When consumption smoothing is possible but not optimal: savings and liquidity constraints with reference-dependent preferences

Thesis submitted in partial fulfillment of the requirements for the Degree of Master of Science in Economics for Development at the University of Oxford

30/05/2017

By

906799

Word count: 9636 piecemeal/9780 by page

Table of Contents

INTRODUCTION 3

SECTION 1 8

1.1 Building the Model 8

1.2 The Basic Model 11

SECTION 2 13

2.1 Introducing Uncertainty 14

2.2 Comparative Statics 16

SECTION 3 17

3.1 Saving Over a Finite Horizon 17

3.2 Rational Expectations 22

SECTION 4 25

4.1 Economic Application to Consumption Smoothing 25

4.2 Rational Expectations Revisited 27

4.3 Discussion and Policy Implications 30

CONCLUSION 32

APPENDICES 33

Appendix 1 34

Appendix 2 37

REFERENCES 39

Abstract

Households in developing countries appear to contradict standard economic theory by holding on to assets that could be used to smooth consumption in times of crisis. Since existing theories seem unsatisfactory in light of current evidence, this essay explores the possibility that individuals resist smoothing consumption due to reference-dependent preferences. A two-period, and then a many-period model are developed to characterise this behaviour. They predict that when households are forced to incur a 'loss', they resist consumption smoothing. The model is applied to Burkina Faso and Zimbabwe, and is better at predicting observed behaviour than a standard model of concave utility. If there are households that exhibit reference-dependent preferences, the model has important policy implications. In particular, it predicts that covariate shocks heterogeneously affect behaviour, resulting in potential for risk-sharing agreements that are significantly welfare enhancing.

Introduction

The poor appear to act in unexpected ways. Such behaviour raises significant doubts for policy, and confounds the economic theory upon which it rests. One such example is the reluctance of poor households to smooth consumption in times of crisis. A wide body of evidence shows that livestock sales are not as responsive to consumption shocks as we would expect (Fafchamps, Udry and Czukas, 1998; Fafchamps and Lund, 2003; Hoogeveen, 2002; Kazianga and Udry, 2006; Lange and Reimers, 2015). Even in the work of Kinsey et al. (1998), there are indications that consumption fell while livestock holdings remained high. Despite the large amount of attention this issue has received, explanations have not always been satisfactory.

Theoretically speaking, there are many possible explanations for this behaviour. Measurement error in livestock sales could bias regression coefficients, indicating that livestock sales have less of an effect on consumption than in reality. A large covariate income shock such as drought would reduce incomes, reducing demand and thus the price of livestock. If the market for livestock collapses, selling livestock will not be an effective way to smooth consumption (indeed, it may not even be possible; See Sen 1981, pp. 46-51). Households could smooth consumption in different ways, such as informal risk-sharing agreements, the use of buffer stocks other than livestock such as grain or durable goods, or through credit markets. High transaction costs or missing markets could prevent the sale of livestock being feasible. Households may over-estimate the probability that a shock will persist (Kazianga and Udry, 2006). This lowers expected future income, thus lowering present consumption. Finally, models in which households are very patient or assets earn high future returns predict that assets will be retained rather than sold.[1]

Explanations in the existing literature are not always satisfactory. Fafchamps and Lund (2003) explain such behaviour through the use of gifts and loans. Households in their study do not sell livestock since they make use of informal gifts and/or loans from other households within the community to smooth consumption. Furthermore, their evidence suggests selling livestock incurs an extremely high transaction cost. However, data from the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT) on Burkina Faso during some of the regions worst droughts do not seem to be reconcilable with any of these explanations. Fafchamps et al. (1998) establish the livestock sales accounted for at most 30% of households' consumption, but do not explain why they did not account for more. Using the same data, Kazianga and Udry (2006) reject the hypothesis that households use risk-sharing agreements to smooth consumption. Furthermore, they state 'livestock holdings reported by most households at the end of the survey were large enough to compensate entirely for their income fall' (Kazianga and Udry, 2006, pp. 3). This suggests that the effects of a large covariate shock on grain and livestock prices are unlikely to have precluded consumption smoothing. Furthermore, with concave preferences, if consumption smoothing is feasible, it is generally optimal (See Section 4). Finally, they argue that the reluctance to sell livestock to smooth consumption cannot be due to high expectations of future prices. If livestock sales are unresponsive to income shocks due to higher expected future prices, this unresponsiveness should dissipate if one controls for them in a village fixed effects model. Households sustained significantly different idiosyncratic income shocks during the drought, but Kazianga and Udry do not observe a higher probability of selling

[1] An increased rate of time preference is the explanation given by Chen, De La Rupelle and Zilibotti (2016) for why households increased savings during the Great Famine in China from 1958-1961.

livestock amongst those most affected by income shocks in the village fixed effects model. This suggests that although consumption smoothing was possible, households deliberately lowered consumption during the drought period.

Secondly, Lange and Reimers (2015) study the effect of drought in Burkina Faso in 2004. The households own sufficient livestock to maintain pre-drought consumption levels if they wanted to. They find significant positive correlations between consumption expenditure, grain stores, cattle sales, and income. Despite this, it seems that consumption during the drought fell, and was not completely offset by sales in cattle or depletion of grain. Surprisingly, households owned 6.04 cattle on average, but had net sales of only 0.2 cattle and 1.33 goats (2015, Table 2). This indicates that livestock holdings were substantial, likely to surpass a critical mass necessary to be used for smoothing consumption, unless the price of livestock had fallen significantly. However, the estimates of rainfall on price indicate a 15% decrease in the price of cattle during the period at most (2015, Figure 3 and Table 8). Indeed, Lange and Reimers argue that the falling price of livestock accounts for why it was not used to smooth consumption. However this explanation appears problematic for standard economic theory, as the value of assets did not appear to drop below the level necessary to induce such immodest falls in consumption.

In Kinsey et al. (1998) there is evidence of households using livestock to smooth consumption but also lowering consumption intentionally. Kinsey writes that 'households decreased food consumption, decreased the frequency of meals, and consumed 'wild foods'' (1998, pp. 94). Although the evidence clearly shows households increased livestock sales to cope with the drought, the extent to which they smoothed consumption is less than what one would expect. This is in spite of drought relief, increasing labour supply to off-farm activities, and the use of cash savings to smooth consumption. Furthermore, around 60% of households in the sample recorded an equal or larger herd size after the 1992-1993 season (1998, Figure 6). In this instance, low livestock prices have significant explanatory power. Kinsey writes 'cattle prices will be low in a drought period both because supply is unusually high and the animals sold will be in a poor condition' (1998, pp. 97). In the presence of relatively well-functioning livestock markets, it would make sense to purchase livestock while prices were temporarily low, rather than use cash to smooth consumption. Despite this, livestock prices only fell by around 10%, suggesting full consumption smoothing was possible but not optimal. Once again, asset worth appears sufficiently high that consumption smoothing was less than would be predicted by standard economic theory.

It is important to note that these households differ significantly from those in Burkina Faso. Those in Burkina Faso own relatively few cattle on average. Despite owning livestock, they are subsistence farmers, and many households earn less than $0.5 a day. By contrast, the sample from Zimbabwe own 10 cattle on average, and are well-established farmers that were given arable land as part of the Zimbabwe Government's relocation programme (Kinsey et al. 1998, pp. 92,90). Although there is less possibility that market frictions prevented the use of livestock sales to smooth consumption, there is a higher possibility that these livestock are seen as a return-earning asset. Indeed, in such cases cattle are used to breed, as a source of draft power, milk, and for commercial sales (Kinsey et al. 1998, pp. 100). This means that returns to in-period saving are potentially higher, which could explain why households retained livestock rather than using it to smooth consumption. However this is not convincing since many of these methods of earning a return would not be relevant during drought years. For example, given that there is no rainfall and thus no land to crop, the use of cattle for draft power would be largely unhelpful. The poor condition of the animals would also lower the returns to using a herd to breed more cattle. Therefore, it is still unclear why consumption smoothing was not optimal when it was clearly possible.

This paper explores the possibility that such behaviour is due to loss aversion within reference-dependent preferences. Such an explanation would be attractive for many reasons. The first is that it would rationalise behaviour rather than produce it from market frictions, which allows consistency with the 'poor but rational' hypothesis (See Schultz (1964), or Duflo (2003)). Secondly, assuming reference-dependency is fairly innocuous on a theoretical level. Such a utility function is convex beneath the reference point, but otherwise identical to a standard concave utility function. Most importantly, there is substantial empirical evidence that supports the existence of these preferences.

Kahneman and Tversky's 'Prospect Theory' (1979) first convincingly identified loss-averse behaviour in a laboratory setting. Their findings have since been tested in the real world by numerous studies, with largely confirmatory results. The most well-known of these relates to the labour supply of New York taxi drivers. While Farber (2008) indicates a lack of importance for reference-dependence, Crawford and Meng (2011) draw more confirmatory conclusions. Card and Dahl (2011) similarly find that some form of reference dependence can explain footballers' attitudes towards domestic violence. Allen et al. (2015) also find evidence for reference-dependent preferences in marathon runners.

Many studies that find evidence for such preferences also try to establish the nature of the reference point itself. Abeler et al. (2011) find empirical evidence in a tightly controlled experimental setting for rational expectations reference points. Gill and Prowse (2012), Ericson and Fuster (2011) and Bartling, Brandes and Schulk (2015), amongst others, obtain similar results. Examples of contrary evidence are Baucells et al. (2011), and Lien and Zheng (2015), who present evidence for lagged status quo expectations. Despite this, not all studies can rule out consistency with more conventional preferences - one example is Baillon et al. (2016). Finally, there is also substantial evidence for loss aversion in different areas, which can be seen as evidence for a necessary condition of reference-dependent preferences.[2] These include real estate (Genesove and Mayer, 2001), investing (Odean, 1998) and trading (Haigh and List, 2005).

Such evidence has provided the justification for a wide range of theoretical literature on reference-dependent preferences, to which this paper contributes. It begins with Kahneman and Tversky (1991) who formalise their 1979 observations in a general model. It has been extended by Bowman et al. (1999), Kremer (2016), but mainly Kőszegi and Rabin (2006, 2007, 2009). Bowman et al. (1999) first state formally the assumptions behind Kahneman and Tversky's original value function. The paper also demonstrates one of the key results of reference-dependent utility, which is that agents prefer certain gains and possible losses to definite losses and possible gains. Building on

[2] Individuals can exhibit loss aversion with concave utility. To this end, such preferences are 'reference-dependent'. They do not, however, exhibit attenuation in 'losses', which is what 'reference dependent preferences' refer to in the context of this paper.

this work, Kőszegi and Rabin have produced three papers detailing various aspects of these preferences. The first develops a general framework where utility is additively separable, consumption utility is linear, and reference points change over time (Kőszegi, Rabin 2006). The second explores risk attitudes, and specifies conditions for personal equilibrium if reference points are stochastic and the agent faces additional income risk (Kőszegi, Rabin 2007). Finally, they explore the notion that information regarding future expectations are themselves carriers of utility, developing a dynamic framework whereby instantaneous consumption utility is a function of not only consumption but also information regarding future period consumption (Kőszegi, Rabin 2009). Kremer (2016) builds on these models to derive optimal consumption when income is stochastic, reference points are endogenous, and the agent faces a liquidity constraint in a two-period model. Agricultural households in developing economies face substantial income risk, difficulty in planning for the future, and display a remarkable level of sophistication. It is therefore appropriate to model their decision-making process within a dynamic stochastic environment. To my knowledge, this is the first paper to adopt such an approach.

Section 1 modifies Bowman et al.'s (1999) model to achieve similar results under assumptions that are more relevant to agricultural households. Section 2 introduces uncertainty into this model. The model is presented differently to the exposition in Bowman et al. (1999), and this is done so that it can be extended into many periods. This extension is carried out in Section 3, alongside a novel result regarding rational expectations in a finite time model. This result suggests that the reference point cannot wholly be constituted by rational expectations. The simulations in Section 3 also mirror the analysis in Deaton (1991), but with reference-dependent preferences. Section 4 demonstrates that when expectations are partly rational, the reference-dependent model is better at explaining observed behaviour than a concave utility model.

Section 1

1.1 Building the Model

This section establishes the basic intuition behind reference-dependent preferences. It then proceeds to build a static, two-period model to illustrate behaviour when income is certain.

Reference-dependent preferences differ from concave preferences in respect of a 'reference point' r. If consumption is above the reference point, it is seen as a 'gain' and a 'loss' if it is below. Furthermore, reference-dependence captures two important features possibly absent from concave utility. The first is 'loss aversion', which states that the disutility from losses exceeds the disutility from failing to consume a similar sized gain. To this end, the utility function is steeper beneath the reference point than above it. The second feature is 'attenuation'. In much the same way concave functions exhibit diminishing marginal utility, reference-dependent functions exhibit 'attenuation' in both losses and gains, which means that the utility function is convex beneath the reference point, and concave above it. This gives rise to the 'S-Shaped' utility function, first described in the famous 'Prospect Theory' (Kahneman and Tversky, 1979). These features can be summarised in assumptions (A1-3):

For: $$U(x,r) = c(x) + u(x,r)$$

(A1) $$u'_x(x,r) > 0\ ,\ \max\{U'(.)\} = U'(r,r)$$

(A2) $$\forall x > r, u''_{xx}(x,r) < 0, \forall x < r, u''_{xx}(x,r) > 0$$

(A3) $$\forall x,y \;\; y > x \geq 0, u(r+y,r) + u(r-y,r) > u(r+x,r) + u(r-x,r)$$

We say that an agent consumes a gain if $x > r$ and a loss if $x < r$. $c(x)$ is consumption utility; a linear function of income x. $u(x,r)$ denotes the 'gain/loss' utility from consuming x units of consumption with reference point r. $U(x,r)$ denotes the total utility from consuming x units of consumption with reference point r. When consumption utility is linear, (A1-3) also hold for $U(x,r)$. A1 is the statement of monotonicity, but also that the utility function is steepest at the reference point. A2 describes attenuation in both losses and gains – as the size of a loss or gain increases the marginal (dis)utility decreases. A3 is equivalent to stating that the marginal utility of a loss is greater than the marginal utility of a comparably sized gain, and captures *loss aversion*. These assumptions are essentially the same as those stated in Bowman et al. (1999). Indeed we can infer A4 from A3 (proof in Appendix 1):[3]

(A4) $$u'_x(r+x) < u'_x(r-x), \forall x > 0$$

There is little dispute over the assumptions necessary to guarantee an 'S-shaped' utility function in consumption, but the behaviour of the reference point is much less clear. How should utility change in response to equal changes in the reference point and income? This paper assumes that someone with more income and higher expectations *is* better off than someone with low income but also low expectations by allowing linear consumption utility. Otherwise, we would be left with the worrying result that utility is *only* gain/loss. Both Kremer (2016) and Bowman et al. (1999) avoid this problem by making consumption utility a function of the reference point rather than consumption. Although theoretically equivalent, this assumption is not intuitive. It implies that consumption utility is increasing in one's reference point (often taken to be expectations), and that the sign of the marginal effect of the reference point on utility is ambiguous. Therefore, this paper also assumes (R1-2):

(R1) $$\forall x, u''_{rr}(x,r) = u''_{xx}(x,r)$$

(R2) $$U(r,r) = c(r) \text{ or } u(r,r) = 0$$

These state that the reference point is treated in the same way as negative income within the gain/loss utility function, and that the gain/loss utility of consuming the reference point is rightly zero. Finally, note that because *U(x,r)* satisfies A1-3, results that are provable for *u(x,r)* are provable for *U(x,r)*. This basic intuition makes analysis much simpler.

With regards to the formation of the reference-point, we have already touched on some of the possibilities. In Kőszegi and Rabin (2009) expectations are wholly rational. Kremer (2016) similarly uses this notion, as well as defining "naïve" expectations, where in-period consumption is not consistent with planned pre-period consumption.

[3] From here on in, this paper replaces *U(x,r)* and *u(x,r)* with *U(x)* and *u(x)*.

Ray (2016) argues that aspirations are environmentally determined, and although aspirations and expectations are not generally equivalent, the argument is applicable to expectations. In Bowman et al. (1999) the reference point is determined in part by the previous period's consumption, and reference point, summarised by (R3):

(R3) $$r_{t+1} = \alpha r_t + (1-\alpha)c_t \ , \ 0 \le \alpha \le 1$$

This paper makes the additional assumption that $\alpha = 1$. The reference point is therefore not at all adaptive, but completely sticky. With regards to the original reference point r_t, this paper assumes for convenience that the reference point is equal to a level of consumption that the agent is accustomed to, which is taken as given. For practical purposes, when income follows a process that is i.i.d, this level will equal expected income.

1.2 The Basic Model

The basic model is a static, two period model where income is certain in both periods. A consumer starts with an income I_1 and receives I_2 the next period, has reference point *r*, and has a rate of time preference β. I begin by assuming $r = I_1 = I_2$. The consumer may save some of his first period income, and does so by purchasing a saving asset this period, that earns a return Φ. The problem can be expressed as

$$Max: V(x_1, x_2) = U(x_1, r) + \beta U(x_2, r).$$

$$\textit{subject to: } x_1 + \frac{x_2}{\Phi} \le I_1 + \frac{I_2}{\Phi}$$

$$x_1, x_2 \ge 0$$

Where $V(x_1,x_2)$ denotes the value of consuming x_1 units of consumption in period 1 and x_2 units in period 2. As this paper is concerned with very poor households who do not usually have access to an interest-earning asset, the rest of this paper assumes $\Phi=1$ unless stated otherwise. The literature widely assumes that an interior solution exists, but this is by no means obvious with our utility function. There are many possible reasons the agent may find it optimal to save all his income, or to save none. To guarantee that an interior solution exists, it is prudent to state the value function in terms of saving:

$$V(s) = U(I_1 - s) + \beta U(I_2 + s)$$
$$V'(s) = -U'(I_1 - s) + \beta U'(I_2 + s)$$
$$V''(s) = U''(I_1 - s) + \beta U''(I_2 + s)$$

Where s denotes the amount saved in period 1 for consumption in period 2. An interior solution will exist if the following two conditions hold:

(B1) $$1.\ V'(s^*) = 0, V''(s^*) < 0$$

(B2) $$2. \lim_{s \to \pm\infty} V(s) < V(s^*)$$

(B1) ensures that s^* is at least a local maximum. (B2) ensures that if *V(s)* is not globally concave, that s^* is a global maximum. Interestingly, (B1) entails A5 when $\beta=1$, and $I_1=I_2=r$:

(A5) $$U''(s^* + r) + U''(-s^* + r) < 0$$

(A5) states that in the neighbourhood of s^*, utility is more concave for gains than it is convex for losses. This property will become useful later, and it is important to note that it does not necessarily hold globally. Indeed, if it doesn't, one can define (A6) that formally describes the shape of *V(s)* as bell shaped:

(A6) $$\exists s_{1,2} \text{ s.t} \begin{cases} s < s_1, V''(.) > 0 \\ s_1 < s < s_2, V''(.) < 0 \\ s < s_2, V''(.) > 0 \end{cases} \text{and } s_1 < s^* < s_2$$

Under Assumptions (A1-3), a maximum exists. Under Assumptions (B1-2), this maximum is global, and constitutes an interior solution. Under (A4), we have $U'(s+r) - U'(-s+r) < 0$, which implies the marginal utility of saving or borrowing is always negative. Hence, it is optimal to consume the reference point when endowed with it in both periods.[4] In order to characterise the model more fully, when income in both states is variable, it is helpful to introduce the concept of the 'coefficient of loss aversion'.

Kahneman and Tversky originally defined this coefficient as the ratio between gains and losses that made the loss just as bad as the gain, using an example of a 50-50 bet (Kahneman and Tversky, 1991). Their studies found that a 50-50 bet to win \$25 or lose \$10 was barely acceptable, implying a coefficient of loss aversion *k* of 2.5 according to $U(-k \times Loss) + U(Loss) = 0$. Differentiating this expression reveals that it is not the same as the coefficient of loss aversion that Bowman et al. (1999) define, which is simply

$$k = \frac{U'(Loss)}{U'(Gain)}, \; Loss = -Gain \; .$$

Formally:

$$k(x) = \frac{U'(r-x)}{U'(r+x)} > 0, \forall x > 0 \qquad \textit{Coefficient of Loss Aversion}$$

In practice, it is convenient to define *k(x)* as a constant and utility as a piecewise function. Utility beneath the reference point is simply the function above the reference point, reflected in both axes, and multiplied by the coefficient of loss aversion *k(x) = k*. Under these assumptions, the first-order condition is

$$U'(I_1 - s) = \frac{\beta U'(I_2 - s)}{k} \; .$$

This equation demonstrates that an interior solution is relatively unlikely – one is very likely to see full consumption or full saving for reference point income in both periods. The full characterisation of the two-period model is in Appendix 1. When average income is low, agents are more likely to resist smoothing consumption, whereas when it is high they are likely to smooth it. These results are achievable in this basic model, and the next section introduces income uncertainty.

Section 2

[4] Imposing $\beta < 1$ changes nothing here.

2.1 Introducing Uncertainty

This section introduces randomness to second-period income. The model is presented differently to those of Bowman et al. (1999) and Kremer (2016), but generates the same results. Using a particular function to illustrate behaviour is not possible with reference-dependent preferences, since in practice one must specify a piecewise function and hence undertake arduous piecewise integration. There is, however, a particular case uncovered in the literature that is relatively straightforward. This is the special 'exponential-normal case', which is excluded here for presentational ease. Instead, a more general model is set out.[5]

The two period value function now introduces the expectations operator 'E', and allows second period income to be normally distributed with mean *r* and variance σ^2:[6]

$$V(s) = U(I_1 - s) + E\beta U(I_2 + s)$$

$$I_2 \sim N(r, \sigma^2)$$

The conditions for the previous case apply here, if s^* is to be a unique solution. Assuming they hold, is s^* positive or negative? The following assumptions on marginal utility simplify the analysis.

(A7a) Marginal utility is bell shaped

$$\exists s_{1,2} \text{ s.t} \begin{cases} s < s_1, U'(.) > 0 \\ s_1 < r < s_2, U'(.) < 0 \\ s < s_2, U'(.) > 0 \end{cases} \text{ and } s_1 < r < s_2$$

(A7b) Marginal utility is convex

$$\forall x, U''(x) > 0 \text{ And} \begin{cases} x < r, U'(x) > 0 \\ x > r, U'(x) < 0 \end{cases}$$

---

[5] The problem is given by max*{f(x)}* s.t

$$x > r, f(x) = e^{-s} + E\beta(\int_{-\infty}^{r-s} 2e^{I_2 - r + s} pdf(I_2)dI_2 + \int_{r-s}^{\infty} e^{-(I_2 - r + s)} pdf(I_2)dI_2$$

$$x \leq r, f(x) = 2e^{s} + E\beta(\int_{-\infty}^{r-s} 2e^{I_2 - r + s} pdf(I_2)dI_2 + \int_{r-s}^{\infty} e^{-(I_2 - r + s)} pdf(I_2)dI_2$$

[6] Income is unlikely to be normally distributed in reality, especially in the presence of droughts. This is accounted for in Section 4.

(A7a) makes marginal utility a continuous, bell-shaped function. In practice, it is very difficult to specify a function that satisfies (A7a), so this paper assumes (A7b) since it corresponds to piecewise functions with well known utility forms, such as logarithmic, power, and exponential. The sufficient conditions for a maximum that is an interior solution are:

(B3) $$\exists s^* \text{ s.t } V''(s^*) = U''(I_1 - s^*) + E\beta U''(I_2 + s^*) = 0$$

(B4) $$V''(s^*) = U''(I_1 - s^*) + E\beta U''(I_2 + s^*) < 0 \text{ , } \forall s, V''(s) < 0$$

These conditions are not satisfied. In fact, conceptually, there may be four possible turning points, for which it is difficult to validate the sign of the second derivative. Fortunately, we can prove Propositions 1 and 2, which allows us to prove Propositions 3 and 4.

Proposition 1: if I_2 is a variable that is a mean preserving spread of r, and is symmetrically distributed, then U(x) obeys Jensen's inequality (Proof in Appendix 1)

Proposition 2: Under similar circumstances, U'(x) also obeys Jensen's inequality (Proof in Appendix 1)

Proposition 3: There exists an $s^*<0$ (Proof in Appendix 1).

Proposition 4: *When k is sufficiently high, s^* is unique (Proof in Appendix 1).*

In the absence of a liquidity constraint, the agent behaves as Bowman et al. (1999) show in Fig. 1(a). In their model, the agent cannot borrow against an income that is uncertain. Therefore, if income is unbounded as assumed in this model, no borrowing is permitted. Hence, it is optimal to consume cash on hand in both periods. Therefore, a 1% fall in expected income growth leads to a 1% decrease in expected consumption growth, as predicted in Fig. 1(a) (Bowman et al. 1999). Furthermore, as long as the agent is sufficiently loss averse, this solution is unique.

If utility is concave, then future uncertainty is a motive for saving, however when utility is reference-dependent, it is a motivation for borrowing. Uncertainty introduces the possibility of sustaining a loss, which the agent enjoys borrowing against. He trades off possibility of second period loss for certain first period gain. Note that because losses attenuate as well as gains, if the agent is not loss averse or very patient, saving all income may be more attractive than consuming the reference point. It will therefore be implicitly assumed that Proposition 4 holds, or that it doesn't but the agent lacks sufficient cash-on-hand to find zero consumption optimal. Under these assumptions, Result 1 holds, which is an important consequence of reference-dependent utility.[7]

Result 1: Agents with reference-dependent preferences prefer risky losses and certain gains when they are loss averse. As loss aversion tends to 0, it may be optimal to save infinitely (Proof in Appendix 1).

2.2 Comparative Statics

Around the optimal solution, all intuitive results hold. Higher first period income increases saving, higher impatience decreases saving, and an increase in the uncertainty of second period income stimulates further borrowing.[8]

$$\frac{ds^*}{dI_1} = \frac{u''(I_1 - s)}{u''(I_1 - s) + E\beta u''(I_2 + s)} > 0$$

$$\frac{ds^*}{d\beta} = -\frac{Eu'(I_1 - s)}{u''(I_1 - s) + E\beta u''(I_2 + s)} > 0$$

$$\frac{ds^*}{dUncertainty} < 0$$

Note that this does not contradict the model of Bowman et al. (1999), in which uncertainty increases or decreases saving. Had we considered the case in which mean

[7] This is similar to Theorem 2 in Bowman et al. (1999).

[8] Proofs of these results are straightforward, and follow immediately from the results from previous sections. They are left out for presentational ease.

income in both periods is higher than the reference point, the response of saving to increased uncertainty would be positive, and we would get the same result as Proposition 5 (Bowman et al., 1999).

This completes the characterisation of the two-period stochastic model, and there are already some implications for policy. The model predicts that when a household receives average income, they have a positive demand for credit. In the same situation, a more risk-averse household with concave preferences would prefer to save. Hence, we see a potential for risk-sharing agreements that are welfare-enhancing even in the absence of idiosyncratic income variation. This is further discussed in Section 4.

Section 3

3.1 Saving Over a Finite Horizon

This section considers the agent's problem when income is independently and identically distributed over many time periods, using the value function methodology. It shows that households accumulate savings to use as a buffer stock to smooth income. It also shows that when households cannot consume a gain, they consume the absolute minimum – a subsistence level of consumption. The model is not fully characterised analytically, but approximated using Matlab.

The setting is finite with T periods. The agent starts with some income I_1, and receives a new stochastic income each period with known distribution. The agent maximises t-period utility by choosing T-t saving values, where such a set of choices constitutes an optimal plan. It is helpful in this case to also specify the Lagrangian, since the optimality conditions are more general, and hence allow corner solutions as well as interior ones.

First period utility is given by:

$$V_1(.) = U(I_1 - s) + E_t \sum_{n=1}^{T} \beta^n U(I_{n+1} - s_{n+1} + s_n)$$

With Lagrangian:

$$L(.) = U(I_1 - s) + E_t \sum_{n=1}^{T} \beta^n U(I_{n+1} - s_{n+1} + s_n) + \sum_{n=1}^{T-1} \lambda_n (I_n - s_n)$$

A set of choices S constitutes an optimal plan at time t if:

$$S_t = \{s_t ... s_T\} = \arg\max V_t(S_t)$$

$$s_T = 0$$

Subject to:

$$s_t(-E_t U'(I_t - s_t + s_{t-1}) + E_t \beta U'(I_{t+1} - s_{t+1} + s_t) - \lambda_t) = 0$$

$$\lambda_t (I_t - s_t) = 0$$

Here there are T-1 equations and T-1 unknowns, so the household is able to solve for in-period saving. This happens in every period, and the model unfolds as income is realised. This paper makes three predictions that draw on the results from Sections 1 and 2:

Prediction 1: When income is persistently above the reference point, consumption smoothing occurs.

Prediction 2: When income is low relative to the reference point, agents avoid consumption smoothing.

Prediction 3: When consuming the reference point is possible, it is optimal.

Before mathematically calculating the value function, the problem is worth attempting to solve analytically. Indeed, one can fully characterise the interior solution when $T = \infty, \beta = 1$.[9] Despite this, there is no guarantee that the interior solution is the optimal one. Furthermore, greater flexibility in assumptions is desirable; therefore it is now prudent to calculate the value function mathematically. The household solves

$$\max V_t(k_t) = \{u(k_t, c_t) + \beta E V_{t+1}(k_{t+1}, c_{t+1})\},$$

subject to

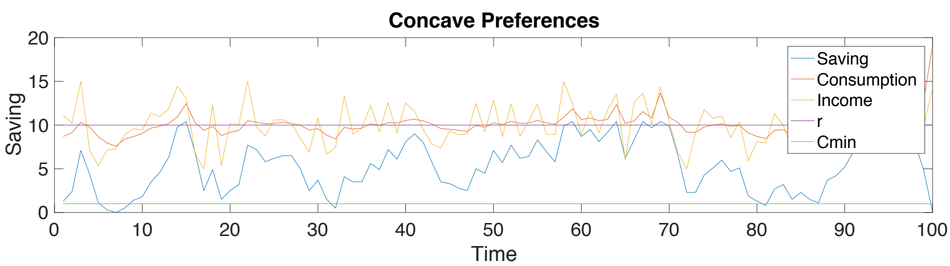
[9] See Appendix 2

$$k_{t+1} = k_t - c_t + I_{t+1}$$
$$0 \le c_t \le I_t$$
$$I_t \sim N(\bar{I}, \sigma_I^2) \quad .$$

Where k_t denotes the cash on hand or capital at time t. Unlike in Sections 1 and 2, sufficient conditions for the existence of an optimal solution are well-known within problems of dynamic programming, and are given by the 'Theorem of the Maximum'. These conditions are satisfied and are not discussed here.[10] The value function is calculated for linear consumption utility, and gain loss utility that is exponential – power and logarithmic utility generate the same results. Households are slightly impatient, and the coefficient of loss aversion is *2* according to the observations made by Kahneman and Tversky (1991). As it is not realistic to ever consume nothing, there is a minimum 'subsistence' level of consumption of 1 for illustrative purposes. Income with mean 10, equal to the reference point *r,* and standard deviation 2.5 is simulated for 100 periods. The parameters are summarised in Table 1, the results in Panels 1-4.

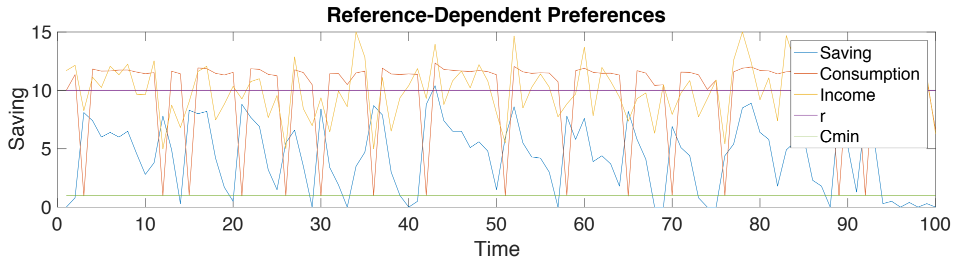
c(x)	*0.1x*
$u(x), x \ge r$	$-e^{x-r}$
$u(x), x < r$	$ke^{x-r} - k - 1$
k (Degree of Loss Aversion)	*2*
Reference Point	*10*
Starting Income	*10*
β (Degree of Impatience)	*0.9*
Income	$I_t \sim N(10, 2.5)$

Table 1

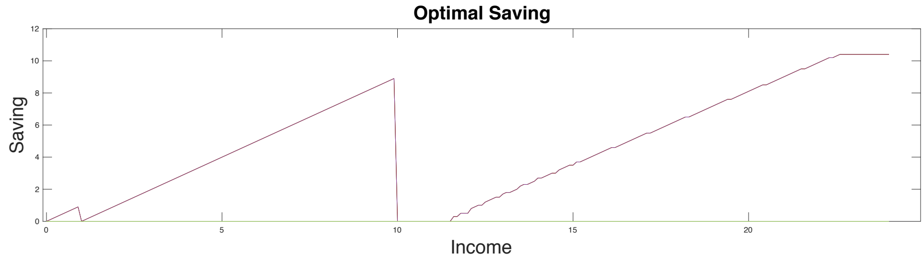


Panel 1

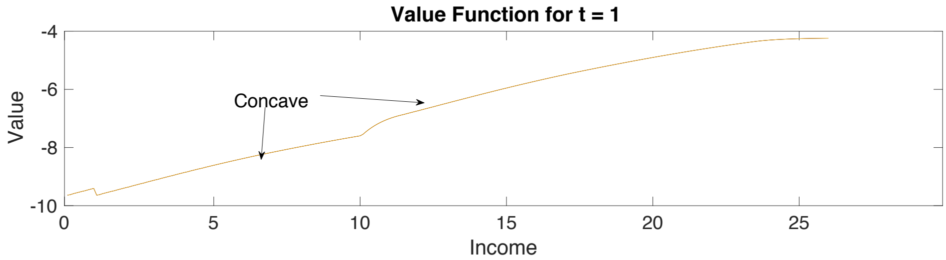
[10] The theorem states that if the constraint set is non-empty, compact and continuous, utility is bounded and continuous, the transition function is continuous, and the cumulative distribution of the stochastic process has the Feller Property, then there exists a solution to the problem and the value function is continuous. For a comprehensive discussion, see Stokey-Lucas, 1989.



Panel 2



Panel 3



Panel 4

These graphs are very informative. The first panel shows behaviour with concave preferences that are exponential, and mirrors Deaton's (1991) albeit with different parameters. As Deaton found, consumption is smoothed effectively with modest asset holdings. It makes for useful comparison because the functional form is the same as the functional form of the reference-dependent function in the domain above the reference point.

The second panel shows the same behaviour under reference-dependent preferences. The red line is relatively smooth above the reference point, indicating that the first prediction was correct – above the reference point, the agent smoothens consumption. Prediction 3 is also confirmed by Panel 3, which shows that when income is above the reference point, so is optimal consumption. Finally, Panels 2 and 3 also confirm Prediction 2. If consuming the reference point is not feasible, the agent endures subsistence consumption, illustrated by the deep red troughs in Panel 2. This means that sometimes, consumption smoothing is possible, but not optimal. Intuitively, if incurring a loss is certain, then it may be optimal to incur a maximal loss since they attenuate in size. Due to loss aversion, they attenuate more quickly than the returns to savings attenuate, making subsistence consumption optimal. This leads to Result 2, which is stated rather than proved, which is an equivalent of Result 1 but for stochastic dynamic environments.

Result 6: An agent in a stochastic dynamic environment will smooth income when it lies above the reference point. If the agent cannot feasibly consume a gain, the agent avoids smoothing consumption, and saves all his income (Unproved).

If we could observe the reference point, it would be relatively straightforward to test for this behaviour using regression discontinuity design. The approach is not completely new. Allen et al. (2015) use regression discontinuity design to test for reference-dependent preferences in marathon runners, and Engström et al. (2011) test loss aversion in Swedish taxpayers. The model predicts that when cash on hand is beneath the reference point, there is a perfect correlation between income and saving, with constant term zero. Given data on a many households over time, estimate the following:

$$\ddot{s}_{it} = s_{it} - r_{it}$$
$$c_{it} = \beta_0 + \tau D_{it} + \beta_1 \ddot{s}_{it} + \beta_2 \ddot{s}_{it} D_{it} + \zeta X_{it} + u_{it}$$

Where D is a binary variable that takes the value 1 if $\ddot{s}_{it}$ is positive, and 0 otherwise. Cash on hand for individual i and time t is given by s_{it} and their reference point by r_{it}. X_{it} is a vector of controls, and u_{it} is the transient error. One then estimates the model round the cut-off $\kappa = 0$. If preferences are concave, then there should be no discontinuity – we would expect consumption to be a continuous linear function of cash on hand. If preferences were reference dependent, we would expect a discontinuity. For a given cut-off equal to the reference point, one can test:

$$H_0 : \tau = 0$$
$$H_a : \tau \neq 0$$

Noting further that under concave preferences we expect $\beta_2 = 0$ and $\beta_1 > 0$, and vice versa for reference-dependent preferences. In practice, this approach may not be easy with 'Large N Small T' panel data, as usually there is no data on the reference point. One would need to impose some additional assumption to estimate r_{it}. For example, under lagged status quo expectations, $r_{it} = c_{it-1}$. If expectations are perfectly rational, then $r_{it} = E_{it-1}c_{it} = c_{it}$.[11] This may raise problems in fixed effects models, since including lags of the dependent variable violate contemporaneous exogeneity necessary for the unbiasedness of OLS. Secondly, saving is invariably auto-correlated, so a test that relies on OLS will have low power since OLS is relatively inefficient. These raise additional considerations for future empirical research.

3.2 Rational Expectations

The model assumes that the reference point is completely determined by the previous period's reference point. In light of the evidence for other sorts of reference-point formation, especially rational expectations, this assumption seems questionable. To address this doubt, this section argues against rational expectations and presents a proof of their inherent unfeasibility.

There are a number of reasons we may not want to assume that households have rational expectations. Firstly, it entails households are abnormally sophisticated. Rational expectations mean that the reference point equals expected consumption. Thus, agents choose not only consumption but also their reference point for the next period. They would need to trade off the benefits of smoothing income, say, and the disutility from having higher expectations, which is arguably unrealistic. Although this paper relies somewhat on the "poor but rational" hypothesis (See Shultz (1964) or Duflo (2003)) this is a step too far, since we do not typically think expectations being wholly under our control. Secondly, the case for sticky or status quo reference points seems strong here. Individuals may get used to a certain level of consumption that serves as their reference point, irrespective of the future. A farmer may forecast a drought months in advance, but may still feel as if he suffers a loss when drought arrives. Reference points seem contingent on the past, which is inconsistent with rational expectations. Finally, in a stochastic setting, agents have been shown to overweight small probabilities and underweight large ones (Kahneman and Tversky, 1979). In this regard, the calculation necessary to work out the true rational expectation will be incorrect, which implies that

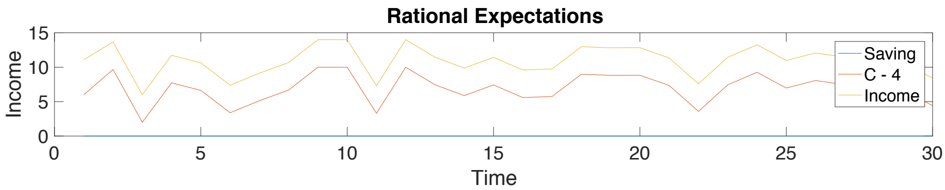
[11] If there is no uncertainty, and expectations are perfectly rational, expected consumption equals actual consumption. This is not necessarily the case if there is income uncertainty.

expectations will be irrational, but 'second-best', in the sense that they are rational given the probability-weighting function.

To test the feasibility of rational expectations, the model was run with rational expectations reference points instead of sticky reference points. According to rational expectations, the reference point in period t should equal the agent's t-1 expectation of consumption in period t.

$$r_t = \int_{Y\min}^{Y\max} g(Y_t + k_{t-1} - g(k_{t-1}, r_{t-1}), r_t) m(Y_t) dY_t$$

Where $m(Y_t)$ corresponds to the probability distribution function of t-period income, and $g(k_t)=c_t^*$. The result is that agents behave like Campbell and Mankiw's (1989) 'rule of thumb' consumers, and consume cash on hand in each period.



To see why, consider the final period.[12] In period T, the agent consumes everything he has. In period T-1, his reference point is therefore given by:

$$r_T = \int_{Y\min}^{Y\max} (Y_t + k_{t-1} - g(k_{t-1}, r_{t-1})) m(Y_t) dY_t$$

$$r_T = \overline{Y}_t + k_{t-1} - g(k_{t-1}, r_{t-1}))$$

Hence, increasing saving in *T-1* increases the reference point in *T* by the same amount. By (R2), T-period utility is invariant to saving in *T-1*, which means that it is not optimal to save anything. However if it is not optimal to save anything in *T-1*, the same argument can be applied to *T-2*, and so on until period 1. Rational expectations predict zero savings, which is not observed empirically, suggesting they are unfeasible. This result is somewhat surprising, especially given some of the rational expectations-based work such as Kőszegi and Rabin (2009), Kremer (2016) and Goette et al. (2014). The proof warrants further discussion because it is an unobvious knife-edge result, but also one that is ambiguously so. There are three reasons I can think of that, if true, would make this a knife-edge result.

[12] Full proof in Appendix 2.

Firstly, it is proven for a finite model. The proof may not be valid if it was applied to an infinite horizon model. However the question of whether or not consumers face finite or infinite horizon has no unambiguous answer. It is feasible that very poor economic agents 'see the end' in the near future, which would support the finite model in this context.

Secondly, it is proven for the case where consumption utility is linear. While I do not suspect we will ever be able to prove the existence of utility functions that are concave, convex, then concave again, such a possibility is widely entertained in the literature (Kőszegi and Rabin 2006, 2007, 2009, Kremer 2016). One should not be surprised that this result does not hold if consumption utility is non-linear, despite the fact that one would be surprised were consumption utility non-linear.

Thirdly, and rather subtly, we should recall the motivation for writing $U(x,r) = c(x) + u(x,r)$. Consumption utility is necessary because otherwise we are left with the intuition that *only* consumption relative to the reference point matters for utility. We therefore wished to make utility increasing in equal increases in consumption and the reference point. This could easily have been done by writing

$$U(x,r) = c(r) + u(x,r) .$$

T-period utility is therefore no longer invariant to saving, so the proof is not necessarily valid. The reason that this objection lacks clout is that it is extremely unintuitive, for the reasons outlined in Section 1. Despite this, as assumptions they are theoretically equivalent.

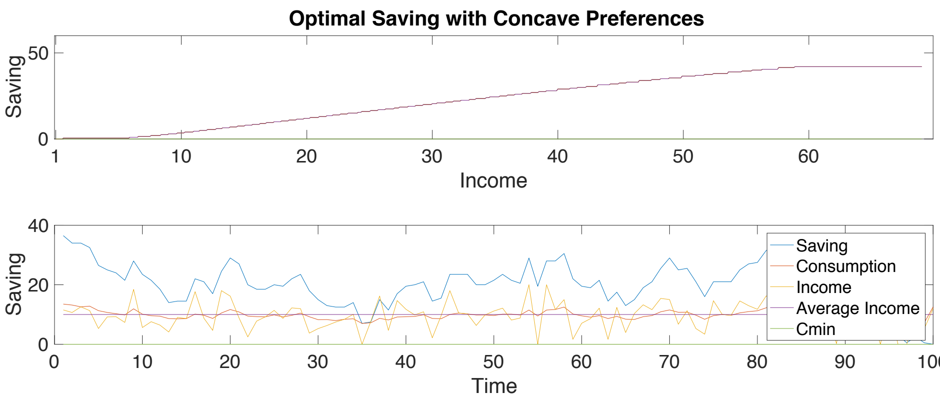
Thus far, this paper has assumed that the reference point is determined by the previous period's. This assumption is questionable given the empirical evidence, however the next best alternatives are unfeasible in a finite model. Therefore, sticky reference points are a reasonable assumption to make in this context. The result may not hold in reality, and to account for this the next section incorporates quasi-rational expectations to explain consumption behaviour.

Section 4

4.1 Economic Application to Consumption Smoothing

This section tests both the reference-dependent model and a standard model against the observations made by Kazianga and Udry (2006), Lange and Reimers (2015) and by Kinsey et al. (1998), to see if they can explain consumption behaviour. It first shows that when preferences are concave and asset stocks are high relative to income, a period of low income should not induce significant falls in consumption. This result is robust to market price effects. I then explore the possibility that consumption could fall if preferences are reference-dependent. By introducing rational expectations to reference point formation, the reference-dependent model can predict consumption-smoothing through asset sales, a retention of assets, or a combination of both. The predictions are therefore consistent with the evidence on livestock sales and consumption reduction during periods of crisis.

Economic theory predicts that when preferences are concave and income is low, if a household owns assets they will be used to smooth consumption. Since the households of interest maintain relatively large asset stocks in reality, this paper now assumes that assets earn a return of 20% and start with a higher asset base.[13] Higher returns increase the incentive to save, so if it still generates behaviour where households smooth consumption, then *a fortiori* it will be valid for more realistic returns on assets. Optimal saving is graphed below, as well as behaviour for 100 time periods.



[13] This assumption is a natural one to make, since livestock earn returns in various ways, such as milk (Masikati, 2010), draft power (Chimonyo et al., 1999), or sales (Kinsey et al., 1998).

Clearly, income is smoothed very well and hardly ever differs significantly from average income. The first panel demonstrates that even when asset worth equals mean income, optimal saving is only 35%. If one imposes a very high level of risk aversion (equal to 10), this number is still on 50%. Even with a function where the coefficient of relative risk aversion is 10, if assets on hand are twice mean income, optimal consumption is still 84% of mean income. If asset worth is high, then optimal consumption is very close to average income. Hence, provided households own assets, agents with concave preferences should not significantly reduce consumption from average income.

Models with concave utility in which livestock, and more generally savings, are used as a buffer stock, are widely studied (Carroll 1997, Deaton and Campbell 1989, Deaton 1989, 1991, 1997, Dercon 2002, Kinsey et al. 1998, amongst others). As discussed at the start of this paper, its predictions are at odds with more recent studies on households during drought. Kazianga and Udry (2006) observe significant drops in consumption in response to income shocks. Indeed, during the droughts in Burkina Faso from 1984-85, Fafchamps et al. (1998) show that livestock sales made up for at most 30% of consumption shortfalls. Lange and Reimers' (2015) evidence suggests that households absorbed income shocks primarily through lowering consumption, rather than other consumption smoothing mechanisms, although they were available. Households own 6.04 cattle on average, but had net sales of only 0.2 cattle and 1.33 goats (Lange and Reimers, 2015, Table 2). Kinsey et al.'s (1998) data also suggests that households reduced food consumption.

This paper now deals with the possibility that the apparent reluctance to smooth consumption is due temporarily low livestock prices. Drought therefore represents both an income shock, and a shock to livestock worth. Consequently, consumption smoothing through the sale of livestock is very costly, especially if households believe expected future prices will be higher.

Within this model, as long as the value of assets relative to income is reasonbly high, consumption decreases will be modest. Price shocks in the aforementioned cases were not large enough to damage livestock worth significantly. For example, in Kinsey et al. (1998), the price of livestock fell only by 10%. Hence, it seems unlikely that the price had fallen so low that the worth of livestock during drought would be less than average income.

Secondly, if households believe expected future prices will be higher, there is an additional incentive to retain livestock. The model accounts for this, since assets earn a large return. When assets earn a large return, the opportunity cost of selling livestock is the same as when the household believes the price is temporarily low. Households still trade-off lower present income for higher future income. In this model, optimal present consumption is still high as long as asset worth is large, so this argument has no traction. It cannot explain why households find consumption smoothing sub-optimal when it is possible. If this analysis is repeated but for a reference-dependent utility function, the results are the same. The next section adds rational expectations in order to explain this behaviour.

4.2 Rational Expectations Revisited

This section proposes a novel explanation based on rational expectations to explain why households do not smooth consumption during drought. Despite the inherent unfeasibility of rational expectations within a finite model, they still retain significant intuitive appeal. This section explores the possibility that drought causes households to revise down their expectations of consumption and future income. Consequently, consumption falls and households find consumption smoothing sub-optimal.

Drought is characterised by prolonged periods of below average rainfall. It is often unexpected, and farmers awaiting the rainy season cannot forecast rainfall levels. However, when rainfall is persistently below average, households may enteratain the possibility of drought more seriosuly, thus revising down their expectations of rainfall and consumption. For households with reference-dependent preferences, a lower reference-point suggests lower consumption, and possibly some dissaving in order to smooth it. Such behaviour would explain why some households lower consumption in response to drought, and why some also increase livestock sales. This section builds a model to test this hypothesis.

Drought is modelled similarly to the 'Two State Markov Process' in Deaton (1991). The household undergoes a period of drought with probability p. This drought lasts for q periods. Income follows the process I_1 which is the same as in Section 3. If the household undergoes drought, income now follows the process I_2. Average drought income is lower and has lower variation. This reflects how households that suffer covariate shocks also experience idiosyncratic income risk. Subsistence consumption is given by C_{min}, which allows these income streams to be summarised as follows

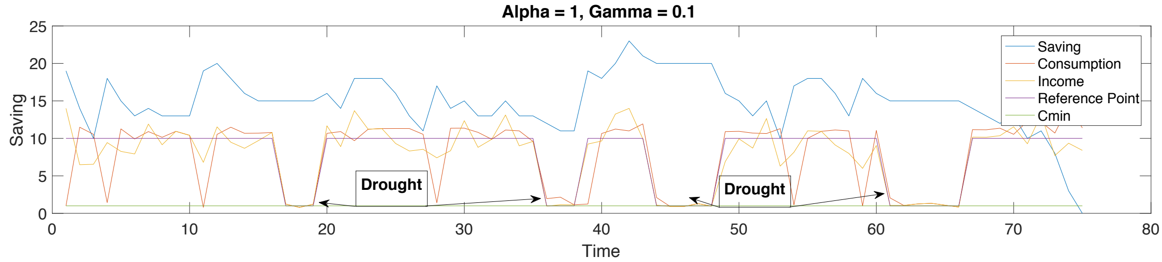
$$I_1 \sim N(\overline{I_1}, \sigma_1^2)$$
$$I_2 \sim N(C_{\min}, \sigma_2^2)$$

If the household is not experiencing drought, the reference point is determined by the first period's reference point and the previous period's consumption. If the household experiences drought, the reference point changes immediately to reflect an instantaneous revision of the household's expectations. These expectations cannot be purely rational, as was argued in the previous section. Instead, I suppose that since the household knows drought income is lower than non-drought income, they revise down their expectations of in-drought consumption by a linear function of $\overline{I_1}$.[14] This can be summarised by

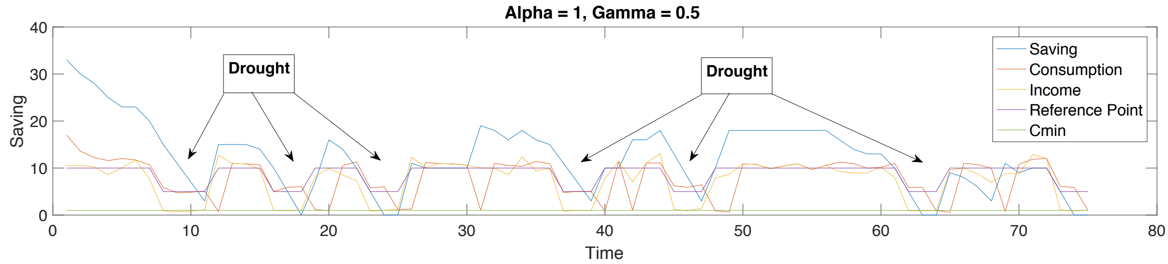
$$r_t = (1 - drought) \times (\alpha r_1 + (1-\alpha)c_{t-1}) + drought \times f(\overline{I_1})\ ,\ f(\overline{I_1}) = \gamma \overline{I_1}\ .$$

Where *drought* is a binary variable that takes the value 1 if the household is experiencing drought, 0 otherwise. $f(\overline{I_1})$ represents the linear function of income that determines the household's drought period reference point. Expectations are not fully rational, as households are naïve. If the household is experiencing drought, it expects to be in drought forever. If the household is not in drought, they expect income to remain as normal. This can be seen as capturing some kind of seasonal optimism or pessimism, and simply allows behaviour to be illustrated more vividly. Panels are presented for a variety of different parameter values:

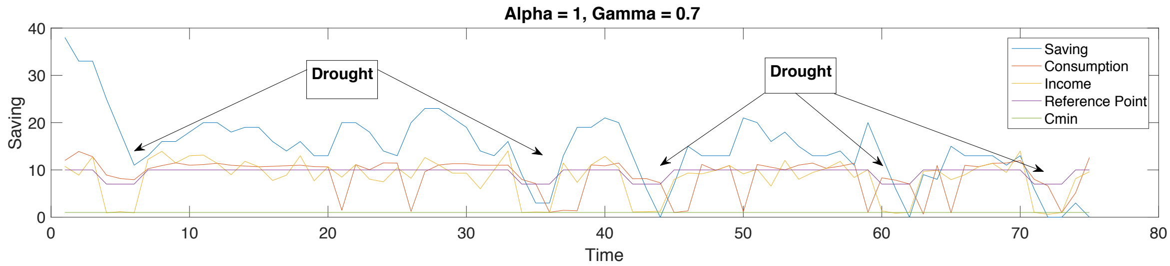
[14] Although unrealistic, it is not problematic since its function is simply to illustrate behaviour when expectations are revised down.



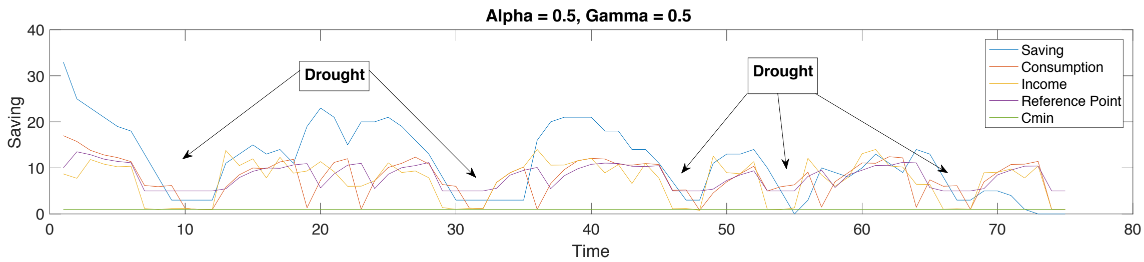
Panel 5



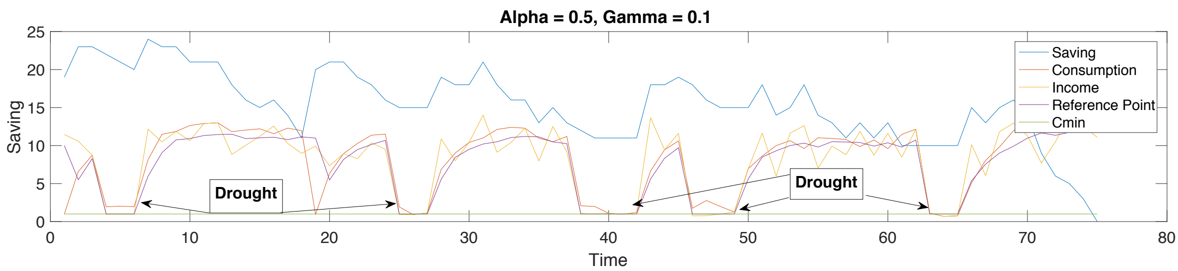
Panel 6



Panel 7



Panel 8



Panel 9

Panels 5-7 illustrate the way in which consumption behaviour is observed. A number of different values for γ are presented to reflect the way drought effects expectations.

When γ is low and households significantly revise down expectations, very little consumption smoothing occurs, if at all. For higher values of γ, consumption is reduced less, and households dis-save more in order to smooth consumption (provided assets are available). Due to the frequency of drought in this model, not enough time is really provided for households to build up assets that they otherwise would, but one can see quite clearly, in Panel 6 for example, that the small red peaks above the purple troughs represent attempts made at smoothing consumption around a lower, 'drought-optimal' level. When assets are available for smoothing, they are used. Finally the last two panels illustrate behaviour when the reference point also depends on the previous period's consumption. There is naturally more movement in the reference point, but during drought, behaviour is roughly similar to the previous three panels. Since the reference point may partly depend on previous consumption, this consistency can be seen as a kind of robustness check for the model.

4.3 Discussion and Policy Implications

Consumption smoothing seems to be an art that is practiced to different degrees by households across developing countries. This model is consistent with all of these degrees, whereas a standard model of concave utility is not. In Kazianga and Udry (2006), households were characterised by small amounts of consumption smoothing using grain stores, and very little through livestock sales. These households were especially poor, and owned few cattle. It therefore makes sense to model these households as viewing the onset of drought particularly badly, and significantly revising their expectations of consumption. This behaviour is consistent with Panel 5, which shows significant reductions in consumption, and small amounts of consumption smoothing, and crucially, the maintenance of a relatively large asset base.

In Lange and Reimers (2015), households were characterised by partial consumption smoothing using both grain and livestock. These households were richer , and owned approximately 6 cows on average (Lange and Reimers, 2015, pp. 14). Furthermore, they had access to larger grain reserves, and well-functioning livestock markets. Consequently, it would make sense for these households to revise their expectations down less, since there was a more abundant range of consumption-smoothing alternatives. Their behaviour is therefore best represented by Panel 6.

In Kinsey et al. (1998), it is suggested that households owning large herds decreased consumption in response to drought, but also made significant use of livestock markets to smooth consumption. This is mirrored in Helgeson et al. (2016), Rozenweig and Wolpin (1993), Swinton (1988) and Watts (1983). In the majority of these cases farmers are well-established and relatively wealthy. As was touched on earlier, the households in Zimbabwe own 10 cows on average and benefit from their own plot of land which was received through the Zimbabwe Governemnt's Land Reform Program. These households suffer from two conflicting effects when drought hits. On the one hand, larger assets livestock holdings serve to protect consumption relatively well from shock to income. On the other, since drought prevents the emergence of crops almost completely, the shocks to income will be especially large in the sense of income foregone. Panel 7 vividly illustrates this sort of situation. During each of the droughts that were simulated in 75 periods, assets are run down significantly to finance consumption needs. Although significant smoothing occurs, we also see households choosing to reduce consumption from pre-drought levels.

From this analysis, one can see that the degree to which consumption smoothing is optimal depends on the degree to which expectations are lowered. Furthermore, this section conjectures that poorer households, in less developed communities, are most likley to do this. As long as some of these households exist, the model has a number of policy implications.

As touched on Section 2, these households will have a positive demand for microcredit even when income is relatively high. It therefore suggests that the provision is microcredit can be welfare enhancing even in the absence of income shocks.

Secondly, if households decrease their expectations then gain/loss utility may not fall in a drought, it could even increase. It therefore suggests that households are not significantly worse off during drought since the reference point is itself a carrier of utility. Worryingly, it suggests that the impact of drought relief measures are probably more limited for this subgroup of the population. Indeed, if they expect to receive drought relief and this is incorporated into their expectations, receiving it will only make them better off in respect of consumption utility. The extent to which welfare is determined by consumption utiltiy is not known, so this would be an important area for future research.

Finally, suppose the other proportion of the population have concave preferences. The correlation between the two groups' savings is likely to be negative during drought, or very low. As was shown in this section, large covariate shocks have heterogenous effects on behaviour. Since the need to run down savings in these situations is not uniform, there is clearly scope for risk-sharing arrangements that are welfare-enhancing. In reality, such arrangements are pervasive in developing countries in the form of rotating credit and savings associations (Roscas). Why people join Roscas is a puzzle. Theories suggest it is to finance the purchase of a lumpy, durable good (Besley et al., 1993), or to enforce saving when preferences are time inconsistent (Gugerty, 2007). An interesting avenue for future research would explore the possibility that such associations exist due to reference-dependent preferences.

Conclusion

This paper attempts to explain why households in developing countries seem reluctant to use livestock as a means to smooth consumption. Existing theories do relatively well at explaining this behaviour, but shed no light on some of the situations that have been considered, where households deliberately reduce consumption during drought. This paper explores the possibility that such behaviour is due to reference-dependent preferences.

To do this, it was necessary to move the existing theoretical literature one step forward by characterising savings behaviour with reference-dependent preferences in a dynamic stochastic environment. The model predicts that households will not smooth consumption if they are forced to incur a loss. It also suggests that if reference points relate to expectations, and drought causes households to lower expectations of consumption, assets will not be used to smooth income. The extent to which consumption smoothing is possible but sub-optimal depends on the extent to which households lower their expectations. Applying this model to Burkina Faso and Zimbabwe, it is significantly better than a model of concave utility at predicting observed behaviour.

That said, the validity of these results is limited by a number of factors. For example, the mechanism by which reference points change in Section 4 is necessarily unrealistic. One would hope, however, that a more sophisticated model that completely internalises rational expectations would generate the same results. Secondly, the model is also not calibrated to data. On inspection, the reference-dependent model picks up the shortfalls of a concave utility model well, however until the models are tested side by side, and fit to data, one should be hesitant to support either model wholeheartedly. Indeed, it is not unlikely that concave utility would fit data as well as, if not better than, a reference-dependent model. Finally, this paper makes no claims to external validity. There is compelling evidence that preferences are reference-dependent, but until these studies are focussed on communities in developing countries, the applicability of these results is still in doubt. It remains to be seen if households' preferences are reference-dependent, and hence, whether or not this paper has any implications for policy at all.

If some proportion of the population do have reference-dependent preferences, there are a number of interesting policy implications. Households will demand microcredit even when income is not particularly high. Therefore, there is more theoretical justification for the provision of microcredit in the absence of covariate shocks. The welfare effects for drought relief may not as significant as hoped. Most interestingly, if preferences are not homogenous, then there is significant scope for risk sharing agreements within communities. Specifically, the model predicts that reference-dependent households should be willing to lend their assets during drought, and households with concave preferences should be willing to borrow these assets. Hence, there is room for a pareto improvement. This highlights the need for future research to focus on establishing the existence of such preferences within communities in developing economies.

Appendices

Appendix 1

Proof of A4 from A3

By the definition of the derivative:

$$u'(r+x)-u'(r-x)=\frac{u(r+x+h)-u(r+x)+u(r-x-h)-u(r-x)}{h}$$

By A3:

$$u(r+y)-u(r+x)+u(r-y)-u(r-x)<0$$

$$\text{Let } y=x+h$$

$$\therefore u'(r+x)-u'(r-x)=\frac{u(r+x+h)-u(r+x)+u(r-x-h)-u(r-x)}{h}<0$$

$$\text{A4: } \therefore u'(r+x)<u'(r-x)$$

Characterisation of the Basic Model

Utility is given by:

$$U=u(I_1-s)+\beta u(I_2+s)$$

Suppose $I_{1,2}<r$

Optimality conditions are:

$$s^*(-u'(I_1-s^*)+\beta u'(I_2+s^*)-\lambda)=0$$
$$(I_1-s^*)\lambda=0$$

Suppose $I_2<I_1$ and $\frac{dU}{ds}(0)>0$

$$\forall s<r-I_1, \frac{dU}{ds}>0$$
$$s^*\geq r-I_1$$

Depending on whether an interior solution exists, s could equal r-I_1, some number greater, or the corner solution I_1. Crucially, saving is positive if next period income is higher than today's, and negative if today's income is higher than tomorrow's.

Similarly for when $\frac{dU}{ds}(0)<0$. Similarly for when $I_2\geq I_1$.

Now suppose $I_2 \le r, I_1 > r$

$$\text{If } \frac{dU}{ds}(0) > 0 \text{ then } \forall s < r - I_2, \frac{dU}{ds} > 0$$

$$\text{And } u'(r+s) > u'(I_1 - s), \forall s \le I_1 - r$$

$$\therefore s^* = \frac{I_1 - I_2}{2}$$

Suppose $\frac{dU}{ds}(0) < 0$ .Then the agent should like to borrow.

$$\text{If } \exists s < 0 \text{ s.t } u'(I_1 - s) = \beta u(I_2 + s)\text{, then } s^* = s,$$

$$\text{else } s^* = -\infty$$

Similarly for $I_2 > r, I_1 \le r$, completing the characterisation of the two-period model.

Proof of Proposition 1

By A3, $U(r+x) + U(r-x) < 2U(r) \quad \forall x > 0$ As Income *I* is distributed symmetrically around *r*, it follows that for any realisation of income, that:
$\Pr(I = i)\{(U(r+i) + U(r-i)\} < 2\Pr(I = i)\{U(r)\}$
Summing over all possible realisations of I shows that $EU(I) < U(r)$.

Proof of Proposition 2

By A1, marginal utility is maximised when *x=r*. Therefore any linear combination of x's s.t their average is equal to r is weakly less than *U'(r)*. At least one element must be different to r in order for the distribution of income *I* to be non trivial. Therefore *U'(.)* obeys Jensen's inequality for random variables symmetrically distributed around the reference point.

Proof of Proposition 3

Suppose only $s^* > 0$. Then $\forall s < 0, -U'(I_1 - s) + E\beta U'(I_2 + s) \ne 0$.

Let $s=-I_a$ Where I_a is large and negative.

$E\beta U'(I_2+s) > U'(I_2+s)$ Since $U'(.)$ is convex in this region.

By Loss Aversion $U'(I_2+s) > U'(I_1-s)$ implying $\exists s$ s.t $-U'(I_1-s)+E\beta U'(I_2+s) > 0$

Consider $E\beta U'(I_2)$. By A1, $U'(I_1) = \max\{U'(.)\}$ implying $-U'(I_1-s)+E\beta U'(I_2+s) < 0$

Both $E\beta U'(.)$ and $U'(.)$ are continuous, which implies $\exists s$ s.t $0 > s > -I_a$ and $-U'(I_1-s)+E\beta U'(I_2+s) = 0$. It is obvious by the previous steps that this point is a global maximum, since the first derivative is positive to the left of the solution, and negative to the right of the solution.

Proof of Proposition 4

Recall the coefficient of loss aversion: $k(x) = \dfrac{U'(r-x)}{U'(r+x)} > 0, \forall x > 0$

Suppose *k(x)* were a constant *k*, implying that *U(x)* is a piecewise function that is concave in gains and convex in losses. If the reference point is equal to the mean of each period's income, the FOC is then:

$$u'(-s) = \beta \int_{-\infty}^{\infty} u'(\varepsilon+s)m(\varepsilon)d\varepsilon$$

$$\frac{u'(s)}{\int_{-\infty}^{\infty} u'(\varepsilon+s)m(\varepsilon)d\varepsilon} = \frac{\beta}{k}\forall s > 0\,,\quad \frac{u'(s)}{\int_{-\infty}^{\infty} u'(\varepsilon+s)m(\varepsilon)d\varepsilon} = k\beta\forall s < 0$$

By inspection $\exists s, f(s) = \left\{ \dfrac{u'(s)}{\int_{-\infty}^{\infty} u'(\varepsilon+s)m(\varepsilon)d\varepsilon} \right\}$ has a local minimum at $s > 0$. Call this s_1.

As both terms of the fraction are strictly positive and finite for all values less than one, $f(s_1) > 0$, but then $\therefore \exists k$ s.t $\dfrac{u'(s)}{\int_{-\infty}^{\infty} u'(\varepsilon+s)m(\varepsilon)d\varepsilon} = \dfrac{\beta}{k}$ has no solution. Namely $k = \dfrac{\beta}{f(s_1)-\varepsilon}$

$\therefore \exists k$ s.t s^* is unique.

Proof of Result 1

Define *f(x)*;

$$f(x) = u(-x) - u(x)$$
$$f'(x) = -u'(-x) - u'(x) < 0$$
$$f''(x) = u''(-x) - u''(x)$$
$$f''(x) > 0 \forall x > 0, f''(x) < 0 \forall x < 0$$

So *f(x)* is concave for negative x and convex for positive *x*. Now suppose an agent faces a sure income this period, and a random period next period, both with mean equal to the reference point. Suppose further that she is forced to either borrow an amount x from next period, or to save an amount *x* for next period. The problem is then;

$$\max U(x) = \{u(-x) + \beta \int u(\varepsilon + x)m(\varepsilon), u(x)d\varepsilon + \beta \int u(\varepsilon - x)m(\varepsilon)\,d\varepsilon\}$$

Borrow if:

$$u(-x) + \beta \int u(\varepsilon + x)m(\varepsilon) < u(x) + \beta \int u(\varepsilon - x)m(\varepsilon)\,d\varepsilon$$
$$u(-x) - u(x) < \beta \int u(\varepsilon - x)m(\varepsilon)\,d\varepsilon - \beta \int u(\varepsilon + x)m(\varepsilon)\,d\varepsilon$$

$$u(-x) - u(x) < \beta \int (u(-\varepsilon - x))m(-\varepsilon)d\varepsilon - \beta \int (u(\varepsilon + x))m(+\varepsilon)d\varepsilon$$

$\int (u(-\varepsilon - x))m(-\varepsilon)d\varepsilon = \int (u(-\varepsilon - x))m(\varepsilon)d\varepsilon$ As income is symmetrically distributed

$$f(x) < \beta \int f(x + \varepsilon)m(\varepsilon)\,d\varepsilon$$

As *f(x)* is convex for positive *x*, by Jensen's inequality it is larger than *f(x)*. As both sides are negative, the imposition of $\beta < 1$ merely affirms that the inequality holds, implying that the agent always finds it optimal to borrow.

Appendix 2

Proof that c*(k)=g(k)=k when expectations are rational in a finite model

Let $c^*(k)=g(k)$ and consider period T:

$$g(Y_T + k_{T-1} - g(k_{T-1}, r_{T-1}), r_T) = Y_T + k_{T-1} - g(k_{T-1}, r_{T-1})$$

Consider period T-1. Knowing that everything will be consumed in T, the r_T is given by:

$$r_T = \int_{Y\min}^{Y\max} g(Y_T + k_{T-1} - g(k_{T-1}, r_{T-1}), r_T)m(Y_T)dY_T$$
$$r_T = \int_{Y\min}^{Y\max} (Y_T + k_{T-1} - g(k_{T-1}, r_{T-1}))m(Y_T)dY_T$$
$$r_T = \overline{Y}_T + k_{T-1} - g(k_{T-1}, r_{T-1}))$$

$V_T(Y_T, r_T) = u(Y_T, r_T)$

$V_T(Y_T + k_{T-1} - g(k_{T-1}, r_{T-1}), \overline{Y}_T + k_{T-1} - g(k_{T-1}, r_{T-1})) = u(Y_T + k_{T-1} - g(k_{T-1}, r_{T-1}), \overline{Y}_T + k_{T-1} - g(k_{T-1}, r_{T-1}))$

More generally:
$$u(Y + z, r + z) = u(Y, r)$$
$$V_T(Y_T, r_T) = V_T(Y_T + k_{T-1} - g(k_{T-1}, r_{T-1}), \overline{Y}_T + k_{T-1} - g(k_{T-1}, r_{T-1}))$$

Therefore V_T is invariant to saving.

$$\therefore g(k_{T-1}, r_{T-1}) = \arg\max V(k_{T-1}, r_{T-1}) \rightarrow g(k_{T-1}, r_{T-1}) = k_{T-1}$$

More generally: $g(k_T, r_T) = k_T \rightarrow g(k_{T-1}, r_{T-1}) = k_{T-1}$

Which means: $g(k_t, r_t) = c^*(k_t) = k_t, \forall t$.

Completing the proof.

Characterisation of the infinite model

The Euler equation for saving is given by:

(E1) $$EU'(I_t - s_t + s_{t-1}) = EU'(I_{t+1} - s_{t+1} + s_t)$$

As marginal utility is a 2:1 mapping (two values of income for every value of marginal utility) it does not follow immediately that the expressions in the brackets are equal, that is, $2s_t = s_{t+1}+s_{t-1}$. But in an infinite horizon problem, expected behaviour in each period is the same. So we may assume that $2s_t = s_{t+1}+s_{t-1}$ which implies predicted saving is constant in each period (See Violante (2000) for a succinct discussion). This being the case, the first order condition for the first period is:

$$U'(I_1 - s_1) = EU'(I_2)$$

By Proposition 4, the optimal solution is given by $s^* < 0$ and all comparative statics results shown in Section 2 apply in the neighbourhood of s^*

References

Abeler, J., Falk, A., Goette, L., Huffman, D. (2011) Reference points and effort provision. American Economic Review 101(2), 470–492.

Allen, E.J., Dechow, P.M.,Pope, D.G., Wu, G. (2014) Reference-Dependent Preferences: Evidence From Marathon Runners. NBER Working Paper no. 20343.

Baillon, A., H. Bleichrodt, and V. Spinu. (2016) Searching for the Reference Point. Working Paper. Available from: https://www.rse.anu.edu.au/media/1829335/-Bleichrodt-Paper-2017.pdf [accessed 16th May 2017].

Bartling,B., Brandes, L., Schunk, D. (2015) Expectations as Reference Points: Field Evidence from Professional Soccer. Management Science 201561:11 , 2646-2661.

Baucells M Weber M, Welfens F. (2011) Reference-Point Formation and Updating. Management Science. 57(3) 506-519.

Besley, T., Coate, S., Loury, G. (1993) The Economics of Rotating Credit and Savings Associations. American Economic Review. 83 (4), pp. 792-810.

Bowman, D., Minehart, D., Rabin, M. (1999) Loss aversion in a consumption/savings model. Journal of Economic Behaviour & Organisation. 38(2), 155–178.

Campbell, J., Deaton, A. (1989) Why is consumption so smooth? Review of Economic Studies. 56(3), 357–373.

Carroll, C. (1997) Buffer-stock saving and the life cycle/permanent income hypothesis. Quarterly Journal of Economics. 112(1), 1–55.

Campbell, J,Y., Mankiw, G.N. (1989) Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence. In: Blanchard, O.J. & Fischer, S. (eds.), NBER Macroeconomics Annual 1989, 185-216, MIT Press.

Card, D., Dahl, G. (2011) Family Violence and Football:
The effect of unexpected Emotional Cues on Violent Behaviour. The Quarterly Journal of Economics. 126(1), 103–143.

Chen, H., De La Rupelle, M., Zilibotti, F. (2016) The Great Famine and Savings Rate in China. Working Paper. Available from: http://www3.grips.-ac.jp/~econseminar/seminar2015/May25_HengChen.pdf . [Accessed on 16th May 2017].

Chimonyo, M., Kusina, N. T., Hamudikuwanda, H., Nyoni, O. (1999) A survey on land use and usage of cattle for draught in a smallholder farming area of Zimbabwe. Journal of Applied Sciences in Southern Africa 5(2),111-121.

Crawford, V.P., Meng, J.J. (2011) New York City Cab Drivers' Labor Supply Revisited: Reference-Dependent Preferences with Rational-Expectations Targets for Hours and Income. American Economic Review, 101(5), 1912-32.

Deaton, A. (1989) Saving in developing countries: theory and review. World Bank Economic Review, Proceedings of the World Bank Annual Conference on Development Economics. 61-96.

Deaton, A. (1991) Saving and liquidity constraints. Econometrica 59(5), 1221–1248 .

Deaton, Angus. 1997. The analysis of household surveys : a microeconometric approach to development policy. Washington, D.C. : The World Bank.

Dercon, S. (2002) Income Risk, Coping Strategies and Safety Nets. World Bank Research Observer. 17, 141-166.

Duflo, E. (2003) Poor but Rational?. In: Banerjee, A.V., Benabou, R., Mookherjee, D. (2006) Understanding Poverty. Oxford University Press.

Engström, P., Nordblom, K.,Ohlsson, H., Persson A. (2015) Tax Compliance and Loss Aversion. American Economic Journal: Economic Policy. 7(4), pp. 132-64.

Ericson, K., Fuster, A. (2011) Expectations as endowments: evidence on reference-dependent preferences from exchange and valuation experiments. Quarterly Journal of Econonomics. 126(4), 1879–1907.

Fafchamps, M. Lund, S. (2003) Risk Sharing Networks in Rural Philippines. Journal of Development Economics. 71(2), pp. 261-287.

Fafchamps, M., Czukas, K., Udry, C. (1998) Drought and Saving in West-Africa: Are Livestock a Buffer Stock. Journal of Development Economics. 55(2), 273–305.

Farber, H.S. (2008) Reference-dependent preferences and labor supply: The case of New York City taxi drivers. American Economic Review. 98(2),1069–1082.

Genesove, D., Mayer, C. (2001) Loss Aversion and Seller Behaviour: Evidence from the Housing Market. NBER Working Paper no. 8143.

Gill, D. Prowse, V. (2012) A Structural Analysis of Disappointment Aversion in a Real Effort Competition. American Economic Review. 102(1), pp. 469-503.

Goette, L., Harms, A., Sprenger, C. (2014) Randomizing endowments: An experimental study of rational expectations and reference-dependent preferences. IZA Discussion Paper no. 8639.

Gugerty, M.K. (2007) You Can't Save Alone: Commitment in Rotating Savings and Credit Associations in Kenya. Economic Development and Cultural Change
55 (2) pp. 251-282.

Haigh, M.S., List, J. A., (2005), Do Professional Traders Exhibit Myopic Loss Aversion? An Experimental Analysis. The Journal of Finance. 60 (1), 523–534.

Helgeson, J., Dietz, S., Hochrainer-Stigler, S., (2012) Vulnerability to weather disasters: the choice of coping strategies in rural Uganda. Grantham Research Institute on Climate Change and the Environment. Working paper no. 91.

Hoogeveen, H. (2002) Evidence on Informal Insurance in Rural Zimbabwe. Journal of African Economies. 11(2), 249–278.

Kahneman, D., Tversky, A. (1979) Prospect theory: an analysis of decision under risk. Econometrica. 47(2), 263–291.

Kahneman, D., Tversky, A. (1991) Loss Aversion in Riskless Choice: A Reference-Dependent Model. The Quarterly Journal of Economics. 106 (4), 1039-1061.

Kinsey, B., Burger, K., Gunning, J. (1998) Coping with drought in Zimbabwe: Survey evidence on responses of rural households to risk. World Development. 26(1), 89-110.

Kőszegi,B., Rabin,M. (2006) A model of reference-dependent preferences.Quarterly Journal of Economics. 121(4),1133–1165.

Kőszegi, B., Rabin, M. (2007) Reference-dependent risk atitittudes. *The American Economic Review*. 97(4), 1047-1073.

Kőszegi, B., Rabin, M. (2009) Reference-dependent consumption plans. American Economic Review. 99(3), 909–936.

Lange, S., Reimers, N. (2015) Livestock as an imperfect Buffer Stock in poorly integrated markets. GlobalFood Discussion Paper no. 65.

Lien, J.W., Zheng, J. (2015) Deciding When to Quit: Reference-Dependence over Slot Machine Outcomes. American Economic Review. 105(5), 366-70.

Leland, H.E. (1968) Saving and Uncertainty: The Precautionary Demand for Saving. The Quarterly Journal of Economics 82(3) 465-473.

Masikati, P. (2010) Improving the water productivity of integrated crop-livestock systems in the semi-arid tropics of Zimbabwe: an ex-ante analysis using simulation modeling. Available from: http://hss.ulb.uni-bonn.de/2011/2463/2463.pdf .[Accessed May 16 2017].

Odean, T. (1998) Are Investors Reluctant to Realize Their Losses?. The Journal of Finance. 53 (5), 1775–1798.

Ray, D. (2016) Aspirations and the Development Treadmill. Journal of Human Development and Capabilities. 17(3), 309–323.

Rosenzweig, M. and K. Wolpin. 1993. "Credit Market Constraints, Consumption Smoothing and the Accumulation of Durable Production Assets in Low-Income Countries: Investments in Bullocks in India." Journal of Political Economy.

Sen, A. (1981) Poverty and Famines: An Essay on Entitlement and Deprivation. Oxford University Press. Ch5, pp. 46-51.

Schultz, T.W. Transforming traditional agriculture. University of Chicago Press. 1964.

Stokey, N. L., Lucas Jnr, R.E. (1989) Recursive Methods in Economic Dynamics. Harvard University Press. Ch3, pp. 55.

Swinton, S. (1988) Drought Survival Tactics of Subsistence Farmers in Niger. Human Ecology. 16 (2), 123-144.

Tversky, A., Kahneman, D.: Advances in prospect theory: cumulative representation of uncertainty. Journal of Risk and Uncertainty. 5(4), 297–323 (1992).

Violante, G., (2000) Violante, Gianluca. 2000. Notes on Discrete Time Stochastic Dynamic Programming, Course Lecture Notes. Department of Economics, New York University. Available from: http://www2.econ.iastate.edu/tesfatsi/-dpbasic.violante.pdf.(Accessed on May 16th 2017).

Watts, M. (1983) Silent Violence: Food, Famine and Peasantry in Northern Nigeria. Berkeley: University of California Press.