Resource allocation with priorities

Hugo Lhuillier*

June 22, 2016

Dosi et al. (2013) have a few economic inconsistencies embedde in their model that we though releveant to remove. Studying the code of Dosi et al. (2013), several inconsistencies have appeared. One of them concerns the way they manage credit demand, debt, and bankruptcy of consumption-good firms. In their model, a firm dies if its net worth is negative. Meanwhile, they introduce a parameter called `repayment_share`, s.t. a firm only has to repay a fraction of its debt in every period. It follows that as long as the value of its liquid assets is above this fraction, plus the interest rate it has to pay on its debt, there is no reason why the firm should go brankrupt.

Requiring firms to pay back their entire debt in every period seems an un-relastic assumption; therefore we kept the `repayment_share` parameter but modified accordingly the condition on the death of firms. Furthermore, firms take into account that they have to repay this debt in their production and investment behaviours. This note presents those modifications. Section 1 presents the adjustments that may be needed depending on the borrowing capacity of firms, while Section **??** focuses on the *a posteriori* adjustments, i.e. when firms know the credit that the bank granted them.

1 A priori adjustment

In Dosi et al. (2013), firms have limited borrowing capacity. Formally, this upper bound on credit demand is $\bar{l}(t) = \lambda S(t-1)$, with $S(\cdot)$ the sale of the firm in the previous period[1]. In consequence, prior to know the loan they will receive from the bank, consumption-good firms may have to adjust their production plan such that the sum of their expenses, plus the fraction of the debt they will have to repay at the end of the current period, will not be greater than what they expect they will be able to pay[2].

Our first departure from the Dosi et al. code is in the way these *a priori* adjustments are done. In their code, firms adjust their production and investment, abstracting from the debt repayment[3]. If firms know they will have to pay back a fraction of their debt, their production and investment plans should take this into account. Furthermore firms should assessed their ability to reimburse their debt not on their current stock of assets, but on their expected stock of liquid assets at the end of the period.

*Collaborative research student at INET, Oxford Martin School; master student at Sciences Po. hugo.lhuillier@sciencespo.fr

[1]Because this note concerns only consumption-good firm at the micro level, subscripts are dropped.

[2]To be concrete, imagine a firm wants to produce and invest for \$2000, has liquid assets worth \$400 and has a borrowing capacith of $\bar{l}(t) = \$500$. Obviously the fund they have at their disposal is lower than their expenses – abstracting from the repayment of their debt. Therefore the firm has to scale down its production and investment plan.

[3]In their final credit demand is eventually added their total debt which is not consistent with the fact that they only have to pay back a fraction of their debt.

Recall some equations

$$\begin{aligned}
nw(t) &= (1-tr)\Pi(t) + nw'(t-1)\\
\Pi(t) &= p(t)D(t) - c(t)Q(t) + \underline{r}nw'(t-1) - \bar{r}(d(t-1)+l(t))\\
\mathbb{E}(D(t)) &= D(t-1) \text{ and } N^d(t) = \iota D(t-1)\\
Q^d(t) &= D(t-1)(1+\iota) - N(t-1) \Leftrightarrow D(t-1) = \frac{1}{1+\iota}\big(Q^d(t) + N(t-1)\big)
\end{aligned}$$

where nw is the stock of liquid aset, nw' is the amount of liquid assets left after having fund production and invesmtent internally, Π is the profit, D is the demand received, Q is the production, Q^d is the desired production, d is the debt, l the loan, N the inventories and N^d the desired inventories. Furthermore, let l^P denote the part of the loan used to fund production and investment.

Lemma 1. *It cannot be that $nw'(t-1)$ and $l^P(t)$ are both positive.*

Lemma 1 follows directly from the preference of firms for internal funds. Consequently, we have

$$\begin{aligned}
nw'(t-1) &= \max\Big\{0, nw(t-1) - c(t)Q(t) - c(A^\tau,t)I(t)\Big\}\\
l^P(t) &= \max\Big\{0, c(t)Q(t)c(A^\tau,t)I(t) - nw(t-1)\Big\} \text{ and } l(t) = l^C(t) + l^P(t) \leq \bar{l}(t)
\end{aligned}$$

where c is the unit cost of production, $c(\cdot)$ the cost of a machine, I the amount of investment in terms of machine and l^C the part of the loan used to fund debt repayment. The expression for l^P follows from Lemma 1.

Assumption 1. *Consumption-good firms cannot infer the loan they will receive from the bank. Hence $\mathbb{E}d(t) = d(t-1) + l(t)$.*

Assumption 1 implies that prior to know their loan, firms act *as if* they will be receiving the loan they asked for. At the end of each period, a firm has to pay a fraction of its debt, denoted κ. Let cf be the cash flow of a firm. Using the previous equations and using Assumption 1, we have

$$cf(t) = \mathbb{E}nw(t) - \kappa\big(d(t-1) + l(t)\big)$$

with

$$\mathbb{E}(nw(t)) = \theta\Big(Q^d(t)(\frac{p}{1+\iota} - c) + \frac{p}{1+\iota}N(t-1) - \bar{r}(d(\cdot)+l)\Big) + nw'(\cdot)(1+\underline{r}\theta)$$

where $\theta = 1 - tr$. Furthermore, firms can use some of their loan to repay their debt.

For a firm not to die, it has to be that $cf(t) + l^C(t) \geq 0$ at the end of the period. Using $l(t) = l^P(t) + l^C(t)$, it is possibe to rewritte the payment condition as

$$l^C(1-\kappa-\bar{r}\theta) + nw'(1+\underline{r}\theta) + \theta\Big(Q^d(p'-c) + p'N\Big) - (d+l^P)(\kappa+\theta\bar{r}) \geq 0$$

with $p' = p/1+\iota$.

Proposition 1. *Algorithm* **??** *allows to achieve a pseudo-optimal allocation of resources such that the payment constraint holds.*

Point **(a)** is implemented in the code presented in Section 2. This `if else` structure looks at whether the optimal plan, Q^d and I^d are achievable with the current resources, $nw(t-1)$ and $\bar{l}$. If not, it adjusts Q^d and I^d. Recall that the firm prefers to use first its internal funds and then only borrow. Furthermore firms value production over investment, such that they adjust first through changes in investment, and only then via changes in production, if indeed. From **(b)** to **(d)**, $nw'(t-1) > 0 \iff l^P(t) = 0 \implies Q^d = Q^*$ and $I^* = I^d$.

(a) Compute Q^*, I^* and l^P as a function of Q^d, I^d and $nw(t-1)$;
Returns $nw'(t-1)$;
if $nw'(t-1) > 0$ **then**
  if $\mathbb{E}(cd \mid l(t) = 0) \geq 0$ **then**
    (b) Stop;
  else
    if $\mathbb{E}(cd + l(t) \mid l(t) \in (0, \bar{l}(t)]) \geq 0$ **then**
      (c) $l^C(t) = l(t) \in (0, \bar{l}(t)]$;
      Stop;
    else
      (d) $l^C(t) = l(t) = \bar{l}(t)$;
      Loop 1;
    end
  end
else
  if $l^P(t) = \bar{l}(t)$ **then**
    if $\mathbb{E}(cd \mid l^P(t) = \bar{l}(t)) \geq 0$ **then**
      (e) Stop;
    else
      (f) Loop 2;
    end
  else
    if $\mathbb{E}(cd + l^C(t) \mid l^C(t) < \bar{l}(t)) \geq 0$ **then**
      (g) Stop;
    else
      (h) $l^C = \bar{l}(t) - l^C(t)$;
      Loop 3;
    end
  end
end

Algorithm 1: Cash flow constraint algorithm

In **(b)**, the firm could fund internally its entire prouction and investment, plus it expects its revenue to be sufficient to repay its due debt. In **(c)**, altough the firm was able to fund its production and investment only through internal funds, it also needs to borrow to be able to make it at the end of the day. Note that $\partial \mathbb{E} cf / \partial l^P > 0$[4]. Hence increasing its credit demand will help the firm to satisfy its payment condition[5]. This rationale also brings **(d)**: if the optimal plans of the firm Q^* and I^* does not allow it to satisfy the payment constraint, then it borrows as much as possible, as this would allow the firm to meet more easily its cash flow constraint, i.e. allows it to depart less from its original plan. Then it re-works its plan until the constraint is met. Loop 1 is described below.

From **(e)** to **(h)**, the firm already had to use external funds to reach its plans. More specifically, in **(e)**, it uses its entire (possible) loan to fund production and investment and its expectation on its revenues satisfy the cash flow constraint. In **(g)**, some of the (possible) loan is still available and the firm uses part (or all) of it to meet the constraint. On the contrary, in **(f)** and **(g)** the constraint is not met and the firm has to go through some adjustments. Loop 2 and 3 are also described below.

[4]This should not be misinterpreted. This is mainly because we are considering this allocation as a static problem. However, the increased loan today will also be a burden tomorrow as it will be part of the debt

[5]The firm could actually try to build new plans that would involved Q^* and I^* satisfying the cash flow constraint. Yet, why the firm would be averse to loan in this case and not when funding production and investment?

1.1 Loop 1

In the first loop, we have $nw' > 0 \implies Q^* = Q^d$, $I^* = I^d$ and $l(t)$ is kept at $\bar{l}(t)$, such that the assets of a firm before paying its debt are[6]

$$\mathbb{E}(cf + l) = (p' - c)Q^* + (1 + \underline{r})\big(nw(t-1) - Q^* - I^*\big) + \bar{l}(1 - \kappa - \bar{r}) - (\bar{r} + \kappa)d(t-1)$$

The firm has a preference for production, such that it first adjusts on investment. It is clear that $\partial(cf + l)/\partial I^* < 0$, hence decreasing investment brings the firm closest to its payment restriction. In the first part of the loop, the firm therefore reduces incrementaly its level of investment. Because $Q^* = Q^d$, the firm does not think that producing more would yield higher revenues – **to check**. Therefore every pound available (from the reduction in investment) is used to increase the stock of liquid asset of the firm. The loop stops when either $\mathbb{E}(cf + l) > 0$ or $I^* = 0$.

If $I = 0$, then the firm is considering reducing its production. Yet, note that $\partial cf + l/\partial Q^* \lesseqgtr 0$ depending on the return to production and the return to savings. The different scenarios are depicted in Figure 1.

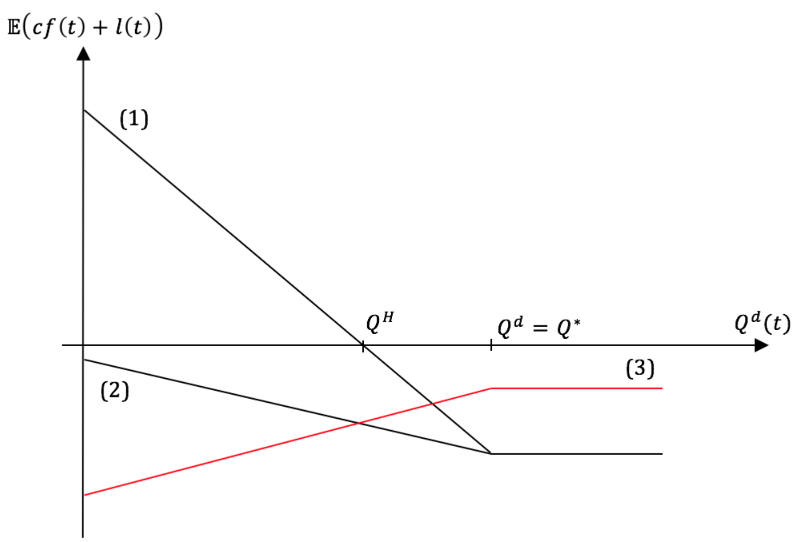

Figure 1: Q^* adjustment with $I^*(t) = 0$, $nw'(t-1) > 0$ and $l^C = \bar{l}$

In **(1)** and **(2)**, $p' - c < 1 + \underline{r}$, s.t. the return to savings is more profitable than the return to production. In this scenario, $\exists\, Q^H : \mathbb{E}(cf + l) = 0$; the question is whether Q^H is above (1) or below zero (2)[7]. In the case of (2), a firm is facing two choices: setting $Q^* = 0$ to be as close as possible to $\mathbb{E}(cf + l) = 0$, or $Q^* > 0$. Yet, it knows that by setting $Q^* = 0$ it would almost surely die while the other scenario relies on future revenues that are uncertain, i.e. $\mathbb{P}(cf + l < 0) < 1$. This leads to Assumption 2.

Assumption 2. *When a firm cannot adjust its production and investment plan such that* $\mathbb{E}(cf + l) \geq 0$, *it opts for the safest strategy that does not imply further leverage.*

Definition 1 (Safest strategy)**.** *The safest strategy is defined as* $l = 0$, $Q^* = \min\{nw(t-1), Q^d\}$ *– **to change with price different than one,** and* $I^* = 0$.

In **(3)**, expected liquid assets are increasing in the quantity produced. Yet, the keynesian spirit of the model makes that your sales are bounded above by the demand you receive, on

[6]For simplicity, assume $tr = 0$ and $N(t-1) = 0$.

[7]A closed form solution of this Q^H exists and is trivial to obtain.

which Q^d is based. Hence the flat part to the right of Q^d. It follows that if $\mathbb{E}(cf + l \mid I^* = 0, nw' > 0) < 0$ and $\partial\mathbb{E}(cf + l)/\partial Q^d > 0$, $\nexists Q^* : \mathbb{E}(cf + l) = 0$. Reducing the production will only worsen the condition of the firm, and increasing it will not improve it. A firm in this situation opts for the safest strategy.

1.2 Loop 3

Loop 3 is studied before loop 2 because it is a simplified verion of the lattest. Here, $l^P < \bar{l} \implies Q^* = Q^d$, $I = I^*$ and $l^C = \bar{l} - l^P$. As in loop 1, the firm is adjusting first through changes in investment. We have $l^P = I^* + Q^* - nw(t-1)$, conditional on $Q^* + I^* > nw(t-1)$ – **once more changes if price are different than one**. Because $Q^* = Q^d$, there is no point for the firm to increase further its production. Hence the money of the investment can be used either to reduce the loan (a), or to reallocate the loan from production funding to debt funding, i.e. $l^P \to l^C$, s.t. $\bar{l}$ is constant (b)[8]. Formally

$$\mathbb{E}(cf + l) \propto (1 - \kappa - \bar{r})l^C - (\kappa + \bar{r})(d + l^P) \tag{a}$$

$$\mathbb{E}(cf + l) \propto (1 - \kappa - \bar{r})(\bar{l} - l^P) - (\kappa + \bar{r})(d + l^P) \tag{b}$$

with $l^P = (I^* + Q^* - nw(t-1))\mathbf{1}_{I^*+Q^*>nw(t-1)}$. It follows that if $1 > \kappa + \bar{r}$, the firm incrementaly reduces its investment while keeping the loan constant until either $I^* = 0$ or $\mathbb{E}(cf + l) > 0$. If l^P reaches zero before, then any further savings from investment reduction are put into the firm deposit.

If $\mathbb{E}(cf + l) < 0$ after the first round of adjustment, then it must be that $I^* = 0$, $Q^d = Q^*$ and $l^P \geq 0$. More specifically, if $l^P = 0$, then the situation is identical to the second part of loop 1. This is not the case if $l^P > 0$. Indeed, while $l^P > 0$, the firm could – given that it is beneficial for it to reduce its production: (a) decrease its production and reallocate the loan to other use than production, (b) decrease its production and reduce its loan. Formally

$$\mathbb{E}(cf + l)_a \propto (p - c)Q^* + (1 + \underline{r})nw' + (1 - \kappa - \bar{r})(\bar{l} - l^P) - (\kappa + \bar{r})(d + l^P) \tag{a}$$

$$\mathbb{E}(cf + l)_b \propto (p - c)Q^* + (1 + \underline{r})nw' + (1 - \kappa - \bar{r})l^C - (\kappa + \bar{r})(d + l^P) \tag{b}$$

with $nw' = (nw(t-1) - Q^*)\mathbf{1}_{l^P=0}$ and $l^P = (Q^* - nw(t-1))\mathbf{1}_{l^P>0}$. Hence we have $\partial\mathbb{E}(cf + l)_a/\partial Q^d = (p' - c) - (1 + \underline{r})\mathbf{1}_{l^P=0} - \mathbf{1}_{l^P>0}$ and $\partial\mathbb{E}(cf + l)_b/\partial Q^d = (p' - c) - (1 + \underline{r})\mathbf{1}_{l^P=0} - (\kappa + \bar{r})\mathbf{1}_{l^P>0}$. Remark that $\partial\mathbb{E}(cf + l)_a/\partial Q^d \leq \partial\mathbb{E}(cf + l)_b/\partial Q^d \iff 1 \geq \kappa + \bar{r}$, which will be satisfied in most of calibration. This in turn implies that if it is indeed beneficial to decrease production, then $\mathbb{E}(cf + l) = 0$ will be reached faster by substituting l^P for l^C and if it is not, then not substituting l^P for l^C will bring the firm further away from $\mathbb{E}(cf + l) = 0$. This is depicted in Figure 2. It follows that firms will never use strategy (b) if $1 > \kappa + \bar{r}$.

Working only with the second strategy, we further have $\{\mathbb{E}(cf + l)/\partial Q^d\}\mathbf{1}_{l^P>0} \geq \{\mathbb{E}(cf + l)/\partial Q^d\}\mathbf{1}_{l^P=0} \iff \underline{r} \geq 0$ which should be satisfied in all calibrations. It follows that if decreasing production is helpful to reach the payment condition when $l^P > 0$, then it is also true when $l^P = 0$ – see Figure 3. Hence, if $\{\mathbb{E}(cf + l)/\partial Q^d\}\mathbf{1}_{l^P>0} < 0$, $\exists\, Q^H : \mathbb{E}(cf + l) = 0$. As before, the question is whether $Q^H > 0$. If not, then the firm turns to the safest strategy. In the case where $\{\mathbb{E}(cf + l)/\partial Q^d\}\mathbf{1}_{l^P>0} > 0$, it could however still be that there exists a solution, namely if $\{\mathbb{E}(cf + l)/\partial Q^d\}\mathbf{1}_{l^P=0} < 0$, with $l^P = \bar{l}$. If this is the case, the firm has once more to check that $Q^H > 0$. In the other cases, it also goes for the safest strategy.

[8] Note that no speculative activity are allowed, i.e. a firm cannot borrow to then put this loan into another asset.

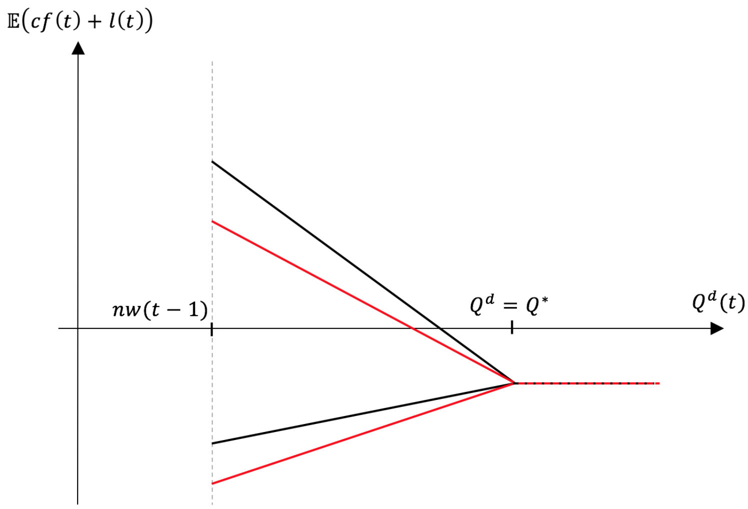

Figure 2: Deleveraging strategy vs. reallocation of the loan
Note: the red lines represent the reallocating strategy while the black lines represent the deleveraging one.

1.3 Loop 2

Loop 2 and 3 are in pratice identical. To see this, note that the only difference between them is that $l^P = \bar{l} \implies \{0 < I^* \leq I^d \cap Q^* = Q^d\} \cup \{I^d = 0 \cap Q^* \leq Q^d\}$. The first case is identical to loop 3. The second implies that adjustment through reduction of investment are impossible. Now if $Q^* = Q^d$, we are once more back to loop 3. Otherwise, if $Q^* < Q^d \iff nw' = 0 \cap l^P = \bar{l}$. It follows that the firm could benefitiate from an increase in production, but has no resource left. Hence the firm can only adjust through reduction in the quantity produced and this is loop 3 as well.

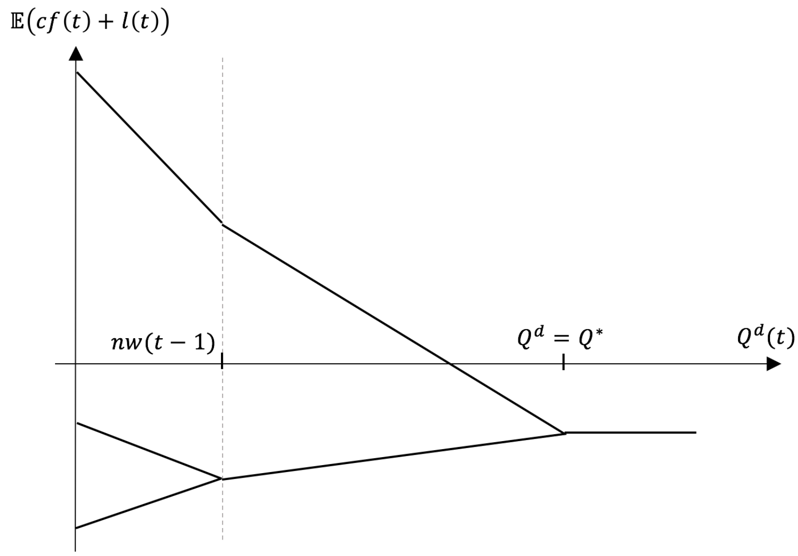

Figure 3: Q^* adjustment with $I^*(t) = 0$, $nw'(t-1) >= 0$ and $l^P > 0$

2 Code

2.1 invUpdated() – (a)

```
public void invUpdated(){
//dInvE : desired exp. investment, dInvR: desired rep. investment
//NOTE: dInvE / dInvR have been rounded down previously
double cInvE = dInvE * supplier.getP()[1]; // cost of exp. inv
double cInvR = dInvR * supplier.getP()[1]; // cost of rep. inv

nwAvail = this.nw[0]; // need nwAvail because do not actually
    withdraw money from their deposit at the bank; simply adjust
    their plan knowing they may have to in the future

// eq. (5) in Dosi et al. (2013)
double maxCred = Parameters.getLambda() * s[0];

// Adjustment process conditional on maxCred
// PRODUCTION
// either able to fund production internally :
if(c[1] * dQ <= nwAvail) { // c[1] : unit cost of production
        nwAvail -= c[1] * dQ;
// or if internal funds are not sufficient, then ask for credit;
    desired production remains id.but unsure of the credit it
    will receive
} else if(c[1] * dQ <= nwAvail + maxCred) {
        maxCred -= c[1] * dQ - nwAvail;
        this.cD +=  c[1] * dQ - nwAvail;  // cD : credit demand
        model.getBank().loan +=  c[1] * dQ - nwAvail; // sum over
            the credit demad of all c-firms
        nwAvail = 0.;
// if not sufficient either, then scale down production plan
} else {
        this.dQ = (nwAvail + maxCred) / c[1];
        this.cD += maxCred;
        model.getBank().loan += maxCred;
        maxCred = 0.;
        nwAvail = 0.;
}

// INV. EXP., ~ if else previous structure
if(cInvE <= nwAvail) {
        nwAvail -= cInvE;
} else if(cInvE <= nwAvail + maxCred){
        maxCred -= cInvE - nwAvail;
        this.cD += cInvE - nwAvail;
        model.getBank().loan += cInvE - nwAvail;
        nwAvail = 0.;
} else {
        this.dInvE = Math.floor((nwAvail + maxCred) /
            supplier.getP()[1]) * Parameters.getDimK();
        this.cD += maxCred;
        model.getBank().loan += maxCred;
        maxCred = 0.;
```

```
        nwAvail = 0.;
}

// INV. REP., ~ id. structure
if(cInvR <= nwAvail) {
        nwAvail -= cInvR;
} else if(cInvR <= nwAvail + maxCred) {
        maxCred -= cInvR - nwAvail;
        this.cD += cInvR - nwAvail;
        model.getBank().loan += cInvR - nwAvail;
        nwAvail = 0.;
} else {
        this.dInvR = Math.floor((nwAvail + maxCred) /
           supplier.getP()[1]) * Parameters.getDimK();
        this.cD += maxCred;
        model.getBank().loan += maxCred;
        maxCred = 0.;
        nwAvail = 0.;  }
}
```

2.2 invUpdated() – (b)