2014

① (a) Formalise the following in predicate logic

(i) There are day and evening classes

"There is X such that X is a day class and there is Y such that Y is an evening class"

∴ in predicate logic:

$\longrightarrow \exists X(day(X)) \land \exists X(evening(X))$.

(ii) All who teach a day class are on salary scheme sd, unless they also teach an evening class, then they are on sde.
Those who teach only evening are on te.

There are X, Y for all P.

"for all X, Y, P such that X are day classes and Y are evening classes, if P teaches only X, then ... if P teaches both X and Y, then ... if P teaches only Y, then ..."

in predicate logic:

$\longrightarrow \forall P \exists X,Y(\ day(X) \land evening(Y) \rightarrow ((teaches(P,X) \land \neg teaches(P,Y) \rightarrow salary(P,sd)) \land$
$(teaches(P,X) \land teaches(P,Y) \rightarrow salary(P,sde)) \land$
$(teaches(P,Y) \land \neg teaches(P,X) \rightarrow salary(P,te)))$.

(iii) The subjects taught in the evening are type: languages and humanities.
There are also day classes teaching these types. But, Latin is not a subject taught in any day/evening class.

"for all X such that X is an evening class, for all Sub such that Sub is taught in evening class X, Sub is a type language, or type humanities. There exists Y such that Y is a day class, there exists Sub taught in day class Y, Sub is a type ...., and Sub is not Latin.

place in front.

in predicate logic:

$\forall X\ (evening(X) \rightarrow \forall S(subject(S,X) \land \neg S = latin \land (type(S,humanities) \lor type(S,languages)))) \land$
$\exists X\ (day(X) \rightarrow \exists S(subject(S,X) \land \neg S = latin \land (type(S,humanities) \lor type(S,languages))))$

(iv) Everyone who is enrolled on a day class will sit an examination on the subject of that class, but no one enrolled on evening classes only will sit any examinations.

There does not exist

"For all student S, there exist X, a day class such that if X X.

"for all X, such that X is a day class, for all students enrolled in X, ~~then~~ and for all Sub taught in X, then S sits an exam for Sub, and there does not exist a student S enrolled in X, X is an evening class, and not enrolled in Y, Y is a day class ...

in predicate logic:

$\forall X, Sub(day(X) \land subject(Sub,X) \rightarrow (\forall S(enrolled(S,X) \rightarrow sitsExam(S,Sub)) \land$
$\exists X,Y,Sub(day(X) \land evening(Y) \land subject(Sub,Y) \rightarrow \neg(\exists S(enrolled(S,Y) \land \neg enrolled(S,X) \rightarrow sitsExam(S,Sub))$.

(b) Store has a Rule for recognising when an item of merchandise is stolen.

If an item of merchandise exits the store, and it has not been paid for by the time it exits then it is stolen at the time of exiting the store.

(i) 2 statements formalise the above. Identify and show they are equivalent.

S1 and S3 formalises the sentence.

$\forall X \forall T(exit(X,T) \land \neg paid\_for(X,T) \rightarrow stolen(X,T)) \equiv \forall X \forall T(\neg stolen(X,T) \rightarrow (\neg exit(X,T) \lor paid\_for(X,T)))$
$\equiv \forall X \forall T(\neg\neg stolen(X,T) \lor (\neg exit(X,T) \lor paid\_for(X,T)))$ (implication Rule)
$\equiv \forall X \forall T(stolen(X,T) \lor \neg(exit(X,T) \land \neg paid\_for(X,T)))$ (De Morgans)
$\equiv \forall X \forall T(\neg(exit(X,T) \land \neg paid\_for(X,T)) \lor stolen(X,T))$ (commutative law)
$\equiv \forall X \forall T(\ exit(X,T) \land \neg paid\_for(X,T)) \rightarrow stolen(X,T))$ (implication Rule).

(ii) For one of the sentences that does not represent the Rule correctly, find an equivalent sentence that has only $\exists, \neg, \vee, \wedge$ as connectives

SQ: $\forall X \forall T(\neg paid\_for(X,T) \vee stolen(X,T) \rightarrow exit(X,T))$

$\equiv \forall X \forall T(\neg(paid\_for(X,T) \vee stolen(X,T)) \vee exit(X,T))$ (implication law)

$\equiv \forall X \forall T(\neg paid\_for(X,T) \wedge \neg stolen(X,T) \vee exit(X,T))$ (DeMorgans)

$\equiv \forall X \neg \exists T \neg(\neg(paid\_for(X,T) \vee stolen(X,T)) \vee exit(X,T))$

$\equiv \forall X \neg \exists T(paid\_for(X,T) \vee stolen(X,T) \wedge \neg exit(X,T))$

$\equiv \neg \exists X, T(paid\_for(X,T) \vee stolen(X,T) \wedge \neg exit(X,T))$

(iii) Represent this more sophisticated rule by sentences of predicate logic...

approved(Y, X, T): removal of X is approved by Y at time T.

authorised(X)

manager(X)

depManager(X) ..

"unless"

$\forall X \forall T(exit(X,T) \wedge \neg paid\_for(X,T) \rightarrow stolen(X,T))$ (from part (i)).

"For all items X, and all times T, if X exits the store at time T and is not paid for by time T, then if there exists A, where A is an authorised person approving X at time T, then not stolen. But, if ---

"The only authorised persons are the manager and deputy manager".

→ "For all X, such that X is a M or DM, then X is authorised" or converse.

- $\forall X(authorised(X) \rightarrow depManager(X) \vee manager(X))$.
- $\forall X, T((exit(X,T) \wedge \neg paid\_for(X,T) \wedge \exists A\ (authorised(A) \wedge approved(A, X, T)) \rightarrow \neg stolen(X,T)) \wedge$

  $(exit(X,T) \wedge \neg paid\_for(X,T) \wedge \neg \exists A(authorised(A) \wedge approved(A, X, T)) \rightarrow stolen(X,T)))$.

② (a) DeMorgan's Rule: $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

To show: $\neg(p \wedge q) \vdash \neg q \vee \neg p$

1. $\neg(p \wedge q)$ Given

  2. $\neg(\neg p \vee \neg q)$ Assume

    3. $\neg p$ Assume

    4. $\neg p \vee \neg q$ 3, $\vee$I

  5. p 2, 3, 4, RAA

    6. $\neg q$ Assume

    7. $\neg p \vee \neg q$ 6, $\vee$I

  8. q 2, 6, 7, RAA

  9. $p \wedge q$ 5, 8, $\wedge$I

10. $\neg p \vee \neg q$ 1, 2, 9, RAA

To show: $\neg p \vee \neg q \vdash \neg(p \wedge q)$

1. $\neg p \vee \neg q$ Given

  2. $p \wedge q$ Assume

  3. p 2, $\wedge$E

  4. q 2, $\wedge$E

  5. $\neg q$ 1, 3, $\vee$E

6. $\neg(p \wedge q)$ 2, 4, 5, RAA

(b) Show that S1, S2, S3, S4 $\vdash \forall X(m(a) \wedge p(a) \rightarrow \exists y\, t(X, y))$

1. S1

2. S2

3. S3

4. S4

given

5. $p(a) \rightarrow \neg(q(a) \wedge r(a))$ 1, $\forall$E

6. $\exists y\, s(a, y) \rightarrow q(a)$ 2, $\forall$E

7. $m(a) \rightarrow \exists y\, s(a, y) \vee \exists y\, t(a, y)$ 3, $\forall$E

8. $\neg(n(a) \wedge r(a)) \rightarrow \exists y\, t(a, y)$ 4, $\forall$E.

  9. $m(a) \wedge p(a)$ Assume

  10. $m(a)$ 9, $\wedge$E

  11. $p(a)$ 9, $\wedge$E

  12. $\neg(q(a) \wedge r(a))$ 5, 11, $\rightarrow$E.

  13. $\exists y\, s(a, y) \vee \exists y\, t(a, y)$ 7, 10, $\rightarrow$E.

  14. $\neg q(a) \vee \neg r(a)$ 12, (a)

    15. $\neg \exists y\, t(a, y)$ Assume

    16. $\exists y\, s(a, y)$ 13, 15, $\vee$E

    17. $q(a)$ 6, 16, $\rightarrow$E.

    18. $\neg r(a)$ 14, 17, $\vee$E

    19. $\neg r(a) \vee \neg n(a)$ 18, $\vee$I

    20. $\neg(n(a) \wedge r(a))$ 19, (a)

    21. $\exists y\, t(a, y)$ 8, 20, $\rightarrow$E.

  22. $\exists y\, t(a, y)$ 15, 21, RAA

23. $m(a) \wedge p(a) \rightarrow \exists y\, t(a, y)$ 9, 22, $\rightarrow$I.

24. $\forall X(m(X) \wedge p(X) \rightarrow \exists y\, t(a, X, y))$, 23, $\forall$I. (QED).

(c) using inference rules ~~are~~ of natural deduction and (b), show that

$S1, S2, S3, S4, S5 \vdash \forall X(\neg\exists Y u(X,Y) \to \neg(m(X) \land p(X))$

1. $S1$
2. $S2$
3. $S3$ given
4. $S4$
5. $S5$
6. $t(a,b) \to u(a,b)$ 5, $\forall E$
7. $\forall X(m(X) \land p(X) \to \exists Y t(X,Y))$ 1-4, part(b)
8. $m(a) \land p(a) \to \exists Y t(a,Y))$ 7, $\forall E$
   9. $\neg\exists Y u(a,Y)$ Assume
      10. $m(a) \land p(a)$ Assume
      11. $\exists Y t(a,Y)$ 8, 10, $\to E$
         12. $t(a,b)$ Assume
         13. $u(a,b)$ 6, 12, $\to E$.
         14. $\exists Y u(a,Y)$ 13, $\exists I$
      15. $\exists Y u(a,Y)$ 11, 12, 14 $\exists E$.
   16. $\neg(m(a) \land p(a))$ 9, 10, 15, RAA.
17. $\neg\exists Y u(a,Y) \to \neg(m(a) \land p(a))$ 9, 16, $\to I$.
18. $\forall X(\neg\exists Y u(X,Y) \to \neg(m(X) \land p(X))$ 17, $\forall I$. QED