equation system 2D phytoplankton & benthic algae

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1 Interior of the lake

Equations governing dynamics in the interior of the lake.

$$\frac{\partial A}{\partial t} = (G(R,I) - l)A - v\frac{\partial A}{\partial z} + d_x\frac{\partial^2 A}{\partial x^2} + d_z\frac{\partial^2 A}{\partial z^2} \tag{1}$$

$$\frac{\partial R}{\partial t} = -q\big((G(R,I) - l)A\big) + d_x\frac{\partial^2 R}{\partial x^2} + d_z\frac{\partial^2 R}{\partial z^2} \tag{2}$$

$$I(z) = I_0 \; exp\left(-k\int_0^z A(s,t)ds - k_{bg}z\right) \tag{3}$$

$$G(R,I) = G_{max}\cdot min\left(\frac{R}{M_L+R}, \frac{I}{H_L+I}\right) \tag{4}$$

2 Top, left, and right boundaries

At the top, left, and right boundaries, the flow of algae and dissolved nutrients in direction parallel to the normal is zero (i.e. no flow in our out of the lake at these boundaries).

$$\nabla A\cdot\hat{n} = 0, \tag{5}$$

$$\nabla R\cdot\hat{n} = 0. \tag{6}$$

3 Bottom boundary

At the bottom border there is a sediment layer of nutrients, as well as a layer of benthic algae. Starting off with the benthic algae, their growth is either limited by light or nutrients. the light limited growth rate is given by

$$g_L = \frac{G_{max,benth}}{k_B}\log\left(\frac{H+I}{H+Ie^{-k_B B}}\right) \tag{7}$$

which is derived by assuming that the benthic algae is a uniform layer with some thickness, and integrating over that layer. H is the half-saturation constant of light dependent benthic algal production, and I is the light intensity at the surface of the benthic layer. k_B is the specific light attenuation coefficient of the benthic algae.

The nutrient limited growth rate of benthic algae is given by

$$g_N = (1/q_B)rR_s + G_{max,benth} \cdot B\frac{R_d}{\mu + R_d} + r_B l_B B \tag{8}$$

Where r is the remineralization rate of the sedimented nutrients, R_s the sediment nutrient concentration. μ is the half saturation constant for the nutrient limited growth, l_B is the death rate of benthic algae and r_B is the portion of the nutrients from the dead benthic algae that the live benthic algae can absorb and use immediately for growth (recycling rate).

The growth of the benthic algae is given by

$$\frac{dB}{dt} = min(g_N, g_L) - l_B B \tag{9}$$

When the phytoplankton sink to the bottom they die and become sedimented nutrients. A portion $(1 - r_B)$ of the dead benthic algae winds up in the sediment, and the live benthic algae absorb nutrients from the sediment. The rate of change of the sediment nutrients is given by

$$\frac{\partial R_s}{\partial t} = -rR_s + qvA(z_{max}(x)) + (1 - r_B)l_B q_B B \tag{10}$$

where $\{z_{max}(x) = max(z)|_x\}$ and q_B is the benthic algae phosphorus-carbon ratio (stochiometry).

The rate of change of the dissolved nutrients on the bottom boundary is given below. nutrient limited flux:

$$NlimFlux = q_B g_N = rRs + q_B G_{max,Benth} \cdot \frac{Rd}{Rd + M_B} + q_B r_B l_B B \tag{11}$$

Light limited flux:

$$LlimFlux = q_B g_L = q_B \frac{G_{max,benth}}{k_B} \log\left(\frac{H + I}{H + Ie^{-k_B B}}\right) \tag{12}$$

the rate of change of the dissolved nutrients at the bottom boundary is given by

$$\frac{dR}{dt}|_{z=z_{max}(x)} = r_B l_B q_B B + rR_s - min(LlimFlux, NlimFlux). \tag{13}$$