LOGIC EXAM QUESTIONS

2011

① (a) Formalise the following in predicate logic.

(i) power g.d are either solar panels OR wind turbines

" For all X, such that X is a pgd, implies that X is a solar panel or a wind turbine, and X cannot be both"

in predicate logic:

$\forall X(pgd(X) \rightarrow (sp(X) \vee wt(X) \wedge \neg(sp(X) \wedge wt(X)))$

(ii) Solar panels weigh at least 150, and wind turbines at least 100.

\ use weigh(X,Y): weight of X is Y.

" For all X, X is a solar panel, implies that there is a weight Y such that Y ≥ 150 ∧ (same for wind turbines"

$\forall X \exists Y(sp(X) \rightarrow weight(X,Y), Y \geq 150) \wedge \forall X \exists Y (wt(X) \rightarrow weight(X,Y) \wedge Y \geq 100)$

(iii) There is a solar panel that weighs more than any wind turbine

" There exists a X, X is a solar panel such that for all Y, Y is a wind turbine, ∃W1 and W2 where weight of X is W1 and weight Y is W2 implies that W1 > W2"

$\exists X \forall Y(sp(X) \wedge wt(Y) \rightarrow \exists w1, w2 (weight(X, w1) \wedge weight(Y, w2) \rightarrow w1 > w2))$

(iv) Some buildings have solar panels, and some have wt, but none has both.

~~XX~~" There exists X, X is a building implies that there exists Y, Z, Y is a solar panel and Z is a wt such that X has Y · OR ? X has Z, but there does not exist any building X has Y and Z."

" There exists X, Y. X is a sp and Y is a wt such that ∃ building B ~~XX~~.

~~$\exists X \; b(X)$~~

$\exists X, Y (sp(X) \wedge wt(Y) \rightarrow (\exists Z (b(Z) \rightarrow has(Z,X) \vee has(Z,Y)) \wedge \neg \exists Z (b(Z) \rightarrow has(Z,X) \wedge has(Z,Y))))$

(v) In any building that has a pgd, weight of device is ≤ maxweight

" For all X, X is a building and for all Y, Y is a pgd such that X has Y, implies that there is a weight W1 and W2 where W1 is the weight of pgd, and W2 is max weight → W1 ≤ W2."

$\forall X, Y((b(X) \wedge pgd(Y) \rightarrow has(X,Y)) \longrightarrow \exists w1, w2(weight(Y, w1) \wedge sup(X, w2) \rightarrow w1 \leq w2)).$

(b) Use equivalences to transform the following to one where negation ··· pred symbols.

$\neg(\forall X (p(X) \rightarrow \exists Y q(X,Y)) \vee \forall X \exists Y r(X,a,Y))$

$\equiv \neg\forall X(p(X) \rightarrow \exists Y q(X,Y)) \wedge \neg\forall X \exists Y r(X,a,Y))$ (De Morgan's Rules)

$\equiv \exists X \neg(p(X) \rightarrow \exists Y q(X,Y)) \wedge \exists X \neg \exists Y r(X,a,Y))$

$\equiv \exists X \neg((\neg p(X) \vee \exists Y q(X,Y)) \wedge \exists X \forall Y \neg r(X,a,Y))$ (implication Rule)

$\equiv \exists X (\neg\neg p(X) \wedge \neg\exists Y q(X,Y)) \wedge \exists X \forall Y \neg r(X,a,Y))$ (De Morgan's Rule)

$\equiv \exists X (p(X) \wedge \forall Y \neg q(X,Y)) \wedge \exists X \forall Y \neg r(X,a,Y)).$

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(c) using natural deduction and part (b) ···

1. $\neg(\forall X(p(X) \rightarrow \exists Y q(X,Y)) \vee \forall X \exists Y r(X,a,Y))$ given

you want to show: $\exists X \neg q(X,a)$.

2. $\exists X(p(X) \wedge \forall Y \neg q(X,Y)) \wedge \exists X \forall Y \neg r(X,a,Y)$ 1, part(b)

3. $\exists X(p(X) \wedge \forall Y \neg q(X,Y))$ 2, $\wedge$E

4. $\exists X \forall Y \neg r(X,a,Y)$ 2, $\wedge$E

 5. $p(b) \wedge \forall Y \neg q(b,Y)$ Assume.

 6. $\forall Y \neg q(b,Y)$ 5, $\wedge$E.

 7. $\neg q(b,a)$ 6, $\forall$E

 8. $\exists X \neg q(X,a)$ 7, $\exists$I

9. $\exists X \neg q(X,a)$ 3, 5, 8, $\exists$E.

QED.

② (ai) $P \otimes Q$: P, but not Q.

definition: $P \wedge \neg Q$

(aii) Show that: $(A \to B) \otimes (A \to C) \equiv (A \otimes C) \otimes (A \otimes B)$.

substituting in the above definition. it is enough to show:

$$(A \to B) \wedge \neg(A \to C) \equiv (A \wedge \neg C) \otimes (A \wedge \neg B)$$

$$\equiv (A \wedge \neg C) \wedge \neg(A \wedge \neg B).$$

starting from LHS:

$(A \to B) \wedge \neg(A \to C) = (\neg A \vee B) \wedge \neg(\neg A \vee C)$ (implication Rule)

$\equiv \neg(A \wedge \neg B) \wedge (\neg\neg A \wedge \neg C)$ (Demorgan)

$\equiv (\neg\neg A \wedge \neg C) \wedge \neg(A \wedge \neg B)$ (Commutativity)

$\equiv (A \wedge \neg C) \wedge \neg(A \wedge \neg B)$ (Negation).

$\equiv$ RHS . QED.

(b) (i) Formalise the following:

S1: $Warm \vee Sunny \to \neg$ ~~Rains~~ $Snows$

S2: $(Sunny \to \neg Rains) \wedge (Cloudy \to (Rains \vee Snows) \wedge \neg(Rains \wedge Snows))$

S3: $(Sunny \wedge Windy \to \neg Warm) \wedge (Sunny \wedge \neg Windy \to Warm)$

S4: $Snows \leftrightarrow Cold$.

(iii) Formalise S2 with at least 2 occurrences of $\otimes$: $P \otimes Q$: P and not Q.

using aii),

~~let A be Sunny, and let B be ¬Rains~~.

~~let second~~

~~Then consider (Cloudy → Q), it can be written as (¬Cloudy ∨ Q)~~ ≡

$(A \to B) \otimes (C \to D)$

\let A be cloudy, B be $(Rains \vee Snows) \wedge \neg(Rains \wedge Snows)$

\C sunny and D: Rainy.

$\Rightarrow$ $Cloudy \otimes ((Rains \vee Snows) \otimes (Rains \wedge Snows)) \otimes (Sunny \otimes Rains)$

(ii) Show that: $\neg(\neg A \wedge \neg B) \vdash A \vee B$.

1. $\neg(\neg A \wedge \neg B)$ given
2. $\neg\neg A \vee \neg\neg B$ 1, lemma 1
3. $A \vee B$ 2, lemma 2.

lemma 1: $\neg(A \wedge B) \vdash \neg A \vee \neg B$

1. $\neg(A \wedge B)$ given
~~2. ¬(A∨B) Assume~~
2. $\neg(\neg A \vee \neg B)$ Assume.
   3. $\neg A$ Assume
   4. $\neg A \vee \neg B$ 3, ∨I.
5. A 2,3,4, RAA
   6. $\neg B$ Assume.
   7. $\neg A \vee \neg B$ 6, ∨I.
8. B 2,6,7, RAA
9. $A \wedge B$ 5,8, ∧I
10. $\neg A \vee \neg B$ 1,2,9, RAA □

lemma 2: $\neg\neg A \vdash A$

1. $\neg\neg A$ given
2. $\neg A$ Assume.
3. A 1, 2, RAA $\square$

$\neg(\neg M \land \neg N)$

$($, $P \lor S$

(ii) L1, L2, L3, L4, L5 $\vdash \neg P \to M \lor N$ and (i): $\neg(\neg A \land \neg B) \vdash A \lor B$.

1. $\neg A \land \neg B \to P$
2. $A \to (C \to P \lor (R \to N))$
3. $A \to (\neg C \to P \lor (R \to N))$
4. $B \to M \land N$
5. $\neg P \lor S \to R$

given.

6. $\neg P$ Assume.
7. $\neg A \land \neg B$ Assume.
8. P 1, 7, $\to$E.
8. $\neg(\neg A \land \neg B)$ 6, 7, 8, RAA
9. $A \lor B$ 8, part i
10. $\neg(M \lor N)$ Assume.
11. $\neg M \land \neg N$ 10, lemma 3.
12. $\neg M$ 11, $\land$E
13. $\neg N$ 11, $\land$E.
14. $\neg A$ Assume
15. B 9, 14, $\lor$E.
16. $M \land N$ 4, 15, $\to$E.
17. M 16, $\land$E.
18. A 12, 14, 17, RAA
19. $C \to P \lor (R \to N)$ 2, 18, $\to$E
20. $\neg C \to P \lor (R \to N)$ 3, 18 $\to$E.
21. $P \lor (R \to N)$ 19, 20, lemma 4.
22. $R \to N$ 6, 21, $\lor$E
23. $\neg P \lor S$ 6, $\lor$I
24. R 23, 5, $\to$E.
25. N 22, 24, $\to$E.
26. $M \lor N$ 10, 13, 25, RAA.
27. $\neg P \to M \lor N$ 6, 26, $\to$I.

lemma 3: $\neg(A \lor B) \vdash \neg A \land \neg B$

1. $\neg(A \lor B)$ given
2. A Assume
3. $A \lor B$ 2, $\lor$I.
4. $\neg A$ 1, 2, 3, RAA
5. B Assume
6. $A \lor B$ 5, $\lor$I.
7. $\neg B$ 1, 5, 6, RAA
8. $\neg A \land \neg B$ 4, 7, $\land$I.

lemma 4 (Dilemma): $A \to B, \neg A \to B \vdash B$

1. $A \to B$
2. $\neg A \to B$

given

3. $\neg B$ Assume.
4. $\neg A$ Assume.
5. B 2, 4, $\to$E.
6. A 3, 4, 5, RAA.
7. B 1, 6, $\to$E.
8. B 3, 7, RAA.

QED.