Práctica 1: Sampling and Quantization

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1 Task 1

Question: Give your interpretation of the resulting graphs. Do the quantization levels correspond with the values you had expected?

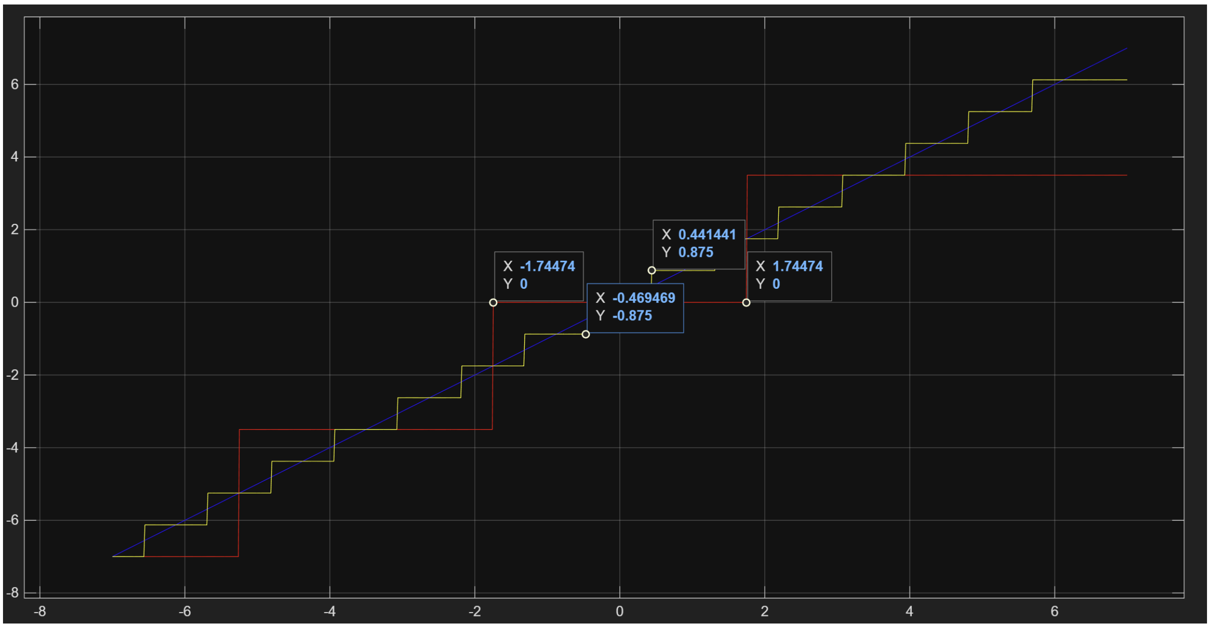
We represent the original continuous signal x (blue) and the 2 quantized version using 2 (red) and 4 (yellow) bits. As expected, the 2 bit quantization produces fewer discrete levels than the 4 bits quantization. Increase the number of bits decreases Δ, resulting in smaller steps and a quantized signal that follows the input more closely.

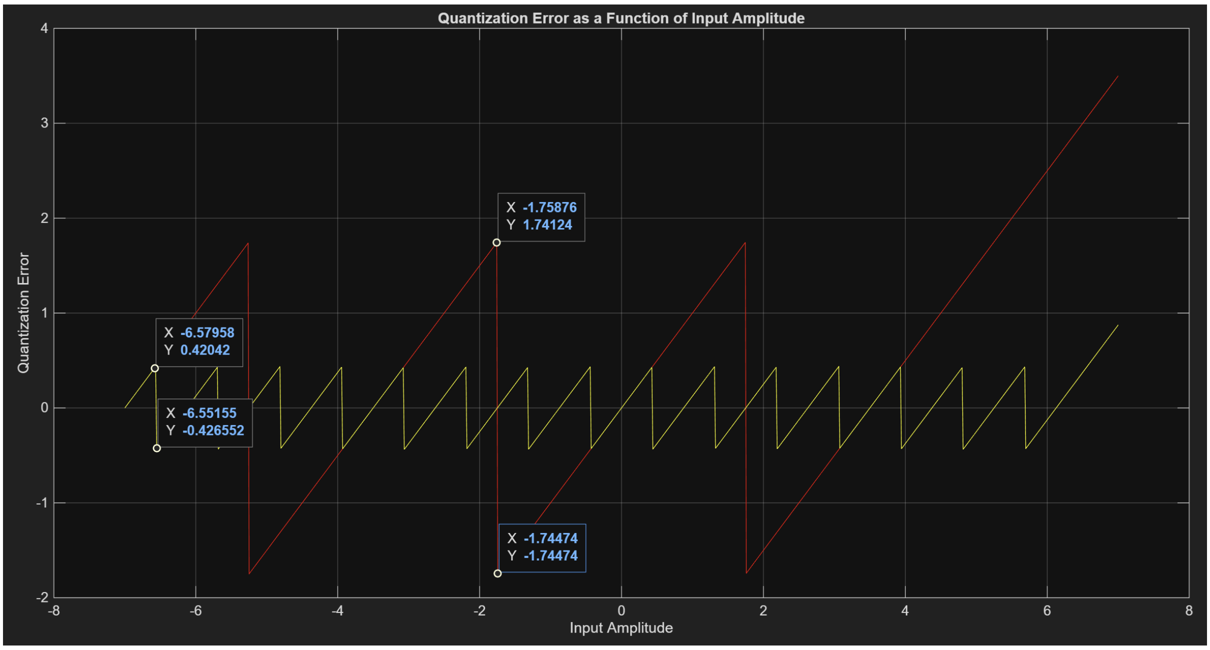
Question: For both cases, represent the quantization error as a function of input amplitude in the range $[-7, +7]$ and comment on your results. Is this error always within the $[-\Delta/2, +\Delta/2]$ interval?

The magnitude of the error decreases as the number of bits increases, since a smaller quantization step Δ reduces the maximum deviation between the input and its quantized version. The $[-\Delta/2, +\Delta/2]$ in each case is as follows:

- For N = 2 the Δ value we get is $\Delta = 3{,}5$, so the interval should be $[-1{,}75, 1{,}75]$.
- For N = 4 the Δ value we get is $\Delta = 0{,}875$, so the interval should be $[-0{,}4375, 0{,}4375]$.

In both cases, the error remains bounded within the theoretical interval $[-\Delta/2, +\Delta/2]$.

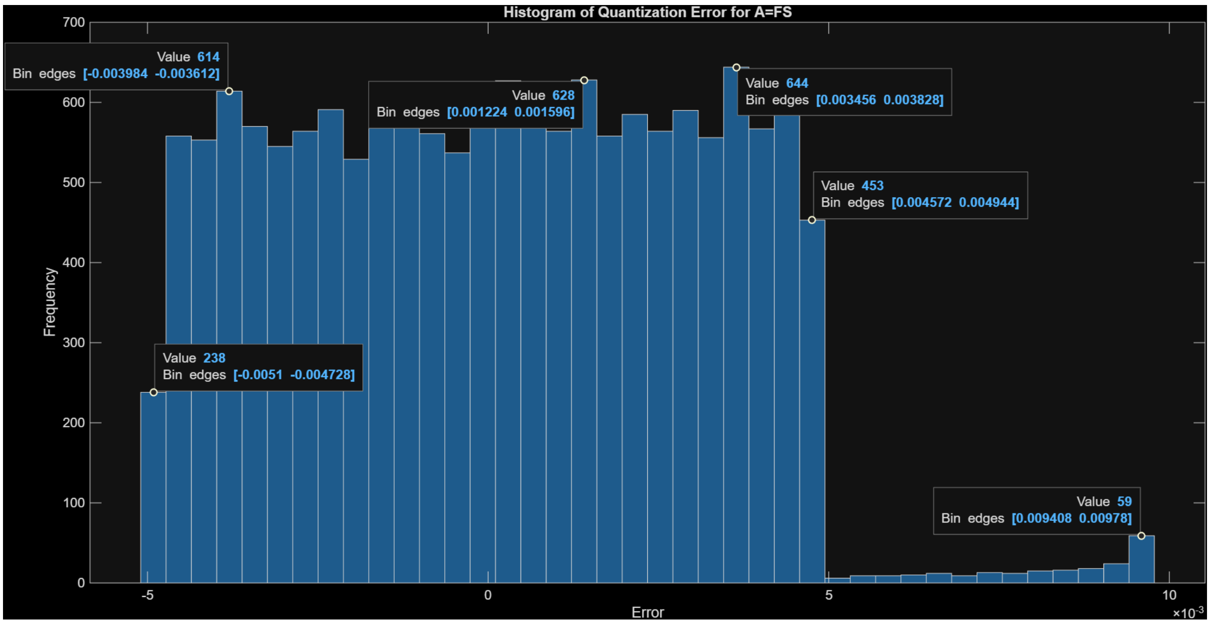




2 Task 2

Question: Assume a full-scale sinusoidal input and plot the histogram of the quantization error. Do you observe what you expected, or not?

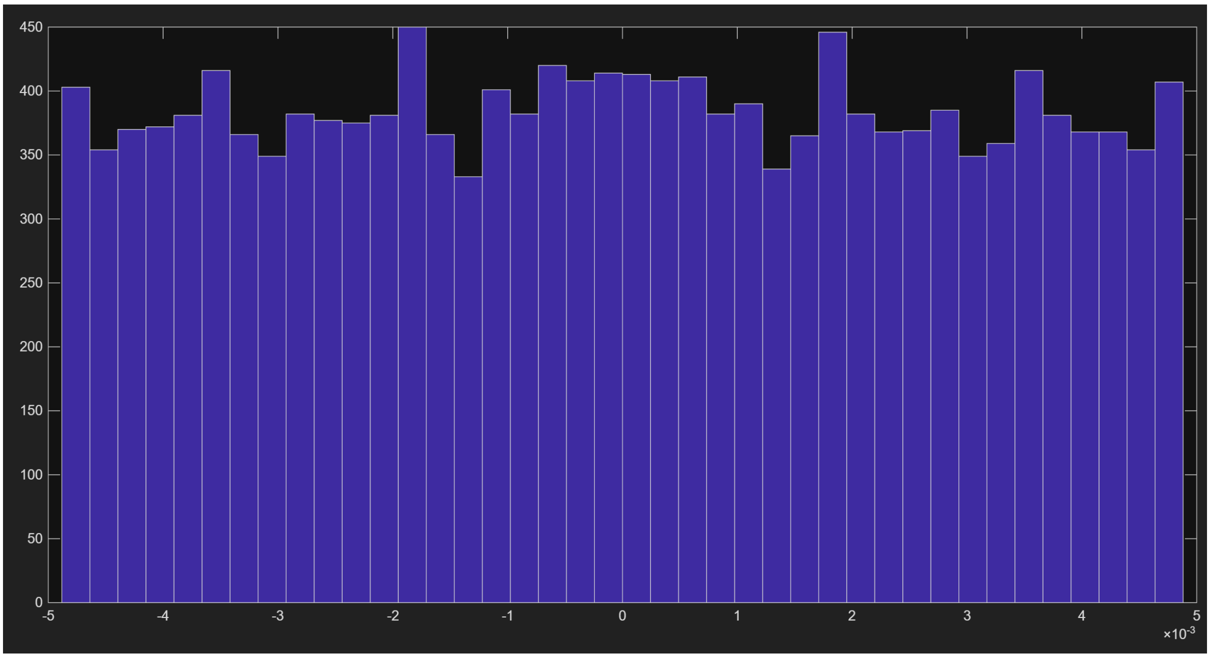
$\Delta = \frac{2*FS}{2^N} = 0{,}0098$, so the $[-\frac{\Delta}{2}, +\frac{\Delta}{2}]$ interval should be $[-0{,}0049, +0{,}0049]$. In the histogram we can see that in that interval the error is uniformly distributed, but there is an error tail in the positive extreme. It means that there is **clipping** in the positive.



Question: Explain the operation of the Matlab command var. Estimate the variance of the quantization error using var, and compare it to its theoretical value. Estimate the value (in dB) of the Signal to-Quantization Noise Ratio (SQNR) and compare it to its theoretical value (1).

The MATLAB command `var` computes the variance of a set of values. For a vector x, it calculates: $\mathrm{var}(x) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, where $\bar{x}$ is the mean of the values in x.

ADD



Question: Repeat the previous steps for sinusoids with different amplitudes, and with decreasing resolutions of 12, 10, 8, 6 and 4 bits, in order to fill Table 1, rounding the SQNR values (in dB) to two decimal places. Comment on your results.

	$A = 0{,}5 \cdot \texttt{FS}$		$A = 0{,}75 \cdot \texttt{FS}$		$A = \texttt{FS}$		$A = 1{,}03 \cdot \texttt{FS}$	
	SQNR (dB)		SQNR (dB)		SQNR (dB)		SQNR (dB)	
N	theory	measured	theory	measured	theory	measured	theory	measured
12	61.96	62.02	67.98	68.03	71.50	71.54	74.00	73.70
10	49.92	50.01	55.94	56.01	59.46	59.53	61.96	61.56
8	37.88	38.07	43.90	44.04	47.42	47.53	49.92	49.15
6	25.84	26.22	31.86	32.13	35.38	35.60	37.88	36.52
4	13.80	14.59	19.82	20.37	23.34	23.78	25.84	23.63

Cuadro 1: Pertaining to Task 2.

3 Task 3

Question: Suppose that you have an N-bit A/D converter with tunable FS, and you know that your input samples follow a symmetric triangular pdf in some interval

$[-x_0, x_0]$. **Intuitively, how would you set the FS value of your converter? What would the resulting rms value σ_x in dBFS be?**

If you set $FS < x_0$ any imput $|x|$ greater than FS will be clipped. If $FS > x_0$,we would be wasting the converter's since the signal would never reach the limits. Therefore, the value of FS should be x_0.

$\sigma_x = \sqrt{var(x)} = \frac{x_0}{\sqrt{6}}$ and in dBFS would be $20\log_{10}(1/\sqrt{6}) = -7,78$ dBFS.

Question: Explain how to generate in Matlab samples of a random variable following a symmetric triangular pdf with zero mean and rms value σ_0. Check the histogram and use the commands mean and var to validate your approach

We have two options to do it:

- Option 1: We can do it using *makedist* function. To do it, we can use the following code:

```
            x0 = 2;
            A = -x0; B = 0; C = +x0; % simetria = media 0

            pd = makedist('Triangular','A',A,'B',B,'C',C);
            N = 100000;
            samples = random(pd, N, 1);

            % comprobaciones rapidas
            emp_mean = mean(samples);
            emp_var = var(samples);
            emp_desv_std = std(samples);

            % valores teoricos
            % theo_mean = 0; % simetria centrado en 0
            theo_var = (A^2 + B^2 + C^2 - A*B - A*C - B*C)/18;
            rms = 20*log10(sqrt(theo_var)/x0);

            fprintf('Theorical mean: 0; emp mean: %.2f\n',emp_mean)
;
            fprintf('Theorical var: %.2f; emp var: %.2f\n',theo_var
,emp_var);
            fprintf('Sigma value: %.2f\n',sqrt(theo_var));
            fprintf('rms value in dBFS: %.2f\n',rms)

            % ver histograma y pdf teorica
            xgrid = linspace(A,C,500)';
            figure
            histogram(samples,100,'Normalization','pdf')
            hold on
            plot(xgrid, pdf(pd,xgrid), 'LineWidth',1.5)
            title('Triangular (media 0) -- muestras vs PDF')
            hold off
```

- Option 2: we can generate samples of a random variable following a symmetric triangular pdf as the sum of two independent random variables X_1 and X_2 from a uniform distribution. we can doit as follows: REVISAR!!

```
            x0=2;
```

```
            sigma0 = x0/sqrt(2);
            N = 100000;

            c = sigma0 * sqrt(3/2);

            x1 = (2 * rand(N, 1) - 1) * c;
            x2 = (2 * rand(N, 1) - 1) * c;

            y = x1 + x2;

            sample_mean = mean(y);
            sample_var = var(y);
            sample_rms = std(y);

            fprintf('--- Validation ---\n');
            fprintf('Target Mean: 0.0\n');
            fprintf('Sample Mean: %f\n\n', sample_mean);

            fprintf('Target Variance (sigma0^2): %f\n', sigma0^2);
            fprintf('Sample Variance: %f\n\n', sample_var);

            fprintf('Target RMS (sigma0): %f\n', sigma0);
            fprintf('Sample RMS: %f\n\n', sample_rms);

            figure;
            histogram(y, 100, 'Normalization', 'pdf', 'DisplayName'
    , 'Generated Samples');
            grid on;
            hold on;

            a = 2*c;
            x_pdf = linspace(-a, a, 400);
            y_pdf = (1/a) * (1 - abs(x_pdf)/a);
            plot(x_pdf, y_pdf, 'r-', 'LineWidth', 2.5, 'DisplayName
    ', 'Theoretical PDF');

            title('Symmetric Triangular Distribution');
            xlabel('Random Variable Value');
            ylabel('Probability Density Function (PDF)');
            legend;
            hold off;
```

Question: Take $10 \cdot 2^{10}$ of these triangularly distributed samples, quantize them, and estimate the SQNR empirically for $N =$ 3, 4, 5 and 6 bits. Do this for σ_x varying in the range $[-50, 0]$ dBFS and in steps of $0{,}1$ dBFS. Plot the resulting curves (SQNR in dB vs. σ_x in dBFS) along with the theoretical expression

$$\mathrm{SQNR} = 6{,}02N + 4{,}77 - 20\log_{10}\frac{\mathrm{FS}}{\sigma_x} \qquad \textbf{(dB).} \tag{1}$$

Are there any differences between the theoretical and empirical curves? If so, how do you explain them? ADD

Question: In view of your results, what are the optimum values (regarding SQNR) of σ_x (in dBFS), and for the different resolutions analyzed (3 to 6 bits)? Does this agree with your intuition (see first point above)?

4 Task 4

Assume a full-scale sinusoidal input with $f_0 = 37{,}1094 MHz$, and let the FFT size be M = 1024. Generate $15 \cdot M$ samples of $x(t)$ (at fs = 100 MHz) and quantize them to N = 12 bits. Break the vector xq of quantized samples into 15 size-M blocks using, e.g., the command reshape:

```
xqblocks = reshape(xq, M, 15);
```

so that each column of the $M \times 15$ matrix xqblocks will contain the corresponding block of size M . Now, since the fft command computes the FFT columnwise, in order to apply an M -point FFT to each block, we simply make

```
X = fft(xqblocks, M);
```

Average the squared magnitude of the DFT coefficients over the 15 blocks and plot the results between 0 and fs/2, in dBFS. Observe the location and peak value of the principal frequency component, as well as the value of the noise floor. Do your observations agree (quantitatively) with what you would expect?