Computer Lab Week 12

STAT221

Bootstrapping: Gamma random variables

1. Generate $n = 100$ observations $(x_1, \ldots, x_n)$ from a Gamma distribution (using function `rgamma()`)

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} \qquad \text{for } x > 0;\ \alpha > 0;\ \beta > 0$$

   with shape parameter $\alpha = 2$ and scale parameter $\beta = 3$.

2. Create a density histogram based on the observed sample.

3. Use a nonparametric bootstrap for the mean and plot a density histogram for the bootstrap distribution.

4. Find the 95% confidence interval for the mean, using the percentile method.

5. Use a nonparametric bootstrap to find the confidence interval for the variance (using function `var()`).

6. Use a nonparametric bootstrap to find the confidence interval for the standard deviation (using function `sd()`).

7. An estimator for the shape parameter is given by

$$\hat{\alpha} = \frac{n \sum_{i=1}^n x_i}{n \sum_{i=1}^n x_i \log(x_i) - \sum_{i=1}^n \log(x_i) \sum_{i=1}^n x_i}$$

   and for the scale parameter the estimator is:

$$\hat{\beta} = \frac{1}{n^2} \left(n \sum_{i=1}^n x_i \log(x_i) - \sum_{i=1}^n \log(x_i) \sum_{i=1}^n x_i \right)$$

   Use a nonparametric bootstrap to find confidence intervals for the shape and the scale parameter.

8. Use a parametric bootstrap, making the assumption that $X \sim Gamma(\alpha, \beta)$, to find a confidence interval for the mean and compare to it to that found using the nonparametric bootstrap.

9. Use a parametric bootstrap to find confidence intervals for the variance, the shape and the scale parameter.