Contents

1 Introduction

paragraphThe objective of this laboratory assignment is to study a circuit composed of eleven branches and eight nodes, arranged in four independant meshes, as detailed bellow. This circuit has seven resistences, numbered from R_1 trough R_7, dependant current and volatage sources, V_C and I_B respectively, and independant current and voltage sources, V_A and I_D respectively. The study of the circuit will be subdivided in two major steps. In Section 2 we will analyse the circuit using the Mesh and Node Analysis, while also discussing the equivalence of both methods in terms of the results they produce. In Section 3 the circuit is going to be simulated with Ngspice, and we will compare the results of that simulation with the ones obtained in Section 2. Finally, the Conclusions of this assignment are detailed in Section 4.

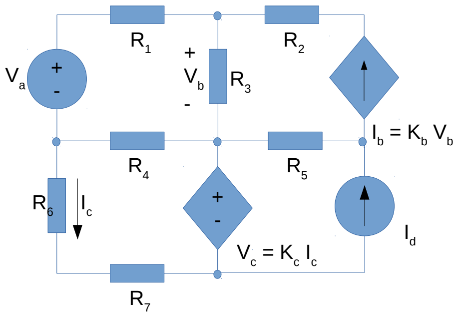

Figure 1: Voltage driven serial RC circuit.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed using the Mesh and Node Methods, after which both results are compared. Those methods are based upon the Kirchoff Laws, and the behaviour of the elements of the circuit, which are detailed bellow.

Kirchoff Laws

The analysis of this circuit is based on the Kirchoff's Circuit Laws, more specifically, the Kirchoff's Current and Voltage laws (KCL and KVL). The first law states that for a given node, the sum of the electrical currents flowing out of the node is equal to the sum of currents flowing into that node. It can be translated into the following equation:

$$\sum_{k=1}^{n} I_k = 0. \tag{1}$$

The second law states that the sum of all the voltages in mesh must be zero, and can be translated into the following equation:

$$\sum_{k=1}^{n} V_k = 0. \tag{2}$$

[INSERIR FONTES]

Resistor

A resistor is an electrical component that poses resistance (as the name implies) to current flow, reducing it. The functioning of a resistor can be described by the following equation:

$$V = R * I \tag{3}$$

Voltage Source

- Imposes a voltage V regardless of current I
- Zero internal resistance

Current Source

- Imposes a current I regardless of voltage V
- Infinite internal resistance

[DESCREVER PROGRAMA DO GNU OCTAVE]

For the simulation and analysis we will use the following values with the numerical values given:

R_1 = 1.00781211614 $k\Omega$ R_2 = 2.00311223204 $k\Omega$ R_3 = 3.04503555589 $k\Omega$ R_4 = 4.17896607062 $k\Omega$ R_5 = 3.10615699135 $k\Omega$ R_6 = 2.06090154363 $k\Omega$ R_7 = 1.00634569025 $k\Omega$ V_a = 5.04864033546 V I_d = 1.02502620056 mA K_b = 7.05958243797 mS K_c = 8.03913881798 $m\Omega$

2.1 Mesh Method Analysis

After the aplication of the Mesh Method to the circuit, we obtainded the following equations:

$$I_D = I_d \tag{4}$$

$$(R_1 + R_3 + R_4)I_A - R_3 I_B - R_4 I_C = -V_A \tag{5}$$

$$(R_4 + R_6 + R_7 - K_C)I_C - R_4 I_A = 0 \tag{6}$$

$$(R_3 K_B - 1)I_B - R_3 K_B I_A = 0 \tag{7}$$

From which we can obtain the following matrix equation:

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ R_1 + R_3 + R_4 & -R_3 & -R_4 & 0 \\ -R_4 & 0 & R_4 + R_6 + R_7 - K_C & 0 \\ -R_3 * K_B & R_3 * K_B - 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_D \end{bmatrix} = \begin{bmatrix} I_d \\ -V_A \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

Finally, solving the problem with Octave we obtain:

Name	Value [mA]
@I_a	-0.186554
@I_b	-0.195655
@I_c	0.983196
@I_d	1.025026

Table 1: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

2.2 Nodal Method Analysis

After the aplication of the Nodal Method to the circuit, we obtainded the following matrix, which in turn was obtained from the equivalent set of equations.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G_1 & (G_1+G_2+G_3) & -G_2 & -G_3 & 0 & 0 & 0 \\ 0 & (G_1+K_B) & -G_2 & -K_B & 0 & 0 & 0 \\ 0 & K_B & 0 & (-G_5-K_B) & G_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (G_6+G_7) & -G_7 \\ 0 & -G_3 & 0 & (G_3+G_4+G_5) & -G_5 & -G_7 & G_7 \\ 0 & 0 & 0 & 1 & 0 & (K_C*G_6) & -1 \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} V_A \\ 0 \\ 0 \\ I_D \\ 0 \\ -I_D \\ 0 \end{bmatrix} \tag{9}$$

Once again, solving the matrix equation with Octave we obtain:

Name	Value [V]
@V_0	0.000000
@V_1	5.048640
@V_2	4.860629
@V_3	4.468710
@V_4	4.888344
@V_5	8.679973
@V_6	-2.026270
@V_7	-3.015705

Table 2: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

2.3 Equivalence of the Mesh and Node Methods