Computer Lab Week 10

STAT221

Hypothesis testing using a parametric t-test

Consider the sample, $x_i \in \{1.5, 3.0, 2.2, 2.5, 4.1, 3.7\}$.

1. Test the hypothesis
$$H_1: \quad \mu > 2$$
by writing the R code needed to perform a t-test, but without using the R function `t.test()`. Can the null-hypothesis be rejected at a type-I-error rate of $\alpha = 0.05$?

2. Modify the above code to test the alternative hypothesis
$$H_1: \quad \mu \neq 2.$$
Compare with the previous one-sided hypothesis and test decision.

Hypothesis testing using a permutation test

Generate two normally distributed random samples

$$X_i \sim Normal(\mu_X = 0, \sigma_X = 1)$$

and

$$Y_j \sim Normal(\mu_Y = 1.5, \sigma_Y = 2)$$

with sample sizes $n_X = n_Y = 4$.

1. Perform a one-sided t-test (using $\alpha = 0.05$) to test if $\mu_X < \mu_Y$ without using the function `t.test()`.
2. Write the R code to perform a permutation test for the t-test statistic and compare the resulting p-value with the p-value of the t-test.
3. By creating a suitable plot, compare the sampling distribution of the test statistic (plotted as a histogram, say) with the density of a t-distribution (plotted by adding a curve to the histogram).
4. Create plots for different sample sizes, n_X and n_Y, e.g. 5, 10, 25, 50, 100, and with a range of different seeds used for the random number generation.