References

Simplifying, Regularizing and Strengthening Sum-Product Network Structure Learning

Antonio Vergari, Nicola Di Mauro and Floriana Esposito {firstname.lastname@uniba.it}

University of Bari "Aldo Moro", Italy
Department of Computer Science

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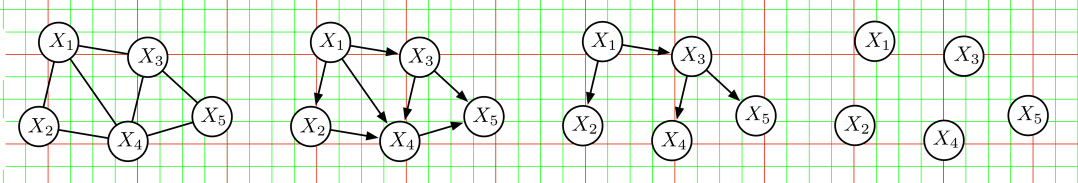
Machine Learning

Sum-Product Networks and Tractable Models

Tractable inference on Probabilistic Graphical Models (PGMs) is at a trade off with model expressiveness.



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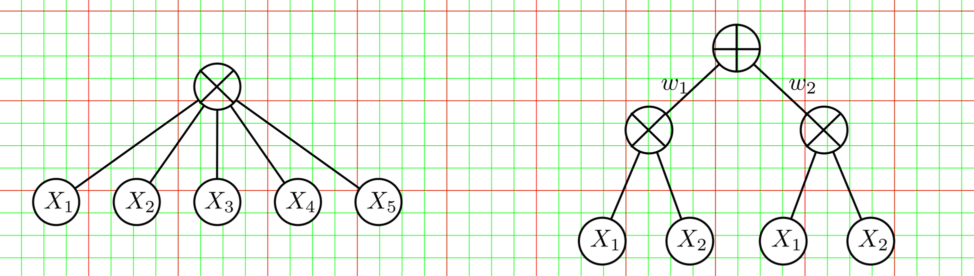
Compiling the partition function of a pdf into a ***deep*** architecture of **sum** and **product** nodes.
Product nodes define factorizations over independent vars, sum nodes mixtures. Leaves are tractable univariate distributions.
Products over nodes with different scopes (*decomposability*) and sums over nodes with same scopes (*completeness*) guarantee modeling a pdf (*validity*).
Considering only valid SPNs of *alternated layers of sum and products*.



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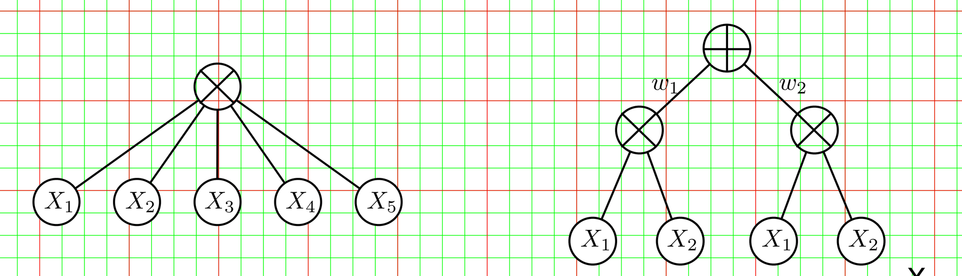

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How and Why to perform Structure Learning

Fixed structures are hard to engineer and train (fully connected layers). Structure learning is more flexible and enables automatic latent features discovery.
Constraint-based search formulation. Discover hidden variables for sum node mixtures and independences for product node components:

- greedy top-down: KMeans on features *[1]*; alternating clustering on instances and independence tests on features, **LearnSPN** *[2]*
- greedy bottom up: merging feature regions by a *Bayesian-Dirichlet independence test*, and reducing edges by maximizing MI *[6]*
- **ID-SPN**: turning LearnSPN in log-likelihood guided expansion of sub-networks approximated by Arithmetic Circuits *[7]*

LearnSPN [2] builds a tree-like SPN by recursively split the data matrix:

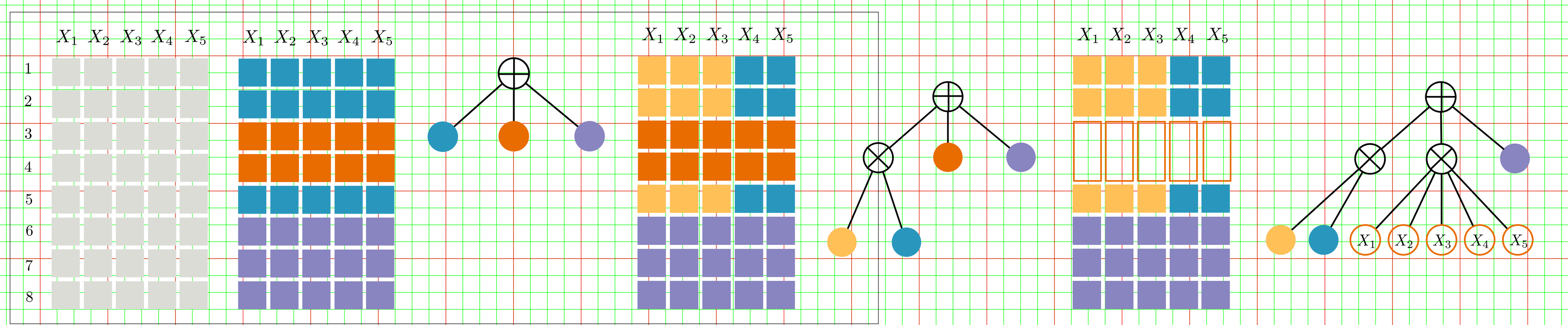
- splitting columns in pairs by a greedy ***G Test*** based procedure with threshold ρ:

$$G(X_i, X_j) = 2\sum_{x_i\sim X_i}\sum_{x_j\sim X_j} c(x_i, x_j)\cdot\log\frac{c(x_i, x_j)\cdot|T|}{c(x_i)c(x_j)}$$

- clustering instances with ***online Hard-EM*** with cluster penalty λ:

$$Pr(\mathbf{X}) = \sum_{C_i\in\mathbf{C}}\prod_{X_j\in\mathbf{X}} Pr(X_j|C_i)Pr(C_i)$$

- if there are less than m instances, put a ***naive factorization*** over leaves
- each univariate distribution get ***ML estimation*** smoothed by α



Simplifying by Limiting Node Splits

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Experiments

Classical setting for ***generative*** graphical models structure learning [2]:

- comparing the ***average log-likelihood*** on predicting instances from a test set
- 19 binary datasets from classification, recommendation, frequent pattern mining...[4] [3]
- Training 75% Validation 10% Test 15% splits (no cv)
- Model selection via *grid search* in the same parameter space:
  - $\lambda \in \{0.2, 0.4, 0.6, 0.8\}$,
  - $\rho \in \{5, 10, 15, 20\}$,
  - $m \in \{1, 50, 100, 500\}$,
  - $\alpha \in \{0.1, 0.2, 0.5, 1.0, 2.0\}$
- comparing our variants against LearnSPN, ID-SPN and MT [5]

Regularizing by introducing tree distributions as leaves

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Strengthening by Model Averaging

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