SWERC NoteBook

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Ensemble d'algorithmes et techniques de programmation



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- 1 Configuration
- 1.1 C/C++
- 2 Chaînes de caractères
- 3 Séquences
- 4 Parcours de graphes
- 4.1 DFS Depth First Search

```
1
   #version iterative pour eviter la recursion limit de python
2
   def dfs_iterative(graph,start,seen):
       seen[start] = True
3
       to_visit = [start]
5
       while to_visit:
6
           node = to_visit.pop()
7
           for neighbour in graph[node]:
8
               if not seen[neighbour]:
9
                  seen[neighbour] = True
10
                  to_visit.append(neighbour)
```

4.2 BFS - Breadth First Search

```
from collections import deque
 1
   def bfs(graph, start=0):
       to_visit = deque()
 3
 4
       dist = [float('inf')] * len(graph)
 5
       prec = [none] * len(graph)
 6
       dist[start] = 0
 7
       to_visit.appendleft(start)
 8
       while to_visit: #evalue a faux si vide
9
           node = to_visit.pop()
10
           for neighbour in graph[node]:
               if dist[neighbour] == float('inf'):
11
12
                  dist[neighbour] = dist[node] + 1
13
                  prec[neighbour] = node
14
                  to_visit.appendleft(neighbour)
15
       return dist, prec
```

- 4.3 Topological Sort
- 4.4 Composantes connexes
- 4.5 Composantes bi-connexe
- 4.6 Composantes fortement connexe
- 4.7 2-SAT
- 4.8 Postier Chinois
- 4.9 Chemin eulérien
- 4.10 Chemin le plus court
- 4.10.1 Poids positif ou nul Dijkstra
- 4.10.2 Poids arbitraire Bellman-Ford
- 4.10.3 Floyd-Warshall

5 Points et polygones

- 5.1 Points
- **5.1.1** Points

```
point = [x,y]
```

5.1.2 Cross-product

```
1 def cross_product(p1, p2):
2  return p1[0] * p2[1] - p2[0] * p1[1]
```

5.1.3 Direction

```
def left_turn(a,b,c):
    return (a[0]-c[0]) * (b[1]-c[1]) - (a[1]-c[1]) * (b[0]-c[0]) > 0
    # If floats are used, instead of 0 test if in [0-10E-7,0+10E-7]
```

5.2 Enveloppe convexe

```
Complexité : \mathcal{O}(n \log(n))
```

```
def andrew(S):
 1
 2
       S.sort()
 3
       top = []
 4
       bot = []
 5
       for p in S:
 6
           while len(top) >= 2 and not left_turn(p,top[-1],top[-2]):
 7
               top.pop()
 8
           top.append(p)
9
           while len(bot) >= 2 and not left_turn(bot[-2],bot[-1],p):
10
               bot.pop()
11
           bot.append(p)
       return bot[:-1] + top[:0:-1]
12
```

5.3 Aire d'un polygone

Uniquement pour les polygones simples. Réduire à des composantes simples sinon. $A = \frac{1}{2} \sum_{i=0}^{n-1} (x_i x_{i+1} - x_{i+1} y_i)$

```
def area(p):
    A = 0
    for i in range(len(p)):
        A += p[i-1][0] * p[i][1] - p[i][0] * p[i-1][1]
    return A/2
```

5.4 Points entiers dans un polygone

5.4.1 Sur le contour

5.4.2 Dans le polygone

```
Théorème de Pick : P = n_i + \frac{n_b}{2} - 1
```

5.5 Paire de points les plus proches

6 Ensembles

6.1 Rendu de monnaie

Problème NP-Complet.

```
def coin(x, R):
    b = [False] * (R+1)
    b[0] = True
    for xi in x:
        for s in range(xi, R+1):
            b[s] |= b[s - xi]
    return b[R]
```

6.2 Sac à dos

Problème NP-Complet.

```
1
   def knapsack(p, v, cmax):
 2
       n = len(p)
 3
       Opt = [[0] * (cmax + 1) for _ in range(n+1)]
       Sel = [[False] * (cmax + 1) for _ in range(n+1)]
 4
 5
       #cas de base
 6
       for cap in range(p[0], cmax +1):
 7
           Opt[0][cap] = v[0]
 8
           Sel[0][cap] = True
9
       # cas d'induction
10
       for i in range(1,n):
11
           for cap in range(cmax+1):
12
               if cap \geq p[i] and Opt[i-1][cap - p[i]] + v[i] \geq Opt[i-1][cap]:
13
                   Opt[i][cap] = Opt[i-1][cap-p[i]] + v[i]
                   Sel[i][cap] = True
14
15
               else:
16
                   Opt[i][cap] = Opt[i-1][cap]
17
                   Sel[i][cap] = False
18
       cap = cmax
19
       sol = []
20
       for i in range(n-1, -1, -1):
21
           if Sel[i][cap]:
22
               sol.append(i)
23
               cap -= p[i]
       return (Opt[n-1][cmax], sol)
24
```

6.3 k-somme

7 Calculs

7.1 PGCD

```
1 def pgcd(a,b):
2 return a if b == 0 else pgcd(b,a%b)
```

7.2 Coefficients de Bézout

```
def bezout(a,b):
1
2
      if b == 0:
3
          return (1,0)
4
      else:
5
          u,v = bezout(b,a%b)
6
          return (v, u - (a//b) *v)
7
  def inv(a,p):
8
      return bezout(a,p)[0]%p
```

7.3 Coefficients binomiaux

```
def binom(n,k,p):
    prod = 1
    for i in range(k):
        prod = (prod * (n-i)) // (i+1) %p
    return prod
    #Enlever le p et mod p pour sans modulo
```