Solutions to Predicate Logic Tutorial 3

Q1.

i) c and d.

ii) You have to show $\vdash c \to d$ and $\vdash d \to c$. I will show the first.

showing $\vdash c \to d$:

~~1~~. $\forall X\,(banker(X) \lor estate\_agent(X) \to unpopular(X))$	assume
~~2~~. $banker(a)$	assume
3. $banker(a) \lor estate\_agent(a)$	2, $\lor$I
4. $banker(a) \lor estate\_agent(a) \to unpopular(a)$	1, $\forall$E
5. $unpopular(a)$	3,4, $\to$E
6. $banker(a) \to unpopular(a)$	2,5, $\to$I
7. $\forall X\,(banker(X) \to unpopular(X))$	6,$\forall$I

In an almost identical way you can show

$\forall X\,(estate\_agent(X) \to unpopular(X))$

Then use $\land$I to derive

$\forall X\,(banker(X) \to unpopular(X)) \land \forall X\,(estate\_agent(X) \to unpopular(X))$

Then by $\to$I you get $c \to d$, discharging 1.

Showing $\vdash d \to c$:

1. $\forall X\,(banker(X) \to unpopular(X)) \land \forall X\,(estate\_agent(X) \to unpopular(X))$	assume
2. $\forall X\,(banker(X) \to unpopular(X))$	1, $\land$E
3. $\forall X\,(estate\_agent(X) \to unpopular(X))$	1, $\land$E
4. $banker(a) \lor estate\_agent(a)$	assume
5. $banker(a) \to unpopular(a)$	2, $\forall$E
6. $estate\_agent(a) \to unpopular(a)$	3, $\forall$E
7. $unpopular(a)$	Proof by cases, 4, 5, 6
8. $banker(a) \lor estate\_agent(a) \to unpopular(a)$	$\to$I, 4, 7
9. $\forall X\,(banker(X) \lor estate\_agent(X) \to unpopular(X))$	$\forall$I, 8

Then by $\to$I you get $d \to c$, discharging 1.

Q2.

a.

1. $\forall X\,(p(X) \to q(X) \land r(X))$	given
2. $p(a)$	assume
3. $p(a) \to q(a) \land r(a)$	1, $\forall$E
4. $q(a) \land r(a)$	3, $\to$E
5. $q(a)$	4, $\land$E
6. $p(a) \to q(a)$	2,5, $\to$I
7. $\forall X\,(p(X) \to q(X))$	6, $\forall$I

Similarly we prove

$\forall X\,(p(X) \to r(X))$

And then apply $\land$I to get:

$\forall X\ (p(X) \rightarrow q(X)) \wedge \forall X\ (p(X) \rightarrow r(X))$

b.

1. $\forall X\ (p(X) \rightarrow (q(X) \rightarrow r(X)))$ given	
2. $p(a) \wedge q(a)$	assume
3. $p(a)$	2, $\wedge$E
4. $q(a) \rightarrow r(a)$	1,3, $\forall\rightarrow$E
5. $q(a)$	2, $\wedge$E
6. $r(a)$	4, 5,$\rightarrow$E
7. $p(a) \wedge q(a)\rightarrow r(a)$	2,6, $\rightarrow$I
8. $\forall X\ (p(X) \wedge q(X)\rightarrow r(X))$	7, $\forall$I

c.

1. $\forall X\ (p(X) \rightarrow \neg q(X))$	given
2. $p(a)$	given
3. $\forall Y(q(Y) \vee s(Y))$	given
4. $\neg q(a)$	1,2,$\forall\rightarrow$E
5. $q(a) \vee s(a)$	3, $\forall$E
6. $s(a)$	4,5, $\vee$E

d. Hint: Think of using proof by cases. Then it is easy.