TÉCNICO LISBOA

Aprendizagem 2021/22

Homework II – Group 117

I. Pen-and-paper

Answer 1

HOMEWORK II

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1- Passo 1 - Design Matrix

$$\phi = \begin{bmatrix} 1 & \phi_1(x_1) & \phi_2(x_2) & \phi_3(x_3) \\ 1 & & & \\ 1 & & & \\ 1 & & & \\ 1 & & & \\ 1 & & & \\ 1 & & & \\ 1 & \phi_1(x_1) & \phi_2(x_2) & \phi_3(x_3) \end{bmatrix} = \begin{bmatrix} 1 & \sqrt{2} & (\sqrt{2})^2 & (\sqrt{2})^3 \\ 1 & \sqrt{27} & (\sqrt{27})^2 & (\sqrt{27})^3 \\ 1 & \sqrt{20} & (\sqrt{20})^2 & (\sqrt{20})^3 \\ 1 & \sqrt{14} & (\sqrt{14})^2 & (\sqrt{14})^3 \\ 1 & \sqrt{53} & (\sqrt{53})^2 & (\sqrt{53})^3 \\ 1 & \sqrt{3} & (\sqrt{3})^2 & (\sqrt{3})^3 \\ 1 & \sqrt{8} & (\sqrt{8})^2 & (\sqrt{8})^3 \\ 1 & \sqrt{85} & (\sqrt{85})^2 & (\sqrt{85})^3 \end{bmatrix} = \begin{bmatrix} 1 & \sqrt{2} & 2 & (\sqrt{2})^3 \\ 1 & \sqrt{27} & 27 & 3 \\ 1 & \sqrt{20} & 20 & (\sqrt{20})^3 \\ 1 & \sqrt{14} & 14 & (\sqrt{14})^3 \\ 1 & \sqrt{53} & 53 & (\sqrt{53})^3 \\ 1 & \sqrt{3} & 3 & (\sqrt{3})^3 \\ 1 & \sqrt{8} & 8 & 2 \\ 1 & \sqrt{85} & 85 & (\sqrt{85})^3 \end{bmatrix}$$

Passo 2 - Target

$$T = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \\ 6 \\ 4 \\ 5 \\ 7 \end{bmatrix}$$

Passo 3 - Erro

$$E(w) = \sum_{i=1}^{m} (t^{(i)} - o^{(i)})^2 = (T - \phi w)^T (T - \phi w)$$

$$\frac{\delta E(w)}{\delta w} = 0 \Leftrightarrow \frac{\delta((T-\phi w)^T (T - \phi w))}{\delta w} = 0 \Leftrightarrow$$

$$\Leftrightarrow (-\phi)^T (T - \phi w) + (T - \phi w)^T (-\phi) = 0 \Leftrightarrow$$

$$\Leftrightarrow w = (\phi^T \phi)^{-1} \phi^T T$$

$$\phi^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \sqrt{2} & \sqrt{27} & \sqrt{20} & \sqrt{14} & \sqrt{53} & \sqrt{3} & \sqrt{8} & \sqrt{85} \\ 2 & 27 & 20 & 14 & 53 & 3 & 8 & 85 \\ \sqrt{2}^3 & \sqrt{27}^3 & \sqrt{20}^3 & \sqrt{14}^3 & \sqrt{53}^3 & \sqrt{3}^3 & \sqrt{8}^3 & \sqrt{85}^3 \end{bmatrix}$$

$$\phi^T \cdot \phi = \begin{bmatrix} 8 & 35{,}8843 & 212 & 1482{,}2811 \\ 35{,}8843 & 212 & 1482{,}2811 & 11436 \\ 212 & 1482{,}2811 & 11436 & 93573{,}5164 \\ 1482{,}2811 & 11436 & 93573{,}5164 & 793976 \end{bmatrix}$$

$$(\phi^T \phi)^{-1} = \begin{bmatrix} 8{,}1955 & -6{,}2313 & 1{,}3049 & -0{,}0793 \\ -6{,}2313 & 5{,}0781 & -1{,}1044 & 0{,}0686 \\ 1{,}3049 & -1{,}1043 & 0{,}2472 & -0{,}0157 \\ -0{,}0793 & 0{,}0687 & -0{,}0157 & 0{,}0010 \end{bmatrix}$$

$$(\phi^T \phi)^{-1} \phi^T = \begin{bmatrix} 1{,}7686 & -0{,}0312 & -0{,}6695 & -1{,}0069 & 1{,}3794 & 0{,}9051 & -0{,}7851 & -0{,}5107 \\ -1{,}0643 & -0{,}6319 & 0{,}5312 & 0{,}9640 & -1{,}3070 & -0{,}3922 & 0{,}8501 & 0{,}5101 \\ 0{,}1933 & 0{,}0436 & -0{,}0907 & -0{,}1868 & 0{,}3232 & 0{,}0524 & -0{,}1954 & -0{,}1345 \\ -0{,}0107 & -0{,}0044 & 0{,}0044 & 0{,}01094 & -0{,}0216 & -0{,}0022 & 0{,}0123 & 0{,}0110 \end{bmatrix}$$

Sendo assim $w = \begin{bmatrix} 4{,}584 \\ -1{,}687 \\ 0{,}338 \\ -0{,}013 \end{bmatrix} = (\phi^T \phi)^{-1} \phi^T T$

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Answer 2

2-

$\hat{o}^{(9)} = w_0 + w_1\phi_1(x_9) + w_2\phi_2(x_9) + w_3\phi_3(x_9)$

$\|x_9\| = 2 \quad \hat{o}^{(9)} = 4{,}584 - 1{,}687\times 2 + 0{,}338\times 2^2 - 0{,}013\times 2^3 = 2{,}458$

$\hat{o}^{(10)} = w_0 + w_1\phi_1(x_{10}) + w_2\phi_2(x_{10}) + w_3\phi_3(x_{10})$

$\|x_{10}\| = \sqrt{6} \quad \hat{o}^{(10)} = 4{,}584 - 1{,}687\times\sqrt{6} + 0{,}338\times\sqrt{6}^2 - 0{,}013\times\sqrt{6}^3 = 2{,}289$

$$RMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(t^{(i)} - \hat{o}^{(i)})^2} = \sqrt{\frac{1}{2}\left[(2-\hat{o}^{(9)})^2 + (4-\hat{o}^{(10)})^2\right]} =$$

$$= \sqrt{\frac{1}{2}\left[(2-2{,}458)^2 + (4-2{,}289)^2\right]} = 1{,}2525$$

Answer 3

3- Após a binarização

	y_3	t_i
x_1	0	N
x_2	1	N
x_3	1	N
x_4	1	N
x_5	1	P
x_6	0	P
x_7	0	P
x_8	1	P
x_9	0	N
x_{10}	0	P

$\rightarrow E_{start}(y_{out}) = E\left(\frac{1}{2},\frac{1}{2}\right) = -\left(\frac{1}{2}\log_2\left(\frac{1}{2}\right) + \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right) = 1$

$\rightarrow E(y_{out}/y_3) = \frac{3}{8}E(y_{out}/y_3=0) + \frac{5}{8}E(y_{out}/y_3=1) =$

$= \frac{3}{8}E\left(\frac{1}{3},\frac{2}{3}\right) + \frac{5}{8}E\left(\frac{3}{5},\frac{2}{5}\right) = \frac{3}{8}\left[-\left(\frac{1}{3}\log_2\left(\frac{1}{3}\right) + \frac{2}{3}\log_2\left(\frac{2}{3}\right)\right)\right] +$

$+ \frac{5}{8}\left[-\left(\frac{3}{5}\log_2\left(\frac{3}{5}\right) + \frac{2}{5}\log_2\left(\frac{2}{5}\right)\right)\right] = 0{,}9512$

$\rightarrow E(y_{out}/y_2) = \frac{2}{8}E(1) + \frac{3}{8}E\left(\frac{2}{3},\frac{1}{3}\right) + \frac{3}{8}E\left(\frac{2}{3},\frac{1}{3}\right) =$

$= \frac{2}{8}\left[-(1\log_2(1))\right] + \frac{3}{8}\left[-\left(\frac{2}{3}\log_2\left(\frac{2}{3}\right) + \frac{1}{3}\log_2\left(\frac{2}{3}\right)\right)\right] +$

$+ \frac{3}{8}\left[-\left(\frac{2}{3}\log_2\left(\frac{2}{3}\right) + \frac{1}{3}\log_2\left(\frac{1}{3}\right)\right)\right] = 0{,}6887$

$\rightarrow E(y_{out}/y_1) = \frac{2}{8}E\left(\frac{1}{2},\frac{1}{2}\right) + \frac{4}{8}E\left(\frac{3}{4},\frac{1}{4}\right) + \frac{2}{8}E(1) = \frac{2}{8}\left[-\left(\frac{1}{2}\log_2\left(\frac{1}{2}\right) + \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right)\right] +$

$+ \frac{4}{8}\left[-\left(\frac{3}{4}\log_2\left(\frac{3}{4}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right)\right)\right] + \frac{2}{8}\left[-(1\log_2(1))\right] = 0{,}6556$

	Information Gain
y_3	$E(y_{out}) - E(y_{out}/y_3) = 1 - 0{,}9512 = 0{,}0488$
y_2	$E(y_{out}) - E(y_{out}/y_2) = 1 - 0{,}6887 = 0{,}3113$
y_1	$E(y_{out}) - E(y_{out}/y_1) = 1 - 0{,}6556 = \boxed{0{,}344}$

O y_1 é o que possui mais information gain.

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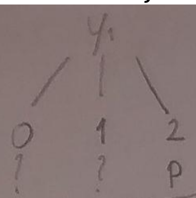
Answer 3 - continuação



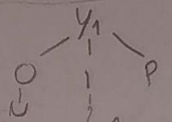
$E(y_{out}|y_1=0) = -\left(\frac{1}{2}\log\left(\frac{1}{2}\right)+\frac{1}{2}\log\left(\frac{1}{2}\right)\right)=1$

→ y_2: para $y_1=0$, y_2 é sempre 2

$E_2 = E\left(\frac{1}{2},\frac{1}{2}\right) = -\left(\frac{1}{2}\log_2\left(\frac{1}{2}\right)+\frac{1}{2}\log_2\left(\frac{1}{2}\right)\right)=1$

→ y_3 para $y_1=0$, y_3 é sempre 1 dada a sua binarização

$E_3 = E\left(\frac{1}{2},\frac{1}{2}\right) = -\left(\frac{1}{2}\log_2\left(\frac{1}{2}\right)+\frac{1}{2}\log_2\left(\frac{1}{2}\right)\right)=1$



Dada as information gain são ambas 0 temos de tomar um decisão e optamos que $y_1=0$ seja N.

$E(y_{out}|y_1=1) = -\left(\frac{3}{4}\log_2\left(\frac{3}{4}\right)+\frac{1}{4}\log_2\left(\frac{1}{4}\right)\right)=0{,}8113$

→ y_2: $E_2 = \frac{3}{4}E\left(\frac{2}{3},\frac{1}{3}\right)+\frac{1}{4}E(1) =$

$= \frac{3}{4}\left[-\left(\frac{2}{3}\log_2\left(\frac{2}{3}\right)+\frac{1}{3}\log_2\left(\frac{1}{3}\right)\right)\right]=0{,}6887$

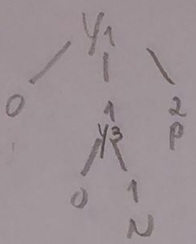
→ y_3: $E_3 = \frac{1}{2}E\left(\frac{1}{2},\frac{1}{2}\right)+\frac{1}{2}E(1) =$

$= \frac{1}{2}\left[-\left(\frac{1}{2}\log_2\left(\frac{1}{2}\right)+\frac{1}{2}\log_2\left(\frac{1}{2}\right)\right)\right]=0{,}5$

Information gain

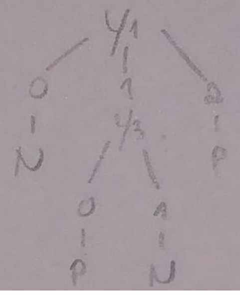
$y_2 = 1-0{,}6887 = 0{,}3113$

$y_3 = 1-0{,}5 = 0{,}5$ ✓ o y_3 é o que tem maior information gain.



Quando $y_1=1$ e $y_3=0$, y_2 é 1 em ambos os casos sendo num caso o output P e no outro N. Face a isto temos de tomar uma decisão e optámos por escolher P para $y_3=0$.

Portanto a decision tree final é esta



Answer 4

4- Para x_9, segundo a decision tree output = P. O output real é N.
Para x_{10}, segundo a decision tree output = . O output real é P.

$$\text{Accuracy} = \frac{\text{output certos}}{\text{casos totais}} = \frac{1}{2} = 0{,}5$$

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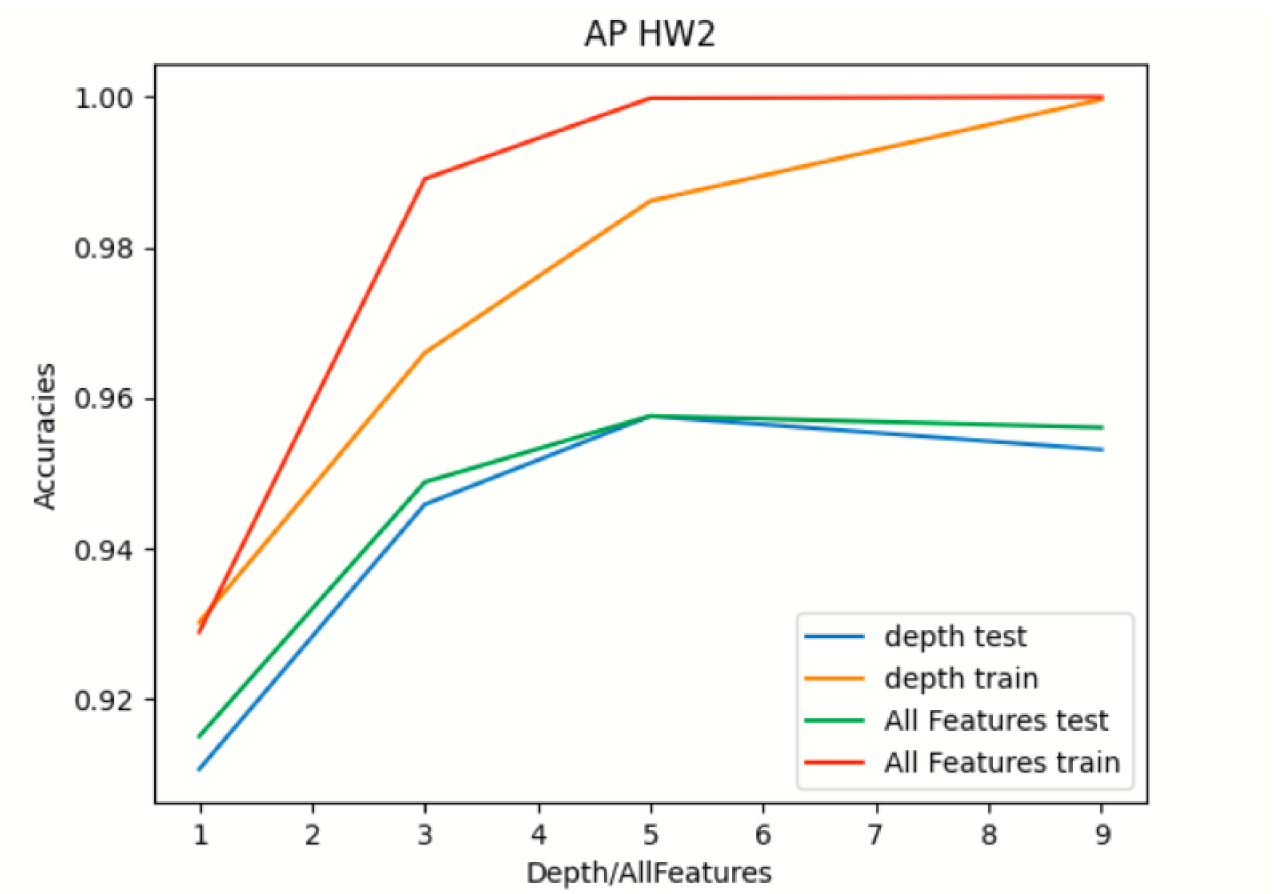
II. Programming and critical analysis

Answer 5



Para a obtenção deste gráfico começámos por separar a data do ficheiro fornecido usando um 10-fold cross validation. Posto isto, selecionámos as melhores features usando a função do python SelectKBest sendo o resultado desta função usado para a alínea i. Assim, para calcular as decision trees usamos a função DecisionTreeClassifier com os parâmetros default, excepto o max depth na alínea ii.

Por fim, fizemos fit da data e predict usando as decision trees para poder comparar resultados.

Answer 6

Uma das razões para a correlação observada deve-se ao facto de apesar de no caso i. se selecionar as max features e no caso ii. se selecionar a max depth. Pode-se verificar que ao selecionar max features = i ∈ [1,3,5,9] estamos a selecionar tambem uma max depth correspondente ao valor de i que selecionamos.

A outra razão passa pelo facto de ao limitarmos tanto a max depth como a max features com valores baixos, teremos pouca informação para testar a data, o que se reflete em accuracies mais baixas. O mesmo efeito acontece para valores altos como k = 9, sendo que nestes casos o facto de existir uma data bastante vasta também leva a uma ligeira diminuição na accuracy.

Answer 7

A depth que selecionamos é k = 5, uma vez que para uma tree com max depth igual a 5 ao testarmos a nossa test data é aí que se atinge um valor máximo, ocorrendo para valores superiores a 5 overfit.

III. APPENDIX

Paste your programming code here using Consolas 9pt or 10pt.

Use **highlighting** or colored text to facilitate the analysis by your faculty hosts.

```
# Grupo 117 Aprendizagem HomeWork 2
# Bernardo Castico ist196845
# Hugo Rita ist196870
from sklearn import tree
from sklearn.model_selection import KFold
from sklearn.feature_selection import SelectKBest
from sklearn.feature_selection import mutual_info_classif
import matplotlib.pyplot as plt

#Res = the 10-fold cross validation with our group number (117)
Res = KFold(n_splits=10, random_state=117, shuffle=True)

def getDataToMatrix(lines):
    realLines = []
    data = []
    toDelete = []
    for i in range(len(lines)):
        if i > 11:
            realLines += [lines[i]]
    for i in range(len(realLines)):
        for j in range(len(realLines[i])):
            if realLines[i][j] == "benign\n":
                realLines[i][j] = 1
            elif realLines[i][j] == "malignant\n":
                realLines[i][j] = 0
            elif realLines[i][j] == '?':
                toDelete += [i]
            else:
                realLines[i][j] = int(realLines[i][j])
    for i in range(len(realLines)):
        if i not in toDelete:
            data += [realLines[i]]
    return data

def splitData(list):
    a = []
    b = []
    for i in list:
        a.append(i[:-1])
        b.append(i[-1])
    return [a,b]
```

```
def main():
    depthTestX, finalAccuraciesAllFeatures, finalAccuraciesDepth, res , AllFeaturesTrainY,
AllFeaturesTrainX = [],[],[],[],[],[]
    depthTestY , AllFeaturesTestY, AllFeaturesTestX, depthTrainY, depthTrainX = [],[],[],[],[]

    with open("HW2.txt") as f:
        lines = f.readlines()
    for line in lines:
        tmp = line.split(',')
        res.append(tmp)
    data = getDataToMatrix(res)

    for i in [1,3,5,9]:
        counter11, counter12, counter21, counter22 = 0,0,0,0
        accuraciesDepth, accuraciesAllFeatures = [],[]

        for train, test in Res.split(data):
            testData, trainData = [],[]
            accuracyAuxDepthTest, accuracyAuxDepthTrain = 0,0
            accuracyAuxAllFeaturesTest, accuracyAuxAllFeaturesTrain = 0,0

            for j in test:
                testData += [data[j]]
            for j in train:
                trainData += [data[j]]
            trainDataSplit = splitData(trainData)
            testDataSplit = splitData(testData)

            decision = SelectKBest(mutual_info_classif, k=i).fit(trainDataSplit[0],
trainDataSplit[1])
            decisionTrainData = decision.transform(trainDataSplit[0])
            decisionTestData = decision.transform(testDataSplit[0])

            resultDepth = tree.DecisionTreeClassifier(max_depth=i, criterion="gini",
max_features=None)
            resultAllFeatures = tree.DecisionTreeClassifier(max_depth=None, criterion="gini",
max_features=None)

            resultDepth.fit(trainDataSplit[0], trainDataSplit[1])
            resultAllFeatures.fit(decisionTrainData, trainDataSplit[1])

            predictionsTest = resultDepth.predict(testDataSplit[0])
            predictionsTrain = resultDepth.predict(trainDataSplit[0])

            predictionsTestFeatures = resultAllFeatures.predict(decisionTestData)
            predictionsTrainFeatures = resultAllFeatures.predict(decisionTrainData)
```

```
            for j in range(len(predictionsTestFeatures)):
                if predictionsTestFeatures[j] == testDataSplit[1][j]:
                    accuracyAuxAllFeaturesTest += 1
                if predictionsTest[j] == testDataSplit[1][j]:
                    accuracyAuxDepthTest += 1
            for j in range(len(predictionsTrainFeatures)):
                if predictionsTrainFeatures[j] == trainDataSplit[1][j]:
                    accuracyAuxAllFeaturesTrain += 1
                if predictionsTrain[j] == trainDataSplit[1][j]:
                    accuracyAuxDepthTrain += 1
            accuraciesAllFeatures += [[accuracyAuxAllFeaturesTest/len(predictionsTestFeatures),
accuracyAuxAllFeaturesTrain/len(predictionsTrainFeatures)]]
            accuraciesDepth += [[accuracyAuxDepthTest / len(predictionsTest), accuracyAuxDepthTrain
/ len(predictionsTrain)]]

        for k in range(len(accuraciesDepth)):
            counter11 += accuraciesDepth[k][0]
            counter12 += accuraciesDepth[k][1]
            counter21 += accuraciesAllFeatures[k][0]
            counter22 += accuraciesAllFeatures[k][1]

        finalAccuraciesDepth += [[counter11 / 10, counter12 / 10]]
        finalAccuraciesAllFeatures += [[counter21 / 10, counter22 / 10]]
    #Plot
    for i in range(4):
        depthTestX = [1,3,5,9]
        depthTestY += [finalAccuraciesDepth[i][0]]
        depthTrainX = [1, 3, 5, 9]
        depthTrainY += [finalAccuraciesDepth[i][1]]
        AllFeaturesTestX = [1,3,5,9]
        AllFeaturesTestY += [finalAccuraciesAllFeatures[i][0]]
        AllFeaturesTrainX = [1, 3, 5, 9]
        AllFeaturesTrainY += [finalAccuraciesAllFeatures[i][1]]

    plt.xlabel('Depth/AllFeatures')
    plt.ylabel('Accuracies')
    plt.title('AP HW2')
    plt.plot(depthTestX, depthTestY, label = "depth test")
    plt.plot(depthTrainX, depthTrainY, label = "depth train")
    plt.plot(AllFeaturesTestX, AllFeaturesTestY, label = "All Features test")
    plt.plot(AllFeaturesTrainX, AllFeaturesTrainY, label = "All Features train")
    plt.legend()
    plt.show()
main()
```

END