Predicate Logic Tutorial 3

1.

i) Which of the following are correct formalisations of the following statement: "Bankers and estate agents are unpopular"?

a. $\forall X$ (banker(X) $\wedge$ estate_agent(X) $\rightarrow$ unpopular(X))
b. $\forall X$ (banker(X)) $\vee$ (estate_agent(X) $\rightarrow$ unpopular(X))
c. $\forall X$ (banker(X) $\vee$ estate_agent(X) $\rightarrow$ unpopular(X))
d. $\forall X$ (banker(X)$\rightarrow$unpopular(X)) $\wedge$ $\forall X$ (estate_agent(X)$\rightarrow$unpopular(X))

ii) Show |- c$\leftrightarrow$d, where c and d refer to the 3rd and 4th wffs above.

2. Prove the following:

a. $\forall X$ (p(X) $\rightarrow$ q(X) $\wedge$ r(X)) |- $\forall X$ (p(X) $\rightarrow$ q(X)) $\wedge$ $\forall X$ (p(X) $\rightarrow$ r(X))
b. $\forall X$ (p(X) $\rightarrow$ (q(X) $\rightarrow$ r(X))) |- $\forall X$ (p(X) $\wedge$ q(X)$\rightarrow$ r(X))
c. $\forall X$ (p(X) $\rightarrow\neg$q(X)), p(a), $\forall Y$(q(Y) $\vee$ s(Y)) |- s(a)
d. $\forall X$ (p(X) $\rightarrow$ m(X) $\vee$ n(X)), $\forall X$ (m(X) $\rightarrow$ $\exists$Yq(X, Y)) , $\forall X$ (n(X) $\rightarrow$ $\exists$Yq(X, Y)), p(a) |- $\exists$Yq(a, Y)