


Ex 4.

i) For these nodes that are not connected (Assume there are n unconnected nodes / we add $n-1$ new additional edges with a cost of infinite, then apply Prim's algorithm.

ii) In the Kruskal's algorithm, we'll first sort all edges in one specific order, in the case that all edges costs are pairwise different, then there are no edges have the same cost.

Consequently, the order is fixed. then we apply Kruskal refer to this order will definitely leading to one unique minimum spanning tree.

iii) If there are no edges with negative costs, then it must be a tree. But if we concern there exist negative costs edges. For the sake of finding minimum spanning tree, the negative cost edge must be connected (more negative edge means smaller total costs). then as the number of negative edges increase, we'll meet the situation that the negative edge are in one closed circle.  ($= e_1, \dots, e_n$ are the edges they form one circle)

Assume that e_n there exist e^* from e_1 to e_n that its cost is positive then this edge should be deleted (after deleting this edge, all the nodes are still connected) then the circle becomes one tree. So if there exist one closed circle, all the edges on the circle should have negative edge.