A)

2:28

Puntos: x_i, x_{i+1}, x_{i-1}, x_{i+2}, x_{i-2}

Estimación de $f'(x_i)$

$x_{i+1} = x_i + h \qquad x_{i+2} = x_i + 2h$

$x_{i-1} = x_i - h \qquad x_{i-2} = x_i - 2h$

$f'(x_i) = A_{x_{i+2}} + B_{x_{i+1}} + C_{x_i}$

$f(x_{i+2}) = f(x_i) + (2h)f'(x_i) + \frac{1}{2!}(2h)^2 f''(x_i) + \frac{1}{3!}(2h)^3 f'''(x_i) + \dots$

$f(x_{i+1}) = f(x_i) + hf'(x_i) + \frac{1}{2!}h^2 f''(x_i) + \frac{1}{3!}h^3 f'''(x_i) + \dots$

$f'(x_i) = A\left(f_i + 2hf_i' + \frac{4h^2}{2}f_i'' + \frac{8h^3}{6}f_i''' + \dots\right)$
$+ B\left(f_i + hf_i' + \frac{h^2}{2}f_i'' + \frac{h^3}{6}f_i''' + \dots\right) + Cf_i$

$f_i'' = (A+B+C)f_i + (2A+B)hf_i' + (2A+\frac{B}{2})h^2 f_i'' + O^3$

$A + B + C = 0$

$2A + B = 0$

$4A + B = 2/h^2$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 4 & 1 & 0 \end{pmatrix}\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2/h^2 \end{pmatrix}$$

$A = 1/h^2 \qquad B = -2/h^2 \qquad C = 1/h^2$

$$f''(x_i) = \frac{f_{x_{i+2}} - 2f_{x_{i+1}} + f_{x_i}}{h^2}$$

B)

One-sided D^{-3} $Av_{i+2} + Bv_{i+1} + Cv_i + Dv_{i-1}$

$$F(x_{i+2}) = F(x_i) + 2h F'(x_i) + \frac{1}{2!}(2h)^2 F''(x_i) + \frac{1}{3!}(2h)^3 F'''(x_i) + \dots$$

$$F(x_{i+1}) = F(x_i) + h F'(x_i) + \frac{1}{2!} h^2 F''(x_i) + \frac{1}{3!} h^3 F'''(x_i) + \dots$$

$$F(x_{i-1}) = F(x_i) - h F'(x_i) + \frac{(-h^2)}{2!} f_i'' + \frac{(-2h)^3}{3!} F'''(x_i) + \dots$$

$$F(x_i) = A\left(f_i + 2h f_i' + \frac{4}{2} h^2 f_i'' + \frac{8}{6} h^3 f_i''' + \dots\right)$$
$$+ B\left(f_i + h f_i' + h^2/2 f_i'' + \frac{h^3}{6} f_i''' + \dots\right)$$
$$+ C f_i + D\left(f_i - h f_i' + \frac{h^2}{2} f_i'' - \frac{h^3}{6} f_i''' + \dots\right)$$

$$f_i = (A+B+C+D) f_i + (2A+B-D) h f_i' + \left(2A + \frac{B}{2} + \frac{D}{2}\right) h^2 f_i''$$
$$+ \left(\frac{8}{6}A + \frac{B}{6} - \frac{D}{6}\right) h^3 f_i''' + O(h^4)$$

$$A+B+C+D=0$$
$$2A+B \quad -D = 0$$
$$2A + \frac{B}{2} \quad + \frac{D}{2} = 0$$
$$8A + B \quad -D = 6/h^3$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & -1 \\ 2 & 1/2 & 0 & 1/2 \\ 8 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 6/h^3 \end{pmatrix}$$

$A = 1/h^3$ $\quad C = 3/h^3$

$B = -3/h^3$ $\quad D = -1/h^3$

$$F'''(x_i) = \frac{f_{i+2} - 3f_{i+1} + 3f_i - f_{i-1}}{h^3}$$