

Mixed Model PDS

Data

```
Datos %>%
  ungroup() %>%
  dplyr::select(Subject,Level,PDS) %>%
  group_by(Subject,Level) %>%
  mutate(mid = 1:n()) %>%
  pivot_wider(names_from=mid,values_from=PDS) %>%
  arrange(Subject,Level) %>%
  kable("latex", booktabs = T) %>%
  kable_styling(latex_options = c("striped", "scale_down"))
```

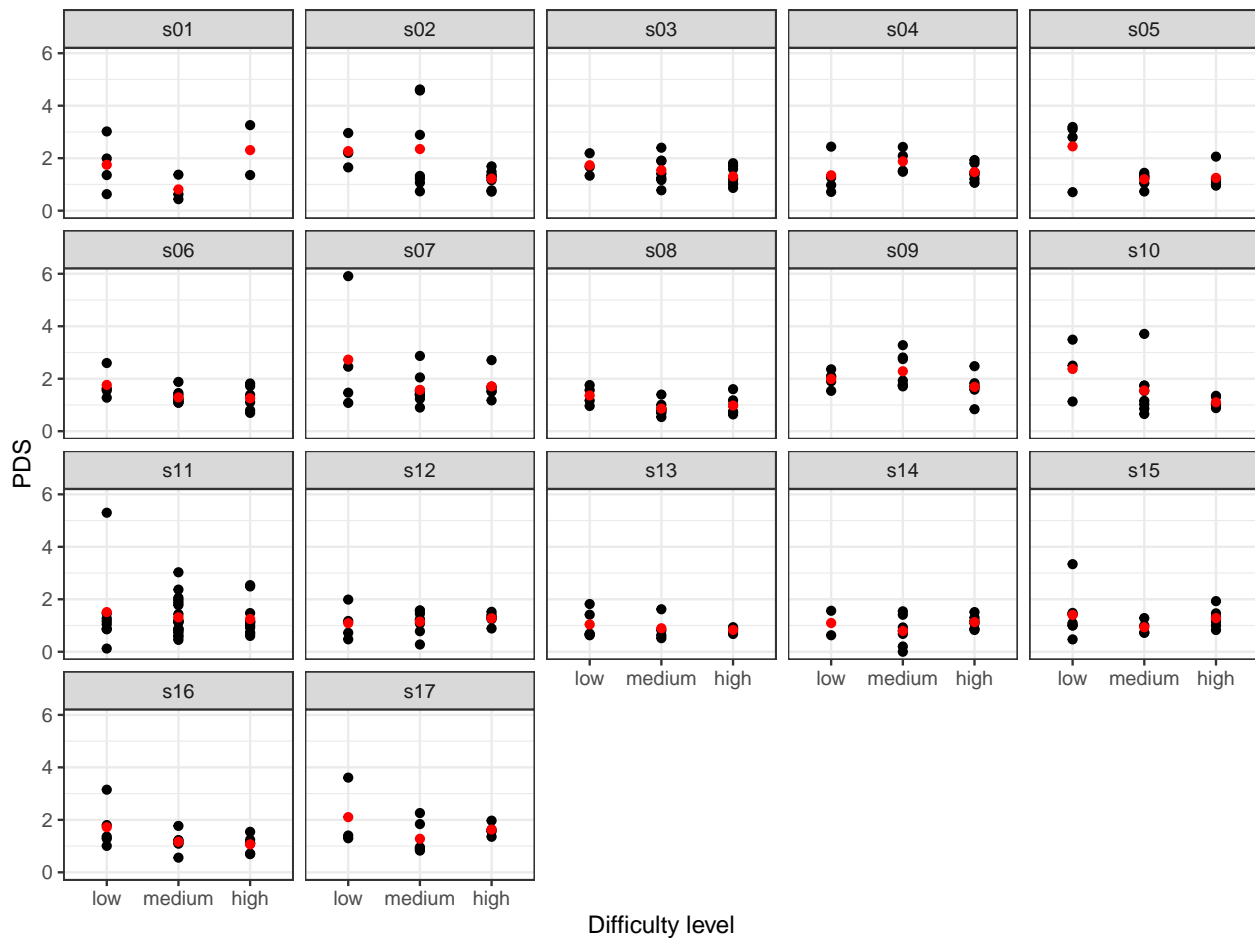
| Subject | Level | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-------|-------|-------|-------|--------|-------|-------|-------|-------|-------|--------|
| s01 | low | 1.3570 | 3.0190 | 0.6290 | 1.9890 | | | | | | | | | | | | | | | |
| s01 | medium | 0.4380 | 1.3710 | 0.6230 | | | | | | | | | | | | | | | | |
| s01 | high | 1.3570 | 3.2590 | | | | | | | | | | | | | | | | | |
| s02 | low | 1.6490 | 2.9590 | 2.1990 | | | | | | | | | | | | | | | | |
| s02 | medium | 1.3220 | 2.8890 | 4.5790 | 0.7360 | 1.0680 | 1.2230 | 4.6190 | | | | | | | | | | | | |
| s02 | high | 1.2570 | 1.4780 | 0.7250 | 1.3410 | 1.3440 | 1.6890 | 1.1610 | 0.7740 | | | | | | | | | | | |
| s03 | low | 2.1890 | 1.6690 | 1.3330 | | | | | | | | | | | | | | | | |
| s03 | medium | 1.4090 | 1.9090 | 1.1670 | 0.7770 | 2.3990 | 1.8890 | 1.2440 | | | | | | | | | | | | |
| s03 | high | 1.2050 | 1.8090 | 1.5870 | 0.8670 | 1.7090 | 1.2150 | 1.0290 | 1.0170 | | | | | | | | | | | |
| s04 | low | 2.4390 | 1.2620 | 0.9740 | 0.7157 | | | | | | | | | | | | | | | |
| s04 | medium | 2.0890 | 2.4290 | 1.5200 | 1.4820 | | | | | | | | | | | | | | | |
| s04 | high | 1.8090 | 1.9290 | 1.4480 | 1.2080 | 1.4590 | 1.3690 | 1.0610 | | | | | | | | | | | | |
| s05 | low | 3.1890 | 3.1090 | 0.7052 | 2.7990 | | | | | | | | | | | | | | | |
| s05 | medium | 1.2820 | 1.2990 | 1.3540 | 1.2270 | 1.4420 | 1.0530 | 0.7310 | | | | | | | | | | | | |
| s05 | high | 0.9530 | 1.0020 | 2.0590 | 1.2180 | 1.0900 | 1.1660 | | | | | | | | | | | | | |
| s06 | low | 2.5990 | 1.6280 | 1.5520 | 1.2780 | | | | | | | | | | | | | | | |
| s06 | medium | 1.1690 | 1.1150 | 1.1990 | 1.3630 | 1.8790 | 1.0920 | 1.4550 | 1.0840 | 1.244 | | | | | | | | | | |
| s06 | high | 1.8190 | 1.7190 | 1.3870 | 0.7990 | 0.6989 | 1.1060 | | | | | | | | | | | | | |
| s07 | low | 5.9090 | 1.4670 | 1.0790 | 2.4590 | | | | | | | | | | | | | | | |
| s07 | medium | 1.5240 | 1.2360 | 1.3940 | 1.3650 | 1.4410 | 2.8690 | 0.9020 | 1.3940 | 1.563 | 2.049 | | | | | | | | | |
| s07 | high | 2.7090 | 1.6390 | 1.1740 | 1.5110 | 1.6990 | 1.5210 | | | | | | | | | | | | | |
| s08 | low | 1.1690 | 1.5650 | 1.7590 | 0.9620 | | | | | | | | | | | | | | | |
| s08 | medium | 1.3960 | 0.6936 | 1.0070 | 0.5407 | 0.7800 | 0.7770 | 0.8640 | 0.8410 | | | | | | | | | | | |
| s08 | high | 1.1760 | 0.7350 | 1.6020 | 0.6399 | 0.9830 | 0.7340 | | | | | | | | | | | | | |
| s09 | low | 1.8990 | 2.3590 | 2.0790 | 2.0890 | 1.5380 | | | | | | | | | | | | | | |
| s09 | medium | 3.2790 | 2.8090 | 1.9290 | 1.7690 | 2.7490 | 1.7490 | 1.7090 | | | | | | | | | | | | |
| s09 | high | 1.7190 | 2.4790 | 1.6890 | 1.8290 | 1.5850 | 0.8380 | | | | | | | | | | | | | |
| s10 | low | 1.1310 | 2.4990 | 3.4890 | | | | | | | | | | | | | | | | |
| s10 | medium | 1.0210 | 1.7390 | 1.5330 | 0.8590 | 1.7390 | 0.6558 | 3.7090 | 1.1580 | | | | | | | | | | | |
| s10 | high | 1.3480 | 0.8810 | 1.0570 | 0.9980 | 1.3130 | 1.0180 | | | | | | | | | | | | | |
| s11 | low | 1.0510 | 0.8780 | 1.2700 | 0.8560 | 1.1650 | 5.2990 | 0.1230 | 1.4610 | | | | | | | | | | | |
| s11 | medium | 1.9690 | 1.1960 | 1.1640 | 0.4530 | 2.3690 | 0.8690 | 1.8690 | 0.7310 | 1.779 | 1.120 | 1.133 | 0.865 | 0.5648 | 1.436 | 3.029 | 0.873 | 0.799 | 2.059 | 0.6242 |
| s11 | high | 2.4890 | 0.9890 | 1.1640 | 1.0850 | 0.7360 | 0.9180 | 0.6044 | 0.6916 | 2.539 | 1.161 | 1.474 | 1.119 | | | | | | | |
| s12 | low | 1.9890 | 1.1730 | 0.4750 | 0.7236 | | | | | | | | | | | | | | | |
| s12 | medium | 1.5720 | 1.2170 | 1.4500 | 1.5720 | 0.2770 | 0.7830 | 1.0780 | 1.2300 | | | | | | | | | | | |
| s12 | high | 1.3070 | 1.5220 | 1.2990 | 1.4000 | 0.8880 | 1.2430 | | | | | | | | | | | | | |
| s13 | low | 0.6290 | 0.6901 | 0.6541 | 1.8190 | 1.4130 | | | | | | | | | | | | | | |
| s13 | medium | 0.5180 | 0.8190 | 1.6200 | 0.6290 | | | | | | | | | | | | | | | |
| s13 | high | 0.6777 | 0.9360 | 0.9000 | 0.7880 | 0.8570 | | | | | | | | | | | | | | |
| s14 | low | 0.6290 | 1.5620 | | | | | | | | | | | | | | | | | |
| s14 | medium | 1.4100 | 0.6802 | 1.5420 | 0.2010 | 0.9270 | 0.0010 | 0.7960 | | | | | | | | | | | | |
| s14 | high | 1.1770 | 0.8720 | 1.5120 | 1.3020 | 0.8300 | 1.0840 | | | | | | | | | | | | | |
| s15 | low | 1.0000 | 1.4810 | 3.3390 | 1.0060 | 0.4710 | 1.4260 | 1.0920 | | | | | | | | | | | | |
| s15 | medium | 0.9970 | 0.7283 | 1.2840 | 0.7246 | | | | | | | | | | | | | | | |
| s15 | high | 1.4640 | 1.1500 | 0.8300 | 1.3250 | 1.9290 | 0.9970 | | | | | | | | | | | | | |
| s16 | low | 1.2850 | 1.0080 | 1.3620 | 1.7990 | 3.1490 | | | | | | | | | | | | | | |
| s16 | medium | 1.7690 | 0.5587 | 1.0910 | 1.1270 | 1.2100 | 1.2280 | | | | | | | | | | | | | |
| s16 | high | 1.5440 | 0.7124 | 1.1120 | 1.1020 | 1.2420 | 0.6924 | | | | | | | | | | | | | |
| s17 | low | 3.6090 | 1.2990 | 1.4030 | | | | | | | | | | | | | | | | |
| s17 | medium | 2.2590 | 0.8280 | 1.8390 | 0.9550 | 0.9180 | 0.8670 | | | | | | | | | | | | | |
| s17 | high | 1.5620 | 1.9690 | 1.6290 | 1.3550 | | | | | | | | | | | | | | | |

| Level | n | MD | SD |
|--------|-----|-------|--------|
| low | 72 | 1.712 | 1.0415 |
| medium | 124 | 1.369 | 0.7588 |
| high | 106 | 1.294 | 0.4703 |

Summary by group

```
Datos %>%
  group_by(Level) %>%
  summarise(n=n(),MD=mean(PDS),SD=sd(PDS)) %>%
  kable()%>%
  kable_styling(latex_options = c("striped"))
```

```
(q <-Datos %>% ggplot(aes(x=Level,y=PDS)) +
  geom_point() + facet_wrap(~ Subject)+
  labs(x="Difficulty level")+theme_bw()+
  stat_summary(fun="mean", geom="point",color="red"))
```



Random Intercept and Slope Model

The following model is used to investigate whether there are significant differences between the study variables:

$$y_{ij} = \mu + l_k + s_j + (sl)_{jk} + \epsilon_{ij}, \quad (1)$$

where y_{ij} is the response variable (PDS) for the i -th observation from the j -th subject, μ is the intercept, l_k is the k -th difficulty level, s_j is the j th subject effect, $(sl)_{jk}$ is the subject-level effect, i.e., the k -th level effect at the j -th subject, ϵ_{ij} is the error term (residual) for the i th observation from the j th subject.

We called *level* l a fixed effect, and ϵ is our *error term* that represent deviations from our predictions due to *random* factors that we cannot control experimentally. However, several measurements were taken for each subject at each difficulty level and that violates the assumption of independence of a linear model. On the other hand, each individual has a different cognitive load capacity, and this will be a characteristic factor that will affect all the responses of the same subject, which will make these responses interdependent instead of independent, see figure ???. The way we approaches this situation is adding a random effect to the subject and to the subject-level interaction. This allows us to solve this lack of independence by assuming a different intercept and slope for each subject. And finally, we assume that the residual, subject and subject-level effects are all relations of separate distributions, all with zero means:

$$\begin{aligned}\epsilon_{ij} &\sim N(0, \sigma^2), \\ s_j &\sim N(0, \sigma_s^2), \\ (sl)_{jk} &\sim N(0, \sigma_{sl}^2).\end{aligned}$$

Hence, s_j and $(sl)_{jk}$ are now random effects, and μ and l_k are fixed effects.

Using the **R** notation the model is

$$PDS = (b_0 + u_{Subject}) + b_{Level}Level + \epsilon$$

In order to evaluate if there is an effect due to the difficulty level we will use the likelihood ratio test of the model with the *Level* effect against the model without the *Level* effect.

```
PDS_mixed_reducido <- lme4::lmer(PDS ~ 1 + (1+Level|Subject),data=Datos,REML=F)

PDS_mixed_lme4 <- lme4::lmer(PDS ~ Level + (1+Level|Subject),data=Datos,REML=F)

anova(PDS_mixed_reducido,PDS_mixed_lme4)

## Data: Datos
## Models:
## PDS_mixed_reducido: PDS ~ 1 + (1 + Level | Subject)
## PDS_mixed_lme4: PDS ~ Level + (1 + Level | Subject)
##          npar AIC BIC logLik deviance Chisq Df Pr(>Chisq)
## PDS_mixed_reducido      8 705 735   -345      689
## PDS_mixed_lme4        10 686 723   -333      666  23.2  2  0.0000091 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The p-value of the ratio test is significant at a level of 0.001.

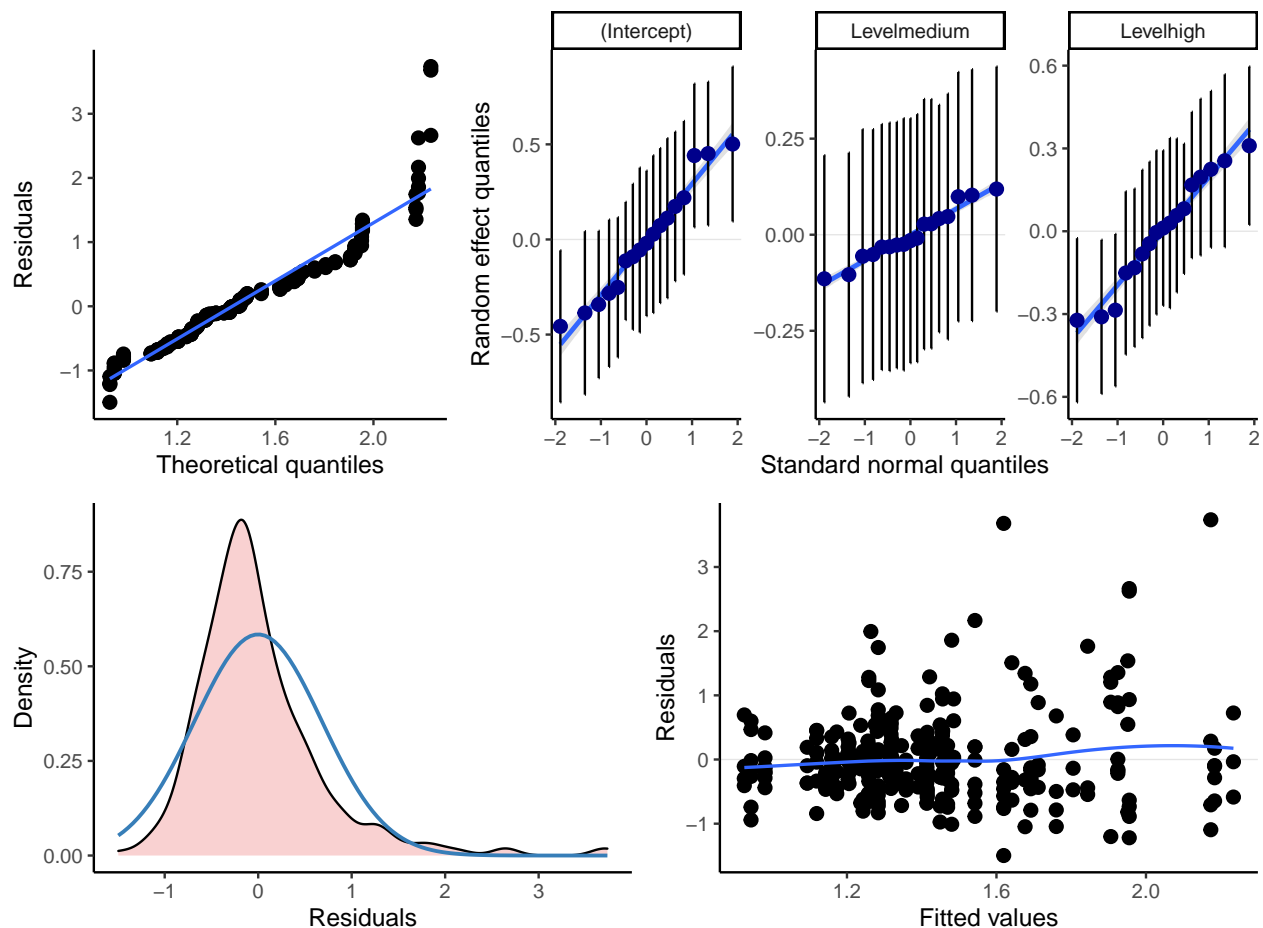
```
PDS_mixed_lme4 <- lme4::lmer(PDS ~ Level + (1+Level|Subject),data=Datos)
summary(PDS_mixed_lme4)

## Linear mixed model fit by REML ['lmerMod']
## Formula: PDS ~ Level + (1 + Level | Subject)
##      Data: Datos
```

```
##
## REML criterion at convergence: 674.7
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.132 -0.584 -0.181  0.342  5.323
##
## Random effects:
##   Groups   Name                Variance Std.Dev. Corr
##   Subject  (Intercept)  0.1274    0.357
##             Levelmedium 0.0315    0.178   -0.21
##             Levelhigh   0.0603    0.245   -1.00  0.31
##   Residual                0.4924    0.702
## Number of obs: 302, groups:  Subject, 17
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)    1.732      0.121   14.35
## Levelmedium   -0.378      0.115    -3.28
## Levelhigh     -0.442      0.124    -3.58
##
## Correlation of Fixed Effects:
##              (Intr) Lvlmdm
## Levelmedium  -0.565
## Levelhigh    -0.818  0.554
## optimizer (nloptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see ?isSingular

p<-plot_model(PDS_mixed_lme4, type = "diag")

({p[[1]]+theme(plot.title=element_blank(),plot.subtitle=element_blank())+scale_x_continuous(name="Theor
```



```
contr <- glht(PDS_mixed_lme4, linfct=mcp(Level="Tukey"))
summary(contr, test = adjusted("holm"))
```

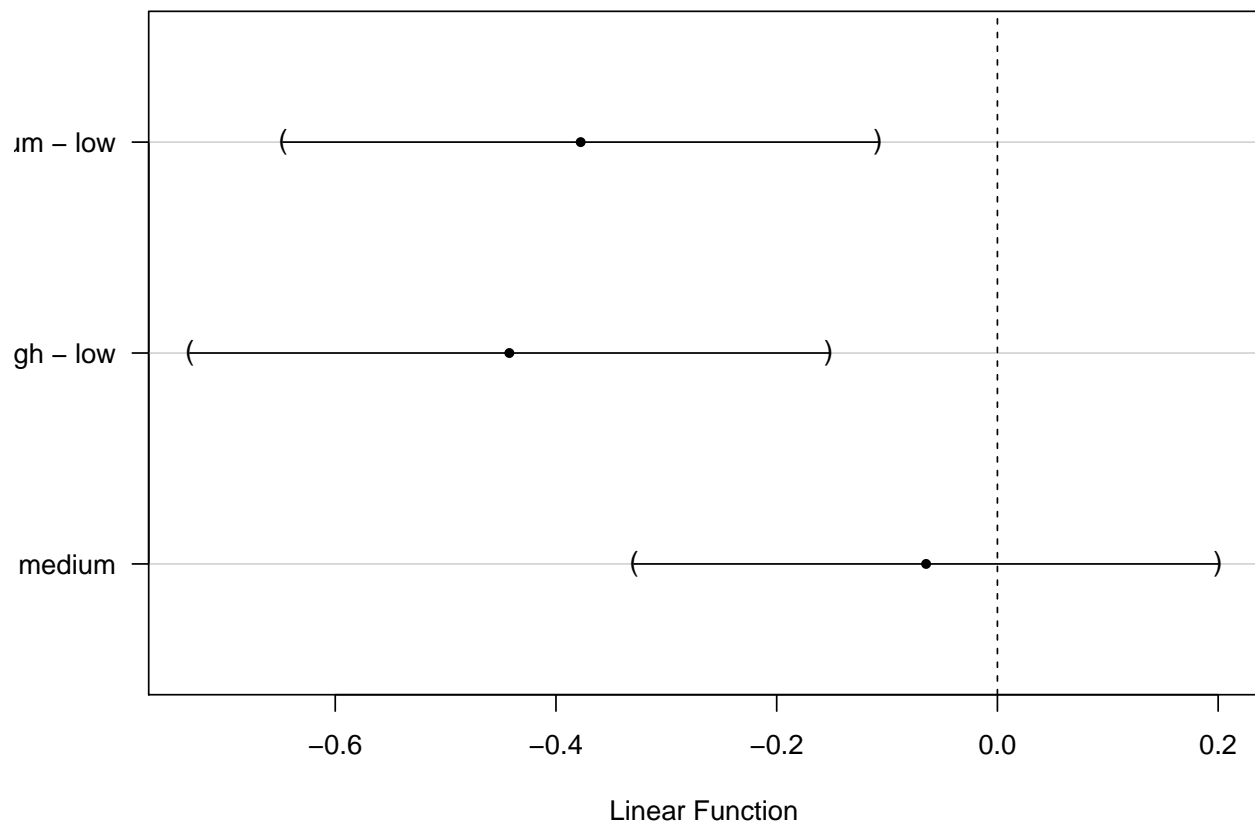
```
##
## Simultaneous Tests for General Linear Hypotheses
##
## Multiple Comparisons of Means: Tukey Contrasts
##
##
## Fit: lme4::lmer(formula = PDS ~ Level + (1 + Level | Subject), data = Datos)
##
## Linear Hypotheses:
##           Estimate Std. Error z value Pr(>|z|)
## medium - low == 0  -0.3776    0.1150  -3.28  0.0021 **
## high - low == 0   -0.4422    0.1235  -3.58  0.0010 **
## high - medium == 0 -0.0646    0.1129  -0.57  0.5669
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- holm method)
```

```
confint(contr)
```

```
##
## Simultaneous Confidence Intervals
##
```

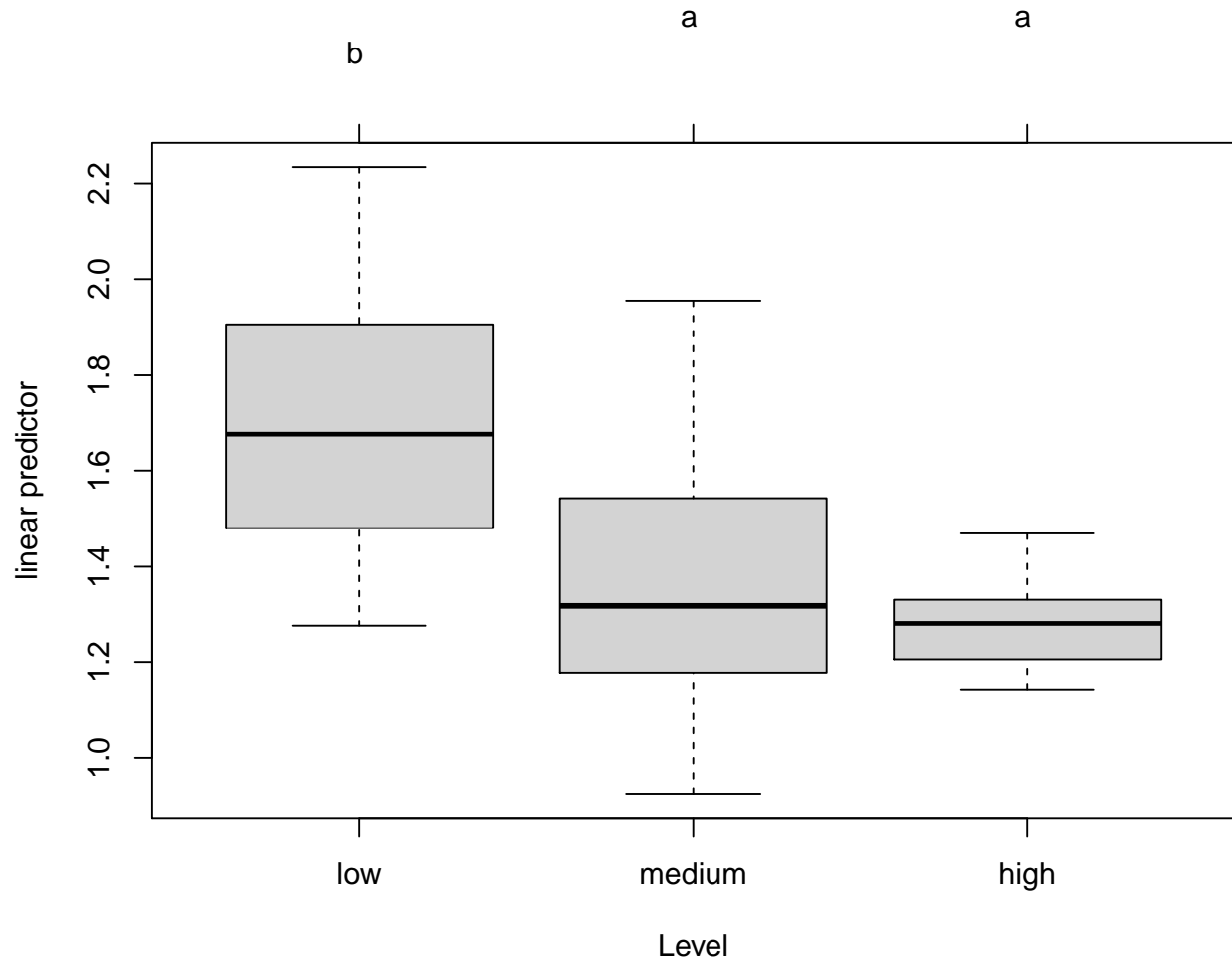
```
## Multiple Comparisons of Means: Tukey Contrasts
##
##
## Fit: lme4::lmer(formula = PDS ~ Level + (1 + Level | Subject), data = Datos)
##
## Quantile = 2.344
## 95% family-wise confidence level
##
## Linear Hypotheses:
##               Estimate lwr      upr
## medium - low == 0  -0.3776 -0.6472 -0.1081
## high - low == 0   -0.4422 -0.7317 -0.1528
## high - medium == 0 -0.0646 -0.3292  0.1999
plot(confint(contr))
```

95% family-wise confidence level



```
contr.cld <- cld(contr)
old.par <- par(mai=c(1,1,1.25,1), no.readonly = TRUE)
plot(contr.cld)
```

| Subject | Name | Training | Nivel | BLPS | MPDC | APCPS | PD | Entropy | TTP | PDS | SequenceMemory | SMN | id | Level | res | fit |
|---------|--------------------|----------|-------|-------|---------|---------|--------|---------|---------|-------|----------------|-----|----|--------|-------|-------|
| s07 | SequenceMemory_r04 | FALSE | 1 | 3.862 | -0.1728 | -0.0447 | 0.2026 | -1.885 | 2451974 | 5.909 | r04 | 4 | 3 | low | 3.736 | 2.174 |
| s11 | SequenceMemory_r21 | FALSE | 1 | 3.555 | -0.0893 | -0.0251 | 0.0867 | -1.794 | 2132932 | 5.299 | r21 | 21 | | low | 3.680 | 1.619 |
| s02 | SequenceMemory_r25 | FALSE | 3 | 4.222 | -0.2127 | -0.0492 | 0.1096 | -2.040 | 2990249 | 4.619 | r25 | 25 | 15 | medium | 2.664 | 1.955 |
| s02 | SequenceMemory_r13 | FALSE | 3 | 4.256 | -0.2476 | -0.0582 | 0.0684 | -2.005 | 2113081 | 4.579 | r13 | 13 | 7 | medium | 2.624 | 1.955 |
| s10 | SequenceMemory_r24 | FALSE | 3 | 2.744 | 1.568 | 0.4215 | 1.6368 | -1.974 | 1776806 | 3.709 | r24 | 24 | 14 | medium | 2.167 | 1.542 |
| s01 | SequenceMemory_r17 | FALSE | 6 | 4.023 | -0.2674 | -0.0665 | 0.1099 | -1.911 | 2292544 | 3.259 | r17 | 17 | 7 | high | 1.995 | 1.264 |




```
shapiro.test(Datos2$res)

##
##  Shapiro-Wilk normality test
##
## data:  Datos2$res
## W = 0.87, p-value = 0.000000000000002

goftest::ad.test(Datos2$res, null="pnorm", mean=mean(Datos2$res), sd=sd(Datos2$res), estimated=TRUE)

##
##  Anderson-Darling test of goodness-of-fit
##  Braun's adjustment using 17 groups
##  Null hypothesis: Normal distribution
##  with parameters mean = 0.00000000000000793745812849067, sd =
##  0.682747266266956
##  Parameters assumed to have been estimated from data
##
## data:  Datos2$res
## Anmax = 3, p-value = 0.4

rstatix::levene_test(data=ungroup(Datos2), res~Level)

## # A tibble: 1 x 4
##   df1 df2 statistic      p
##   <int> <int>   <dbl>   <dbl>
## 1     2   299     8.37 0.000291
```

The same model without the outliers

We repeat the analysis without the outlier

```
# we exclude the outlier
Datos <- Datos %>% filter(!(Subject=="s07"&SMN==4),
                          !(Subject=="s11"&SMN==21),
                          !(Subject=="s02"&SMN==25),
                          !(Subject=="s02"&SMN==13))

PDS_mixed_lme4 <- lme4::lmer(PDS ~ Level + (1+Level|Subject), data=Datos)
summary(PDS_mixed_lme4)

## Linear mixed model fit by REML ['lmerMod']
## Formula: PDS ~ Level + (1 + Level | Subject)
## Data: Datos
##
## REML criterion at convergence: 554.4
##
## Scaled residuals:
##   Min       1Q   Median       3Q      Max
## -2.400 -0.603 -0.170  0.405  3.899
##
## Random effects:
##   Groups   Name                Variance Std.Dev. Corr
##   Subject (Intercept)  0.142      0.376
##           Levelmedium  0.109      0.330   -0.61
```

```
##           Levelhigh    0.104    0.322   -0.92  0.87
## Residual              0.329    0.574
## Number of obs: 297, groups: Subject, 17
##
## Fixed effects:
##           Estimate Std. Error t value
## (Intercept)    1.637      0.115   14.17
## Levelmedium   -0.327      0.120   -2.72
## Levelhigh     -0.347      0.119   -2.91
##
## Correlation of Fixed Effects:
##           (Intr) Lvlmdm
## Levelmedium -0.686
## Levelhigh   -0.839  0.736
## optimizer (nloptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see ?isSingular
```

```
anova(PDS_mixed_lme4)
```

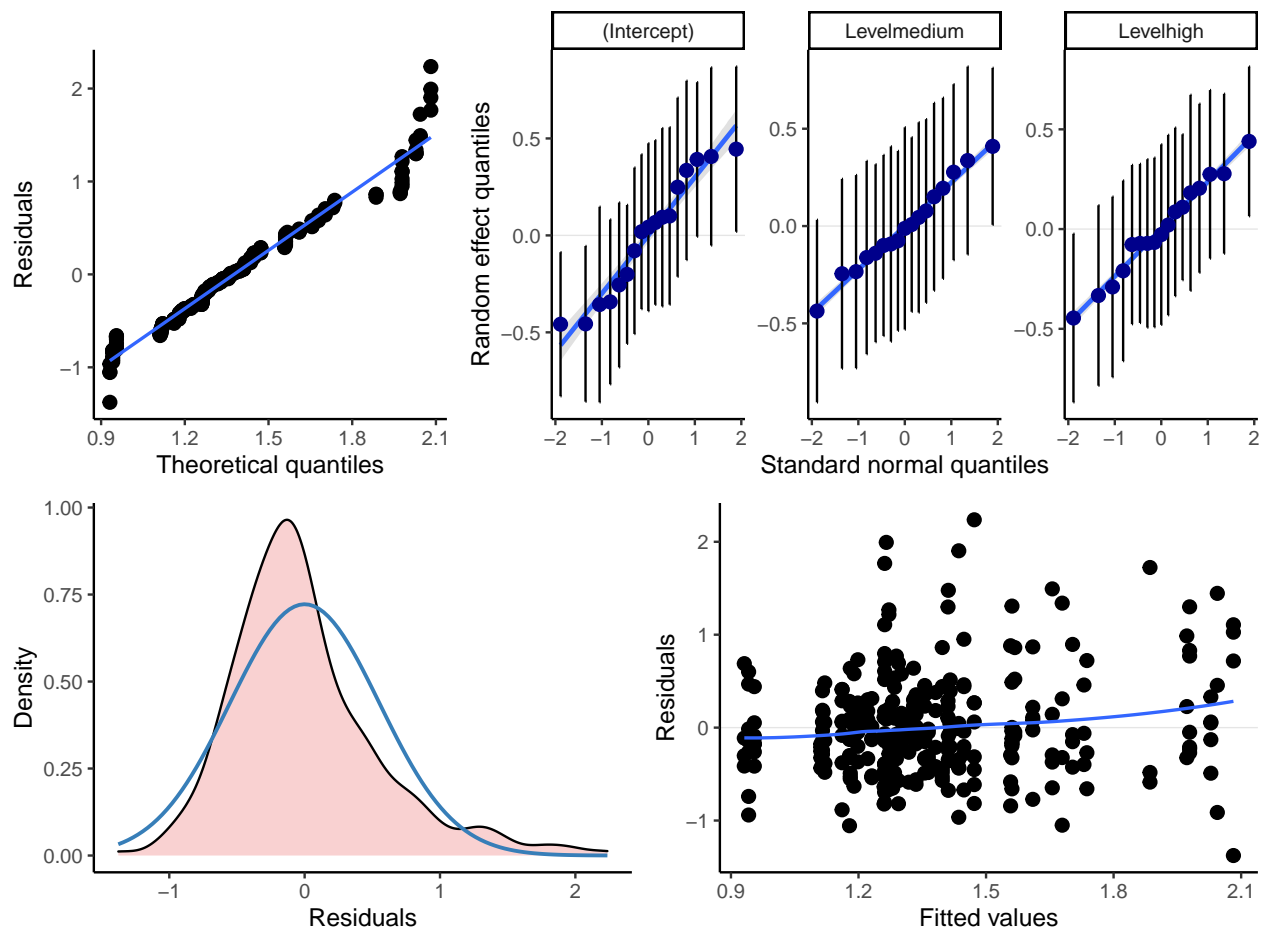
```
## Analysis of Variance Table
##           npar Sum Sq Mean Sq F value
## Level    2    3.02    1.51    4.6
```

```
coef(PDS_mixed_lme4)
```

```
## $Subject
##           (Intercept) Levelmedium Levelhigh
## s01          1.679    -0.418902  -0.41379
## s02          1.972    -0.560987  -0.63690
## s03          1.730    -0.282396  -0.37452
## s04          1.558     0.009823  -0.14203
## s05          2.082    -0.763502  -0.79276
## s06          1.704    -0.402666  -0.41873
## s07          1.737    -0.176216  -0.32664
## s08          1.381    -0.426423  -0.26074
## s09          2.029    -0.049890  -0.41880
## s10          2.044    -0.571699  -0.67984
## s11          1.179     0.082709   0.09270
## s12          1.294    -0.132521  -0.07224
## s13          1.181    -0.249272  -0.06926
## s14          1.280    -0.338774  -0.16509
## s15          1.436    -0.319033  -0.23734
## s16          1.656    -0.466351  -0.42451
## s17          1.886    -0.488540  -0.55618
##
## attr(,"class")
## [1] "coef.mer"
```

```
p<-plot_model(PDS_mixed_lme4, type = "diag")
```

```
(q<-{p[[1]]+theme(plot.title=element_blank(),plot.subtitle=element_blank())+scale_x_continuous(name="Th
```



```
# muy importante Tukey para lme4.
contr <- glht(PDS_mixed_lme4, linfct=mcp(Level="Tukey"))
summary(contr, test = adjusted("holm"))

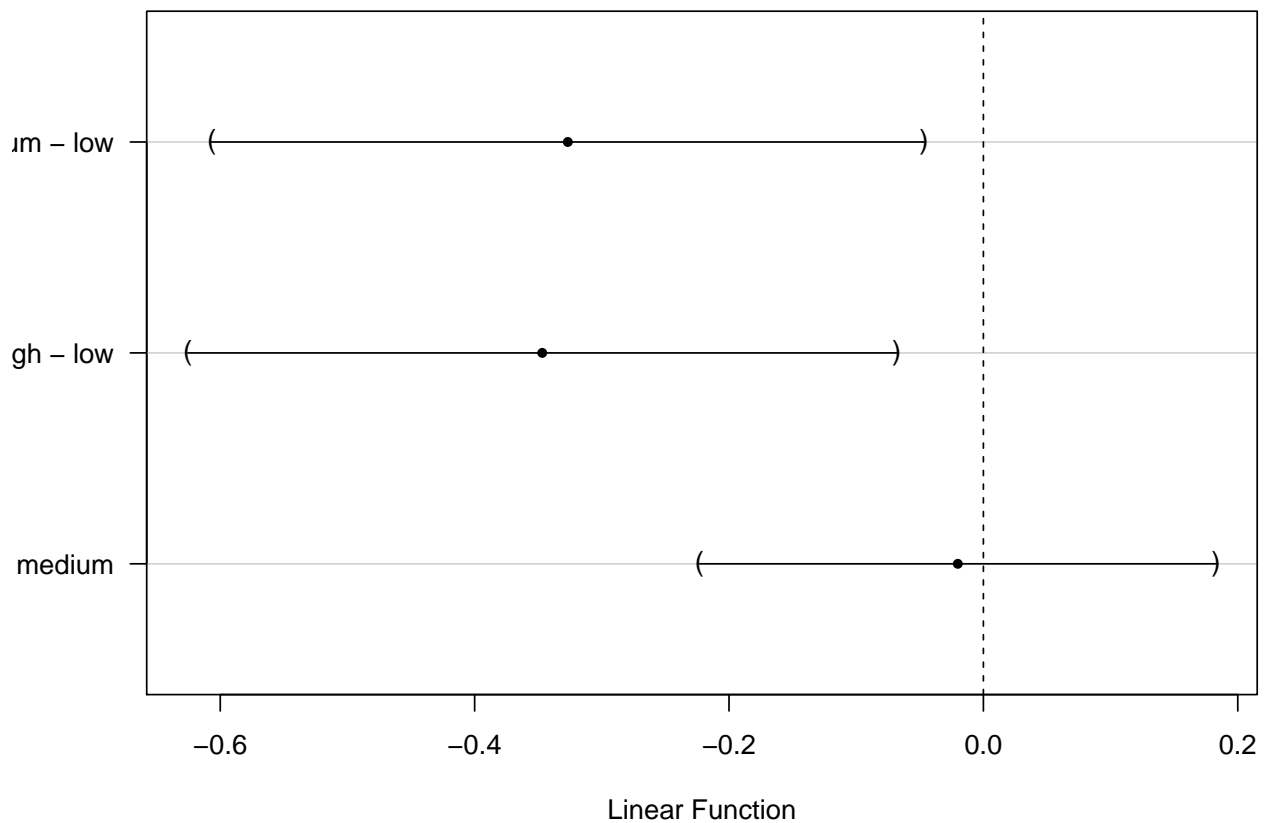
##
## Simultaneous Tests for General Linear Hypotheses
##
## Multiple Comparisons of Means: Tukey Contrasts
##
##
## Fit: lme4::lmer(formula = PDS ~ Level + (1 + Level | Subject), data = Datos)
##
## Linear Hypotheses:
##               Estimate Std. Error z value Pr(>|z|)
## medium - low == 0   -0.3267    0.1200   -2.72   0.013 *
## high - low == 0     -0.3469    0.1193   -2.91   0.011 *
## high - medium == 0  -0.0201    0.0870   -0.23   0.817
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- holm method)

confint(contr)

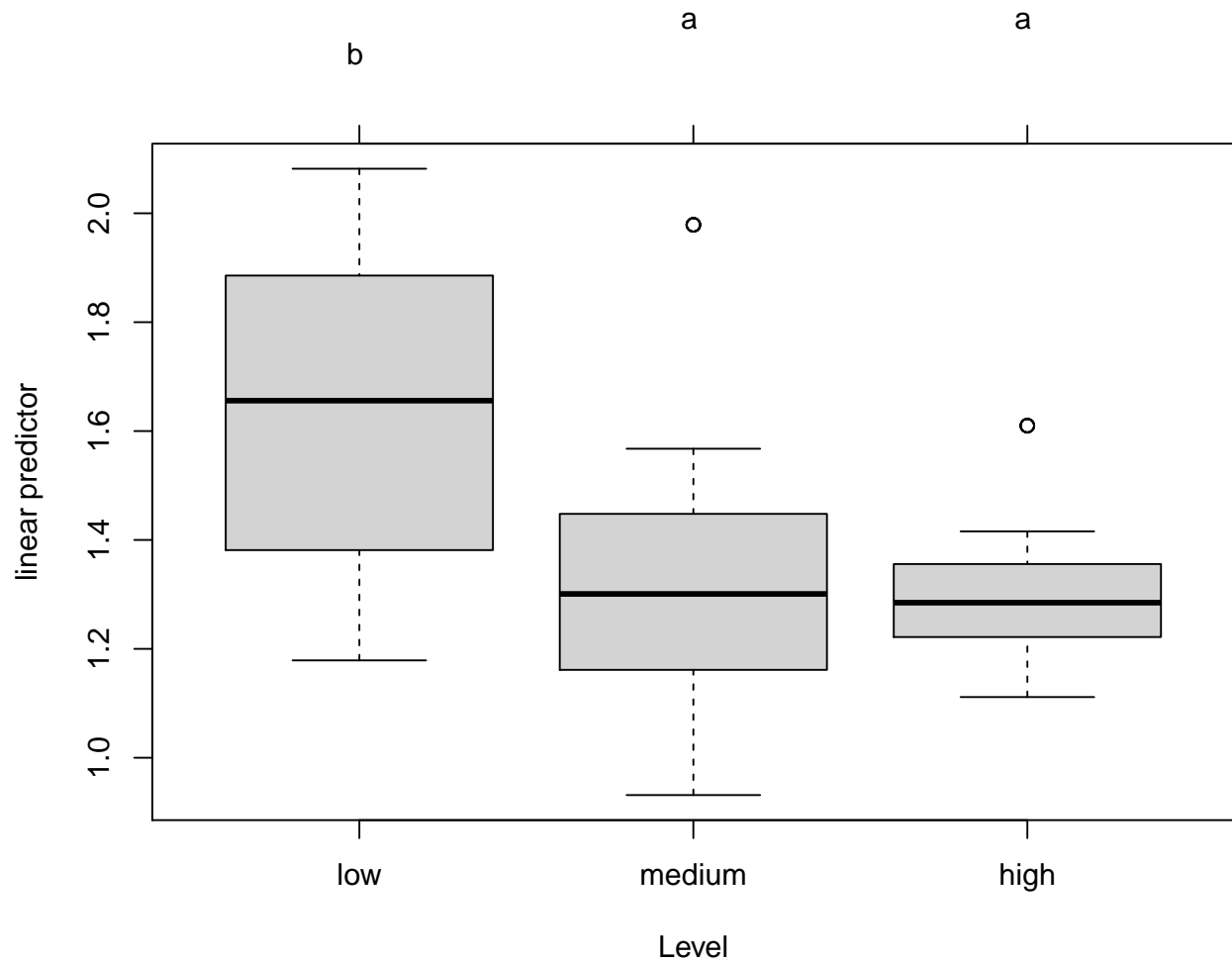
##
## Simultaneous Confidence Intervals
```

```
##
## Multiple Comparisons of Means: Tukey Contrasts
##
##
## Fit: lme4::lmer(formula = PDS ~ Level + (1 + Level | Subject), data = Datos)
##
## Quantile = 2.334
## 95% family-wise confidence level
##
## Linear Hypotheses:
##           Estimate lwr      upr
## medium - low == 0  -0.3267 -0.6068 -0.0467
## high - low == 0   -0.3469 -0.6254 -0.0683
## high - medium == 0 -0.0201 -0.2231  0.1829
plot(confint(contr))
```

95% family-wise confidence level



```
contr.cld <- cld(contr)
### use sufficiently large upper margin
old.par <- par(mai=c(1,1,1.25,1), no.readonly = TRUE)
### plot
plot(contr.cld)
```



```
par(old.par)
```

```
Datos2=Datos
Datos2$res = residuals(PDS_mixed_lme4,type="pearson")
Datos2$fit = fitted(PDS_mixed_lme4,type="pearson")
shapiro.test(Datos2$res)
```

```
##
## Shapiro-Wilk normality test
##
## data: Datos2$res
## W = 0.94, p-value = 0.0000000004
```

```
goftest::ad.test(Datos2$res,null="pnorm",mean=mean(Datos2$res), sd=sd(Datos2$res), estimated=TRUE)
```

```
##
## Anderson-Darling test of goodness-of-fit
## Braun's adjustment using 17 groups
## Null hypothesis: Normal distribution
## with parameters mean = 0.00000000000000671390181825922, sd =
## 0.552432179624567
## Parameters assumed to have been estimated from data
##
## data: Datos2$res
```

```
## Anmax = 4, p-value = 0.1
rstatix::levene_test(data=ungroup(Datos2),res~Level)
```

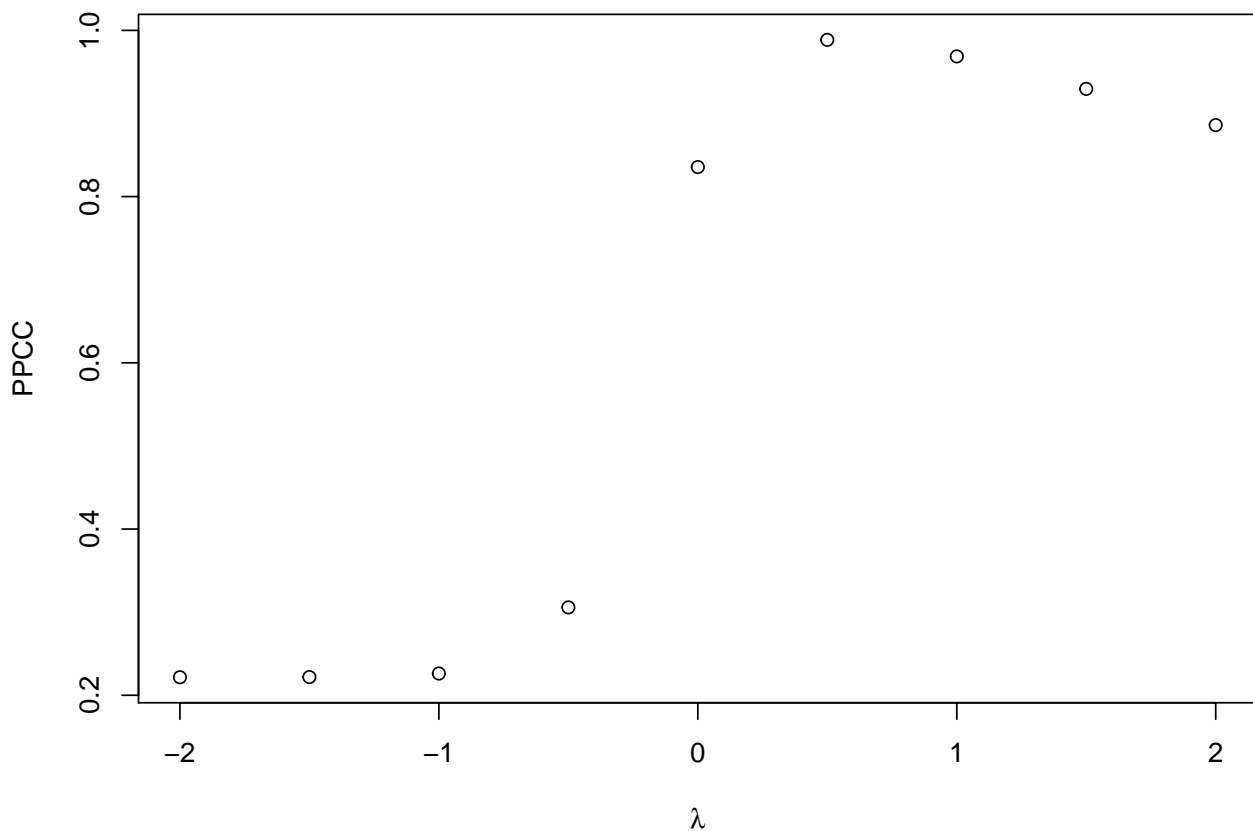
```
## # A tibble: 1 x 4
##   df1 df2 statistic      p
##   <int> <int>   <dbl>   <dbl>
## 1     2  294     7.10 0.000976
```

Box-Cox transformation

```
(PDS_lm <- lm(PDS ~ Level ,data=Datos))
```

```
##
## Call:
## lm(formula = PDS ~ Level, data = Datos)
##
## Coefficients:
## (Intercept) Levelmedium Levelhigh
##          1.607         -0.291         -0.313
boxcox.list <- EnvStats::boxcox(PDS_lm)
plot(boxcox.list)
```

Box-Cox Transformation Results: PPCC vs. lambda for PDS_lm



| Level | n | MD | SD |
|--------|-----|--------|--------|
| low | 69 | 0.4608 | 0.6562 |
| medium | 122 | 0.2312 | 0.5626 |
| high | 106 | 0.2444 | 0.3958 |

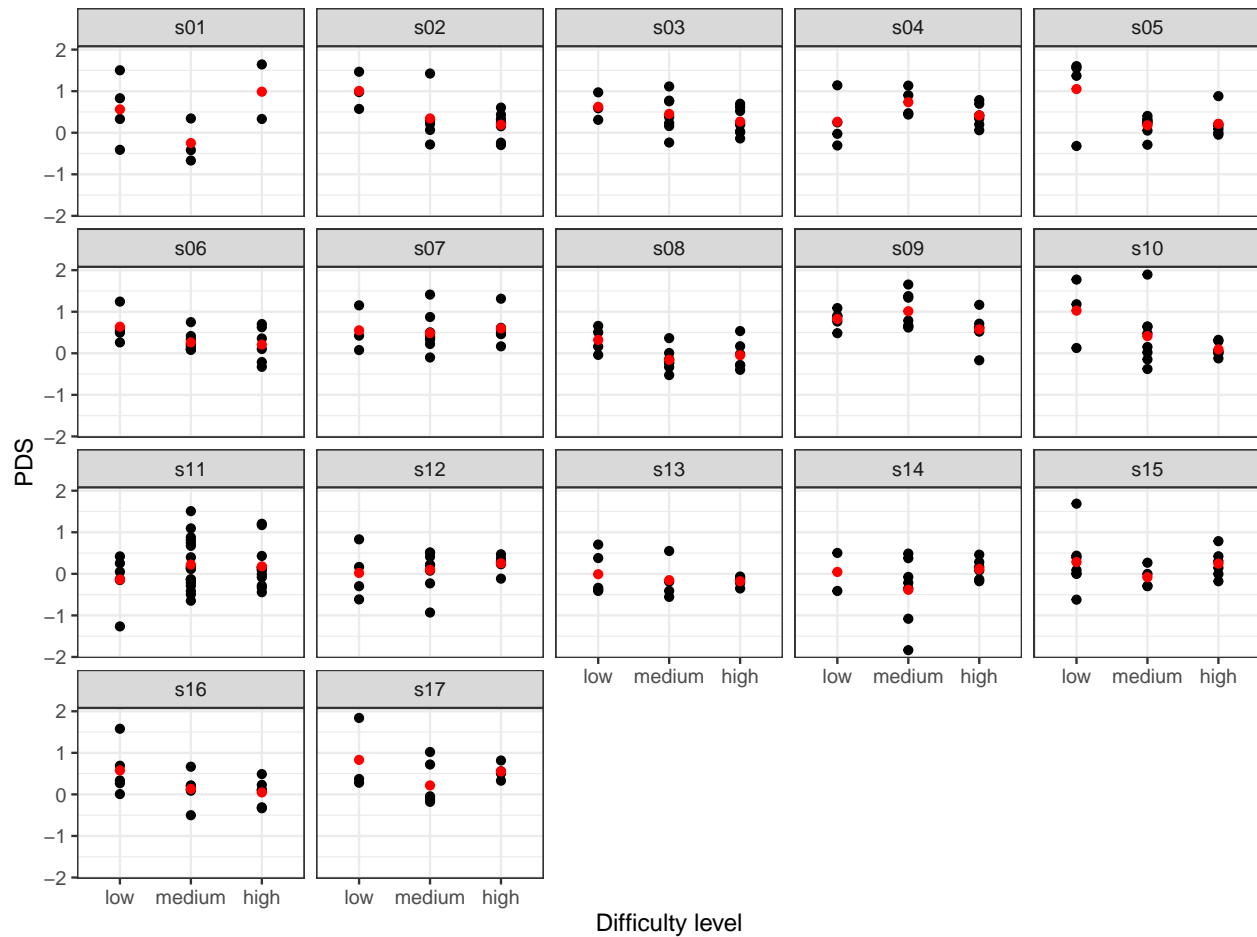
```
(boxcox.list <- EnvStats::boxcox(PDS_lm,optimize = TRUE))

## $lambda
## [1] 0.5307
##
## $objective
## [1] 0.9888
##
## $objective.name
## [1] "PPCC"
##
## $optimize
## [1] TRUE
##
## $optimize.bounds
## lower upper
##      -2      2
##
## $eps
## [1] 0.0000000000000000222
##
## $lm.obj
##
## Call:
## lm(formula = PDS ~ Level, data = Datos, y = TRUE, qr = TRUE)
##
## Coefficients:
## (Intercept)  Levelmedium  Levelhigh
##          1.607         -0.291         -0.313
##
##
## $sample.size
## [1] 297
##
## $data.name
## [1] "PDS_lm"
##
## attr("class")
## [1] "boxcoxLm"

Datos$PDS = (Datos$PDS^(0.5307)-1)/(0.5307)
Datos %>%
  group_by(Level) %>%
  summarise(n=n(),MD=mean(PDS),SD=sd(PDS)) %>%
  kable() %>%
  kable_styling(latex_options = c("striped"))

(q <- Datos %>% ggplot(aes(x=Level,y=PDS)) +
```

```
geom_point() + facet_wrap(~ Subject)+
labs(x="Difficulty level")+theme_bw()+
stat_summary(fun="mean", geom="point",color="red"))
```



```
PDS_mixed_lme4 <- lme4::lmer(PDS ~ Level + (1+Level|Subject),data=Datos)
summary(PDS_mixed_lme4)
```

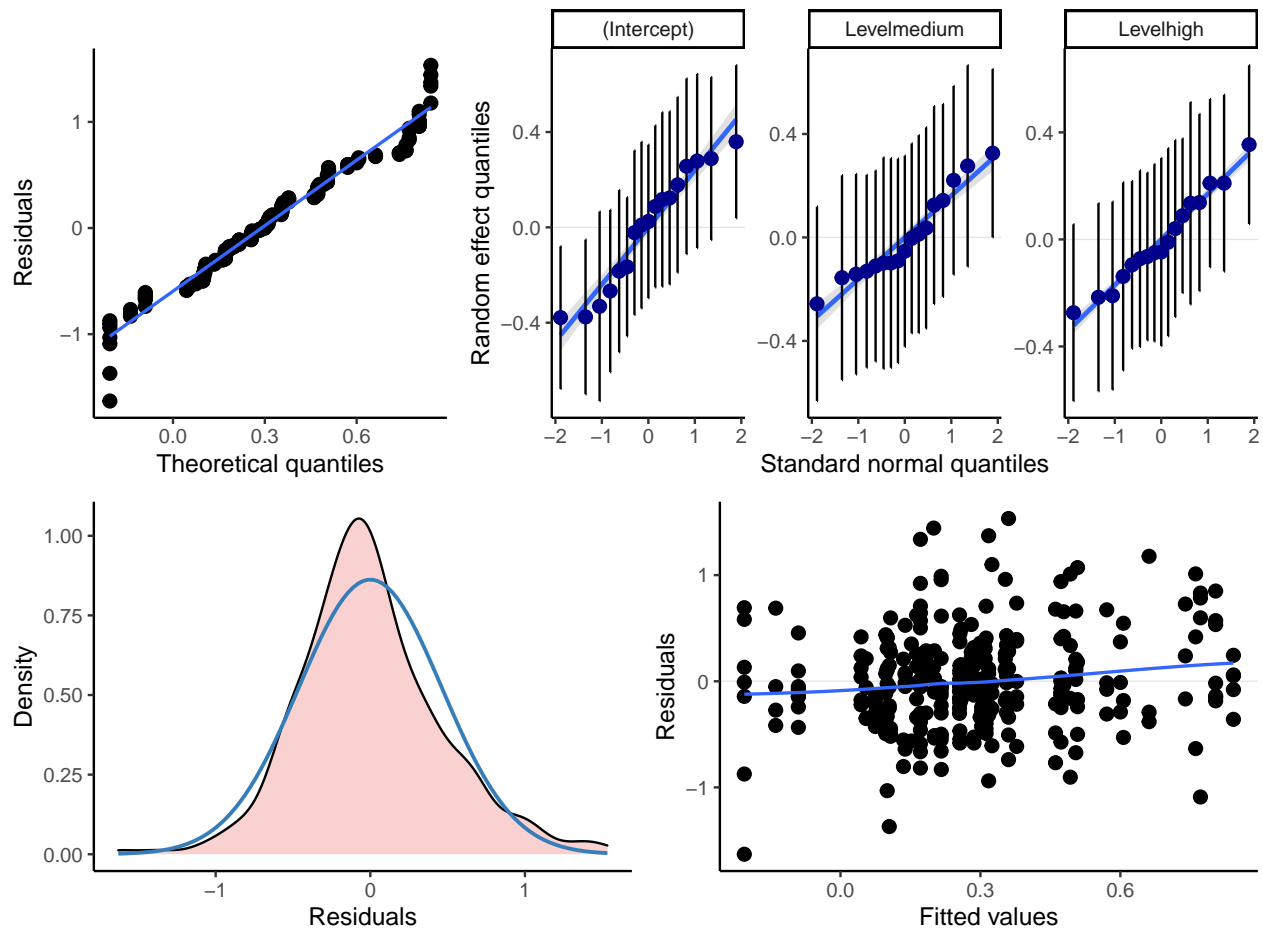
```
## Linear mixed model fit by REML ['lmerMod']
## Formula: PDS ~ Level + (1 + Level | Subject)
## Data: Datos
##
## REML criterion at convergence: 450.1
##
## Scaled residuals:
## Min 1Q Median 3Q Max
## -3.395 -0.611 -0.090 0.510 3.194
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## Subject (Intercept) 0.0904 0.301
## Levelmedium 0.0650 0.255 -0.47
## Levelhigh 0.0581 0.241 -0.91 0.80
## Residual 0.2305 0.480
## Number of obs: 297, groups: Subject, 17
```



```
##
## Fixed effects:
##           Estimate Std. Error t value
## (Intercept)  0.4831    0.0939    5.15
## Levelmedium -0.2590    0.0971   -2.67
## Levelhigh   -0.2429    0.0954   -2.54
##
## Correlation of Fixed Effects:
##           (Intr) Lvlmdm
## Levelmedium -0.617
## Levelhigh   -0.821  0.695
## optimizer (nloptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see ?isSingular
```

```
p<-plot_model(PDS_mixed_lme4, type = "diag")
```

```
({p[[1]]+theme(plot.title=element_blank(),plot.subtitle=element_blank())+scale_x_continuous(name="Theoretical quantiles")})
```



```
contr <- glht(PDS_mixed_lme4, linfct=mcp(Level="Tukey"))
summary(contr, test = adjusted("holm"))
```

```
##
## Simultaneous Tests for General Linear Hypotheses
##
## Multiple Comparisons of Means: Tukey Contrasts
```

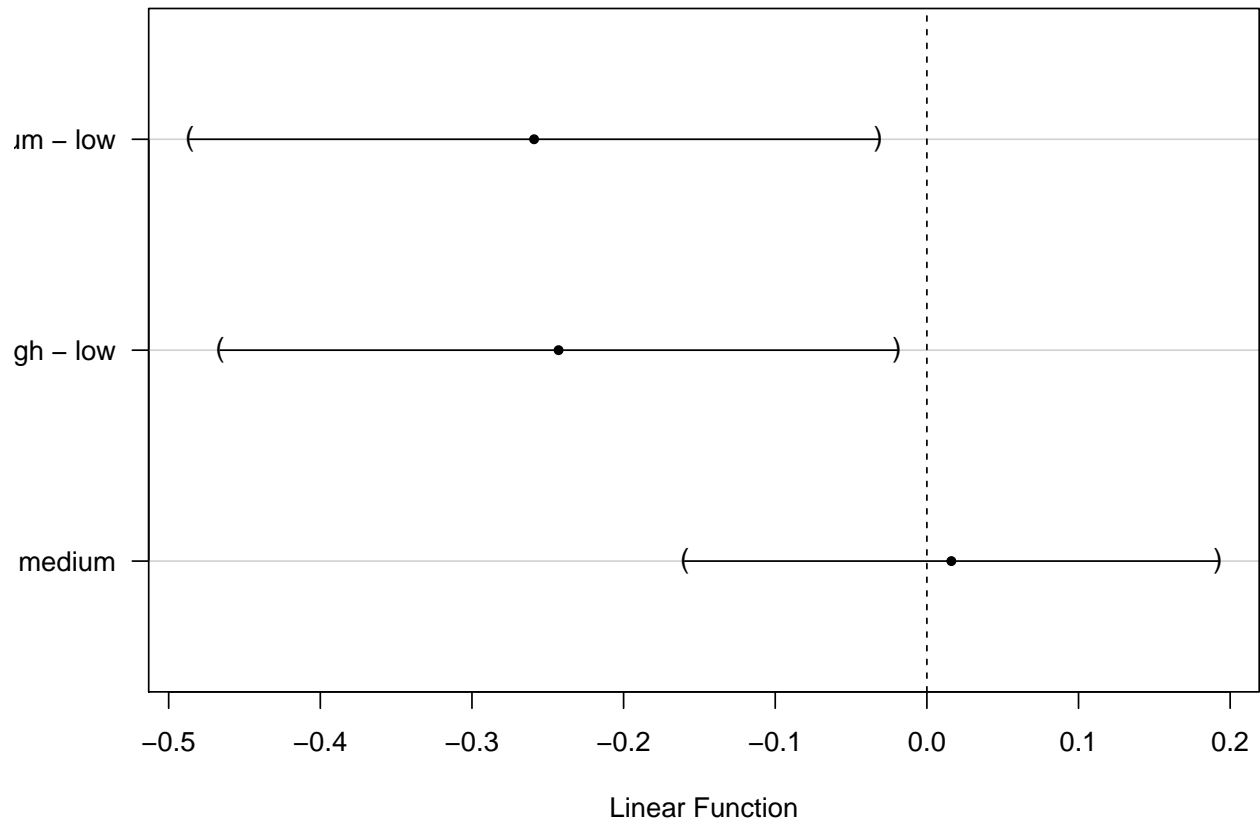
```
##
##
## Fit: lme4::lmer(formula = PDS ~ Level + (1 + Level | Subject), data = Datos)
##
## Linear Hypotheses:
##           Estimate Std. Error z value Pr(>|z|)
## medium - low == 0   -0.2590    0.0971   -2.67   0.023 *
## high - low == 0     -0.2429    0.0954   -2.54   0.023 *
## high - medium == 0   0.0161    0.0752    0.21   0.830
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- holm method)

confint(contr)

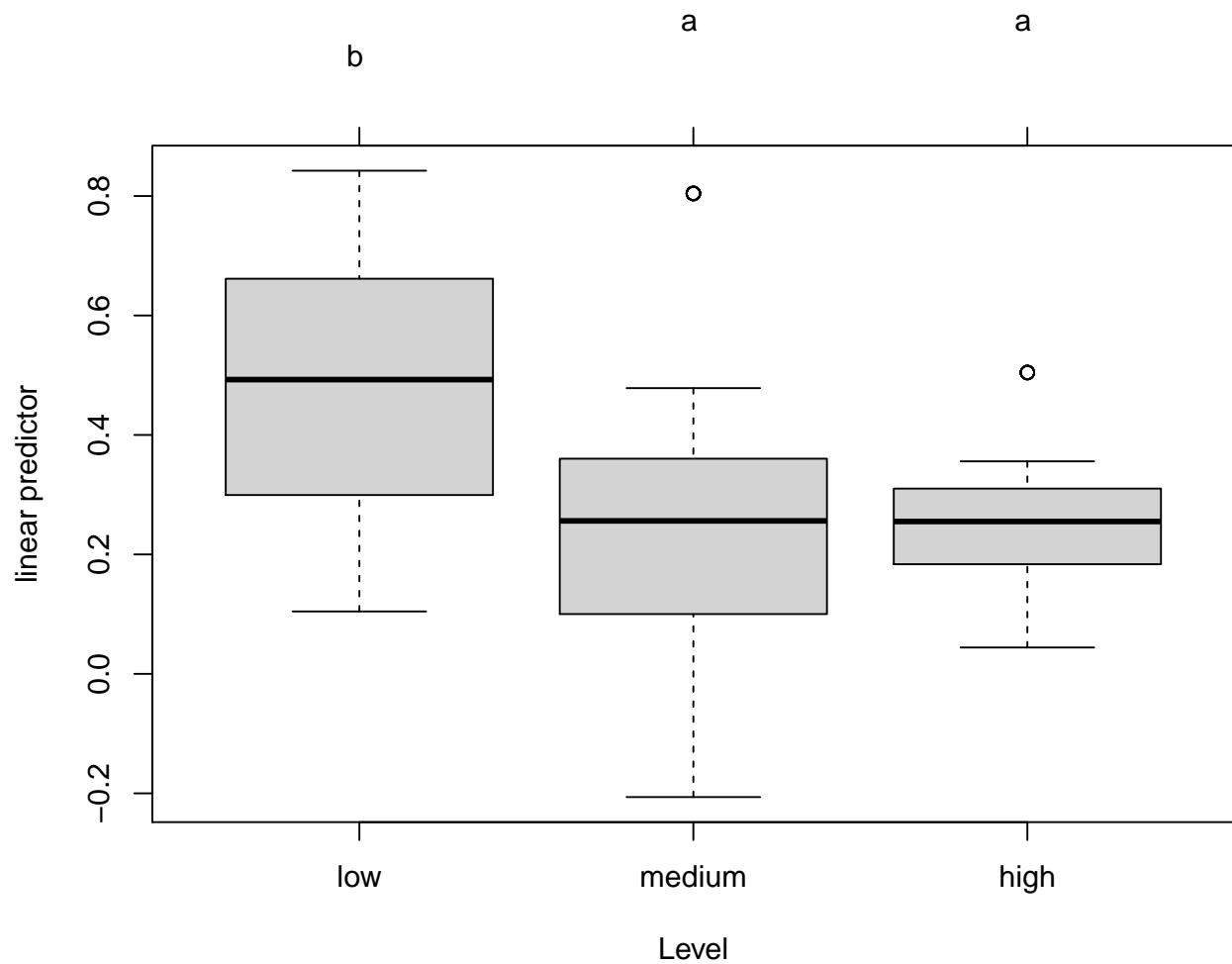
##
## Simultaneous Confidence Intervals
##
## Multiple Comparisons of Means: Tukey Contrasts
##
##
## Fit: lme4::lmer(formula = PDS ~ Level + (1 + Level | Subject), data = Datos)
##
## Quantile = 2.337
## 95% family-wise confidence level
##
## Linear Hypotheses:
##           Estimate lwr      upr
## medium - low == 0  -0.2590 -0.4860 -0.0321
## high - low == 0    -0.2429 -0.4659 -0.0198
## high - medium == 0  0.0161 -0.1597  0.1920

plot(confint(contr))
```

95% family-wise confidence level



```
contr.cld <- cld(contr)
old.par <- par(mai=c(1,1,1.25,1), no.readonly = TRUE)
plot(contr.cld)
```



```
par(old.par)
```

Non parametric tests

```
kruskal.test(PDS ~ Level, data=Datos)
```

```
##
##  Kruskal-Wallis rank sum test
##
## data:  PDS by Level
## Kruskal-Wallis chi-squared = 6.7, df = 2, p-value = 0.03
```

```
PMCMR::posthoc.kruskal.nemenyi.test(data=Datos,PDS~Level, dist="Tukey")
```

```
##
##  Pairwise comparisons using Tukey and Kramer (Nemenyi) test
##           with Tukey-Dist approximation for independent samples
##
## data:  PDS by Level
##
##      low  medium
## medium 0.040 -
## high   0.067 0.986
```

```
##  
## P value adjustment method: none  
PMCMRplus::tukeyTest(data=Datos,PDS~Level)  
  
##          low   medium  
## medium 0.013 -  
## high   0.025 0.981
```