Vehicle dynamics of cars with trailers

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1 Introduction

The significance of the presence of a trailer attached to a vehicle for vehicle dynamics will be illustrated in this contribution. Basic analyses usually comprise firstly of simple steady-state straight-line braking and acceleration analysis of a vehicle-trailer system and further of the directional stability analysis of the vehicle-trailer system utilizing a simplified (bicycle) model of the vehicle.

Due to the geometrical and mechanical complexity of the vehicle-trailer system, where eventual large displacements and rotations inevitably lead to high non-linearity, we believe the presented approach to be more appropriate for detailed analyses.

In this contribution, we present an approach utilizing the MBS modelling of a vehicle-trailer system dynamics. The mechanical model of the vehicle-trailer system shall be described as a system of rigid bodies interconnected through rotational, translational and spherical kinematic constraints, springs and dampers all of which govern the relative motion of bodies in the system. The modelled vehicle shall be a four wheeled passenger car with front wheels individually suspended (MacPherson strut), and the rear wheels suspended by a semi-independent *twist axle* system[1]. Damping and bump stop elements of suspensions have non-linear characteristics and are also modelled as such. The steering mechanism of a *rack and pinion* type is implemented exclusively on the front axis.

The mechanical model of the vehicle shall consist of 11 bodies. Each body with its own mass and inertia characteristics obtained either by measuring or geometrical modelling. All mass and inertia characteristics of the bodies in the mechanical model of the vehicle, except the characteristics of a self-supporting body, are obtained by geometrical modelling. The spring

[1] www.carbibles.com

is modelled with a linear spring characteristic, whereas the damper is modelled with the use of a measured force velocity relationship and has a non-linear characteristic. Two additional elements that contribute significantly to the vehicle response are the bump stop and the work stroke limiter of the damper. The bump stop is also modelled through the use of a measured non-linear force deflection relationship.

Several simulations of the vehicle-trailer system shall be made with the implementation of computer codes developed for general purpose MBS dynamics modelling. Vehicle-trailer system dynamic responses shall be compared against each other and against measurement results, where appropriate. The use of simulation tools enables us to carry out even simulations of situations not generally executable because of the threat of damage to researchers and the equipment. Simulations of events that pose no threat to researchers and the equipment shall be verified experimentally. Presented shall be the simulation examples of vehicle-trailer combination with the trailer loaded in unsuitable way.

2 Yaw-plane dynamic model

Several papers covering the yaw plane vehicle-trailer system dynamics have been published [1, 2] describing theoretical analysis, simulation and vehicle testing.

Particularly the work done in [1] explains the theoretical analysis of yaw plane dynamics of single track vehicle-trailer model in quite some detail. The model stated to have also been used previously is illustrated in Figure 1.

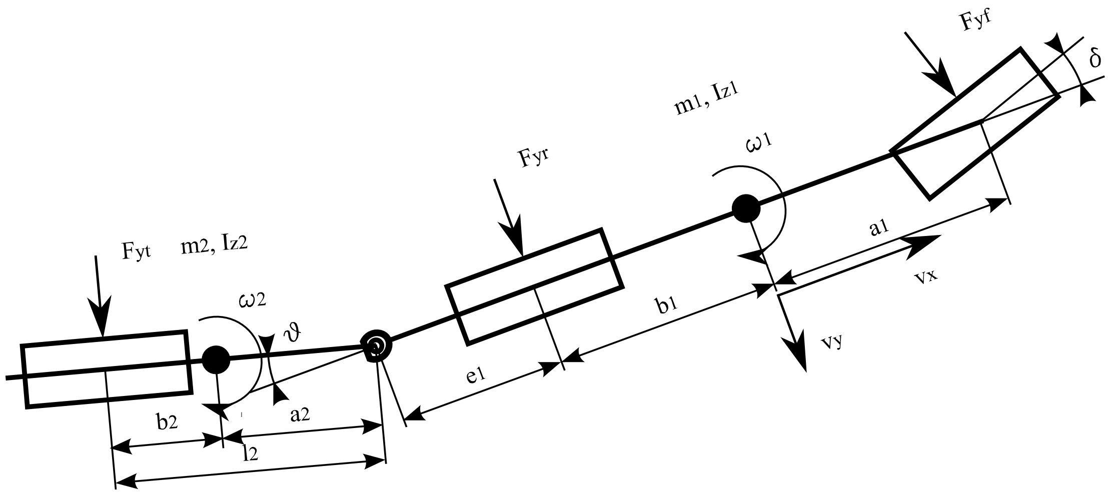

Figure 1: Simplified model of a vehicle with trailer in the yaw plane

The equations of lateral and yaw motions of the vehicle-trailer system, considering the appertaining kinematic relationships as well as tire forces that are presumed to be proportional to the tire slip angles, yield a system of linear equations written in a matrix form:

$$\mathbf{M}\dot{\mathbf{x}} = \mathbf{D}\left(v_x\right)x + \mathbf{E}u + \mathbf{F}\delta \tag{1}$$

From the perspective of the Control System Theory, the equation (1) can be read with $\mathbf{x}$ being the state vector, u the control input and δ the steering input. For the purposes of this text, the control input shall not be considered and the state equation obtained by pre-multiplying the above equation with $\mathbf{M}^{-1}$ thus reads:

$$\dot{\mathbf{x}} = \mathbf{A}\left(v_x\right)\mathbf{x} + \mathbf{G}\delta \tag{2}$$

Let us again emphasize that the model described above is used to study primarily the effect of vehicle and trailer parameters on the stability of the system and the onset of potential instability when amplitudes of motion are small and the linear assumptions are approximately satisfied [1].

In order to consider the dynamic stability of the vehicle-trailer combination, we may consider the state equation with steering inputs equal to 0. Under the assumption of constant velocity, the equation (2) reads:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}\,. \tag{3}$$

It is well known from the control system theory that the system modelled with such an equation is asymptotically stable if all the eigenvalues of matrix $\mathbf{A}$ have negative real parts. Furthermore, the matrix eigenvalues can be represented in the form of

$$s_{1,2} = -d \pm j\omega_{\mathrm{d}}\,, \tag{4}$$

where j is the imaginary unit, d is the damping coefficient, and ω_{d} is the damped natural frequency. The damping ratio can be expressed as

$$\zeta = \frac{-\mathrm{d}}{\sqrt{\mathrm{d}^2 + \omega_d^2}}\,. \tag{5}$$

A negative damping ratio ζ indicates an unstable system.

The damping ratio ζ is vehicle velocity dependent and can be readily evaluated for various system parameters with the use of mathematics software such as Wolfram Mathematica.

We have used the above equations coupled with actual test vehicle-trailer system parameters to determine the stability properties of the system. Firstly, the stability of the vehicle-trailer combination was considered for the trailer loaded evenly, as recommended by the trailer manufacturer. The plot in Figure 2 suggests that the system stays stable up to velocities exceeding 150 km/h.

Additionally, the stability of the system with an incorrectly loaded trailer was also considered. The plot below suggests that the system in such a case becomes unstable at a far lower velocity of approximately 105 km/h.

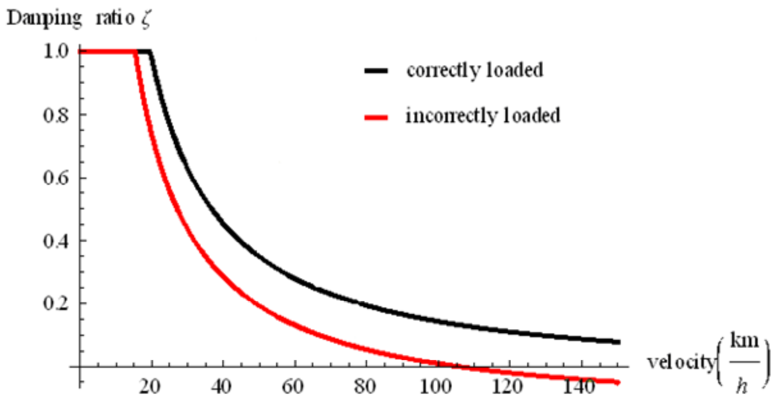

Figure 2: System damping ratio for a correctly and incorrectly loaded trailer

Once the stability of the vehicle-trailer system has been evaluated with the approach described above, the MBS model of the system was built, and analyses were carried out on both correctly and incorrectly loaded systems at below and above critical velocities.

3 Detailed MBS model

Figure 3: Instrumented test Vehicle-Trailer Combination

The implemented MBS mechanical model is fairly complex and highly non-linear as suspension characteristics, apart from the bushings, were taken into consideration with their actual characteristics (measured data) rather than their linearised values, whenever possible.

Figure 4: Geometric model of the front suspension

Figure 5: Geometric model of the rear suspension

Figure 6: Geometric model of the vehicle

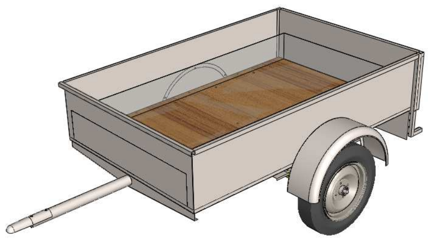

Figure 7: Geometrical model of the trailer

Geometric model (vehicle, trailer)

Detailed geometric models were prepared for the test vehicle as well as the test trailer. The models obtained by labour-intensive measuring of physical dimensions provide us with geometry parameters as well as mass and inertia properties of modelled parts. The sprung weight and inertia values were obtained from data in the CARAT database [3].

Suspension parameters (Vehicle front and rear)

There are many elements that determine the vehicle dynamic response. The ones considered in the MBS model of the vehicle are a spring, damper and bumpstop element and are all depicted in Figure 8. To determine parameters of each element that were later used in mathematical models of the elements, several measurements were carried out. The measured data are shown in Figure 11 - Figure 13.

Figure 8: Elements of vehicle suspension

Suspension parameters (Trailer)

Torsional suspension with rubber element, Figure 12, provides both springing and damping. It is thus not as straightforward to determine the characteristics as it was with the vehicle suspension.

Figure 9: Bumpstop spring characteristic

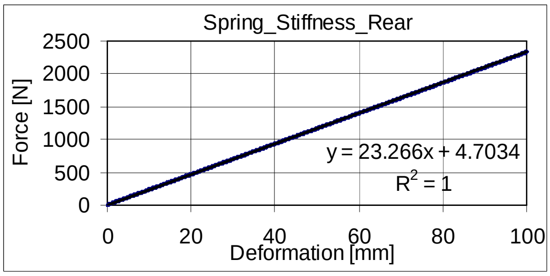

Figure 10: Spring stiffness

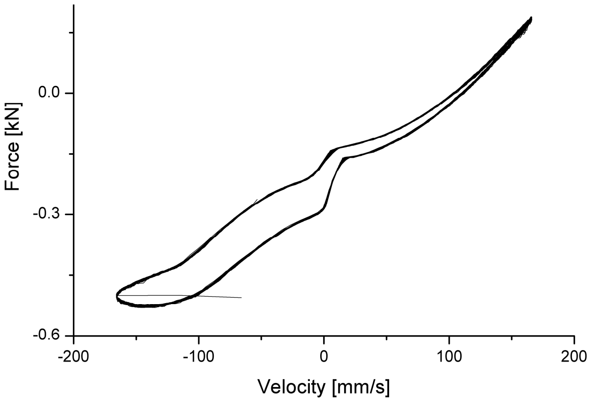

Figure 11: Damper characteristic

We decided to use a linear mathematical model of the trailer suspension. We can describe it by using the following equation

$$J\ddot{\phi} + 2\beta J\dot{\phi} + k\phi = 0\,. \tag{6}$$

The factor in the second term of equation (6) can also be expressed as $c = 2\beta J$.

Several simple experiments were carried out in order to obtain the parameters needed for the mathematical model of the trailer suspension described by the equation (6).

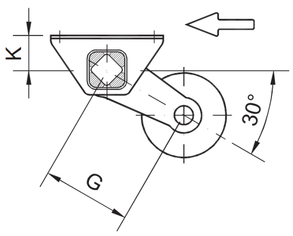

Figure 12: Torsional suspension with rubber element

To determine the spring coefficient k, we applied several values of torsional moment to the trailing arm (via a set of weights applied on a lever arm) and measured the rotation induced by it, Figure 13.

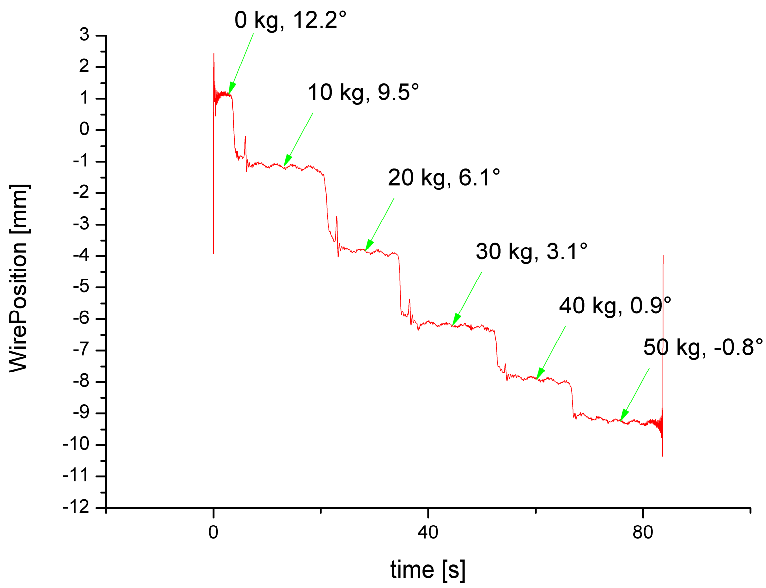

Figure 13: Results of spring characteristic measurement

To determine the damping coefficient, we induced oscillation of the suspension by deflecting it out of the equilibrium position and letting it oscillate till standstill, Figure 14. A wire of a wire transducer was wound around a circular part of the trailing arm that is coaxial with the trailing arms main axis. By measuring the length of wire wound on this circular part and by knowing its diameter, the angle of rotation of the trailing arm is readily determinable.

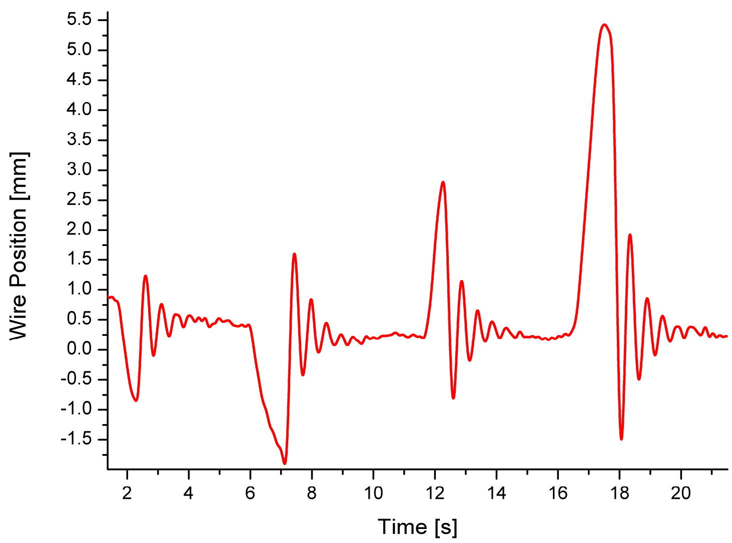

Figure 14: Deflected suspension oscillating to standstill

Then we proceeded in the following manner. First the natural frequency of damped oscillation was determined by measuring the time between maximums of individual oscillation waves

$$\omega' = 2\pi / t_0 \ . \tag{7}$$

Next, a damped sine function, equation (8), was fitted over the measured data and thereby the damping ratio β was determined.

$$y = y_0 + Ae^{(-x\beta)} \sin\left(\pi \frac{x - x_c}{w} \right) \tag{8}$$

Next, the natural frequency of the un-damped oscillation was determined

$$\boldsymbol{\omega}_0'^2 = \boldsymbol{\omega}'^2 - \beta^2 \ . \tag{9}$$

We further determined the moment of inertia from un-damped natural frequency and the spring coefficient

$$\omega_0 = \sqrt{k/J} \ . \tag{10}$$

At last, we determined the damping coefficient

$$c = 2\beta J \ , \tag{11}$$

to be utilized in equation (6).

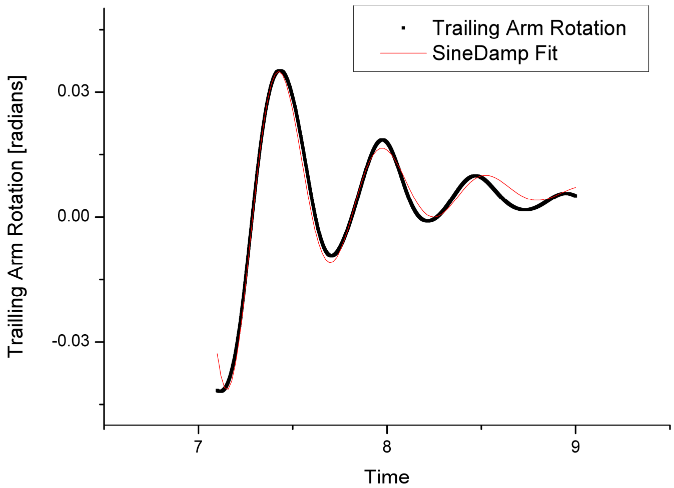

Figure 15: Fitting a damped sine curve to measured data

In addition to suspension parameters many other parameters such as tyre model parameters and vehicle engine torque curve also needed to be determined and provided to the MBS modeller.

As all the pertinent data was now gathered a MBS model was built in the MSC.ADAMS application [4] that is a well known and widely used MBS dynamics software.

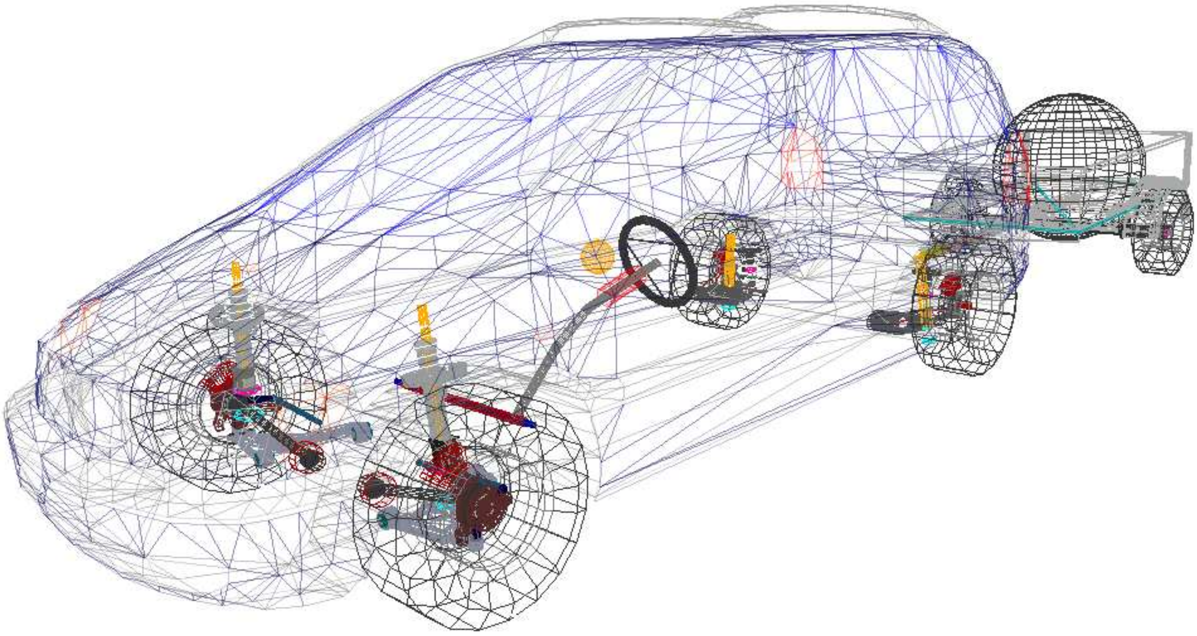

Figure 16: Screen shot of the vehicle-trailer system MBS model

Comparison of MBS model results and yaw-plane model results

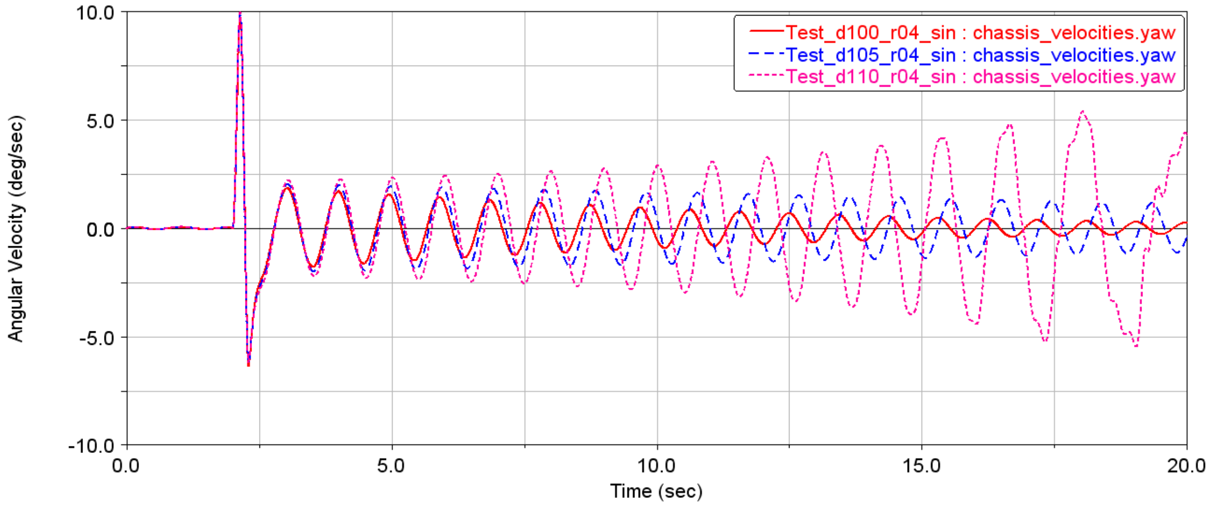

Figure 17: Simulation resultas at under and over critical velocities

Various simulations of the vehicle-trailer system can be carried out with the use of the MBS model described above. To compare the results of the yaw-plane model to the MBS model, several simulations of the vehicle-trailer system travelling at constant speeds were carried out at speeds below, at and above that predicted by the yaw plane model to be the critical speed, Figure 17.

It is clear that MBS model critical velocity coincides closely with results predicted by the yaw-plane model.

4 Experimental verification

In order to verify the MBS model as well as the yaw-plane model, an experiment was devised in which an instrumented test vehicle would be brought up to some predetermined velocity and a disturbance would be applied to the system in the form of a brief impulsive driver action, simulating perhaps an avoidance manoeuvre or a manoeuvre carried out by an inattentive driver when realizing the vehicle trailer system has slightly drifted off-course.

Instrumentation on the vehicle was used to measure the vehicle steering wheel angle, the angle between the vehicle and the trailer, longitudinal and lateral velocities of the trailer at trailer tire and three-axial accelerations at trailer CM, trailer wheel and vehicle CM. A plot of typical data acquired during a single test run is shown in Figure 21.

The vehicle steering wheel sensor was used to record the driver action. This was crucial to the experiment from the point of synchronization of multiple test runs carried out and from the point of control of drivers actions as we needed to be sure that only the staring disturbance was caused by the driver and that drivers actions did not continue during the rest of the experimental run.

The angle between the vehicle and the trailer needed to be measured as it is one of the most evident parameters that speaks clearly of the system stability. If the amplitudes of angle oscillations continue to reduce with time, the system is clearly stable, while if they increase, the system is clearly unstable.

Longitudinal vehicle-trailer system velocity sensor is crucial in determining the velocity the vehicle has gained at the onset of the drivers impulsive action. The lateral velocity of the trailer is another parameter that can be used to determine the level of stability of the system.

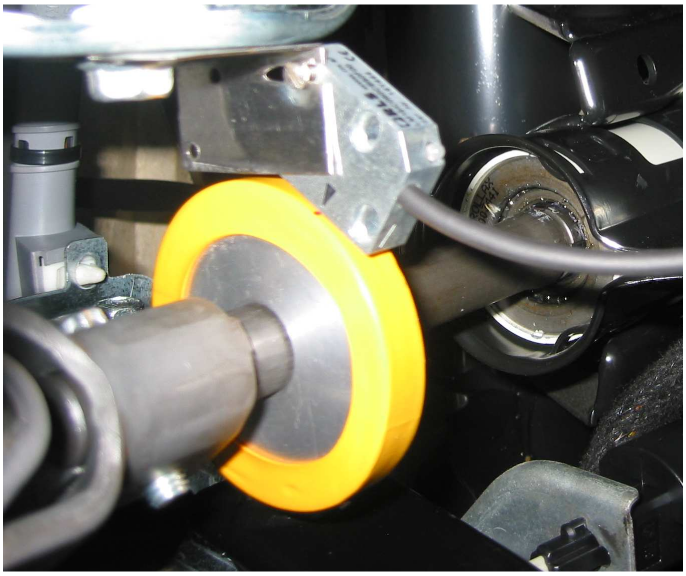

Figure 18: Magnetic steering wheel angle sensor mounted on the steering wheel post

Figure 19: Velocity sensor mounted on a trailer axis

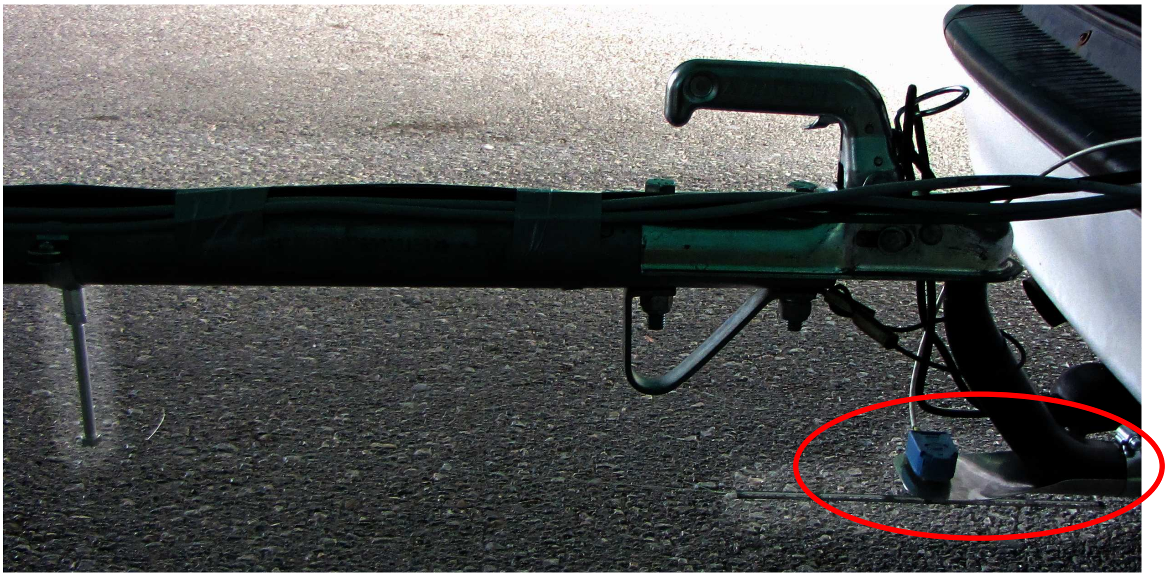

Figure 20: A simple but ingenious device to measure the trailer angle

Vehicle and trailer accelerations can be utilized for the same purpose as well. Data from the accelerometers measuring lateral accelerations of vehicle and trailer carry most significance.

Due to safety considerations, the experiments were not carried out at velocities close to critical. Due to limitations of the test course, the test velocities were even considerably lower than critical velocity. It was thus not enough to predict whether the vehicle was stable or not, but we had to determine how stable it was as well.

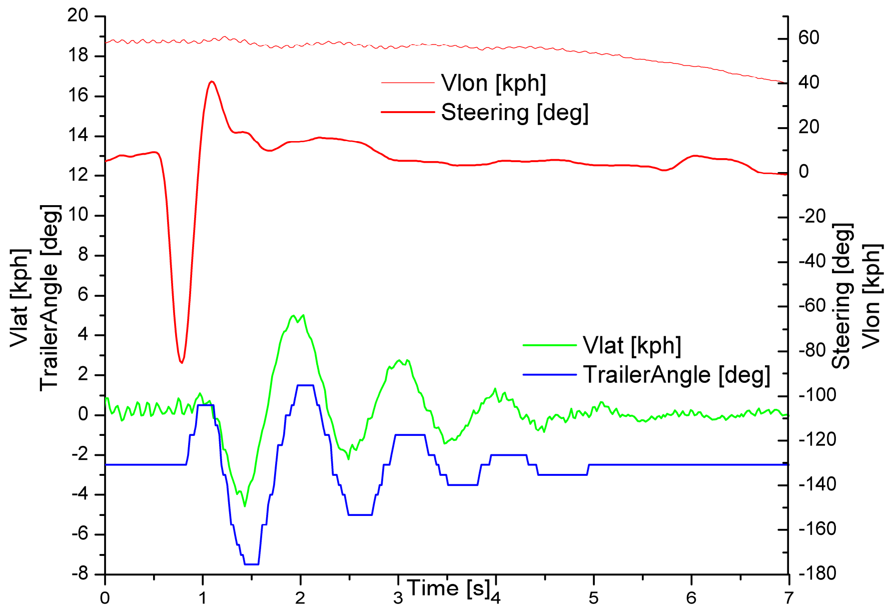

Figure 21: Some of measured data during a single test run

A concept already discussed in the yaw-plane model can be utilized for this purpose as well. Namely, the damping ratio already evaluated for the yaw-plane model can be evaluated for the MBS model and the measured data as well. The tool implemented for this purpose is the so called *logarithmic decrement*.

Logarithmic decrement, δ, is used to find the damping ratio of an under damped system in the time domain [5]. The logarithmic decrement is the natural log of the amplitudes of any two successive peaks:

$$\delta = \frac{1}{n}\ln\frac{x_0}{x_n} \tag{12}$$

where x_0 is the greater of the two amplitudes and x_n is the amplitude of a peak n periods away. The damping ratio is then found from the logarithmic decrement:

$$\zeta = \frac{1}{\sqrt{1+\left(\frac{2\pi}{\delta}\right)^2}} \tag{13}$$

Comparison of yaw-plane model, MBS model and measured data damping ratio can be made by gathering the calculated and measured data in the following table.

	$v_{critical}$ [km/h]	***ζ_{60}***	***ζ_{50}***	***ζ_{40}***
yaw-plane model	*105*	*0.130*	*0.191*	*0.284*
MBS model	*105*	*0.129*	*0.179*	*0.223*
Measurement	*N/A*	*0.132*	*0.165*	*N/A*

The critical velocities determined by both mathematical models are identical, while the experiment was not carried out at the critical velocity and the experimental data is thus not available. The damping rations at 60 km/h also show high coincidence, while the models seem to overestimate the damping ratios at lower velocities. It must be kept in mind that as the velocities keep getting lower, the oscillations keep getting fewer and the determination of the damping ratio harder and more prone to error.

5 Conclusions

Several approaches to vehicle dynamics modelling can be implemented with the purpose of vehicle dynamics modelling. Two of them have been described in this contribution and used to model vehicle-trailer system dynamic instability. While both give useful results, they do have their own advantages and drawbacks. While the yaw-plane model is fairly simple in comparison to the MBS model, the latter can be readily implemented for various other analyses as well. Results of both models were compared against each other and against measured results.

A measurement system for measuring the relevant kinematic quantities on a vehicle-trailer system has been devised and used to acquire the data. It is aplicable in research of various scenarios of driving a vehicle-trailer system and can thus be used in future research in this field.

Both models as well as measurements show that trailer load position and arrengement greatly influence the vehicle-trailer system stability. When incorrectly loaded, it is perfectly possible for the vehicle-trailer combination to become unstable at road legal velocities.

6 References

[1] Aleksander, H., Daniel, F., Hsien, C. (2008). Stability and control considerations of vehicle-trailer combination. *SAE TECHNICAL PAPER SERIES*, (2008-01-1228).

[2] Christopher J., K. (2003). The dynamics of towed vehicles: Final year project. Bath: University of Bath, 71 p.

[3] CARAT (Computer Aided Reconstruction of Accidents in Traffic). [computer program] (1997-2010). Burg, H. IbB Informatik GmbH: Wiesbaden.

[4] MSC.ADAMS. [computer program] (2007). MSC.Software Corporation: Santa Ana, CA, USA.

[5] Logarithmic decrement. (2009). [Online], [cited Available at: http://en.wikipedia.org/wiki/Logarithmic_decrement

[6] Control systems. (2010). [Online], [cited 22.06.2010]. Available at: http://en.wikibooks.org/wiki/File:Control_Systems.pdf