Constrained portfolio optimization

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November 15, 2025

1 Mathematical description

We need to solve the minimization problem at a given time t

$$\min_{z_i \in \{-1,0,1\}} \left[\lambda \sum_{ij=1}^{n} \sigma_{ij}(t) z_i z_j - (1-\lambda) \sum_{i=1}^{n} \overline{r_i}(t) z_i \right] \tag{1}$$

with the additional constraint

$$\sum_{i=1}^{n} z_i = A \ . \tag{2}$$

The variables encode the following information

- Each asset i has an associated position $z_i \in \{-1, 0, 1\}$, corresponding to short, hold or buy.
- $\lambda \in [0, 1]$ is the risk avertion.
- σ_{ij} is the covariance describing the correlation between assets i and j.
- $r_i(t) \in \mathbb{R}$ are the returns at time t and $\overline{r_i}(t) = \sum_{\tau=1}^{t} r_i(\tau)/t$ are the average returns up to time t.
- $A \in \mathbb{Z}$ is the net position of the portfolio, interpreted as the net confidence in the stock market.

The returns can be calculated as

$$r_i(t) = \frac{p_i(t) - p_i(t-1)}{p_i(t-1)} \ , \tag{3}$$

where $p_i(t)$ is the price of asset i at time $t \in \{0, 1, 2, \dots\}$. The covariance matrix can be derived from historical data as

$$\sigma_{ij}(t) = \frac{1}{t-1} \sum_{\tau=1}^{t} \left[r_j(\tau) - \overline{r_j}(t) \right] \left[r_i(\tau) - \overline{r_i}(t) \right] \ , \tag{4}$$

2 Finding a corresponding dataset

We will consider the following use cases:

- Yearly time series of publicly traded companies in Qatar