Circuit Theory and Electronics Fundamentals

Integrated Masters in Aerospace Engineering, Instituto Superior Técnico, University of Lisbon

Hugo Tavares dos Santos, 86639

Ricardo Esteves Rodrigues, 95841

Víctor Negrini Liotti, 94774

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1 Introduction

The objective of this laboratory assignment is to study a circuit composed of eleven branches and eight nodes, arranged in four independant meshes, as detailed bellow. This circuit has seven resistences, numbered from R_1 trough R_7, dependant current and volatage sources, V_C and I_B respectively, and independant current and voltage sources, V_A and I_D respectively. The study of the circuit will be subdivided in two major steps. In Section 2 we will analyse the circuit using the Mesh and Node Analysis. In Section 3 the circuit is going to be simulated with Ngspice, and we will compare the results of that simulation with the ones obtained in Section 2. Finally, the Conclusions of this assignment are detailed in Section 4.

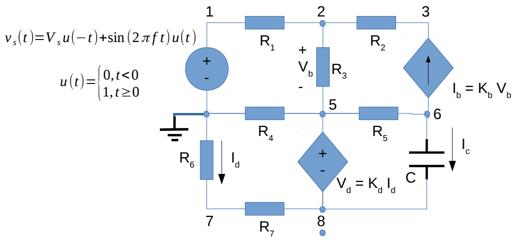

Figure 1: Circuit to be analyzed in the laboratory assignment.

2 Theoretical Analysis

The values used in both analysis are the following:
R_1 = 1.00781211614 $k\Omega$ R_2 = 2.00311223204 $k\Omega$ R_3 = 3.04503555589 $k\Omega$ R_4 = 4.17896607062 $k\Omega$ R_5 = 3.10615699135 $k\Omega$ R_6 = 2.06090154363 $k\Omega$ R_7 = 1.00634569025 $k\Omega$ V_s = 5.04864033546 V C = 1.02502620056 mA K_b = 7.05958243797 mS K_c = 8.03913881798 $m\Omega$

2.1 Analysis for t¡0

For t¡0 we can see that we are working in the steady state, given that in that time interval $v_S = V_S$. In a DC circuit, the capacitor charges up to it's full capacity, blocking the flow of electricity. Taking this into account we can replace the capacitor with an open circuit. With this information, and knowing that the the tension in V_4, since it is connected to ground, we can obtain the following equations.

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1-G_2-G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & -K_b-G_2 & G_2 & K_b & 0 & 0 & 0 \\ G_1 & -G_1 & 0 & -G_4 & 0 & -G_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6+G_7 & -G_7 \\ 0 & 0 & 0 & -1 & 0 & -G_6*K_d & 1 \\ 0 & G_3 & 0 & -G_3-G_4-G_5 & G_5 & -G_6 & 0 \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} -V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

After solving them with the Octave sofware, we obtained the following results:
[INSERIR TABELA]

2.2 Equivalent Resistance

In order to determine the equivalent resistance we need to determine V_X, this is, the difference between V_6 - V_8. This is made to ensure that the voltage in the capacitor is continuous.

Making use of the matrix below:

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1-G_2-G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & -K_b-G_2 & G_2 & K_b & 0 & 0 & 0 \\ G_1 & -G_1 & 0 & -G_4 & 0 & -G_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6+G_7 & -G_7 \\ 0 & 0 & 0 & -1 & 0 & -G_6*K_d & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -V_x \end{bmatrix} \quad (2)$$

And knowing that:

$$V_x = V_6 - V_8 \tag{3}$$

$$I_x = \frac{V_6 - V_5}{R_5} + \frac{V_3 - V_2}{R_2} \tag{4}$$

$$R_e q = \frac{V_x}{I_x} \tag{5}$$

$$\tau = R_e q * C \tag{6}$$

We can obtain the following values:
[INSERIR TABELA]

2.3 Natural Solution

The natural solution doesn't take into account idependent power sources. Using V_x as the initial condition the natural solution can be obtained with the following equation

$$V_{6n}(t) = V_x e^{(-\frac{t}{\tau})} \tag{7}$$

After computing this data on Octave, we can plot the results of the interval [0, 20]ms.

Figure 2: Natural solution for t [0, 20]

2.4 Forced Solution

In order to obtain the forced solution we must be able to solve the following equation.

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1 - G_2 - G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & -K_b - G_2 & G_2 & K_b & 0 & 0 & 0 \\ G_1 & -G_1 & 0 & -G_4 & 0 & -G_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6 + G_7 & -G_7 \\ 0 & 0 & 0 & -1 & 0 & -G_6 * K_d & 1 \\ 0 & G_3 & 0 & -G_3 - G_4 - G_5 & G_5 - jwC & -G_6 & jwc \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} -j \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

From which we obtain the following values:

Name	Value [mA]

Table 1: Mesh currents expressed in mA

2.5 Final Total Solution

The final total solution is obtained by superimposing both natural and forced responses:

$$V_6(t) = V_{6f}(t) + V_{6n}(t) \tag{9}$$

We can see the plot of $V_s(t)$ and $V_6(t)$ from -5ms to 20ms [Observações].

2.6 Frequency Response

We can see that for low frequencies (¡1Hz) every voltage is in phase. The capacitor is given enough time to charge up to the same voltage as the voltage source.

Figure 3: Total solution for V_6 and V_s

Figure 4: Variation of frequency of V_s, V_6 and V_c voltage magnitudes

Figure 5: Variation of frequency of V_s, V_6 and V_c voltage phases

3 Simulation Analysis

In this section we analised the circuit making use of the NGSpice software. Given that this circuit has a sinusoidal voltage source, the current that flows trough the circuit components varies with time. As such, in order to simulate the circuit's total response we must run a transient analysis. We also ran an operating point analysis for $t < 0$ and $t = 0$.

3.1 Operating Point Analysis

Below we can find the simulated operating point results for $t < 0$ and $t = 0$:

[INSERIR TABELAS]

For $t < 0$ we replaced the capacitor with a voltage source V_x in order to determine the equivalent resistor.

[INSERIR CÁLCULO DA Req]

3.2 Natural Response

The graphic bellow is the plot of $v_6(t)$ for the $[0, 20]ms$ time interval, using transient analysis simulation.

3.3 Total Response

The previous step was repeated, this time considering $v_s(t)$ and f = 1kHz. In the plot below we have both the stimulus and the response.

3.4 Frequency Response

Analogous to the theoretical analysis we can find bellow:

4 Conclusion

In this laboratory assignment we analysed the circuit indicated in Figure 1 successfully. This analysis was done both theoretically, making use of the

The theoretical estimates were compared with the simulation results, yielding very similar results. The plots obtainted from performing the nodal method in GNU Octave and the simulation in NGSpice are mostly identical, with errors being of such low order of magnitude that

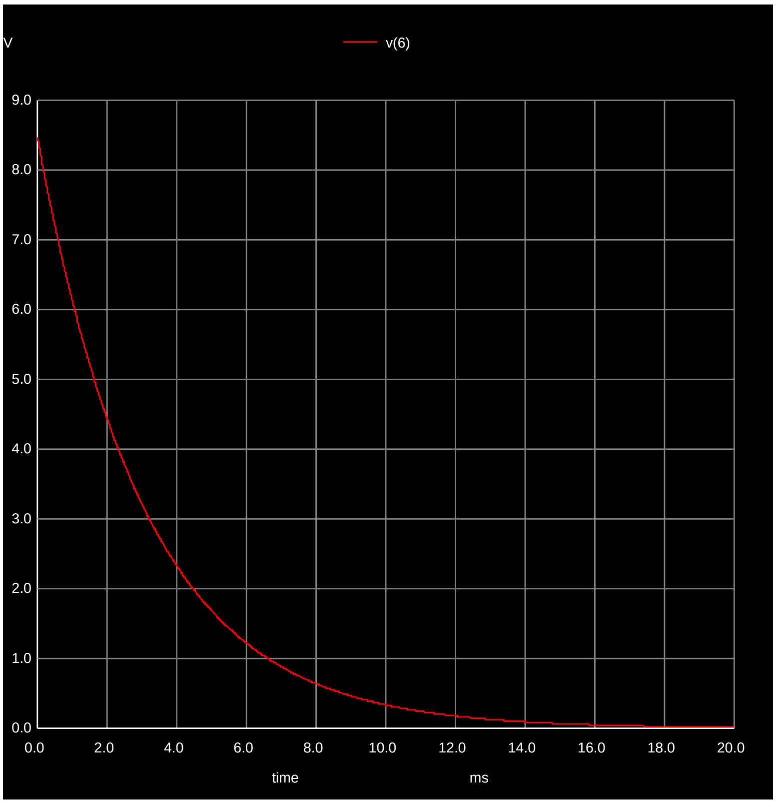

Figure 6: Natural response simulated for the V_6 voltage

they become negligible, and are most likely attributable to rounding errors. This outcome was expected, since the circuit is composed of linear components, and hence NGSpice likely used the same methods as Octave did in the theoretical part. Because the results are coherent and within expectation, we consider this task to be successful.

Kirchoff's Laws and the Octave math tool to perform the calculations, and by circuit simulation, using the Ngspice tool. Unlike in theory, in real life we could expect several experimental errors, due to the measurement instruments, the way the measurement is performed, internal resistance of the components of the circuit, temperature, etc.

However, using the computer simulation, we came to the conclusion that the results we obtained matched the ones we calculated theoretically precisely. This is because this is a relatively simple circuit with few components. This corroborates with what was taught in theory classes, that theoretical and simulation models cannot differ; unless we are talking about more complex circuits, which is clearly not the case.

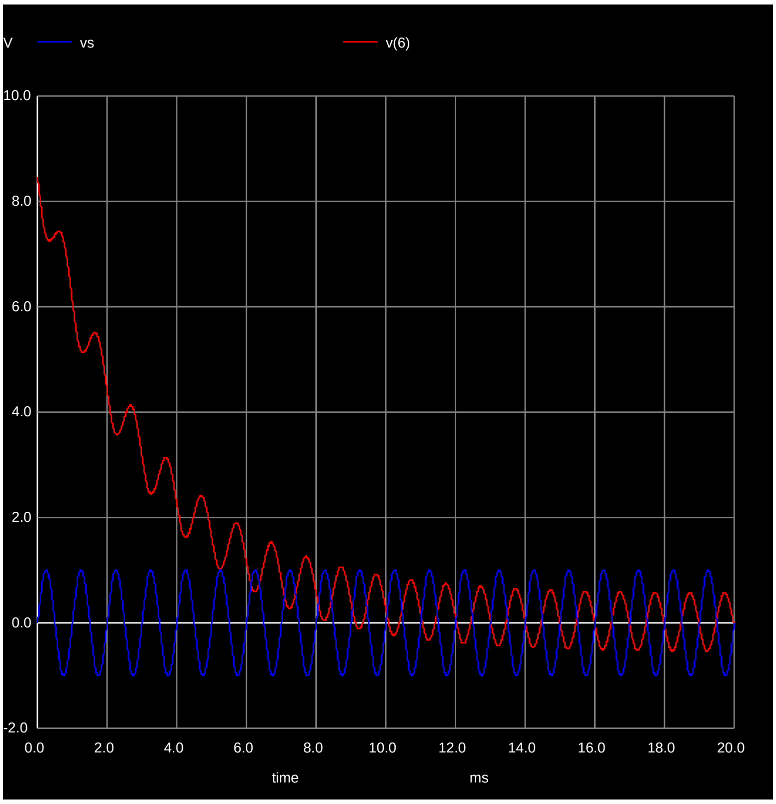

Figure 7: V_6 and V_S voltages simulated responses

Finally, this lab assignment was useful to put to test our knowledge of circuits and the ”laws” that they obey to and familiarize ourselves with sophisticated softwares, like Git, Makefile, Octave and Ngspice.

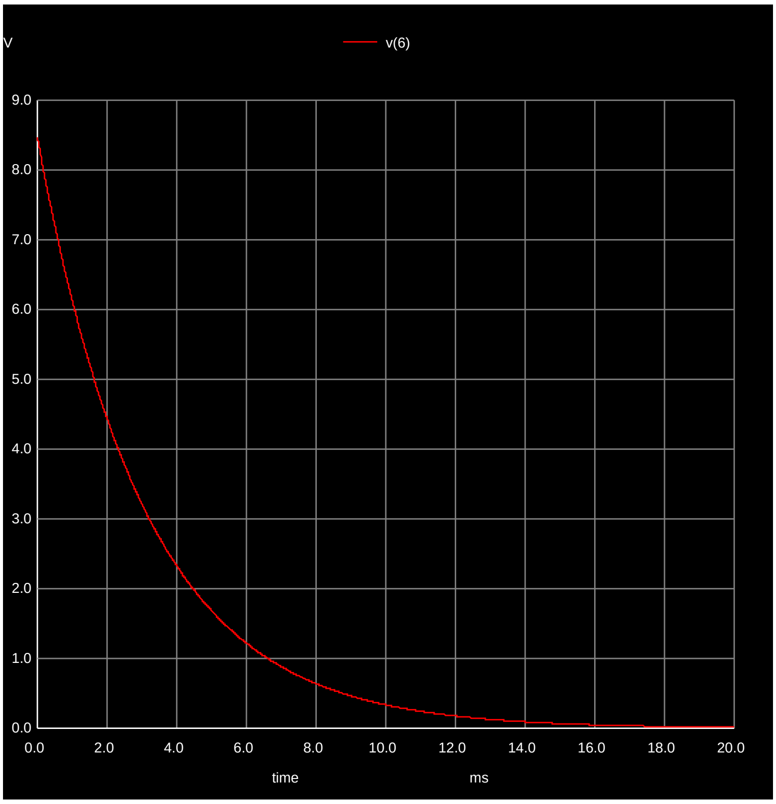

Figure 8: Magnitude of V_S, V_c and V_6 in function of frequency

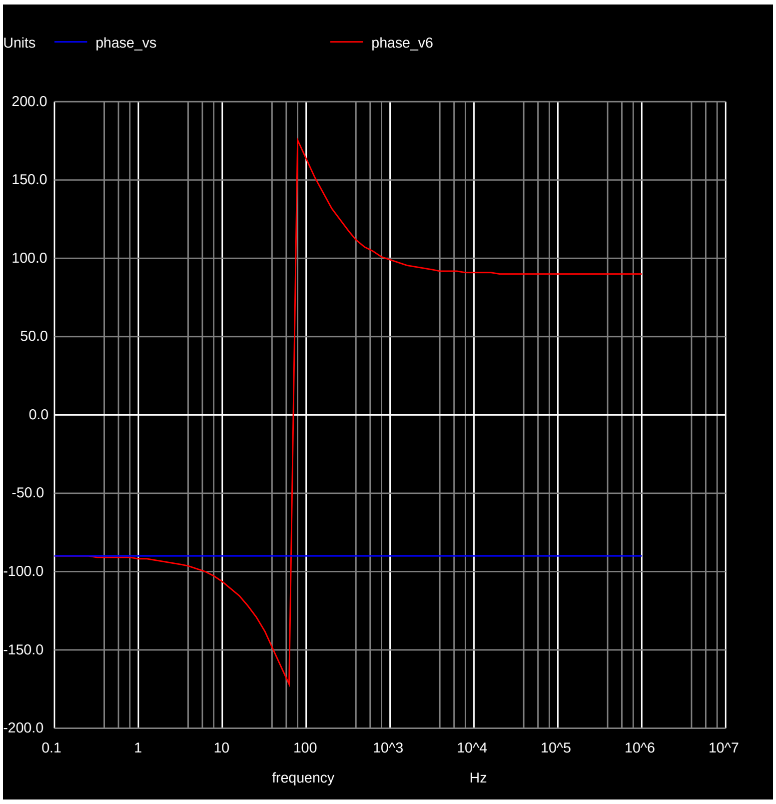

Figure 9: Phase of V_S, V_c and V_6 in function of frequency