SWERC NoteBook

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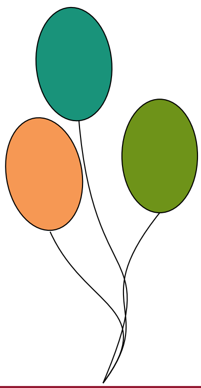
Ensemble d'algorithmes et techniques de programmation

Et quelques notions de Mathématiques



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1 Configuration

1.1 C/C++

1.1.1 Vecteur

```cpp
#include <vector>
//Declaration :
std::vector<int> name;
//Debut /Fin (element)
name.begin();
name.back();
//Debut / Fin
name.front();
name.end();
//Ajout /Deletion (fin)
name.push_back(val);
name.pop_back();
//Ajout /Suppression(index)
name.insert(index, val);
name.erase(index);
name.erase(index1,index2);
//Taille
name.size();
//Destruction
name.clear();
```

2 Chaînes de caractères

3 Séquences

3.1 Anagramme

```python
def anagrams(w):
    w = list(set(w)) #retire les doublons
    d = {}
    for i in range(len(w)):
        s = ''.join(sorted(w[i]))
        if s in d:
            d[s].append(i)
        else:
            d[s] = [i]
    answer = []
    for s in d:
        if len(d[s]) > 1:
            answer.append([w[i] for i in d[s]])
    return answer
```

3.2 Distance de Levenshtein

```python
def levenshtein(x,y):
    n = len(x)
    m = len(y)
    A = [[i+j for j in range(m+1)] for i in range(n+1)]
    for i in range(n):
        for j in range(m):
            A[i+1][j+1] = min(A[i][j+1] + 1, A[i+1][j] + 1, A[i][j] + int(x[i] != y[j]))
    return A[n][m]
```

3.3 Plus grand facteur commun

```
def longest_common_subsequence(x,y):
    n = len(x)
    m = len(y)
    A = [[0 for j in range(m+1)] for i in range(n+1)]
    for i in range(n):
        for j in range(m):
            if x[i] == y[j]:
                A[i+1][j+1] = A[i][j] + 1
            else:
                A[i+1][j+1] = max(A[i][j+1], A[i+1][j])
    sol = []
    i, j = n, m
    while A[i][j] > 0:
        if A[i][j] == A[i-1][j]:
            i -= 1
        elif A[i][j] == A[i][j-1]:
            j -= 1
        else:
            i -= 1
            j -= 1
            sol.append(x[i])
    return ''.join(sol[::-1])
```

3.4 Rabin-Karp — Recherche d'un pattern

```
#a mettre en C++ !
PRIME = 72057594037927931  # < 2^{56}
DOMAIN = 128
def roll_hash(old_val, out_digit, in_digit, last_pos):
    """roll_hash """
    val = (old_val - out_digit * last_pos + DOMAIN * PRIME) % PRIME
    val = (val * DOMAIN) % PRIME
    return (val + in_digit) % PRIME
def matches(s, t, i, j, k):
    for d in range(k):
        if s[i + d] != t[j + d]:
            return False
    return True
def rabin_karp_matching(s, t):
    hash_s = 0
    hash_t = 0
    len_s = len(s)
    len_t = len(t)
    last_pos = pow(DOMAIN, len_t - 1) % PRIME
    if len_s < len_t:           # substring too long
        return -1
    for i in range(len_t):      # preprocessing
        hash_s = (DOMAIN * hash_s + ord(s[i])) % PRIME
        hash_t = (DOMAIN * hash_t + ord(t[i])) % PRIME
    for i in range(len_s - len_t + 1):
        if hash_s == hash_t:             # hashes match
            # check character by character
            if matches(s, t, i, 0, len_t):
                return i
        if i < len_s - len_t:
            # shift window and calculate new hash on s
            hash_s = roll_hash(hash_s, ord(s[i]), ord(s[i + len_t]),
                               last_pos)
    return -1                            #no match
def rabin_karp_factor(s, t, k):
    last_pos = pow(DOMAIN, k - 1) % PRIME
    pos = {}
```

```
    assert k > 0
    if len(s) < k or len(t) < k:
        return None
    hash_t = 0
    # First calculate hash values of factors of t
    for j in range(k):
        hash_t = (DOMAIN * hash_t + ord(t[j])) % PRIME
    for j in range(len(t) - k + 1):
        # store the start position with the hash value
        if hash_t in pos:
            pos[hash_t].append(j)
        else:
            pos[hash_t] = [j]
        if j < len(t) - k:
            hash_t = roll_hash(hash_t, ord(t[j]), ord(t[j + k]), last_pos)
    hash_s = 0
    # Now check for matching factors in s
    for i in range(k):      # preprocessing
        hash_s = (DOMAIN * hash_s + ord(s[i])) % PRIME
    for i in range(len(s) - k + 1):
        if hash_s in pos:   # is this signature in s?
            for j in pos[hash_s]:
                if matches(s, t, i, j, k):
                    return (i, j)
        if i < len(s) - k:
            hash_s = roll_hash(hash_s, ord(s[i]), ord(s[i + k]), last_pos)
    return None
```

4 Parcours de graphes

4.1 DFS - Depth First Search

```
#version iterative pour eviter la recursion limit de python
def dfs_iterative(graph,start,seen):
    seen[start] = True
    to_visit = [start]
    while to_visit:
        node = to_visit.pop()
        for neighbour in graph[node]:
            if not seen[neighbour]:
                seen[neighbour] = True
                to_visit.append(neighbour)
```

4.2 BFS - Breadth First Search

```
from collections import deque
def bfs(graph, start=0):
    to_visit = deque()
    dist = [float('inf')] * len(graph)
    prec = [none] * len(graph)
    dist[start] = 0
    to_visit.appendleft(start)
    while to_visit: #evalue a faux si vide
        node = to_visit.pop()
        for neighbour in graph[node]:
            if dist[neighbour] == float('inf'):
                dist[neighbour] = dist[node] + 1
                prec[neighbour] = node
                to_visit.appendleft(neighbour)
    return dist, prec
```

4.3 Topological Sort

```
def topological_order(graph):
    V = range(len(graph))
    indeg = [0 for _ in V]
    for node in V:          # compute indegree
        for neighbor in graph[node]:
            indeg[neighbor] += 1
    Q = [node for node in V if indeg[node] == 0]
    order = []
    while Q:
        node = Q.pop()      # node without incoming arrows
        order.append(node)
        for neighbor in graph[node]:
            indeg[neighbor] -= 1
            if indeg[neighbor] == 0:
                Q.append(neighbor)
    return order
```

4.4 Composantes connexes

```
#connex : 4-connex
connex = [(i,j+1),(i,j-1),(i+1,j),(i-1,j)]
#connex : 8-connex
connex = [(i,j+1),(i,j-1),(i+1,j),(i-1,j),(i+1,j+1),(i-1,j-1),(i+1,j-1),(i-1,j+1)]
def dfs_grid(grid, i, j, mark, free):
    grid[i][j] = mark
    height = len(grid)
    width = len(grid[0])
    for ni, nj in connex:
        if 0 <= ni < height and 0 <= nj < width:
            if grid[ni][nj] == free:
                dfs_grid(grid, ni, nj, mark, free)
def nb_connected_components(grid, free='#'):
    nb_components = 0
    height = len(grid)
    width = len(grid[0])
    for i in range(height):
        for j in range(width):
            if grid[i][j] == free:
                nb_components += 1
                dfs_grid(grid, i, j, str(nb_components), free)
    return nb_components
```

4.5 Composantes bi-connexe

```
def cut_nodes_edges(graph):
    n = len(graph)
    time = 0
    num = [None] * n
    low = [n] * n
    parent = [None] * n      # parent[v] = None if root else parent of v
    critical_children = [0] * n # cc[u] = #{children v | low[v] >= num[u]}
    times_seen = [-1] * n
    for start in range(n):
        if times_seen[start] == -1:           # init DFS path
            times_seen[start] = 0
            to_visit = [start]
            while to_visit:
                node = to_visit[-1]
                if times_seen[node] == 0:     # start processing
                    num[node] = time
                    time += 1
```

```
                low[node] = float('inf')
            children = graph[node]
            if times_seen[node] == len(children): # end processing
                to_visit.pop()
                up = parent[node]          # propagate low to parent
                if up is not None:
                    low[up] = min(low[up], low[node])
                    if low[node] >= num[up]:
                        critical_children[up] += 1
            else:
                child = children[times_seen[node]] # next arrow
                times_seen[node] += 1
                if times_seen[child] == -1: # not visited yet
                    parent[child] = node    # link arrow
                    times_seen[child] = 0
                    to_visit.append(child) # (below) back arrow
                elif num[child] < num[node] and parent[node] != child:
                    low[node] = min(low[node], num[child])
    cut_edges = []
    cut_nodes = []                          # extract solution
    for node in range(n):
        if parent[node] is None:            # characteristics
            if critical_children[node] >= 2:
                cut_nodes.append(node)
        else:                               # internal nodes
            if critical_children[node] >= 1:
                cut_nodes.append(node)
            if low[node] >= num[node]:
                cut_edges.append((parent[node], node))
    return cut_nodes, cut_edges
```

4.6 Composantes fortement connexe

4.6.1 Kosaraju

```
def kosaraju_dfs(graph,nodes,order,sccp):
    times_seen = [-1] * len(graph)
    for start in nodes:
        if times_seen[start] == -1:
            to_visit = [start]
            times_seen[start] = 0
            sccp.append([start])
            while to_visit:
                node = to_visit[-1]
                children = graph[node]
                if times_seen[node] == len(children):
                    to_visit.pop()
                    order.append(node)
                else:
                    child = children[times_seen[node]]
                    times_seen[node] += 1
                    if times_seen[child] == -1:
                        times_seen[child] = 0
                        to_visit.append(child)
                        sccp[-1].append(child)
def reverse(graph):
    rev_graph = [[] for node in graph]
    for node in range(len(graph)):
        for neighbour in graph[node]:
            rev_graph[neighbour].append(node)
    return rev_graph
def kosaraju(graph):
    n = len(graph)
    order = []
```

```
    sccp = []
    kosaraju_dfs(graph, range(n), order, [])
    kosaraju_dfs(reverse(graph),order[::-1], [], sccp)
    return sccp[::-1]
```

4.7 2-SAT

```
def vertex(lit):
    if lit > 0:
        return 2 * (lit - 1)
    else:
        return 2 * (-lit -1) +1
def two_sat(formula):
    n = max(abs(clause[p]) for p in (0,1) for clause in formula)
    graph = [[] for node in range(2*n)]
    for x,y in formula:
        graph[vertex(-x)].append(vertex(y))
        graph[vertex(-y)].append(vertex(x))
    sccp = kosaraju(graph)
    comp_id = [None] * (2*n)
    affectations = [None] * (2*n)
    for component in sccp:
        rep = min(component)
        for vtx in component:
            comp_id[vtx] = rep
            if affectations[vtx] == None:
                affectations[vtx] = True
                affectations[vtx ^ 1] = False
    for i in range(n):
        if comp_id[2*i] == comp_id[2*i+1]:
            return None
    return affectations[::2]
```

4.8 Postier Chinois

4.9 Chemin eulérien

4.9.1 Dirigé

```
def eulerian_tour_directed(graph):
    P = []
    Q = [0]
    R = []
    next = [0] * len(graph)
    while Q:
        node = Q.pop()
        P.append(node)
        while next[node] < len(graph[node]):
            neighbour = graph[node][next[node]]
            next[node] += 1
            R.append(neighbour)
            node = neighbour
        while R:
            Q.append(R.pop())
    return P
```

4.9.2 Non Dirigé

```
def eulerian_tour_undirected(graph):
    P = []
    Q = [0]
    R = []
```

```python
    next = [0] * len(graph)
    seen = [set() for _ in graph]
    while Q:
        node = Q.pop()
        P.append(node)
        while next[node] < len(graph[node]):
            neighbour = graph[node][next[node]]
            next[node] += 1
            if neighbour not in seen[node]:
                seen[neighbour].add(node)
                R.append(neighbour)
                node = neighbour
        while R:
            Q.append(R.pop())
    return P
```

4.10 Chemin le plus court

4.10.1 Poids positif ou nul - Dijkstra

```python
from heapq import heappop, heappush
def dijkstra(graph, weight, source=0, target=None):
    """single source shortest paths by Dijkstra
       :complexity: O(|V| + |E|log|V|)"""
    n = len(graph)
    assert all(weight[u][v] >= 0 for u in range(n) for v in graph[u])
    prec = [None] * n
    black = [False] * n
    dist = [float('inf')] * n
    dist[source] = 0
    heap = [(0, source)]
    while heap:
        dist_node, node = heappop(heap)   # Closest node from source
        if not black[node]:
            black[node] = True
            if node == target:
                break
            for neighbor in graph[node]:
                dist_neighbor = dist_node + weight[node][neighbor]
                if dist_neighbor < dist[neighbor]:
                    dist[neighbor] = dist_neighbor
                    prec[neighbor] = node
                    heappush(heap, (dist_neighbor, neighbor))
    return dist, prec
def dijkstra_update_heap(graph, weight, source=0, target=None):
    """single source shortest paths by Dijkstra
       :complexity: O(|V| + |E|log|V|)"""
    n = len(graph)
    assert all(weight[u][v] >= 0 for u in range(n) for v in graph[u])
    prec = [None] * n
    dist = [float('inf')] * n
    dist[source] = 0
    heap = OurHeap([(dist[node], node) for node in range(n)])
    while heap:
        dist_node, node = heap.pop()   # Closest node from source
        if node == target:
            break
        for neighbor in graph[node]:
            old = dist[neighbor]
            new = dist_node + weight[node][neighbor]
            if new < old:
                dist[neighbor] = new
                prec[neighbor] = node
                heap.update((old, neighbor), (new, neighbor))
```

```
    return dist, prec
```

4.10.2 Poids arbitraire - Bellman-Ford

```

def bellman_ford2(graph, weight, source):
    """    :complexity: O(|V|*|E|)"""
    n = len(graph)
    dist = [float('inf')] * n
    prec = [None] * n
    dist[source] = 0

    def relax():
        for nb_iterations in range(n-1):
            for node in range(n):
                for neighbor in graph[node]:
                    alt = dist[node] + weight[node][neighbor]
                    if alt < dist[neighbor]:
                        dist[neighbor] = alt
                        prec[neighbor] = node
    relax()
    intermediate = dist[:] # is fixpoint in absence of neg cycles
    relax()
    for node in range(n):
        if dist[node] < intermediate[node]:
            dist[node] = float('-inf')
    return dist, prec, min(dist) == float('-inf')
```

4.10.3 Floyd-Warshall

```
def floyd_warshall(weight):
    """"O(|V|^3)"""
    for k, Wk in enumerate(weight):
        for _, Wu in enumerate(weight):
            for v, Wuv in enumerate(Wu):
                alt = Wu[k] + Wk[v]
                if alt < Wuv:
                    Wu[v] = alt
    for v, Wv in enumerate(weight):
        if Wv[v] < 0:    # negative cycle found
            return True
    return False
```

5 Points et polygones

5.1 Points

5.1.1 Points

```
point = [x,y]
```

5.1.2 Cross-product

```
def cross_product(p1, p2):
  return p1[0] * p2[1] - p2[0] * p1[1]
```

5.1.3 Direction

```
def left_turn(a,b,c):
  return (a[0]-c[0]) * (b[1]-c[1]) - (a[1]-c[1]) * (b[0]-c[0]) > 0
  # If floats are used, instead of 0 test if in [0-10E-7,0+10E-7]
```

5.2 Enveloppe convexe

Complexité : $\mathcal{O}(n \log(n))$

```
def andrew(S):
    S.sort()
    top = []
    bot = []
    for p in S:
        while len(top) >= 2 and not left_turn(p,top[-1],top[-2]):
            top.pop()
        top.append(p)
        while len(bot) >= 2 and not left_turn(bot[-2],bot[-1],p):
            bot.pop()
        bot.append(p)
    return bot[:-1] + top[:0:-1]
```

5.3 Aire d'un polygone

Uniquement pour les polygones simples. Réduire à des composantes simples sinon. Voir partie Mathématiques.

```
def area(p):
    A = 0
    for i in range(len(p)):
        A += p[i-1][0] * p[i][1] - p[i][0] * p[i-1][1]
    return A/2
```

5.4 Polygone simple

```
def is_simple(polygon):
    """complexity: O(n log n) for n=len(polygon)"""
    n = len(polygon)
    order = list(range(n))
    order.sort(key=lambda i: polygon[i])     # lexicographic order
    rank_to_y = list(set(p[1] for p in polygon))
    rank_to_y.sort()
    y_to_rank = {y: rank for rank, y in enumerate(rank_to_y)}
    S = RangeMinQuery([0] * len(rank_to_y)) # sweep structure
    last_y = None
    for i in order:
        x, y = polygon[i]
        rank = y_to_rank[y]
        right_x = max(polygon[i - 1][0], polygon[(i + 1) % n][0])
        left = x < right_x
        below_y = min(polygon[i - 1][1], polygon[(i + 1) % n][1])
        high = y > below_y
        if left:                 # y does not need to be in S yet
            if S[rank]:
                return False     # two horizontal segments intersect
            S[rank] = -1         # add y to S
        else:
            S[rank] = 0          # remove y from S
        if high:
            lo = y_to_rank[below_y] # check S between [lo + 1, rank - 1]
            if (below_y != last_y or last_y == y or
                    rank - lo >= 2 and S.range_min(lo + 1, rank)):
                return False     # horiz. & vert. segments intersect
        last_y = y               # remember for next iteration
    return True
```

5.5 Paire de points les plus proches

6 Ensembles

6.1 Rendu de monnaie

Problème NP-Complet.

```
def coin(x, R):
    b = [False] * (R+1)
    b[0] = True
    for xi in x:
        for s in range(xi, R+1):
            b[s] |= b[s - xi]
    return b[R]
```

6.2 Sac à dos

Problème NP-Complet.

```
def knapsack(p, v, cmax):
    n = len(p)
    Opt = [[0] * (cmax + 1) for _ in range(n+1)]
    Sel = [[False] * (cmax + 1) for _ in range(n+1)]
    #cas de base
    for cap in range(p[0], cmax +1):
        Opt[0][cap] = v[0]
        Sel[0][cap] = True
    # cas d'induction
    for i in range(1,n):
        for cap in range(cmax+1):
            if cap >= p[i] and Opt[i-1][cap - p[i]] + v[i] > Opt[i-1][cap]:
                Opt[i][cap] = Opt[i-1][cap-p[i]] + v[i]
                Sel[i][cap] = True
            else:
                Opt[i][cap] = Opt[i-1][cap]
                Sel[i][cap] = False
    cap = cmax
    sol = []
    for i in range(n-1, -1, -1):
        if Sel[i][cap]:
            sol.append(i)
            cap -= p[i]
    return (Opt[n-1][cmax], sol)
```

6.3 k-somme

6.4 Points les plus proches

```
def dist(p, q):
    return hypot(p[0] - q[0], p[1] - q[1]) # Euclidean dist.
def cell(point, size):
    """ returns the grid cell coordinates containing the given point.
    size is the side length of a grid cell
    beware: in other languages negative coordinates need special care
    in C++ for example int(-1.5) == -1 and not -2 as we need
    hence we need floor(x / pas) in C++ using #include <cmath>
    """
    x, y = point                    # size = grid cell side length
    return (int(x // size), int(y // size))
def improve(S, d):
    G = {}                          # maps grid cell to its point
    for p in S:                     # for every point
        a, b = cell(p, d / 2)       # determine its grid cell
        for a1 in range(a - 2, a + 3):
            for b1 in range(b - 2, b + 3):
```

```
                if (a1, b1) in G:     # compare with points
                    q = G[a1, b1]     # in surrounding cells
                    pq = dist(p, q)
                    if pq < d:        # improvement found
                        return pq, p, q
        G[a, b] = p
    return None
def closest_points(S):
    shuffle(S)
    assert len(S) >= 2
    p = S[0]              # start with distance between
    q = S[1]              # first two points
    d = dist(p, q)
    while d > 0:          # distance 0 cannot be improved
        r = improve(S, d)
        if r:             # distance improved
            d, p, q = r
        else:             # r is None: could not improve
            break
    return p, q
```

6.5 Valeurs les plus proches

```
def closest_values(L):
    assert len(L) >= 2
    L.sort()
    valmin, argmin = min((L[i] - L[i - 1], i) for i in range(1, len(L)))
    return L[argmin - 1], L[argmin]
```

7 Calculs

7.1 PGCD

```
def pgcd(a,b):
    return a if b == 0 else pgcd(b,a%b)
```

7.2 Coefficients de Bézout

```
def bezout(a,b):
    if b == 0:
        return (1,0)
    else:
        u,v = bezout(b,a%b)
        return (v, u - (a//b) *v)
def inv(a,p):
    return bezout(a,p)[0]%p
```

7.3 Coefficients binomiaux

```
def binom(n,k):
    prod = 1
    for i in range(k):
        prod = (prod * (n-i)) // (i+1)
    return prod
def binom_modulo(n,k,p):
    prod = 1
    for i in range(k):
        prod = (prod * (n-1) * inv(i+1,p)) %p
    return prod
```

7.4 Inverse

```
def inv(a,p):
    return bezout(a,p)[0] %p
```

7.5 Nombres premiers

```
def eratosthene(n):
    """O(n loglog n)"""
    P = [True] * n
    answ = [2]
    for i in range(3, n, 2):
        if P[i]:
            answ.append(i)
            for j in range(i * i, n, i):
                P[j] = False
    return answ
def gries_misra(n):
    """Prime numbers by the sieve of Gries-Misra
    Computes both the list of all prime numbers less than n,
    and a table mapping every integer 2 < x < n to its smallest prime factor
    :param n: positive integer
    :returns: list of prime numbers, and list of prime factors
    :complexity: O(n)
    """
    primes = []
    factor = [0] * n
    for x in range(2, n):
        if not factor[x]:    # no factor found
            factor[x] = x    # meaning x is prime
            primes.append(x)
        for p in primes:     # loop over primes found so far
            if p > factor[x] or p * x >= n:
                break
            factor[p * x] = p # p is the smallest factor of p * x
    return primes, factor
```

8 Mathématiques

8.1 Géométrie

8.1.1 3D

- Sphère : Volume : $\frac{4}{3}\pi r^3$ — Surface : $4\pi r^2$
- Cylindre droit : Volume $\pi r^2 h$ — Surface : $2\pi r(r+h)$
- Cone circulaire droit : Volume $\frac{1}{3}\pi r^2 h$ — Surface : $\pi r(r+s)$
- Prisme triangulaire : Volume Al ou $\frac{1}{2}bhl$ — Surface : $bh + 2ls + lb$
- Prisme : Volume Ah — Surface : $2A + (h \times p)$
- Pyramide : Volume : $\frac{1}{3}Ah$
- Tétrahèdre : Volume : $\frac{b^3}{6\sqrt[2]{2}}$ — Surface : $\sqrt[2]{3}b^2$
- Pyramide carré : Volume : $\frac{1}{3}s^2 \times h$ — Surface : $s^2 + 2sh$
- Cuboide : Volume : $l \times w \times h$ — Surface : $2lh + 2lw + 2wh$ (Cube : $6s^2$)

8.1.2 2D

- Polygone simple : Aire : $A = \frac{1}{2}\sum_{i=0}^{n-1}(x_i y_{i+1} - x_{i+1} y_i)$
- Cercle : Aire : πr^2 — Périmètre : $2 \times \pi \times r$
- Losange : Aire : $\frac{D \times d}{2}$
- Trapèze : Aire : $\frac{(B+b) \times h}{2}$
- Parralélogramme : Aire : $B \times h$

8.1.3 Points entiers dans un polygone

Sur le contour :
Dans le polygone :
Théorème de Pick : $P = n_i + \frac{n_b}{2} - 1$

8.1.4 Théorème de la galerie d'art

Pour garder un polygone simple à n sommets, $\lfloor \frac{n}{3} \rfloor$ gardiens suffisent.

8.2 Approximations

8.2.1 Méthode de Newton

$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$ — $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$

8.2.2 Méthode de la sécante

Cette méthode est à appliquer quand le calcul de la dérivée est couteux $\frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}$

8.2.3 Plus forte pente — Méthode du gradient

Cette méthode peut être assez couteuse (ZigZag)
Algorithme du gradient — On se donne un point initial $x_0 \in \mathbb{E}$ et un seuil de tolérance $\epsilon \geq 0$. L'algorithme du gradient donne une suite d'itérés $x_1, x_2 ... \in \mathbb{E}$, jusqu'à ce qu'un test d'arrêt soit satisfait. Il passe de x_k à x_{k+1} par les étapes suivantes :

- 1. *Simulation* : Calcul de $\nabla f(x_k)$
- 2. *Test d'arrêt* : Si $\|\nabla f(x_k)\| \leq \epsilon$, arrêt
- 3. *Calcul du pas* : $\alpha_k > 0$ par un règle de recherche linéaire sur f en x_k le long de la direction $-\nabla f(x_k)$
- 4. *Nouvel itéré* : $x_{k+1} = x_k - \alpha_k \nabla f(x_k)$

8.3 Probabilités et Statistiques

8.3.1 Lois de probabilités

Discrètes :

- Poisson : $\mathbb{P}(X = k) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad \mathbb{E}[X] = \lambda, \quad \mathbb{V}[X] = \lambda$
- Binomiale : $\mathbb{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad \mathbb{E}[X] = np, \quad \mathbb{V}[X] = np(1-p)$
- Géométrique : $\mathbb{P}(X = k) = (1-p)^{k-1} p, \quad \mathbb{E}[X] = \frac{1}{p}, \quad \mathbb{V}[X] = \frac{1-p}{p^2}$
- Uniforme : $\mathbb{P}(X = k) = \frac{1}{n}, \quad \mathbb{E}[X] = \frac{1}{n}\sum_{k=1}^{n} x_k$

8.3.2 Techniques statistiques

Théorème de la limite centrale :
Soit $X_1, X_2, ...$ une suite de variable aléatoires réelles définies sur le même espace de probabilités, i.i.d et suivant la même loi $\mathcal{L}$. De plus, l'espérance μ et l'écart-type σ de $\mathcal{L}$ existent et soient finis avec $\sigma \neq 0$.
Soit la somme $S_n = X_1 + X_2 + ... + X_n$
Alors l'espérance de S_n est $n\mu$ et l'écart-type est $\sigma\sqrt{n}$
Quand n est assez grand, la Loi Normale $\mathcal{N}(n\mu, n\sigma^2)$ est une bonne approximation de S_n
On pose $\overline{X_n} = \frac{S_n}{n}$ et $Z_n = \frac{S_n - n\mu}{\sigma\sqrt{n}} = \frac{\overline{X_n} - \mu}{\sigma / \sqrt{n}}$

9 Techniques de programmation

9.1 Programmation dynamique

Résoudre le problème en le divisant en sous-problèmes, résoudre les sous-problèmes, stocker les résultats intermédiaires ("mémoisation")

9.2 Diviser pour régner

Diviser un problème en sous-problèmes; Résoudre les sous-problèmes; Combiner : calculer la solution grâce aux solutions des sous-problèmes.

9.3 Floyd's Hare and Tortoise

L'objectif de cette méthode est de détecter des cycles. L'idée est de parcourir la liste chaînée avec deux pointeurs : un lent (tortoise) et un deux fois plus rapide (hare). Si les deux pointeurs s'intersectent, il y a un cycle.