

Title: Classification - I

Course: Machine Learning

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Master: Data Science and Business Analytics

Master: Artificial Intelligence and Innovation

Master: Finance and Financial Technologies

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- Introduction to Classification

Unsupervised Classification

- the unsupervised mining techniques which can be in some way related to classification are usually known in literature with names different from classification
- for this reason in this course with the term *classification* we will always mean *supervised classification*



Supervised Classification 1/2

In the following, simply classification

Consider the "soybean" example shown in the introduction (link to the dataset)

- ullet The data set ${\mathcal X}$ contains N individuals described by D attribute values each
- We have also a $\mathcal Y$ vector which, for each individual x contains the class value y(x)
- The class allows a finite set of different values (e.g. the diseases),
 say C
- The class values are provided by experts: the supervisors



Supervised Classification 2/2

- We want to learn how to guess the value of the y(x) for individuals which have not been examined by the experts
- We want to learn a classification model



2	Classification model
3	C4.5 – Classification



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Classification model

- An algorithm which, given an individual for which the class is not known, computes the class
- The algorithm is *parametrized* in order to optimize the results for the specific problem at hand
- Developing a classification model requires
 - choose the learning algorithm
 - let the algorithm learn its parametrization
 - assess the quality of the classification model
- The classification model is used by a run—time *classification* algorithm with the developed parametrization



Classification model or, shortly, classifier

A bit of formality

ullet a decision function which, given a data element x whose class label y(x) is unknown, makes a prediction as

$$\mathcal{M}(x,\theta) = y(x)_{pred}$$

where θ is a set of values of the parameters of the decision function

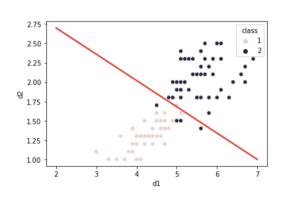
- the prediction can be true or false
- the learning process for a given classifier $\mathcal{M}(.,.)$, given the dataset \mathcal{X} and the set of supervised class labels \mathcal{Y} determines θ in order to reduce the prediction error as much as possible

Example of decision function

- supervised dataset with two dimensions, two classes
- use as decision function a straight line

$$\theta_1 * d_1 + \theta_2 * d_2 + \theta_3 \geqslant 0 \Rightarrow c_1$$

$$\theta_1 * d_1 + \theta_2 * d_2 + \theta_3 < 0 \Rightarrow c_2$$





All models are wrong, but some are useful

George Box

- The model (decision function) of the previous page makes some errors
 - even the best choice of parameters cannot avoid errors
- Different models can have different power to shatter the dataset into subsets with homogeneous classes
 - e.g. what about a quadratic function? $\theta_1 * d_1^2 + \theta_2 * d_2^2 + \theta_3 * d_1 * d_2 + \theta_4 * d_1 + \theta_5 d_2 + \theta_6$



A workflow for classification - I

- 1. Learning the model for the given set of classes
 - 1.1 a training set is available, containing a number of individuals
 - 1.2 for each individual the value of the class label is available (also named ground truth)
 - 1.3 the training set should be *representative* as much as possible 1.3.1 the training set should be obtained by a random process
 - 1.4 the model is fit learning from data the best parameter setting

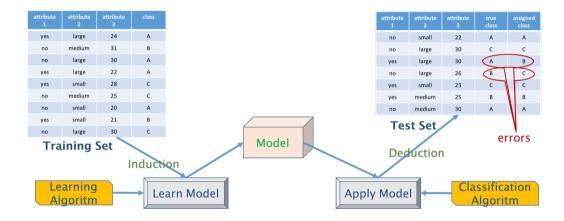


A workflow for classification - II

- 1. Estimate the accuracy of the model
 - 1.1 a test set is available, for which the class labels are known
 - 1.2 the model is run by a *classification algorithm* to assign the labels to the individuals
 - 1.2.1 the classification algorithm implements the model with the parameters
 - 1.3 the labels assigned by the model are compared with the true ones, to estimate the accuracy
- 2. The model is used to label new individuals
 - 2.1 possibly, after the labeling, the true labels may become available and the true accuracy can be compared with the estimated one



A workflow for Learning and Estimation





Question Optional

- is there a hidden assumption in the description of the soybean example of page 4?
- is there a workaround to this hidden assumption?



Two flavors for classification

Crisp

• the classifier assigns to each individual *one label*

Probabilistic

• the classifier assigns a probability for each of the possible labels



Pruning a Decision TreeImpurity functionsFinal remarks on DTs

Conclusion

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Decision Trees

C4.5 and beyond [Buntine(1992)]

- Good compromise: decent performance, fast to train and execute, easy to understand
- History
 - 1966 ID3 [Hunt et al.(1966)Hunt, Marin, and Stone]
 - 1979 CLS [Quinlan(1979)]
 - 1993 C4.5 [Quinlan(1979)]
- Generate classifiers structured as decision trees



Using a Decision Tree $1/2^1$

- A run-time classifier structured as a decision tree is a tree-shaped set of tests
- the decision tree has inner nodes and leaf nodes

-BBS 🌘

1 Syntetic description in IWu et al. (2008) Wu, Kumar, Quinlan, Ghosh, Yang, Motoda, McLachlan, Ng, Liu, Yu, Zhou, Steinbach, Hand, and Steinberg

Using a Decision Tree 2/2

```
Inner nodes:
```

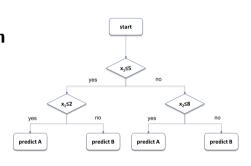
if test on attribute *d* of element *x* **then** execute node'

else

execute node"

Leaf nodes:

predict class of element x as c"



Learning a decision tree – Model generation

Given a set ${\mathcal X}$ of elements for which the class is known, grow a decision tree as follows

- ullet if all the elements belong to class c or ${\mathcal X}$ is small generate a leaf node with label c
- otherwise
 - choose a test based on a single attribute with two or more outcomes
 - make this test the root of a tree with one branch for each of the outcomes
 of the test
 - ullet partition ${\cal X}$ into subsets corresponding to the outcomes and apply recursively the procedures to the subsets



Learning a decision tree

Problems to solve:

- 1. which attribute should we test?
- 2. which kind of test?
 - 2.1 binary, multi-way, ..., depends also on the domain of the attribute
- 3. what does it mean \mathcal{X} is small, in order to choose if a leaf node is to be generated also if the class in \mathcal{X} is not unique?

A supervised dataset: Iris

sepal length (cm)	sepal width (cm)	petal length (cm)	petal width (cm)	class
6.2	2.2	4.5	1.5	1
5.2	3.5	1.5	0.2	0
5.6	3.0	4.5	1.5	1
6.0	2.9	4.5	1.5	1
7.7	3.0	6.1	2.3	2
5.1	3.8	1.5	0.3	0
5.9	3.2	4.8	1.8	1
5.7	4.4	1.5	0.4	0
6.7	3.1	5.6	2.4	2
6.5	3.2	5.1	2.0	2

. . .

. . .

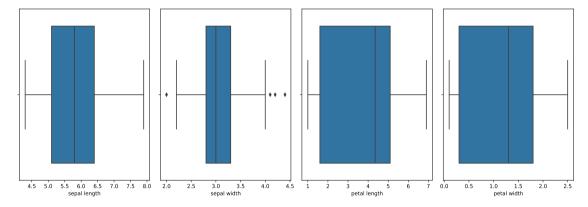
. . .

Dataset description

- 150 examples of iris flowers
- 4 attributes describing sizes of petals and sepals, class is the target
 - class has three values
- we could be interested in predicting the class for a new individual, given the measures

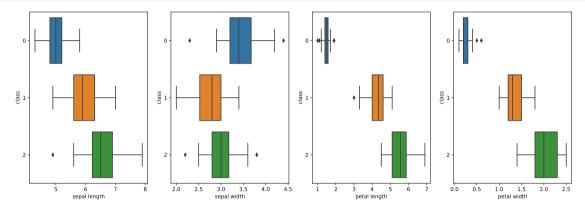


Exploration of the dataset - Boxplot - General



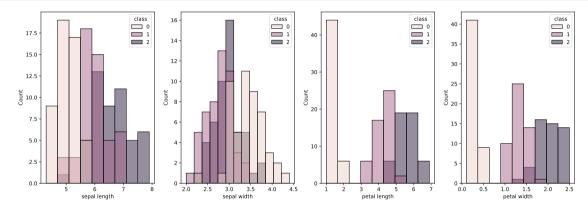


Exploration of the dataset - Boxplot - Inside classes



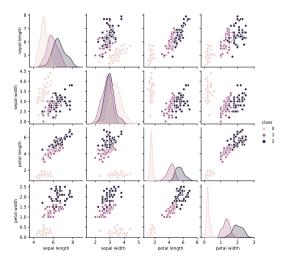


Exploration of the dataset - Histograms





Exploration of the dataset - Pairplots





Supervised learning goals

- design an algorithm to find interesting patterns, in order to forecast the values of an attribute given the values of other attributes
 - in our case, find patterns to guess the class given the other values
- distinguish real patterns from illusions
- choose useful patterns
- in real life, we could have millions of rows and thousands of columns
 - looking at plots could be very hard



Evaluate how much a pattern is interesting

- several methods, one of them is based on *information theory*
 - information theory is primarily used in telecommunications
 - it is based on the concept of entropy
 - information content, surprise, ...
 - Claude Shannon, "A Mathematical Theory of Communication", 1948



The bit transmission example

 given a variable with 4 possible values and a given probability distribution

$$P(A) = 0.25, P(B) = 0.25, P(C) = 0.25, P(D) = 0.25$$

- an observation of the data stream could return BAACBADCDADDDA . . .
- the observation could be transmitted on a serial digital line with a two-bit coding

$$A = 00, B = 01, C = 10, D = 11$$

• the transmission will be 010000100100111110111111100 . . .



Less bits

- What if the probability distributions are uneven? P(A) = 0.5, P(B) = 0.25, P(C) = 0.125, P(D) = 0.125
- of course, the coding shown above is possible, requiring two bits per symbol
- is there a coding requiring only 1.75 bit per symbol, on the average?

Less bits

- What if the probability distributions are uneven? P(A) = 0.5, P(B) = 0.25, P(C) = 0.125, P(D) = 0.125
- of course, the coding shown above is possible, requiring two bits per symbol
- is there a coding requiring only 1.75 bit per symbol, on the average?

$$A = 0, B = 10, C = 110, D = 111$$



Even less bits

- What if there are only three symbols with equal probability? P(A) = 1/3, P(B) = 1/3, P(C) = 1/3
- of course, the two-bit coding shown above is still possible
- is there a coding requiring less than 1.6 bit per symbol, on the average?

Even less bits

- What if there are only three symbols with equal probability? P(A) = 1/3, P(B) = 1/3, P(C) = 1/3
- of course, the two-bit coding shown above is still possible
- is there a coding requiring less than 1.6 bit per symbol, on the average?

A=0, B=10, C=11 or any permutation of the assignment

General case

• Given a source X with V possible values, with probability distribution

$$P(v_1) = p_1, P(v_2) = p_2, \dots, P(v_V) = p_V$$

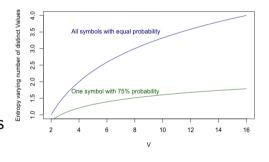
 the best coding allows the transmission with an average number of bits given by

$$H(X) = -\sum_{i} p_{j} \log_{2}(p_{j})$$

H(X) is the *entropy* of the information source X

Meaning of entropy of an information source

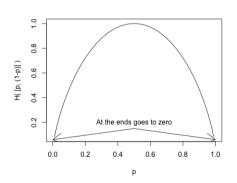
- high entropy means that the probabilities are mostly similar
 - the histogram would be flat
- low entropy means that some symbols have much higher probability
 - the histogram would have peaks
- higher number of allowed symbols (i.e. of distinct values in an attribute) gives higher entropy





Entropy of a binary source

In a binary source with symbol probabilities p and (1-p) when p is 0 or 1 the entropy goes to 0



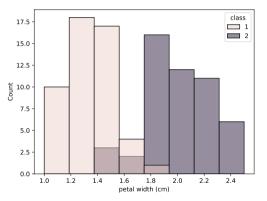
From entropy in transmission to entropy in classification

- In transmission, information sources with lower entropy can be represented (and transmitted) with shorter codes
 - b.t.w. this is the basis of information compression, i.e. the basis of modern audio and video telecommunications
- In classification, low entropy of the class labels of a dataset means that it is there is low diversity in the labels (i.e. the dataset has high purity, there is a majority class)
- We look for criteria that allow to split a dataset into subsets with higher purity
 - with *criteria* we mean logical formulas to be used as decision function to partition the set elements into the subsets



Entropy for the target column **class** in the reduced Iris dataset

A subset: only the fourth data column and the target, only the rows with classes 1 or 2



petal width (cm)	class
2	2
1.7	1
1.3	1
2.2	2
1.5	1
1.5	2
2.3	2
2	2
2.5	2
1.7	2

$$N = 100$$
 $p_{class=1} = 0.5, p_{class=2} = 0.5$

$$H_{class} = -(p_{class=1} * log_2(p_{class=1}) + p_{class=2} * log_2(p_{class=2})) = 1$$

Entropy after a threshold-based split

- Splitting the dataset in two parts according to a threshold on a numeric attribute the entropy changes, and becomes the weighted sum of the entropies of the two parts
 - the weights are the relative sizes of the two parts
- Let $d \in \mathcal{D}$ be a real-valued attribute, let t be a value of the domain of d, let c be the class attribute
- We define the entropy of c w.r.t. d with threshold t as $H(c|d:t) = H(c|d < t) * P(d < t) + H(c|d \ge t) * P(d \ge t)$

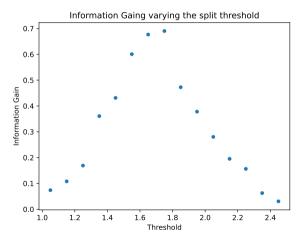


Information Gain for binary split

- It is the reduction of the entropy of a target class obtained with a split of the dataset based on a threshold for a given attribute
- We define IG(c|d:t) = H(c) H(c|d:t)
 - it is the information gain provided when we know if, for an individual, *d* exceeds the threshold *t* in order to forecast the class value
- We define $IG(c|d) = \max_t IG(c|d:t)$

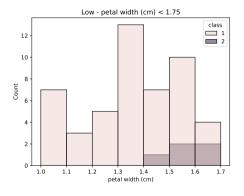


Change the threshold to find the best split

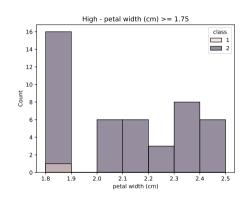




Let's split the reduced Iris with threshold 1.75



	low	high
1	49	1
2	5	45
	54	46



H(class|petalwidth: 1.75) = 0.31 = $- (49/54 * log_2(49/54) + 5/54 * log_2(5/54)) * 0.54 +$ $(1/46 * log_2(1/46) + 45/46 * log_2(45/46)) * 0.46$



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How can we use the information gain?

Predict the probability of long life given some historical data on person characteristics and life style

- IG(LongLife|HairColor) = 0.01
- IG(LongLife|Smoker) = 0.2
- IG(LongLife|Gender) = 0.25
- IG(LongLife|LastDigitSSN) = 0.00001

Correlations between attributes is an important issue: it is not considered here

Back to DT generation

Choosing the attribute to test

A decision tree is a tree—structured plan generating a sequence of tests on the known attributes (predicting attributes) to predict the values of an unknown attribute.

Consider question 1 of page 21: which attribute should we test?

- ullet test the attribute which guarantees the maximum IG for the class attribute in the current data set ${\cal X}$
- ullet partition ${\mathcal X}$ according to the test outcomes
- recursion on the partitioned data



Train/Test split³

- The supervised data set will be split in (at least) two parts:
 - Training set used to learn the model
 - Test set used to evaluate the learned model on fresh data
- The split is done randomly
- Assumption: the parts have similar characteristics
- The proportion of the split is decided by the experimenter
 - Common solutions: 80-20, 67-33, 50-50
- The following slides consider a 50-50 split of the Iris dataset²
 - For this specific split, entropies for the class column in training and test turns out to be both 1.58

³ You can read this link for a short discussion

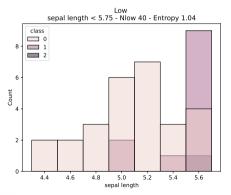


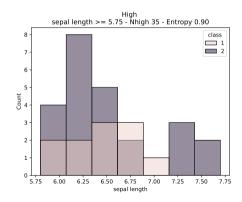
² In the example, the split has been done using sklearn.model_selection_train_test_split and random_state =10

Iris Dataset - Predicting attribute: Sepal Length

Best threshold: 5.75

Information Gain: 1.58 - (40 * 1.04 + 35 * 0.90)/75 = 0.61

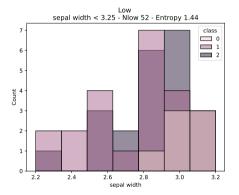


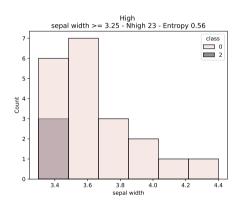




Iris Dataset - Predicting attribute: Sepal Width

Best threshold: 3.25 - Information Gain: 1.58 - (52*1.44 + 23*0.56)/75 = 0.41

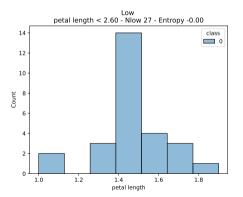


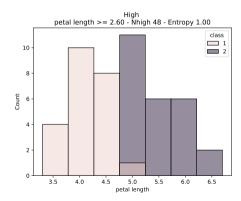




Iris Dataset - Predicting attribute: Petal Length

Best threshold: 2.6 - Information Gain: 0.94

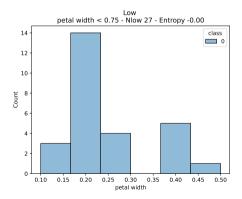


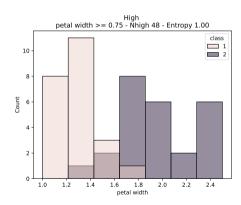




Iris Dataset - Predicting attribute: Petal Width

Best threshold: 0.75 - Information Gain: 0.94



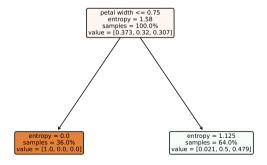




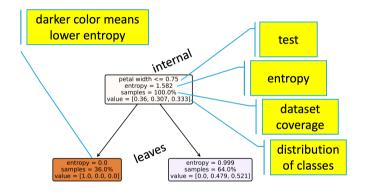
One-stump decision

Now on the entire training set with the three classes

- choose the attribute giving the highest IG
- partition the dataset according to the chosen attribute
- choose as class label of each partition the majority

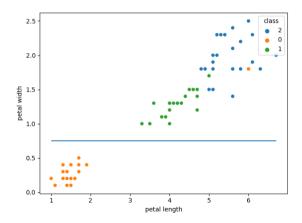


What's in a node



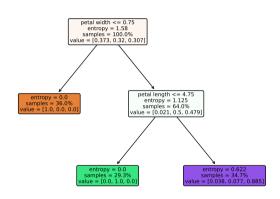


First split



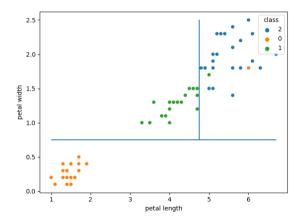
Recursion step

Build a new tree starting from each subset where the minority is non-empty



Learning the Decision Tree

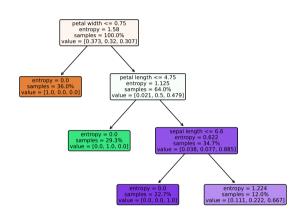
Second split





Recursion step

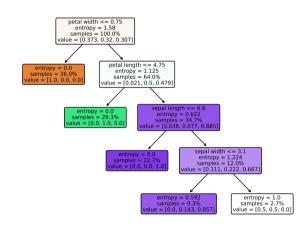
Build a new tree starting from each subset where the minority is non-empty



Recursion step

Observation

- The weighted sum of the entropy of the descendant nodes is always smaller than the entropy in the ancestor node, even if one of the descendant has higher entropy.
- Consider the three bottom right nodes (a=ancestor, Id=left descendant, rd=right descendant)

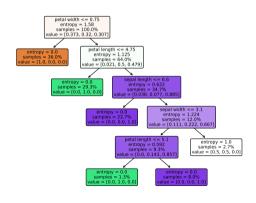




 $ent_a = 0.622 > (ent_{ld} * samp_{ld} + ent_{rd} * samp_{rd}) / samp_a = (0*22.7 + 1.224*12) / 34.7 = 0.39$

Recursion ends

- Most of the leaves are pure, recursion impossible
- One of the leaves is not pure, but no more tests are able to give positive information gain, recursion impossible
 - it is labelled with the majority class, or, in case of tie, with one of the non-empty classes
- The error rate on the training set is 1.35%
 - 1 of the 75 examples in the training set is not correctly classified by the learned decision tree
 - it is one of the two items in the rightmost leaf



Building a *Decision Tree* with binary splits

```
procedure BUILDTREE(dataset \mathcal{X}, node p)
    if all the class values of \mathcal{X} are c then
         return node p as a leaf, label of p is c
    if no attribute can give a positive information gain in \mathcal{X} then
         say that the majority of elements in \mathcal{X} has class c
         return node p as a leaf, label of p is c
    find the attribute d and threshold t giving maximum information gain in \mathcal{X}
    create two internal nodes descendant of p, say p_{left} and p_{right}
    let \mathcal{X}_{left} = \text{selection on } \mathcal{X} \text{ with } d < t
    BUILDTREE(\mathcal{X}_{left}, p_{left})
    let \mathcal{X}_{right} = selection on sdata with d \ge t
    BUILDTREE(\mathcal{X}_{right}, p_{right})
```

Decision tree for the Iris classifier

Internal representation

	ChLeft	ChRight	Feature	Threshold	NNodeSamples	Impurity
0	1	2	petal width	0.750000	75	1.579659
1	-	-	-	nan	27	0.000000
2	3	4	petal length	4.750000	48	1.124941
3	-	-	-	nan	22	0.000000
4	5	6	sepal length	6.600000	26	0.621904
5	-	-	-	nan	17	0.000000
6	7	10	sepal width	3.100000	9	1.224394
7	8	9	petal length	5.100000	7	0.591673
8	-	-	-	nan	1	0.000000
9	-	-	-	nan	6	0.000000
10	-	-	-	nan	2	1.000000



Training Set Error

- execute the generated decision tree on the training set itself
 obviously the class attribute is hidden
- count the number of discordances between the true and the predicted class
- this is the *training set error*



Causes of non-zero training set error

https://app.wooclap.com/COFEKT

The training set error can be greater than zero because of...



Causes of non-zero training set error

https://app.wooclap.com/COFEKT

The training set error can be greater than zero because of. . .

- the limits of decision trees in general:
 - a decision tree based on tests on attribute values can fail
- insufficient information in the predicting attributes



Training Set Error

• Is this 1.35% interesting? What is its *meaning*?



Training Set Error

- Is this 1.35% interesting? What is its *meaning*?
- It is the error we make on the data we used to generate the classification model
- It is probably the lower limit of the error we can expect when classifying new data
- We are much more interested to an upper limit, or to a more significant value



Test set error

- The test set error is more indicative of the expected behaviour with new data
- Additional statistic reasoning can be used to infer error bounds given the test set error
- We have available 75 additional labelled records in the Iris dataset



Iris classification error

	Num Errors	Set Size	%Wrong
Training Set	1	75	1.35
Test Set	13	75	17.33



Iris classification error

	Num Errors	Set Size	%Wrong
Training Set	1	75	1.35
Test Set	13	75	17.33

Why the test set error is so much worse?

Overfitting 1/2

Definition: overfitting happens when the learning is affected by noise When a learning algorithm is affected by noise, the performance on the test set is (much) worse than that on the training set



Overfitting 2/2

OPTIONAL

More formally

A decision tree is a *hypothesis* of the relationship between the predictor attributes and the class. Some definitions:

- *h* = hypothesis
- \bullet error_{train}(h) = error of the hypothesis on the training set
- $error_{\mathcal{X}}(h) = error$ of the hypothesis on the entire dataset

h overfits the training set if there is an alternative hypothesis h' such that

$$error_{train}(h) < error_{train}(h')$$

 $error_{\mathcal{X}}(h) > error_{\mathcal{X}}(h')$

Causes for overfitting

1. Presence of noise

- individuals in the training set can have bad values in the predicting attributes and/or in the class label, or can represent unusual cases
- in this case, the model is influenced from partly wrong or unusual training data

2. Lack of representative instances

- some situations of the real world can be underrepresented, or not represented at all, in the training set
- this situation is quite common

A good model has low *generalization* error i.e. it works well on examples different from those used in training



Occam's Razor 4

OPTIONAL

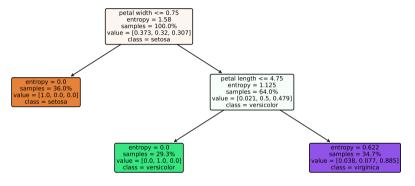
Everything should be made as simple as possibile, but not simpler

- all other things being equal, simple theories are preferable to complex ones
- a long hypothesis that fits the data is more likely to be a coincidence
- pruning a decision tree is a way to simplify it
 - we need to find precise, quantitative guidelines for effective pruning

4 William of Ockham, an english franciscan philosopher of the 14-th century



Example: Iris classification with pruned tree

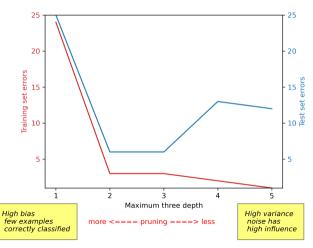


	Num Errors	Set Size	%Wrong
Training Set	3	75	4.00
Test Set	6	75	8.00



Effect of adjusting pruning

Pruning is the way to simplify the model when you are using a decision tree



Hyperparameters

- Every model generation algorithm can be adjusted by setting specific hyperparameters
- Each model has its own hyperparameters
- One of the hyperparameters of decision tree generation is the maximum tree depth



Choice of the attribute to split the dataset

- Looking for the split generating the maximum purity
- We need a measure for the purity of a node
 - a node with two classes in the same proportion has low purity
 - a node with only one class has highest purity



Impurity functions

Measures of the impurity of a node

Entropy – already seen ⁵
Gini Index
Misclassification Error

OPTIONAL

5 it is available in *Scikit-Learn*

Gini Index⁶ – Intuition

- Consider a node p with C_p classes
- Which is the frequency of the wrong classification in class *j* given by a random assignment based only on the class frequencies in the current node?
- For class *j*
 - frequency $f_{p,j}$
 - ullet frequency of the other classes $1-f_{p,j}$
 - ullet probability of wrong assignment $f_{p,j}*(1-f_{p,j})$
- the Gini Index is the total probability of wrong classification

$$\sum_{i} f_{p,j} * (1 - f_{p,j}) = \sum_{i} f_{p,j} - \sum_{i} f_{p,j}^{2} = 1 - \sum_{i} f_{p,j}^{2}$$

6 This is the default impurity measure in Scikit-Learn



Gini Index – Discussion

- ullet the maximum value is when all the records are uniformly distributed over all the classes: ${f 1}-{f 1}/{m C_p}$
- the minimum value is when all the records belong to the same class: 0

Splitting based on the Gini Index

- Used by CART, SLIQ, SPRINT
- When a node p is split into ds descendants, say p_1, \ldots, p_{ds}
- Let $N_{p,i}$ and N_p be the number of records in the i-th descendant node and in the root, respectively
- We choose the split giving the maximum reduction of the Gini Index

$$GINI_{split} = GINI_p - \sum_{i=1}^{ds} \frac{N_{p,i}}{N_p} GINI(p_i)$$



Misclassification Error

OPTIONAL

- If a node is a leaf, we find the highest label frequency; this
 frequency is the accuracy of the node and this label is the output of
 the node
- The misclassification error is the complement to 1 of the accuracy
- Since the most frequent class determines the node label, the complement is the error
 - ullet The maximum value is when all the records are uniformly distributed over all the classes: $1-1/C_p$
 - The minimum value is when all the records belong to the same class: 0
- The choice of the split is done in the same way as for the Gini index

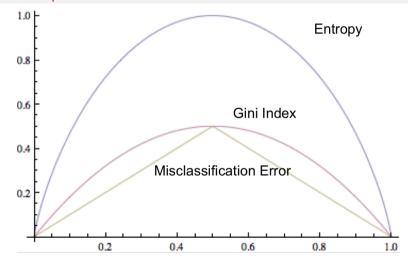
$$ME(p) = 1 - \max_{i} f_{p,j}$$



Comparison of the impurity functions

OPTIONAL

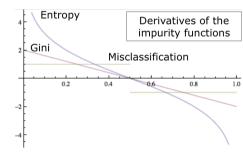
For two classes with frequencies f and 1 - f



Comparison of the impurity functions – Discussion

OPTIONAL

- The behavior of ME is linear, therefore an error in the frequency is completely transferred into the impurity computation
- Entropy and Gini have varying derivative, with the minimum around the center
 - they are more robust w.r.t. errors in the frequency, when the frequencies of the two classes are similar





Complexity of DT induction (C4.5 algorithm) OPTIONAL I

- ullet N instances and D attributes in ${\mathcal X}$
 - tree height is $\mathcal{O}(\log N)$
- Each level of the tree requires the consideration of all the dataset (considering all the nodes)
- Each node requires the consideration of all the attributes
 - overall cost is $\mathcal{O}(DN \log N)$



Complexity of DT induction (C4.5 algorithm) OPTIONAL II

In addition

- binary split of numeric attributes costs $\mathcal{O}(N \log N)$, without increment of complexity
- pruning requires to consider globally all instances at each level, generating an additional $\mathcal{O}(N \log N)$, which does not increase complexity.



Characteristics of DT Induction

OPTIONAL I

- 1. It is a non-parametric approach to build classification models
 - it does not require any assumption on the probability distributions of classes and attribute values
- 2. Finding the best DT is NP-complete, the heuristic algorithms allow to find sub-optimal solutions in reasonable times
- 3. The run-time use of a DT to classify new instances is extremely efficient: $\mathcal{O}(h)$, where h is the height of the tree
- 4. Robust w.r.t. noise in the training set (i.e. wrong class labels), if the overfitting is avoided with appropriate pruning
- 5. Redundant attributes do not cause any difficulty

Characteristics of DT Induction

OPTIONAL II

- In case of strong correlation between two attributes, if one is chosen for a split, most likely the other will never provide a good increment of node purity, and will never be chosen
- 6. The nodes at a high depth are easily irrelevant (and therefore pruned), due to the low number of training records they cover
- 7. In practice, the impurity measure has low impact on the final result
- 8. In practice, the pruning strategy has high impact on the final result



Algorithms for building DTs

OPTIONAL

- Several variants, depending on
 - tree construction strategy
 - partition strategy
 - pruning strategy
- Tests based on linear combinations of numeric attributes
- Multivariate tests (e.g. a = x and b = y)
- . . .



Conclusion

- Decision trees are usually the best starting point to learn supervised machine learning
 - easy to understand
 - easy to implement
 - easy to use
- Overfitting can be controlled by adjusting the maximum tree depth
 - $\bullet\,$ other adjustments are possible, depending on the implementation
 - e.g. lookup the scikit-learn description of the DecisionTreeClassifier
- Are able to predict discrete values (the class) on the basis of continuous or discrete predictor attributes⁷

⁷ The Scikit-Learn implementation of Decision Trees do not allow discrete attributes, therefore in these cases a *data transformation* is necessary



Important concepts

- Impurity functions: entropy, Gini, misclassification
- The recursive greedy algorithm for building a decision tree
- Training error and test error
- Why the test error can be much greater than the training error
- Why the pruning can improve the performance
- How to deal with continuous attributes



Questions

- Why maximising the Information Gain and the Gini Index gain should be, in general, better than minimising the Misclassification Error?
- Why do we prefer a greedy algorithm instead of trying all the possibile trees?
- Consider the Adult dataset. If the decision tree to predict wealth has the marital status near to the top, can we say that the marital status is a major cause for wealth?
- Can we say that the attributes which are not mentioned in the tree are not a cause for wealth?

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