Assume S is the set of entries. So we get

$$|S| = m$$

Use the Bernoulli- distributed random variables X0, . . . , Xm−1, we can set

$$X_i = \begin{cases} 1, A[i] = \phi \\ 0, else \end{cases}$$

$$E(X) = E\left(\sum_{i \in S} X_i\right) = \sum_{i \in S} P(A[i] = \phi)$$

As h is a random hash function, it maps e to each h(e) ∈ {0,...,m−1} with the same probability. So the probability of h(e) doesn't belong to A[i] is (1-1/m) and there are n elements so the probability of A[i] is a null set is,

$$P(A[i] = \phi) = \left(\frac{m-1}{m}\right)^n$$

So the expected number of empty entries in the hash table is

$$E(x) = m \cdot \left(\frac{m-1}{n}\right)^n = \frac{(m-1)^n}{m^{n-1}}$$