Outsourcing, Inequality and Aggregate Output*

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Abstract

Outsourced workers experience large wage declines, yet domestic outsourcing may raise aggregate productivity. To study this equity-efficiency trade-off, we contribute a framework in which firms either hire many imperfectly substitutable worker types in-house by posting wages along a job ladder, or rent labor services from contractors who hire in the same frictional labor markets. Three implications arise. First, more productive firms are more likely to outsource to save on higher wage premia. Second, outsourcing raises output at the firm level. Third, contractors endogenously locate at the bottom of the job ladder, implying that outsourced workers receive lower wages. Using firm-level instruments for outsourcing and revenue productivity, we find empirical support for all three predictions in French administrative data. After structurally estimating the model, we find that the rise in outsourcing in France between 1996 and 2007 raised aggregate output by 3% and reduced the labor share by 0.7 percentage points. A 9% minimum wage increase stabilizes the labor share and maintains two thirds of the output gains.

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Introduction

Outsourcing is fundamentally changing the nature of the labor market. During the last two decades, firms have been increasingly contracting out a vast array of labor services, such as security guards, food and janitorial services. Workers in these occupations receive much lower wages at contractor firms than at traditional employers. This relative wage gap suggests that rising domestic outsourcing redistributes away from workers. At the same time, firms presumably scale up more efficiently by contracting out. Outsourcing may thus generate aggregate output gains that all workers share in. Despite the prevalence of outsourcing in the labor market, there is little guidance to trace out its determinants and effects. Why do firms outsource? How can low-paying contractor firms co-exist with high-paying traditional employers? How does outsourcing change aggregate production and its split between workers and firms?

In this paper, we propose answers to these questions in three parts. First, we build a theory of domestic outsourcing based on imperfect rent-sharing in the labor market. Second, we offer new reduced-form evidence that confirms the distributional and productivity effects of outsourcing that our theory highlights using administrative data from France. Third, we structurally estimate our general equilibrium model and quantify the effects of outsourcing on aggregate output and the labor share.

Specifically, in the first part of the paper, we contribute a framework to study the emergence of outsourcing. To set the stage for our analysis of outsourcing, we start with an environment that features three necessary properties, but no outsourcing yet. First, firms have heterogeneous productivities and well-defined boundaries due to decreasing returns to scale in revenue. Second, not all workers are equally exposed to outsourcing. Firms hire workers of different skills in segmented labor markets, who enter as imperfect substitutes in the production process and search for employment opportunities on and off the job. Third, seemingly identical workers earn different wages at different employers. Frictions in skill-specific labor markets give rise to rent-sharing between workers and firms. Firms exploit their monopsony power over workers, leading to wage dispersion, and firms with a larger target size post higher wages.

We then introduce contractor firms in our environment. Contractor firms hire labor in the same frictional labor markets as traditional firms. Contractor firms then sell labor services of their employees in a competitive labor service market. In the aggregate, contractor firms effectively expand resources available for recruiting. Traditional firms may buy labor services at an equilibrium price to sidestep labor market frictions, or hire workers in-house directly in the frictional labor markets. The price of outsourced labor services reflects both wages paid to workers at contractor firms, as well as a markup which we call the cost of outsourcing. This cost may reflect either the cost of capital or a trade cost encapsulating communication and coordination costs.

Three main implications emerge. First, traditional firms select into outsourcing. More productive firms have the strongest incentives to outsource and save on labor costs because they pay higher wages to attract and retain a larger workforce. Less productive firms who pay lower wages prefer to hire in-house and avoid the outsourcing markup. Second, outsourcing lets traditional firms sidestep labor market frictions and scale up: employment and revenues increase when traditional firms outsource. Thus, outsourcing leads to a positive productivity effect at the firm level. Third, large wage gaps emerge between traditional and contractor firms. In an equilibrium with outsourcing, the price of outsourcing services must be such that both contractors and traditional firms that outsource gain from trading. Thus, contractor firms pay lower wages than the marginal outsourcing firm. When a traditional firm outsources its workforce and its former workers transition to a contractor firm, their wages drop from the top to the bottom of the job ladder. The resulting wage losses for outsourced workers capture the distributional effect.

To reach these conclusions, we required a setup that departed from the assumptions of constant returns to scale and perfect substitution between workers that are traditionally imposed to retain traction in wage-posting models. We overcome this technical difficulty with two sufficient conditions. First, the revenue function exhibits a single-crossing property in firm productivity and employment of each worker skill. This condition ensures that more productive firms always prefer to hire more and nests most standard revenue functions. Our second condition is more technical in nature. It consists in a trembling-hand equilibrium refinement that rules out non-smooth equilibria.

In the second part of the paper, we test the main implications of our theory using administrative data from France.[1] We combine matched employer-employee data from employer tax returns, balance sheet and income statement records for the universe of firms, firm-level customs data and a firm-level survey that details outsourcing information. We measure expenditures on outsourcing at the firm level as expenditures on workers who are not employees of the firm, but are at least partially under the legal authority of the purchasing firm. Our main analysis starts in 1996 and stops in 2007 due to a substantial change in data collection procedures. In the aggregate, expenditures on outsourcing represented 6% of the aggregate wage bill in 1996, before rising to almost 11% in 2007. Extrapolating beyond 2007 using slightly coarser data, we find that outsourcing represents 19% of aggregate wages by 2015.

First, we test whether firms select into outsourcing. To do so, we investigate the relationship between firm-level value added and the outsourcing share, defined as outsourcing expenditures out of all labor costs including outsourcing. To isolate the effect of revenue productivity from confounding factors such as Information Technologies (IT) improvements that may affect a

[1] We also rule out alternative explanations such as demand volatility and comparative advantage of contractors in production.

particular firm's propensity to outsource, we construct an instrument at the firm level. We interact initial firm-level export shares with changes in foreign demand across 4-digit industries and countries (Hummels et al., 2014). We find that a 10% increase in value added leads to a 0.35 percentage points increase in the outsourcing share. We conclude that firms indeed select into outsourcing.

Second, we test whether a decline in the cost of outsourcing leads firms to scale up. To do so, we investigate the reverse relationship, running from the outsourcing share to firm-level value added. To isolate a decline in firm-level outsourcing costs from revenue productivity, we need a second instrument for firms' outsourcing share. We interact initial firm-level occupation shares with changes in aggregate spending on outsourcing at the occupation level. We find that a 1 percentage point increase in the outsourcing share leads to a 5% increase in value added. We conclude that outsourcing indeed has a positive productivity effect at the firm level

Third, we confirm the wage penalty from outsourcing in France with an event study design. We define outsourcing events building on Goldschmidt and Schmieder (2017), based on changes in occupational shares, increases in outsourcing expenditures and joint mobility of clusters of workers. We find that job switchers in an outsourcing event lose 12% of their pre-event wage relative to workers at the firm who are not in the outsourcing event but also switch employers. We conclude that outsourcing indeed redistributes away from workers due to a relative wage gap.

In the third part of the paper, we develop and structurally estimate a quantitative version of the framework, before investigating the role of the rise in outsourcing for inequality and output. The main additions are a flexible curvature in traditional firms' vacancy cost function, and firm-level outsourcing costs leading to mixing in the outsourcing decision at all scales, while preserving selection into outsourcing on average. There are two skill types, and only low-skill workers are exposed to outsourcing. We estimate the model with a Method of Simulated Moments (MSM) estimator. We use indirect inference and target cross-sectional moments.

We start with a set of validation exercises using non-targeted moments that closely follow the three key implications we highlighted in our reduced-form exercises: selection into outsourcing, the productivity and distributional effects. We reproduce our reduced-form regressions in the estimated model. First, we find that the within-firm Ordinary Least Squares (OLS) estimate of selection into outsourcing aligns quantitatively with the data. As in the data, the Two Stage Least Squares (2SLS) estimate is also larger because of correlation between revenue productivity and idiosyncratic outsourcing costs. Quantitatively, the 2SLS estimate for a 10% increase in value added is somewhat lower in the model than in the data, between 0.07 and 0.25 percentage points. Second, our model-based estimate of the productivity effect for a one percentage point increase in the outsourcing share lies between 2% and 4%, moderately lower than its empirical counterpart. Third, the estimated model also matches well moments related to the distributional effect. It matches the standard deviation of firm wage premia: 0.13 in the model, against 0.14 in the data. The predicted outsourcing wage penalty is 11% in the estimated model, against 12% in the data. Together, these observations support the estimated model's ability to account for the productivity and distributional effects of outsourcing in the cross-section.

We then quantify the race between the productivity and the distributional effects of outsourcing in the aggregate. Our main counterfactual changes the cost of outsourcing such that outsourcing expenditures track the rise seen in France between 1996 and 2007. Aggregate output rises by 3.3%. High productivity firms expand production most thanks to outsourcing. By improving aggregate Total Factor Productivity (TFP), this reallocation of labor in the economy accounts for two fifths of aggregate output gains. In addition, outsourcing effectively expands resources available for hiring in the aggregate and more workers enter employment. This extensive margin accounts for the remaining three fifths of output gains.

Low-skill workers' exposure to outsourcing has three components. The first component is a partial equilibrium impact. Low-skill workers are twice as likely to work for contractor firms by 2007 relative to 1996. This reallocation of workers towards low-paying contractor firms depresses expected earnings by 2%. The second component is a general equilibrium effect. Wages of low-skill workers decline even in-house, because traditional employers now face weaker labor market competition for workers. Traditional employers can easily poach workers from contractor firms, while shielded from wage competition from firms previously at the top of the job ladder, who now outsource and left the labor market. This general equilibrium response of traditional firms' wages is substantial and depresses expected earnings further by 2%. The third component is also a general equilibrium effect. Cheaper outsourcing increases labor demand at high-productivity firms. This labor demand is met by contractor firms, improving the aggregate search efficiency of the economy. Low-skill workers' employment rate increases, leading to a 3% increase in expected earnings. Adding up all components, expected earnings decline by 1%.

Our results thus indicate that low-skill workers are worse off with a higher degree of outsourcing since in our setup, welfare coincides with expected earnings. The distributional effects from outsourcing dominate the productivity effects in the aggregate. Our decomposition highlights that reduced-form approaches that can only pick up the first, partial equilibrium impact would miss important general equilibrium feedback mechanisms. The partial equilibrium impact would overstate low-skill workers' welfare losses twofold.

By contrast, high-skill workers benefit unambiguously from rising outsourcing. The demand for low-skill labor rises as the cost of outsourcing falls. Skills being complements in production, demand for high-skill workers increases too. For high-skill workers that cannot be outsourced, rising demand materializes as increasing wages. On net, average earnings across all skills rise by just over 0.3%, but the labor share declines by 0.7 percentage points. We conclude that, in the absence of any policy intervention, outsourcing leads to positive aggregate productivity effects that primarily benefit firms' shareholders and high-skill workers. At the same time, low-skill workers' labor market prospects deteriorate.

The final part of the paper investigates whether simple labor market policies can ensure that both workers and firms gain equally from outsourcing. We focus on the minimum wage as the main policy instrument. By pushing up the overall level of wages, the minimum wage alleviates the adverse impact of outsourcing on workers' wages. At the same time, it raises the cost of labor relative to the counterfactual economy without a binding minimum wage. We consider a 9% increase in the minimum wage between 1996 and 2007, chosen so that the labor share remains constant. Output rises by 2%, two thirds of its baseline increase. Expected earnings of low-skill workers increase by 2%. We also evaluate a 3% increase in the minimum wage that leaves low-skill workers indifferent, leading to a 2.7% rise in output. Overall, we conclude that increases in the minimum wage can ensure that outsourcing benefits firms and workers equally, but that it diminishes gains in output and firm profits.

This paper relates to several strands of literature. The first is the empirical literature that studies the wage and employment effects of outsourcing. Goldschmidt and Schmieder (2017) and Drenik et al. (2020) document that outsourced workers experience large wage declines in Germany and Argentina, respectively. Katz and Krueger (2017) document a rise in alternative work arrangements in the U.S. Bergeaud et al. (2020) highlight that internet broadband expansion lead firms to concentrate on their core activities in France. Relatedly, LeMoigne (2020) highlights that the consequences of fragmentation events for workers resemble those of outsourcing events. Bertrand et al. (2020) show that an increase in the supply of contract labor helped Indian firms scale up. We contribute to this literature by providing a micro-founded theory of outsourcing, and testing its firm-level implications using direct measures of outsourcing expenditures.

Second, our paper connects to the large literature studying how labor market frictions give rise to wage premia across employers. We build on the wage-posting tradition, starting with Burdett and Mortensen (1998), and enriched with multiple worker types by Engbom and Moser (2021). We contribute to this literature by providing sufficient conditions to depart from constant returns to scale and perfect substitutability between workers in production.[2]

Finally, our paper relates more broadly to the literature on trade in intermediate inputs and international offshoring—see for instance Feenstra and Hanson (1999) and Grossman and Rossi-Hansberg (2008).[3] When firms trade intermediate inputs, they contract on a physical good. Under domestic outsourcing, firms contract on a worker's flow of services, thereby leading

[2]See Gouin-Bonenfant (2018) for a wage-posting model with productivity fluctuations. Models based on compensating differentials such as Card et al. (2018) imply that individuals working at high-paying firms attain exactly the same expected utility as individuals working at low-paying firms, making any distributional effects difficult to interpret.

[3]See also Acemoglu et al. (2015) and Antràs et al. (2017).

to distinct implications for wage inequality. When firms offshore internationally, they take advantage of lower wages in other countries. Domestic outsourcing reflects similar forces, but requires first to understand how to break the law of one price in the domestic labor market.

The rest of this paper is organized as follows. Section 1 lays out the basic framework without outsourcing. Section 2 introduces outsourcing in the economy. Section 3 details the reduced-form results supporting our theory. Section 4 lays out the quantitative extensions of the model and the structural estimation. Section 5 presents our counterfactuals. The last section concludes. Proofs and further details can be found in the Appendix and the Online Supplemental Material.

1 A theory of wage premia with large firms

We start the exposition of our environment with an economy in which wage premia across employers arise endogenously, as the result of monopsony power that employers exert over workers.

1.1 Setup

Time is continuous, and we focus on a steady-state equilibrium. There is a unit measure of workers. Each worker is characterized by its exogenous and permanent skill type $s \geq 0$. We assume that types are distributed in the population according to the measure $m_s ds$ with respect to a base measure denoted by ds.[4] Workers have linear preferences in income at every point in time, inelastically provide one unit of labor per time period, and discount future utility at rate r. They can be either employed or unemployed, in which case they earn real skill-specific unemployment benefits b_s.

There is a measure M of active firms in the economy. They are indexed by productivity z with support $[\underline{z}, \overline{z}]$, $\overline{z} \leq +\infty$. The corresponding cumulative distribution function Γ admits a finite and continuous density. Assume for simplicity that $\underline{z}$ is large enough relative to $\sup_s b_s$ so that all matches are viable. A firm with productivity z that hires a measure n_s of workers of skill $s \geq 0$ generates revenue $R(z, \boldsymbol{n})$, where $\boldsymbol{n} = \{n_s\}_s$ denotes the vector of employment across worker types. Assume that R is twice continuously differentiable and increasing in each argument.

Labor markets are segmented by skill s. A labor market consists of an equilibrium distribution of skill-specific wage offers, and job searchers. Unemployed workers of skill s sample wage offers randomly at Poisson intensity λ_s^U. Employed workers of skill s sample wage offers ran-

[4]This notation allows us to capture both continuous and discrete type distributions without loss of generality.

domly at Poisson intensity $\lambda_s^E \leq \lambda_s^U$ from the same distribution. Employed workers can break their current contract to accept a new wage offer. Existing matches are destroyed at Poisson rate δ_s. Thus, a match ends either when it is exogenously destroyed, or when the worker accepts a wage offer.

Firms optimally post wage offers in every skill-specific labor market to attract and retain workers. As in Burdett and Mortensen (1998), firms fully commit to a single wage per skill. Wages are fixed, are not state-contingent, and cannot be renegotiated throughout employment spells. Every firm is endowed with a unit measure of vacancies for every skill s to which they attach the same skill-specific wage offer.

1.2 Labor market transitions and the labor supply curve

To understand the labor supply curve faced by each firm, we must first characterize the job search behavior of workers. This subsection follows closely Burdett and Mortensen (1998). Given the equilibrium distribution of wage offers for skill s, denoted $F_s(w)$, the value of unemployment and the value of being employed at a given wage w satisfy:

$$
\begin{aligned}
rU_s &= b + \lambda_s^U \int \max\{V_s(w) - U_s, 0\} dF_s(w) \\
rV_s(w) &= w + \lambda_s^E \int \max\{V_s(w') - V_s(w), 0\} dF_s(w') + \delta_s(U_s - V_s(w)).
\end{aligned}
$$

The value of being employed at wage w, $V_s(w)$, is increasing with the wage w, so that workers behave as income maximizers: they always accept higher wage offers while employed. Equating the value of being employed to the value of being unemployed defines the reservation wage $\underline{w}_s$, given in Appendix A.1.

The equilibrium distribution of wages of employed workers, denoted $G_s(w)$, determines the labor supply curve of each firm. We relate the wage offer distribution $F_s(w)$ to the wage distribution of employed workers $G_s(w)$ using worker flows. We show in Appendix A.2 that the distribution of wages of employed workers writes

$$
G_s(w) = \frac{F_s(w)}{1 + k_s(1 - F_s(w))}, \qquad k_s = \frac{\lambda_s^E}{\delta_s}. \tag{1}
$$

From equation (1) we may characterize the number $N_s(w)$ of employed workers per wage offer in the interval $(w - \varepsilon, w]$ for skill s.[5] We obtain

$$
N_s(w) = \frac{(1 + k_s)e_s}{\big(1 + k_s(1 - F_s(w))\big)\big(1 + k_s(1 - F_s(w^-))\big)}, \tag{2}
$$

[5] $N_s(w)$ is equal to the limit of the ratio $\frac{G_s(w) - G_s(w-\varepsilon)}{F_s(w) - F_s(w-\varepsilon)}$ when $\varepsilon \to 0$, times the number of employed workers $m_s - u_s$. This limit implies that the left-limit $F_s(w^-)$ of the distribution appears in equation (2).

where $e_s = \frac{\lambda_s^U m_s}{\delta_s + \lambda_s^U}$ is the measure of employed workers of skill s.

Crucially, $N_s(w)$ is non-decreasing in the wage w. $N_s(w)$ thus defines the upward-sloping labor supply curve faced by firms. It depends on the equilibrium distribution of wage offers in the economy, $F_s(w)$. We now turn to firms' decision problem to characterize this distribution.

1.3 Wage and employment distributions

The number of workers per firm posting w, $n_s(w)$, is simply related to the number of workers employed at every wage by $n_s(w) = N_s(w)/M$ since firms post a unit measure of vacancies. When the discount rate is low enough, firms choose their wage offers $\{w_s(z)\}_s$ to maximize their flow profits.[6] Firms take as given how their size depends on their wage offer through their upward-sloping labor supply curve given in equation (2). Flow profits are given by:

$$\pi(z) \quad = \quad \max_{\{w_s, n_s\}_s} R\big(z, \{n_s\}_s\big) - \int w_s n_s ds \ \text{ s.t. } \ n_s \leq n_s(w) = \frac{N_s(w)}{M}. \tag{3}$$

Unless the distribution $F_s(w)$ can be characterized more precisely, the problem in equation (3) is intractable in general equilibrium. The wage-posting literature—from Burdett and Mortensen (1998) to Engbom and Moser (2021)—has leveraged a key simplifying assumption to make progress. Under constant returns and perfect substitutability of workers in production, $R(z, \boldsymbol{n}) = z \int n_s ds$, the problem (3) can be split at the match level. Once decoupled across matches, it is straightforward to see that (3) exhibits a single-crossing property. This structure implies that wages are increasing in productivity z, which in turn allows to solve for the distribution of wage offers in terms of the equilibrium wage policy and the exogenous productivity distribution, $F_s(w_s(z)) = \Gamma(z)$.

Studying outsourcing requires however a well-defined boundary of the firm as well as possible interactions between workers in production. We overcome the challenges that come with this departure from linearity with two sufficient conditions. Our first and main sufficient condition imposes minimal structure on the revenue function R that lets us rank wages by firm productivity.

Assumption (A). $(z, \boldsymbol{n}) \mapsto R(z, \boldsymbol{n})$ is strictly supermodular in all its arguments.

Given that R is twice continuously differentiable, Assumption (A) is equivalent to imposing strictly positive cross-derivatives between all arguments. It amounts to a form of complementarity between productivity and every labor type, as well as between any two types of labor. Assumption (A) guarantees that as productivity rises, firms always prefer to hire more labor of every type, through two channels. First, the direct effect of a productivity increase incentivizes more hiring of a given labor type because productivity and labor of any type are complements

[6] We fully derive the formulation in equation (3) from the dynamic problem of the firm in Appendix E.1.

in levels. Second, Assumption (A) also ensures that the firm never prefers to lower employment of a given labor type as it hires more labor of another type, holding productivity fixed. Assumption (A) is much weaker than and nests the traditional linearity assumption.

Importantly, the complementarities built in Assumption (A) stand in productivity and employment levels, as opposed to the usual notion of complementarity between worker types that stands in proportional deviations. Our supermodularity assumption is thus compatible with a relatively general class of revenue functions and allows for workers to be complements or substitutes in production in the usual sense. For instance, consider the revenue function $R(z, \boldsymbol{n}) = z \left(\int (a_s n_s)^{1-\frac{1}{\eta}} ds \right)^{\frac{\eta(\sigma-1)}{\sigma(\eta-1)}}$. Such a revenue function arises when workers have CES demand over M differentiated varieties with elasticity of substitution $\sigma > 1$, and firms produce with a CES production function with elasticity of substitution η. This revenue function also arises if there are technological decreasing returns to scale in production.

Supermodularity then requires $\sigma > \eta$. By comparing the curvature in the revenue function to the substitutability between worker types, this condition ensures that the marginal revenue gain from rising employment of one skill type does not incentivize the firm to lower employment of another skill type. Since typical estimates of σ lie above 3 to 5, while most estimates of η lie below 2, the condition for supermodularity is compatible with standard parametrizations.

We impose Assumption (A) in the remainder of this paper. We start by focusing attention on equilibria in which the wage offer distribution is continuous, and show that there exists a unique equilibrium with a simple structure under this restriction. We discuss how our second sufficient condition—an equilibrium refinement concept—ensures uniqueness without the smoothness restriction at the end of this section.

Assumption (B). The distribution of wage offers $F(w)$ is continuous.

Proposition 1 formalizes how these assumptions ensure that more productive firms post higher wages.

Proposition 1. *(Wage ranking)*
Under Assumption (B), wages $w_s(z)$ are strictly increasing with firm productivity z. The wage function is continuous in z. The wage offer distribution satisfies $F_s(w_s(z)) = \Gamma(z)$.

Proof. See Appendix A.3. □

With Proposition 1 at hand, the distribution of workers across firms is fully determined.

Proposition 2. *(Employment distribution)*
Under Assumption (B), the number of workers of skill s hired by firm z is given by

$$n_s(z) = \frac{(1 + k_s)e_s}{M[1 + k_s(1 - \Gamma(z))]^2}.$$

Proof. See Appendix A.4. □

Firm size in Proposition 2 depends only on the ranking of firms, $\Gamma(z)$, because firm size is fully determined by worker flows up the job ladder. This stark result arises because we do not let firms choose how many vacancies to post—which is equivalent to a vacancy cost with infinite curvature. We introduce endogenous vacancies in our quantitative extensions in Section 4, so that firm size also reflects the marginal product of labor.

To understand why $k_s = \lambda_s^E/\delta_s$ enters in Proposition 2, recall that workers start from unemployment and initially accept relatively low-paying jobs. While searching on the job, they accept any offer above their current wage. The speed at which they climb the job ladder in their market depends on the frequency at which they receive wage offers, the job-finding rate λ_s^E. They fall down the job ladder back into unemployment at rate δ_s. On net, the allocation of workers along the job ladder depends on the ratio $k_s = \lambda_s^E/\delta_s$.

Building on Propositions 1 and 2, we solve explicitly for the wage distribution.

Proposition 3. *(Wage distribution)*
Under Assumption (B), equilibrium wages are given by

$$n_s(z)w_s(z) = n_s(\underline{z})\underline{w}_s + \int_{\underline{z}}^{z} \frac{\partial R}{\partial n_s}\big(z, \boldsymbol{n}(x)\big)n_s'(x)dx.$$

Proof. See Appendix A.5. □

Proposition 3 captures the logic of the job ladder. Productive firms raise their wages to poach workers from lower-productivity firms in order to attain their target size. The equilibrium value of a worker to these lower-productive firms is given by their marginal product of labor $\frac{\partial R}{\partial n_s}$. Competitive wage pressure for a firm with productivity z then builds up from below. Wages at a firm with productivity z are pushed up, starting from the reservation wage, and integrating up to productivity z. Proposition 3 nests the wage equations with linear revenue in e.g. Burdett and Mortensen (1998) and Engbom and Moser (2021).

Proposition 3 characterizes wages having assumed that the wage offer distribution was continuous to begin with. Proposition 1 highlights that the wage offer distribution that results from firms' choices is internally consistent with continuity. Thus, we have guessed and verified existence of an equilibrium with a smooth wage offer distribution. In the presence of decreasing returns to scale, wage-posting models can however exhibit multiple equilibria. If a positive measure of firms happens to coordinate on posting exactly the same wage, it may be optimal for other firms to post that same wage since deviating away from that mass point would imply too large a change in size given decreasing returns to production. Thus, equilibria with a smooth wage distribution may in principle co-exist with equilibria with mass points, as highlighted in Mortensen and Vishwanath (1991).

Our second sufficient is an equilibrium refinement concept to overcome equilibrium multiplicity. We show that a trembling-hand refinement eliminates equilibria with mass points. If firms make small mistakes in their wage-setting policy, no mass point can arise. When dispersion in mistakes vanishes asymptotically so that we recover the maximization problem in (3), the only equilibrium that survives is the one with a smooth wage distribution. We formalize our trembling-hand refinement in Appendix A.6 and call it Assumption (C).

Proposition 4. *(Existence and uniqueness)*
There exists a unique equilibrium among equilibria with a continuous wage offer distribution. Under the equilibrium refinement in Assumption (C), Appendix A.6, this equilibrium is unique among all possible equilibria

Proof. See Appendix A.6. □

Having characterized the emergence of wage premia across firms in our baseline economy, we are now ready to introduce outsourcing.

2 A theory of outsourcing

In this section, we enrich our basic environment with contractor firms that provide outsourcing services and characterize how they affect the economy.

2.1 Contractor firms

There is a continuum of identical contractor firms in every skill market s. To make the distinction clear, we now call firms that produce a consumption good 'traditional firms.' Contractor firms hire workers in the same frictional labor market as final good producers, also by posting wages. A given contractor firm hires in a single skill market s, and faces constant returns in production for simplicity. There is perfect competition in rental markets for labor services. The equilibrium rental price of one efficiency unit of labor of skill s is denoted p_s.

We model a wedge $\tau_s < 1$ between the price p_s and the revenue that contractor firms earns when a traditional firm buys one unit of labor from them. Contractor firms earn profits

$$\pi^C(w) = \big(\tau_s p_s - w\big) n_s(w).$$

We propose two micro-foundations for τ_s, detailed in Appendix A.7. Regardless of the micro-foundation, the wedge τ_s is an exogenous parameter that captures how costly it is to outsource workers. In our first micro-foundation, τ_s simply reflects a parameter, the inverse of an iceberg trade cost between contractor firms and traditional firms. This trade cost captures the idea that communication, monitoring and coordination between the traditional firm and outsourced workers may be more difficult when workers are employees of another firm. As a result, some efficiency units of labor are lost. In our second micro-foundation, contractor firms combine a small amount of capital and labor according to a Cobb-Douglas production function. τ_s then simply encapsulates the equilibrium price of capital. Equivalently, $1/\tau_s$ may be interpreted as the markup charged by contractor firms.

Finally, we specify how the supply of outsourcing services is determined. To keep the exposition as simple as possible in the main text, we assume that contractor firms face a free entry condition:

$$0 \geq \max_{w} \big(\tau_s p_s - w\big) n_s(w). \tag{4}$$

Given constant returns for contractor firms, free entry is equivalent to costless recruitment activities. Thus, free-entry (4) is equivalent to contractor firms operating a different hiring technology than traditional firms. This assumption is natural. The core activity of contractor firms is precisely to hire workers and sell their labor services to other firms. In contrast, the core activity of traditional firms is to produce a particular good. For traditional firms, hiring activities divert resources away from production because of span of control limits in recruiting. Although intuitive, our free entry condition (4) is not necessary for our main results to hold. We show in Appendix A.9 that relaxing (4) does not affect our main conclusions.

2.2 Traditional firms and outsourcing

Traditional firms now face an additional possibility to buy labor. They may still hire workers in-house in a frictional labor market. The other possibility is now to rent labor services. Their decision problem becomes

$$\pi(z) = \max_{\{n_s\}_s, \{w_s\}_s, \{o_s\}_s \in \{0,1\}^S} R\big(z, \{n_s\}_s\big) - \int \Big[(1-o_s)w_s + o_s p_s\Big] n_s ds \quad \text{s.t.} \quad n_s \leq n_s(w_s) \text{ if } o_s = 0. \tag{5}$$

The indicators $o_s \in \{0,1\}$ indicate whether a traditional firm outsources skill s. n_s denotes in-house labor if $o_s = 0$, and denotes outsourced labor if $o_s = 1$.

If the traditional firm hires in-house ($o_s = 0$), it effectively faces an upward-sloping labor supply curve embedded in the function of $n_s(w)$. Thus, a traditional firm with a large target size n_s ends up paying high wages in-house. In contrast, if the traditional firm outsources ($o_s = 1$), it faces a flat labor supply curve at price p_s. In that case, outsourcing is more advantageous due to the upward-sloping wage premia curve $n_s(w)$. However, when traditional firms target a small size n_s, the price of outsourcing exceeds in-house wages since the price p_s reflects both the wage paid to employees of contractor firms as well as the wedge τ_s. We formalize this discussion in Proposition 5.

Proposition 5. *(Outsourcing)*
Suppose that τ_s is high enough that there is some outsourcing in equilibrium for skill s. Then:

1. *Contractor firms pay the reservation wage $\underline{w}_s$.*
2. *The price of outsourcing is $p_s = \underline{w}_s/\tau_s$.*
3. *There exists a threshold productivity $\hat{z}_s$, such that outsourcing occurs if and only if $z \geq \hat{z}_s$.*
4. *The highest wage in the economy is capped by the price of outsourcing: $w_s(\hat{z}_s) \leq \underline{w}_s/\tau_s$.*
5. *If $\frac{\partial^2 R}{\partial n_s^2} < 0$, the previous inequality is strict, and revenue and labor inputs discretely jump up as firms outsource: $R(\hat{z}_s^+, \boldsymbol{n}^*(\hat{z}_s^+)) > R(\hat{z}_s^-, \boldsymbol{n}^*(\hat{z}_s^-))$ and $n_s(\hat{z}_s^+) > n_s(\hat{z}_s^-)$.*

Proof. See Appendix A.8. □

Proposition 5 characterizes how outsourcing shapes the labor market. First, contractor firms pay the lowest wage in the economy, the reservation wage $\underline{w}_s$. For outsourcing to arise in equilibrium, the price of outsourcing services must induce contractor firms and traditional firms that outsource to gain from trading. Therefore, wages paid to contractor workers must be lower than wages paid to in-house workers at the marginal outsourcing traditional firm. How much lower? The zero-profit condition (4) for contractor firms pins down the wages of contractors to the reservation wage $\underline{w}_s$.[7] If any wage $w > \underline{w}_s$ was posted in equilibrium by a contractor firm, it could make positive profits by lowering their wage offer by a small amount, thereby contradicting the zero-profit condition. This result reveals that outsourcing has distributional consequences by reallocating workers towards the lowest-paying firms in the labor market.

Second, the price of outsourcing is a simple markup $1/\tau_s > 1$ over the wage that contractor firms pay, $\underline{w}_s$. This result again follows directly from the free-entry condition of contractor firms (4).

Third, we obtain selection into outsourcing. More productive pay higher wage premia if they hire in-house since they have a larger target size. Thus, highly productive firms have the strongest incentives to outsource.

Fourth, the price of outsourcing is an effective wage cap in the labor market. Firms never pay wages above the price of outsourcing because they always have the option to outsource. Only the most productive firms outsource. Thus, outsourcing removes the highest-paying jobs from the job ladder. This result highlights that not only does outsourcing reallocates workers towards low-paying jobs, outsourcing also removes workers' best options from the labor market, thereby reinforcing its distributional consequences.

Fifth, the amount of efficiency units of labor hired by a traditional firm jumps up upon outsourcing. This increase in effective firm size arises because traditional firms equate the

[7] We show in Appendix A.9 that wages paid by contractors firms remain lower than wages of the marginal traditional firm without (4). We also show that implications 3, 4 and 5 of Proposition 5 continue to hold.

marginal cost of labor to its marginal product. When outsourcing, traditional firms switch from a convex to a linear labor cost curve. Thus, they are able to scale up, leading to a productivity effect.

2.3 Equilibrium

To complete the description of the equilibrium, consider the wage offer distribution. Proposition 5 states that every contractor firm posts the reservation wage. With some outsourcing in equilibrium, the wage offer distribution thus starts with a mass point at the reservation wage. Traditional firms $z \in [\underline{z}, \min_s \hat{z}_s]$ then behave similarly to the no-outsourcing economy. For these firms, $o_s = 0$, and the only change relative to Propositions 1, 2 and 3 stems from the number of workers they can attract and retain with a wage offer. These traditional firms are able to poach workers from contractor firms at the bottom of the job ladder, and no longer face competition from the most productive traditional firms who now outsource their labor services. Thus, their equilibrium rank in the job ladder is given by

$$\Upsilon_s(z) = \frac{\mathcal{V}_s + M\Gamma(z)}{\mathcal{V}_s + M\Gamma(\hat{z}_s)}, \quad \forall z \leq \hat{z}_s, \tag{6}$$

where $\mathcal{V}_s$ denotes the measure of wage offers by contractor firms. Their equilibrium size and wage then follows from replacing $\Gamma(z)$ in Propositions 2 and 3 by $\Upsilon_s(z)$.

The market clearing condition for labor services determines how many contractor firms find it profitable to operate:

$$M \int_{\hat{z}_s}^{\overline{z}} n_s(z) d\Gamma(z) = \kappa_s n_s^{\text{Out}} \frac{\mathcal{V}_s}{\mathcal{V}_s + M\Gamma(\hat{z}_s)}, \tag{7}$$

where, when evaluated at $z > \hat{z}_s$, $n_s(z)$ is the demand for outsourced labor services which follows from the traditional firm's first-order condition. $n_s^{\text{Out}} = \frac{e_s}{1+k_s}$ indicates how many workers contractor firms manage to recruit and retain per wage offer, from equation (2). κ_s is a dummy-type variable that depends on the type of micro-foundation we choose for outsourcing.[8]

2.4 Testable implications

The theory of outsourcing we have laid out delivers several testable implications that follow from Proposition 5. We test these implications in Section 3 below.

Our first implication is that firms with larger revenue spend relatively more on outsourcing services. Indeed, selection into outsourcing (Proposition 5.3) implies that more productive firms are more likely to outsource.

[8] κ_s is equal to τ_s when we micro-found the cost of outsourcing with an iceberg trade cost. κ_s is equal to 1 when we micro-found the cost of outsourcing with capital use.

Our second implication is that firms with lower costs of outsourcing produce more: the productivity effect of outsourcing. To understand how the productivity effect emerges in our theory, consider a firm z just below an outsourcing threshold $\hat{z}_s$. We anticipate Section 4 where we introduce idiosyncratic outsourcing costs $\tau_s(\varepsilon) \equiv \tau_s \times \varepsilon$. ε is a firm-specific outsourcing cost shock. Now consider a small shock $d\varepsilon$ so that the cost of outsourcing of the marginal firm drops to $\tau_s \times (\varepsilon - d\varepsilon)$. The marginal firm outsources skill s. As per Proposition 5.5, its share of expenditures on outsourcing increases together with its revenue. Together, these observations imply that when firms are subject to outsourcing cost shocks leading to a rise in outsourcing expenditures, their revenues rise too.

Our third implication is the wage penalty of outsourcing that we set out to rationalize with our theory: the distributional effect of outsourcing. Proposition 5.1 implies that contractor firms pay the lowest wages in the labor market. When traditional firms experience an idiosyncratic productivity shock dz and cross the outsourcing threshold $\hat{z}_s$, they outsource. When some of their workers transition into contractor firms, the wages of these workers drop.

3 Reduced-form evidence

In this section, we first describe our data. Second, we discuss aggregate trends in outsourcing in France. Then, we proceed to test our three main predictions: selection into outsourcing, the productivity effect, and the distributional effect. Finally, we rule out alternative explanations for outsourcing. We provide additional details in Supplemental Material D.

3.1 Data

We use a combination of administrative and survey data for France between 1996 and 2007. Our first data source is the near-universe of annual tax records of French firms (FICUS) that report balance sheet and income statement information. Among others, we observe total employment and wage bill, sales and purchases of intermediate inputs, from which we construct value added. However, this dataset does not break down purchases of intermediate inputs finely enough to isolate outsourcing expenditures.

Our second data source is a large, mandatory annual firm-level survey that provides a more detailed breakdown of firms purchases of intermediate inputs (EAE). Surveyed firms report expenditures on 'external workers'. External workers are employees of another firm, but that fall under a contracting agreement with the surveyed firm. Importantly, these workers are at least partially under the authority of the surveyed firm. We use expenditures on external workers as our measure of expenditures on outsourced workers. Finally, firms remain in the survey once they enter, which allows us to leverage the panel dimension of this data.

Our third data source consists of employer tax records that provide information on labor market outcomes for French workers (DADS). We use repeated cross-sections that cover the universe of French workers to construct employment and wages at the firm-occupation-year level. We also use a 4% representative panel that tracks workers throughout their labor market histories to study the wage penalty of outsourcing.

Our fourth data source are customs records for the universe of trade transactions. We observe imports and exports crossing the French border, at the product-country-firm-year level. We use this data to construct export demand shocks at the firm-level and exploit variation in firm scale.

We link these four data sources together using a common firm identifier. For our main empirical exercises at the firm level, we aggregate years into three periods 1997-1999, 2000-2002, 2003-2007 and keep only firms with at least ten in-house employees to limit measurement error in outsourcing expenditures. We stop our main analysis in 2007 because of a large change in classification, including a change in the structure of the firm-level survey that prevents us from measuring outsourcing expenditures directly in subsequent years. Our final sample consists of 173,547 firm-periods.

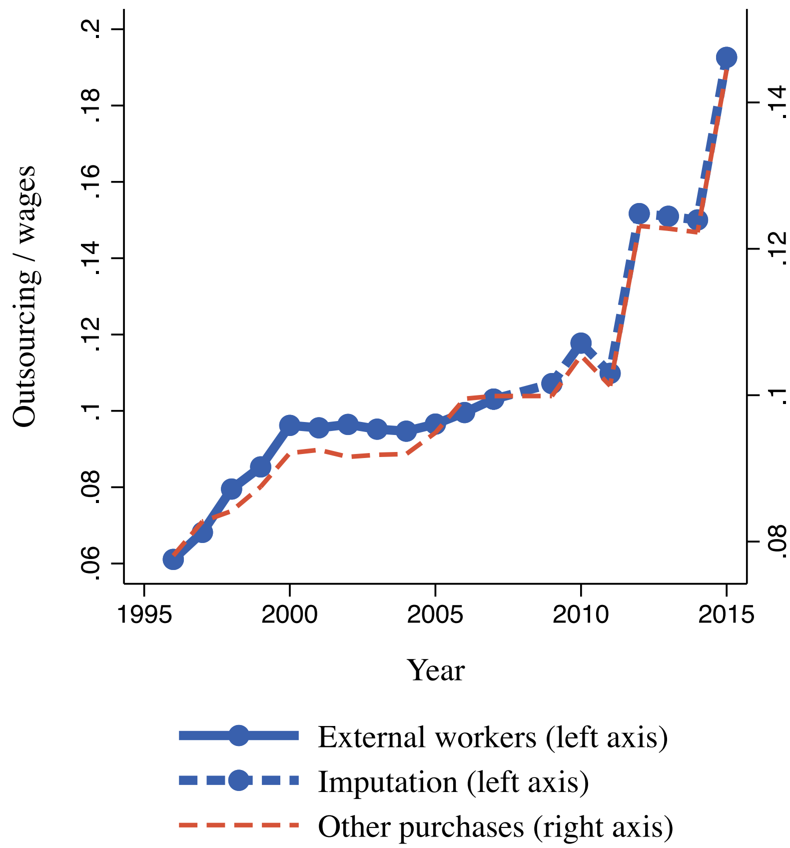
3.2 Aggregate trends in outsourcing

We start by asking how much did outsourcing rise in France. Figure 1 shows that outsourcing expenditures as a fraction of the aggregate wage bill almost doubled in the decade that we study: it increased from 6% in 1996 to 11% in 2007. We cannot reliably measure outsourcing expenditures directly in the following years. To infer whether the upward trend in outsourcing continues past 2007, we extrapolate using a slightly more aggregated income statement category that nests outsourcing expenditures. Figure 1 reveals that outsourcing may account for almost 20% of France's aggregate wage bill by 2015.

Does this rise in aggregate outsourcing expenditures translate into an aggregate increase in the employment share of contractor firms? To identify contractor firms in the data, we follow Goldschmidt and Schmieder (2017) and rely on industry and occupation codes to detect contractor firms and service workers at contractor firms.[9] Figure 5(a) in Appendix B shows the fraction of workers employed at contractor firms. The employment share at low-skill contractor firms rises from 5% in 1996 to 9% in 2007. Figure 5(b) in Appendix B reveals this increase is driven specifically by service workers reallocating towards contracting firms over time. We conclude that the rise in outsourcing arises not only in expenditures, but also in employment.

[9]To define a low-skill contractor firm, we use industry codes that specifically label firms as providing food, security, cleaning or general administrative services to other firms. To define a high-skill contractor firm, we use industry codes that label firms as providing accounting, law or consulting services to other firms. An important caveat to that approach is that it may miss any firm that is a contractor firm, but does not fall into those specific industry codes. Our measure using outsourcing expenditures is not subject to this limitation.

Figure 1: Outsourcing expenditures in France.



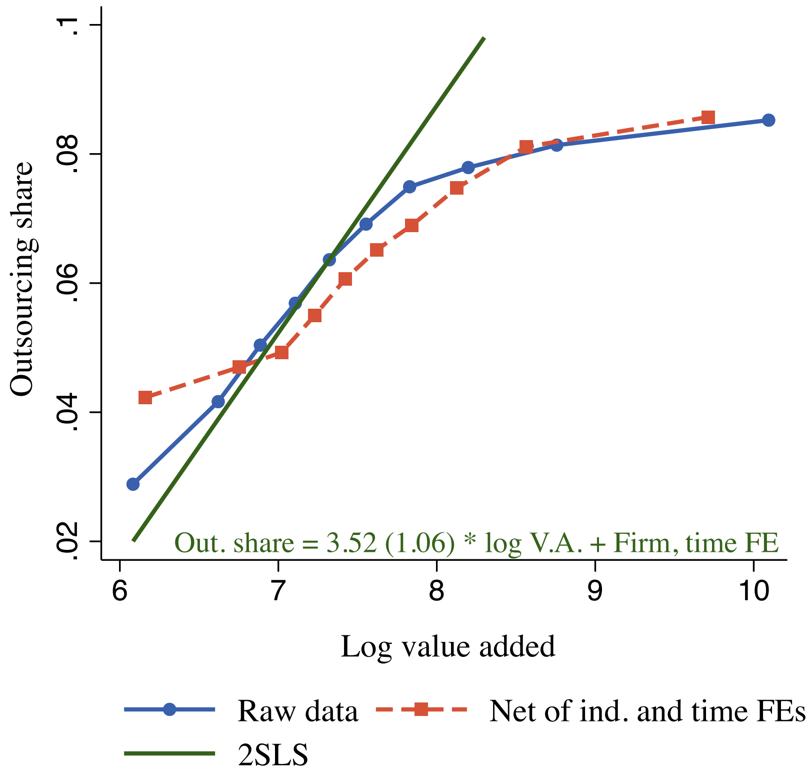
3.3 Selection into outsourcing

We start by testing the first core prediction of our theory: selection into outsourcing. Following Section 2.4, firms with larger revenues spend relatively more on outsourcing services. Consistently with our model that abstracts from intermediate inputs, we use value added as our main measure of firm revenues. We define the outsourcing share S_{ft} of firm f in time period t as its expenditures on external workers $\mathcal{E}_{ft}$ divided by the sum of its expenditures on labor $\mathcal{W}_{ft} + \mathcal{E}_{ft}$, where $\mathcal{W}_{ft}$ denotes the gross wage bill.

Figure 2 plots the outsourcing share by decile of value added. In both the raw data and the residualized relationship, Figure 2 reveals that high value-added firms tend to outsource more. A firm in the first decile of value added spends less than 4% of its labor costs on outsourced labor, while a firm in the tenth decile of value added spends over 8%. The residualized relationship reveals an S-shaped pattern, steeper at intermediate levels of value added and flatter at the extremes.

Of course, the relationship depicted in Figure 2 could be the result of unobserved firm-level heterogeneity that would affect both productivity and the ability to outsource. IT-intensity would be an obvious example. Therefore, we turn to a regression design in order to assess the

Figure 2: Outsourcing share by value added.



Note: Solid blue line : raw data. Dashed orange line: after removing 3-digit industry and time period fixed effects outsourcing share and log value added. Green line: 2SLS estimate using the export demand shift-share instrument in equation (9).

robustness of our results. We consider econometric specifications of the following form:

$$S_{ft} = \alpha_t + \beta_f + \gamma \log \mathrm{VA}_{ft} + \varepsilon_{ft}, \tag{8}$$

where α_t is a time period fixed effect, β_f a firm fixed effect, and ε_{ft} a mean zero residual.

Conditioning on firm fixed effects removes time-invariant unobserved confounders. To address time-varying confounders and exploit quasi-experimental variation in firm value added, we leverage the granularity of our customs data. We follow Hummels et al. (2014) and first construct firm-level export shares in the first time period, $\pi_{f,t_0,j}$, across 4-digit industry-country pairs j. We then interact those shares with export demand growth $\Delta \log X_{j,t,-f}$ in industry-country pair j between time periods t_0 and t, excluding firm f's exports. Our instrument is thus defined as

$$Z_{f,t} = \sum_j \pi_{f,t_0,j} \, \Delta \log X_{j,t,-f}. \tag{9}$$

To the extent that export demand growth at the industry-country level is orthogonal to firms' idiosyncratic ability to outsource, $Z_{f,t}$ is a valid instrument for firm value added.

Table 1: Relationship between firm-level outsourcing shares and firm-level value added.

	All				Exporters			
	(1) OLS	(2) OLS	(3) OLS	(4) OLS (P25,P75)	(5) OLS (P25,P75)	(6) 2SLS	(7) 2SLS	(8) 2SLS
Log Value Added	1.38***	1.50***	0.64***	2.05***	1.49*	3.52***		
	(0.03)	(0.03)	(0.06)	(0.31)	(0.61)	(1.06)		
Log Size							5.81***	
							(1.74)	
Log Labor Prod.								12.72**
								(4.34)
Fixed Effects								
Year	✓	✓	✓	✓	✓	✓	✓	✓
Industry		✓						
Firm			✓	✓	✓	✓	✓	✓
Obs.	173547	173540	173547	61549	14561	46152	46152	46152
Stand. coef.	0.18	0.19	0.08	0.27	0.19	0.46	0.75	1.65
1^{st}-stage F-stat.						267.88	193.09	53.75

Standard errors in parenthesis, clustered by firm. $^+$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Dependent variable: spending on external workers as a fraction of labor costs, in p.p. Variables winsorized at 5% level. Instrument: shift-share of export demand growth by 4-digit industry, projected by firm using firm-level export shares in first period. Restriction in columns (4-5): residual value added (net of year and firm fixed effects) between 25^{th} and 75^{th} percentile of its distribution. All regressions weighted by firm value added. Unweighted regressions in Table 7 in Appendix B.

Table 1 displays our results. Columns (1-2) show the regression analog of Figure 2. The estimate in columns (2) implies that a 100 log points increase in value added is associated with a 1.50 percentage point increase in the outsourcing share. Our estimate drops to 0.64 when we focus on within-firm variation in column (3). This reduction is consistent with either an important role for time-invariant confounders, or with a nonlinear relationship. Column (4) restricts attention to firm-time period pairs that are within the 25^{th} to 75^{th} percentiles of the distribution of changes in value added. Our point estimate rises to 2.05. We conclude that nonlinearities are a more likely driver of the lower coefficient in column (3) rather than time-invariant confounders.

Nonetheless, we then introduce our firm-level instrument. Since our instrument only affects exporters, we confirm that exporters exhibit a similar relationship between value added and outsourcing in column (5). Column (6) then shows our 2SLS estimate. The estimate rises to 3.52, but is not statistically different from the one in column (4). The solid green line in Figure 2 depicts our 2SLS estimate graphically. Similarly to column (4), the larger value of our 2SLS estimate is likely to also reflect the local nature of the 2SLS estimate that may concentrate the identifying variation in the steep part of the relationship between value added and the outsourcing share.[10]

[10] We verify the robustness of our results in several ways. We confirm our results with other metrics of firm

We thus conclude that firms select into outsourcing, with more productive firms outsourcing more. We now turn to our test of the productivity effect of outsourcing.

3.4 The productivity effect

Following Section 2.4, firms that outsource more for idiosyncratic reasons should be able to scale up and generate more revenues. Similarly to equation (8) in Section 3.3, we consider econometric specifications of the following form:

$$\log \mathrm{VA}_{ft} = \alpha'_t + \beta'_f + \gamma' S_{ft} + \varepsilon'_{ft}. \tag{10}$$

Relative to equation (8), equation (10) interchanges the dependent and independent variables. Of course, comparing the OLS estimates of equations (8) and (10) would simply amount to rescale the conditional correlation between both variables. Relative to equation (8), equation (10) conveys content only if we can isolate variation in the outsourcing share S_{ft} that arises due to exogenous shocks to firms' idiosyncratic cost of outsourcing.

Thus, we construct an instrument for the outsourcing share. Our goal is that it be orthogonal to firm-level changes in revenue productivity z. We leverage differential exposure of firms to service occupations: food, security, cleaning or general administrative occupations. We compute the within-firm employment share for each of these services occupations o in the initial period, ω_{f,o,t_0}. We then interact these firm-level initial employment shares with a measure of the change in aggregate outsourcing spending on occupation o, $\Delta \log \Omega_{o,t,-f}$, net of firm f's spending.[11] Our instrument is therefore defined by

$$Z'_{f,t} = \sum_o \omega_{f,o,t_0} \, \Delta \log \Omega_{o,t,-f}$$

Table 2 displays our results. Columns (1-2) first present OLS estimates for comparability. Column (2) again focuses on non-extreme values of residual changes in the outsourcing share. We find that a 1 percentage point increase in the outsourcing share is associated with a 2.7% increase in value added. Column (3) shows our 2SLS estimate. We find that a 1 percentage point increase in the outsourcing share implies a 5.1% increase in value added.[12]

From Table 2, we thus conclude that outsourcing has a positive productivity effect at the firm level. We now turn to our tests of the distributional effect of outsourcing.

performance such as size and value added per worker in columns (7-8) of Table 1. We demonstrate that weighting regressions by firm scale or not has virtually no impact on the results with Table 7 in Appendix B.

[11] To construct aggregate outsourcing spending on occupation o, we first infer outsourcing expenditures by occupation at the firm level by interacting initial employment shares ω_{f,o,t_0} with firm-level outsourcing expenditures $\mathcal{E}_{f,t}$. We then sum across firms to define $\Omega_{o,t,-f} = \sum_f \omega_{f,o,t_0} \mathcal{E}_{f,t}$. We then difference in time to remove the contribution of time t_0 expenditures and remove firm f's expenditures to obtain $\Delta \log \Omega_{o,t,-f}$.

[12] Columns (4-5) reveal that this increase in value added is approximately equally spread between an increase in labor productivity and an increase in in-house employment.

Table 2: Relationship between firm-level outsourcing shares and firm-level value added.

	Log VA			Log VA/N	Log N
	(1) OLS	(2) OLS (P25,P75)	(3) 2SLS	(4) 2SLS	(5) 2SLS
Outsourcing Share	0.007***	0.027***	0.051***	0.025***	0.023***
	(0.001)	(0.004)	(0.002)	(0.001)	(0.001)
Fixed Effects					
Firm	✓	✓	✓	✓	✓
Obs.	94336	47168	94336	94336	94336
Stand. coef.	0.030	0.117	0.224	0.109	0.103
1^{st}-stage F-stat.			18206.439	18206.439	18206.439

Standard errors in parenthesis, clustered by firm. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Variables winsorized at 5% level. Instrument: shift-share of outsourcing spending growth, projected by firm using firm-level occupation shares in first period. Restriction in column (2): residual outsourcing share (net of firm fixed effects) between 25^{th} and 75^{th} percentile of its distribution. Due to changes in the occupation classification in 2002, we lose the time period 1997-1999 and only have two time periods and thus no time period fixed effects. All regressions weighted by firm value added. Unweighted regressions in Table 8 in Appendix B.

3.5 The distributional effect

We propose three exercises that confirm the links between the structure of our theory and the distributional effect. First, we verify that service workers are indeed paid more at larger firms. Second, we show that contractor firms locate at the bottom of the job ladder by computing the contractor wage premium. Third, we use an event study design to measure the wage penalty of outsourcing.

We start by verifying that larger firms pay service workers more. We project log wages at the worker level on their employer's measure of scale, controlling for worker fixed effects to absorb worker-level heterogeneity that may be correlated with firm scale. Table 9 in Appendix B shows that larger firms indeed pay their service workers more. Conditional on worker effects, firms with 1,000 employees pay wages that are on average 9.6% higher than firms with 10 employees. We confirm our findings with other metrics of firm performance. We conclude that traditional employers with larger scale pay higher wages and locate at the top of the job ladder.

Second, we test whether contractor firms locate at the bottom of the job ladder. We measure the wage premium paid by contractor firms, using the industry codes we associate with contractor firms as in Section 3.2. We run a two-way fixed effects regression in the spirit of Abowd et al. (1999):

$$\log w_{i,t} = \varphi_i + \psi_{J(i,t)} + \eta_{i,t}. \tag{11}$$

i indexes workers, $J(i,t)$ the employer of worker i in quarter t, and η is a mean-zero residual. $\log w_{i,J(i,t),t}$ denotes the log wage, φ_i is a worker fixed effect, and $\psi_{J(i,t)}$ a firm fixed effect.[13]

We estimate the standard deviation of firm effects to be 0.14. We compute the mean firm effect for contractor firms that we identify based on industry codes. We find that the mean contractor firm wage premium is -0.12 relative to the mean firm wage premium normalized to 0. Thus, contractor firms pay wages that are almost one standard deviation of firm wage premia below the average firm wage premium. We conclude that, consistently with our theory, contractor firms indeed pay wages towards the bottom of the job ladder.

Third, we measure the outsourcing wage penalty directly with an event study. In contrast to our measure of contractor wage premia above, the goal of our event study is to identify workers who explicitly transition between a traditional employer and a contractor. Similarly to Goldschmidt and Schmieder (2017), we define outsourcing events as a combination of declines in in-house employment, rise in outsourcing expenditure and movements of cluster of workers to a new employer.[14] We consider an econometric specification of the form:

$$\Delta^2 \log w_{i,t} = \delta_{o(i)} + \mu \mathcal{O}_{i,t_0(i)} + \nu \mathcal{J}_{i,t_0(i)} + \upsilon_{i,t}. \tag{12}$$

t_0 denotes the year of the outsourcing event, $\mathcal{O}_{i,t_0(i)}$ is a dummy variable that equals one if the worker is an outsourcing event in year $t_0(i)$. $\mathcal{J}_{i,t_0(i)}$ denotes an indicator that equals one if worker i changes employer in year $t_0(i)$ to control for the possible common effect of switching employer. $\Delta^2 \log w_{i,t}$ denotes wage growth between year t and year $t_0(i)$. We use the notation Δ^2 to indicate that we also remove worker-specific linear trends in wages that may confound our results. We estimate those trends using only years preceding the outsourcing event $t < t_0$. Finally, $\delta_{o(i)}$ are two-digit occupation fixed effects that capture occupation-specific trends.

[13] As in Engbom and Moser (2021), when λ^E_s/δ_s is independent from s, $b_s = ba_s$, and the revenue function is linear, the wage formula in Proposition 3 is log-additive in a worker effect and a firm effect. In addition, worker mobility is then conditionally random as in Card et al. (2013). Together, these observations imply that equation (11) can be consistently estimated by OLS. Yet, when the aforementioned assumptions are relaxed, our theory's wage equation is not log-additive. In this case, while no longer exact, equation (11) still provides a useful diagnostic device to measure the average wage premium paid by a firm. Estimating the full distribution of worker and firm effects in equation (11) leads to the well-known limited mobility bias. Hence, we follow the clustering approach developed in Bonhomme et al. (2019). We group workers and firms each in 50 equally populated groups, based on the unconditional mean worker and mean firm wage. We then estimate equation (11) with OLS at the group level. Our results are virtually identical when varying the number of groups between 10 and 200.

[14] Specifically, we define an 'outsourcing event' at firm f and occupation o in year t for worker i if the following conditions are met. First, the employment share of occupation o at firm f drops by at least 25% between year t and year $t+1$. Second, outsourcing expenditures rise at firm f rise by at least 50% of the corresponding wage bill reduction, between t and $t+1$. Third, worker i transitions to a firm f' within one year. Fourth, at least 10% of firm switchers from firm f also move to f' within the year. The first two conditions ensure that firm f undergoes a large enough change change in its occupational structure at the same time as its spends more on outsourced labor. The third and fourth conditions isolate events in which a large enough group of workers transitions to the same destination employer as in Goldschmidt and Schmieder (2017). We use a lower threshold than them to maximize statistical power, as we must work with a 4% panel to track workers across years.

Table 3: Wage penalty from outsourcing.

	Post-Outsourcing		Pre-Outsourcing	
	(1)	(2)	(3)	(4)
Outsourcing Event	-0.140***	-0.119**	-0.006	-0.005
	(0.037)	(0.039)	(0.006)	(0.006)
Employer Switch		-0.026***		-0.001
		(0.007)		(0.001)
Fixed Effects				
2-digit occupation	✓	✓	✓	✓
Obs.	874650	874650	1060032	1060032

Note: Standard errors in parenthesis, clustered by 2-digit occupation. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Due to changes in the occupation classification in 2002, we lose the time period 1997-1999 and only have two time periods and thus no time period fixed effects.

Table 3 displays our results. In column (1), we find that an outsourcing event is associated with a 14% wage decline on average over the subsequent four years. Since some of that estimate may be driven by displacement-like effects, we also control for the common effect of switching employers in column (2). Our coefficient of interest declines modestly to 12% and remains statistically significant. Interestingly, it coincides almost exactly with the negative contractor firm wage premium that we estimated in the two-way fixed effect specification (11). Columns (3-4) show that there is no evidence of pre-outsourcing effects. We conclude that outsourcing is associated with a substantial wage penalty.

3.6 Alternative explanations

In principle, mechanisms absent from the model may also lead firms to outsource. In this final subsection, we rule out two prominent alternative explanations as key drivers of outsourcing.

The first alternative explanation is that firms outsource because contractor firms have a comparative advantage in producing services of a particular type. For instance, security contractor firms may thrive because they are more efficient at providing security services than traditional firms with the same workers. However, Proposition 5 in combination with the observed outsourcing wage penalty rules out this comparative advantage motive. In the model, this conjecture manifests itself by imposing $\tau_s > 1$. In this case, the price of outsourcing services is $p_s = \underline{w}_s/\tau_s < \underline{w}_s$, leading to three counterfactual implications. First, all traditional firms would outsource their workers of skill s. By contrast, in the data, only few firms outsource. Second, there is no selection into outsourcing. This implication is at odds with our reduced-form results in Section 3.3 that find strong evidence of selection into outsourcing. Third, the outsourcing wage penalty is zero at best. As τ_s rises to become larger than 1, low-productivity traditional firms are the last to hire in-house. As τ_s approaches 1 from below, the outsourcing wage penalty vanishes. This implication contradicts our reduced-form results in Section 3.5 that find a substantial outsourcing wage penalty.

The second explanation is that firms outsource to avoid labor hoarding in the face of volatile demand. If demand fluctuates strongly quarter-to-quarter, contract labor may be more easily adjusted to meet this time-varying demand than in-house employment. Perhaps surprisingly, this explanation is at odds with the data along two dimensions. First, small firms are more volatile as shown in Figure 6(a) in Appendix B. If anything, we should then expect a negative relationship between firm scale and outsourcing. The data clearly points to the opposite. Second, we rank industries by value added volatility, and check whether more volatile industries rely more on outsourcing. Figure 6(b) in Appendix B shows that, if anything, the opposite holds in the data.

To summarize, this section has proposed reduced-form evidence supporting the key predictions of our theory: selection into outsourcing, productivity and distributional effects. Having validated the core structure of our theory of outsourcing, we now turn to our general equilibrium quantitative exercises.

4 Extended model and estimation

In this section, we start by presenting the extended version of the model from Section 2 that we take to the data. We then discuss our estimation strategy.

4.1 Quantitative setup

We start by discussing the elements we add to the setup of Section 2. Next, we describe the functional forms we use.

We start by introducing firm-skill-specific costs of outsourcing, ε_s, that enter the profit-maximization problem of the firm (16) below. We assume that these costs are independent across firms and skills, but may be correlated with productivity z. Denote by $\Gamma(z, \boldsymbol{\varepsilon})$ the joint cumulative distribution function of $(z, \boldsymbol{\varepsilon} \equiv \{\varepsilon_s\}_s)$. We also let traditional firms post any number of vacancies v in market s at a convex cost $c(v) = c_{0s} v^{1+\gamma}$ for $\gamma > 0$. When $\gamma \to +\infty$ we recover the model of Section 2. Since not all traditional firms post the same number of vacancies in equilibrium, the number of workers a traditional firm attracts and retains reflects its vacancy share:

$$n_s(w, v) = \frac{(1 + k_s) e_s}{\big(1 + k_s(1 - F_s(w))\big)^2} \cdot \frac{v}{V_s}, \tag{13}$$

where the equilibrium number of vacancies in market s satisfies

$$V_s = \mathcal{V}_s + M \sum_{\boldsymbol{o}:o_s=0} \int v_s(z, \boldsymbol{\varepsilon}, \boldsymbol{o}) \Omega(\boldsymbol{o}|z, \boldsymbol{\varepsilon}) \Gamma(dz, d\boldsymbol{\varepsilon}). \tag{14}$$

$v_s(z, \boldsymbol{\varepsilon}, \boldsymbol{o})$ denotes the equilibrium number of vacancies posted by a firm with productivity z, outsoucing costs $\boldsymbol{\varepsilon}$ and decision $\boldsymbol{o} = \{o_s\}_s \in \{0,1\}^S$. $\Omega(\boldsymbol{o}|z, \boldsymbol{\varepsilon})$ denotes the share of firms with productivity z and outsoucing costs $\boldsymbol{\varepsilon}$ that choose the outsourcing bundle $\boldsymbol{o}$.

Equation (13) is the analog of equation (2) when vacancies vary across firms. The first term on the right-hand-side is identical to that in equation (2), having anticipated that the wage offer distribution has no mass points except at the reservation wage. The second term on the right-hand-side replaces $1/M$ in equation (2), and is the firm's vacancy share.

As opposed to the simpler setup in Section 2, the equilibrium number of workers that traditional firm z attracts and retains cannot be expressed as a function of $\Gamma(z, \boldsymbol{\varepsilon})$ alone because traditional firms with different outsourcing bundles $\boldsymbol{o}$ may post different wages even at the same productivity z. Instead, this number of workers is a function of $\Upsilon_s(z, \boldsymbol{\varepsilon}, \boldsymbol{o}) = F_s(w_s(z, \boldsymbol{\varepsilon}, \boldsymbol{o}))$, where the wage offer distribution F_s solves

$$F_s(w) = \sum_{\boldsymbol{o}:o_s=0} \int \mathbb{1}\{w_s(z, \boldsymbol{\varepsilon}, \boldsymbol{o}) \leq w\} v_s(z, \boldsymbol{\varepsilon}, \boldsymbol{o}) \Omega(\boldsymbol{o}|z, \boldsymbol{\varepsilon}) \Gamma(dz, d\boldsymbol{\varepsilon}). \tag{15}$$

With the additions discussed above, traditional firms solve:

$$\begin{aligned} \pi(z, \{\varepsilon_s\}_s) \quad &= \max_{\{n_s\}_s, \{v_s\}_s, \{w_s\}_s, \{o_s\}_s} R\big(z, \{n_s\}_s\big) - \sum_{s=1}^{S} \Big\{ \big[(1-o_s)w_s + o_s p_s \varepsilon_s\big] n_s + (1-o_s)c(v_s) \Big\} \\ &\text{s.t.} \quad n_s = n_s(w_s, v_s) \text{ as per (13) if } o_s = 0. \end{aligned} \tag{16}$$

Idiosyncratic outsourcing costs ε_s enter in the cost of hiring n_s units of labor services. This cost becomes $p_s \varepsilon_s n_s$. We interpret expenditures on outsourcing as the value of the transaction with the contractor firm, $p_s n_s$. We interpret the remainder of the cost $p_s(\varepsilon_s - 1)n_s$ as a running capital cost for the outsourcing firm.

We now specify functional forms. We impose a Cobb-Douglas revenue function nested in a decreasing returns upper tier $R\big(z, \{n_s\}_s\big) = \Big(z \prod_{s=1}^{S} n_s^{a_s}\Big)^{\rho}$, where $\sum_{s=1}^{S} a_s = 1$. We assume that the matching function is Cobb-Douglas:

$$\mathcal{M}_s = \mu_s (m_s(u_s + \zeta_s(1-u_s))^{\xi} V_s^{1-\xi}. \tag{17}$$

The parameter ζ_s denotes the relative search intensity of employed workers, so that $\lambda_s^E = \zeta_s \lambda_s^U = \frac{\zeta_s \mathcal{M}_s}{m_s(u_s + \zeta_s(1-u_s))}$. μ_s is the matching efficiency in market s.

We parametrize the joint distribution of productivity and outsourcing costs $(z, \boldsymbol{\varepsilon})$ as jointly lognormal with respective standard deviations ν, σ and correlation ι. We normalize the log means to zero as they are not separately identified from $\{\tau_s\}_s$ and $\{b_s\}_s$.

To limit the strength of congestion externalities in the matching function that are not the focus of this paper, we assume that the shifter of the vacancy cost is proportional to the ratio of in-house to total vacancies: $c_{0s} = \frac{1}{1+\gamma}\left(\frac{V_s}{V_s - \mathcal{V}_s}\right)^{1+\gamma}$.[15] This specification captures that labor market competition may be somewhat segmented between contractors and traditional firms. It also encapsulates the advertising view of recruiting activities. As traditional firms lower their amount of recruiting effort, it becomes easier for any other firm to attract workers.

We solve the model for two skill types. To focus on low-skill outsourcing, we impose that high-skill workers are never outsourced $\tau_2 = +\infty$. We relegate additional derivations to Supplemental Material F.1 and computation details to Supplemental Material F.2. Having laid out the structure of the extended framework, we turn to the estimation strategy.

4.2 Estimation strategy

We set a quarterly frequency. We then define skill groups as revealed by the occupations of workers. We rank 2-digit occupations by their average wage. We compute the mean predicted wage of a worker by interacting the mean occupational wages and the time spent by the worker in each occupation. A worker is low-skill if their predicted wage is below median, and high-skill if above. We use non-employment as our primary measure of 'unemployment' in the model, since a large fraction of steady-state flows into employment stem from individuals officially out of the labor force.

We then estimate the model in three steps. We estimate a first group of parameters that can be directly mapped to data. Second, we set one parameter to a specified value. In the third step, we estimate the remaining parameters jointly with a MSM estimator.

First, we identify the parameters $\{\delta_s, \zeta_s\}_{s=1}^{2}$ from labor market flows and the measure of firms M from firm size. The employment-to-non-employment transition rate EN_s is equal to the job losing rate parameter δ_s. In addition, in the extended model of Section 4.1, all matches are viable due to the Inada property of the revenue function. Therefore, all meetings from non-employment result in a viable match, implying that the time-aggregated non-employment-to-employment transition rate NE_s is equal to the endogenous offer rate λ_s^U from non-employment. We then relate the employment-to-employment transition rate EE_s to underlying arrival rates. We show in Supplemental Material F.3 that

$$\frac{\mathrm{EE}_s}{\mathrm{EN}_s} = \frac{(1+k_s)\log(1+k_s) - k_s}{k_s}.$$

Since $k_s = \lambda_s^E/\delta_s$, this relationship immediately determines the endogenous offer rate from employment, λ_s^E. Thus, we recover $\zeta_s = \lambda_s^E/\lambda_s^U$. We choose M to match average firm size.

[15]For a discussion of congestion externalities in a wage-posting setting, see Fukui (2020).

Second, we set the elasticity of the matching function ξ to a pre-specified value. Since our data does not let us credibly estimate it, we set to $\xi = 0.5$, a central value found in the literature as reviewed by Petrongolo and Pissarides (2001).

Third, we jointly estimate the remaining parameters $\{\mu_s\}_{s=1}^2, \{b_s\}_{s=1}^2, a_2, \rho, \gamma, \tau_1, \nu, \sigma, \iota$ by MSM. While the parameters are of course jointly identified, we provide an heuristic argument that describes how the moments we choose inform parameters. We confirm our argument numerically in Figure 7 in Appendix C.

Inspection of equation (17) reveals that the matching function efficiency for skill s, μ_s, has a direct impact on the non-employment-to-employment transition rates NE_s which we target for each skill. By shifting unemployment benefits conditional on wages, the parameters $\{b_s\}_{s=1}^2$ affects the replacement rate for each skill type which we target. We inform the relative demand for high-skill workers a_2 using the skill premium. The curvature in the revenue function ρ shifts average profits in the economy conditional on wages, and so we target the aggregate labor share.

Given ρ, γ and ν then jointly determine the dispersion in size and value added. When there is less curvature in the vacancy cost, productive firms are able to hire more and dispersion in firm size increases. Conditional on size, the dispersion in productivity raises dispersion in value added. Thus, we target the standard deviation of log firm size to inform γ, and the standard deviation of log value added to inform ν.

The outsourcing cost τ_1 pins down how much outsourcing there is in the economy by directly entering the price of outsourcing as per Proposition 5. We target aggregate outsourcing expenditures as a fraction of wages in 1996 to pin down the common level of τ_1. The dispersion in outsourcing costs σ is related to the OLS coefficient in column (2) in Table 1: more dispersion implies a steeper relationship. Importantly, we target a cross-sectional moment as our model is a steady-state model. To inform the correlation between productivity and outsourcing costs ι, we target the outsourcing employment share. The outsourcing expenditure share does not directly involve outsourcing costs ε. By contrast, when traditional firms purchase labor services, their idiosyncratic outsourcing costs ε directly enter their decision through their first-order condition. A higher cost ε leads to less labor services. As the correlation between productivity z and costs ε rises, productive firms who would otherwise purchase a large amount of labor services reduce their demand for labor services. Overall demand for outsourcing services falls, and so does the outsourcing employment share.

We use a loss function with squared proportional deviations for each moment, weight moments equally and use a gradient descent algorithm to find the minimum. We provide more details in Supplemental Material F.4.

Table 4: Parameter estimates and empirical targets.

Parameter	Interpretation	Target	Empirical Moment	Simulated Moment	Parameter Estimate
		A. Parameters from direct inversion.			
δ_1	Job loss rate low-skill	EN rate low-skill	0.04		0.04
δ_2	Job loss rate high-skill	EN rate high-skill	0.03		0.03
ζ_1	Rel. search. emp. low-skill	EE rate low-skill	0.04		0.75
ζ_2	Rel. search. emp. high-skill	EE rate high-skill	0.03		0.68
ζ_2	Rel. search. emp. high-skill	EE rate high-skill	0.03		0.68
M	Measure of firms	Average firm size	8.31		0.10
		B. Parameters from MSM estimator.			
μ_1	NE rate low-skill	NE rate low-skill	0.17	0.17	0.61
μ_2	NE rate high-skill	NE rate high-skill	0.17	0.17	0.57
a_2	Rel. prod. high-skill	Skill premium	1.74	1.73	1.90
b_1	Unemp. benefits low-skill	Replacement rate low-skill	0.70	0.71	0.19
b_2	Unemp. benefits high-skill	Replacement rate high-skill	0.70	0.70	0.32
ρ	Curvature in revenue	Labor share	0.70	0.68	0.80
γ	Curvature vac. cost	St dev. log firm size	0.98	0.97	4.69
τ	Outsourcing cost	Out. share (spending)	0.06	0.06	0.20
ν	Standard dev. prod. z	St. dev. log VA	1.14	1.12	0.26
σ	Standard dev. out. costs ε	VA elasticity of out. share	1.50	1.53	0.58
ι	Correlation btw. z and ε	Out. share (employment)	0.05	0.05	0.13

4.3 Estimation results and identification

Table 4 summarizes our estimation results. Our parameter estimates fall within conventional ranges found in the literature. The revenue function curvature parameter $\rho = 0.80$ translates into an elasticity of substitution between varieties of about 5. Our estimate of a_2 implies that high-skill workers' marginal product of labor is twice as large as low-skill workers' at equal employment shares. The curvature in the vacancy cost $\gamma = 4.69$ lies within conventional values.

Our estimate of outsourcing costs $\tau_1 = 0.2$ implies that, for the average firm with $\varepsilon = 1$, outsourcing low-skill labor implies a markup around 5 over the wages paid to the workers providing the labor services. Of course, only firms with a particularly low idiosyncratic outsourcing cost ε will select into outsourcing. The effective markup $\mathbb{E}[p\varepsilon/\underline{w}_1|o = 1]$ is much lower and equal to 1.15.

How well are parameters identified? To answer this question, Figure 7 in Appendix C plots both the simulated moment (univariate identification) and the loss function (multivariate identification) as we vary the parameter close to its estimated value. Most of the parameters are well identified locally, the moment deviation being steep as a function of parameter deviations, and the loss function being peaked around 0.[16] Overall, Figure 7 confirms our identification argument and supports our estimation strategy.

4.4 Over-identification

We propose three over-identification checks that relate to our theory's three main predictions. The first relates to selection into outsourcing. The second relates to the productivity effect. The third relates to the distributional effect.

We start by verifying whether the estimated model accounts for selection into outsourcing. We have targeted the cross-sectional OLS coefficient in column (2) from Table 1 to inform σ. Our reduced-form analysis has however shown that focusing on within-firm changes (column (3)) as well as instrumenting for firm revenue productivity (column (6)) affects the coefficient. While these are non-targeted moments, can the estimated model rationalize these differences?

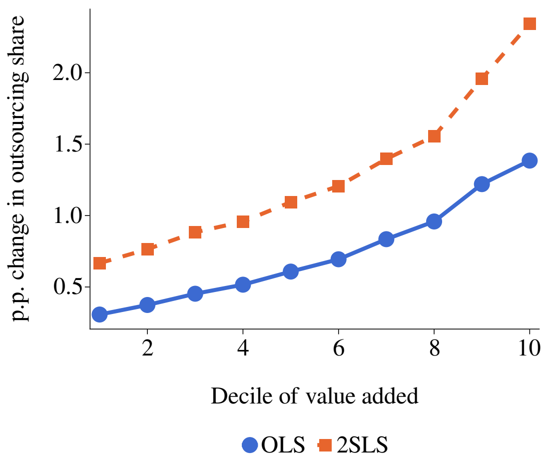
The blue line in Figure 3(a) displays the model equivalent of the coefficient in column (3), from the following experiment. Consider a one standard deviation shock to revenue productivity z. Because z and ε are positively correlated—more productivity firms face larger outsourcing costs—the increase in z is also associated with an average increase in ε. For every firm, we compute the change in the outsourcing share following the joint change in (z, ε) and project it on the associated change in value added. We then display the resulting OLS coefficient in the model by decile of initial value added. When aggregating across all deciles, we obtain an average coefficient of 0.7, nearly identical to our point estimate of 0.6 in Table 1.

In the model, the within-firm OLS coefficient conflates the change in revenue productivity z with the associated change in outsourcing costs ε. We mimick the instrumental variable strategy from Table 1, column (6) as follows. We interpret the export demand instrument as removing the increase in ε associated with the increase in z. We then only shift z to compute the change in the outsourcing share and value added instead of the joint shift in (z, ε). The orange line displays our results. Consistently with the data, the model counterpart of the 2SLS estimate is much larger than the OLS estimate. This ordering occurs because of the positive correlation between z and ε. When revenue productivity z rises alone, firms are more inclined to increase outsourcing than when the cost of outsourcing increases simultaneously. Quantitatively, the model somewhat under-predicts the magnitude of the 2SLS coefficient which lies between 0.07 and 0.25 depending on the incidence of shocks. Given that these moments are non-targeted, we nonetheless conclude that the estimated model provides an empirically plausible account of

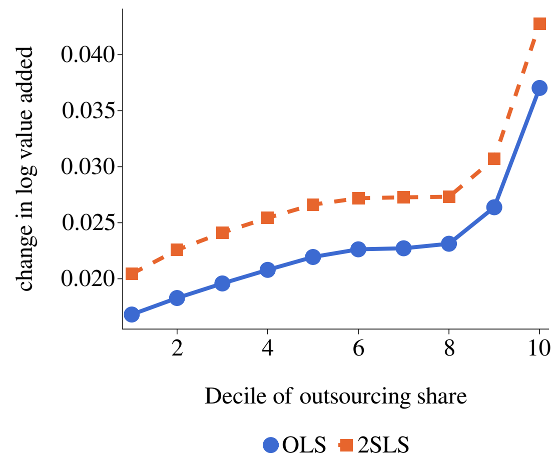
[16] Lest we simulate the model on a full twelve-dimensional hypercube, we cannot guarantee global identification.

Figure 3: Selection into outsourcing and the productivity effect in the estimated model.

(a) Selection into outsourcing.



(b) Productivity effect.



Note: Panel (a): OLS (solid blue) and 2SLS (dashed orange) coefficients in the estimated model. OLS coefficient computed by projecting the change in the outsourcing share on the change in log value added, following a one standard deviation Δz increase in z and the corresponding change in $\Delta\varepsilon = \frac{\iota\sigma}{\nu}\Delta z$. 2SLS coefficient computed by only increasing z by one standard deviation. Panel (b): OLS (solid blue) and 2SLS (dashed orange) coefficients in the estimated model. OLS coefficient computed by projecting the change in log value added on the change in the outsourcing share, following a one standard deviation $\Delta\varepsilon$ decrease in ε and the corresponding change in $\Delta z = \frac{\iota\nu}{\sigma}\Delta\varepsilon$. 2SLS coefficient computed by only increasing ε by one standard deviation.

selection into outsourcing.

Our second over-identification exercise asks whether the estimated model accounts for the untargeted productivity effect. We mimick the OLS and 2SLS coefficients from Table 2 similarly to selection into outsourcing. We consider a negative one standard deviation shock to firms' idiosyncratic outsourcing cost ε. We then decrease revenue productivity z accordingly for the OLS coefficient, or leave it unchanged for the 2SLS coefficient.

Figure 3(b) displays the within-firm model counterparts of the OLS and 2SLS coefficients from Table 2, columns (1) and (3), by initial decile of firm value added. Qualitatively, the estimated model aligns with the data. Quantitatively however, the estimated model somewhat overpredicts the OLS coefficient, while under-predicting the 2SLS coeffient. In the model, the OLS coefficient is just over 2%, while closer to 1% in the data. The 2SLS coefficient lies between 2% and 4% in the model, while reaching 5% in the data. Nevertheless, we conclude that the estimated model accounts for the productivity effect.

Our third over-identification exercise verifies whether the estimated model accounts for the distributional effects of outsourcing. The distributional effect depends on two key moments. The first moment is the steady-state dispersion in firm wage premia. Our estimation strategy has imposed no direct restrictions on wage dispersion. To estimate firm wage premia consistently with our theory, we run equation (11) similarly to Section 3.5 but separately by skill group. We estimate the within-skill standard deviation of firm wage premia to be 0.14 in the data. In our estimated model, the within-skill standard deviation of log wage premia is 0.13. We conclude that the estimated model accounts for observed residual wage dispersion.

The second moment that determines the distributional effect is the outsourcing wage penalty. Our estimation strategy only targets expenditures on outsourcing by traditional firms, and does not restrict the wage gap between traditional and contractor firms. Using an event study, we highlighted in Section 3.5 that outsourced workers lose on average 12% relative to pre-outsourcing wages. We replicate the event study in the estimated model. Doing so necessarily involves additional assumptions. We consider traditional firms that start in an initial period with an idiosyncratic outsourcing cost ε. We then assume that they draw a new cost ε' in a second period, and choose their preferred production structure anew. We assume that the new cost ε' is independent from the initial draw ε and their productivity z. Depending on the new cost, traditional firms may lay off workers to rent labor services from contractor firms. Finally, we assume that the measure of firms who re-draw these shocks is small enough that workers who lose their job due to an outsourcing event transition into contractor firms.

Armed with those assumptions, we compute the outsourcing wage penalty in the estimated model. We find that outsourced workers' wages drop by 11% on average. We conclude that the estimated model accounts for the outsourcing wage penalty. Having shown that the estimated model replicates our three key implications, we turn to our main counterfactual exercises.

5 The effects of outsourcing on inequality and output

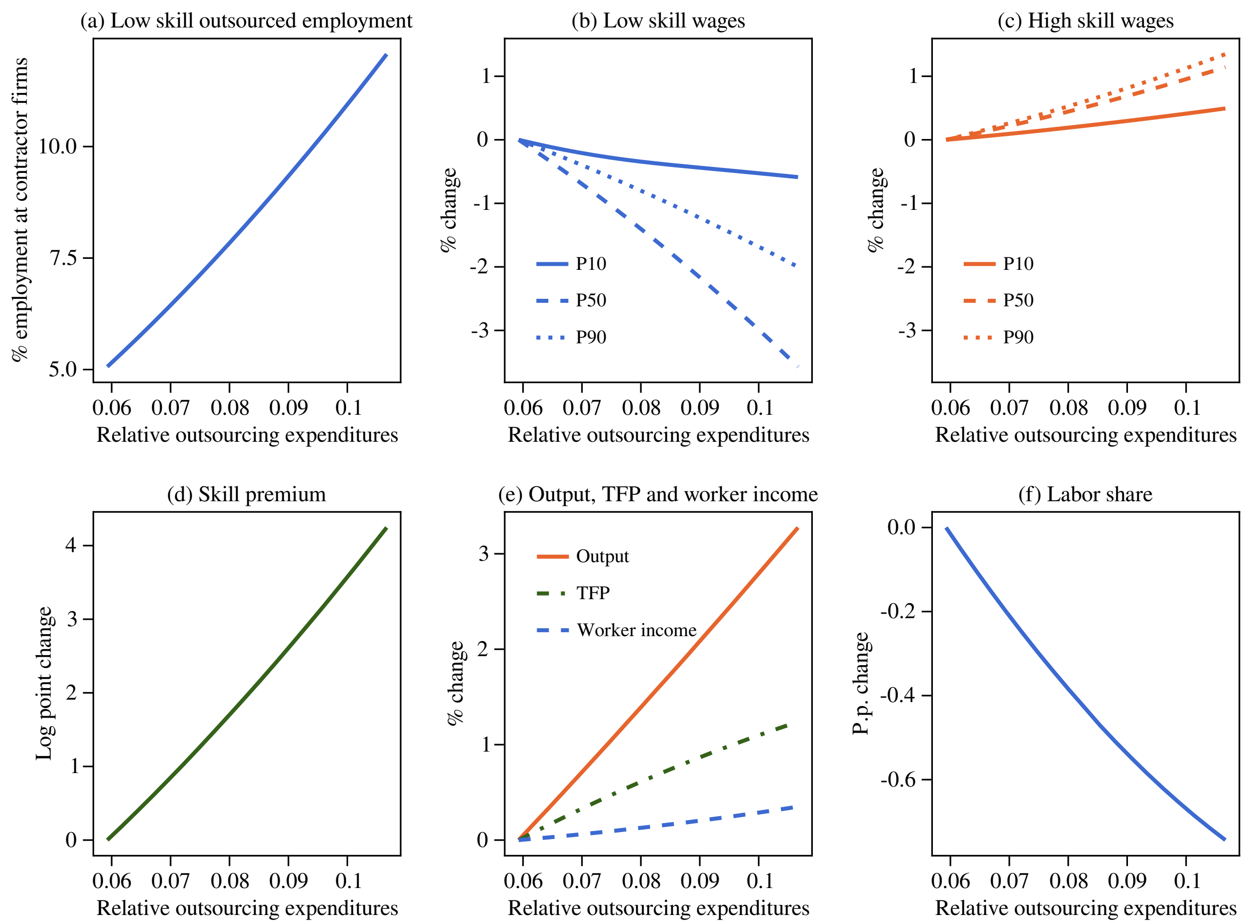
In this section, we present our general equilibrium results describing the aggregate effects of outsourcing.

5.1 The aggregate effects of outsourcing

We investigate the effect of the rise in outsourcing on inequality and aggregate output. To do so, we lower the cost of outsourcing $1/\tau$ in order to replicate the increase in the aggregate expenditure share on outsourcing from Figure 1 between 1996 and 2007. We compare steady-states of the estimated model, and interpret our results as the effect of outsourcing on the French labor market in that decade. Figure 4 displays the results.

Panel (a) shows the fraction of workers employed at contractor firms. As the cost of outsourcing falls, the fraction of low-skill workers at contractor firms rises from 5% to almost 12%. While non-targeted, this increase tracks the one observed in the data in Figure 5(a). This reallocation of workers towards contractor firms preludes to the adverse partial equilibrium impact of outsourcing on workers' expected earnings.

Figure 4: The effects of outsourcing on inequality and output.



Note: Counterfactual steady-state economies as the outsourcing cost $1/\tau$ falls. Change in τ calibrated to match the rise in outsourcing expenditure share in France from 1996 to 2007. All outcomes are shown as a function of aggregate outsourcing expenditures relative to the aggregate wage bill on the x-axis. TFP in panel (e) calculated as $M \int R(z, \boldsymbol{n}(z, \boldsymbol{\varepsilon}))\Gamma(dz, d\boldsymbol{\varepsilon})/R(1, \boldsymbol{N})$, where $\boldsymbol{N}$ denotes aggregate employment of each skill.

Panels (b) reveals that general equilibrium effects further deteriorate workers' prospects. Wages of low-skill workers fall across the distribution, due to two effects. First, the fall in the price of outsourcing implies a direct compression effect on top wages highlighted in Proposition 5. This first effect results in a decline of wages at the 90$^{\text{th}}$ percentile of the distribution (dotted blue line). Second, the reallocation of workers away from high-paying traditional firms and towards contractor firms at the bottom of the job ladder weakens labor market competition for workers from both ends of the job ladder. Middle-productivity firms that hire in-house are now shielded from losing their workers to a high-productivity firm with more generous wage offers. In addition, middle-productivity firms can now poach workers more easily from low-paying contractor firms. At the same time, expected revenues and thus target size for middle-productivity firms do not change. Together, these observations imply that middle-productivity firms lower pay and still recruit as many workers as they need. This effect is particularly potent for in-house low-skill workers around the median of the wage distribution, who lose over 3% in wages. As a result of the general equilibrium decline in wages throughout the distribution, the reservation wage also falls and workers at the bottom decile also lose, albeit more modestly.

Panel (c) reveals that high-skill worker benefit modestly from the rise in outsourcing. As firms outsource and increase their low-skill employment as per Proposition 5, the marginal product for high-skill labor rises due to complementarity in the production function. As demand for high-skill workers increases, so do their wages. Combining the decline in low-skill wages from panel (b) with the rise in high-skill wages from panel (c), the skill premium rises by 4%.

Together, panels (a) through (d) reveal that outsourcing deteriorates the prospects of low-skill workers conditional on being employed. At the same time, rising outsourcing is akin to an improvement in the aggregate search technology of the economy. As a result, low-skill workers may be more likely to be employed by 2007 relative to 1996. Thus, it is a priori unclear whether low-skill workers lose on net from the rise in outsourcing.

Figure 4(e) plots changes in expected earnings (worker income) averaged across skill types. We show in Supplemental Material E.3 that expected earnings coincide with welfare given our assumption of a small discount rate. On average, workers gain moderately from more outsourcing. However, this average increase masks an unequal incidence between skills.

Table 5 reports how these gains are split between the outsourcing wage penalty, in-house wages and employment rates, for low-skill and high-skill workers. The reallocation of low-skill workers towards contractors firms that pay low wages highlighted in Figure 4(a) results in a 2% decline of expected earnings for low-skill workers. This decline is the partial equilibrium impact of outsourcing.

General equilibrium effects add two layers to changes in earnings. First, Figure 4(b) highlights that low-skill workers employed in-house experience wage declines. Table 5 indicates that this channel results in an additional 2% decline in earnings. Second, the employment rate of low-skill workers increases by 6%. This general equilibrium response arises because contractor firms effectively expand available resources for hiring in the aggregate. Traditional firms can then scale up without being constrained by labor market frictions when they outsource. This employment rate increase benefits workers proportionally to the replacement rate of unemployment insurance, leading to a 3% rise in expected earnings. On net, low-skill workers lose 1% in expected earnings and welfare.

This decomposition highlights that general equilibrium effects are critical to evaluate the impact of outsourcing on the economy. The partial equilibrium impact alone would overstate the magnitude of low-skill workers' welfare losses twofold. For high-skill workers, earnings gains primarily occur through wage gains.

Figure 4(e) also reveals that aggregate output rises by 3.3%. This increase is partly driven by the extensive margin of worker employment. As contractors take over a larger market share

Table 5: The Impacts of Outsourcing on Expected Earnings of Workers.

	Low skill	High skill
Expected earnings	-0.01	0.01
Expected outsourcing wage penalty	-0.02	
In-house wage	-0.02	0.01
Expected unemployment earnings penalty	0.03	0.00

Note: Exact accounting decomposition of expected earnings I into three factors. Table reports log change in I and log change in each of the three factors. $I = (1 - p_{\text{out}} + p_{\text{out}}\underline{w}/w_{\text{in}})\, w_{\text{in}}\, ((1-u) + ub/\, ((1-p_{\text{out}})w_{\text{in}} + p_{\text{out}}\underline{w}))$, where w_{in} denotes the average in-house wage, p_{out} the probability that an employed worker is working at a contractor. Expected outsourcing wage penalty: $(1 - p_{\text{out}} + p_{\text{out}}\underline{w}/w_{\text{in}})$. In-house wage: w_{in}. Last factor is the unemployment earnings penalty.

and the economy becomes less constrained by labor market frictions, recruiting becomes less costly in the aggregate and employment rises. Three fifths of output gains are driven by the rise in employment.

Reallocation of employment across firms is the second margin leading to a rise in output. High productivity traditional firms are those that are most constrained by labor market frictions when hiring in-house. When the price of outsourcing falls, these high productivity firms start outsourcing and scale up. This reallocation of labor improves aggregate TFP, accounting for two fifths of overall output gains.

Combining these aggregate impacts, Figure 4(f) shows that outsourcing depresses the labor share by 0.7 percentage points between 1996 and the counterfactual 2007 economy. We conclude that outsourcing has positive productivity effects in the aggregate that benefit firms' shareholders and high-skill workers. However, outsourcing deteriorates labor market prospects and welfare of low-skill workers on average.

5.2 The impact of outsourcing by minimum wage policies

Given the redistributive effects of outsourcing, a natural question is whether standard labor market policy instruments can ensure that both workers and firms benefit equally from outsourcing. We focus on the minimum wage as our main policy instrument. The minimum wage is a natural candidate since it maintains wages at any desired level. The counterpart is that the minimum wage may push up the price of labor so much that it deters vacancy creation and reduces employment, ultimately lowering output. We use our estimated model to explore whether outsourcing can benefit workers and firms equally with a simple minimum wage reform.

We conduct three minimum wage experiments, in conjunction with the same reduction in the cost of outsourcing as in Section 5.1. First, we impose a constant minimum wage equal to the 1996 reservation wage. The minimum wage is only binding for low-skill workers in all

Table 6: The Impacts of Outsourcing by Minimum Wage.

	Minimum Wage			
	None	Constant	Increase (1)	Increase (2)
Minimum wage (%)		0.00	3.29	9.38
Average wage (%)				
Low-skill	-3.08	-2.89	-1.27	1.62
High-skill	1.09	1.07	0.89	0.58
Skill premium (log points)	4.25	4.01	2.03	-1.41
Non-employment rate (p.p.)				
Low-skill	-4.52	-4.41	-3.52	-2.04
High-skill	-0.67	-0.65	-0.52	-0.29
Worker expected earnings (%)	0.35	0.38	0.62	1.05
Low-skill	-1.29	-1.15	0.00	2.03
High-skill	1.14	1.12	0.92	0.58
Outsourcing (p.p.)				
Spending share	4.74	4.67	4.10	3.15
Employment share	7.01	6.89	5.95	4.44
Output (%)	3.28	3.22	2.77	2.01
Labor share (p.p.)	-0.74	-0.71	-0.46	0.00

Note: Change in aggregate statistics following a fall in the outsourcing cost $1/\tau$ calibrated to the 1996-2007 rise in the aggregate outsourcing expenditure share in France, under three minimum wage scenarios. 'None': no biding minimum wage. This column corresponds to the change between the right-most and left-most points in Figure 4. 'Constant' imposes a minimum wage equal to the low-skill's reservation wage in the baseline calibration. 'Increase (1)' imposes a 3.29% increase in the mandatory minimum wage starting from the reservation wage in the baseline calibration. 'Increase (2)' similarly imposes a 9.38% increase in the mandatory minimum wage.

experiments. Second, we increase the minimum wage by just over 3%, a value chosen to ensure constant expected earnings for low-skill workers. Third, we increase the minimum wage by 9% to ensure a constant labor share. These magnitudes also turn out to be of comparable to the increase in the real minimum wage in France during that decade (28%).

Table 6 summarizes our results. The first column reports results for the case without any minimum wage, and coincides with the results in Section 5.1. The second column shows that a constant minimum wage only moderately attenuates the redistributive effects of outsourcing. Output remains virtually identical to its baseline change, while the decline in the labor share is only marginally reduced relative to the baseline counterfactual.

The third column shows that a 3% increase in the minimum wage maintains low-skill worker earnings and welfare at their 1996 value. Output still increases by 2.7%—four fifths of its baseline increase—reflecting the smaller increase in both expenditure and employment outsourcing shares. In the fourth column, we increase the minimum wage by 9% to keep the labor share constant at its 1996 value. Expected earnings of low-skill workers increase by 2%. Aggregate output rises by 2%—three fifths of the baseline increase. We thus interpret the lack of decline in the aggregate labor share in France between 1996 and 2007 as consistent with the rise in outsourcing given the simultaneous increase in the real minimum wage during that period. Overall, we conclude that a moderate increase in the minimum wage can ensure that outsourcing benefits both workers and firms' shareholders. Such a policy would reduce shareholders' gains from outsourcing.

Conclusion

This paper started with a theory of domestic outsourcing. We have argued that it is useful to conceptualize firms' outsourcing decision in the context of frictional labor markets giving rise to firm wage premia. More productive firms are then more likely to outsource. Outsourcing raises output at the firm level. Contractors endogenously locate at the bottom of the job ladder, implying that outsourced workers receive lower wages. Together, these observations characterize the tension between productivity enhancements and redistribution away from workers that is tied to outsourcing. Using firm-level instruments for outsourcing and revenue productivity, we have proposed new reduced-form evidence that confirms the productivity and redistributive effects of outsourcing. Finally, equipped with a structurally estimated model, we have shown that outsourcing benefits both shareholders and high-skill workers, but deteriorates low-skill workers' labor market prospects. Accompanied by a minimum wage increase, outsourcing can benefit both workers and shareholders at the cost of lower gains for shareholders.

There are at least two natural directions along which to expand this research agenda. First, the productivity and distributional effects of outsourcing could be more fully contrasted with those from trade in intermediate goods and services than the stark perspective we have taken to highlight the unique features of outsourcing. Second, due to its tractability under parsimonious assumptions, our framework is naturally equipped to study questions with an efficiency-equity trade-off that involve workers' wages and scale-biased aggregate transformations, such as trade liberalizations or the rise of Artificial Intelligence.

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Appendix

A Proofs

A.1 Reservation wage

Omit s indices. Suppose without loss of generality that F admits a density f. Then $\Big[r + \delta + \lambda^E(1 - F(w))\Big]V(w) = w + \delta U + \lambda^E \int_w^\infty V(x)f(x)dx$. Differentiate w.r.t. w to obtain $\Big[r+\delta+\lambda^E(1-F(w))\Big]V'(w) = 1$. Integrate back to $V(w) = U + \int_{\underline{w}}^{w} \frac{dx}{r+\delta+\lambda^E(1-F(x))}$. Substituting into the value of unemployment, $rU = b + \lambda^U \int_{\underline{w}}^\infty \frac{(1-F(x))dx}{r+\delta+\lambda^E(1-F(x))}$. Since $V(\underline{w}) = U$, $(r + \lambda^U)U = b+\lambda^U \int_{\underline{w}}^\infty V(x)f(x)dx$ and $(r+\lambda^E)U = \underline{w}+\lambda^E \int_{\underline{w}}^\infty V(x)f(x)dx$. Thus, $rU = \frac{\lambda^U \underline{w}-\lambda^E b}{\lambda^U-\lambda^E}$. Therefore,

$$\lambda^U \underline{w} = \lambda^E b + (\lambda^U - \lambda^E) \left[b + \lambda^U \int_{\underline{w}}^\infty \frac{(1 - F(x))dx}{r + \delta + \lambda^E(1 - F(x))}.\right] \tag{18}$$

A.2 Proof of equation (1)

The flow of workers out of any wage interval $[\underline{w}_s, w)$ must be equal to the flow of workers into that wage interval: $\lambda_s^U F_s(w)u_s = \big(\delta_s + \lambda_s^E(1 - F_s(w))\big)(m_s - u_s)G_s(w)$, where u_s denotes the skill-specific unemployment rate. The left-hand-side is the flow of workers out of unemployment into the wage interval $[\underline{w}_s, w)$, while the right-hand-side is the flow of workers out of that wage interval. It consists of workers who exogenously lose their job, and those who transition into higher wages. A similar argument guarantees that $u_s = \frac{m_s\delta_s}{\delta_s+\lambda_s^U}$. Re-arranging delivers the expression in equation (1).

A.3 Proof of Proposition 1

Impose Assumption (B). Then $n_s(w) = \frac{n_{0s}}{[1+k_s(1-F_s(w))]^2}$ for a constant n_{0s}, and $n_s'(w) = \frac{2kn_{0s}F_s'(w)}{[1+k_s(1-F_s(w))]^3}$. Re-write (3) as

$$\pi(z) = \max_{v_s\in[0,1]^S, w_s} R(z, \{n_s(w)v_s\}_s) - \int w_s n_s(w_s)v_s ds. \tag{19}$$

Start from the FOC for wages in (3). We obtain $R_{n_s}n_s' - n_s - w_s n_s' = 0.$[17] Differentiating the objective in (19) w.r.t. v_s and using the FOC for wages, we obtain $\frac{\partial\big(R(z,\{n_s(w)v_s\}_s)-\int w_s n_s(w_s)v_s ds\big)}{\partial v_s} =$

[17]This equality implies $(R_{n_s} - w_s)n_s' = n_s > 0$. Thus, $R_{n_s} > w_s$ and $n_s' > 0$.

$(R_{n_s} - w_s)n_s > 0$. Thus, firms are always at the corner $v_s = 1$. Hence, (3) coincides with

$$\pi(z) = \max_{w_s} \Pi[z, \{w_s\}_s] \equiv R(z, \{n_s(w)\}_s) - \int w_s n_s(w_s) ds. \tag{20}$$

Since n_s is increasing in w, Π is continuously differentiable and strictly supermodular in any pair (z, w_s). In addition, the profit function is supermodular in $\{w_s\}_s$, and exhibits increasing differences in (z, w_s) for all s. In addition, the set of $\{w_s\}_s$ forms a lattice with the element-wise order. Therefore, we can apply Theorem 2.8.5. p. 79 in Topkis (1998). Thus, the set of maximizers $\{w_s(z)\}_s$ are strictly increasing in z for each s. Given the ordering of the ordering of wages, $F(w_s(z)) = \Gamma(z)$

A.4 Proof of Proposition 2

Given Proposition 1, it is immediate to verify that $n_s(z) = \frac{(1+k_s)e_s}{M[1+k_s(1-\Gamma(z))]}$.

A.5 Proof of Proposition 3

Because wages are strictly increasing in z, they are continuous almost everywhere and we may take first-order conditions for almost every productivity z. Hence:

$$\frac{d\big(n_s(w)w\big)}{dw}\Big|_{w=w_s(z)} = \frac{dR(z, \boldsymbol{n}_{-s}(w_s(z)), n_s(w))}{dw}\Big|_{w=w_s(z)} = \frac{\partial R}{\partial n_s}(z, \boldsymbol{n}(z)) \cdot n_s'(w_s(z))$$

where $\boldsymbol{n}_{-s}$ denotes the vector $\boldsymbol{n}$ without its entry s. Multiplying both sides by $w_s'(z)$ and changing variables to $n_s(w_s(z)) \equiv n_s(z)$ delivers

$$n_s(z)w_s'(z) = n_s'(z)\big(R_{n_s}(z, \{n_t(z)\}_t) - w_s(z)\big). \tag{21}$$

Integrating over z delivers the formula in Proposition 3.

A.6 Proof of Proposition 4

Existence and uniqueness among equilibria with continuous F. Together, Propositions 1, 2 and 3 suffice to complete a guess and verify strategy to exhibit an equilibrium with a continuous wage offer distribution. The last condition to verify is whether a reservation wage compatible with those results exists. Omitting s subscripts, re-write (18) as

$$\lambda^U \underline{w} = \lambda^E b + (\lambda^U - \lambda^E)\left[b + \lambda^U \int_{\underline{w}}^{\infty} \frac{(1-\Gamma(x))w'(x)dx}{r + \delta + \lambda^E(1-\Gamma(x))}\right]. \tag{22}$$

$w'(x)$ is a function of the reservation wage $\underline{w}$ through the ODE (21). To explicit its dependence, denote $d(z) = \frac{\partial w(z)}{\partial \underline{w}}$ where the partial derivative is understood as a derivative w.r.t. the initial condition of the ODE (21). Differentiating (21), we obtain $n(z)d'(z) = -n'(z)d(z)$. Solving this ODE explicitly and using $d(\underline{z}) = 1$ by definition, we obtain $d(z) = \frac{n(\underline{z})}{n(z)}$. Hence, $d'(z) = -\frac{n(\underline{z})n'(z)}{n(z)^2} < 0$. Thus, $w'(x)$ is a decreasing function of $\underline{w}$.

Hence, the right-hand-side of (22) is a decreasing function of $\underline{w}$ that goes to $\lambda^E b \leq \lambda^U b$ as $\underline{w}$ goes to infinity. Its left-hand-side is an increasing function of $\underline{w}$ that spans $\lambda^U b$ to $+\infty$. Therefore, there exists a unique reservation wage $\underline{w}$.

Existence and uniqueness among all possible equilibria. For expositional simplicity and without loss of generality, we present our trembling-hand refinement with a single skill. The maximization problem (3) becomes:

$$w(z) = \operatorname*{argmax}_{w,n} R(z,n) - wn, \quad n \leq n(w) \equiv \frac{n_0}{[1 + k(1 - F(w))][1 + k(1 - F(w^-))]}.$$

Suppose that firms make mistakes ε after choosing their target wage, i.e. firms post $w(z) + \varepsilon$ while having chosen $w(z)$ for an i.i.d. shock ε across firms. Firm z does not expect to make a mistake, but takes into account the equilibrium wage offer distribution inclusive of other firms' mistakes.

The distribution F that enters the constraint is the vacancy-weighted distribution of posted wages $w + \varepsilon$ in the economy. We impose the following assumptions on the distribution of mistakes ε, H_σ. First, H_σ has a $\mathcal{C}^\infty$ density with compact support. Second, the variance of H_σ (or any relevant measure of dispersion such as the size of its support) is given by σ. Third, convergence of H_σ is uniform: $|H_\sigma(\varepsilon) - \mathbb{1}\{\varepsilon \geq 0\}| \leq h_0\sigma$, where h_0 is a constant independent from σ. Fourth, H_σ is strictly increasing. Standard results on convolution kernels imply that there exists such a distribution. Our trembling-hand refinement is to consider the limiting economy when $\sigma \downarrow 0$.

Assumption (C). When $\sigma = 0$, we restrict attention to decentralized equilibria that are the limit of a sequence of decentralized equilibria when $\sigma \downarrow 0$.

The remainder of this section is structured as follows. First, we show that the wage distribution is smooth when $\sigma > 0$. Second, we show that wages are strictly increasing in z when $\sigma > 0$. Third, we show that the wage rank converges to the productivity rank as $\sigma \to 0$. Fourth, we show that wages converge to our candidate equilibrium when $\sigma \to 0$.

1. Smooth wage distribution when $\sigma > 0$. F is a convolution between the distribution of chosen wages $w(z)$ and an i.i.d. shock ε. Therefore, standard results on regularizing convolutions ensure that F admits a $\mathcal{C}^\infty$ density when $\sigma > 0$. This conclusion follows from $F(w) = \int H_\sigma(w - \omega)d\Omega(\omega)$ together with dominated convergence, where $\omega = w(z)$ is a random variable that denotes chosen wages, and Ω is its c.d.f. In addition, F is strictly increasing: $F'(w) > 0$. Since F is smooth, for any $\sigma > 0$, $n(w) = \frac{n_0}{[1+k(1-F(w))]^2}$ and $n'(w) = \frac{2kn_0F'(w)}{[1+k(1-F(w))]^3}$.

2. Binding constraint and increasing wages when $\sigma > 0$. Conditional on F being smooth, the argument is identical to Section A.3. Crucially, this conclusion would not be valid in general if there were a mass point in the distribution F.

3. Wage rank and productivity rank when $\sigma \downarrow 0$. Denote by $w_\sigma(z)$ the wage function for a given σ. Write $F(w(z_0)) = \mathbb{P}[\varepsilon \le w(z_0) - w(z)] = \int H_\sigma\big(w(z_0) - w(z)\big)d\Gamma(z)$. Then

$$\int H_\sigma\Big(w_\sigma(z_0) - w_\sigma(z)\Big)d\Gamma(z) = \underbrace{\int \mathbb{1}\Big\{z \le z_0\Big\}d\Gamma(z)}_{\text{By wage ranking}} + \int \underbrace{\Big[H_\sigma\Big(w_\sigma(z_0) - w_\sigma(z)\Big) - \mathbb{1}\Big\{w_\sigma(z_0) - w_\sigma(z) \ge 0\Big\}\Big]}_{\le h_0\sigma \text{ by assumption}} d\Gamma(z)$$

Therefore, for all z, $F(w(z)) \to \Gamma(z)$ uniformly, and $n(w(z)) \to n(z) \equiv \frac{n_0}{[1+k(1-\Gamma(z))]^2}$ uniformly.

4. Wages when $\sigma \downarrow 0$. We go back to the maximization problem (19) and use an argument that resembles Berge's maximum theorem that we cannot apply directly. Re-write (19) as choosing the wage $w_\sigma(Z)$ of a firm with productivity Z. The wage function w_σ must hence satisfy $z = \text{argmax}_Z\, R(z, n(w_\sigma(Z))) - w_\sigma(Z)n(w_\sigma(Z))\big]$. In particular, $Z^*(z) = z$ for all σ. Suppose for a contradiction that w_σ was discontinuous in σ at $\sigma = 0$ for some z_0. Since $n(w_\sigma(Z)) \to n(Z)$, it must be that $Z^*(z)$ jumps down at $\sigma = 0$ since firms downscale due to higher costs of labor. This contradicts $Z^*(z_0) = z_0$. Therefore, w_σ is continuous in σ at $\sigma = 0$. At $\sigma = 0$, w_0 satisfies $z = \text{argmax}_Z\, R(z, n(Z)) - w_0(Z)n(Z)$. w_0 thus solves $(R_n(z, n(z)) - w_0(z))n'(z) = w_0'(z)n(z)$, which coincides with the wage ODE in our candidate equilibrium. Thus, the limit of any equilibrium under Assumption (C) as $\sigma \downarrow 0$ converges to the candidate equilibrium.

A.7 Micro-foundations for the cost of outsourcing

1. Iceberg trade cost. To sell one unit of labor services to a traditional firm, contractor firms must hire $1/\tau_s$ units of labor.

2. Capital. Assume that contractor firms for skill s combine capital, in exogenous supply K_s, and labor to produce one unit of efficiency unit of labor services of a given skill s. The decision problem of the contractor firm is

$$\pi^C(w) = \max_k p_s k^{1-\beta} n_s(w)^\beta - r_s k - w n_s(w). \tag{23}$$

The optimality condition for capital is then $k = \left(\frac{(1-\beta)p_s}{r_s}\right)^{\frac{1}{\beta}} \cdot n_s(w)$. Market clearing for capital leads to $\frac{r_s}{1-\beta} = p_s(Q_s^{\text{Out}}/K_s)^{\beta}$ where Q_s^{Out} is aggregate employment in contractor firms. Substituting back into (23), we obtain $\pi^C(w) = p_s\left(\frac{K_s}{Q_s^{\text{Out}}}\right)^{1-\beta} n_s(w) - wn_s(w)$. Assume further that $K_s = \tau_s^{\frac{1}{1-\beta}}$, and take $\beta \to 1$. Then, (23) becomes

$$\pi^C(w) = (\tau_s p_s - w)n_s(w).$$

A.8 Proof of Proposition 5

1. Contractor firm wage. The free-entry condition (4) immediately implies that contractor firms pay the reservation wage. If they posted $w > \underline{w}_s$, they could deviate to $w - \varepsilon$ for a small ε and make positive profits, a contradiction.

2. Price of outsourcing. The price of outsourcing also follows immediately from the free-entry condition (4) when there is some outsourcing in equilibrium.

3. Outsourcing threshold. We immediately see that the profit function in (5) is supermodular in $(z, \{n_s\}_s, \{o_s\}_s)$. We again use Theorem 2.8.1. p. 76 in Topkis (1998) to obtain that wages and size are rising in productivity. In addition, we also obtain that the outsourcing decision is rising in productivity. Since it is binary, there must be a threshold productivity $\hat{z}_s$ such that firms outsource if and only if $z \geq \hat{z}_s$.

4. Wage cap. Cost-minimization immediately implies that $p_s \geq w_s(\hat{z}_s)$.

5. Size and revenue jump and strict wage cap. For notational simplicity and without loss of generality, focus on the case with a single worker type in the remainder of this proof and drop s indices. Denote outsourced employment by q.

Size jump. We start by showing that $q(\hat{z}) > n(\hat{z})$. Because of the theorem of the maximum, profits must be continuous at the outsourcing cutoff $\hat{z}$. Then, the indifference condition at $\hat{z}$ becomes

$$R(\hat{z}, n(\hat{z})) - w(n(\hat{z}))n(\hat{z}) = R(\hat{z}, q(\hat{z})) - pq(\hat{z}) \tag{24}$$

where $w(n)$ is an increasing function. In addition, the first-order condition for in-house employment is $R_n(\hat{z}, n(\hat{z})) = w(\hat{z}) + n(\hat{z})w'(n(\hat{z}))$. For outsourcing it is $R_n(\hat{z}, q(\hat{z})) = p$. Substituting both into the indifference condition (24):

$$R(\hat{z}, n(\hat{z})) - R_n(\hat{z}, n(\hat{z}))n(\hat{z}) + n(\hat{z})^2 w'(n(\hat{z})) = R(\hat{z}, q(\hat{z})) - q(\hat{z})R_n(\hat{z}, q(\hat{z}))$$

Since we assumed that R be strictly concave in n, the function $R - nR_n$ is strictly increasing in n. To finish the proof, we simply note that $w'(n(\hat{z})) > 0$.

Revenue jump. Since labor inputs jump up at the outsourcing cutoff, so does revenue.

Strict wage cap. Since the marginal in-house producer could have chosen to outsource the same-sized workforce, it must be that $R(\hat{z}, n(\hat{z})) - w(n(\hat{z}))n(\hat{z}) \geq R(\hat{z}, n(\hat{z})) - pn(\hat{z})$. Given that $n(\hat{z}) < q(\hat{z})$, we obtain $p > w(n(\hat{z}))$.

A.9 Relaxing contractor free-entry

In this section we show that our main implications are robust to relaxing contractor free-entry, i.e. endowing contractors with the same hiring technology than traditional firms. Our comparative statics are conditional on the price of outsourcing. Thus, conditional on some outsourcing arising in equilibrium, we still obtain that firms select into outsourcing and that outsourcing firms' size and revenues jump up.

The only implication left to verify is the outsourcing wage penalty. For simplicity, we focus on a the single skill type case, and an extreme form of decreasing returns to scale. Suppose that the revenue function is $R(z, n) = z \min\{n, \bar{n}\}$, so that no firm ever hires more than $\bar{n}$ workers. Suppose that $\bar{n} > n_0$ so that there is a motive to outsource. Denote by $\mathcal{V}$ the exogenous measure of contractors, and let $q = p/\tau$.

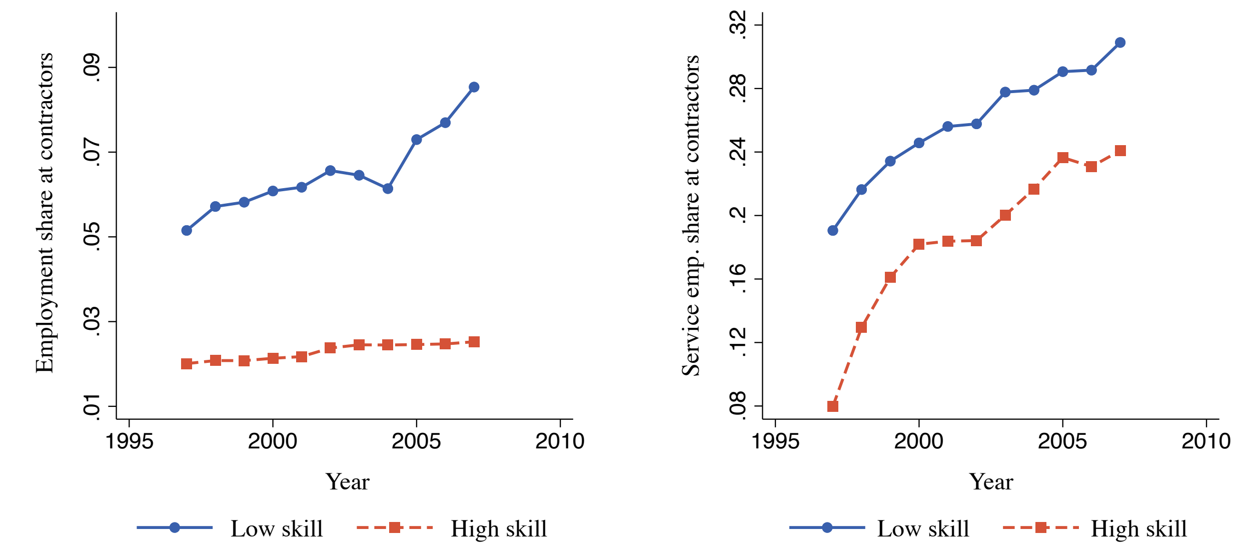
Due to constant returns conditional on hiring in-house, the wage-posting decision coincides with a standard Burdett and Mortensen (1998) economy, with a measure $\mathcal{V}$ of firms with effective productivity q. The wage distribution is described in more detail in Supplemental Material E.2. In particular, wages are strictly increasing in effective productivity: z for traditional firms, and q for contractors.

Now consider a traditional firm of productivity $z = p$. If that firm outsourced, it makes zero profits: $(p-p)\bar{n} = 0$. If it hires in-house, it makes strictly positive profits $\max_w (p-w)n(w) > 0$. Hence, traditional firm p hires in-house. Because of selection into outsourcing, the marginal outsourcer $\hat{z}$ is such that $\hat{z} > p \geq q$. Wages being increasing in effective productivity implies that $w(\hat{z})$ is larger than any wage paid by contractors: a positive outsourcing wage penalty.

B Additional reduced-form results

Figure 5: Outsourced employment in France.

(a) Fraction of employment at contractors. (b) Fraction of service employment at contractors.



Note: Low-skill contractor firm defined by industry codes specifically labeling firms as providing food, security, cleaning or general administrative services to other firms. High-skill contractor firm defined by industry codes specifically labeling firms as providing accounting, law or consulting services to other firms.

Figure 6: Outsourcing by industry volatility.

(a) Volatility by value added and industry. (b) Outsourcing by and industry volatility.

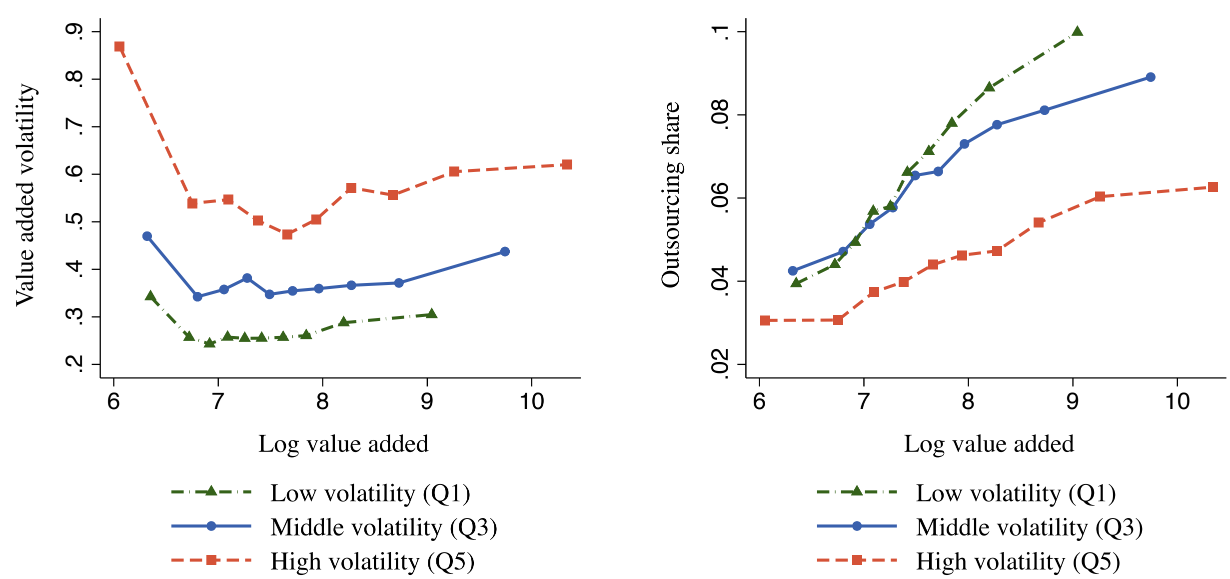


Table 7: Relationship between firm-level outsourcing shares and firm-level value added. Dependent variable: spending on external workers as a fraction of labor costs, in p.p.

	All				Exporters			
	OLS	OLS	OLS	OLS (P25,P75)	OLS (P25,P75)	2SLS	2SLS	2SLS
Log Value Added	1.457***	1.535***	0.679***	2.131***	1.579**	3.348**		
	(0.027)	(0.027)	(0.055)	(0.312)	(0.609)	(1.029)		
Log Size							5.662**	
							(1.733)	
Log Labor Prod.								11.461**
								(3.939)
Fixed Effects								
Year	✓	✓	✓	✓	✓	✓	✓	✓
Industry		✓						
Firm			✓	✓	✓	✓	✓	✓
Obs.	173547	173540	173547	61549	14561	46152	46152	46152
Stand. coef.	0.189	0.200	0.088	0.277	0.205	0.435	0.736	1.491
1^{st}-stage F-stat.	.	.	.	.	.	273	193	61

Standard errors in parenthesis, clustered by firm. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Variables winsrized at 5% level. Includes period dummies. Instrument: shift-share of export demand growth by 4-digit industry, projected by firm using firm-level export shares in first period. Restriction in columns (4-5): residual value added (net of year and firm fixed effects) between 25^{th} and 75^{th} percentile of its distribution. Unweighted regressions.

Table 8: Relationship between firm-level outsourcing shares and firm-level value added.

	Log VA			Log VA/N	Log N
	OLS	OLS (P25,P75)	2SLS	2SLS	2SLS
Outsourcing Share	0.007***	0.027***	0.050***	0.026***	0.022***
	(0.001)	(0.004)	(0.002)	(0.001)	(0.001)
Fixed Effects					
Firm	✓	✓	✓	✓	✓
Obs.	94336	47168	94336	94336	94336
Stand. coef.	0.031	0.119	0.220	0.114	0.095
1^{st}-stage F-stat.			16837	16837	16837

Standard errors in parenthesis, clustered by firm. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Variables winsorized at 5% level. Includes period dummies. Instrument: shift-share of outsourcing spending growth, projected by firm using firm-level occupation shares in first period. Restriction in column (2): residual outsourcing share (net of firm fixed effects) between 25^{th} and 75^{th} percentile of its distribution. Due to changes in the occupation classification in 2002, we lose the time period 1997-1999 and only have two time periods and thus no time period fixed effects.

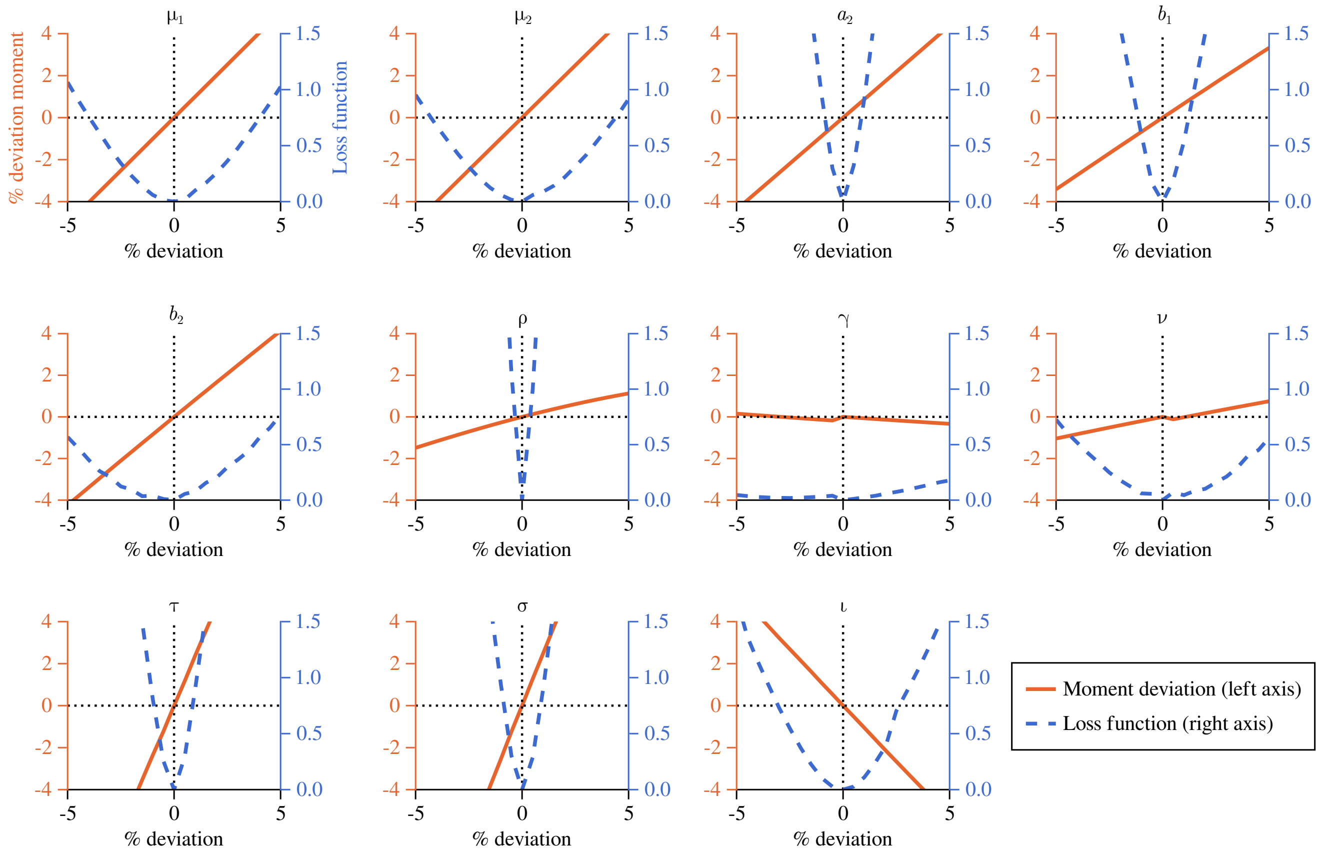
Table 9: Firm size wage premium in France.

	(1)	(2)	(3)	(4)
Log Firm In-house Employment	0.032***	0.021***		
	(0.004)	(0.005)		
Log Firm Value Added			0.023***	
			(0.004)	
Log Firm Mean Wage				0.036*
				(0.018)
Fixed Effects				
Year & 3-digit Industry	✓	✓	✓	✓
Worker		✓	✓	✓
Obs.	96697	94316	94316	94316

Note: Dependent variable: log worker daily wage. Standard errors in parenthesis, clustered by 3-digit industry. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Regression for service workers only, defined as in Section 3.2. In-house firm employment, value added and mean wage computed from firm-level data. Regression equation: $\log w_{i,t} = \varphi_i + \psi_{I(i,t)} + \beta X_{J(i,t)} + \eta_{i,t}$. i indexes workers, t indexes year-quarters. φ_i is a worker fixed effect. $\psi_{I(i,t)}$ is a fixed effect for the workers' employer's 3-digit industry $I(i,t)$. $J(i,t)$ denotes the worker's employer. X denotes either log employment, log value added or log mean wage.

C Identification

Figure 7: Simulated moments and loss function across parameter values.



Note: Numerical local identification of parameters. Solid orange line: percentage deviation of targeted moment relative to moment at estimated parameters, as a function of percentage deviation of parameter. Mapping as per Table 4. Dashed blue line: percentage deviation of loss function relative to loss function at estimated parameters, as a function of percentage deviation of parameter.

Supplemental Material

For Online Publication Only

Outsourcing, Inequality and Aggregate Output

Adrien Bilal & Hugo Lhuillier

D Data description

Firm-level balance sheet data. We use the FICUS data (*"Fichier Complet Unifié de Suse"*) which covers the near universe of nonfarm French businesses. The unit of observation is a firm-year, and firms are identified by their tax identifier (*"siren"*). It details balance sheet information. We construct value added by substracting purchases of intermediate goods and other intermediate purchases from firm sales.

Firm-level survey data. We use the EAE data (*"Enquête Annuelle d'Entreprise*). It covers a random sample of firms and tracks them across years. We link it to other sources using the common tax identifier (*"siren"*). The unit of observation is a firm-year. Among others, the dataset breaks down intermediate purchases of goods and services. In particular, we use expenditures on external workers (*"Dépenses de personnel extérieur"*) as our main measure of outsourcing expenditures.

DADS panel. We use the 4% sample of the DADS panel, between 1996 and 2007. Once a worker enters the dataset in any year after 1976, all her subsequent employment spells are recorded. Individuals' employment history is recorded in the dataset if (a) they have at least one employment spell, and (b) they are born in October in even years. The dataset provides start and end days of each employment spell, the job's wage, four-digit occupation and industry, as well as establishment and firm tax identifiers that can be linked to other datasets. We follow Bilal (2020) to set sample restrictions and define unemployment.

DADS cross-section. The DADS *Postes*, are used by the French statistical institute to construct the DADS *Panel*. They cover the universe of French workers, but in the version available to researchers, worker identifiers are reshuffled every two years. The DADS Postes allow to compute employment, wages, occupational mix for the near universe of French establishments.

Firm-level customs data. We use customs data for the universe of French importers and exporters. The unit of observation is at the firm-product-year-country-export/import level. We aggregate French exports for every firm, year and destination country at the 4-digit industry level to construct our firm-level instrument.

E Additional proofs

E.1 Dynamic firm problem

We first show that the size constraint in (3) is consistent with the firm-level decision. Omit s indices whenever unambiguous. Denote by q the vacancy contact rate. Without loss of generality, we use a continuous offer distribution $F(w)$ to lighten notation. Start from the firm-level Kolmogorov Forward Equation:

$$\frac{dn(w,t)}{dt} = q[\phi + (1-\phi)G(w)] - [\delta + \lambda^E(1-F(w))]n$$

where $\phi = \frac{u}{u + \frac{\lambda^E}{\lambda^U}(1-u)} = \frac{1}{1+k}$ is the probability of meeting an unemployed worker. In steady-state $dn/dt = 0$. Hence, from (1), $\phi + (1-\phi)G(w) = \frac{1}{1+k(1-F(w))}$, and so $n(w) = \frac{q}{\delta}\frac{1}{[1+k(1-F(w))]^2}$. Then, from a constant returns matching function, $\lambda^U = \theta q(\theta) = \frac{M}{m[u+(1-u)\lambda^E/\lambda^U]}q(\theta)$ where θ is labor market tightness. Re-arranging leads to $q = \frac{e\delta(1+k)}{M}$. Therefore,

$$n(w) = \frac{1}{M}\frac{(1+k)e}{[1+k(1-F(w))]^2}$$

We now turn to showing that the decisions from the firm's dynamic profit-maximization problem coincides with those from the static firm profit maximization problem (3) when the discount rate is low enough.

Consider the dynamic problem of a firm which may be out of its long-run size, while the rest of the economy is in steady-state. Assume that firms may freely adjust their wage each instant, but face an equal-pay constraint within worker type. Without loss of generality, we consider a single worker type to make notation lighter. Firms solve

$$rJ(z,n) = \max_w R(z,n) - wn + [q(\phi + (1-\phi)G(w)) - n(\delta + \lambda^E(1-F(w)))J_n(z,n)$$

Using $\phi = \frac{1}{1+k}$,

$$rJ(z,n) = \max_w R(z,n) - wn + \delta(1+k(1-F(w))(n(w)-n)J_n(z,n)$$

The first-order condition implies $-n + \delta(1+k(1-F))n'(w)J_n + kF'(n(w)-n)J_n = 0$. Evaluated at long-run size $n = n(w)$,

$$n(w) = \delta(1+k(1-F))n'(w)J_n(z,n(w))$$

The envelope condition then yields $rJ_n = R_n - w + \delta(1+k(1-F))[-J_n + (n(w)-n)J_{nn}]$ which again evaluated at long-run size $n = n(w)$ leads to

$$rJ_n(z,n(w)) = R_n(z,n(w)) - w - \delta(1+k(1-F(w)))J_n(z,n(w))$$

When the discount rate goes to zero $r \to 0$,

$$J_n(z,n(w)) = \frac{R_n(z,n(w)) - w}{\delta(1+k(1-F(w)))}$$

Substituting into the first-order condition, we obtain

$$n(w) = n'(w)(R_n - w)$$

which coincides with the static first-order condition.

E.2 Wage distribution without contractor free-entry

Let $P = \mathcal{V}/M$. Thus, the wage distribution is given by $F(w(z)) = \Upsilon(z) = \frac{1}{1+P}\left[\Gamma(z) + \mathbf{1}\{z \geq q\}\right]$. For $\underline{z} \leq z < q$ and $z > q$, the wage is the solution to the differential equation $w'(z) = \frac{2kd\Upsilon(z)}{1+k\bar{\Upsilon}(z)}(z - w(z))$. On the first segment, the boundary condition is $w(\underline{z}) = \underline{w}$. On the second segment, the boundary condition is $w(q) = \overline{w}_q = q - (p - w(q^-))\left[\frac{1+k(1-\Upsilon(q))}{1+k(1-\Upsilon(q^-))}\right]^2$. For $w \in (\underline{w}_q, \overline{w}_q)$, the wage offer distribution is $1 + k\bar{F}(w) = \left[\frac{p-w}{q-\underline{w}_q}\right]^{1/2}\left[1 + k\bar{\Upsilon}(q^-)\right]$.

E.3 Welfare and expected earnings

The value function of a worker with state $x \in \{b, w\}$, where b denotes unemployment, satisfies $V(x) = r\mathbb{E}_0 \int_0^\infty e^{-rt} x_t dt$. We rescale values by the discount rate r as we require values to remain finite in the limit $r \to 0$. We show that, when $r \to 0$, the value of any worker, regardless of their state, converges to steady-state expected earnings $\mathbb{E}[x]$ when the process x_t has a unique invariant distribution. Denote by $h(t, x)$ the solution to the time-dependent KFE satisfied by the density of x_t. Then $V(x) = \int xr \int_0^\infty e^{-rt} h(t, x) d\mu(x)$ where μ is a base measure that has a Dirac mass point at b and is the Lebesgue measure for all $x \geq \underline{w}$. If we can show that $r \int_0^\infty e^{-rt} u(t) dt \to \lim_{t\to\infty} u(t)$ for any smooth and bounded function u, we can apply this last result x by x and obtain $V(x) \to \int xh(\infty, x)dx \equiv \mathbb{E}[x]$. To show $r \int_0^\infty e^{-rt} u(t) dt \to \lim_{t\to\infty} u(t)$ for any smooth function u, change variables $\tau = rt$: $r \int_0^\infty e^{-rt} u(t) dt = \int_0^\infty e^{-\tau} u(\tau/r) d\tau$. $u(\tau/r) \to u(\infty)$ for all $\tau > 0$. We conclude the proof by dominated convergence.

F Simulation and estimation

F.1 Additional derivations for quantitative model

We drop the s subscript in ε and p_s for notational simplicity since only low skill workers can be outsourced. We define by $\pi^o(z, \varepsilon)$ the profit of a firm that outsource its low skill workers, and by $\pi^i(z, \varepsilon)$ the profit of a firm that hires in-house. When the firm hires the low-skill in-house, its profits are independent of ε, so that

$$\pi^i(z) \equiv \pi^i(z, \varepsilon) = \max_{\{v_s, w_s\}_{s=1}^2} \left(z \prod_{s=1}^2 n_s(w_s, v_s)^{a_s}\right)^\rho - \sum_{s=1}^2 \left(w_s n_s(w_s, v_s) + \frac{1}{1+\gamma}\left(\frac{V_s^i}{V_s} v_s\right)^{1+\gamma}\right). \quad (25)$$

When a firm is outsourcing, maximizing out n_1 yields

$$\pi^o(z, \varepsilon) = \max_{v_2, w_2} \kappa \left(z\varepsilon^{-a_1} n_2(w_2, v_2)^{a_2} \left[\frac{\rho a_1}{p}\right]^{a_1}\right)^{\rho/\kappa} - w_2 n_2(w_2, v_2) - \frac{1}{1+\gamma}\left(\frac{V_2^i}{V_2} v_2\right)^{1+\gamma}, \quad (26)$$

where $\kappa = (1 - \rho a_1)$. Inspection of (26) reveals that wages and vacancies depend only on the productivity index $\hat{z}(z, \varepsilon) \equiv z\varepsilon^{-a_1}$. We define accordingly $\pi^o(\hat{z}) \equiv \pi^o(z, (z/\hat{z})^{1/a_1})$ for any z. This definition provides an efficient way to compute the solutions to (25) and (26). Given wage offer distributions $\{F_s\}_{s=1}^2$, we can solve both the in-house and outsourcing wage posting problem independently on a unidimensional grid of z and $\hat{z}$ respectively: we solve the model entirely in $(z, \hat{z})$ space. Only for estimation do we return to (z, ε) space. Let Ψ be the law of $(z, \hat{z})$, obtained as a transformation of Γ. Since (z, ε) is jointly log-normal, we alternate between the (z, ε) and $(z, \hat{z})$ space without relying on numerical integration and simply use standard results for conditional normal laws

We obtain wage offer distributions by combining wage posting behaviors of firms with their outsourcing decisions. A firm outsources if and only if it is profitable to do so, that is $o = 1 \iff \pi^o(\hat{z}) \geq \pi^i(z)$. Both π^i and π^o are increasing in their respective arguments, so that we can define the strictly increasing indifference curve $\varphi(z)$ by $\pi^i(z) = \pi^o(\varphi(z))$, and firms outsource if and only if $\hat{z} \geq \varphi(z)$. Firms with $\hat{z} \geq \varphi(z)$ do not post a wage for low skill workers and post a wage for high-skill according to the solution to (26), while firms with $\hat{z} < \varphi(z)$ post a wage both for low and high skills according to (25). The wage offer distribution of low skill then reads

$$F_1(w) = \int_Z \frac{v_1^i(z)}{V_1} \mathbf{1}\{w_1^i(z) \leq w\} \Psi_{\hat{z}|z}(\varphi(z) \mid z) \mathrm{d}\Psi_z(z), \tag{27}$$

where Ψ_z is the marginal of Ψ with respect to z, and $\Psi_{\hat{z}|z}$ the conditional distribution of $\hat{z}$ given z. The wage offer distribution of high skill workers reads

$$F_2(w) = \int_Z \frac{v_2^i(z)}{V_2} \mathbf{1}\{w_2^i(z) \leq w\} \Psi_{\hat{z}|z}(\varphi(z) \mid z) \mathrm{d}\Psi_z(z) + \int_{\hat{Z}} \frac{v_2^o(\hat{z})}{V_2} \mathbf{1}\{w_2^o(\hat{z}) \leq w\} \Psi_{z|\hat{z}}(\varphi^{-1}(\hat{z}) \mid \hat{z}) \mathrm{d}\Psi_{\hat{z}}(\hat{z}). \tag{28}$$

We use equations (27) and (28) to express firms' first order conditions as ODEs. Define $\Upsilon_s^\theta(z) \equiv F_s(w_s^\theta(z))$ as the wage offer distribution of skill s evaluated at the wage posted by firm z under outsourcing strategy $\theta \in \{i, o\}$. Taking the first order conditions of (25) and (26) and evaluating these optimality conditions at z returns

$$\partial w_s^\theta(z) = \frac{2k_s \mathrm{d}\Upsilon_s^\theta(z)}{1 + k\bar{\Upsilon}_s^\theta(z)} \left[\partial_{n_s} R^\theta(z, \boldsymbol{n}^\theta(z)) - w_s^\theta(z)\right] \tag{29}$$

with respect to w_s^θ, and

$$\left(\frac{V_s^i}{V_s} v_s^\theta(z)\right)^{1+\gamma} = n_s^\theta(z) \left[\partial_{n_s} R^\theta(z, \boldsymbol{n}^\theta(z)) - w_s^\theta(z)\right] \tag{30}$$

with respect to v_s^i, where

$$n_s^\theta(z) \equiv \frac{e_s}{M} \frac{1 + k_s}{[1 + k_s \bar{\Upsilon}_s^\theta(z)]} \frac{v_s^\theta(z)}{V_s}.$$

Computing (29) requires knowledge of Υ_s^θ. For low skill workers, only in-house firms are posting wages and that these wages are strictly increasing in z. Thus, we can evaluate (27) at $w_1^i(z)$ and differentiate with respect to z to obtain

$$\mathrm{d}\Upsilon_1^i(z) = \frac{v_1^i(z)}{V_1} \Psi_{\hat{z}|z}(\varphi(z) \mid z) \mathrm{d}\Psi_z(z). \tag{31}$$

For high skill workers, we first define the function ω that equalizes the wages posted by in-house and outsourcing firms, $w_2^i(z) = w_2^o(\omega(z))$. By definition, we obtain $\Upsilon_2^i(z) = \Upsilon_2^o(\omega(z))$ and therefore $n_2^i(z) = n_2^o(\omega(z))$. In addition, $\omega'(z) = \partial w_2^i(z)/\partial w_2^o(\omega(z)) = \mathrm{d}\Upsilon_2^i(z)/\mathrm{d}\Upsilon_2^o(\omega(z))$. Evaluating (29-30) at z and $\omega(z)$ for $\theta = i$ and $\theta = o$ respectively and combining them returns an explicit expression for $\omega(z)$,

$$\omega(z) = \left(\left[\frac{\rho}{\kappa}\right]^{1/\rho} z n_1^i(z)^{a_1} n_2^i(z)^{a_2(1-1/\kappa)}\right)^\kappa. \tag{32}$$

Plugging (32) in (28) and differentiating returns expressions for $\mathrm{d}\Upsilon_2^i$ and $\mathrm{d}\Upsilon_2^o$, respectively

$$\mathrm{d}\Upsilon_2^i(z) = \frac{v_2^i(z)}{V_2} \left[\Psi_{\hat{z}|z}(\varphi(z) \mid z) \mathrm{d}\Psi_z(z) + \Psi_{z|\hat{z}}[\varphi^{-1}(\omega(z)) \mid \omega(z)] \mathrm{d}\Psi_{\hat{z}}(\omega(z))\omega'(z)\right] \tag{33}$$

and

$$\mathrm{d}\Upsilon_2^o(\hat{z}) = \frac{v_2^o(\hat{z})}{V_2}\left[\Psi_{\hat{z}|z}(\varphi[\omega^{-1}(\hat{z})] \mid \omega^{-1}(\hat{z}))\mathrm{d}\Psi_z(\omega^{-1}(z))(\omega^{-1}(z))' + \Psi_{z|\hat{z}}[\varphi^{-1}(\hat{z}) \mid \hat{z}]\mathrm{d}\Psi_{\hat{z}}(\hat{z})\right]. \tag{34}$$

Given the indifference function φ, we solve for the policy functions and wage offer distributions by iterating forward on the ODEs for w and Υ while solving simultaneously for v and ω (see Section F.2 for more details).

To close the model, it remains to compute the aggregate quantities. First, aggregate vacancies posted by final producers, V_1 and V_2, satisfy

$$V_1^i = \int_Z v_1^i(z)\Psi_{\hat{z}|z}(\varphi(z) \mid z)\mathrm{d}\Psi_z(z) \tag{35}$$

$$V_2 = V_2^i = \int_Z v_2^i(z)\Psi_{\hat{z}|z}(\varphi(z) \mid z)\mathrm{d}\Psi_z(z) + \int_{\hat{Z}} v_2^o(\hat{z})\Psi_{z|\hat{z}}(\varphi^{-1}(\hat{z}) \mid \hat{z})\mathrm{d}\Psi_{\hat{z}}(\hat{z}). \tag{36}$$

The reservation wage is obtained from (18) for each s. Contact rates are obtained from the matching function (17): $\lambda_s^U = \mu_s\left(\frac{V_s}{m_s(u_s+\zeta_s(1-u_s))}\right)^{1-\chi}$, and $\lambda_s^E = \zeta_s\lambda_s^U$. The price of outsourcing is pin down by the free entry condition, $p = \underline{w}_1/\tau$, and the number of service providers $\mathcal{V}$ adjusts so as to clear the market for the outsourcing service,

$$\frac{e_1}{1+k_1}\frac{\mathcal{V}}{\mathcal{V}+V_1^i} = M\int_{Z\times\hat{Z}} n_1^o(z,\hat{z})\mathrm{d}\Psi(z,\hat{z}) \quad , \quad \frac{n_1^o(z,\hat{z})}{R^o(\hat{z})} = \frac{\rho a_1}{p}\left(\frac{\hat{z}}{z}\right)^{1/a_1}, \tag{37}$$

where the left-hand side is the supply, the right-hand side is the demand, and n_1 follows from the first order condition of the outsourcing problem. Equations (25-37) and the definition of $\varphi(z)$ fully summarize the model.

F.2 Algorithm

The algorithm is made of three loops. The inner loop solves jointly for $z \to \{w_s^\theta(z), v_s^\theta(z), \Upsilon_s^\theta(z)\}_{s,\theta}$ as well as $\omega(z)$ and the aggregate in-house vacancies $\{V_s^i\}_s$. The middle loop computes the indifference function $\varphi(z)$ and updates the reservation wages $\{\underline{w}_s\}_s$. Finally, the outer loop calculates the number of service providers in the economy $\mathcal{V}$.

Inner loop. Given the indifference function $\varphi(z)$, the inner loop solves for the firms' policy functions and wage offer distributions on a grid of $\{z_j\}_{j=1}^N$.[18] Specifically, we start by iterating on the z_j's to solve for the in-house problem. Both w and Υ are solved by iterating forward on their respective ODEs, which we approximate by explicit finite difference. At z_1, the boundary conditions $w_s^i(z_1) = \underline{w}_s$ and $\Upsilon_s^i(z_1) = 0$ must hold for both s. Then, for each $j \geq 1$, we compute the vacancies $\{v_s^i(z_j)\}_s$ by solving (30) jointly for both s.[19] Given $\{N_j^i(z_j)\}_s$, we compute $\omega(z_{j+1})$ from (32) by linear extrapolations, which we can then use to obtain $\Upsilon_s^i(z_{j+1})$ from $\Upsilon_s^i(z_{j+1}) = \Upsilon_s^i(z_j) + \mathrm{d}\Upsilon_s^i(z_j)$, where the differential is given by (31) and (33). Specifically, we compute $\omega(z_{j+1})$ as

$$\omega(z_{j+1}) = \left(\left[\frac{\rho}{\kappa}\right]^{1/\rho} z_{j+1}\tilde{n}_1^i(z_{j+1})^{a_1}\tilde{n}_2^i(z_{j+1})^{a_2(1-1/\kappa)}\right)^\kappa$$

where $\tilde{n}_s^i(z_{j+1})$ is obtained from linearly extrapolating $n_s^i(z_{j-1})$ and $n_s^i(z_j)$. Given $\omega(z_{j+1})$, we then

[18] We impose an identical grid for z and $\hat{z}$.

[19] Note that, since we are iterating forward on the z_j's, we know $\Upsilon_s^i(z_j)$ and can therefore compute $n_s^i(z_j)$.

update the wage offer distribution

$$\Upsilon_2^i(z_{j+1}) = \Upsilon_2^i(z_j) + \frac{v_2^i(z_j)}{V_2}\left[\Psi_{\hat{z}|z}(\varphi(z_j) \mid z_j)\Delta\Psi_z(z_{j+1}) + \Psi_{z|\hat{z}}[\varphi^{-1}(\omega(z_j)) \mid \omega(z_j)]\Delta\Psi_{\hat{z}}(\omega(z_j))\frac{\Delta\omega(z_{j+1})}{\Delta z_{j+1}}\right]$$

To find $\varphi^{-1}(\omega(z_j)$, we take the numerical inverse of φ and use linear interpolation to compute its value off grid. Obtaining $\Upsilon_1^i(z_{j+1})$ is a straightforward application of (31).

Finally, we compute $w_s^i(z_{j+1})$ from $w_s^i(z_{j+1}) = w_s^i(z_j) + \mathrm{d}w_s^i(z_j)$, where the differential is (29). Once this iteration is completed, we iterate a second time on the z_j's to compute the policy functions of the outsourcing firms. As for in-house firms, we first start by computing the vacancies, then updating the wage offer distribution, and finally solving for wages. At the end of these two iterations, we use (35) and (36) to compute the aggregate in-house vacancies, and we update the contact rates according to the matching function.

Middle loop. The middle loop solves for the indifference function and computes the reservation wages. Given π^i and π^o obtained in the inner loop, φ is found by numerically computing $(\pi^o)^{-1}[\pi^i(z_j)] = \varphi(z_j)$, where the inverse of π^o is approximated by linear interpolation. Given φ and the wage and policy functions, we can then compute the wage offer distributions $\{F_s\}_s$ from (27) and (28), that we then use to find the reservation wages from (18).

Outer loop. The outer loop solves for the number of service providers by inverting the market clearing condition (37). In particular, the log-normal assumption allows us to express the demand for outsourcing as a uni-dimensional integral, and the market clearing reads

$$\frac{e_1}{1+k_1}\frac{\mathcal{V}}{\mathcal{V}+V_1^i} = \frac{M\rho a_1}{p}\exp\left(\frac{\sigma_{z|\hat{z}}^2}{2a_1^2}\right)\int R^o(\hat{z})\exp\left(\frac{\log(\hat{z})-g(\hat{z})}{a_1}\right)\ N\left(\frac{\log[\varphi^{-1}(\hat{z})]-g(\hat{z})+\sigma_{z|\hat{z}}^2/a_1}{\sigma_{z|\hat{z}}}\right)\mathrm{d}\Psi_{\hat{z}}(\hat{z})$$

where $g(z) \equiv \mu_1 + \rho\Sigma_{11}[\log(z)-\mu_2]/\Sigma_{22}$, $\sigma_{z|\hat{z}}^2 \equiv (1-[\Sigma_{12}/(\Sigma_{11}\Sigma_{22})]^2)\Sigma_{11}^2$, and N denotes the cdf of a standard normal distribution.

F.3 Estimation: expression for the employment-employment transition rate

Omit s indices for simplicity. Our argument requires only that the economy be stationary. Index firms by their wage offer w and thie vacancy decision v. Denote $H(v|w)$ the conditional c.d.f. of vacancies given the wage offer. Then

$$EE = \frac{\lambda^E\iint n(w,v)(1-F(w))dF(w)H(dv|w)}{\iint n(w,v)dF(w)H(dv|w)}.$$

The integral over $H(dv|w)$ produces the vacancy share of traditional firms in the numerator and denominator, and hence drops out. Hence,

$$EE = \frac{\lambda^E\int\frac{(1+k)e}{(1+k(1-F(w)))^2}(1-F(w))dF(w)}{\int\frac{(1+k)e}{(1+k(1-F(w)))^2}dF(w)} = \frac{\lambda^E\int_0^1\frac{(1-F)dF}{(1+k(1-F))^2}}{\int_0^1\frac{dF}{(1+k(1-F))^2}}$$

after changing variables to $F = F(w)$. Both integrals admit closed-form expressions, and thus:

$$EE = \lambda^E\frac{\big((1+k)\log(1+k)-k\big)/\big(k^2(1+k)\big)}{1/(1+k)} = \delta\frac{(1+k)\log(1+k)-k}{k}.$$

F.4 Estimation: MSM routine

To estimate the model, we define the loss function

$$\mathcal{L}(\boldsymbol{\theta}) = \sqrt{\frac{1}{M_m}\sum_{m=1}^{M_m}\left[h_m(\boldsymbol{\theta}) - \hat{h}_m\right]^2},$$

where $\boldsymbol{\theta}$ is the vector of parameter to be estimated, $\{\hat{h}_m\}_m$ is the set of empirical moments we are targeting, and $h : \mathbb{R}^{M_m} \mapsto \mathbb{R}^{M_m}$ maps parameters into simulated moments from our model. The simulated moments are computed as exact analogs of the empirical moments. To compute the simulated moments, we simulate a dataset in the (z, ε) space, projecting onto this space the policy functions found in the $(z, \hat{z})$ space using linear interpolations.

To find the minimum of $\mathcal{L}$, we use a gradient descent algorithm. That is, starting from $\boldsymbol{\theta}^0$, we obtain a sequence of parameters $\{\boldsymbol{\theta}^n\}_n$ by iterating on $\boldsymbol{\theta}^{n+1} = \boldsymbol{\theta} - \gamma_n \nabla\mathcal{L}(\boldsymbol{\theta})$, where the endogenous step size follows the Barzilai–Borwein method. Namely, for $n > 1$,

$$\gamma_n = \frac{\max\left\{|\boldsymbol{\theta}^n - \boldsymbol{\theta}^{n+1}|^T \cdot |\nabla\mathcal{L}(\boldsymbol{\theta}^n) - \nabla\mathcal{L}(\boldsymbol{\theta}^{n-1})|, 10^{-3}\right\}}{\|\nabla\mathcal{L}(\boldsymbol{\theta}^n) - \nabla\mathcal{L}(\boldsymbol{\theta}^{n-1})\|^2}.$$

We impose a maximal step size as in Burdakov et al. (2019) to stabilize the descent. The gradient of the loss function is approximated with central finite difference to maximize accuracy. Given that we use $M_m = 11$ parameters, the loss function $\mathcal{L}(\boldsymbol{\theta})$ is high-dimensional and we cannot check for the existence of local minima. To avoid those, we first search manually to start the algorithm from a $\boldsymbol{\theta}^0$ with a relatively low loss function, in practice $\mathcal{L}(\boldsymbol{\theta}^0) \in [0.3, 0.5]$. The gradient descent attains its minimum at $\mathcal{L}(\boldsymbol{\theta}^*) = .009$. The gradient descent is implemented in Julia and parallelized over 6 CPUs. The descent is run on a standard laptop and takes about one hour to converge.