Back Driving Assistant for Passenger Cars with Trailer

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Back Driving Assistant for Passenger Cars with Trailer

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ABSTRACT

This paper focuses on control strategies that are needed to stabilise a backing trailer and steer it into the desired direction. A model of the trailer and the car is used in order to calculate the desired steering wheel angle. An important constraint is that the driver should not be disturbed by the steering intervention. Measurements are done with a prototype car with an active front steering system. The results show that the drivers manage to fulfill the given driving task faster and with less steering wheel activity than without the assistant.

INTRODUCTION AND MOTIVATION

Active front steering is a newly developed technology for passenger cars that realises an electronically controlled superposition of an angle to the hand steering wheel angle that is prescribed by the driver [1].

Driving backwards with a trailer connected to a passenger car usually poses some challenge to untrained drivers, who use their trailer only rarely. Having an active front steering on board, the car automatically corrects the mistakes of the driver and steers the car and the trailer to the desired location.

Outline of the paper First, a car and trailer model is derived. Some nonlinear control strategies are derived next. Several approaches to make the steering feeling smoother to the driver are described. Results with measurement data from a prototype vehicle equipped with active front steering are presented and discussed thereafter.

CAR AND TRAILER MODEL

In this section the model equations of the car and trailer system will be derived. The derivation of the model equations in differential algebraic equation (DAE) form is followed by the constraint equations written in implicit form. Finally, implicit differential equations (DE) in the independent coordinates are obtained by elimination of the dependent coordinates using the explicit constraint equations.

Ackermann Steering Geometry

To allow all wheels of the car to roll without lateral slip, the inner front wheel needs to turn more than the outer. The ideal geometry, often called the Ackermann steering geometry, is shown in Figure 1. All wheels are aligned to move in circles around a common centre, see also [2] or [3].

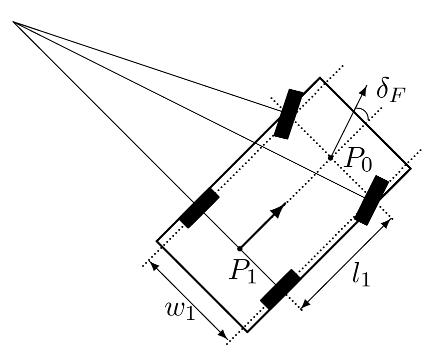

Figure 1: The Ackermann steering geometry.

The relation between the left front wheel angle, δ_{FL}, and the right, δ_{FR}, is

$$l_1 \cot \delta_{FR} - l_1 \cot \delta_{FL} = w_1$$

Assuming all wheels move without slipping, the movement of the car can obviously be derived from any of the front wheel angles δ_{FL} and δ_{FR} or from the angle δ_F, of an imagined middle wheel. For symmetry we choose the latter. The value of δ_F can easily be calculated from measurements of δ_{FL} or δ_{FR}:

$$l_1 \cot \delta_F = l_1 \cot \delta_{FL} + w_1/2 = l_1 \cot \delta_{FR} - w_1/2$$

For later use we also note that the point P_1 moves in the forward direction of the car.

This model for the car movement is sometimes called a bike model, because the same model is attained by assuming the car has only two wheels.

Coordinate Frames

Car and the trailer are modelled as rigid bodies connected by a joint, see also [4]. From now on the front wheel(s) will be referred to as body 0, the car as body 1 and the trailer as body 2.

We introduce a (global) inertial frame R with coordinates x^R and y^R as well as (local) body-fixed frames L_i with coordinates x^{L_i} and y^{L_i}, see Figure 2. The body-fixed frames are defined with the x-axis in the direction of the body. That is x^{L_2} points in the forward direction of the car.

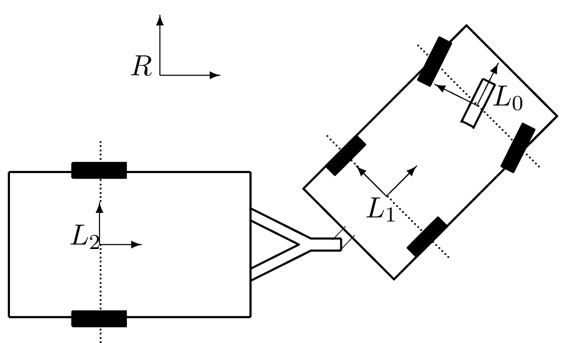

Figure 2: The different coordinate frames.

The position of body i in the plane is specified by the global coordinates of the origin P_i of the body-fixed frame L_i and the orientation of body i is specified by $\psi_i = \psi_{L_i R}$, the angle of rotation of L_i with respect to R, see Figure 3.

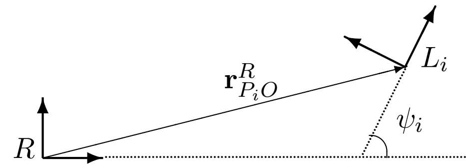

Figure 3: The relationship between two points in the system

To uniquely specify the position of all parts of the system, we use a set of coordinates, $\mathbf{p} \in \mathbb{R}^9$ defined as

$$\mathbf{p} = \begin{pmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \end{pmatrix} \quad \text{with} \quad \mathbf{p}_i = \begin{pmatrix} \mathbf{r}^R_{P_iO} \\ \psi_i \end{pmatrix} \quad \text{and} \quad \mathbf{r}^R_{P_iO} = \begin{pmatrix} x^R_{P_iO} \\ y^R_{P_iO} \end{pmatrix}$$

Constraint Equations

The angle between the front wheel and the car is controlled by the driver together with the regulator. This angle is called δ_F,

$$\psi_0 - \psi_1 - \delta_F = 0.$$

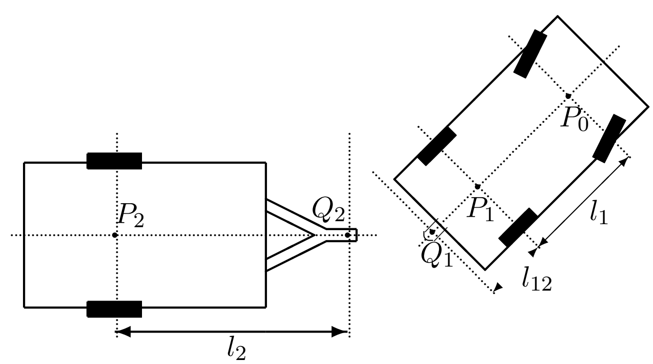

Figure 4: Points in the different frames.

The point P_0 on body 0 is fixed in the L_1 frame,

$$\mathbf{r}^R_{P_0O} - \mathbf{A}^{RL_1} \cdot \mathbf{r}^{L_1}_{S_1O} - \mathbf{r}^R_{P_1O} = \mathbf{0},$$

see Figure 4. Using

$$\mathbf{A}^{RL_1} = \begin{pmatrix} \cos\psi_1 & -\sin\psi_1 \\ \sin\psi_1 & \cos\psi_1 \end{pmatrix} \quad \text{and} \quad \mathbf{r}^{L_1}_{S_1O} = \begin{pmatrix} l_1 \\ 0 \end{pmatrix}$$

we get the constraint equations

$$x^R_{P_0O} - l_1 \cdot \cos\psi_1 - x^R_{P_1O} = 0$$
$$y^R_{P_0O} - l_1 \cdot \sin\psi_1 - y^R_{P_1O} = 0.$$

The point Q_1 on body 1 coincide with the point Q_2 on body 2,

$$\mathbf{r}^R_{P_1O} + \mathbf{r}^R_{Q_1P_1} - (\mathbf{r}^R_{P_2O} + \mathbf{r}^R_{Q_2P_2}) = \mathbf{0}.$$

Using the geometric relations

$$\mathbf{r}^R_{Q_1P_1} = \mathbf{A}^{RL_1} \cdot \begin{pmatrix} -l_{12} \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{r}^R_{Q_2P_2} = \mathbf{A}^{RL_2} \cdot \begin{pmatrix} l_2 \\ 0 \end{pmatrix}$$

we get

$$x^R_{P_1O} - l_{12} \cdot \cos\psi_1 - x^R_{P_2O} - l_2 \cdot \cos\psi_2 = 0$$
$$y^R_{P_1O} - l_{12} \cdot \sin\psi_1 - y^R_{P_2O} - l_2 \cdot \sin\psi_2 = 0.$$

So far, we have only considered the geometry of the car and trailer system. To get further, we need to make some assumptions about the dynamic properties of the system. Because we are only interested in low speed behaviour a natural assumption is that all wheels roll without slipping. Because the orientation of the wheels coincide with the orientation of one body-fixed frame, this is easily expressed analytically,

$$\mathbf{A}^{L_iR} \cdot \dot{\mathbf{r}}^R_{P_iO} = \begin{pmatrix} \cdot \\ 0 \end{pmatrix} \quad , \quad i = 0, 1, 2$$

$$\Rightarrow$$

$$-\sin\psi_0 \cdot \dot{x}^R_{P_0O} + \cos\psi_0 \cdot \dot{y}^R_{P_0O} = 0$$
$$-\sin\psi_1 \cdot \dot{x}^R_{P_1O} + \cos\psi_1 \cdot \dot{y}^R_{P_1O} = 0$$
$$-\sin\psi_2 \cdot \dot{x}^R_{P_2O} + \cos\psi_2 \cdot \dot{y}^R_{P_2O} = 0.$$

Finally, the speed of the car can be measured. Because there is no slipping, we get

$$\mathbf{A}^{L_1R} \cdot \dot{\mathbf{r}}^R_{P_1O} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$
$$\Rightarrow$$
$$\cos\psi_1 \cdot \dot{x}^R_{P_1O} + \sin\psi_1 \cdot \dot{y}^R_{P_1O} = v.$$

Assuming that initial conditions are known we now have enough equations to decide the behaviour of the system as a result of the steering angle, δ_F, and vehicle speed, v.

Differential Equation

To make our model easier to handle we want to rewrite it as a differential equation. Because we have four non-algebraic constraint equations, our differential equation will have four independent variables. We choose these to be $x_1 = x^R_{P_1O}$, $y_1 = y^R_{P_1O}$, ψ_1 and $\gamma = \psi_1 - \psi_2$. With some effort we arrive at

$$\begin{aligned} \dot{x}_1 &= v \cdot \cos\psi_1 \\ \dot{y}_1 &= v \cdot \sin\psi_1 \\ \dot{\psi}_1 &= \frac{v}{l_1} \tan\delta_F \\ \dot{\gamma} &= \left(\frac{v}{l_1} + \frac{v\, l_{12}}{l_1\, l_2} \cos\gamma \right) \tan\delta_F - \frac{v}{l_2} \sin\gamma. \end{aligned} \quad (1)$$

For obvious reasons the input δ_F is bounded. We have

$$|\delta_F| \leq B_{\delta_F}.$$

Model Validation

The attained model was validated using data from test drives at low speeds (0-15 kph) without controller. The tests showed a fairly good compliance between simulated and measured output, see Figure 5.

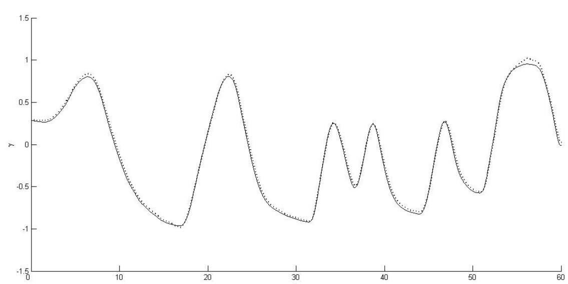

Figure 5: Validation of the DE, the solid line is the simulated γ and the dashed line is the measured angle.

The purpose of our model is to help constructing an effective and robust controller for the system. Our model is accurate enough for this purpose.

CONTROLLER DESIGN

Optimal Feedback

In optimal control we seek a control law that minimizes criterion. Choosing this criterion is described in for example [5].

For shorter writing, we replace our input δ_F with $u = \tan\delta_F$. Assuming constant speed, the system we want to control (1) can now be written

$$\dot{\gamma} = (c_1 + c_3\cos\gamma)\, u - c_2\sin\gamma = f(u,\gamma).$$

We start with the general expression

$$\min_u \int_0^\infty L(u,\gamma(t))dt.$$

Since the interval is infinite and all functions are time invariant the optimal return function $V(t,\gamma)$ must be independent of time

$$V_t(t,\gamma) \equiv 0.$$

We get the Hamilton-Jacobi equation

$$\begin{aligned} 0 &= \min_u V_\gamma\, f(\gamma) + L(u,\gamma) = \\ &= \min_u V_\gamma\left((c_1 + c_3\cos\gamma)u - c_2\sin\gamma\right) + L(u,\gamma). \end{aligned}$$

We want a controller that steers γ to its reference value and the steering to feel smooth. To ensure this we introduce the criterion function L

$$L(u,\gamma) = \frac{K^2}{2}(\gamma - r_\gamma)^2 + \frac{1}{2}(u - u_e)^2, \quad K > 0$$

where the ratio between following the reference and smoothness in the steering wheel can be adjusted with K and u_e refers to the current equilibrium angle, that is

$$u_e = u_e(\gamma) = \frac{c_2\sin\gamma}{c_1 + c_2\cos\gamma}.$$

In this case minimum is attained for

$$u = \frac{c_2\sin\gamma}{c_1 + c_3\cos\gamma} - V_\gamma(c_1 + c_3\ \cos\gamma)$$

giving the equation

$$-\frac{1}{2}\, V_\gamma^2(c_1 + c_3\cos\gamma)^2 = K^2(\gamma - r_\gamma)^2$$

with the solutions

$$V_\gamma = \pm K\frac{\gamma - r_\gamma}{c_1 + c_3\cos\gamma}$$

Which gives us

$$u = \frac{c_2\sin\gamma}{c_1 + c_3\cos\gamma} \pm K(\gamma - r_\gamma)$$

Recognizing that only the positive sign gives a stabilizing controller we conclude that the optimal feedback must be

$$u = \frac{c_2\sin\gamma}{c_1 + c_3\cos\gamma} + K(\gamma - r_\gamma)$$

Linearising γ-Controller

A approach is to use the steering wheel angle as a reference signal for the trailer angle γ. To control γ we linearize the relation between the front wheel angle δ_F and γ and then use proportional control on the linearised system

$$\tan\delta_F = \frac{-K_p(r_\gamma - \gamma) + \frac{1}{l_2}\ \sin\gamma}{\frac{1}{l_1} + \frac{l_{12}}{l_1\, l_2}\ \cos\gamma}.$$

Because of the bounded front wheel angle δ_F, the trailer angle cannot be controlled if it gets larger than the critical angle C_γ. This angle can be found by examining the sign of the derivatives $\dot{\gamma}$. Since the system is symmetrical it is sufficient to consider the case $\gamma > 0$. Because $v < 0$ and $|\gamma| < \pi/2$ it holds for (1) that

$$\dot{\gamma} \geq \left(\frac{v}{l_1} + \frac{v\, l_{12}}{l_1\, l_2}\ \cos\gamma\right)\tan B_{\delta_F} - \frac{v}{l_2}\ \sin\gamma$$

which is positive when

$$l_1\ \sin\gamma > (l_2 + l_{12}\ \cos\gamma)\tan B_{\delta_F}$$

Rewriting this inequality

$$a\ \sin\gamma - b\ \cos\gamma > c$$

we find the critical trailer angle

$$\gamma > \arcsin\left(\frac{c}{\sqrt{a^2+b^2}}\right) + \arctan\left(\frac{b}{a}\right) = C_\gamma$$

To simplify discussions somewhat, we introduce a region of controllability, $\mathcal{D}$.

$$\mathcal{D} = \left\{(\delta_F,\ \gamma,\ v)\ :\ |\delta_F| \leq B_{\delta_F}\ ,\ |\gamma| \leq C_\gamma\ ,\ v < 0\right\}$$

Controlling the Turning Radius

A drawback of controlling γ is that the transient behaviour of the trailer can be somewhat uncomfortable for the driver.

This is because the actual movement of the trailer is not regarded. In this section a controller for the turning radius of the trailer is defined.

An expression for the turning radius, R_T, or rather an expression for the quotient q is derived,

$$q = \frac{l_2}{R_T} = \frac{l_1 \sin\gamma - l_{12} \tan\delta_F \cos\gamma}{l_1 \cos\gamma + l_{12} \tan\delta_F \sin\gamma}$$

This quotient is not measured so it cannot be controlled using the standard methods. Trying to estimate its value from measured wheel velocities, would not be a good idea as these measurements tend to be very noisy at low speeds. Instead we use a kind of predictive control choosing the control signal that, according to the model, should give the desired output. Choosing

$$\tan\delta_F = \frac{l_1}{l_{12}} \frac{\tan\gamma - r_q}{1 + r_q \tan\gamma} \tag{2}$$

the trailer will move in a circle of the desired radius $\frac{l_2}{R_T}$. However, this can sometimes make the trailer angle γ larger than desirable. We check this by inserting (2) in the differential equation (1), seeking the equilibriums γ_E.

$$\dot{\gamma} = \left(\frac{v_R}{l_{12}} + \frac{v_R}{l_2} \cos\gamma\right) \frac{\tan\gamma - r_q}{1 + r_q \tan\gamma} - \frac{v_R}{l_2} \sin\gamma$$

Assuming $r_q \cdot \tan\gamma \neq -1$,

$$\dot{\gamma} = 0 \quad \Rightarrow$$

$$\frac{1}{l_{12}}(\sin\gamma_E - r_q \cos\gamma_E) - \frac{r_q}{l_2} \cos^2\gamma_E = \frac{r_q}{l_2} \sin^2\gamma_E \quad \Leftrightarrow$$

$$r_q = \frac{l_2 \sin\gamma_E}{l_{12} + l_2 \cos\gamma_E}$$

Inserting the maximum controllable trailer angle, C_γ we find the maximum controllable q, C_q.

Another problem is that more equilibria have the same turning radius. We want to make sure the controller steers towards an equilibrium in $\mathcal{D}$. When r_q gets too small the controller output turns negative, steering the trailer towards an equilibrium with $|\gamma_E| > \pi/2$. The discontinuity occurs when the denominator in (2) turns zero, that is when

$$r_q \tan\gamma = -1$$

One way to check this is to examine the stability of an equilibrium

$$\gamma_E \in [-C_\gamma, C_\gamma] \quad , \quad r_q = q_E = \frac{l_2 \sin\gamma_E}{l_{12} + l_2 \cos\gamma_E}$$

also assuming $|\delta_F| \leq B_{\delta_F}$.

For this purpose we introduce the Lyapunov function

$$V(\gamma) = \frac{(\gamma - \gamma_E)^2}{2}.$$

The system we need to examine can be written

$$\dot{\gamma} = \left(\frac{v_R}{l_1} + \frac{v_R\, l_{12}}{l_1\, l_2} \cos\gamma\right) g(\gamma) - \frac{v_R}{l_2} \sin\gamma = f(\gamma)$$

$$g(\gamma) = \tan\left(\delta_F(\gamma)\right) = \frac{l_1}{l_{12}} \frac{\tan\gamma - q_E}{1 + q_E \tan\gamma}$$

The criterion for stability is

$$V'(\gamma)\, f(\gamma) < 0 \quad \forall \gamma \neq \gamma_E$$

To check when this holds, we first note that

$$f(\gamma_E) = 0.$$

Next, we differentiate $f(\gamma)$ and find that for $v_R < 0$ holds that

$$f'(\gamma) \leq \frac{v_R}{l_{12}} - \frac{v_R\, l_{12}}{l_1\, l_2} \sin\gamma\, g(\gamma) + \frac{v_R}{l_2} \cos\gamma - \frac{v_R}{l_2} \cos\gamma =$$

$$= \frac{v_R}{l_{12}}(1 - \frac{l_{12}^2}{l_1\, l_2}\, g(\gamma) \sin\gamma).$$

Using

$$\frac{l_{12}^2}{l_1\, l_2} \tan\delta_F \sin\gamma \leq \frac{l_{12}^2}{l_1\, l_2} \tan B_{\delta_F} \sin C_\gamma < 1$$

we finally arrive at

$$f'(\gamma) < 0 \quad \forall \gamma.$$

That means the control algorithm stabilizes the system when $q_E \cdot \tan\gamma > -1$.

Changing this and also considering that δ_F is bounded we get the modified controller

$$\delta_F = B_{\delta_F} \quad \text{when} \quad r_q \geq \frac{l_1 \tan\gamma - l_{12} \tan B_{\delta_F}}{l_1 + l_{12} \tan\gamma \tan B_{\delta_F}}$$

$$\delta_F = -B_{\delta_F} \quad \text{when} \quad r_q \leq \frac{l_1 \tan\gamma + l_{12} \tan B_{\delta_F}}{l_1 - l_{12} \tan\gamma \tan B_{\delta_F}}$$

else

$$\delta_F = \arctan\left(\frac{l_1}{l_{12}} \frac{\tan\gamma - r_q}{1 + r_q \tan\gamma}\right).$$

STEERING WHEEL AS REFERENCE

When steering a car, the steering wheel angle is used to indicate the desired route. Our control system is meant to assist drivers not used to backwards driving with trailer. Our aim is therefore to make it as similar as possible to backwards driving without trailer. That means that turning the steering wheel to the left should cause the trailer to go left. However this turned out to be problematic.

The torque needed to turn the front wheels will cause an opposing torque in the steering wheel. Normally, of course, turning the steering wheel to the left will cause the wheels to go left and the opposing torque in the steering wheel will act to retard the steering wheel movement.

However, when driving backwards with a trailer the front wheels initially have to turn right for the trailer to move left. The torque in the steering wheel will therefore tend to accelerate the steering wheel movement. This makes it very hard to steer. For the upcoming discussion, let T be the reaction torque in the steering wheel.

Inverted Steering

There is a simple solution to this problem. Changing the sign of the reference signal will bring back the normal stability of the steering wheel. However this means that the driver must turn the steering wheel to the right to get the trailer to turn left and vice versa. Apart from stability, this has the advantage that it is not so confusing for drivers used to driving with a trailer. For inexperienced drivers, left-left-steering should be preferable.

To analyse the problem with the steering wheel feedback more accurately, we construct a linear model of the system.

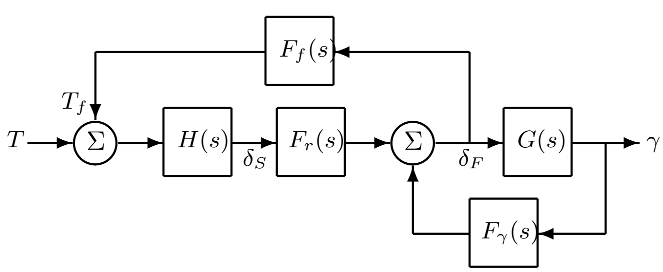

Figure 6: The unwanted torque feedback, T_f.

We start by linearizing our car-trailer model (1) around $(\gamma, \delta_F) = (0, 0)$, assuming constant negative speed.

$$\dot{\gamma} \approx \left(\frac{v}{l_1} + \frac{v\, l_{12}}{l_1\, l_2}\right)\delta_F - \frac{v}{l_2}\,\gamma = -b\,\delta_F + a\,\gamma$$

Laplace transformation gives us the transfer function

$$G(s) = -\frac{b}{s-a}$$

Next, we need a model of the steering. The feedback torque, T_f, in the steering wheel is approximately proportional to the angular velocity of the front wheels, $\dot{\delta}_F$.

$$T_f = -k\,\dot{\delta}_F \quad \Rightarrow \quad F_f(s) = -k\,s$$

Assuming that friction in the steering shaft can be ignored, the steering wheel angle, δ_S, follows

$$\ddot{\delta}_S = J\,(T + T_f) \quad \Rightarrow \quad H(s) = \frac{J}{s^2}$$

where J is the moment of inertia. Now the transfer function, $G_C(s)$, from driver torque, T, to trailer angle, γ, can be calculated.

$$G_C(s) = \frac{F_r\,G(s)\,H(s)}{1 - F_r\,H(s)\,F_f(s) - F_\gamma\,G(s)}$$

From test drives we know the nice properties of inverted steering. Are they also reflected in this linearized model? First look at proportional control, $F_r = K_r > 0$, $F_\gamma = K_\gamma > 0$. Note that a positive steering wheel angle will give a negative trailer angle. We get the transfer function

$$G_C(s) = \frac{-K_r\,b\,J}{s^2 + (K_\gamma\,b - K_r\,J\,k - a)\,s + K_r\,Jkb}\,\frac{1}{s}$$

and the system poles (not counting the integration)

$$s = -\frac{1}{2}(K_\gamma\,b - K_r\,Jk - a) \pm \sqrt{\frac{1}{4}(K_\gamma\,b - K_r\,Jk - a)^2 - K_r\,Jkb}$$

For small positive K_r, the system is stable (apart from the integration), but if K_r gets too large the system will become oscillating or even unstable.

To get left-left-steering instead, we use the controller $F_r = -K_r$. Obviously, at least one of the poles will always be in the right half plane.

Local Stability

The nice properties of the inverted steering inspired the following compromise. Allowing locally inverted steering, it should be possible to stabilize the steering wheel while keeping the left-left-steering. An example of a locally inverted steering is the following relation between steering wheel angle δ_S and desired trailer angle r_γ.

$$k\, r_\gamma = \delta_S + \sin(10\delta_S)$$

Major drawbacks of this method is that small changes of the steering wheel angle will have inverse effect, and that the steering feels discretised.

Limited Instability

Another approach is to limit how fast the wheels turn, thereby limiting the steering wheel torque. This could be done by low pass filtering of the control signal or by punishing a large derivative. In both cases the system will react slower to the drivers actions.

RESULTS FROM DRIVING TESTS

All presented controllers and ways to stabilise the reference signal were tested in a prototype car. As an example driving results from the linearised γ-controller with limited instability are shown in this section.

In Figure 7 a track beginning with a slalom and ending with parking is shown. The solid line is driven with the linearised γ-controller and the dashed line is without controller. The reference signal, the steering wheel angle, is low pass filtered in order to limit the unwanted torque feedback. The steering wheel angle is shown in Figure 8, as can be seen the measurement with controller is shorter and the steering activity is lower. These tests were done by a driver used to back with trailer.

In Figure 9, the same track was driven by a person who never has reversed with trailer. As can be seen, he has problems to come around the first cone in the slalom, the dashed line. With the γ-controller he manage to arrive in the parking lot. In Figure 10 the steering activity is shown.

CONCLUSIONS AND FUTURE WORK

Possibilities of utilizing active front steering to assist drivers when reversing the car with a trailer have been shown.

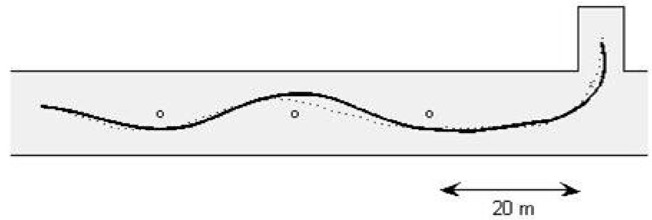

Figure 7: Trailer maneuver with slalom and parking driven by an experienced driver. The solid line is with linearised γ-controller and the dashed line is without controller

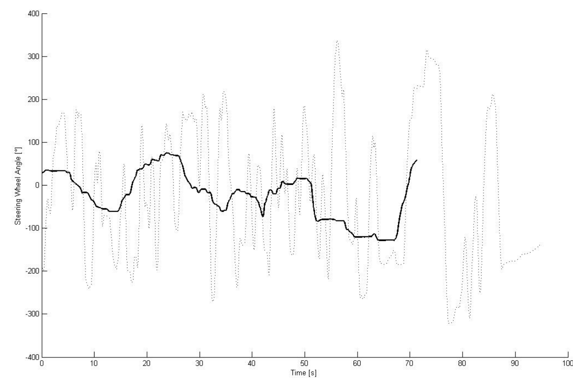

Figure 8: The measurement shows the steering wheel angle. The solid line is with linearised γ-controller and the dashed without controller. With the controller the maneuver is managed faster and with less steering wheel activity.

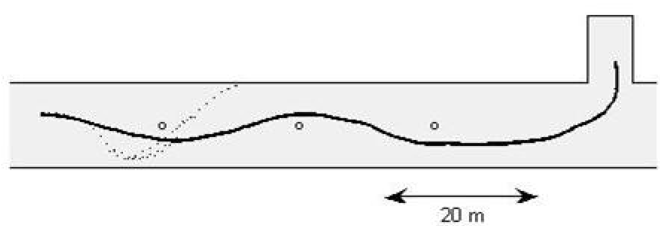

Figure 9: Trailer maneuver with slalom and parking driven by a person who have never backed a trailer. The solid line is with linearised γ-controller and the dashed line, ending between pylon one and two, is without controller.

Starting off with a theoretical derivation of different strategies, practical constraints such as reaction torque an human machine interface in general have been discussed. Test drives maesured in a vehicle equipped with active front steering have been shown and discussed as well. Clearly,

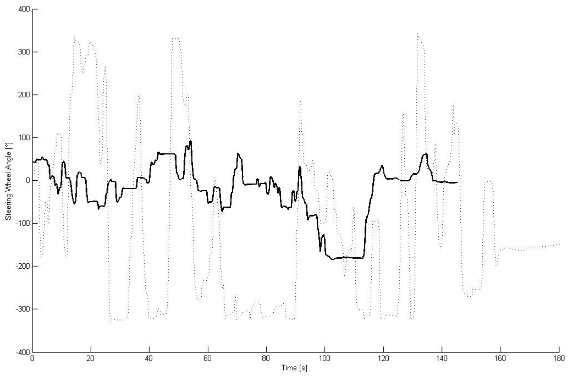

Figure 10: The measurement shows the steering wheel angle for the person not used to trailers. The solid line is with linearised γ-controller and the dashed without controller. With the controller the maneuver is managed faster and with less steering wheel activity.

the approach suffers from the fact that one single actuator has to deal with contradicting control strategies, namely moving the wheel in one direction and minimizing the effect of the reaction torque at the same time. Obvious possibiliies for improvement are to ock the hand wheel and implement automatic steeting or to use two actuators.

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CONTACT

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