Problem Chosen	2025	Team Control Number
B	MCM/ICM Summary Sheet	9555

Submersible Rescue

Summary

The 2023 Wimbledon [1] Gentlemen's final witnessed a wonderful batter, a Spanish player ended great tennis player Novak Djokovic's Grand Slam. Not only did the match see a rising star apparence, but also revealed the crucial role of “momentum" in tennis. In this regard, based on relevant data, we conduct in-deep studies of “momentum" in tennis.

Firstly, we did the data cleaning. We determined the cumulative score ratio of a particular game as the label vector, then fused the physical time-series feature and the temporal features of the match phase as the input vector. After processing the data, we chose the LSTM model with the RNN framework as the basic framework of the model. Considering its objective position in the overall game, we used its variant BiLSTM. At the same time, we introduced residual structure, L2 regularization, and the dropout mechanism to avoid the overfitting problem and stacked a 3-layer network to increase the expressive power. We used a deep learning approach to train the model and visualized the results. By comparing the real data and the predicted data, we found our model can well capture the rapidly changing situation in the field. We calculated the probability of "winning under one's serve" as an indicator of a player's performance and compared it with a player's probability. This method can quantify player performance and remove the effect of factors like servers.

For correlation between “momentum" and “swings" analysis, we quantify “momentum". To construct a quantitative model, we use the physical definition and formula for “momentum”$P = mv$ for reference and construct the initial model. Next, with the purpose of making our model in line with the actual situation, we constantly try introducing parameters to optimize the initial model for the most suitable model.

Based on the final model, we denote “swings" and make it harmonize with “ momentum". Owing to the ternary discrete nature of “swings" data, we adopt image analysis to explore the correlation between “momentum" and “swings". After analyzing corresponding images and comparing with actual match scores, we conclude that there is indeed a correlation between “momentum" and “swings" and it may even determine the direction of the game to a certain extent.

We determined which places are the nodes where the momentum shifts. Since we took an approach similar to smoothing relative momentum, we only need to look at the places where the momentum crosses the zero point or drops off sharply. We looked at all the labels in the dataset, coded the features, such as one-hot coding, and numericalized some of the symbols to get some computable vectors of information about the features and used them as inputs to fit a simple multiple regression model to the "momentum". After the training, we sorted the features, and then analyzed them one by one to give suggestions to the athletes using these features.

Keywords: AHP; SAR(submersible search and rescue); Monte Carlo sampling; Runge-Kuta; Sensitivity Analysis.

Contents

1 Introduction

According to historical research, the reservoir was first built around 600 BC. With the devel-opment of science and technology, the function of reservoirs has gradually expanded to irrigating farmland, supplying domestic water, preventing floods and developing hydropower to help maintain people's normal life.

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1.1 Background

1.2 Problem Restatement and Analysis

1.3 Overview of our work

2 Assumptions and Notaions

2.1 List of Notaion

Table 1: Symbols and explanations

Symbols	Description	Unit
$S_{A/B}(i)$	The total score won by player A/B at time step i	m/s
$\theta_{A/B}$	The ability of player A/B to maintain "momentum"	m/s
$M_A(i)$	The relative "momentum" of A at the time step i	s
J	Loss function for the multiple regression model	m/s
α	Multiple regression weights for each feature	m/s

2.2 Assumptions

To simplify the problem and make it convenient for us to simulate real-life conditions, we make the following basic assumptions, each of which is properly justified.

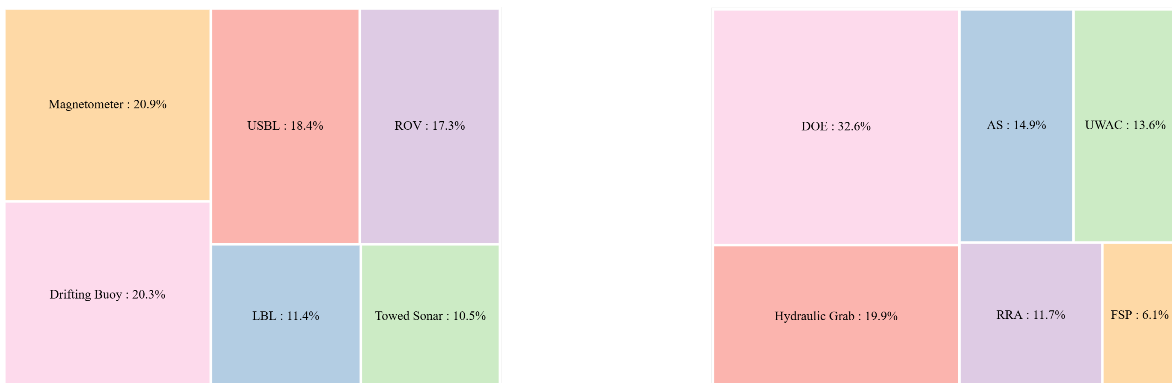
- **Assumption 1:** The sea surface of the Ionian Sea remained stable during our study period.
Justification: The Ionian Sea is located in the central Mediterranean Sea, bordered by the Adriatic Sea to the north, Calabria and Sicily in Italy to the west, and Albania as well as many Greek islands to the east, making it a vast area. Therefore we assume that the sea area will not change significantly and that there is no risk of the submersible and the host ship running aground.

- **Assumption 2:** Climate types in the Ionian Sea remained constant over the study period.
Justification: The climate type of the Ionian Sea is Mediterranean, influenced by the subtropical high and the westerly wind belt, the salinity of the seawater, the air pressure, and the seawater

temperature are roughly stable, so that the size of the wind, the strength of the ocean currents and the density of the seawater are approximately the same.

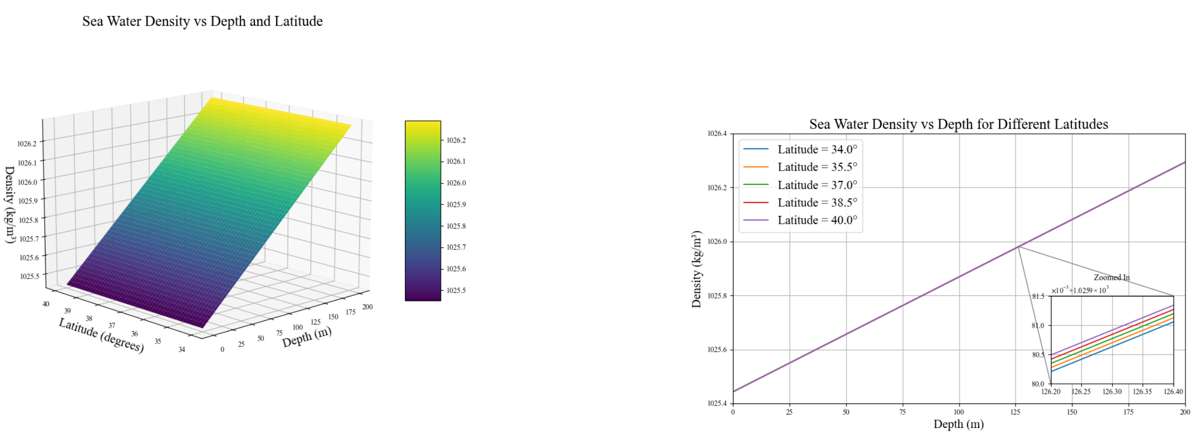
- **Assumption 3:** The Ionian Sea will not experience large natural disasters during the study period.
Justification: Large natural disasters such as earthquakes, tsunamis, and typhoons can

3 Model I: Analysis Model of Submersible Loss of Contact Process



(a) tree map 1 (b) tree map 2

Figure 1: two examples of tree map



(a) density 1 (b) density 2

Figure 2: density of sea

4 Model II: AHP-based Rescue Equipment Selection Model

$$P(0,0,0) = 1, i = 0, j = 0$$
$$P(i,0,k) = P(i-1,0,k-1) \times \frac{1}{3}, j = 0, i \geq 1$$
$$P(0,j,k) = P(i,j-1,k-1) \times \frac{1}{3}, i = 0, j \geq 1$$
$$P(i,j,k) = [P(i-1,j,k-1) + P(i,j-1,k-1) + P(i-1,j-1,k-1)] \times \frac{1}{3}; i \geq 1, j \geq 1$$

5 Model III: Rescue strategy and survival probability model

5.1 Calculation of Ocean Current Velocity at Corresponding Depth

In force equilibrium:

$$M_{\text{sub}} g = \rho(z_{\text{eq}}) g V_{\text{sub}} \tag{1}$$

Solving for z_{eq}:

$$z_{\text{eq}} = \rho^{-1} \left(\frac{M_{\text{sub}}}{V_{\text{sub}}} \right) \tag{2}$$

To solve for z_{eq}, methods such as graphical methods, bisection method, or Newton's method can be used, as the function is monotonic in the range $30m \sim 200m$. The following graph illustrates the density as a function of depth.

Algorithm 1 Fixed Point Iteration to Solve $\rho(z) - \frac{M}{V} = 0$

Input: Tolerance ϵ, Maximum iterations n_{iter}, known constants M, V
Output: Solution z_{eq}
$n \leftarrow 0$
$z \leftarrow z_0 \sim \mathcal{U}(30, 200)$
while $|z_{\text{new}} - z| > \epsilon$ **and** $n < n_{\text{iter}}$ **do**
    $z_{\text{new}} \leftarrow \rho(z) + z - \frac{M}{V}$
    $z \leftarrow z_{\text{new}}$
    $n \leftarrow n + 1$
end while
Return z

Then, the ocean current velocity can be calculated using the formula:

$$v(z_{\text{eq}}) = v(z_0) \cdot e^{-\frac{z_{\text{eq}} - z_0}{\delta}} \tag{3}$$

Where $\delta = \sqrt{\frac{v}{\Omega \sin(\phi)}}$.

In this case, we use $\delta = 112m$ (assumed without questioning the reasoning behind it).

5.2 Calculating the Drift Distance of the Submersible During One Search Round

Assuming the ship quickly reaches a steady drift state, the speed of the submersible's drift is the ocean current velocity. The distance the submersible will drift during one search period is:

$$s = t_{\text{search}} \cdot v(z_{\text{eq}})$$

Typically, this distance s is several times the length d, and the number of steps Δn is:

$$\Delta n = \lceil \frac{s}{d} \rceil$$

This means: within one search cycle, the object might move several grid steps.

5.3 Search process

since we assume the time it takes for the rescue equipment to return to the starting search location is much smaller than the search time, we establish a grid starting from the initial loss of contact location at $(0, 0, 0)$, where the last 0 represents the step.

ignoring seasonal variations, we start with the assumption that the drifting probability for a grid point is:

$$\begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix}$$

at step k, the probability of finding the lost submersible at a given location is represented as:

$$p(i, j, k)$$

the state transition equations are:

$$p(i, j, k) = \begin{cases} 1, & \text{if } i = 0, j = 0, k = 0 \\ p(i-1, 0, k-1) \times \frac{1}{3}, & \text{if } j = 0, i \geq 1, k \geq 1 \\ p(0, j-1, k-1) \times \frac{1}{3}, & \text{if } i = 0, j \geq 1, k \geq 1 \\ [p(i-1, j, k-1) + p(i, j-1, k-1) + p(i-1, j-1, k-1)] \times \frac{1}{3}, & \text{if } i \geq 1, j \geq 1, k \geq 1 \end{cases}$$

we calculate the probability matrix $p[:, :, T_0]$, which represents the probability of finding the submersible at different locations at time t.

next, we find the coordinates corresponding to the maximum probability:

$$i^*, j^* \leftarrow \arg\max_{i,j} p(i, j, T_0)$$

then, we set:

$$p(i^*, j^*, T_0) \leftarrow 0$$

this indicates that the submersible is not found at that location. we update the matrix $p[:, :, T_0 + \Delta n]$ and repeat the process.

thus, by the k-th search round, the probability of finding the submersible is:

$$p(k) = \max_{i,j} p(i, j, T_0 + k\Delta n)$$

we can compute this for multiple values of $k = 1, 2, \ldots, 20$ and store the results in an array.

Additionally, we can compare this strategy with a random search approach. in the random search, the probability of finding the submersible at time k is:

$$p(k) = \frac{1}{(T_0 + k\Delta n)^2}$$

finally, we can generate a comparison plot between the two strategies.

6 Sensitivity Analysis

7 Model Evaluation and Further Discussion

7.1 Strengths

7.2 Weaknesses

7.3 Further Discussion

7.3.1 Model Improvements

7.3.2 Model Extensions

References

[1] A. Vaswani *et al.*, “Attention is all you need,” *Advances in neural information processing systems*, vol. 30, 2017.

Memo for Greek Government

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