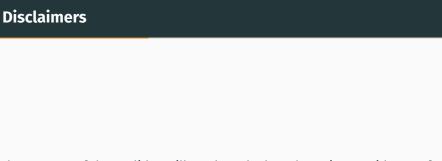
Comments on the implementation of the simplex algorithm

Disclaimers



The content of these slides will not be asked on the quizzes, midterm of final.



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I am not a specialist of the the implementation of the simplex method. Information that follows may contain inaccuracies.

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Caracteristics of "real life" examples:

- Large scale (number of variables and constraints from 10^3 to 10^8).
- The matrix A is sparse (no more than 1 to 5% of non zero entries).
- The data is structured: you don't solve a "random" LP.
- · (You may want to do computations in a distributed fashion).

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Number of operations by pivot: the bottleneck is inverting B.

Inverting the matrix B

The main bottleneck is about inverting the matrix B, or at least solving systems of the form $(x, y \text{ unknown}, b, c_B \text{ data})$

$$B\mathbf{x} = \mathbf{b}$$
 or $B^{\mathsf{T}}\mathbf{y} = \mathbf{c}_B$

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If B_k is the basis matrix after k iterations we can write it

$$B_k = B_0 E_1 E_2 \dots E_k$$
 hence $B_k^{-1} = E_k^{-1} E_{k-1}^{-1} \dots E_1^{-1} B_0^{-1}$

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But numerical instabilities accumulate and more and more E_k need to be stored and used: a new factorization of the matrix B is computed from scratch every 100 iterations or so.

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- "Partial pricing": compute $\mathbf{c}_N^\top \mathbf{c}_B^\top B^{-1} A_N$ one coefficient after another and pick among the first ones which are negative as entering.