Circuit Theory and Electronics Fundamentals

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T2: RC Circuit Analysis

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Contents

1 Introduction

The objective of this laboratory assignment is to study an RC circuit as described in Figure 1.

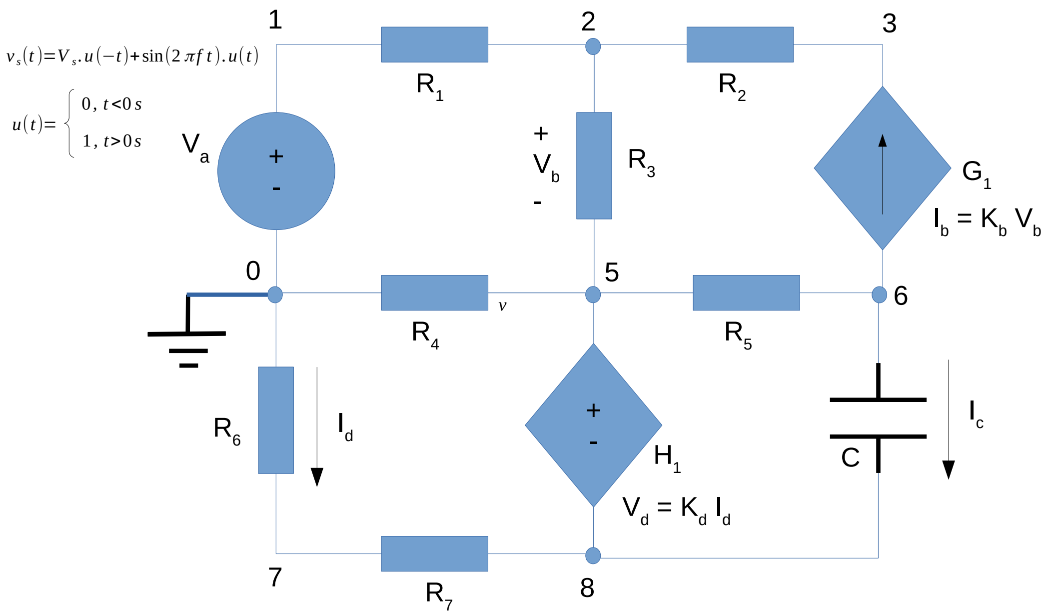

Figure 1: T2 RC circuit.

The objective of this laboratory assignment is to study a circuit composed of eleven branches and eight nodes, arranged in four independant meshes, as detailed bellow. This circuit has seven resistences, numbered from R_1 trough R_7, dependant current and volatage sources, V_C and I_B respectively, and independant current and voltage sources, V_A and I_D respectively. The study of the circuit will be subdivided in two major steps. In Section 2 we will analyse the circuit using the Mesh and Node Analysis. In Section 3 the circuit is going to be simulated with Ngspice, and we will compare the results of that simulation with the ones obtained in Section 2. Finally, the Conclusions of this assignment are detailed in Section 4.

2 Theoretical Analysis

The values used in both analysis are the following:

Name	**Value**
R_1	1.007812 $k\Omega$
R_2	2.003112 $k\Omega$
R_3	3.045036 $k\Omega$
R_4	4.178966 $k\Omega$
R_5	3.106157 $k\Omega$
R_6	2.060902 $k\Omega$
R_7	1.006346 $k\Omega$
V_S	5.048640 V
Cn	1.025026 uF
K_b	7.059582 mS
K_d	8.039139 mS

Table 1: Circuit data generated by inputing 86639 to the $t2\_datagen.py$ file.

2.1 Analysis for t¡0

For t¡0 we can see that we are working in the steady state, given that in that time interval v_S = V_S. In a DC circuit, the capacitor charges up to it's full capacity, blocking the flow of electricity. Taking this into account we can replace the capacitor with an open circuit. With this information, and knowing that the the tension in V_4, since it is connected to ground, we can obtain the following equations.

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1-G_2-G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & -K_b-G_2 & G_2 & K_b & 0 & 0 & 0 \\ G_1 & -G_1 & 0 & -G_4 & 0 & -G_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6+G_7 & -G_7 \\ 0 & 0 & 0 & -1 & 0 & -G_6*K_d & 1 \\ 0 & G_3 & 0 & -G_3-G_4-G_5 & G_5 & -G_6 & 0 \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} -V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

After solving them with the Octave sofware, we obtained the following results:

2.2 Equivalent Resistance and Initial Conditions

In order to determine the equivalent resistance we need to determine V_X, this is, the difference between V_6 - V_8. This is made to ensure that the voltage in the capacitor is continuous.

Making use of the matrix below:

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G1 & -G1-G2-G3 & G2 & G3 & 0 & 0 & 0 \\ 0 & -Kb-G2 & G2 & Kb & 0 & 0 & 0 \\ G1 & -G1 & 0 & -G4 & 0 & -G6 & 0 \\ 0 & 0 & 0 & 0 & 0 & G6+G7 & -G7 \\ 0 & 0 & 0 & -1 & 0 & -G6*Kd & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \\ V5 \\ V6 \\ V7 \\ V8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -Vx \end{pmatrix} \tag{2}$$

Name	Value
$V1$	5.048640 V
$V2$	4.860629 V
$V3$	4.468710 V
$V5$	4.888344 V
$V6$	5.496080 V
$V7$	-2.026270 V
V8	-3.015705 V
$I(R_1)$	0.186554 mA
$I(R_2)$	0.195655 mA
$I(R_3)$	0.009102 mA
$I(R_4)$	1.169750 mA
$I(R_5)$	0.195655 mA
$I(R_6)$	0.983196 mA
$I(R_7)$	0.983196 mA
$I(V_s)$	0.186554 mA
I_b	-0.195655 mA
I_c	0.000000 mA
$I(K_d)$	0.983196 mA

Table 2: Nodal Analysis for $t < 0\ s$. Currents are expressed in milliAmpere and Voltages are expressed in Volt.

And knowing that:

$$V_x = V_6 - V_8 \tag{3}$$

$$I_x = \frac{V_6 - V_5}{R_5} + \frac{V_3 - V_2}{R_2} \tag{4}$$

$$R_eq = \frac{V_x}{I_x} \tag{5}$$

$$\tau = R_eq * C \tag{6}$$

We can obtain the following values:

Name	Value
V_x	8.511786 V
I_x	2.740295 mA
R_{Eq}	3.106157 $k\Omega$
τ	0.003184 s

Table 3: Second nodal analysis and calculations for R_{Eq} and τ. Currents are expressed in $milliAmpere$, Voltages are expressed in $Volt$, Resistances are expressed in $kiloOhm$ and Time Constant is expressed in $second$.

2.3 Natural Solution

The natural solution doesn't take into account independent power sources. Using V_x as the initial condition the natural solution can be obtained with the following equation

$$V_{6n}(t) = V_x e^{(-\frac{t}{\tau})} \tag{7}$$

After computing this data on Octave, we can plot the results of the interval [0, 20]ms.

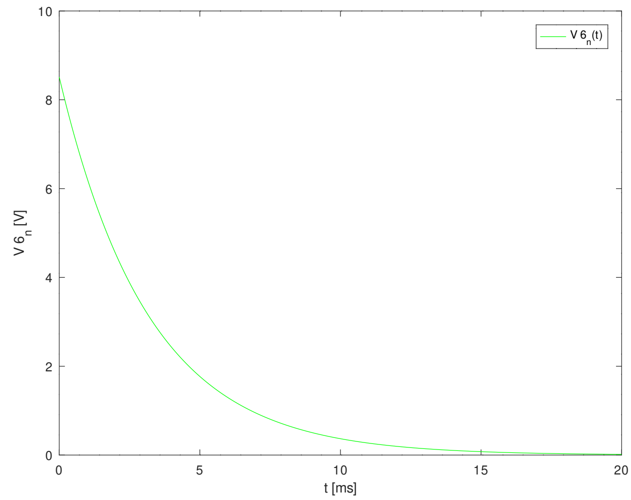

Figure 2: Natural solution V_{6n} in the interval $[0, 20]\ ms$ plot.

2.4 Forced Solution

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In order to obtain the forced solution we must be able to solve the following equation.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G1 & G1+G2+G3 & -G2 & -G3 & 0 & 0 & 0 \\ 0 & Kb+G2 & -G2 & -Kb & 0 & 0 & 0 \\ -G1 & G1 & 0 & G4 & 0 & G6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -G6-G7 & G7 \\ 0 & 0 & 0 & 1 & 0 & G6*Kd & -1 \\ 0 & -G3 & 0 & G3+G4+G5 & -G5-jwC & G6 & jwC \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \\ V5 \\ V6 \\ V7 \\ V8 \end{pmatrix} = \begin{pmatrix} -j \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{8}$$

The solution to this system is presented in the table below:

These values are needed to determine the forced solution V_{6f}, which is given by the following formula:

$$V_{6f}(t) = V_{6r} cos(\omega t + V_{6\phi}); \tag{9}$$

Where

$$\omega = 2\pi f; \tag{10}$$

Name	Complex Amplitude [V]	Phase [Degrees]
V_1	1.000000	-90.000000
V_2	0.962760	-90.000000
V_3	0.885131	-90.000000
V_5	0.968250	-90.000000
V_6	0.599056	98.067060
V_7	0.401350	90.000000
V_8	0.597330	90.000000

Table 4: Nodal analysis for phasor voltage in forced state.

2.5 Final Total Solution

The final total solution $V_6(t)$ is achieved by superimposing the natural and forced solutions.

$$V_6(t) = V_{6n}(t) + V_{6f}(t); \tag{11}$$

By converting the phasors to real time functions for $f = 1kHz$ and superimposing the natural and forced solutions, we can plot both $V_S(t)$ and $V_6(t)$ in the interval $[-5, 20]\ ms$, as shown in the figure below:

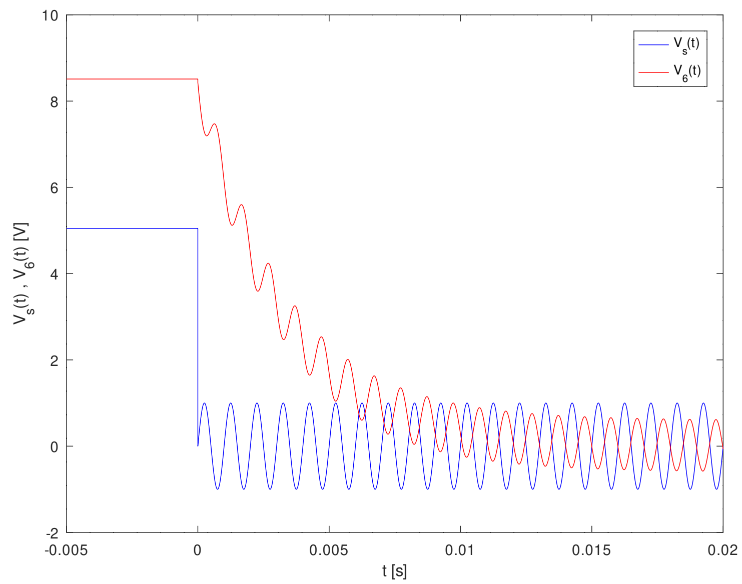

Figure 3: Total solution of V_6 and V_s plot.

As expected, both curves shown in Figure 3 are constant for $t < 0\ s$. For $t > 0\ s$, we can see an evident negative exponencial behavior and an induced frequency in V_6.

2.6 Frequency Responses

Because $V_S(t) = sin(\omega t)$, the magnitude and phase are independent of the frequency f. This means that both the magnitude and phase are expected to be constant in the plots that follow.

The magnitude frequency response is given in Figure 4. The phase response for frequencies ranging from $0.1\ Hz$ to $1\ MHz$ is given in Figure 5. The apparent discontinuity showed in the phase of $V(6)$ is in fact caused by the domain of the arctan function that is used to determined the phase, which is, in reality, continuous.

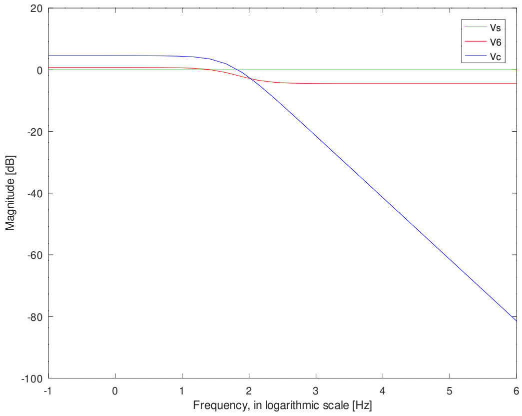

Figure 4: Magnitude frequency response plot, in dB, of V_c, $V(6)$ and V_s. Frequencies ranging from $0.1\ Hz$ to $1\ MHz$ in logarithmic scale.

The circuit being analysed can be used as a low-pass filter. This means that, for low frequencies, the capacitator has time to charge up to almost the same voltage provided as input (it approximates to an open-circuit behavior), which translates to a proximity in phase between the voltage in the capacitor and the voltage source. Having said that, high frequencies, on the other hand, will give the capacitor a small time to charge up before a change in the input direction occurs (it approximates to a short-circuit behavior), which translates to a growing difference in phase between the voltage in the capacitor and the voltage source. This difference of phase is noticeable for frequencies greater than the cut-off frequency, f_c, which is given by $f_c = \frac{1}{2\pi\tau}$. In this particular case, the cut-off frequency value is approximately $50\ Hz$. This is why we see a significant voltage drop in Fig 4 at around $10\ Hz$ and $10^2\ Hz$. The phase difference also starts to increase in that same range, as seen in Fig 5.

By simplifying this circuit to an equivalent one composed by a voltage source, capacitor and equivalent resistor, we reach the following equations that help us to better comprehend the magnitude drop and the phase difference rise with the increase of frequency:

$$V_c = \frac{V_s}{\sqrt{1 + (R_{Eq} \times C \times 2\pi \times f)^2}} \tag{12}$$

$$\phi_{V_c} = -\frac{\pi}{2} + arctan(R_{Eq} \times C \times 2\pi \times f) \tag{13}$$

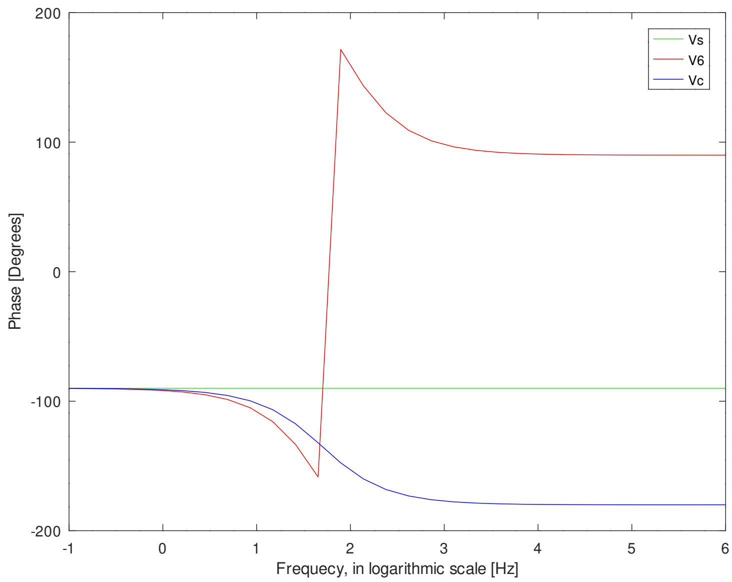

Figure 5: Phase response plot, in degrees, of V_c, $V(6)$ and V_s. Frequencies ranging from $0.1\ Hz$ to $1\ MHz$ in logarithmic scale.

3 Simulation Analysis

In this section, the circuit shown in Figure 1 is simulated with the use of NGSpice. Using the operating point analysis for both $t < 0\ s$ and $t = 0\ s$, we determine the initial conditions needed for the transient analysis, which in turn simulates the circuit's total response. Because of the use of NGSpice for this simulation, there was a need to create a "dummy" voltage source, between nodes 7 and 8, that provided $0\ V$ to the circuit (thus not changing the behavior of the original circuit). Because this is just a technical issue that does not affect the original circuit, the one shown in Figure 1 can be used for illustrative purposes.

3.1 Operating Point Analysis

Table 5 shows the simulated operating point results for $t < 0\ s$, where it's assumed that no current flows through the capacitor (open circuit). Table 6 shows the simulated operating point results for $t = 0\ s$, where V_S is short-circuited and the capacitor is replaced with a voltage source $V_x = V(6) - V(8)$ (with $V(6)$ and $V(8)$ as obtained in Table 5.

Compared to the theoretical analysis results, one notices that the simulated data matches almost perfectly the theoretical values. This is expected, as the circuits being simulated in both scenarios are exclusively composed by linear components. Moreover, the slight discrepancies (of the order of $1e-15$) can be associated to the precision of ngspice and to some approximations made by octave because of the precision of the floating point used.

Name	Value [A or V]
@gib[i]	4.336393e-18
@r1[i]	-4.13467e-18
@r2[i]	-4.33639e-18
@r3[i]	-2.01724e-19
@r4[i]	8.501418e-19
@r5[i]	2.740295e-03
@r6[i]	8.673617e-19
@r7[i]	-1.75404e-18
v(1)	0.000000e+00
v(2)	4.166970e-15
v(3)	1.285325e-14
v(5)	3.552714e-15
v(6)	8.511786e+00
v(7)	-1.78755e-15
v(8)	-3.55271e-15
v(9)	-1.78755e-15

Table 5: Operating point data for $t < 0\ s$. A variable preceded by @ is of type Current and is expressed in Ampere; other variables are of type Voltage and are expressed in Volt.

Name	Value [A or V]
@gib[i]	4.336393e-18
@r1[i]	-4.13467e-18
@r2[i]	-4.33639e-18
@r3[i]	-2.01724e-19
@r4[i]	8.501418e-19
@r5[i]	2.740295e-03
@r6[i]	8.673617e-19
@r7[i]	-1.75404e-18
v(1)	0.000000e+00
v(2)	4.166970e-15
v(3)	1.285325e-14
v(5)	3.552714e-15
v(6)	8.511786e+00
v(7)	-1.78755e-15
v(8)	-3.55271e-15
v(9)	-1.78755e-15

Table 6: Operating point data for $t = 0\ s$. A variable preceded by @ is of type Current and is expressed in Ampere; other variables are of type Voltage and are expressed in Volt.

3.2 Transient Analysis

3.2.1 Natural Response

Figure 6 shows the plot of the simulated transient analysis results in the interval $[0, 20]\ ms$, using the boundary conditions of $V(6)$ and $V(8)$ as determined before. Once again, the simulation data matches with the theoretical natural response prediction, and one can clearly see the negative exponential behavior of $V(6)$, as was expected.

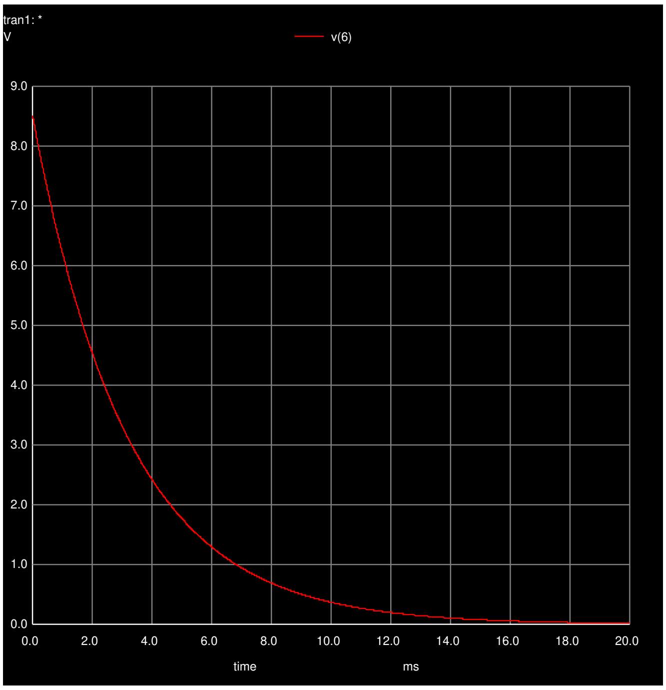

Figure 6: Natural response of V_6 in the interval $[0, 20]\ ms$.

3.2.2 Total Response

Figure 7 shows the plot of the simulated transient analysis results in the interval $[0, 20]\ ms$, by using $V_S(t)$ as given in Figure 1 and $f = 1\ kHz$. Once again, the simulation data matches with the theoretical total response prediction, as was expected.

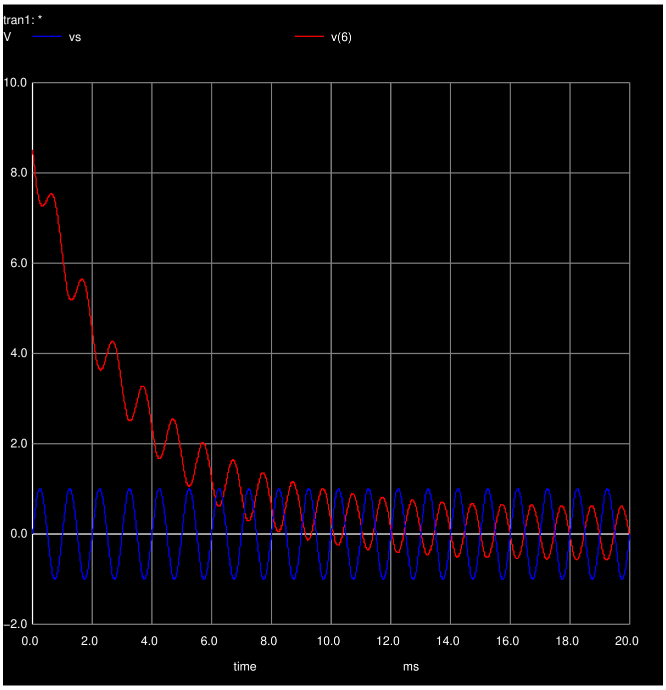

Figure 7: Total response of V_6 and V_S in the interval $[0, 20]$ ms.

3.3 Frequency Analysis

In this section, the frequency response in node 6 is simulated, with the frequency in logscale, magnitude in dB and phase in $degrees$, for the frequency range of 0.1 Hz to 1 MHz.

3.3.1 Magnitude Response

Figure 8 shows the magnitude of the frequency response for the circuit under analysis. Compared to the theoretical analysis results, one notices a clear match between plots. Thus, the reasons for how $V(6)$ and V_S differ from each other are the same as explained in the theoretical analysis above.

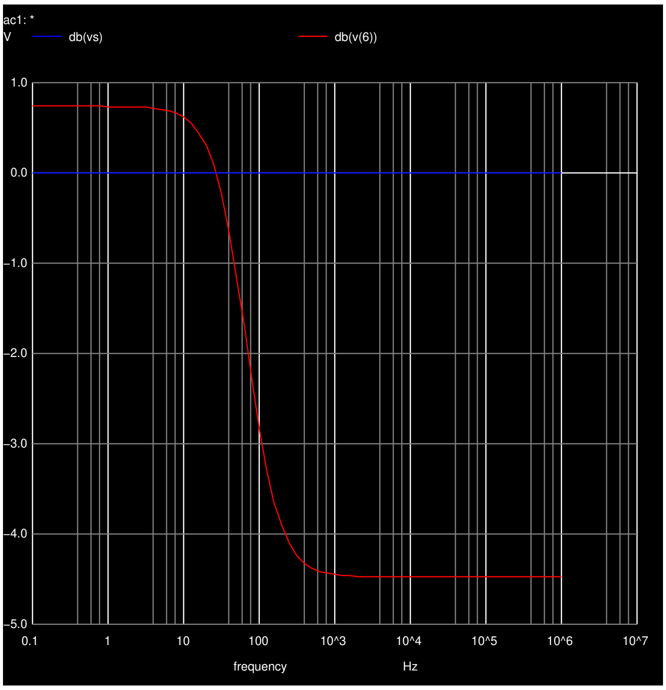

Figure 8: Magnitude of frequency response of $V(6)$ and V_S plot.

3.3.2 Phase Response

Figure 9 shows the magnitude of the frequency response for the circuit under analysis. Compared to the theoretical analysis results, one notices a clear match between plots. Thus, the reasons for how $V(6)$ and V_S differ from each other are the same as explained in the theoretical analysis above.

4 Conclusion

In this laboratory assignment the objective of analysing an RC circuit has been achieved. Static, time and frequency analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the NGSpice tool. The simulation results matched the theoretical results precisely, with the slight differences (in the order of magnitude of $1e-15$) being associated to software precision and roundings done both by Octave and NGSpice. The reason for this almost perfect match is the fact that this is a fairly simple circuit, with only one capacitor and the remaining linear components. Any differences will relate to the capacitor model used by NGSpice. For more complex circuits, these differences are expected to be greater.

In conclusion, the results presented in this report show that this model can be used to simulate this circuit with great accuracy.

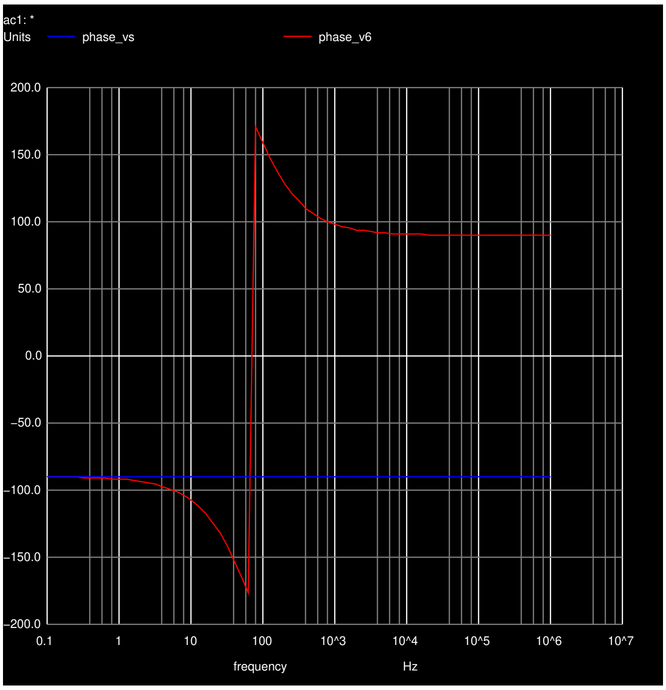

Figure 9: Phase response of $V(6)$ and V_S plot.