- p0 = 5
- a0 = 4
- b0 = 6
- a1 = 0.65
- a2 = 0.2
- b1 = 0.65
- b2 = 0.2
- np = 0.4
- wa = 0.05
- wb = 0.05

We also calculated the respective fixed points, which resulted in

- p = a = b = 0
- a + b = 8 and p = 3.25

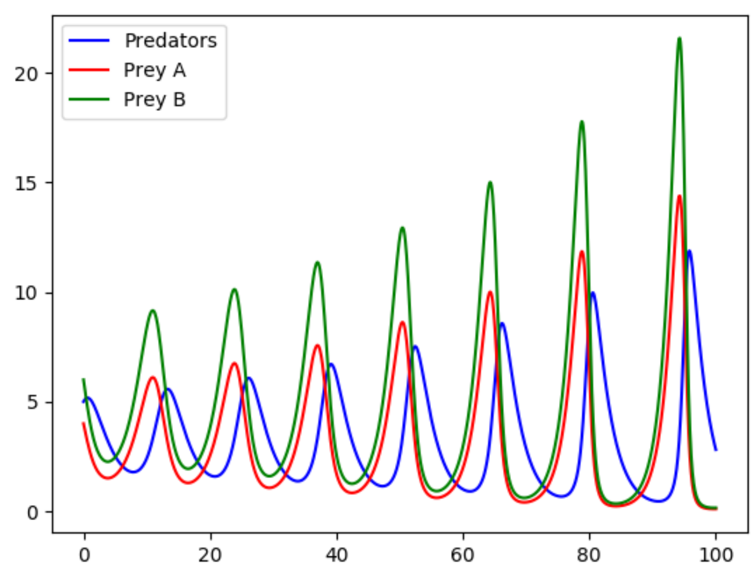

Fig. 4. Experiment Graph 2 - Red and Green depict Preys' A and B population respectively, while Blue depicts the Predator Population

4.3 Phase Space Diagrams

To analyse the a space in which all possible states of our system are represented we plotted a phase space diagram using the various preys states as independent variables and the predator as a dependent variable, since the predator population completely depends on the preys. To make simulations with a specific amount of steps we used again the library *matplotlib*.

– On the first experiments' parameters, our phase planes were the following:

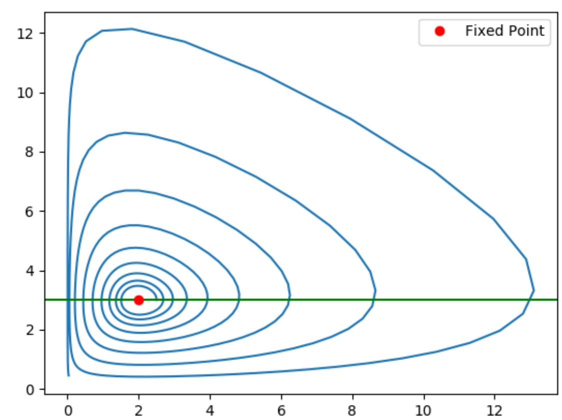

Fig. 5. Phase Plane 1- For the first experiment, with the number of predators being equal to the formula *preyA + 0.5*preyB*. This graph represents on the dependent variable the total number of preys on that exact step.

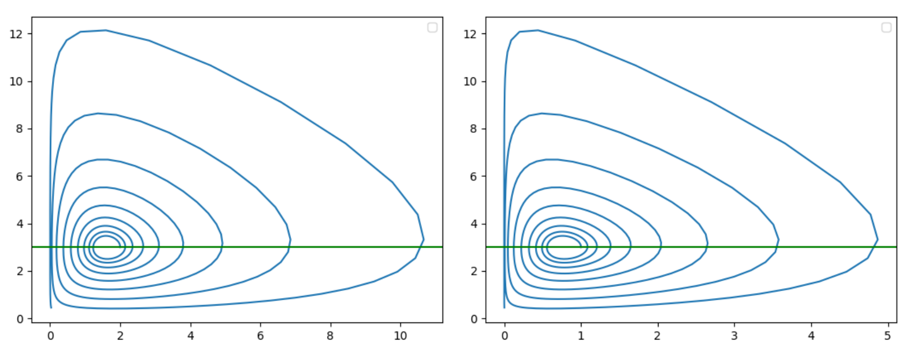

Fig. 6. Prey Phase Plane 1- For the first experiment, the first graph represents prey A, while the second represents prey B

– And on the second experiments' parameters, our phase planes were the following:

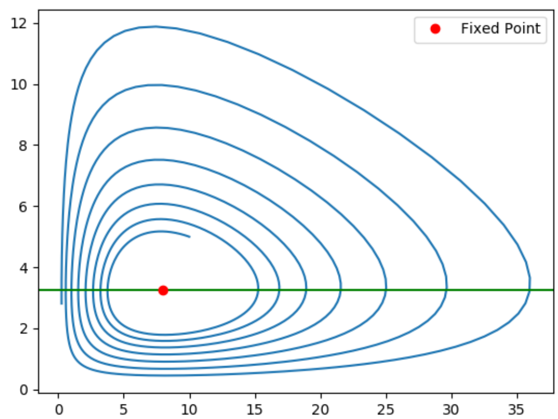

Fig. 7. Prey Phase Plane 2- For the first experiment, with the number of preys being equal to the sum of prey A and prey B. This graph represents on the dependent variable the total number of preys on that exact step.

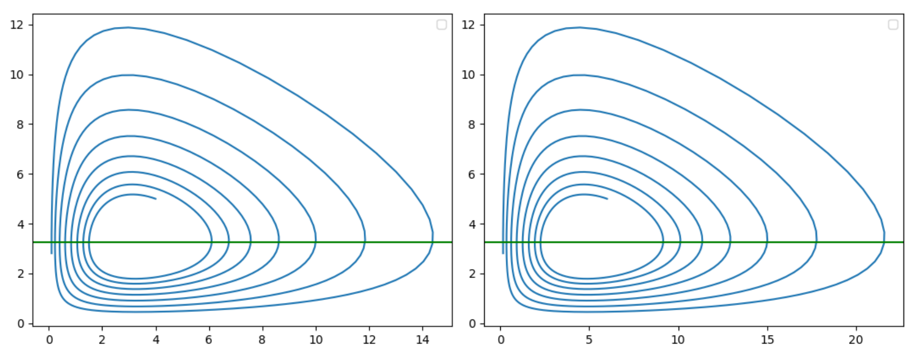

Fig. 8. Prey Phase Plane 2- For the first experiment, the first graph represents prey A, while the second represents prey B

4.4 Comment on the results of the Experiments

After extensive simulation, we were confident enough to comment on the results of our experiments. On our experiments on the behaviour of the stability and proportionality of the populations, we were able to effectively mimic the real world dynamics of the Canadian Lynx and Hares to a satisfactory degree, however with a bit more instability, as while on the real-life example the peak of preys and predators kept going up and down,in a cyclic manner, our experiments gave constant increasing peaks, with less amplitude.

In both graphs, the preys experience an exponential growth, followed by a growth from the predators, where in graph 1 the predators overtake the maximum number of preys on the first oscillations, with that gap getting shorter and shorter as the simulation runs more steps. In graph 2, the preys fit more with the real example, by constantly being more numerous than the following predators, in terms of maximum number. Both of these growths are explained by the fact that when there exists a considerable amount of prey the predators will start preying on them, creating said growth. However, once their prey decrease to low levels, they will start to die due to starvation, eventually being reduced to a level where the prey can flourish again, continuing the cycle.

On the phase spaces subject, we can again see the unstable nature of both systems, since they continuously diverge from the starting conditions outwards, with an exponential increase, still similar shape of a phase space of a real life example. We can also verify that in terms of fixed points, both situations went according to expected, where the phase space graphs would be centered around the fixed point that didn't correspond to the extinction of the species.

5 Conclusion

Modelling has attracted the attention of many researchers and has been applied with success to many different domains, being particularly well suited to problems that require the prediction of future trends and behaviour. And one well known type of model is the Continuous type, that is well suited for continuously tracking system responses according to a set of differential equations. In this paper we described an experiment which had us implement, test and simulate three different equations with the goal of analysing their behaviour over time. As discussed in our review of the results, we can affirm that we succeeded in putting the equations tested in this paper through a fair and consistent examination. We also agree that, given more time, an even more in-dept analysis of the behaviours exhibited by the system could have been made. Even so, we believe the primary objective of this paper was successfully achieved. By the end of the paper, we were able to model and display several different behaviours, with the resulting graphs being comparable to similar works of single-prey models. It is also worthy to note that due to the nature of the system, the equations must work in conjunction with each other for the behaviour to be accurate, therefor it is a necessity to use a Continuous Model. Should the nature of the equations used to represent the system change, other models could have been more appropriate for this experiment. In terms of future work, one needs to consider alternative and more diverse methods of testing, such as adding additional stimuli to the populations represented in the system; For example, the human footprint, a cap to an habitats maximum population, and interaction between different preys.

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