DARK
Niels Bohr Institute

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Python for Astronomy

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Ben Bar Or (youtube)

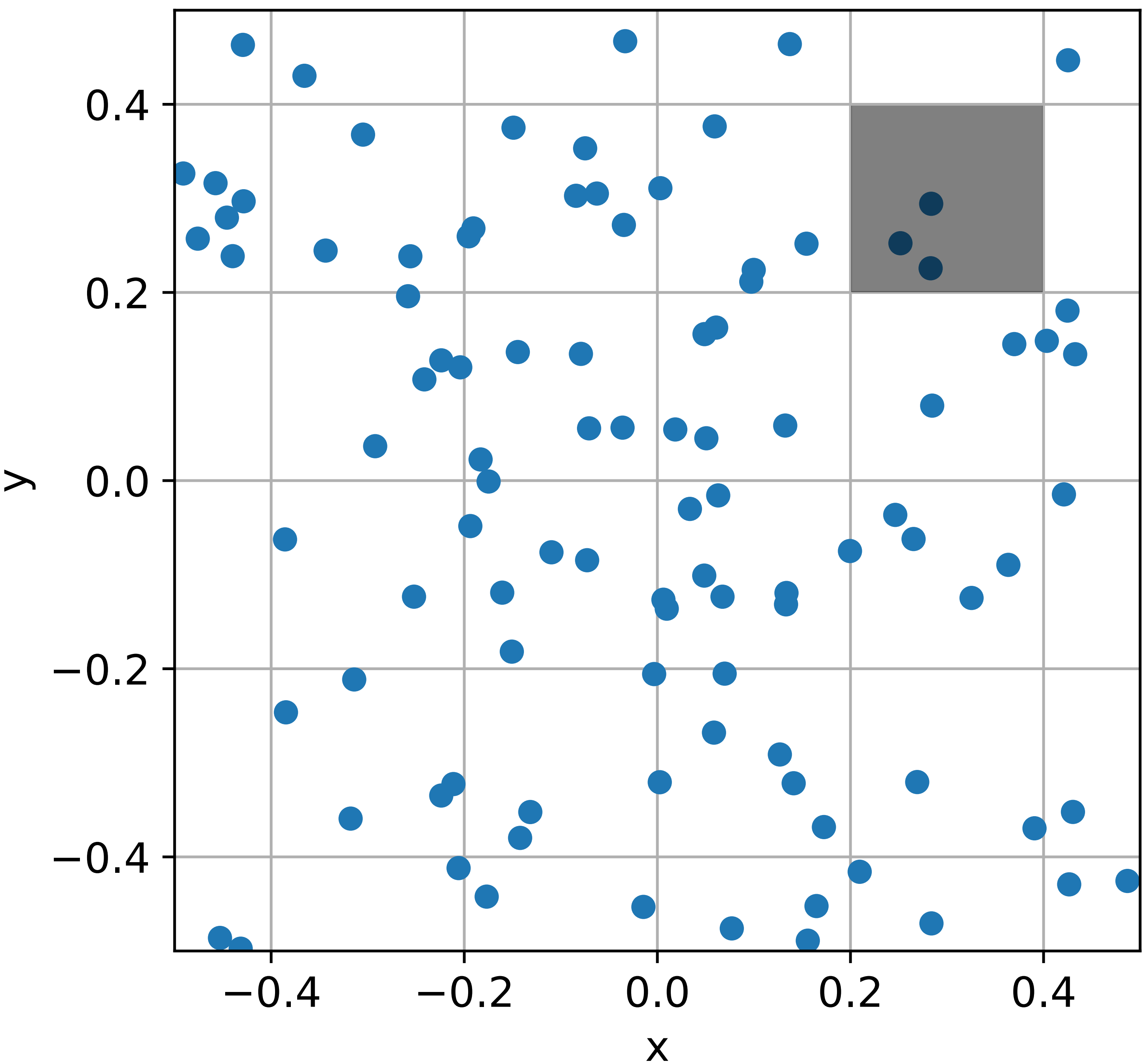
Outline

I. Density profile

II. Numerical fitting

Density profile

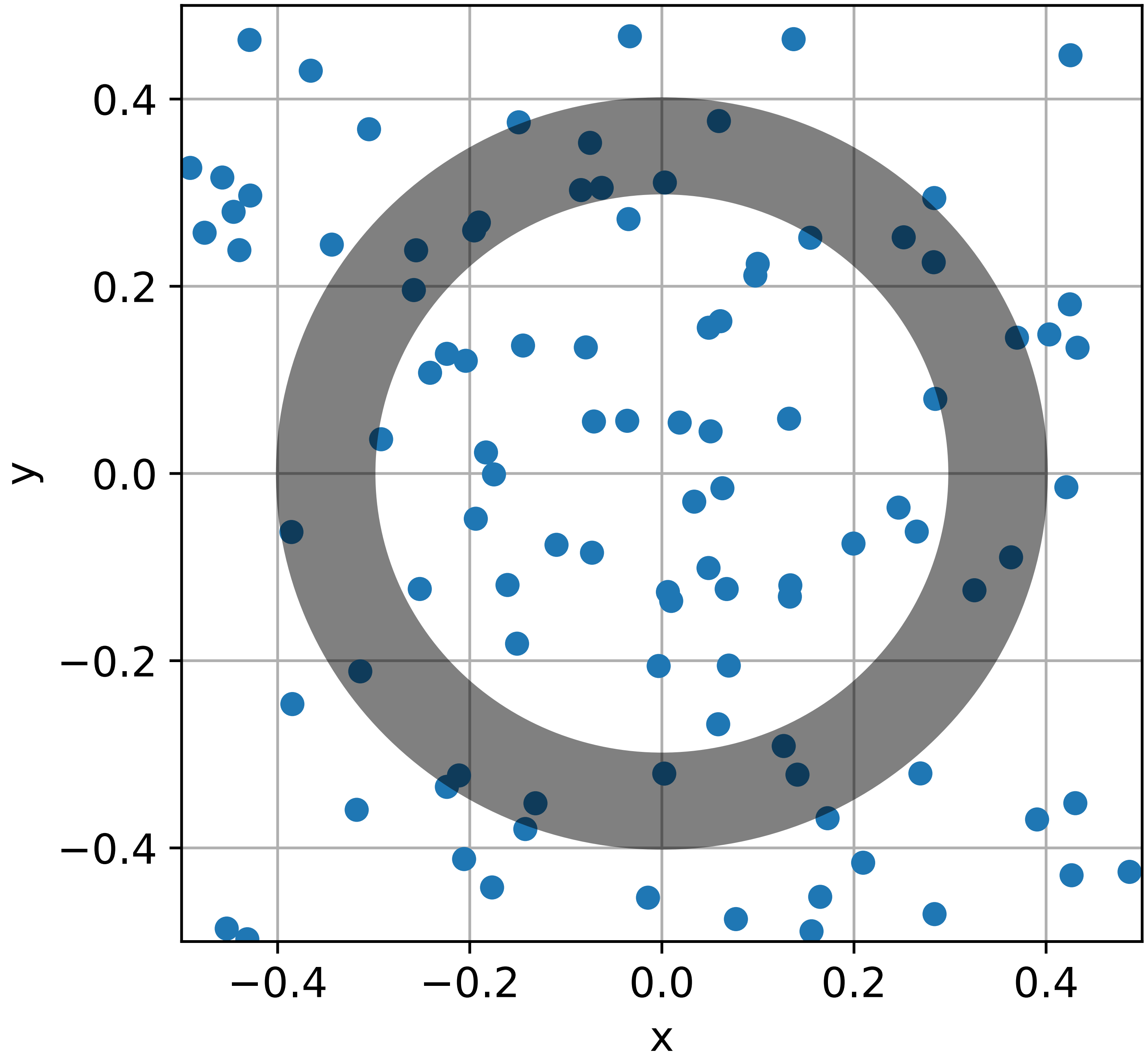
By definition, $\Sigma = \dfrac{\mathrm{d}M}{\mathrm{d}S}$, where $\mathrm{d}M$ is the mass in the elementary surface $\mathrm{d}S$



- In a computer, we have to use finite numbers, so we will use *small* squares between $[x; x+\mathrm{d}x]$ and $[y; x+\mathrm{d}y]$ with surface $\mathrm{d}S = \mathrm{d}x\mathrm{d}y$
- We initiate the 2D matrix $\mathrm{d}M$ to 0
- We loop on all particles i. We find the associated square, and we update $\mathrm{d}M \leftarrow \mathrm{d}M + m_i$
- The built-in numpy function for this loop is **np.histogram2d** on the x_i and y_i, weighted by m_i

Density profile

By definition, $\Sigma = \dfrac{\mathrm{d}M}{\mathrm{d}S}$, where $\mathrm{d}M$ is the mass in the elementary surface $\mathrm{d}S$



- We now use small shells at $[r; r + \mathrm{d}r]$ with surface $\mathrm{d}S = 2\pi r \mathrm{d}r = \mathrm{d}(\pi r^2)$
- We initiate the **1D vector** $\mathrm{d}M$ to 0
- We loop on all particles i. We find the associated shell, and we update $\mathrm{d}M \leftarrow \mathrm{d}M + m_i$
- The built-in numpy function for this loop is **np.histogram** on the r_i, weighted by m_i

Outline

I. Density profile

II. Numerical fitting

   1. Basics

   2. The logarithmic issue

Basics

We have a dataset (x_i, y_i). We think that a function $f_\lambda(x)$, which takes x as input and depends on a set of parameters λ, could model well the data. Our goal is to find the best λ so that the data are well reproduced, that is $f_\lambda(x_i) \sim y_i$.

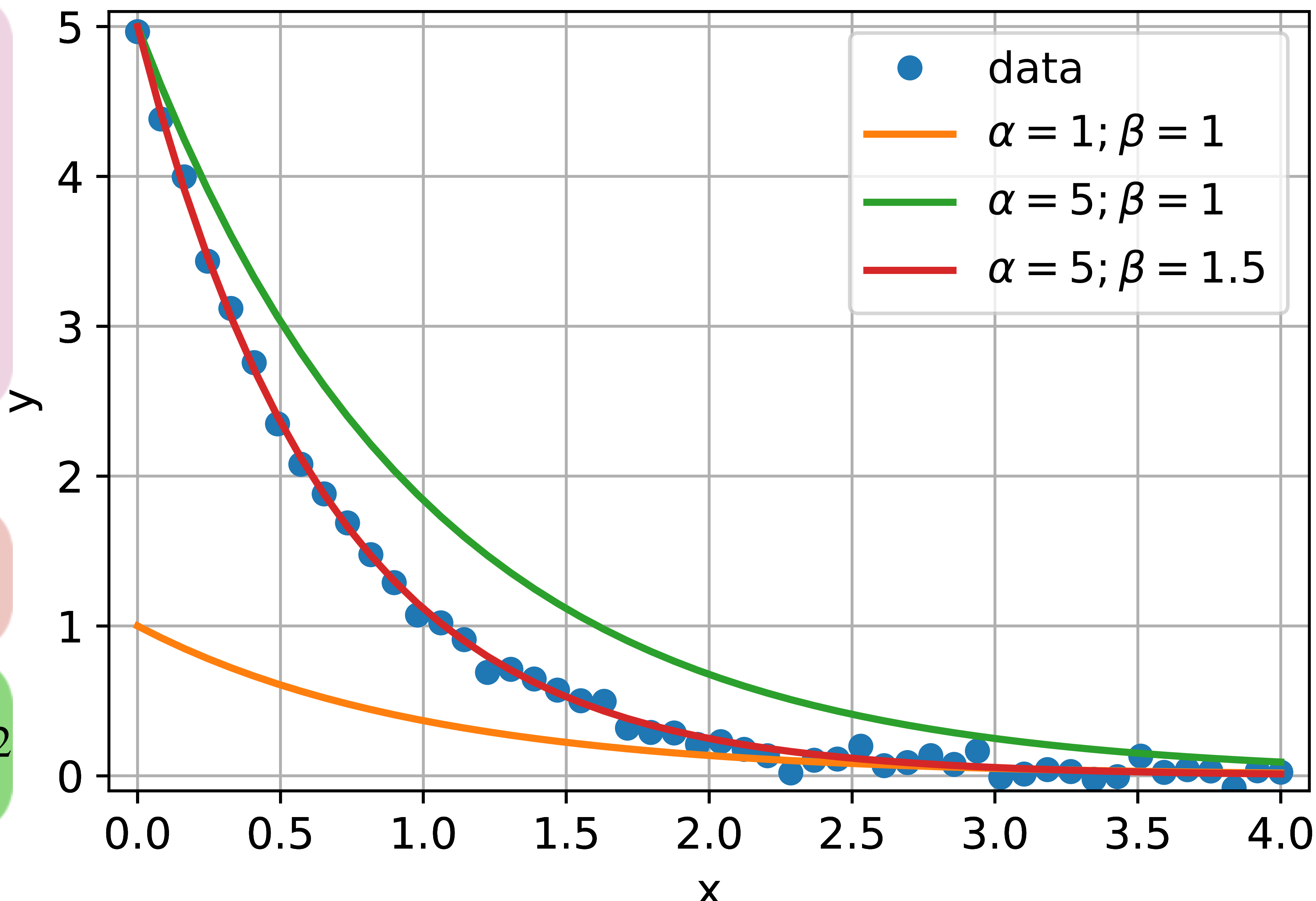
$$f_{(\alpha,\beta)} = \alpha \exp\left(-\frac{x}{\beta}\right)$$

This functional is a *choice*, someone else may have chosen a power law…

Which is the best model?

minimizes

$$\chi^2 = \sum \left(f_\lambda(x_i) - y_i\right)^2$$



Outline

I. Density profile

II. Numerical fitting

   1. Basics

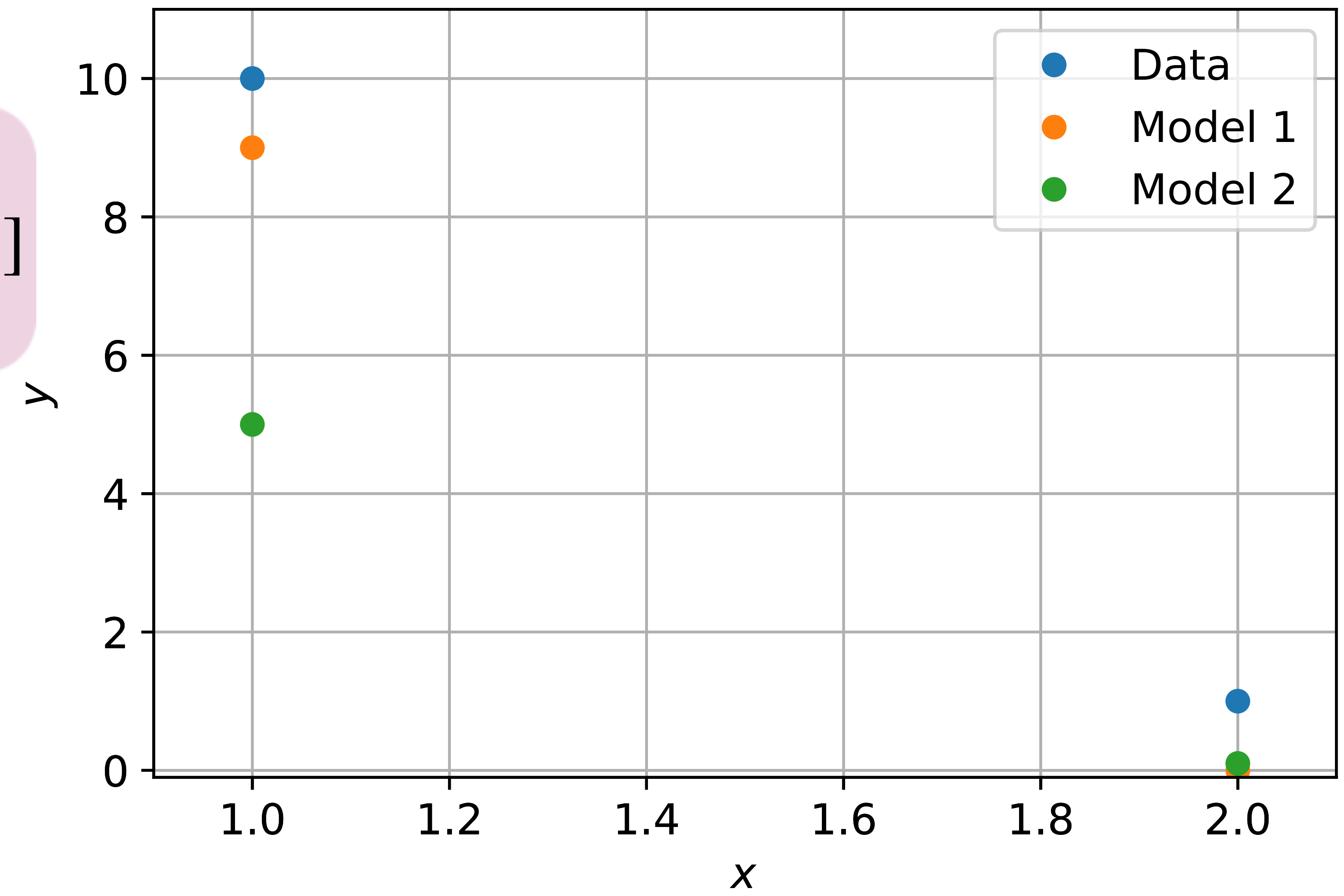
   2. The logarithmic issue

The logarithmic issue

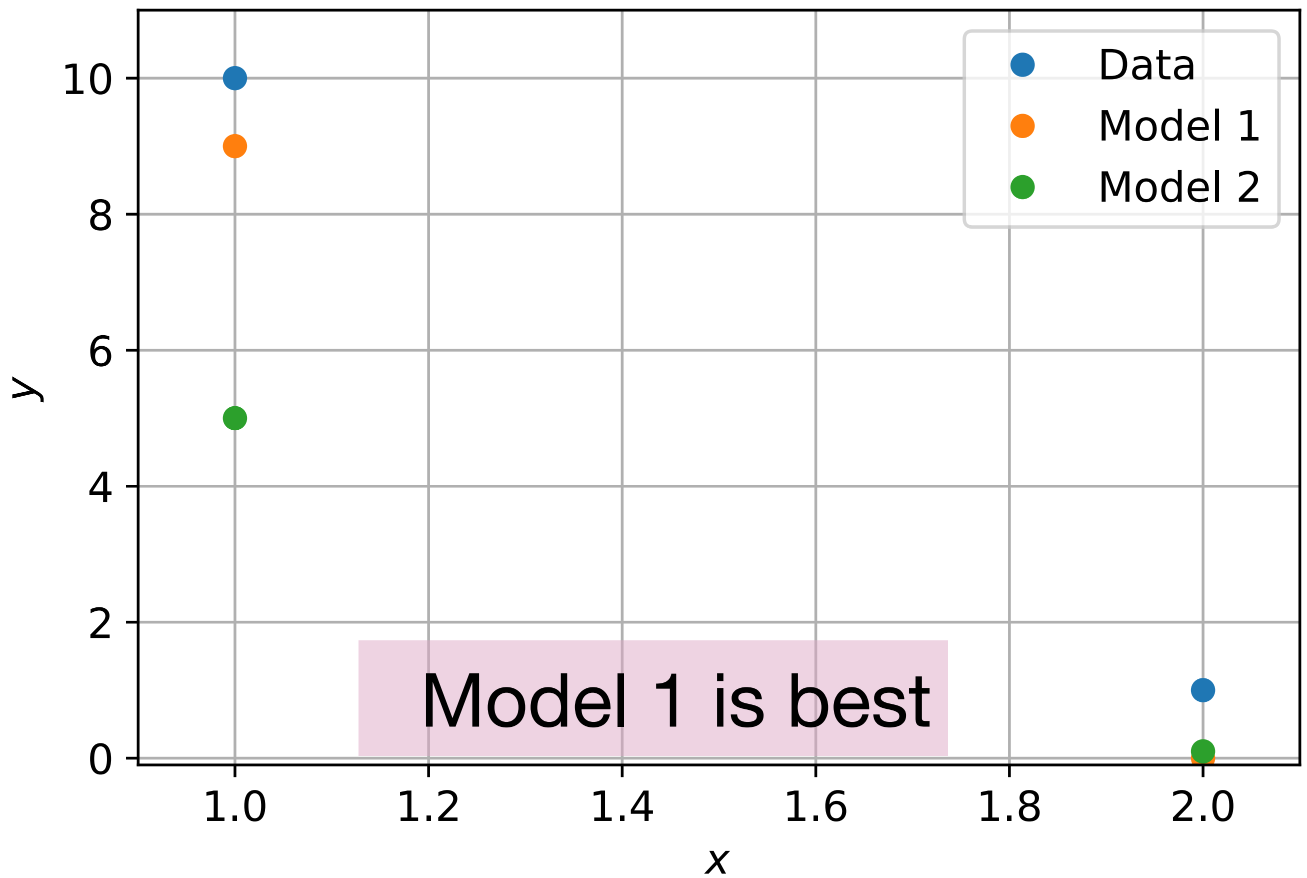
- Data: $[(1,10),(2,1)]$
- Model 1: $[(1,9),(2,10^{-10})]$
- Model 2: $[(1,5),(2,10^{-1})]$

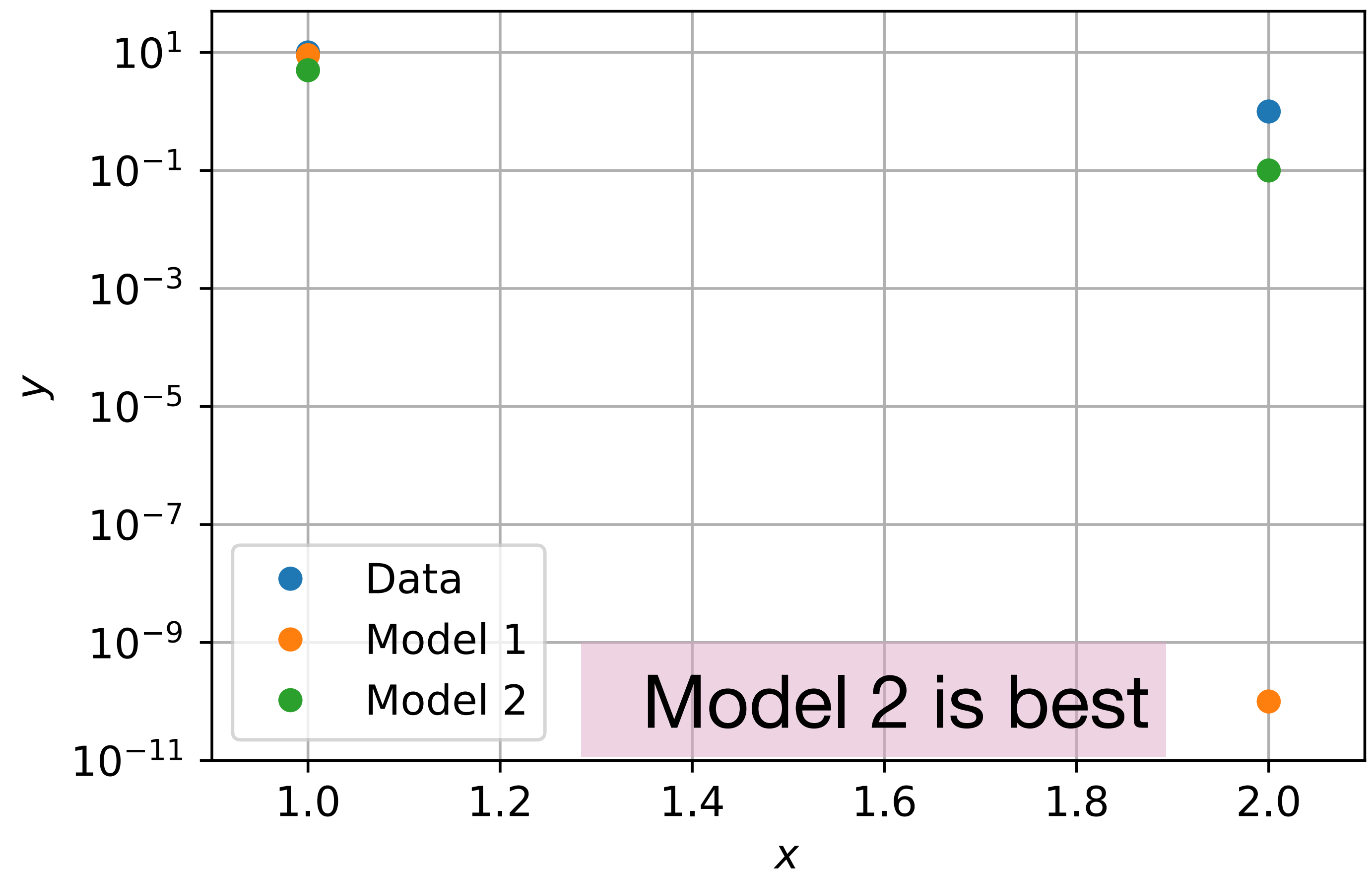
Which model is best?

It depends!



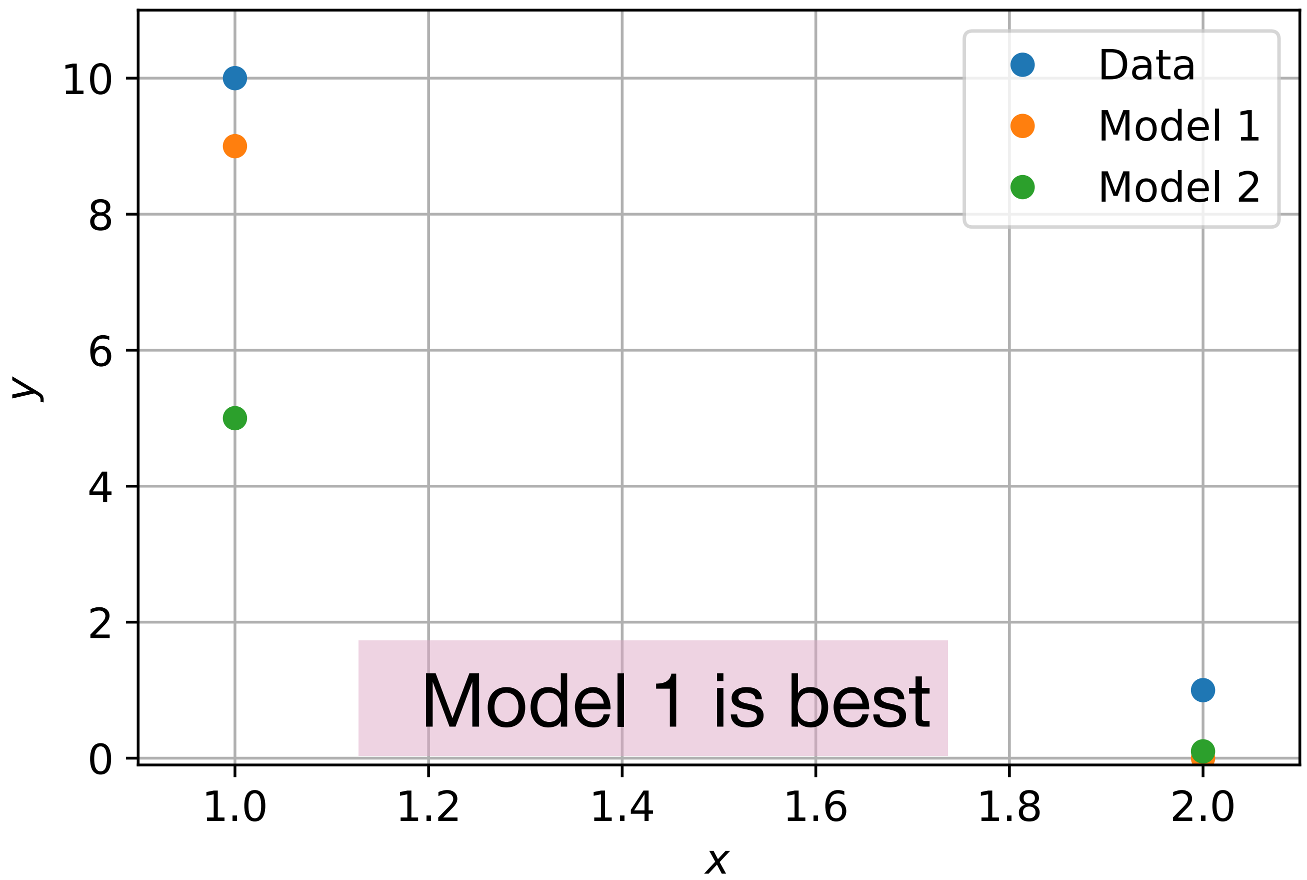
The logarithmic issue





... but what about our mathematical definition of best?

The logarithmic issue



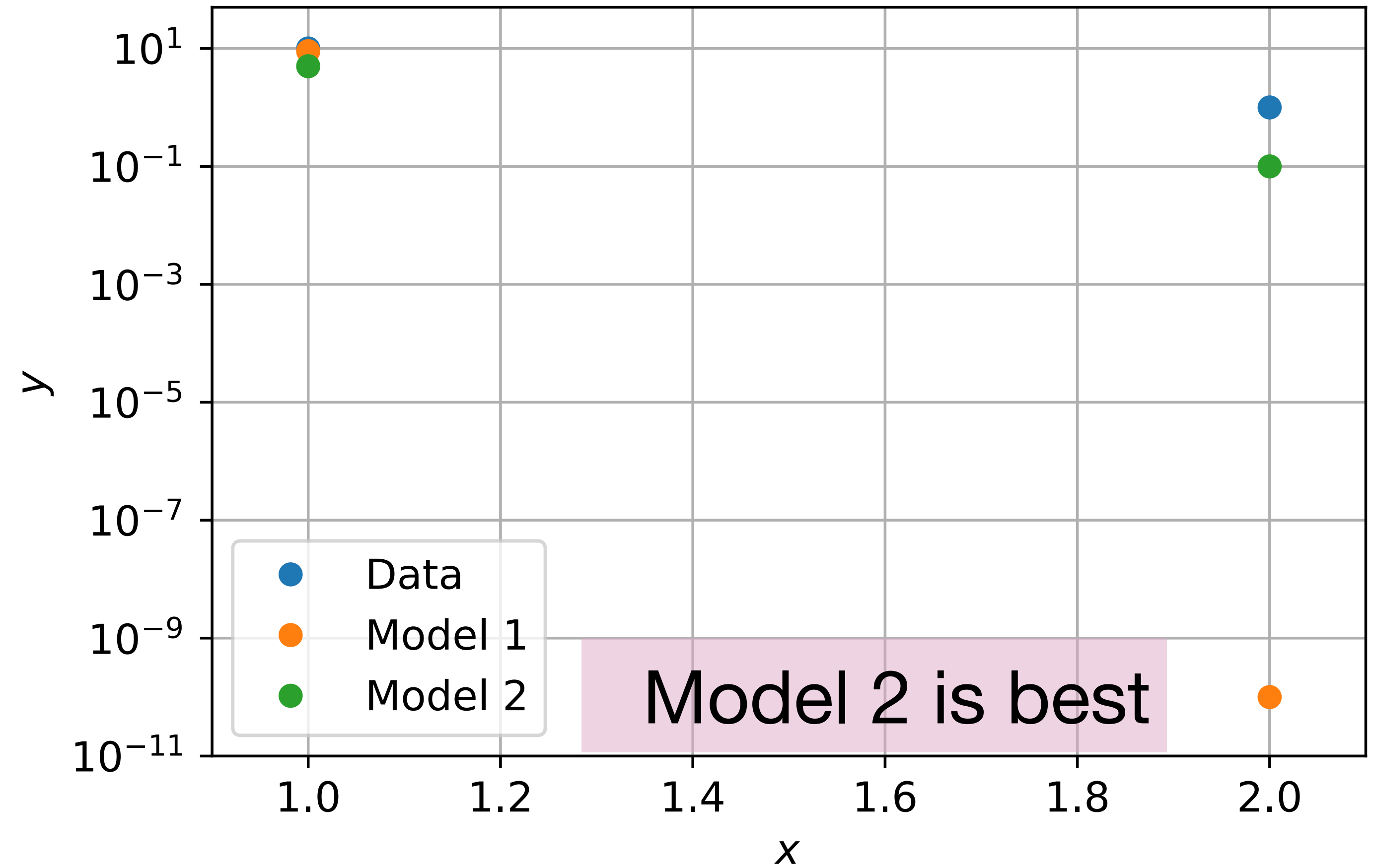
Model 1:

$$\chi^2 = (9 - 10)^2 + (10^{-10} - 1)^2 \sim 2$$

Model 2:

$$\chi^2 = (5 - 10)^2 + (0.1 - 1)^2 \sim 26$$

The logarithmic issue



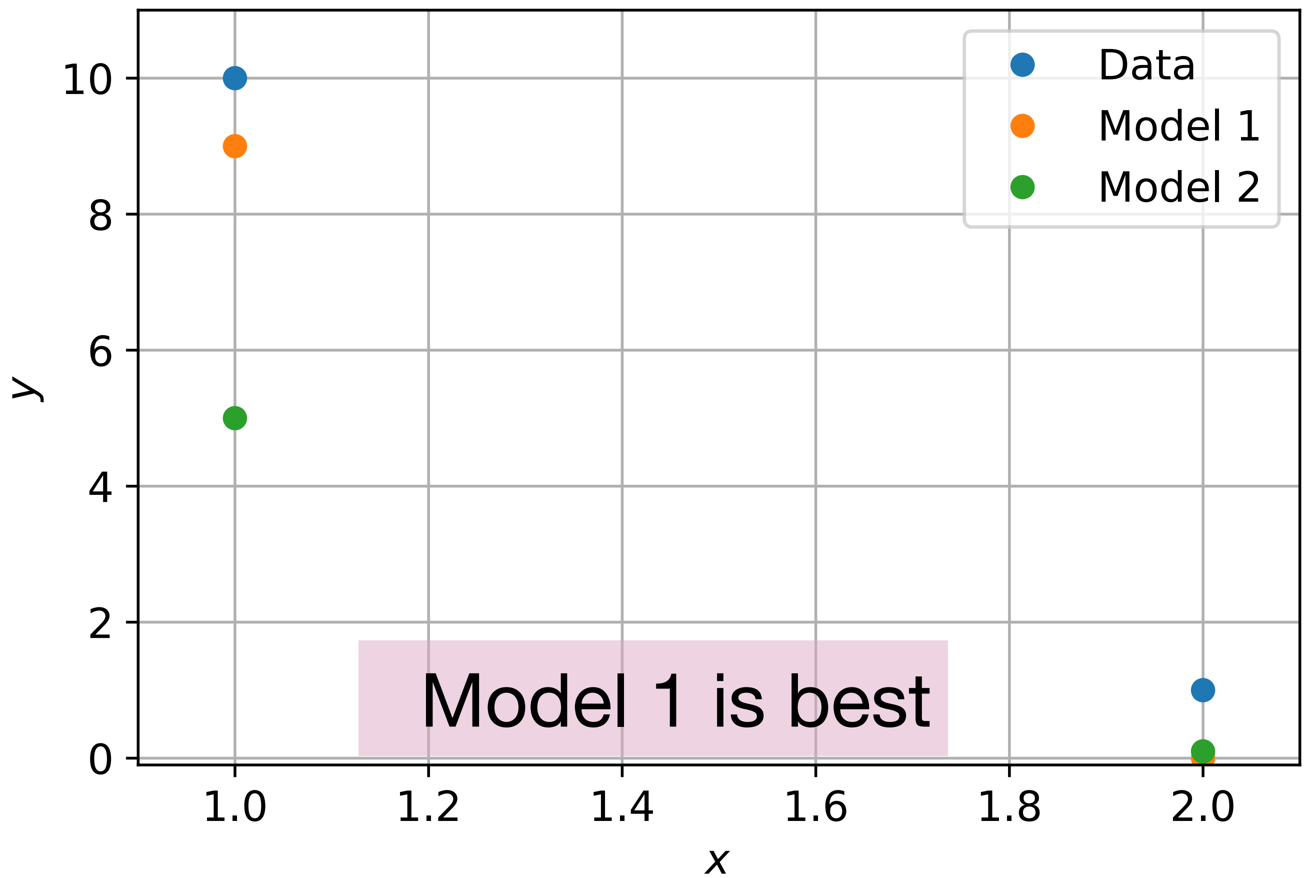
Model 1:

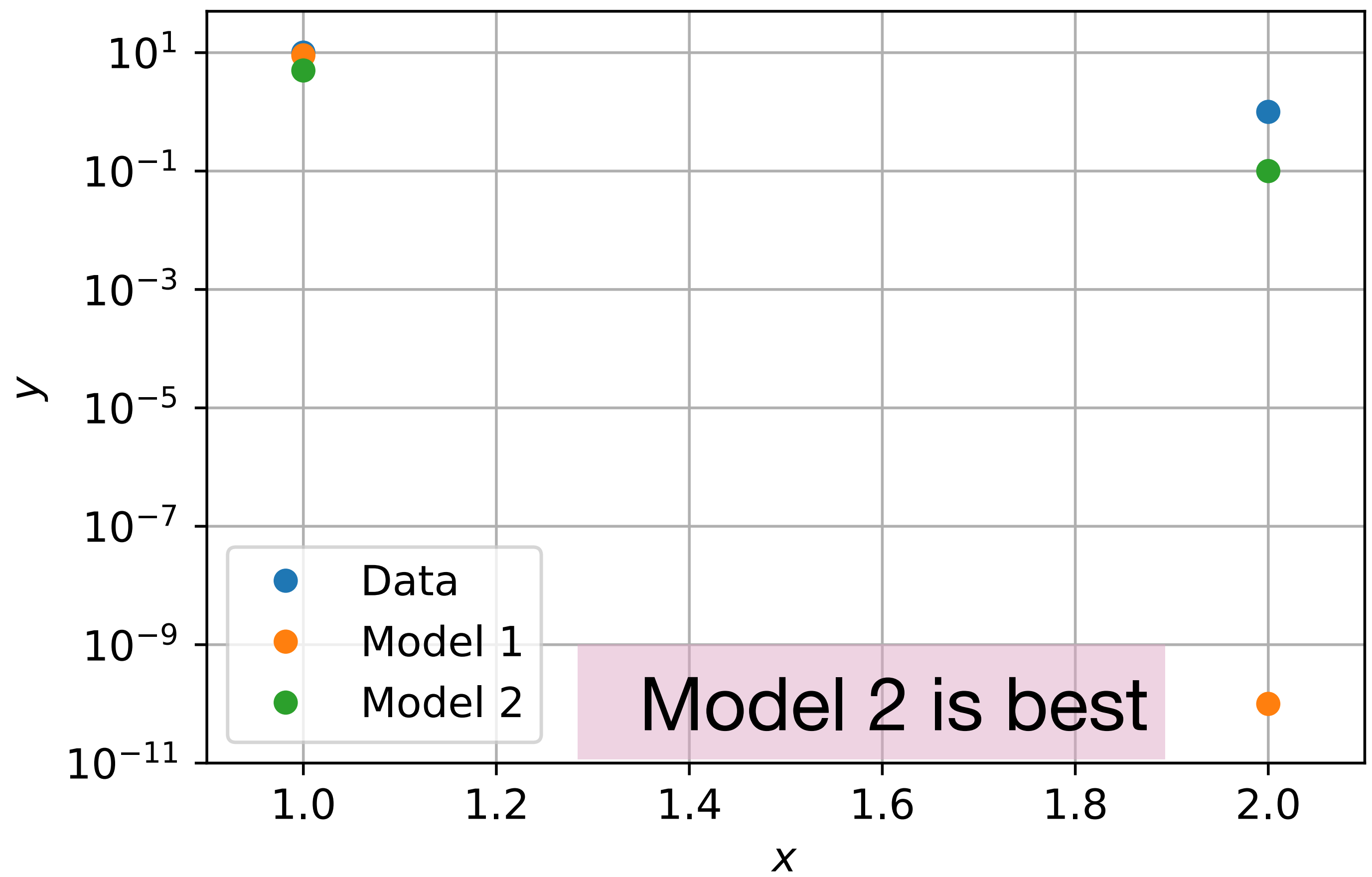
$$\chi^2 = (\log 9 - \log 10)^2 + (\log 10^{-10} - \log 1)^2 \sim 100$$

Model 2:

$$\chi^2 = (\log 5 - \log 10)^2 + (\log 0.1 - \log 1)^2 \sim 1$$

The logarithmic issue





Your eyes are doing the fit in log... so you have to do the fit in log mathematically too!