



Practical exercises

Group I. Calculus [14.6v]

Considering the following dataset, where $y_1 \in [0,3]$, y_2 is ordinal and z is a nominal target.

	y_1	y_2	z
x_1	1.2	C	X
x_2	0.2	B	X
x_3	3	A	Y
x_4	0.5	B	Y
x_5	0.3	A	Y

1. [1.5v] Considering y_2 numerical encoding, {A: 0, B: 1, C: 3}, Manhattan distance, and fully unsupervised setting. Draw the dendrogram under complete linkage.

	x_1	x_2	x_3	x_4	x_5
x_1	0	3	4.8	2.7	3.9
x_2		0	3.8	0.3	1.1
x_3			0	3.5	2.7
x_4				0	1.2
x_5					0

Dendrogram: {{{{x2,x4}[0.3], x5}[1.2], x3}[3.8], x1}[4.8]

2. [1v] Are there multivariate outliers in accordance with DBSCAN ($p=3, \varepsilon=3$)? Which? $\{x_1, x_3\}$

3. Assuming a solution with maximal purity against the output variable z .

a) [1v] Identify the medoid of the larger cluster

Larger cluster $\{x_3, x_4, x_5\}$

Average distances q to other observations in the cluster: $q(x_3) = 3.1, q(x_4) = 2.35, q(x_5) = 1.95$

b) [1.5v] Identify the silhouette of the smaller cluster

$$\text{silhouette}(x_1) = 1 - \frac{3}{3.8} = 0.21, \quad \text{silhouette}(x_2) = \frac{\frac{5.2}{3}}{3} - 1 = -0.42,$$

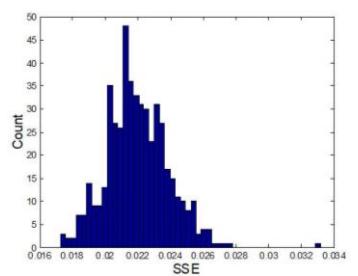
Note that when $a(x) > b(x)$ then a better proxy to the silhouette of an observation is $\frac{b(x)}{a(x)} - 1$

$$\text{silhouette}(x_1) = \frac{0.21 - (-0.42)}{2} = -0.1$$

4. [1v] Consider the following analysis of sum squared errors (SSE) gathered from a thousand of randomized datasets using k-means.

Identify the correct statement:

- i. A SSE in $[0.02, 0.023]$ is statistically significant
- ii. A SSE above 0.34 is statistically significant
- iii. A SSE below 0.017 is statistically significant ↵
- iv. None of above



5. Under the same numerical encoding, consider the biclustering task on $\{y_1, y_2\}$.

- a) [1v] Identify the largest perfect constant with coherence strength $\delta = 0.5$

Constant: $B = \{I=\{x2, x4\}, J=\{y1, y2\}\}$

- b) [1.1v] Compute the quality of order-preserving bicluster ($I=\{x1, x2, x4, x5\}, J=\{y1, y2\}$)

Quality = $7/8$ (mode orders are $y1 \leq y2$)

6. [1v] Binarize y_1 after standard scaling using equal-range discretization.

Standardly scaled $y1|X = (0.14, -0.72, 1.69, -0.46, -0.64)$

Two bin equal-range of $N(0,1)$ is $]-\infty, 0[$ and $]0, \infty[$

As a result, binarized $y1 = (U, L, U, L, L)$

7. Assuming the binarization yields $y_1=(Q, Q, R, Q, Q)$ and $\{y_1, y_2\}$ variables:

- a) [1.5v] Identify the set of closed co-occurrences satisfying $\min_{sup} = 0.4$

All patterns: Q, A, B, QB

Closed patterns: Q, A, QB

- b) [2.4v] Compute the statistical significance of pattern QB and the lift of rule $Q \Rightarrow B$

$$p_{null}(QD) = \frac{4}{5} \times \frac{2}{5} = \frac{8}{25} = 0.32$$

$$P(X \geq 2 | X \sim \text{Binomial}(p = 0.32, N = 5) = 0.51$$

$$\text{lift} = \text{sup}(QB) / (\text{sup}(Q) \times \text{sup}(B)) = (2/5) / (4/5 \times 2/5) = 1.25$$

8. [1.6v] Consider the covariance matrix C obtained from the given dataset and the corresponding eigenvectors. Which of the eigenvectors should be considered to reduce the dimensionality?

$$C = \begin{pmatrix} 1.353 & -0.225 \\ -0.225 & 1.5 \end{pmatrix}, \quad v_1 = \begin{pmatrix} -0.8 \\ -0.59 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0.59 \\ -0.8 \end{pmatrix}$$

Given $Cv_1 = \lambda_1 v_1$, then $\lambda_1 \approx 1.189$

Given $Cv_2 = \lambda_2 v_2$, then $\lambda_2 \approx 1.66$

The second eigenvector is used to reduce the space has it explains more variability

Group II. True-and-false [5.4v] (+0.45v for correct, -0.2v for incorrect)

1. An outlier can be inconsistent with the remaining data or just its neighbourhood. **True**
2. In supervised outlier analysis, assessing the sensitivity of a classifier is often preferred over its accuracy. **True**
3. Contextual outliers only deviate on a compact subset of variables. **False**
4. Hamming distance is adequate to handle ordinal variables with high cardinality. **False**
5. k -means does not adequately identify spherical clusters. **False**
6. Purity is biased when the number of found clusters approaches the total number of observations. **True**
7. Spearman correlation is preferred over Pearson correlation if the order of values is more relevant than their absolute value. **True**
8. Given a m -dimensional dataset, PCA can reconstruct any data point using $m - 1$ components of PCA with zero reconstruction error. **False**

9. The lift measure of an association rule $A \Rightarrow B$ does not change if we add a new transaction that does not either contain A or B. **False**
10. Sequential pattern mining can be applied to find frequent co-occurrences in symbolic multivariate time series data. **True**
11. A false negative tricluster is a statistically significant subspace that was not retrieved. **True**
12. The search for triclusters with low variance allows the discovery of additive subspaces. **False**

END